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SOME ASPECTS OF ASTEROID MASS DETERMINATION

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Abstract. There is great variety of astronomical objects in the Universe. Each of these classes of objects follows a certain distribution function in size, luminosity or mass. Most individual mass distributions approximately follow a power law of the form $f(M)\propto M^{-2}$. A notable exception are planets and small bodies which seem to obey a flatter distribution. In spite of the rapidly growing number of newly detected extrasolar planets, our knowledge of the mass function of planetary and small bodies relay entirely on the our Solar System. If is there a 'universal' mass distribution for astronomical objects on all scales, it will be very important to know mass distribtuion of small solar system bodies. Having in mind mentioned reasons we will present methods for asteroid mass determination as well as some of most interesting results.

1. INTRODUCTION

As it is well known, the architecture of the Universe is made up by a great variety of astronomical objects. Usually, they are classified by increasing average size or mass: from asteroids (as the smallest objects) to clusters of galaxies (as the largest entities).

Each class follows, generaly not well known, distribution function in size, luminosity or mass of objects. It could be noticed that the most common feature of all distribution of objects is that smaller objects of given kind are more abundant than larger ones. For example, there are about dozen of asteroids with radius greater than 200 km, and about $2\cdot10^6$ of these bodies with radius greater than 1km.

Also, it is obvious for different class of objects per unit volume of space that there is more asteroids than planets, more planets than stars, etc.

Mass is the most fundamental property of an astronomical object (luminosity is inappropiate because of the existence of dark objects). Astronomical methods for its determination depend on objects dimensions, structure and distance. For example, the mean distance R of the line emitting clouds and their velocity dispersion v derived from the mean width of the rms emission line profiles (FWHM) are needed for computing a central black hole mass in active galaxies (Koratkar and Gaskell 1991), while for objects in our Solar system, could be used the motion of their natural satellites, perturbations on neighboring bodies (direct methods), or values of their diameters and densities (indirect methods), etc.

The astrometric technique of mass determination of small Solar system bodies, in which the deflection of a smaller bodie's trajectory enables us to calculate the mass of a larger perturbing body, may be entering a particularly fruitful period, as near-Earth asteroid (NEA) surveys coincidentally produce a flood of high-precision main-belt asteroid observations.

The mass of an asteroid, when combined with its volume, yields information on its composition and structure. Densities of minor planets are valuable from the point of view of their evolution. Very low density can be consequence of 'rublle pile' structure which body obtained after fragmentation and accretion. Also, another explanation could be referred to cometary origin of some minor planets. The evaluation of percent of such objects in minor planet population is an important task for understanding main asteroid belt as a complex entity in our solar system.

These data are needed for precise modeling of motion of Solar system bodies as well as for accurate navigation of space missions and their successful landing, particularly on Mars. As it is well known, the inability to accurately model asteroid perturbations due to their unknown masses represents the single greatest source of error in planetary ephemerides (Standish 2000). While indirect methods of mass calculation, such as assuming a given density based on taxonomic class, have proven extremely useful in dynamical modeling, such assumptions must be calibrated against direct observation.

As is well known, the method of minor planet mass determination that considers gravitational perturbations produced by an asteroid on other bodies (major planets, minor planets, spacecrafts) during mutual close encounter was developed first. However, this method is affected by significant formal errors of mass derivation. For example, adopted masses of only five bodies in main asteroid belt: Ceres, Pallas, Vesta, Parthenope and Mathilde were determined with formal errors smaller than 5%. This may be a consequence of inhomogeneous distribution of observations of perturbed bodies, their insufficient number and accuracy or low gravitational effects. Nevertheless, the majority of asteroid mass determination was based on single asteroid close encounters.

In order to avoid problems of reliability of asteroid mass determination, Sitarski and Todorovic-Juchniewicz (1992) used the method of asteroid gravitational perturbations on the orbits of many other perturbed bodies. A simplified method was aplied to asteroid mass determination by Kuznetsov (1999) and Michalak (2000).

The mentioned reasons imply that new asteroid mass determinations (especially based on new recorded close encounters) are needed. In this paper we present a modified method of asteroid mass determination and dynamical masses of some largest bodies in main asteroid belt.

2. OVERVIEW OF MASS DETERMINATION METHODS

The most used astrometric mass determination method is a modification of conventional least-squares orbit determination, in which the mass of the perturbing body is added as a seventh solve-for parameter. Ideally, the process is applied to relatively close encounters between a large target asteroid and a small test asteroid, where precise observations of the test asteroid exist before, during, and after the encounter.

According to this method, the system of linear equations could be expressed in the matrix space as:

$$L\Delta E = R \qquad (1)$$

where the matrix L depends on the partial derivatives of the coordinates (right ascensions and declinations) of the perturbed asteroid with respect to seven parameters (six osculating elements of the perturbed body and the perturbing mass). Further, $\Delta E = \Delta E_1 ... \Delta E_6, \Delta E_7$ is a 7×1 matrix belonging to the space of system solutions, which contains the corrections of six orbital elements of the perturbed body and correction of the mass of the perturbing body. Finally, R is the matrix depending on (O-C) residuals in coordinates of the perturbed body. Elements of matrices L, R were computed for each epoch of observation.

As it is well known, the procedure of solving the system (1) is an iterative one. At the first iteration, elements of matrices L and R were calculated using previously selected observations of perturbed bodies (based on 3σ criterion described later in this section). Obtained corrections, the matrix ΔE, produced a new solution which was used as the initial condition for the next iteration. Only two iterations were performed until convergence. This technique, applied on Keplerian orbital elements, produces a correlation matrix with a large correlation between the mass of the perturbing body and the mean motion (or the semimajor axis) of the perturbed one. On the other hand, if the calculation is performed using Cartesian coordinates (initial position and velocity) such a characteristic is not common. A metric which could parameterize the uncertainty in the mass of the perturbing asteroid (Bowell et al. 1994) depends on the RMS of orbital residuals, semimajor axis and eccentricity of perturbed asteroid orbit, mass of perturbing asteroid, number of pre and post encounter observations used, length of corresponding orbital arcs covered by them, as well as the impact parameter and relative velocity of the close encounter.

On the other hand, in our work we tried to find out is it possible to determine correction of perturbing mass separately from corrections of six osculating elements of perturbed asteroid. As a consequence we introduce the modified method of asteroid mass determination. The idea of our modification is to separate preencounter and postencounter sets of observations (parts of orbit) of perturbed asteroid. During this process it is not necessary to know the mass of the perturbing asteroid, because its perturbing effects are negligible. These two orbits are separated by an impulsive change due to the close encounter and have to be connected by properly accounted gravitational effects of the perturbing body. If the pre and post encounter orbits are accurately determined, the same mass of the perturbing body will give the best representation of the postencounter observations with the preencounter orbit and vice-versa. Similarly to classical least squares method, solution of system of linear equations of modified method can be expressed in the matrix space as:

$$\Delta m = A^{-1}B \qquad (2)$$

where the matrix A depends on the partial derivatives of the coordinates of postencounter observations (right ascensions and declinations) of the perturbed asteroid with respect to the perturbing mass. Δm is the correction of the perturbing mass and B is the matrix depending on (O-C) residuals in postencounter coordinates of the perturbed body. Elements of matrices A, B were computed for each epoch of observation.

The procedure of solving the system (2) is an iterative one. At the first iteration, elements of matrices A and B were calculated using previously selected observations of perturbed bodies (based on 3 σ criterion). Obtained correction for the perturbing mass produced a new solution which was used as initial condition for the next iteration. Only two iterations were necessary until convergence. The formal error of calculated mass can be described as follows:

$$\sigma_m = \frac{\sigma_0}{\sqrt{\sum_1^n \frac{\partial c_i}{\partial m}}}$$, n is the number of observations

and σ_0 is:

$$\sigma_0 = \sqrt{\frac{(O-C)^2}{2n-1}}$$

Since the mass determination calculations are a time consuming process, it is first necessary to conduct a survey for encounters likely to result in significant deflection of the perturbed asteroid. Furthermore, since the mass determination process is based upon perturbations in the trajectory of a perturbed asteroid, it is absolutely essential to employ an accurate forcemodel that accounts for all other known perturbations upon that asteroid. And since newly-calculated asteroid masses and orbits improve the accuracy of the force model, the processes of mass determination and force model refinement are intertwined

3. SELECTION OF CANDIDATE CLOSE ENCOUNTERS

As described by many authors (Michalak, 2000; Baer and Chesley, 2008) perhaps the most direct method of selecting suitable mass determination encounters involves integrating the orbit of a small perturbed asteroid through the period covered by observations, both with and without the influence of the large perturbing asteroid; cases in which the perturbed and unperturbed trajectories result in significant differences in predicted right ascension and declination are obvious candidates for detailed analysis.

However, with over 100,000 numbered asteroids catalogued at the time of our analysis, it is obvious that the integration required to test all of the possible encounters for even a limited number of large asteroids would be robust. We therefore used a computationally-efficient method for probing asteroids close encounters.

A two-body approximation of the deflection angle θ in the trajectory of a small perturbed asteroid due to the gravitational perturbation of an encounter with a perturbing asteroid is given by

$$tg\frac{\theta}{2}=\frac{G(M+m)}{v^2 b}$$

where m and M represent the masses of the test and subject asteroids, v is the unperturbed relative velocity, and b is the unperturbed distance of closest approach.

However, there are limits to relying only upon the deflection angle as the survey criterion. First, it is unclear whether the direction of deflection will result in an easily-observable change in trajectory; a deflection that largely impacts the inclination of the test asteroid's orbit, for instance, would not be as easily noted as a deflection that significantly perturbs its semi-major axis. Second, even a relatively small change in the test asteroid's semi-major axis may provide useful data if several decades of observations exist both before and after the encounter.

With intentions to optimize our selection of candidate encounters, we therefore applied combined procedure: the suitable asteroids were found combining traditional approach and procedure introduced by Kuzmanoski and Kovačević (1992). The outcome of this procedure was the list of dates of the closest encounters of seven largest perturbing bodies in main asteroid belt with suitable perturbed asteroids as well as the absolute value of the maximum difference in right ascension and declination between two trajectories of perturbed body: the first one takes into account perturbation of perturbing body, whereas the second does not. If the difference was large (typically, larger than 15 arcsec in right ascension) and if the available observations covered long enough period before and after the encounter, the perturbed asteroid was selected as a good candidate for the mass determination. The distribution of indentified candidate events could be seen in Table 1.

Table 1. Close encouters used for mass determinations: T is the total number of used close encounters, N is the number of newly found close encounters.

Perturbing asteroid	T	N
Ceres	21	4
(2) Pallas	4	0
(4) Vesta	12	4
(10) Hygiea	8	1
(52) Europa	2	0
(511) Davida	3	1
(704) Interamnia	1	1

4. THE DYNAMICAL MODEL

The main asteroid belt is a chaotic system where mutual gravitational perturbations of bodies are expressed; to make an analogy with radio, successful mass determination therefore requires isolating a very weak signal from a very noisy background environment. For example, in the case of Ceres, signal to noise ratio was -1.05. Clearly, we needed to account for every other significant perturbation on a perturbed asteroid, so that the perturbations due to the perturbing asteroid could be isolated by the least-squares algorithm.

Bearing in mind that some other minor planets could perturb the motion of the chosen perturbed asteroids, the 9 largest asteroids have been included in the dynamical model, as well as all major planets. The mass values of perturbing asteroids used are given in Table 2. The gravitational influence of the perturbed asteroids on the perturber is negligible due to their small diameters.

Table 2. Masses of perturbing bodies from main asteroid belt.

Asteroid	Mass (10^{-10}) M_{sun}	
Ceres	4.76	adopted
(2) Pallas	1.08	adopted
(4) Vesta	1.35	adopted
(10) Hygiea	0.47	Scholl et al. (1987)
(11) Parthenope	0.0256	Viateau and Rapaport (2001)
(16) Psyche	0.34	Kuzmanoski and Kovačević (2002)
(52) Europa	0.011	IRAS
(511) Davida	0.014	IRAS
(704) Interamnia	0.013	IRAS

The numerical integration of differential equations of motion of perturbed bodies is carried out by Addams-Bashforth-Moulton predictor-corrector method (Moshier, 1992). The initial osculating orbital elements for the epoch JD 2452600.5 were taken from the Edward Bowell database, http://www.lowell.edu/users/elgb/. In order to analyze the motion of perturbed asteroids, sets of observational data were downloaded from the public database AstDys (http://hamilton.dm.unipi.it/astdys).

4. RESULTS

As can be seen from Table 1, we used 51 close encounters in order to determine masses of 7 largest asteroids in the main belt using modified and standard method. The largest asteroid in the main belt has the largest number of efficient close encounters (already known as well as newly found).

The range of values for Ceres mass, determined by other authors, is (4.6-5.0)10^{-10} M_{sun}. After applying modified and standard method we obtained results which are presented in Table 3. As it can be seen, both methods provided results which differ from each other by no more then 3σ (their own formal error). Also, they are within the historical range of determined masses of Ceres. Also, the results for the Ceres mass based on newly found close encounters, that occurred with asteroids: (2051) Chang, (6010) Lyzenga, (6594) Tasman and (34755) 2001QW120, are presented. In addition, we found that weighted mean of the values of the Ceres mass obtained by the modified (4.63±0.07)$10^{-10}M_{sun}$ and the standard method (4.70±0.05)$10^{-10}M_{sun}$ satisfied 3σ criterion with respect to the adopted value of the mass of Ceres.

Because of the highly inclined and eccentric orbit of (2) Pallas, its close encounters with other asteroids are rare. In Table 4 we present the solutions for the mass of (2) Pallas with formal errors not greater than approximately half the mass of this minor planet. Its weighted mean value obtained using modified method is (1.23±0.11)10^{-10} M_{sun}, while standard method produced (0.95± 0.08)10^{-10} M_{sun}.

Many authors emphasized the importance of reliable level of accuracy of the mass of (4) Vesta, since this asteroid is the second most massive body in the main belt. The results are listed in Table 5. The final values of the mass of Vesta, determined as a weighted mean are (1.28±0.03) 10^{-10} M_{sun} (using modified method) and (1.35±0.18)$10^{-10}M_{sun}$ (using standard method). These values agree with all determinations made so far. Also, we presented results based on newly found close encounters with (5205) 1988CU7, (21225) 1995GQ1.

From 8 close encounters with (10) Hygiea we obtained results presented in Table 6. Difference between the weighted mean mass obtained by the modified method ((4.72±0.16)10^{-11} M_{sun}) and standard method ((4.68±0.24)$10^{-11}M_{sun}$) are not larger than 3 σ.

The mass of (52) Europa was determined for the first time by Michalak (2000). Both our methods gave results for its mass only in the case of the close encounters with (306) Unitas and (1023) Thomana. We did not calculate the weighted mean value of the mass of (52) Europa, because we have only two cases.

Table 3. Geometrical and kinematical parameters of close encounters: ρ is the minimum distance, Vr is relative velocity and θ is deflection angle of perturbed asteroid. Masses of Ceres obtained using standard and modified method are given in columns SM and MM.

Perturbing body	**Date (d.m.y)**	ρ [AU]	**Vr [km/s]**	θ [“]	**SM [10^{-10} M_{sun}]**	**MM [10^{-10} M_{sun}]**
(2) Pallas	16.05.1825	0.188	12.61	0.01	4.45±0.05	4.22±0.04
(32) Pomona	25.11.1975	0.025	4.75	0.31	5.32(0.16	5.18(0.05
(76) Freia	05.08.1957	0.212	4.08	0.05	4.27(0.08	4.14(0.06
(91) Aegina	13.09.1973	0.033	3.28	0.49	4.91(0.04	5.00(0.02
(203) Pompeja	22.08.1948	0.016	4.12	0.63	4.73(0.04	4.79(0.02
(348) May	02.09.1984	0.046	0.79	6.07	4.74±0.05	4.77±0.01
(347) Pariana	29.05.1943	0.078	1.48	1.02	4.80±0.09	4.72±0.05
(454) Mathesis	23.11.1971	0.021	2.93	0.97	4.48±0.06	4.33±0.01
(488) Kreussa	17.07.1963	0.282	3.00	0.07	4.64±0.16	4.26±0.11
(534) Nassovia	24.12.1975	0.023	2.75	1.00	4.83±0.07	5.12±0.04
(548) Kressida	13.07.1982	0.049	2.95	0.41	5.28±0.24	4.89±0.10
(621) Werdandi	01.05.1962	0.050	3.04	0.38	4.35±0.15	4.56±0.20
(792) Metkalfia	25.07.1950	0.013	5.78	0.40	5.81±1.10	5.22±0.35
(850) Altona	22.02.1970	0.026	3.84	0.45	4.91±0.16	4.68±0.11
(1642) Hill	25.11.1925	0.012	5.54	0.47	4.81±0.06	4.81±0.08
(1847) Stobbe	07.09.1958	0.094	1.76	0.60	3.94±0.23	4.10±0.17
(3344) Modena	27.09.1980	0.021	2.39	1.45	4.34±0.38	4.36±0.11
(8) Flora	09.03.1963	0.2265	3.02	0.02	1.35±0.05	1.64±0.02
(17) Thetis	19.06.1996	0.0194	1.18	1.83	1.35±0.02	1.288±0.001
(56) Melete	14.11.1923	0.1122	4.62	0.02	1.34±0.09	1.57±0.04
(67) Asia	20.01.1991	0.0311	4.45	0.08	1.18±0.07	1.21±0.01
(77) Frigga	07.06.1955	0.0249	4.62	0.09	1.38±0.06	1.29±0.04
(109) Felicitas	15.04.1959	0.0191	8.16	0.04	1.52±0.06	1.77±0.03
(163) Erigone	24.04.1934	0.1960	5.64	0.01	1.16±0.09	1.05±0.06
(197) Arete	14.05.1885	0.0181	2.22	0.55	1.32±0.02	1.34±0.01
(5205) 1988CU7	12.05.1977	0.0029	3.73	1.22	1.31±0.02	1.25±0.02
(21225) 1995GQ1	12.01.1981	0.0141	1.66	1.28	1.35±0.18	1.28±0.02

Table 4. Mass of Pallas obtained using standard and modified method.

Perturbing body	**Date (d.m.y)**	ρ [AU]	**Vr [km/s]**	θ [“]	**SM [10^{-10} M_{sun}]**	**MM [10^{-10} M_{sun}]**
Ceres	02.01.1830	0.188	12.61	0.01	1.32±0.05	1.57±0.03
(582) Olimpia	14.07.1936	0.033	3.19	0.12	0.90±0.08	1.04±0.21
3131) Mason-Dixon	04.12.1984	0.012	10.84	0.03	1.66±0.32	1.23±0.22
(5930) Zhiganov	17.06.1977	0.015	12.01	0.02	1.17±0.44	1.36±0.17

Table 5. Masses of (4) Vesta obtained using standard and modified method.

Perturbing body	**Date (d.m.y)**	**ρ [AU]**	**Vr [km/s]**	**θ [“]**	**SM [10^{-10} M_{sun}]**	**MM [10^{-10} M_{sun}]**
(8) Flora	09.03.1963	0.2265	3.02	0.02	1.35±0.05	1.64±0.02
(17) Thetis	19.06.1996	0.0194	1.18	1.83	1.35±0.02	1.288±0.001
(56) Melete	14.11.1923	0.1122	4.62	0.02	1.34±0.09	1.57±0.04
(67) Asia	20.01.1991	0.0311	4.45	0.08	1.18±0.07	1.21±0.01
(77) Frigga	07.06.1955	0.0249	4.62	0.09	1.38±0.06	1.29±0.04
(109) Felicitas	15.04.1959	0.0191	8.16	0.04	1.52±0.06	1.77±0.03
(163) Erigone	24.04.1934	0.1960	5.64	0.01	1.16±0.09	1.05±0.06
(197) Arete	14.05.1885	0.0181	2.22	0.55	1.32±0.02	1.34±0.01
(5205) 1988CU7	12.05.1977	0.0029	3.73	1.22	1.31±0.02	1.25±0.02
(21225) 1995GQ1	12.01.1981	0.0141	1.66	1.28	1.35±0.18	1.28±0.02

Table 6. Mass of (10) Hygiea obtained using standard and modified method.

Perturbing body	**Date (d.m.y)**	**ρ [AU]**	**Vr [km/s]**	**θ [“]**	**SM [10^{-10} M_{sun}]**	**MM [10^{-10} M_{sun}]**
(7)Iris	18.01.1928	0.0724	4.26	1.31	5.86±0.4	5.29±0.40
(20)Massalia	05.11.1933	0.1499	2.15	2.48	4.69±0.46	6.49±0.5
60)Echo	07.05.1867	0.2111	3.24	0.78	5.30±1.00	5.08±0.9
(69)Hesperia	05.09.1951	0.0862	5.23	0.73	5.76±1.1	5.10±0.8
(111) Ate	11.02.1878	0.0942	1.76	5.90	5.88±0.6	5.60±0.2
(209)Dido	09.05.1958	0.2463	2.22	1.42	4.30±1.0	4.98±0.7
(829)Academia	19.05.1927	0.0064	3.22	25.92	2.65±1.0	2.49±1.23
(3946) Shor	30.03.1998	0.0144	0.91	1.44	3.10±0.4	2.52±0.26

Table 7. Mass of (52) Europa obtained using standard and modified method.

Perturbing body	**Date (d.m.y)**	**ρ [AU]**	**Vr [km/s]**	**θ [“]**	**SM [10^{-10} M_{sun}]**	**MM [10^{-10} M_{sun}]**
(306) Unitas	14.01.1945	0.0980	2.2	0.01	2.12±0.56	2.76±0.21
(1023) Thomana	31.05.1971	0.0066	3.76	0.04	0.78±0.49	1.17±0.70

We used 3 close encounters for mass determination of (511) Davida.Weighted mean values are: (2.21±0.18) 10^{-10} M_{sun} obtained by the standard method and (2.72±0.02) 10^{-10} M_{sun} based on the modified method.

Table 8. Mass of (511) Davida obtained using standard and modified method.

Perturbing body	**Date (d.m.y)**	**ρ [AU]**	**Vr [km/s]**	**θ [“]**	**SM [10^{-10} M_{sun}]**	**MM [10^{-10} M_{sun}]**
(89) Julia	27.10.1957	0.0389	9.90	0.001	2.09±0.53	2.75±0.02
(532) Herculina	14.04.1963	0.0307	4.24	0.01	2.20±0.20	2.35±0.07
(7191) 1993MA1	16.07.1969	0.0046	5.98	0.03	2.88±1.08	2.47±0.47

In the case of (704) Interamnia we have only one useful close encounter. As it can be seen from Table 9, formal errors of the masses of Interamnia are larger then 50 % of the calculated values for the masses. Further observations of this asteroid are highly desirable to enable a more exact and reliable estimation of Interamnia's mass.

Table 9. Mass of (704) Interamnia obtained using standard and modified method.

Perturbing body	**Date (d.m.y)**	**ρ [AU]**	**Vr [km/s]**	**θ [“]**	**SM $[10^{-10} M_{sun}]$**	**MM $[10^{-10} M_{sun}]$**
(7461) Kachmokiam	31.05.1997	0.0075	5.29	0.02	2.23±1.00	0.97±0.52

Since our mass determination survey was intentionally centered on the largest asteroids, and since we observed non-uniform densities within taxonomic classes, we investigated whether there might be a relationship between asteroid radius and bulk density.

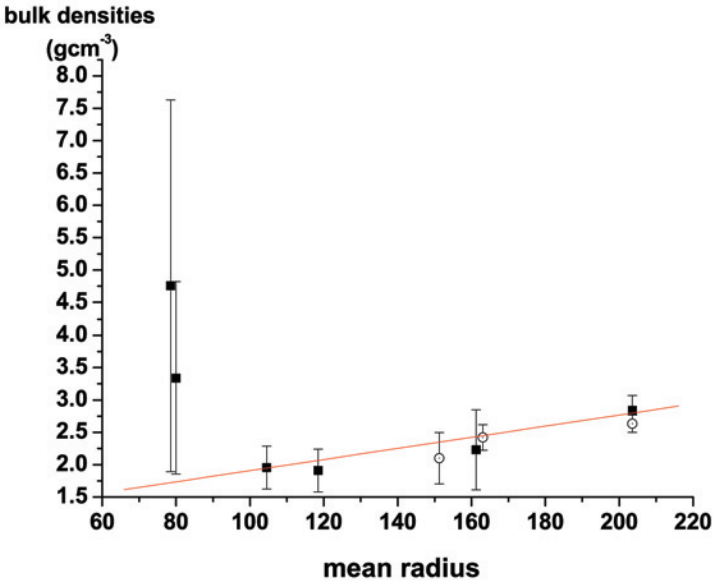

Figure 1: Mean radius versus bulk density for C-class asteroids. Circles denote our results (for densites of (10) Hygiea, (52) Europa, (511) Davida). Results of other authors are marked by squeres.

Figure 1 includes all of the C-class asteroids which masses were determined by other authors (Baer and Chesley, 2008) and by us ((10) Hygiea, (52) Europa, (511) Davida); the correlation coefficients of 0.84 suggest a fairly strong relationship between mean radius and bulk density. The best-fit line for C-class is

$$\rho=1.046+0.086r$$

Note that the best-fit line in Fig. 1 predicts that large C-class asteroids should have bulk densities of approximately 3 g/cm^3, however, as the mean radius is decreasing, corresponding reductions in bulk density suggest increasing levels of porosity. One possible explanation (Baer and Chesley 2008) could be that, while most asteroids began as relatively solid objects with low porosity, subsequent collisions have resulted in varying degrees of structural change.

One could expect that the largest asteroids survive most impacts with little damage their bulk densities should therefore remain similar to the grain densities of their constituent minerals. Medium-sized asteroids might suffer from fractures; and the resulting voids would produce a moderate degree of porosity, and a reduced bulk density. The smaller asteroids might suffer catastrophic damage, even to the point of disruption; the fragments would subsequently collapse under their own weak gravitational attraction, leaving extensive voids between them. Such "rubble piles" would have high porosity, and low bulk density. This rough model cannot precisely account for the structural evolution of every C asteroid because some medium-sized asteroids, such as 52 Europa, may also have involved in particularly severe collisions, resulting in high levels of porosity that leave them below the trend lines.

4. CONCLUSIONS

We calculated the masses of 7 largest asteroids independently for all perturbed asteroids using standard and modified method. Some of these perturbed asteroids were never used before for this purpose, nevertheless giving quite good estimates of the mass of the massive minor planets. Generally, the masses we found agree with recent results of other authors and indicate that the mass of (1) Ceres appears to be equal to the adopted value as well as the mass of (4) Vesta. Results for the masses of other six asteroids are in good agreement with results obtained by other authors.

However, most of the available observations used for their mass determination have high errors and uneven distribution. As it can be seen from numerical tests, modified method provided results which are in good agreement with standard method and adopted values of asteroid's masses. It can be used as a tool for asteroid mass determination. It is obvious that refining the dynamical model will improve the accuracy of the mass determination of massive asteroids.

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