

Сриниваса Ађангар Рамануџан - "Човек који је
познавао бесконачност"



Ове године се навршава 130 година од рођења С. Рамануџана

A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with *ideas*. (GH Hardy)

Mathematics is the study of mental objects with reproducible properties. (A. Borovik)

Srinivasa Ramanujan Said:

*“An equation for me
has no meaning unless
it expresses a thought of
God”*

While asleep, I had an unusual experience. There was a red screen formed by flowing blood, as it were. I was observing it. Suddenly a hand began to write on the screen. I became all attention. That hand wrote a number of elliptic integrals. They stuck to my mind. As soon as I woke up, I committed them to writing."



Lakshmi

Goddess of Namagiri



Ramanujan's home on Sarangapani Sannidhi Street, Kumbakonam



Ramanjan's Home, Kumbakonam



Da li je veće $\frac{2}{3}$ ili $\frac{3}{5}$?

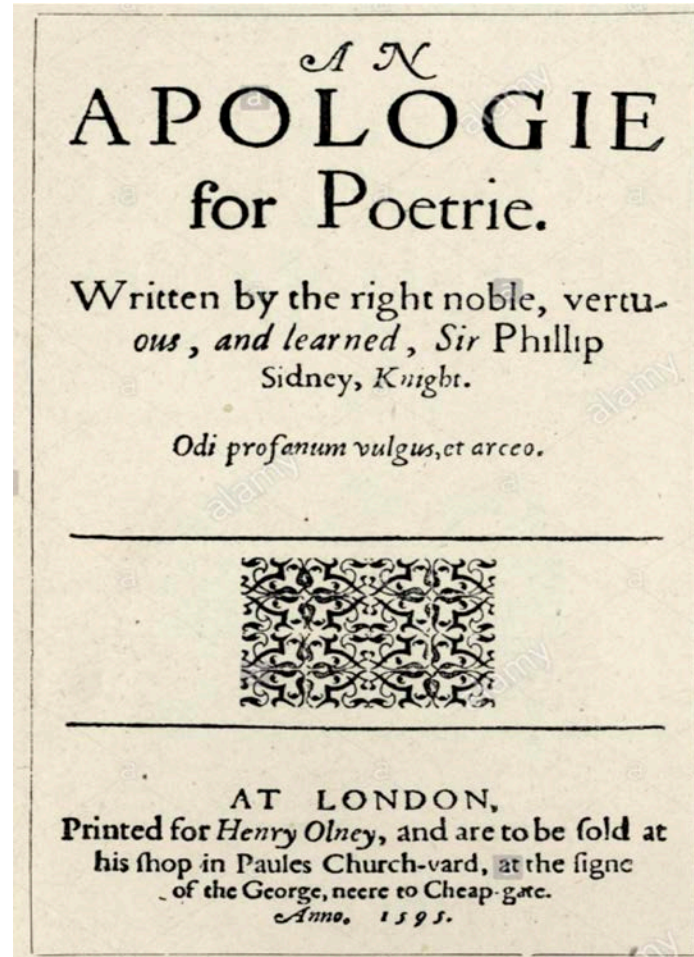
LILI LALAUNA (1)

Lala lula luna lina
Ala luna lani lana
Ana lili ula ina
Nali ilun liliana

Lila ani ul ulana
Lani linu ul nanula
Anali ni nina nana
Ila ala una nula

Alauna lui il lala
Alilana, lan, lu, li, la
Nalu nilu nun ninala
Nala una an anila.
(1950)



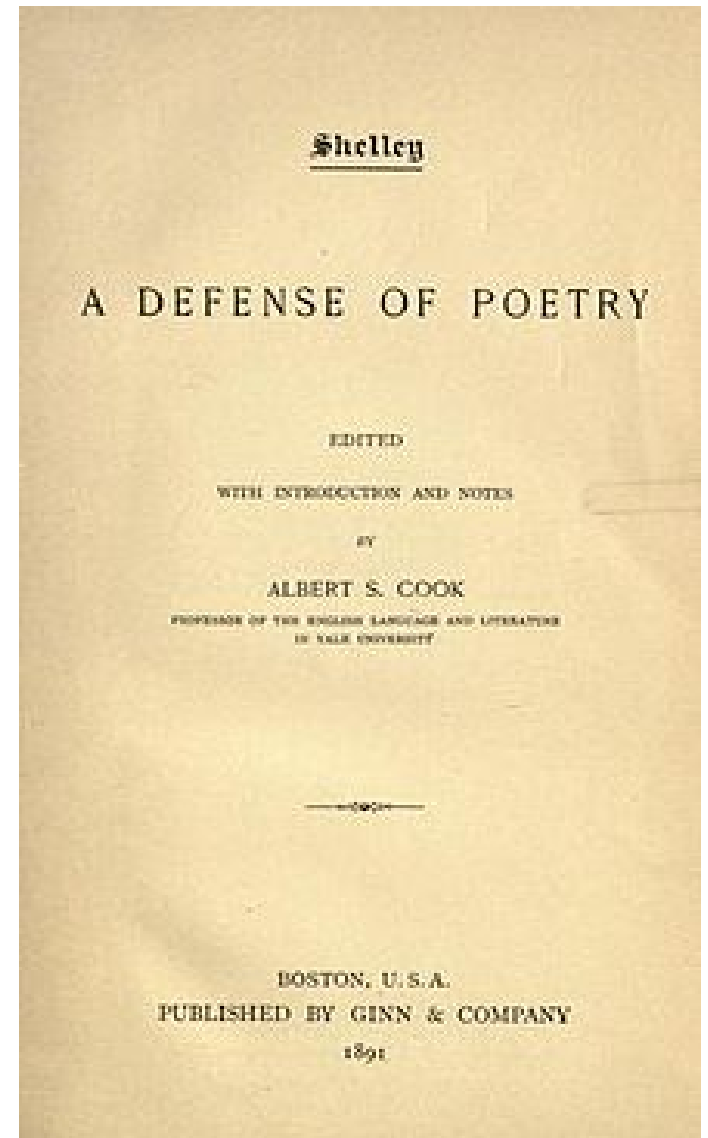


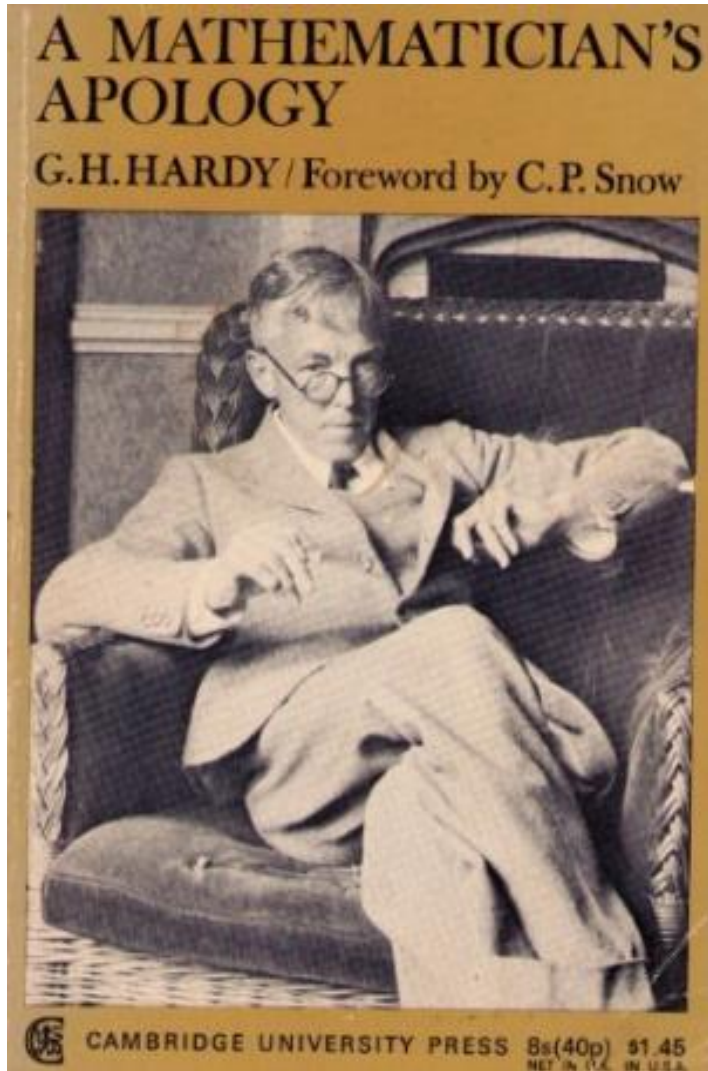
Odi profanum vulgus, et arceo = I hate the common masses and avoid them (Horacio)

The essence of his defense is that poetry, by combining the liveliness of history with the ethical focus of philosophy, is more effective than either history or philosophy in rousing its readers to virtue. (1595)



Shelley (1792-1822)





Hardy argued that the point of mathematics was the same as the point of art: the creation of intrinsic beauty.

I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."

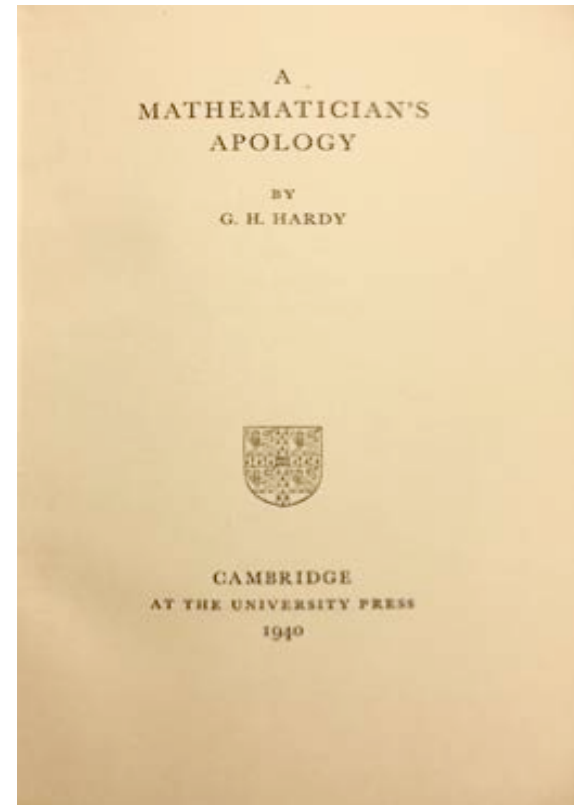




Fig. 1. G. H. Hardy around 1900.

A COURSE
OF
PURE MATHEMATICS

BY
G. H. HARDY, M.A., F.R.S.
FELLOW OF NEW COLLEGE
SAVILIAN PROFESSOR OF GEOMETRY IN THE UNIVERSITY
OF OXFORD
LATE FELLOW OF TRINITY COLLEGE, CAMBRIDGE

THIRD EDITION

Cambridge
at the University Press
1921

less than
an expression which very nearly
to the real results, the error being negligible I
would request you to go through the enclosed
pages. Being poor, if you are convinced that
there is anything of value I would like to
have my theorems published. I have not given
the actual investigation nor the expression
which I got but I have indicated the lines
which I proceed. Being inexperienced I
value any advice you give
very highly and am
requesting to be excused for the trouble
you.

I remain, Dear Sirs
yours truly

S. Ramanujan

input
is very great
From the forms of
 $O\left\{\frac{x}{(\log x)^\Delta}\right\}$, $O(x^\epsilon)$
that from partici
have been guess
Even in regular
idea of the for
a complicated
an idea ev
even if
to be found
I have
that it
0 and

"Dear Sir, I am very much gratified on perusing your letter of the 8th February 1913. I was expecting a reply from you similar to the one which a Mathematics Professor at London wrote asking me to study carefully Bromwich's Infinite Series and not fall into the pitfalls of divergent series. ... I told him that the sum of an infinite number of terms of the series: $1 + 2 + 3 + 4 + \dots = -1/12$ under my theory. If I tell you this you will at once point out to me the lunatic asylum as my goal. I dilate on this simply to convince you that you will not be able to follow my methods of proof if I indicate the lines on which I proceed in a single letter. ...

Another way of finding the constant is as follows - 41

Let us take the series $1 + 2 + 3 + 4 + 5 + \dots$. Let C be its constant. Then

$$C = 1 + 2 + 3 + 4 + \dots$$

$$\therefore 4C = 4 + 8 + \dots$$

$$\therefore -3C = 1 - 2 + 3 - 4 + \dots = \frac{1}{(1+1)^2} = \frac{1}{4}$$

$$\therefore C = -\frac{1}{12}$$

So what was Hardy's reaction? First he consulted Littlewood. Was it perhaps a practical joke? Were these formulas all already known, or perhaps completely wrong? Some they recognized, and knew were correct. But many they did not. But as Hardy later said with characteristic clever gloss, they concluded that these too "must be true because, if they were not true, no one would have the imagination to invent them."



Fig. 1. G.H. Hardy around 1900.

The seven partitions of 5 are:

5

4 + 1

3 + 2

3 + 1 + 1

2 + 2 + 1

2 + 1 + 1 + 1

1 + 1 + 1 + 1 + 1

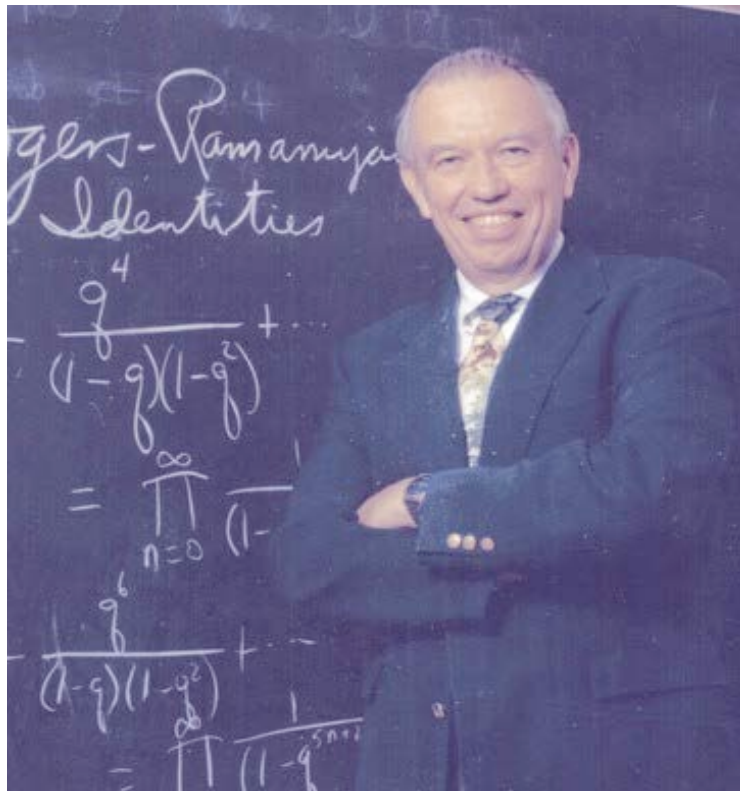
The first few values of the partition function are (starting with $p(0) = 1$):

1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231, 297, 385, 490, 627, 792, 1002, 1255, 1575, 1958, 2436, 3010, 3718, 4565, 5604, ...

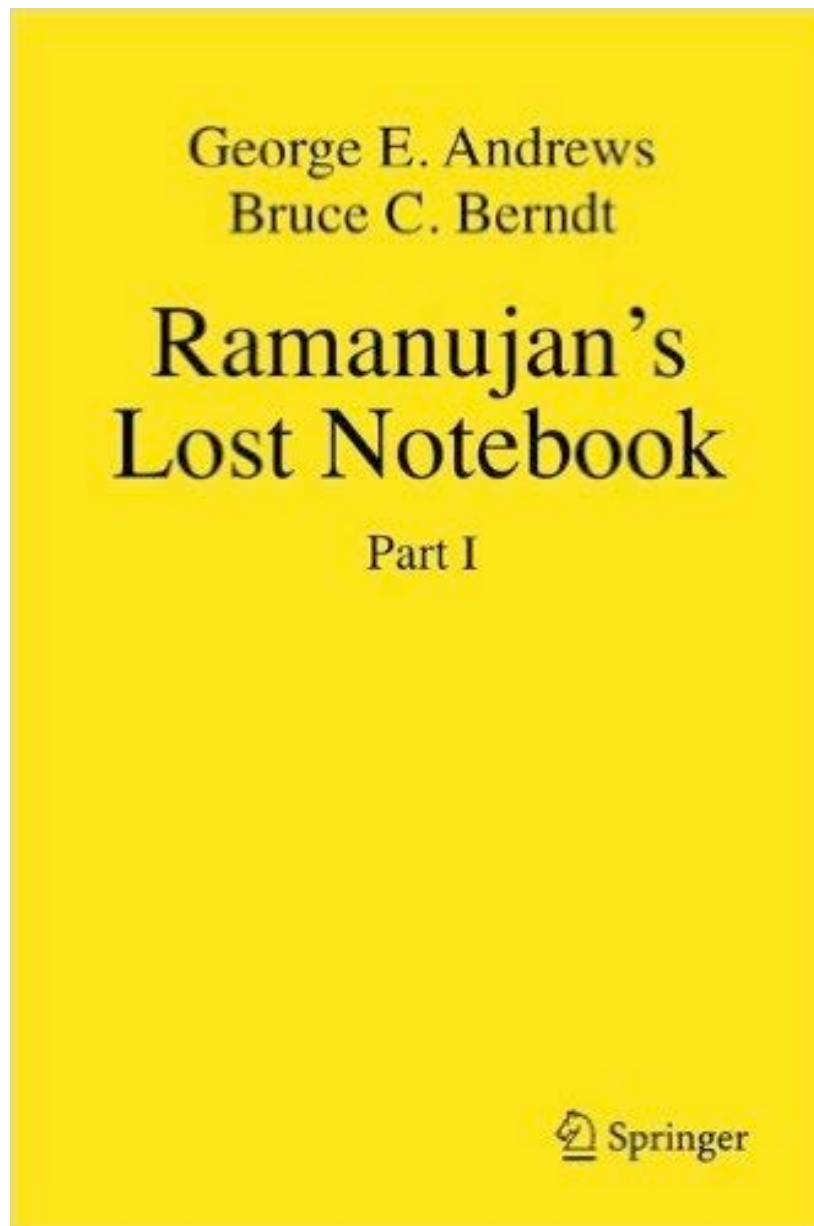
$p(100) = 190,569,292$, $p(1000)$ is 24,061,467,864,032,622,473,692,149,727,991 or approximately 2.40615×10^{31}

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} \sqrt{k} A_k(n) \frac{d}{dn} \left(\frac{1}{\sqrt{n - \frac{1}{24}}} \sinh \left[\frac{\pi}{k} \sqrt{\frac{2}{3} \left(n - \frac{1}{24} \right)} \right] \right).$$

Hardy-Ramanujan-Rademacher formula



George Andrews



ff

$$(i) \frac{1 + 53x + 9x^2}{1 - 82x - 82x^2 + x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$

$$\text{or } \frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$$

$$(ii) \frac{2 - 26x - 12x^2}{1 - 82x - 82x^2 + x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$$

$$\text{or } \frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$$

$$(iii) \frac{2 + 8x - 10x^2}{1 - 82x - 82x^2 + x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$$

$$\text{or } \frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } d_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$65601^3 + 67402^3 = 83802^3 + 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

$$1+2+3+4+\dots = -\frac{1}{12}$$

$$1^3+2^3+3^3+\dots = \frac{1}{120}$$

+1 -1 +1 -1

+1	+1	-1	+1	-1
-1	-1	+1	-1	+1
+1	+1	-1	+1	-1
-1	-1	+1	-1	+1



$$1729 = 1^3 + 12^3 = 9^3 + 10^3.$$

$$\begin{aligned} \text{Ta}(2) &= 1729 = 1^3 + 12^3 \\ &= 9^3 + 10^3 \end{aligned}$$

$$\begin{aligned} \text{Ta}(3) &= 87539319 = 167^3 + 436^3 \\ &= 228^3 + 423^3 \\ &= 255^3 + 414^3 \end{aligned}$$

$$\begin{aligned} \text{Ta}(4) &= 6963472309248 = 2421^3 + 19083^3 \\ &= 5436^3 + 18948^3 \\ &= 10200^3 + 18072^3 \\ &= 13322^3 + 16630^3 \end{aligned}$$

$$\begin{aligned} \text{Ta}(5) &= 48988659276962496 = 38787^3 + 365757^3 \\ &= 107839^3 + 362753^3 \\ &= 205292^3 + 342952^3 \\ &= 221424^3 + 336588^3 \\ &= 231518^3 + 331954^3 \end{aligned}$$

$$\begin{aligned} \text{Ta}(6) &= 24153319581254312065344 = 582162^3 + 28906206^3 \\ &= 3064173^3 + 28894803^3 \\ &= 8519281^3 + 28657487^3 \\ &= 16218068^3 + 27093208^3 \\ &= 17492496^3 + 26590452^3 \\ &= 18289922^3 + 26224366^3 \end{aligned}$$

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The first few values of the partition function are (starting with $p(0) = 1$):

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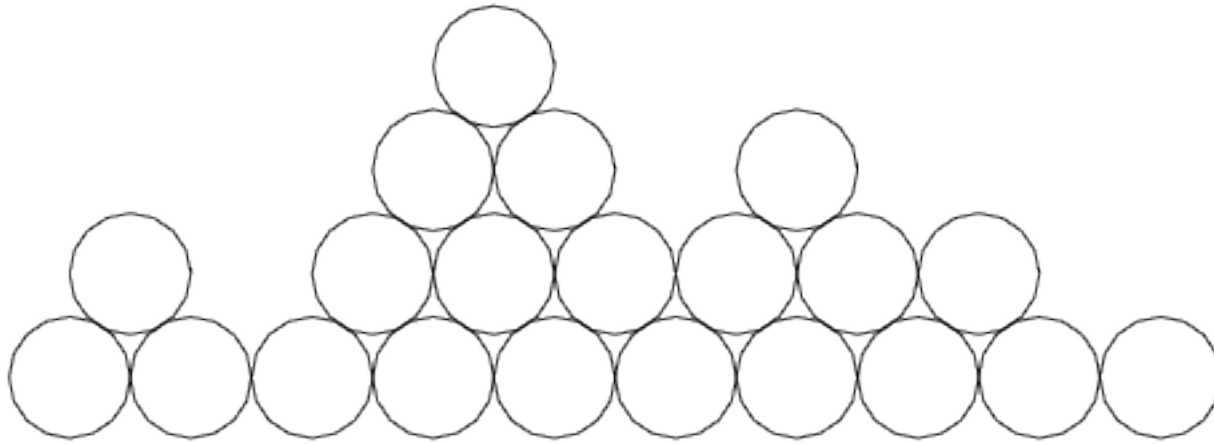
$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} \sqrt{k} A_k(n) \frac{d}{dn} \left(\frac{1}{\sqrt{n - \frac{1}{24}}} \sinh \left[\frac{\pi}{k} \sqrt{\frac{2}{3} \left(n - \frac{1}{24} \right)} \right] \right).$$

George E. Andrews
Bruce C. Berndt

Ramanujan's Lost Notebook

Part I

 Springer



$$R(q) = \sqrt[5]{q} \times \frac{1}{1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{1 + \frac{q^4}{1 + \dots}}}}}$$



Relations (5) essentially sum up the possible configurations of the 60° row above and to the right of the 60° base row with the origin site included (see figure 1). For example, the 1 in (5) corresponds to no second 60° row (the $N = k$ animal). The zF_1 term corresponds to one site, at $(x, y) = (1, 0)$, in the second 60° row, etc. Configurations of the more distant 60° rows are summed up in the appropriate F_m . Except for the $k = 1$ 'boundary condition'

$$F_1(z) = 1 + zF_1(z) + z^2F_2(z) \quad (6)$$

relations (5) can be replaced by their differences:

$$F_{k+1}(z) - F_k(z) = z^{k+2}F_{k+2}(z) \quad k \geq 1 \quad (7)$$

which form a homogeneous set of equations. Recursions of the general type (7) have been encountered in the theory of q series; see, for example, Adiga *et al* (1985, p 26). (In the present context the role of q is played by the variable z .) However, none of the particular forms there are suitable for our problem (see below). Thus, we devised the q series

$$\phi_k(z) = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{z^{n(n+k+1)}}{q_n} \quad q_n \equiv \prod_{j=1}^n (1 - z^j) \quad (8)$$

which satisfy the recursions

$$\phi_{k+1}(z) - \phi_k(z) = z^{k+2}\phi_{k+2}(z) \quad (9)$$

Let us also point out that by the Pincherle theorem (see, e.g., Gautschi 1967), a continued fraction analogue of (11) can be obtained by standard methods. Specifically

$$\frac{\phi_2(z)}{\phi_1(z)} = \frac{1}{1 - \frac{z^3}{1 - \frac{z^4}{1 - \frac{z^5}{1 - \dots}}}}$$

Exact Generating Function for Fully Directed Compact Lattice Animals

V. Privman and N. M. Švrakić

Department of Physics, Clarkson University, Potsdam, New York 13676

(Received 14 December 1987)

The fully directed compact lattice-animal model on the square lattice is solved exactly. An explicit expression is obtained for the cluster-number generating function which has a complicated complex-plane singularity pattern including a natural boundary of essential singularities. However, the cluster numbers are controlled by a simple pole singularity and grow proportionally to λ^N , where N is the number of sites, and $\lambda \approx 2.66$.

PACS numbers: 05.50.+q, 36.20.Ey, 64.60.-i

Models of lattice animals (connected clusters) have received much attention in recent years as simply systems with "geometrical" critical phenomena. Various aspects of the isotropic lattice animals have been reviewed, e.g., by Bovier, Froehlich, and Glaus,¹ Privman,² and Sykes.³ Directed lattice animals have also been studied extensively, numerically and by analytic methods.⁴⁻⁶ Several conjectured exact critical-exponent values and relations have been found for both types of animals.⁴⁻¹² When an additional compactness constraint is imposed with directedness, a new universality class is obtained.¹³⁻¹⁵ Partially directed compact lattice-animal models on the square and triangular lattices can be solved exactly.¹⁴⁻¹⁶ The generating functions for the numbers c_N of distinct N -site clusters,

$$G(z) = \sum_{N=1}^{\infty} c_N z^N, \quad (1)$$

turn out to be rational functions of z . The singularity nearest to the origin is a simple pole at $z_c = \lambda^{-1} < 1$, where λ is model dependent. Thus, the generic lattice-animal cluster-number asymptotic form for large N ,

$$c_N \approx AN^{-\theta} \lambda^N, \quad (2)$$

applies with the critical exponent $\theta=0$. For the fully directed square-lattice model, numerical studies^{13,14} suggest that the leading singularity is similar ($\theta=0$). However, a recent exact calculation¹⁷ for a related circle-stacking model¹⁸ indicates that the generating function for the fully directed compact animals may have a far

richer complex-plane structure.

In this work, we report exact results for the fully directed compact square-lattice animals. First, we define the model and present an exact expression for $G(z)$. We then discuss the complex-plane structure of this generating function, focusing on the new features not present in the previously studied partially directed models. Lastly, we outline our method of solution.

A fully directed lattice animal on the square lattice of unit spacing is defined as follows. Let the xy coordinate axes coincide with the principal lattice directions, with the origin at a lattice site. This origin site is always part of a cluster. The different directed N -site clusters are defined by all the possible selections of the remaining $N-1$ sites in such a way that each can be reached from the origin, by a walk of $+\hat{x}$ and $+\hat{y}$ steps, through other occupied cluster sites. Obviously, the directed axis for this problem is defined by the unit vector $(\hat{x}+\hat{y})/\sqrt{2}$. The compactness condition^{13,14} is then added by the requirement that all cluster sites at a given "time," i.e., for fixed values of $x+y$, form a continuous chain of diagonal neighbors (there is no restriction in the case of a single site at a given $x+y$). Note that the sites at each time level (fixed $x+y > 0$) are connected to the origin site (at $x=y=0$) via the sites in the preceding time level. The compactness condition has therefore an effect of suppressing branchings in a cluster. However, the terminology is somewhat misleading: The clusters can be quite sparse.

Our results for the generating function are summarized below:

$$G(z) = z \frac{(1-z^2)T_{12}(z) - z^3 T_{13}(z)}{(1+z)(1-3z+z^2)T_{12}(z) - z^3(1-2z)T_{13}(z) + z^5 T_{23}(z)}, \quad (3)$$

where

$$T_{ij}(z) = A_i(z)B_j(z) - A_j(z)B_i(z), \quad (4)$$

$$A_k(z) = \sum_{n=0}^{\infty} q_n^{-2}(z) z^{n(3n+2k+3)/2}, \quad (5)$$

$$B_k(z) = k + \sum_{n=1}^{\infty} q_n^{-2}(z) z^{n(3n+2k+3)/2} \left[k+n+2 \sum_{m=1}^n \frac{1}{1-z^m} \right], \quad (6)$$

$$q_n(z) \equiv \prod_{m=1}^n (1-z^m), \quad \text{for } n \geq 1, \quad (7)$$

Lecture Notes in Physics

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H. A. Weidenmüller, Heidelberg, J. Wess, Karlsruhe and J. Zittartz, Köln

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V. Privman
N.M. Švrakić

Directed Models of Polymers,
Interfaces, and Clusters:
Scaling and
Finite-Size Properties

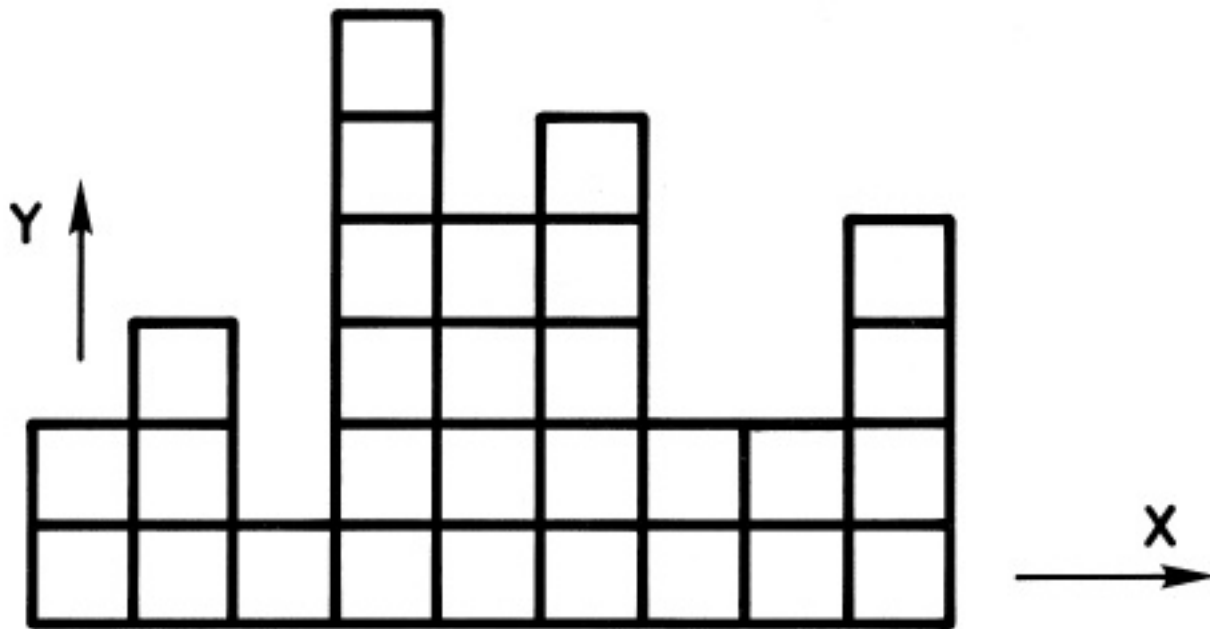


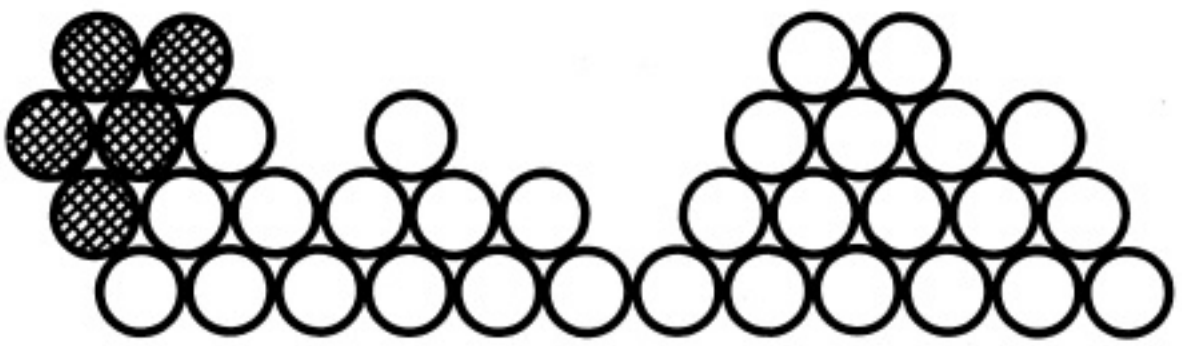
Springer-Verlag

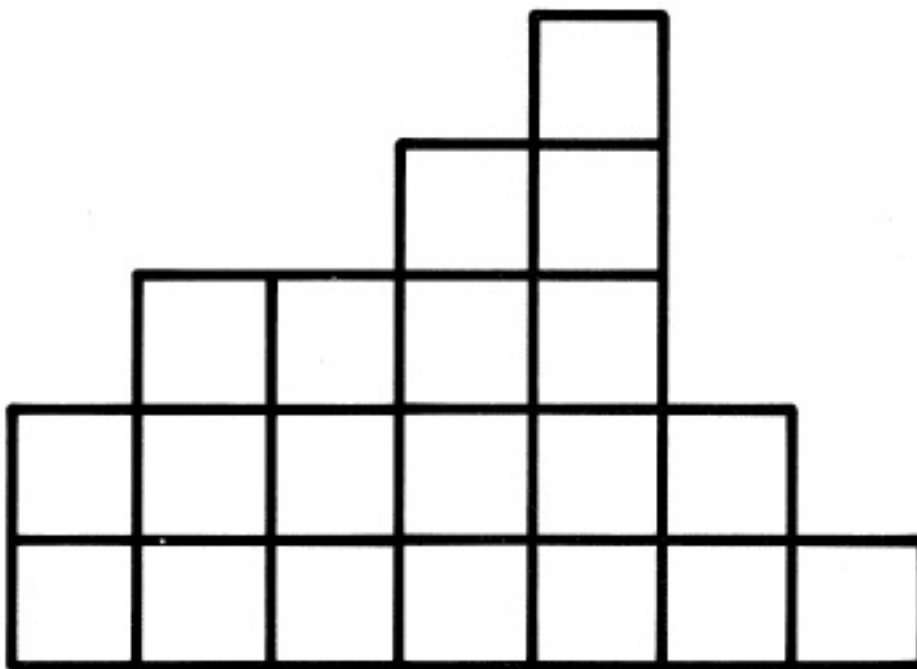
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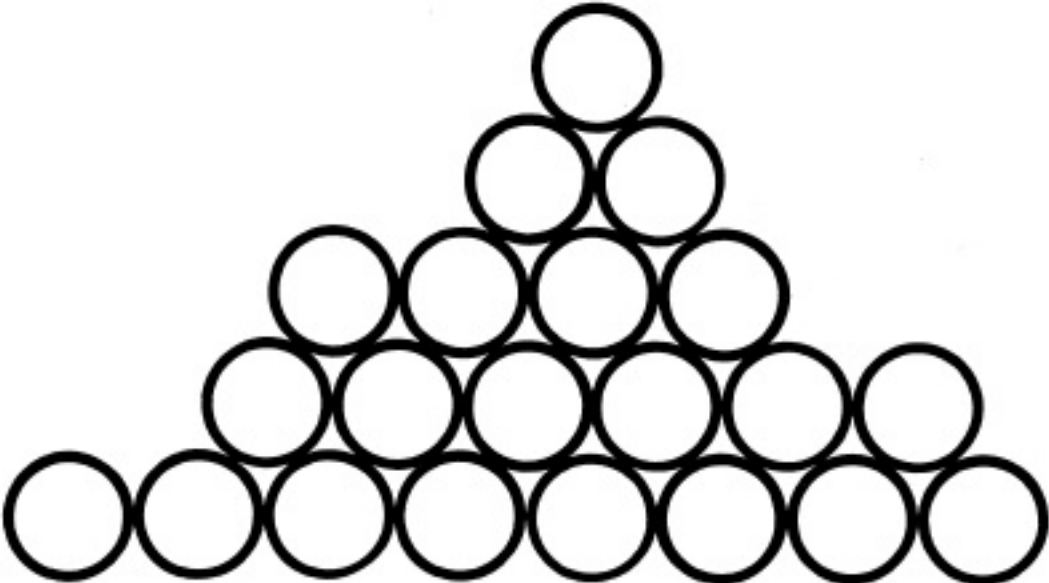
Privman · Švrakić

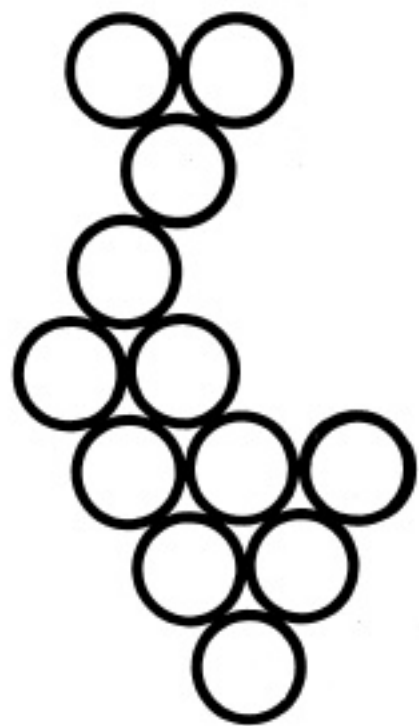
Directed Models of Polymers, Interfaces, and Clusters:
Scaling and Finite-Size Properties











$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + 5\sqrt{1 + \dots}}}}} = ?$$

$$3 = \sqrt{9}$$

$$= \sqrt{1 + 8}$$

$$= \sqrt{1 + 2 * 4}$$

$$= \sqrt{1 + 2\sqrt{16}}$$

$$= \sqrt{1 + 2\sqrt{1 + 15}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3 * 5}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3\sqrt{25}}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 24}}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4 * 6}}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{36}}}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + \dots}}}$$

For estimating π

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}$$

$$e^{\pi\sqrt{19}} \approx 12^3(3^2 - 1)^3 + 744 - 0.22$$

$$e^{\pi\sqrt{43}} \approx 12^3(9^2 - 1)^3 + 744 - 0.00022$$

$$e^{\pi\sqrt{67}} \approx 12^3(21^2 - 1)^3 + 744 - 0.0000013$$

$$e^{\pi\sqrt{163}} \approx 12^3(231^2 - 1)^3 + 744 - 0.00000000000075$$

$$e^{\pi\sqrt{19}} \approx 96^3 + 744 - 0.22$$

$$e^{\pi\sqrt{43}} \approx 960^3 + 744 - 0.00022$$

$$e^{\pi\sqrt{67}} \approx 5280^3 + 744 - 0.0000013$$

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744 - 0.00000000000075$$

$$e^{\pi\sqrt{163}} = 262537412640768743.999999999992500725971981856888793538563373369908627075374103782\dots$$

We have to do mathematics using the brain which evolved 30 000 years ago for survival in the African savanna. (S. Dehaene)

$$\text{sinc}(x) = \sin(x)/x; x \neq 0$$

$$\int_0^{\infty} \text{sinc}(x) = \pi/2$$

$$\int_0^{\infty} \text{sinc}(x)\text{sinc}(x/3) = \pi/2$$

$$\int_0^{\infty} \text{sinc}(x)\text{sinc}(x/3)\text{sinc}(x/5) = \pi/2$$

$$\int_0^{\infty} \text{sinc}(x)\text{sinc}(x/3)\text{sinc}(x/5)\text{sinc}(x/7) = \pi/2$$

$$\int_0^{\infty} \text{sinc}(x)\text{sinc}(x/3)\text{sinc}(x/5)\text{sinc}(x/7)\text{sinc}(x/9) = \pi/2.$$

Izvesna pravilnost se nazire...Ako nastavimo

$$\int_0^{\infty} \operatorname{sinc}(x)\operatorname{sinc}(x/3)\operatorname{sinc}(x/5)\operatorname{sinc}(x/7)\operatorname{sinc}(x/9)\operatorname{sinc}(x/11) = \pi/2$$

$$\int_0^{\infty} \operatorname{sinc}(x)\operatorname{sinc}(x/3)\operatorname{sinc}(x/5)\operatorname{sinc}(x/7)\operatorname{sinc}(x/9)\operatorname{sinc}(x/11)\operatorname{sinc}(x/13) = \pi/2.$$

Then, out of blue:

$$\int_0^{\infty} \operatorname{sinc}(x)\operatorname{sinc}(x/3)\operatorname{sinc}(x/5)\operatorname{sinc}(x/7)\operatorname{sinc}(x/9)\operatorname{sinc}(x/11)\operatorname{sinc}(x/13)\operatorname{sinc}(x/15) \\ = \frac{467807924713440738696537864469}{935615849440640907310521750000} \cdot \pi.$$

A NEW YORK TIMES NOTABLE BOOK OF THE YEAR

THE MAN WHO KNEW INFINITY



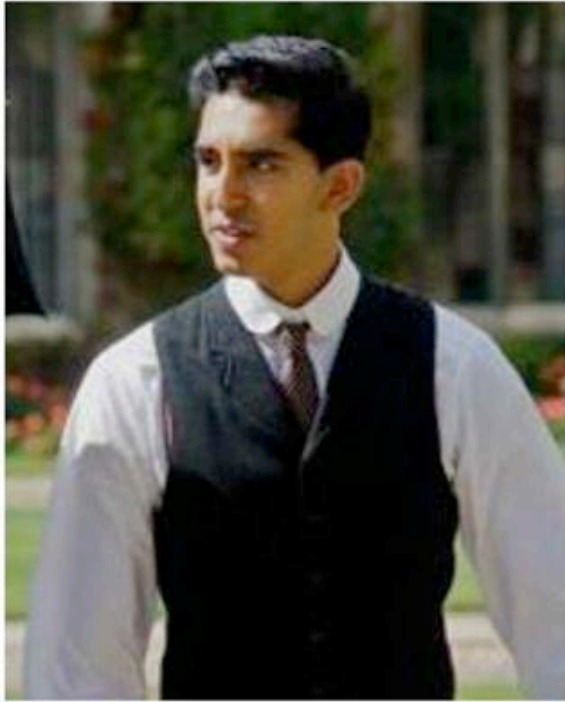
A LIFE OF
THE GENIUS
RAMANUJAN

"A masterpiece,"—*The Washington Post Book World*

ROBERT KANIGEL

Author of *The
One Best Way*







Novi razvoji

Another way of finding the constant is as follows - 41

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$$\therefore 4C = 4 + 8 + \dots$$

$$\therefore -3C = 1-2+3-4+\dots = \frac{1}{(1+1)^2} = \frac{1}{4}$$

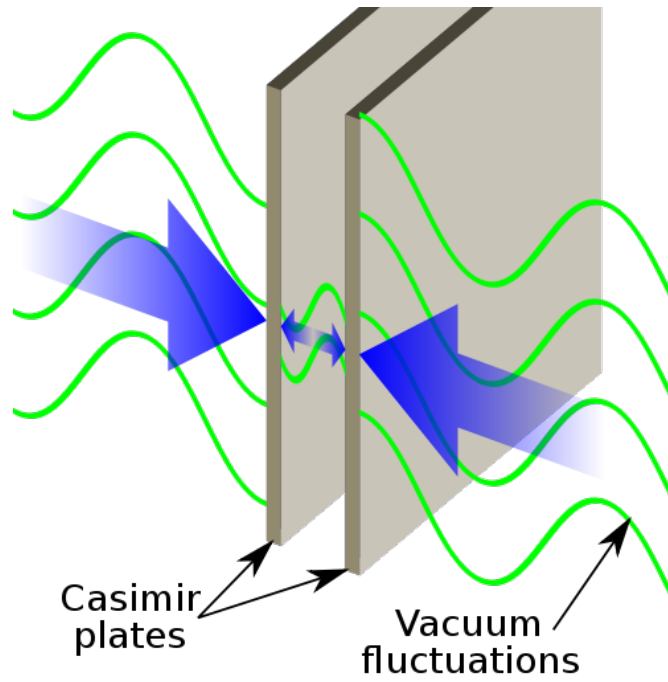
$$\therefore C = -\frac{1}{12}$$

$$1 + r + r^2 + r^3 + r^4 + \dots = 1/(1-r) \text{ where } -1 < r < 1.$$

$$1 - 1 + 1 - 1 + 1 - \dots = 1/(1+1).$$

$$1 - 2 + 3 - 4 + 5 - 6 + \dots = (1 - 1 + 1 - 1 + \dots)^2$$

	+1	-1	+1	-1
+1	+1	-1	+1	-1
-1	-1	+1	-1	+1
+1	+1	-1	+1	-1
-1	-1	+1	-1	+1



$$\zeta(-3) = \frac{1}{120}$$

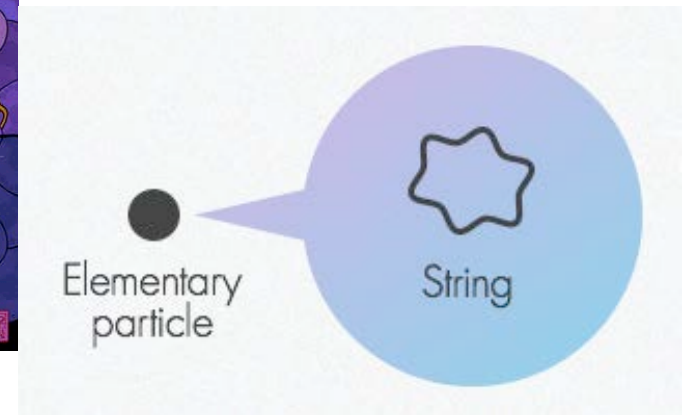
$$J(q) = q^{-1} + 196884 q + 21493760 q^2 + \dots$$

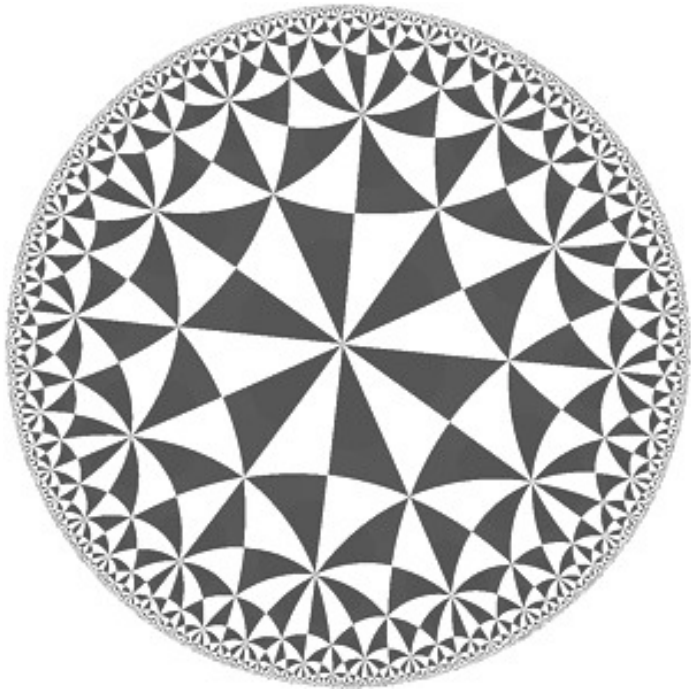
Suffice it to say that the lightest possible black hole turns out to have 196883 quantum states. (Witten)

Mathematicians Chase Moonshine's Shadow

Researchers are on the trail of a mysterious connection between number theory, algebra and string theory. (2015)

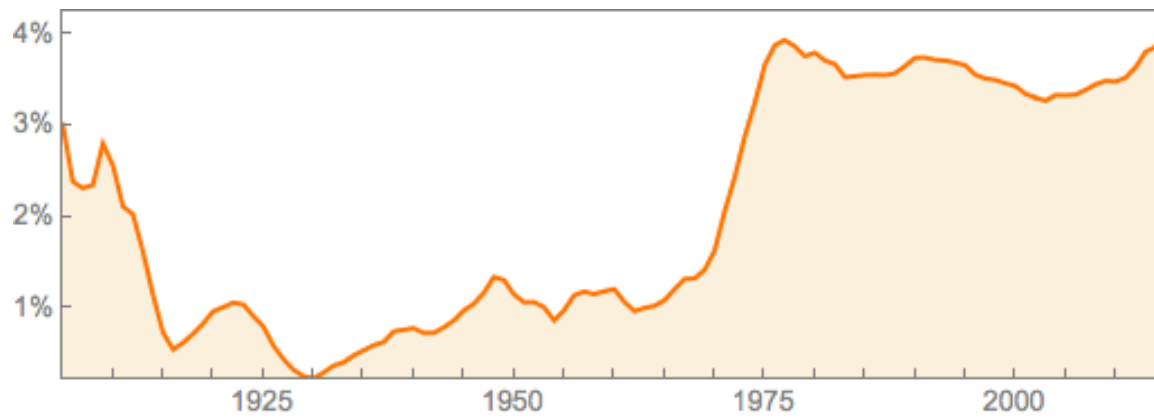
$$\begin{aligned} 1 &= r_1 \\ 196884 &= r_1 + r_2 \\ 21493760 &= r_1 + r_2 + r_3 \\ 864299970 &= 2r_1 + 2r_2 + r_3 + r_4 \\ 20245856256 &= 3r_1 + 3r_2 + r_3 + 2r_4 + r_5 \\ &= 2r_1 + 3r_2 + 2r_3 + r_4 + r_6 \\ 333202640600 &= 5r_1 + 5r_2 + 2r_3 + 3r_4 + 2r_5 + r_7 \\ &= 4r_1 + 5r_2 + 3r_3 + 2r_4 + r_5 + r_6 + r_7 \end{aligned}$$





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 If we consider a B -function, in (1)
 the transformed Eulerian, e.g.
 (A) $1 + \frac{v}{(1-v)^2} + \frac{v^2}{(1-v)^2(1-v^2)^2} + \frac{v^3}{(1-v)^2(1-v^2)^2(1-v^3)^2}$
 (B) $1 + \frac{v}{1-v} + \frac{v^2}{(1-v)(1-v^2)} + \frac{v^3}{(1-v)(1-v^2)^2}$
 and similarly determine the nature of
 the singularities at the points $v=1$,
 $v^2=1$, $v^3=1$, $v^4=1$, ... we know how
 beautifully the asymptotic nature
 form of this function can be expressed
 in a very neat and closed form ex-
 ponential form. For instance
 when $v = e^{-t}$ and $t \rightarrow 0$
 (A) $= \sqrt{\frac{1}{2\pi}} e^{\frac{\pi^2}{6t} - \frac{5}{24}t}$
 (B) $= \frac{e^{\frac{\pi^2}{6t} - \frac{5}{24}t} + o(1)}{\sqrt{2\pi t}}$
 and similar results at other singu-
 -larities. * It is not necessary that
 there should be only one term like this.
 There may be many terms but the number
 of terms must be finite. † Also $o(1)$
 may turn out to be $O(1)$. That is all.
 For instance when $v \rightarrow 1$ the function
 $\frac{1}{(1-v)(1-v^2)(1-v^3)(1-v^4)(1-v^5)(1-v^6)(1-v^7)(1-v^8)(1-v^9)(1-v^{10})}$
 is equivalent to the sum of five
 terms like (*) together with $O(1)$ in-
 stead of $o(1)$.
 If we take a number of functions
 like A and (B) it is only in a limited
 number of cases the terms close as
 above; but in the majority of cases they
 never close as above. For instance,
 when $v = e^{-t}$ and $t \rightarrow 0$
 (C) $1 + \frac{v}{(1-v)^2} + \frac{v^2}{(1-v)^2(1-v^2)^2} + \frac{v^3}{(1-v)^2(1-v^2)^2(1-v^3)^2}$
 $= \sqrt{\frac{1}{2\pi}} e^{\frac{\pi^2}{6t} + a_1 t + a_2 t^2 + \dots} + o(a_n t^n)$
 where $a_1 = \frac{1}{8\sqrt{5}}$, and so on.

In 2007, the physicist Edward Witten, of the Institute for Advanced Study in Princeton, N.J., speculated that the string theory in monstrous moonshine should offer a way to construct a model of three-dimensional quantum gravity, in which 194 natural categories of elements in the monster group correspond to 194 classes of black holes.



Radovi iz teorije brojeva