ПУБЛИКАЦИЈЕ АСТРОНОМСКОГ ДРУШТВА “РУЂЕР БОШКОВИЋ” Св. 21
PUBLICATIONS OF THE ASTRONOMICAL SOCIETY “RUDJER BOŠKOVIĆ” No. 21

COLLECTED WORKS OF NADEŽDA PEJOVIĆ

Ω

BELGRADE, 2020

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COLLECTED WORKS OF NADEŽDA PEJOVIĆ

Part I: Biography and Doctoral dissertation
Part II: Astronomy
Part III: History of Science and Digitization
Part IV: Bibliography

Ω series

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Professor Nada Pejović wishes to dedicate
this book to her daughter Ana and son Aleksandar

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PART I

Biography and doctoral dissertation

Overview

This collection of books presents scientific and professional work of Nadežda Nada Pejović, professor of astronomy at the Department of Astronomy of the Faculty of Mathematics of the University of Belgrade. It consists of four parts. The first part includes biography of professor Pejović, her doctoral dissertation and her master of science thesis. The second part contains all her scientific papers, 41 of them, including the coauthor ones. The third volume comprises all works of professor Pejović in history of astronomy, in total 43 articles. The fourth and the last part gives her complete bibliography.

The reader will find that scientific works of professor Pejović in astronomy are related to several branches of astronomy. They range from geodynamics, astrometry and celestial mechanics to cosmology. In history of science she is mostly interested in the development of astronomy in Serbia, but she also has a great merit for digitization of first Serbian astronomical books and books of Serbian astronomers from the past. These books make an important and valuable part of the Virtual Library of the Faculty of Mathematics in Belgrade. Her wide interest in several, obviously disparate disciplines of astronomy and her very dedicated work in history of science connected with the modern tools such as the use of information technologies in digitization put her in a group of rare contemporary scientist with a very broad and universal approach to science.

1 Biography

Prepared by Slobodan Ninković
November 2020, Belgrade

This is a somewhat extended version of the biography of Nadežda Nada Pejović, professor of astronomy at the Faculty of mathematics of the University of Belgrade. It does not cover only her scientific and professional work in astronomy, but also her activities in teaching, popularization of science and some details from her biography not directly connected to science and academic life. Among other things, in this text the reader will find the rise of a young and talented schoolgirl from the vicinity of Belgrade to the first woman astronomer at the prestigious Department of Astronomy at the Faculty of Mathematics in Belgrade. As sources for this text, the referee report for her election for full professor is used, but also other documents as well, such as yearbooks, news-paper clippings and various reports. Details from professor Pejović personal life were given by herself.



Nadežda Nada Pejović

NADEŽDA PEJOVIĆ

Profesossor Nadežda Nada Pejović belongs to a somewhat older generation among still active Serbian astronomers. Coming from the territory of Belgrade through her talent and activity she has succeeded to distinguish herself as a respected astronomy teacher at the Belgrade University. The text given below, together with the appendices, confirms that her scientific and technical bibliography is rich, as well as that her scientific, academic, teaching and pedagogical activities in astronomy have contributed significantly to the development of that science in Serbia.[1]

Curriculum Vitae

Nadežda Nada Pejović was born on March 10, 1952 in Pudarci (Municipality of Grocka, Belgrade). Her maiden name is Matorčević, her mother's first name is Zlata, her father's first name is Radomir. Lower forms of the primary school she finished in her birthplace, higher forms in a neighbouring settlement of Umčari. Her secondary education took place in XIII Beogradska gimnazija which she finished in 1971 in the framework of the curriculum with mathematics and natural sciences pointed out. During her schooling she was among the best pupils; she was awarded diplomas "Vuk Stefanović Karadžić" (general success) and "Mihailo Petrović Alas" (distinguishing in mathematics and natural sciences). She was especially interested in mathematics and the subjects of natural sciences. In the final form there was the subject of astronomy, which together with that of mathematics had a decisive influence on her choice of university course. Having finished the secondary education she enrolled herself at the Faculty of Sciences of the University of Belgrade to study astronomy where she took degree in 1976. Her further professional progress was continued through postgraduate studies at the same faculty. In 1984 she obtained the Master-of-Science (MSC) Degree, the title of the thesis was "*Prilog izučavanju kretanja Zemljinih polova*"(A Contribution to the Study of Motion of the Terrestrial Poles), the supervisor was Dr Dragutin Đurović (Belgrade University). After that she continued her studies in Czechoslovakia (Astronomical Institute of the Czechoslovak Academy of Sciences in Prague and Ondřejov near Prague) where she spent the school year of 1986/87 as a fellow of the Ministry of Science of the Republic of Serbia for the purpose of forming PhD thesis. The thesis was defended on February 2, 1989 in Belgrade at the Faculty of Sciences, its title is "*Globalna atmosferska cirkulacija i Zemljina rotacija*" (Global Atmospheric Circulation and Earth's Rotation), the supervisor was Dr Jan Vondrak from the Astronomical Institute of the Czechoslovak Academy of Sciences; at that time President of IAU Commission Earth's Rotation. The former Section of Mathematics, Mechanics and Astronomy of the Faculty of Sciences became thirty years ago a new faculty, Faculty of Mathematics (FM), as it is named also today.

Prof. Pejović's career at the Belgade University started in 1977 when she at the Faculty of Sciences (Section of Mathematics, Mecahanics and Astronomy) became assistant trainee. In 1985 she became assistant. In 1989 she became a teacher, at first as a Lecturer or Senior Lecturer (UK), i. e. Assistant-Professor (US), then in 1996 as a Reader (UK), i. e. Associate Professor (US), to finally become Professor (UK), i. e. Full Professor (US). In the last two cases the institution was already FM. Prof. Pejović was retired on October 1, 2017. She lives in the Municipality of Novi Beograd, her husband's name is Milovan, they have two issues, a son Aleksandar and a daughter Ana.

[1] The data for curriculum vitae of Professor Pejović and the appendices for her collected Works were prepared during 2020, at the time when the pandemic caused by covid 19 paralysed the whole world.

Teaching Activity

During her career at the Astronomy Department of FM Prof. Pejović taught more than ten subjects. Already as an MSC student, then as an assistant, she led exercises on the following subjects *General Astronomy*, *Spherical Astronomy*, *Stellar Astronomy*, *Practical Astronomy*, *Mathematical Treatment of Astronomical Observations*. From the time when she became a teacher in 1989 her courses were *General Astronomy*, *Stellar Systems*. In the first decade of the XXI century she initiated teaching astronomy for students of mathematics so that she taught to them *General Astronomy* and *Spherical Astronomy*. She also taught at other faculties, for instance *Fundamentals of Astrophysics* to physics students at the Faculty of Physics in Belgrade and *Geodetic Astronomy* to geodesy students at the Faculty of Civil Engineering in Sarajevo between 1990 and 1992. At her own Department she taught or held consultations from a few subjects and special courses for MSC students, as well as later on to master and PhD students, such as: *Selected Chapters in Astronomy*, *Theory of Earth's Rotation*, *Stellar Systems* II, *Astronomy*. After the Bologna reform of teaching at the Belgrade University at FM her subjects were *General Astronomy* I, *General Astronomy* II for astronomy students, *Fundamentals of Astronomy* and *Introduction to Astronomy* for mathematics students, *Fundamentals of Astronomy* for informatics students.

Prof. Pejović was supervisor for the PhD of Goran Damljanović, an astronomer from the Astronomical Observatory in Belgrade. This thesis was in the framework of international Project Hipparcos. Prof. Pejović was also supervisor to a few MSC theses. She was a member in several commissions for both inspection and evaluation and examination of PhD theses, MSC theses, as well as later on, when the master degree was introduced, master works. She composed manuscript entitled Opšta astronomija (General Astronomy, reviewed in 1996), translated with colleagues Stevo Šegan and Zlatko Ćatović university textbook *Spherical Astronomy* by Robin Green. She also wrote together with Professor Šegan university textbook *Fundamentals of Astronomy* (e-edition http://matf.bg.ac.rs/astronomija/spe07.htm).

Through her work on digitisation of university textbooks sold out long ago and hardly available and of collections of astronomical problems Professor Pejović has contributed these books in a digital form to have become a standard part of additional literature in teaching astronomy. The e-copies of these books are available now in the MF Virtual Library at http://elib.matf.bg.ac.rs. For instance, the following books have been digitised: *Celestial Mechanics, History of Astronomical Science from Its First Commencements to 1727*, as well as *Astronomical Theory of Climate Changes and Its Application in Geophysics* by Milutin Milanković, *General Astronomy* I part - *Spherical Trigonometry*, *Collection of Solved Problems from General Astronomy* I part (printed in 1956) and manuscript *Collection of Solved Problems from General Astronomy* II part by Vojislav Mišković. It should be mentioned that the second part of Mišković's problem collection had existed as a manuscript only which the author had prepared in the middle of the last century and had never been printed. Through digitisation this book became for the first time available to the students and broader public. These books by Mišković are still the only collections of astronomical problems written in Serbian, which emphasizes the importance of their digitisation. She has also digitised university textbooks *Opšta astronomija* (General Astronomy) by Branislav Ševarlić and Zaharije Brkić, *Teorijska astronomija* (Theoretical Astronomy) by Jovan Simovljević, *Osnovi teorije kretanja Zemljinih veštačkih satelita* (Fundamentals of Theory of Motion of Terrestrial Artificial Satellites) by Jovan Lazović and *Obrada astronomskih posmatranja* (Treatment of Astronomical Observations) by Dragutin Đurović which have been used and are still used in teaching astronomy at the Astronomy Department. Open access to the e-copies of these books has significantly contributed to the teaching and has enabled the students to better prepare and pass their exams.

Prof. Pejović has an exceptional talent for the pedagogical work. She was very keen and devoted to the work with students. In addition to the recollections of her students this can be also seen from a high presence to the lessons from optional subjects for students of mathematics and informatics to whom she

taught. About her popularity as a teacher says the fact that she was always among the best ranked teachers in the sutveys wherein the students evaluate the work of their teachers.

Scientific and Professional Activity

In the beginning of Prof. Pejović's research work her field of interest was geodynamics, discipline wherein mathematico-dynamical models of the Earth are studied. Prof. Pejović's interest was focused on non-uniformities in the terrestrial rotation and motion of the poles. The themes of her MSC and PhD theses cover these subjects. The theoretical results obtained by her by analysing the mathematical models agree well with the observational results. These results attracted international attention and have been cited several times. The papers of Prof. Pejović have been cited in Serbian journals and abroad. There are more than 200 citations in Google Scholar, out of this at least 30 in the leading international journals and international monographs.

Prof. Pejović has been also active in the fields of astrodynamics, astrometry, cosmology and Galactic astronomy. The characteristics of the recent years are her interest in history of astronomy where she published several papers.

She has published 84 scientific and professional papers in Serbia and abroad, also in conference proceedings, until the end of the year 2020. She has presented her results at scientific meetings in her country and abroad about 60 times. The professional and scientific activities have also occurred through her participation in the scientific projects. In this publication which consists of three volumes there are all papers, scientific and professional, published by Prof. Pejović, as well as her MSC and PhD theses.

MSC thesis *Prilog izučavanju kretanja Zemljinih polova* (A Contribution to the Study of Motion of the Terrestrial Poles) was defended in 1984 at the Faculty of Sciences of Belgrade University. The members of the examination commission were: Dr Dragutin Đurović (supervisor), Professor Branislav Ševarlić and Professor Jovan Simovljević. In this work the author carried out an extensive analysis of the coordinates of the instantaneous pole of the terrestrial rotation with respect to the Conventional International Conventional Pole (CIO), based on the data of the International Latitude Service (ILS) at Mizusawa for the period 1899.9-1979.0. On the basis of this analysis Prof. Pejović inferred that there exist secular variations in the pole coordinates, but that there was still uncertain whether that was due to the intrinsic polar motion or to the precession, proper motions of stars, etc. For the direction and speed of polar motion she obtained results close to those of some well known authors. By means of spectral analysis of Kimura's Z-term she established, in addition to the already known annual cyclical variation, another semiannual variation, which is a novel discovery.

PhD thesis *Globalna atmosferska cirkulacija i Zemljina rotacija* (Global Atmospheric Circulation and Earth's Rotation) was defended on February 2, 1989 at the Faculty of Sciences of Begrade University. The members of the examination commission were: Professor Jan Vondrak (supervisor), Professor Dragutin Đurović and Professor Jovan Simovljević. The thesis concerns the mathematico-dynamical models of the Earth. The work is an extensive study of the influence of the global atmospheric circulation on the terrestrial rotation. The theme was very up-to-date, because for a precise guidance of space vehicles, as well as for reliable geophysical research, it was necessary to track the space orientation of the Earth to an accuracy which nowadays is of order of 10^{-4} arcseconds. In the solving of these tasks actual deviations of the global atmospheric circulation from the known models used in the computation of the Earth rotation parameters appear as a big problem. An important progress in the solving of this problem was achieved in the eighties, in the same decade when the thesis was defended. This occurred when the functions of the effective angular momentums of the atmosphere became available. They were computed from the global atmospheric data at three world meteorological centres following the algorithm proposed

by Barnes et al. in 1983. From that time the contribution of the atmospheric motion in the variations of the terrestrial rotation parameters was determined more precisely. Nevertheless, the following questions were still insufficiently studied:

- **a.** How significant is the ocean response to the changes of air pressure and if the inverse barometer hypothesis is sufficient to describe this response?
- **b.** What is the role of the wind term in the polar motion and if the available data have a sufficient density and precision for a polar motion computation of a high quality?
- **c.** Is the geophysical Earth model used by Barnes et al. adequate for the computation of the Earth response to the atmospheric extinction?
- **d.** Is the atmospheric circulation self-sufficient to excite the observed polar motion or there exist another excitation sources?

In her PhD thesis Prof. Pejović answers just these very up-to-date questions. After a short presentation of the state of the art she gives a short review of the theoretical background based on the Liouville equations for the terrestrial rotation. The atmospheric excitation was estimated by taking into account that the Earth is a deformable body consisting of: liquid core, viscous and elastic mantle and dynamical ocean. A special attention was focused on the question if a non-linear ocean response to the polar motion can cause a dependence of the Chandler nutation frequency on the amplitude of the total motion of the terrestrial poles. By comparison of the most contemporaneous theory (for that time) supplemented with new hypotheses with the observations interesting and novel results were obtained.

Published Papers

The scientific interests of Prof. Pejović are various covering several fields. Though her main research has been done in astronomy, she has been able to go beyond that science. Therefore, the published papers of Prof. Pejović can be divided into four groups.

In the first group there are papers concerning geodynamics. They are related to the mathematico-dynamical model of the Earth, with a special indication to the theory of motion of the terrestrial poles and non-uniformities in the rotation of the Earth. Prof. Pejović considers a geodynamical model of the Earth being composed of: liquid core, viscous and elastic mantle and dynamical ocean. A special attention in Pejović's research is focused on the influence of the global atmospheric circulation on the terrestrial rotation. She studies the ocean influence on the change of the air pressure, as well as the wind influence on the polar motion. She shows how a non-linear ocean response to the polar motion can cause the dependence of the Chandler nutation on the amplitude of the total motion of the terrestrial poles. In the papers of this group Prof. Pejović always insisted on theoretical models as good and as applicable as possible used for the purpose of solving particular problems, as well as on comparisons of the obtained results with the astronomical observations. These papers have been cited by many foreign authors in the well known journals. Some of these papers she published together with Jan Vondrak, who is an astronomer of world reputation. To the first group belong the papers resulting from the joint project *Dinamika Zemljimih večtačkih satelita i geodinamika - poređenje teorije i posmatranja* (Dynamics of Earth Artificial Satellites and Geodynamics – Comparison of theory and Observations) of the Serbian Academy of Sciences and Arts and the Czechoslovak Academy of Sciences for the period 1986-1990. Some of these papers from astrodynamics and geodynamics she published together with her colleague Stevo Šegan, who taught at FM.

In the second group there are papers from astrometry. The topic is the computation of corrections to proper motions of stars from the Hipparcos Catalogue for the epoch 1991.25. This catalogue was compiled on the basis of the data collected during the observational mission on board of the Hipparcos satellite (HIPPARCOS - High Precision PARallax COllecting Satellite) launched in August 1989. The

satellite finished its work after almost four years. Since the Hipparcos mission lasted shortly, it was necessary to correct the proper motions of the stars. For the purpose of impoving the accuracy of the Hipparcos Catalogue two million classical optical observations were used. They were collected during the XX century at observatories all over the world. In the treatment of this vast observational material Prof. Pejović worked successfully in the framework of a project of the International Astronomical Union (IAU). The project was accepted at the IAU General Assembly in Baltimore,1988, its leader was Dr Jan Vondrak from Prague, then President of the IAU First Division (out of 12) IAU. The proper motions in declination were corrected for 2347 stars. The results were presented in the papers which Prof. Pejović wrote as coauthor with Dr Goran Damljanović from the Astronomical Observatory in Belgrade. Damljanović's PhD is from this field, the supervisor was Prof. Pejović.

The third group contains cosmology papers wherein a mathematical model of the Universe is studied. The subject is the standard, so-called ΛCDM model of the Universe. The fundamentals of this theory lie in the Einstein field equations the derivative of which are the Friedmann equations. In Prof. Pejović's papers the solutions of this system of differential equations satisfying a generalised power law are examined. This mathematical model corresponds to the era of dominating of the baryonic and dark matter in the overall evolution of the Universe. The fact that in these studies the theory of regularly varying functions founded by a well known Serbian mathematician, Jovan Karamata, is applied does them specially interesting. The papers of this group Prof. Pejović has chiefly written together with Professor Žarko Mijajlović (FM). They have been published in highly ranked journals of applied mathematics and theory of gravitation. Prof. Pejović has also published papers from Galactic astronomy with a coauthor, Dr Slobodan Ninković (Astronomical Observatory Belgrade).

The fourth group consists of papers covering history of astronomy wherein Prof. Pejović deals with pioneer articles and books in astronomy and related sciences written by Serbian authors in the past. Some of these rare books Prof. Pejović digitised and in her articles presented their contents, analysed their significance and indicated interesting facts concerning their publishing. Electronic copies of these books are in the FM Virtual Library (Virtual library, http://elib.matf.bg.ac.rs) , the access is free so that the contents of this database are easily available to any person interested in them. Prof. Pejović specially studied and presented a collection of old books of Serbian astronomers which were written in the XVIII and XIX centuries. The oldest of them were written in the languages of Ancient Slavs or Serbian before Vuk Karadžić, or in Latin; they include: *Elementi matematike* (Elements of Mathematics) by Ruđer Bošković (Venice, 1757, in Latin), *Večni kalendar* (Eternal Calendar) by Zaharije Stefanović Orfelin (Vienna, 1783), *Fizika* (Physics) by Atanasije Stojković (Buda, 1810), *Zvezdano nebo nezavisne Srbije* (Starry Sky of Independent Serbia) by Đorđe Stanojević (1882, Belgrade), *Kosmografija* (Cosmography) by Milan Andonović (1888, Belgrade), *Atomistika – Jedan deo iz filozofije Ruđera Boškovića* (Atomistics – A Part from Philosophy of Ruđer Bošković) by Kosta Stojanović (Niš, 1892), *Nebeska mehanika* (Celestial Mechanics) by Milutin Milanković (Belgrade, 1935), *Osnovi matematične i fizičke geografije* (Fundamentals of Mathematical and Physical Geographies) by Pavle Vujević (Belgrade, 1924). In the Virtual Library there are also *Aritmetika* (Arithmetics) by Vasilije Damjanović, the first book on mathematics printed in Serbian (Venice, 1767), and *Čislenica* (Reckoning) by Jovan Došenović (Budapest,1809). As Prof. Pejović notes, every written or printed word in the past, especially in a farther one, is part of the cultural heritage which is sufficient for these books to deserve attention and a special way of keeping. A part of these papers she published together with Prof. Žarko Mijajlović, in the framework of Project "Digitisation of Scientific and Cultural Heritage" of the Ministry of Science of the Republic of Serbia (RS). To be mentioned is that these papers have met a very positive reaction on the Internet. They have been cited more than 150 times, which can be tracked on Scholar Google. That for this kind of papers there exists interest in the world can be seen on the example of a collection of articles dealing with history of mathematics in India, http://www.pdfqueen.com, where one can find the following article: *Digitization of mathematical textbooks used in Serbia in the past*, NCD Review, vol. 12, 55-64, 2008.

Other Activities

Prof. Pejović has delivered public lectures at People's University Kolarac, has appeared on radio and television. From 2002 she prepared talented pupils of secondary and primary schools for the participation in the national and international contests in astronomy. She founded in 2005 a facultative class of astronomy in the secondary school in Prokuplje, with the approval of the Ministry of Education of RS. She received in 2006 from the Municipality of Prokuplje *Zahvalnicu* (Certificate of Gratitude) for the contribution to the building of the mountain station of the Belgrade Observatory situated in the Mountain of Vidojevica near the town of Prokuplje. Another certificate of gratitude she got in 2019 from the Centre for the Promotion of Science of the Republic of Serbia for public lectures held in the framework of manifestation "May Month of Mathematics".

During the academic year of 2008/2009 she took an active part in the formation of new curricula in accordance with the Bologna process for the astronomy studies at FM. She has merits for the establishing of the study direction Astroinformatics at FM. She actively engaged in reincluding astronomy as a special subject in the secondary schools.

All mentioned activities of Prof. Pejović required much time, engagement, devotion and assiduity. Taking into account that her scientific work is in this book presented in detail, the following aspects of Prof. Pejović's activity will be subjected to additional treatment.

The first ideas to construct a mountain station of the Belgrade Observatory in the mountain named Vidojevica came from FM. The proposers were Prof. Stevo Šegan and Prof. Žarko Mijajlović. Prof. Pejović, then the head of FM Astronomy Department, joined this project in 2001. After the foundation of Astronomical Society "Magelanov oblak" (Magellanic Cloud) in Prokuplje it became possible to start looking for a suitable location, because it was intented to build an observational pavilion of this society. The society members were so much delighted with the top of the Vidojevica mountain as a possible location of the observational pavilion so that they proposed to FM and Observatory in Belgrade to build there a new professional observational station, the Astronomical Station of Vidojevica (ASV). The proposal was accepted and sponsored by the Ministry of Science and the construction could statrt. It was arranged to build a school pavilion within the ASV area for the purpose of teaching the students of astronomy, as well as a small pavilion for the necessities of the Society. In the gathering of the necessary documents Prof. Pejović took part as Head of Astronomy Department. She sent proper requests to the local authorities in Prokuplje. From the Municipality of Prokuplje she received in 2006 a certificate of gratitude for the contribution to the construction of ASV.

Simultaneously with the construction of the new station other activities in Prokuplje started. Namely many public lectures from astronomy, mathematics and informatics were organised by the Society "Magelanov oblak" in the rooms of the society, in the secondary school and in the Prokuplje primary schools. The lectures were delivered by teachers from FM and Faculty of Sciences in Niš, as well as by astronomers from the Belgrade Observatory. Prof. Pejović was among them.

Another important activity of Prof. Pejović in Prokuplje should be mentioned. Namely for a few years she led projects of one-year duration in the framework of the Programme of stimulating, promotion and dissemination of science, which were financed by the Ministry of Science of the Republic of Serbia. Among the project objectives was to form a facultative class for astronomy in the Prokuplje Secondary School. The projects made it possible to finance the coming of lecturers from Belgrade, as well as assistants who for the facultative class at the Secondary School dealt in addition to lectures also with the preparations of pupils for national and international contests in astronomy. For the purpose of preparations of contestants not living in Belgrade Prof. Pejović digitised secondary school astronomy

textbooks sold out long ago to include them in the FM Virtual Library. Exercises for the contestants were carried out by most active memebers of the Society ("Magelanov oblak"), then geography students at Belgrade University Aleksandar Valjarević and Aleksandar Simonović. An important support was given by Secondary School Director Dragan Krstić who is a mathematics teacher. All these activities were registered on the local television and radio of Prokuplje, local papers, but also in other daily papers (Politika, Večernje novosti, Blic).

Prof. Pejović organised the first participation of junior contestants (age 14-15) in the X International Astronomy Olympiad (IAO) held in Beijing in 2005. At the two previous IAO form Serbia there had been only senior constestants (age 16-17). Prof. Pejović in October 2005 became chairperson of the National Astronomical Olympic Committee (NAOC) of Serbia. This body was founded in 2002 by Prof. Jelena Milogradov Turin who was its first chairperson. Having become NAOC chairperson Prof. Pejović did many efforts to improve the preparations of the motivated pupils. Thanks to her activity it was possible for her succcessors as NAOC chairpersons Dr Slobodan Ninković and Dr Sonja Vidojević to continue this important work successfully. In 2007 another international contest in astronomy appeared – International Olympiad on Astronomy and Astrophysics (IOAA). In the years 2009, 2010 and 2011 contestants from Serbia participated in both contests – IAO and IOAA. From 2012 Serbia has been participating in IOAA only. IOAA is foreseen for secondary school pupils (age 15-20) with no subdivision as to their age. Contestants from Serbia in both IAO and IOAA have been successful, they have got many medals and even, special prizes.

At Petnica Science Centre (PSC) Prof. Pejović actively worked on dissemination of astronomy. For almost two decades, during the eighties and nineties, she was a regular lecturer at the Winter Astronomy Seminar at PSC. It is well known that PSC is a unique institution in Serbia which has engaged in extra school education for pupils attending higher forms of the primary school and secondary school. As such it has become recognisible in Europe and outside of Europe. Prof. Pejović enthusiastically promoted the astronomical theory of planet insolation whose author is our and world great man Milutin Milanković. She promoted interesting facts and beautiful places of astronomical science through lectures devoted to themes, such as Solar System, Sundials, mathematicco-dynamical model of the Earth, motion of terrestrial poles, time systems, differential equations of planetary motion and the biggest telescopes of the world.

Prof. Pejović is an active member of Association "Milutin Milanković", which was founded in 2007 with the objective of promoting the image and achievements of that great man of ours and of the entire humanity. She has been a member of the Directory Board and secretary of this association. In 2011 she organised the cooperation between FM and Association "Milutin Milanković" in order to form a digital legacy of Milutin Milanković, http://legati.matf.bg.ac.rs/milankovic. The purpose of the legacy is to collect, unify and make easily accessible the entire scientific contribution to the world science of the great Serbian scientist, as well as to offer the facts from his biography. At that site one can find digital copies of his books, monographs, university textbooks, also the ones of all scientific papers published by him, to add the manuscripts of Milanković's university lectures. This portal contains many photographs and other documents which more closely throw light on his life, but also the books and papers written about Milanković by others. As a member of the Association Prof. Pejović has visited many schools throughout Serbia, where she promoted exhibitions and films about Milanković and delivered lectures like: Milanković's Cycles, Digital Legacy of Milutin Milanković, Milutin Milanković – Mysteries of Ice Periods. She has been several times guest of the primary school in Rabrovo near Kučevo named after the Serbian great man. At the Faculty of Economics, then Faculty of Mathematics in Belgrade and other institutions she delivered several lectures on Milutin Milanković and his astronomical theory of climate changes. In the rooms of the Association, Pop Lukina street 1b, she has organised several times a School of Young Talents in astronomy, preparations of pupils for participation in international astronomy contests, student workshops and taking pictures with astronomical contents for various TV programmes.

Prof. Pejović is an active member of Astronomical Society "Ruđer Bošković" (ASRB). This society published between 1935 and 1940 science popular journal *Saturn*, to start a new journal with the same purpose in 1953 named *Vasiona* (Universe). This journal had regularly appeared by 2011, after a break caused by finances, its publishing started again in 2017. Thanks to Prof. Pejović who has organised digitisation of a few old Serbian astronomical journals the Serbian astronomical community has become aware of their existence. For instance, together with Prof. Mijajlović she contributed to collect, digitise and put to the Virtual Library of FM all issues of journal *Saturn* (whole period 1935-1940). On the other hand, the digitisation for "Vasiona" is taking place. All issues for the period 1953-2004 have been already digitised and their digital copies are already in the Virtual Library of FM, so that they are publicly available. The most recent issues of Vasiona, concerning the period 2017-2020, put to the disposal by People's Observatory Belgrade, also have their e-copies in the Virtual Library of FM. At the People's Observatory (Kalemegdan, Belgrade), under the auspices of ASRB with interval of two years about Easter are organised conferences under the common title *Razvoj astronomije kod Srba* (Development of Astronomy among Serbs) by Serbian astronomer of high reputation Milan Dimitrijević. For more than a decade Prof. Pejović is a permanent participant of these meetings with communications chiefly concerning the digitisation of rare astronomical books from the XVIII, XIX and XX (first half only) centuries wherein the history of astronomy among Serbs is written out. Some of these books have been already mentioned: *Elementi matematike* (Elements of Mathematics) by Ruđer Bošković, *Večni calendar* (Eternal Calendar) by Zaharije Stefanović Orfelin, *Zvezdano nebo nezavisne Srbije* (Starry Sky of Independent Serbia) by Đorđe Stanojević and *Kanon osunčavanja* (Cannon of Insolation) by Milutin Milanković.

Accreditation of curricula is a new term which arose in 2005 when tertiary schools were formally recognised in Serbia. In that year a grand reform of teaching in tertiary schools and its adjusting to the Bologna declaration at the University of Belgrade and other universities in Serbia was carried out. Prof. Pejović as Department Head was a member of the accreditation team for teaching reform at FM from the first day. That was a hard, requiring and exhausting work followed with unavoidable frequent meetings, which lasted long time. For instance, it was necessary to form two separate one-semestre subjects instead of every two-semester subject, also one third of the mandatory subjects ought to have become optional, professional subjects had to be reduced to two thirds in order to get time for new subjects corresponding to general education, to add a number of other changes. From the academic year 2006/2007 FM has carried out reformed teaching and a new direction – informatics – was introduced. Bearing in mind this fact and following the new trends and technologies Prof. Pejović for the Astronomy Department proposed astroinformatics as a new direction for the studies, which was accepted. Also, for students of mathematics and informatics in addition to the already existing optional subject *Osnovi astronomije* (Fundamentals of Astronomy) new ones were introduced: *Uvod u astronomiju* (Introduction to Astronomy), *Uvod u mehaniku* (Introduction to Mechanics), *Mehanika* (Mechanics), *Opšta teorija relativnosti i kosmološki modeli* (General Relativity and Cosmological Models). In the framework of master studies for students of mathematics a new modulus *Astronomija i mehanika* (Astronomy and Mechanics), which included a number of new subjects concerning mechanics, was introduced.

In Serbia astronomy was taught as a special subject in schools already in the late XIX century till the beginning of the Second World War. From 1880 at the Grand School in Belgrade astronomy was taught by Milan Nedeljković, who founded the Astronomical Observatory in Belgrade. After him at the Belgrade University (in 1905 Grand School became University) astronomy was taught by famous scientist Milutin Milanković and Vojislav Mišković, who founded the new Astronomical Observatory at a new site in Belgrade. Invited

After the Second World War astronomy was a special subject in the secondary schools for direction of mathematics and natural sciences for 25 years, between 1969 and 1994. Unfortunately, by a decision of the Education Council of Serbia of June 28, 1990 astronomy ceased to exist as a special subject and

astronomy lessons were assigned to physics. So astronomy has been no longer taught in the secondary schools and then a painful struggle started to get it back, which has shown to be a Sisyphys work. Prof. Pejović together with professor Olga Atanacković and other her colleagues from the Department and from the Observatory took part in that struggle. From 2000 she has sent many requests to various addresses, such as the Ministry of Education and Sport of the Republic of Serbia, the National Education Council of the Republic of Serbia, Institution for Improving Education and Rearing, Institution for Evaluating Quality of Rearing and Education, Faculty of Physics, daily papers, even Television. Unfortunately, all these efforts have had no success. This was the reason to publish an article in daily papers "Politika" (Politics) on May 23, 2013 entitled: *Zahtev da astronomija ponovo bude poseban predmet* (Requirement for Astronomy to be a Special Subject Again).

In the framework of Project *Digitalizacija naučnog i kulturnog nasleđa* (Digitisation of Scientific and Cultural Heritage) Prof. Pejović was a member of a team consisting of colleagues from FM whose aim was to create the Virtual Library of FM, http://elibrary.matf.bg.ac.rs. This is the largest base of digitised books in Serbia with a free access. At the moment the Virtual Library contains more than 5000 books. Collections of PhD theses defended at FM and of rare books from the XVIII and XIX centuries are important items of this library. How rare these old books are can be seen from the fact that for some of them there exists only one copy so that such books are, practically, unavailable to the public. Only after digitisation they became easily available to pupils, students and other interested persons. Prof. Pejović has digitised almost all old Serbian books on astronomy. She has also made with the same team and within the same project digital legacies, http://legati.matf.bg.ac.rs. This digital portal has as its objective to make available to the public the great legacies of the most significant Serbian scientists from distant or near past who worked in mathematics. The first digital legacy to be created was that of Milutin Milanković, created in 2012. This is the first digital monument to that great man of the Serbian and world science. In addition to the legacy of Milutin Milanković digital legacies have been also created for Bogdan Gavrilović, Anton Bilimovič, Mihailo Petrović Alas, Zaharije Brkić, Đuro Kurepa and Slaviša Prešić. Currently another three digital legacies for three namesakes are being created - Jovan Karamata, Jovan Simovljević and Jovan Lazović, all taught at FM. Prof. Pejović engaged in creating digital legacies for astronomers Milutin Milanković and Zaharije Brkić, now she is working on the legacies of Jovan Simovljević and Jovan Lazović.

Prof. Pejović is the first woman who became Professor at the Astronomy Department of FM coming from the Subdepartment of Astrometry and Celestial Mechanics which officially existed from 1974 to 1991. To hear this we could from the talk of Prof. Olga Atanacković *Žene na Katedri za astronomiju* (Women at Astronomy Department) presented during the meeting *Žene u astronomiji* (Women in Astronomy) held on March 8, 2019 at FM. This meeting was dedicated to the International Day of women in Science (February 11) and to the jubilee of the hundredth anniversary of foundation of International Astronomical Union. Prof. Pejović was very proud of it, the fact that she became a part of history of Astronomy Department and that she taught at the same place where before renowned Milutin Milanković, Vojislav Mišković, Milan Nedeljković and others had taught.

APPENDICES

Scientific Meetings

Prof. Pejović has participated in more than 60 meetings organised in Serbia (or former Yugoslavia), as well as international ones. To emphasize are those meetings being done regularly within intervals between one and four years wherein Prof. Pejović has participated regularly or very often. Those are:

Nacionalna konferencija jugoslovenskih astrónoma (National conference of Yugoslav astronomers), she has been perticipating from 1977. After the final dissolution of the common state in 2006 this kind of meeting has been held every three years, it changed its name and now it is National Conference of Astronomers of Serbia. The continuity of the ordinal number is conserved and the last conference held in October 2020 was the 19th one. It is a conference with international participation, especially in the recent times, when the contribution of foreign participants is almost 50%.

Congress of mathematicians of Yugoslavia (held once in four years).
Congress of mathematicians, physicists and astronomers of Yugoslavia (held eight times, between 1949 and 1985, interval five years, some national astronomical conferences were part of its, in 1975, in 1980, in 1985)
Bulgarian–Serbian Conference on Astronomy and Space science (interval two years, even year).
Serbian-Belorussian Symposium on Physics and Diagnostics of Laboratory and Astrophysical Plasmas.
Serbian Conference on Spectral Line Shapes in Astrophysics.
Konferencija: Razvoj astronomije kod Srba (Development of Astronomy among Serbs, two years).
SEEDI International Conference: Digitization of cultural and scientific heritage (interval three years).
NCD konferencija Tehnologije i standardi: digitalizacija nacionalne baštine (Confrence Technologies and Standards: Digitization of National Heritage, once a year). Prof. Pejović has participated in each of these meetings from 2003.

Prof. Pejović has also participated in important international meetings:
International Symposium Geodesy and Physics of the Earth. Potsdam, 1988.
IAU (International Astronomical Union) General Assembly, Buenos Aires, 1991.
Earth Rotation, Reference System in Geodynamics and Solar System, Varšava, 1995.
Three conferences *IAU Colloquium*, Belgrade 1987 and 2004, Cambridge 2005.
Three conferences which were held under the auspices *AIP* (American Institute of Physics), Bucharest 2006, Sinaia 2007, Sremski Karlovci 2007.
Two conferences held under the auspices of *Astronomical Society of Pacific*, Side/Antalia 2006. and Sinaia 2006.

Prof. Pejović has also participated in jubilee meetings where the participants were by invitation:

Conference Fifty Years of Romanian Astrophysics, Bukurešt 2006.
International Conference 60 years of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, 2007, Sofia.
Konferencija: Đordje Stanojević- život i delo - povodom 150 godina od rodjenja (Confrence: Đorđe Stanjević – Life and Work – dedicated to 150 years after his birth), 2008, Novi Sad.

Prof. Pejović has delivered a few invited lecturs: „*Project ASV*“ in international meeting: The 5th Bulgarian–Serbian Conf. on Astron. and Space science, May 9–12, 2006, Sofia/Bulgaria and also „*Teaching of Astroinformatics at the University of Belgrade*“ na VII Bulgarian-Serbian Astronomical Conference: Astroinformatics, 1-4 June 2010, Chepelare-Rozen, Bulgaria.

Prof. Pejović has often directly engaged in organising scientific meetings. For instance, she was member of the Scientific Organising Committee (SOC) of XI (1996) and XIII (2002) National Conference of Yugoslav Astronomers with international participation, organised by Astronomy Department of FM and under the auspices of the Federal Secretariat of Science of Yugoslavia and and Ministry of Science, Technology and Development of the Republic of Serbia. She was a SOC member of the XIV (2005) National Conference organised by the Astronomical Observatory in Belgrade. She was co-chairperson, together with Dr Slobodan Jankov of the SOC for the XVI National Conference of Astronomers of Serbia in 2011. She has also participated in the organisation and programme committees of international conferences and those held in Serbia: Serbian-Bulgarian Conference of Astronomers, NCD and SEEDI conferences with subject of digitisation of scientific and cultural heritage and conference Development of Astronomy among Serbs.

Textbooks and translations.

Prof. Pejović has written alone or as a coauthor a few textbooks and translated one book.

1. N. Pejović, *Opšta astronomija* (General Astronomy), 1996, manuscript reviewed.
2. Translation of university textbook *Astronomija – klasika u novom ruhu*, Vesta Company, Beograd, 1998. (original: Spherical Astronomy, Robin Green, Cambr. Univ. Press, 1985, IX+520pp). Translated by: S. Šegan, N. Pejović i Z. Ćatović.
3. N. Pejović, S. Šegan, university textbook *Osnovi astronomije* (Fundamentals of Astronomy) Matematički fakultet, 2006, e-version is on the Internet page http:\\matf.bg.ac.rs/astronomija/spe07.htm Matematičkog fakulteta.

Citations.

Prof. Pejović's publications have been cited in both Serbian journals and international ones. In leading scientific journals and international monographs she has at least 30 citations, in each of the four fields where she has been active. According to Scholar Google she has more than 200 citations. About ten citations important enough are worth mentioning.

1. *Earth's model with variable Chandler's frequency*, Astron. Instit. Czech., Bulletin, **41**(1990). Citations:
   - D. Gilbert, M. Holschneider, J.L. Le Mouel, *Wavelet analysis of the Chandler wobble,* Jour. Geophys. Res. **103**(1998).
   - J. Vondrak, *Earth Rotation Parameters* 1899.7–:1992.0, Survays in Geophys. **20**(1999).
   - L. T. Liu, H. T. Hsu, B. X. Gao, B. Wu, *Wavelet analysis of the variable Chandler wobble*, Geophys. Res. Letters, **27**(2000).
   - J. Kaschenz, S. Petrović, *A methodology for the identification of periodicities in twodimensional time series*. Zeitschr. f. Vermessungswes., **134**(2009).
2. *Atmospheric excitation of polar motion - Comparison of the polar motion spectrum with spectra of effective atmospheric angular momentum functions*, Astron. Instit. Czech., Bulletin, **39** (1990). Citations:
   - T. M. Eubanks, Variations in the Orientation of the Earth, Contributions of Space Geodesy to Geodynamics: Earth. Dynamics, 24(1993, 2013).
   - B. Fong Chao , A. Y. Au, *Atmospheric excitation of the Earth's annual wobble: 1980–1988*, Jour. Geophys. Res. **96**(1991).
   - R. Hide, *Fluctuations in the Earth's rotation and the topography of the core-mantle interface*, Phyl. Trans. Royal. Soc. **328**(1989).
3. *Polar motion: observations and atmospheric excitation*, Veroff. Zentralinst. Phys., Erde, 1988. Citations:
   - *S. Segan, I. Damjanov and B. Surlan, Earth's rotation irregularities derived from uti$_{bli}$ by method of multi-composing of ordinates*, Serb. Astron. Jour. **167**(2003).
   - H. P. Plag, *Chandler wobble and pole tide in relation to interannual atmosphere-ocean dynamics*, Tidal Phenomena, Springer 2005
4. *Astronomical heritage in NCD Virtual Library*, NCD Review **19**(2011). Citation:
   - A.Martocchia, S. Marchionni, *Djordje Nikolić' "Yugoslavs in Astronomy",* Semantic Scholar, Corpus ID: 219332281
5. *On asymptotic solutions of Friedmann equations*, Appl. Math. Computation, **219**(2012). Citation:

- S. Hossain, M. M. Moheuddin, M. S. A. Titu, *A New Mathematical Approach based on the Friedmann Equation*, IOSR Jour. Appl. Phys. **12**(2020).

6. *Twenty four manuscripts in the Virtual Library of the Faculty of mathematics in Belgrade*, NCD Review **25**(2014). Citation:
   - V Todorčević, M Šegan-Radonjić, *Mihailo Petrović Alas: Mathematician and Master Fisherman*, The Mathematical Intelligencer, 2019 – Springer.
7. *Scientific papers of Milutin Milanković in his digital legacy*, NCD Review **25**(2015). Citation:
   - Z. Ognjanović, B. Marinković, M. Šegan-Radonjić, *Cultural Heritage Digitization in Serbia: Standards, Policies, and Case Studies*, Sustainability **11**(2019)

Participation in Projects

During her academic career Prof. Pejović has taken part in several projects, national and international. The projects were in the field of astronomy, for some of them Prof. Pejović was the leader. She led from 1986 to 1990 the project Motion of Earth's Artificial Satellites and Geodynamics – Comparison between Theory and Observations in the framework of the cooperation between the Czechoslovak Academy of Sciences and Serbian Academy of Sciences and Arts. Then she together with colleague Stevo Šegan published a few common papers. In the period 2006-2009 she led the project Programme of Promotion and Stimulation of Science of the Ministry of Science of the Republic of Serbia. Between 2014-2017 she was also leader for the subproject of Project III44006 at MF, Digitisation of Scientific and Cultural Heritage. She has taken part in 10 scientific and professional projects, 7 within Serbia and 3 international.

1. Project from Astro-Geo Sciences of the Ministry of Science of Serbia, 1986-1990.
2. *Dynamics of Earth's Artificial Satellites and Geodynamics – Comparison between Theory and Observations*, cooperation between Serbian Academy of Sciences and Arts and Czechoslovak Academy of Siences, 1986-1990. Project leader: N. Pejović.
3. *Physics and Motion of Celestial Bodies and Artificial Satellites*, Proj. 23, subproject *Geodynamics and Artificial Satellites*, 1990-1995. The project was financed by the Ministry of Science and Technology of the Republic of Serbia; project leader: Milan Dimitrijević.
4. *Astrometric, Astrodynamical and Astrophysical Investigations*, 1996-2000, financed by the Ministry of Science and Technology of the Republic of Serbia; project leader: M. Dimitrijević.
5. *WG ERHRF – IAU Working Group on Earh Rotation in the Hipparcos Reference Frame* (founded by Baltimor 1998, XX IAU General Assembly). Leader: Jan Vondrak.
6. *Digitisation of Scientific and cultural Heritage*, proj. 6201, period 2005-2007, financed by the Ministry of Sience and Technology of the Republic of Serbia. Leader: Žarko Mijajlović.
7. *Programme of Stimulation, Promotion and Popularisation of Science*, 2006-2009, financed by the Ministry of Sience and Technology of the Republic of Serbia. Through this project astronomy at schools was promoted, especially in Prokuplje, near which the construction of the observational station was being carried out. Leader: N. Pejović.
8. *Databases of Digital Images, Compression and Fast Search* Realisation: Serb. Acad. Sci. and Art i Bulg. Acad. Sci, 2007-2009, 2010-2013. Leaders: Žarko Mijajlović and Ognyan Kounchev.
9. *Application of Information Technologies in Digitisation of Scientific and Cultural Heritage*, projekt 13017, 2007-2010, financed by the Ministry of Sience and Technology of the Republic of Serbia. Leader: Žarko Mijajlović
10. *Digitisation of Scientific and Culturne Heritage*, III44006, period 2011-2020, by means of annexes continued from 2014 to 2020, financed by the Ministry of Education, Science and Technological Development of the Republic of Serbia. Project leader: Zoran Ognjanović, Director of the Mathematical Institute of the Serbian Academy of Sciences and Arts, leader of the subproject at FM was Žarko Mijajlović, after his retirement new subproject leader became Prof. Pejović, after her retirement the leader has been Nenad Mitić also from FM.

Administrative and Official Duties. Memberships

Almost during all her active work at FM, but also after retirement, Prof. Pejović has had important functions in the work of FM and professional societies. She was secretary of the Astronomy Department, member of the Council of FM, member of FM Directory Board and head of Astronomy Department (1999-2007, 2012-2016). She was also a member of the Scientific Council of the Astronomical Observatory in Belgrade and a member of the FM Commission for construction of ASV near Prokuplje. She is member of Astronomical Society „Ruđer Bošković", Association „Milutin Milanković" and of Astronomical Society „Magelanov oblak"from Prokuplje, of Society of Astronomers of Serbia. She was a member of the National Astronomical Committee of Serbia, is member of European Astronomical Sosiety (EAS) and International Astronomical Union (IAU). She was the chairwoman of the National Astronomical Olympic Committee of Serbia and Deputy of President of the Society of Astronomers of Serbia.

Newspapers Articles

Prof. Pejović has written several articles for newspapers about astronomy: *U spomen Gagarinu* (In the memory of Gagarin), Blic daily papers, April 12, 2003, p. 24; *Beograđani u Manavgatu u Turskoj - fotografije potpunog pomračenja Sunca* (Belgraders in Manavgat in Turkey – Photographs of a Total Solar Eclipse), Politika daily papers March 31, 2006, p. 26; *Zahtev da astronomija ponovo bude poseban predmet* (Requirement Astronomy to be a Special Subject again) Politika daily papers May 23, 2013, p. 8*; Zašto naši najugledniji naučnici tvrde da HAARP ne može da deluje na vremenske prilike* (Why our Most Respectable Scientists Claim that HAARP can have No Influence on Weather), Politika daily papers May 25, 2014, p.10; *Prof. Jovan Lazović*, Politika daily papers, July 18, 2020, p. 9. She has also written articles and reviews for journal of science dissemination *Vasiona*.

Appearing on TV

RTS Programme for the diaspora, Magazin Srbija na vezi, *Digitalni legati Matematičkog fakulteta* (Digital legacies of Faculty of Mathematics), September 16, 2020,
https://www.youtube.com/watch?v=1x9MD23uKnE

Studio B Jutro sa Sanjom (A morning with Sanja), *Astronom Jovan Lazović - ko je bio i na čemu je radio?* (Astronomer Jovan Lazović – who he was and on what he worked), June 5, 2020,
https://www.youtube.com/watch?v=K-Sdrmml8Mc

She was a few times guest of the local TV of Prokuplje. The topic was the building of the Astronomical Observational Station in the Mountain of Vidojevica near Prokuplje.

2 Biografija

Napisao Slobodan Ninković
Novembar 2020, Beograd

Ovaj tekst predstavlja donekle proširenu verziju biografije Nadežde Nade Pejović, profesorke astronomije na Matematičkom fakultetu Univerziteta u Beogradu. Biografija se ne odnosi samo na naučni i stručni rad u astronomiji, već takođe na njene aktivnosti u nastavi, popularizaciji nauke, a isto tako daje detalje iz njenog života koji nisu neposredno vezani za nauku i akademski život. Između ostalog, čitalac će ovde naći kako je jedna mlada i talentovana učenica iz okoline Beograda postala prva žena astronom na prestižnoj Katedri za astronomiju Matematičkog fakulteta u Beogradu. Prilikom pisanja ovog teksta koristili smo referat za njen izbor u zvanje redovnog profesora, ali takođe i dokumenta druge vrste, kao što su Spomenica Matematičkog fakulteta, novinski isečci i razni izveštaji. Detalje iz ličnog života profesorke Pejović dala nam je ona sama.



Nadežda Nada Pejović

NADEŽDA PEJOVIĆ

Profesorka Nadežda Nada Pejović pripada nešto starijoj generaciji još uvek aktivnih srpskih astronoma. Poreklom iz okoline Beograda uspela je da se svojim talentom i radom afirmiše kao uvažena profesorka astronomije Beogradskog Univerziteta. Tekst koji sledi i prilozi svedoče da je njena naučna i stručna biografija bogata, a da su njene naučne, akademske, nastavne i pedagoške aktivnosti u astronomiji dale važan doprinos u razvoju ove nauke u Srbiji.[1]

Biografija

Nadežda Nada Pejović rodjena je 10. marta 1952. u porodici Matorčević od oca Radomira i majke Zlate u Pudarcima kod Grocke. Niže razrede osnovne škole završila je u rodnom mestu, dok je više razrede završila u susednom mestu Umčari. Školovanje je nastavila 1967. u Beogradu, gde 1971 godine završava XIII Beogradsku gimnaziju prirodnomatematičkog smera. Tokom školovanja bila je jedan od najboljih đaka i tim povodom dobila diplome Vuk Stefanović Karadžić i Mihailo Petrović Alas. Posebno je volela matematiku i predmete iz prirodnih nauka. U četvrtom razredu gimnazije imala je predmet astronomija, koji je zajedno sa predmetom matematika uticao na izbor njenih studija. Po završetku gimnazije upisala je studije astronomije na Odseku za matematiku, mehaniku i astronomiju Prirodno-matematičkog fakulteta (PMF) u Beogradu, gde je diplomirala 1976. Ovaj odsek je pre 30 godina prerastao u Matematički fakultet (MF). Dalje usavršavanje je nastavila upisom magistarskih studija astronomije na istom fakultetu. Magistarsku tezu *Prilog izučavanju kretanja Zemljinih polova* odbranila je 1984. pod rukovodstvom profesora Dragutina Đurovića. Posle magistrature dalje usavršavanje nastavila je u Pragu. Kao stipendista Ministarstva nauke Republike Srbije školske 1986/87 godine radi na doktorskoj disertaciji na Astronomskom institutu Čehoslovačke akademije nauka u Pragu, kao i na Opservatoriji Ondrejov kod Praga. Na Matematičkom fakultetu u Beogradu 2. februara 1989. odbranila je doktorsku disertaciju *Globalna atmosferska cirkulacija i Zemljina rotacija* pod rukovodstvom profesora Jana Vondraka, naučnog savetnika Astronomskog instituta Čehoslovačke akademije nauka. U to vreme profesor Vondrak bio je predsednik Komisije za Zemljinu rotaciju Međunarodne astronomske unije (Interntional Astronomical Union, IAU).

Karijera prof. Pejović na Beogradskom univerzitetu počela je 1977. godine kada je ona izabrana za asistenta-pripravnika na Odseku za matematiku, mehaniku i astronomiju PMF. Za asitenta izabrana je 1985, u zvanje docenta 1989. i za vanrednog profesora 1996. U zvanje redovnog profesora izabrana je 2010. Spomenimo da se u vreme njenog izbora u profesorsko zvanje Matematički fakultet već izdvojio u zasebnu univerzitetsku instituciju. Profesorka Pejović otišla je u penziju 1. oktobra 2017. Živi na Novom Beogradu, udata je za Milovana i majka je dvoje dece, Aleksandra i Ane.

[1] Podaci za biografiju profesorke Pejović i prilozi za njena sabrana dela pripremani su tokom 2020. godine, u vreme kada je pandemija izazvana Korona virusom paralisala čitav svet.

Nastavna delatnost

Kao profesor Matematičkog fakulteta u okviru svoje katedre profesorka Pejović predavala je preko deset predmeta. Već kao poslediplomac, a zatim kao asistent držala je vežbe iz predmeta *Opšta astronomija*, *Sferna astronomija*, *Zvezdana astronomija*, *Praktična astronomija*, *Matematička obrada astronomskih posmatranja*. Od izbora za nastavnika 1989. predavala je predmete *Opšta astronomija*, *Zvezdani sistemi*. U prvoj dekadi dvehiljaditih oživela je nastavu astronomije za studente matematike pa je na tim studijama predavala predmete *Opšta astronomija* i *Sferna astronomija*. Predavala je i na drugim fakulteima, na primer predmete *Osnovi astrofizike* za studente fizike na Fizičkom fakultetu u Beogradu i *Geodetsku astronomiju* za studente geodezije na Građevinskom fakultetu u Sarajevu, 1990-1992. Na svojoj katedri predavala je i držala konsultacije iz nekoliko predmeta i specijalnih kurseva na magistarskim studijama, a kasnije na master i doktorskim studijama: *Odabrana poglavlja astronomije*, *Teorija Zemljine rotacije*, *Zvezdani sistemi* II, *Astronomija*. Posle Bolonjske reforme nastave na Beogradskom univerzitetu na matičnom fakultetu držala je predmete *Opšta astronomija* I, *Opšta astronomija* II za studente astronomije, *Osnovi astronomije* i *Uvod u astronomiju* za studene matematike, *Osnovi astronomije* za studente informatike.

U svojstvu mentora profesorka Pejović rukovodila je izradom doktorske disertacije Gorana Damljanovića, astronoma sa Astronomske opservatorije u Beogradu. Rad na ovoj disertaciji bio je u okviru međunarodnog projekta Hiparkos. Profesorka Pejović takođe je rukovodila izradom nekoliko magistarskih teza. Učestvovala je u više komisija za pregled ocenu i komisija za odbranu doktorskih disertacija, magistarskih teza, a kasnije, po uvođenju ove kategorije studija, i master radova.

Napisala je rukopis Opšta astronomija (recenziran 1996. godine), prevela je sa kolegama Stevom Šeganom i Zlatkom Ćatovićem univerzitetski udžbenik *Sferna astronomija* Robina Grina. Takođe je napisala zajedno sa profesorom Šeganom univerzitetski udžbenik *Osnovi astronomije* (el. izdanje http://matf.bg.ac.rs/astronomija/spe07.htm).

Svojim radom na digitalizaciji davno rasprodatih i teško dostupnih univerzitetskih udzbenika i zbirki zadataka iz astronomije, profesorka Pejović doprinela je da ove knjige, u digitalnoj formi, postanu standardni deo dopunske literature za nastavu astronomije. Elektronske kopije ovih knjiga sada su dostupne u Virtuelnoj biblioteci MF, http://elib.matf.bg.ac.rs. Na primer, digitalizovane su sledeće knjige: *Nebeska mehanika*, *Istorija astronomske nauke od njenih prvih početaka do 1727* kao i *Astronomska teorija klimatskih promena i njena primena u geofizici* Milutina Milankovića, *Opšta astronomija* I deo - *Sferna trigonometrija*, *Zbirka rešenih zadataka iz Opšte astronomije* I deo (štampana 1956) i rukopis *Zbirka rešenih zadataka iz Opšte astronomije* II deo Vojislava Miškovića. Spomenimo da je II deo Miškovićeve zbirke postojao samo u rukopisu koji je autor pripremio sredinom prošlog veka i nikada nije štampan. Digitalizacijom ova knjiga je po prvi put postala dostupna studentima i široj javnosti. Ove Miškovićeve knjige su do sada dve jedine zbirke zadataka iz astronomije napisane na srpskom, što posebno ističe značaj njihove digitalizacije. Takođe je digitalizovala univerzitetske udzbenike *Opšta astronomija* Branislava Ševarlića i Zaharija Brkića, *Teorijska astronomija* Jovana Simovljevića, *Osnovi teorije kretanja Zemljinih veštačkih satelita* Jovana Lazovića i *Obrada astronomskih posmatranja* Dragutina Đurovića koji su se koristili i još uvek se koriste u nastavi na Katedri za astronomiju. Otvoren dostup elektronskim kopijama ovih udžbenika znatno je doprineo nastavi, a studentima boljoj pripremi i uspešnosti polaganja ispita.

Profesorka Nada Pejović ima izuzetno izražen smisao za pedagoški rad. U nastavi bila je veoma angažovana i posvećena radu sa studentima. Pored direktnog svedočenja studenata to se takođe moglo videti iz velike posećenosti nastavi izbornih predmeta za studente matematike i informatike koje je ona predavala. O njenoj popularnosti kao nastavnika govori i činjenica da se uvek nalazila pri vrhu studentskih anketa za ocenu rada profesora.

Naučni i stručni rad

U početku svog naučno istraživackog rada profesorka Pejović bila je orijentisana na geodinamiku, oblast koja se bavi matematičko-dinamičkim modelom Zemlje. Njenu užu specijalnost činile su neravnomernosti Zemljine rotacije i kretanje Zemljinih polova. Teme njenog doktorata i magistarske teze su iz ovih oblasti. Teorijski rezultati koje je dobila analizom matematičkih modela saglasni su sa rezultatima dobijenim iz astronomskih posmatranja. Ovi rezultati privukli su pažnju naučne javnosti u ovoj oblasti i citirani su više puta. Radovi profesorke Pejović citiraju se u domaćim i inostranim časopisima. Ima preko 200 citata u Google Scholar, od toga bar 30 u vodećim međunarodnim časopisima i međunarodnim monografijama.

Profesorka Pejović takođe je radila i u oblastima astrodinamike, astrometrije, kosmologije i galaktičke astronomije. Poslednjih godina intenzivno radi i objavljuje radove iz istorije astronomije. Objavila je 84 naučnih i stručnih radova u domaćim i stranim časopisima i zbornicima sa konferencija. Imala je oko 60 saopštenja na naučnim skupovima u zemlji i inostranstvu. Stručni i naučni rad takođe ostvaruje kao učesnik u naučnim projektima. U ovom izdanju sabrani su svi njeni objavljeni naučni i stručni radovi, magistarski rad i doktorska disertacija.

Magistarski rad *Prilog izučavanju kretanja Zemljinih polova*, odbranila je 1984. na Matematičkom fakultetu Univerziteta u Beogradu pred Komisijom: profesor Dragutin Đurović (mentor), profesor Branislav Ševarlić i profesor Jovan Simovljević. U ovom radu autor daje opširnu analizu koordinata trenutnog pola Zemljine rotacije u odnosu na Međunarodni konvencionalni pol (CIO), baziranu na podacima Medjunarodne službe širine (ILS) u Micusavi za period 1899.9-1979.0. Na osnovu te analize Pejović je zaključila da postoje sekularne promene koordinata pola, ali je i dalje neizvesno da li je to posledica stvarnog kretanja pola ili precesije, sopstvenih kretanja zvezda, itd. Za pravac i brzinu pomeranja pola dobila je rezultate bliske rezultatima nekih poznatih autora. Spektralnom analizom Kimurinog Z-člana našla je, pored poznate godišnje ciklične varijacije i polugodišnju varijaciju, što predstavlja sasvim novo otkriće.

Doktorsku disertaciju *Globalna atmosferska cirkulacija i Zemljina rotacija*, odbranila je 2. februara 1989. na Matematičkom fakultetu Univerziteta u Beogradu, pred komisijom: profesor Jan Vondrak (mentor), profesor Dragutin Đurović i profesor Jovan Simovljević. Disertacija se odnosi na matematičko-dinamički modele Zemlje. Rad sadrži opširnu studiju uticaja globalne atmosferske cirkulacije na Zemljinu rotaciju. Tema je bila vrlo aktuelna, jer je za precizno upravljanje kosmičkim letilicama, kao i za pouzdana geofizička istraživanja neophodno pratiti orijentaciju Zemlje u prostoru sa tačnošću koja se danas nalazi u granicama od nekoliko desetohiljaditih delova lučne sekunde. U rešavanju ovih zadataka veliki problem predstavljaju realna odstupanja globalne atmosferske cirkulacije od poznatih modela koji se koriste u računu parametara Zemljine rotacije. Značajan napredak u rešavanju ovog problema postignut je u to vreme ne tako davno, kad su postale dostupne funkcije efektivnih ugaonih momenata atmosfere, računate iz globalnih atmosferskih podataka u tri svetska meteorološka centra, po algoritmu koji su dali Barnes i drugi 1983. Od tada se preciznije određuje doprinos kretanja atmosfere u promenama parametara Zemljine rotacije. Ipak, ostala su nedovoljno proučena sledaća pitanja :

- **a.** Koliko je velika reakcija okeana na promene vazdušnog pritiska i da li je hipoteza invertnog barometra dovoljna da opiše ovu reakciju?
- **b.** Kolika je uloga člana vetra u polarnom kretanju i da li su raspoloživi podaci dovoljne gustine i preciznosti za kvalitetan račun polarnog kretanja?
- **c.** Da li je geofizički model Zemlje koji su koristili Barnes i drugi adekvatan za računanje Zemljine reakcije na atmosfersku ekscitaciju?

d. Da li je atmosferska cirkulacija sama dovoljna da ekscituje posmatrano polarno kretanje ili postoje i drugi izvori ekscitacije?

U svojoj disertaciji Nada Pejović daje odgovore baš na ova aktuelna pitanja. Posle ukratko prikazanog stanja problema navodi i kratak pregled teorijskih osnova, baziranih na Liuvilovim jednačinama Zemljine rotacije. Procenjena je atmosferska ekscitacija uzimajući u obzir da je Zemlja deformabilno telo koje čini: tečno jezgro, viskozno-elastični omotač i dinamički okean. Posebna pažnja posvećena je pitanju da li nelinearna reakcija okeana na polarno kretanje može uzrokovati zavisnost frekvencije Čendlerove nutacije od amplitude ukupnog kretanja Zemljinih polova. Upoređujući u to vreme najsavremeniju teoriju dopunjenu novim hipotezama sa posmatranjima, dobijeni su interesantni i novi rezultati.

Objavljeni radovi

Naučna interesovanja profesorke Pejović su dosta široka i zahvataju nekoliko oblasti. Mada su njena glavna istraživanja bila u astronomiji, umela su i da izađu izvan okvira ove nauke. Otuda se objavljeni radovi profesorke Pejović mogu podeliti u četiri grupe.

U prvoj grupi su radovi iz oblasti Geodinamike. Radovi se u ovom slučaju odnose na matematičko-dinamički model Zemlje, s posebnim osvrtom na teoriju kretanja Zemljinih polova i neravnomernosti Zemljine rotacije. Nada Pejović razmatra geodinamički model Zemlje koji čine: tečno jezgro, viskozni omotač i dinamički okean. Posebnu pažnju u svojim istraživanjima posvećuje uticaju globalne atmosferske cirkulacije na Zemljinu rotaciju. Izučava uticaj okeana na promenu vazdušnog pritiska, kao i uticaj vetra na polarno kretanje. Pokazuje kako nelinearna reakcija okeana na polarno kretanje može uzrokovati zavisnost frekvencije Čendlerove nutacije od amplitude ukupnog kretanja Zemljinih polova. U radovima iz ove grupe Pejović je uvek vodila računa o što boljim i primenljivijim teorijskim modelima za rešavanje konkretnih problema kao i poređenju dobijenih rezultata sa astronomskim posmatranjima. Ovi radovi citirani su od većeg broja stranih autora u poznatim časopisima. Neke od ovih radova objavila je sa Janom Vondrakom, svetski uglednim astronomom. U prvoj grupi su radovi urađeni u okviru zajedničkog projekta *Dinamika Zemljimih večtačkih satelita i geodinamika - poređenje teorije i posmatranja* Srpske i Čehoslovačke akademije nauka za period 1986-1990. Neke od ovih radova iz astrodinamike i geodinamike objavila je sa kolegom Stevom Šeganom, profesorom Matematičkog fakulteta u Beogradu.

U drugoj grupi nalaze se radovi iz astrometrije. Radovi se bave izučavanjem popravki sopstvenih kretanja zvezda Hiparkos kataloga za epohu 1991.25. Ovaj katalog je dobijen na osnovu podataka sakupljenih posmatračkom misijom Hiparkos satelita (HIPPARCOS - High Precision PARallax COllecting Satellite) lansiranog avgusta 1989. Satelit je za nešto manje od četiri godine završio sa radom. Kako je posmatračka misija Hiparkos satelita kratko trajala, bila je neophodna popravka sopstvenih kretanja zvezda. Za povećanje tačnosti sopstvenih kretanja zvezda Hiparkos kataloga korišćeno je dva miliona klasičnih optičkih posmatranja sakupljenih tokom XX veka na opservatorijama širom sveta. Na obradi ovog opsežnog posmatračkog materijala profesorka Pejović je uspešno radila u okviru Projekta Međunarodne astronomske unije (IAU General Assembly, Baltimor 1988) kojim je rukovodio Jan Vondrak iz Praga, u to vreme predsednik Prve divizije (od dvanaest) IAU. Popravljena su sopstvena kretanja u deklinaciji za 2347 zvezda. Rezultati su predstavljeni u radovima koje je pisala u koautorstvu sa Goranom Damljanovićem, naučnim saradnikom Astronomske opservatorije u Beogradu. Damljanović je iz ove oblasti uradio doktorsku disertaciju pod rukovodstvom profesorke Pejović.

Treću grupu čine radovi iz kosmologije i bave se matematičkim modelom Kosmosa. Ovi radovi se odnose na standardni, tzv. ΛCDM model Kosmosa. Osnove ove teorije leže u Ajnštajnovim jednačinama

polja čiji izvod predstavljaju Fridmanove jednačine. U radovima profesorke Pejović istražuju se rešenja ovog sistema diferencijalnih jednačina koja zadovoljavaju uopšteni stepeni zakon. Ovaj matematički model odgovara eri dominacije barijonske i tamne materije u sveukupnoj evoluciji kosmosa. Posebnu zanimljivost ovih istraživanja predstavlja činjenica da se u njima koristi teorija pravilno promenljivih funkcija koju je zasnovao poznati srpski matematičar Jovan Karamata. Ove radove profesorka Pejović je uglavnom pisala u koautorstvu sa Žarkom Mijajlovićem, profesorom Matematičkog fakulteta. Radovi su objavljeni u visokorangiranim časopisima iz primenjene matematike i teorije gravitacije. Takođe je sa kolegom Slobodanom Ninkovićem, naučnim savetnikom Astronomske opservatorije u Beogradu, objavila radove iz galaktičke astronomije.

U četvrtoj grupi su radovi iz istorije astronomije u kojima se profesorka Pejović bavi pionirskim radovima i knjigama iz astronomije i srodnih nauka srpskih autora pisanih u prošlosti. Neke od ovih retkih knjiga profesorka Pejović je digitalizovala i u svojim radovima predstavila njihove sadržaje, analizirala njihov značaj i ukazivala na zanimljivosti vezane za ta izdanja. Elektronske kopije ovih knjiga nalaze se u Virtuelnoj biblioteci (Virtual library, http://elib.matf.bg.ac.rs) Matematičkog fakulteta sa slobodnim pristupom i otuda su sadržaji ove Internet baze javno i lako dostupni zainteresovanoj publici. Profesorka Pejović posebno istražuje i predstavlja kolekciju starih knjiga srpskih astronoma koje su pisane u XVIII i XIX veku. One najstarije pisane su na starosrpskom jeziku i predvukovskim pismom, ili na latinskom, i uključuju: *Elementi matematike* Ruđera Boškovića (Venecija, 1757, na latinskom), *Večni kalendar* Zaharija Stefanovića Orfelina (Beč, 1783), *Fizika* Atanasija Stojkovića (Budim, 1810), *Zvezdano nebo nezavisne Srbije* Djorđa Stanojevića (1882, Beograd), *Kosmografija* Milana Andonovića (1888, Beograd), *Atomistika – Jedan deo iz filozofije Ruđera Boškovića* Koste Stojanovića (Niš, 1892), *Nebeska mehanika* Milutina Milankovića (Beograd, 1935), *Osnovi matematične i fizičke geografije* Pavla Vujevića (Beograd, 1924). U Virtuelnoj biblioteci nalaze se takođe *Aritmetika* Vasilija Damjanovića, prva kjiga iz matematike štampana na srpskom (Venecija, 1767) i *Čislenica* Jovana Došenovića (Budimpešta,1809). Kako profesorka Pejović primećuje, svaka pisana ili štampana reč u prošlosti, naročito u daljoj, deo je kulturnog nasleđa, i već time ove knjige zaslužuju ovakvu pažnju i posebnu vrstu njihovog čuvanja. Deo ovih radova objavila je u koautorstvu sa kolegom Žarkom Mijajlovićem, u okviru projekta Digitalizacija naučne i kulturne baštine Ministarstva za nauku RS. Spomenimo da su ovi radovi doživeli veliki odziv na Internetu i citarni su više od 150 puta, što se može pratiti na Scholar Google. Da za ovu vrstu radova postoji zanimanje i u svetu, može se videti na primeru jedne kolekcije članaka iz istorije matemike u Indiji, http://www.pdfqueen.com, gde je postavljen članak *Digitization of mathematical textbooks used in Serbia in the past*, NCD Review, vol. 12, 55-64, 2008.

Ostale aktivnosi

Profesorka Pejović držala je predavanja u Istraživačkoj stanici Petnica, na Kolarčevom narodnom univerzitetu i govorila na radiju i televiziji. Od 2002. godine pripremala je talentovane učenike srednjih i osnovnih škola za učešće na međunarodnim olimpijadama iz astronomije. Osnovala je 2005. fakultativno odeljenje astronomije u Gimnaziji u Prokuplju, uz saglasnost Ministarstva prosvete Srbije. Dobila je 2006. od Opštine Prokuplje *Zahvalnicu* za doprinos izgradnji nove opservatorije na planini Vidojevica kod Prokuplja. Od Centra za promociju nauke Republike Srbije dobila je 2019. zahvalnicu za predavanja održana u okviru manifestacije "Maj mesec matematike".

Tokom školske 2008/2009 aktivno je učestvovala u izradi novih programa prilagođenih Bolonjskom procesu za studije astronomije na Matematičkom fakultetu. Zaslužna je za uvodjenje studijskog usmerenja Astroinformatika na Matematičkom fakultetu. Godinama je bila aktivno angažovana na vraćanju astronomije kao posebnog predmeta u gimnazije i srednje tehničke škole.

Sve navedene aktivnosti profesorke Nade Pejović iziskivale su mnogo vremena, truda, posvećenosti i predanosti. S obzirom da je naučni rad detaljno predstavljen u ovoj knjizi, osvrnimo se nešto više na sledeće oblasti njenog delovanja.

Pomenimo da su prve ideje oko izgradnje nove Astronomske stanice na Vidojevici (ASV) krenule sa Matematičkog fakulteta u Beogradu. Projekat su pokrenuli profesori Stevo Šegan i Žarko Mijajlović, kojima se pridružila Nada Pejović, tada šef Katedre za astronomiju MF. Naime, 2001. posle osnivanja Astronomskog društva "Magelanov oblak" u Prokuplju, krenulo se u potragu za odgovarjućom lokacijom povodom izgradnje posmatračkog paviljona Društva. Članovi Društva bili su toliko oduševljeni vrhom planine Vidojevice kao mogućom lokacijom za posmatrački paviljon, da su predložili Matematičkom fakultetu i Astronomskoj opservatoriji u Beogradu da se na tom mestu izgradi nova profesionalna opservatorija, odnosno Astronomska stanica Vidojevica. Predlog je prihvaćen i uz pomoć Ministarstva za nauku gradnja nove opservatorije je započeta. Dogovoreno je da se u krugu ASV izgradi školski paviljon za studente MF, kao i mali paviljon za članove Društva. Pri sakupljanju odgovarajuće dokumentacije za ASV, Nada Pejović se aktivno uključila šaljući, kao šef Katedre za astronomiju, odgovarajuće dopise nadležnim vlastima u Prokuplju. Od Opštine Prokuplje 2006. dobila je Zahvalnicu za doprinos u izgradnji Astronomske stanice na Vidojevici.

Uporedo sa izgradnjom nove opservatorije, krenule su i druge aktivnosti u Prokuplju. Naime, Astronomsko društvo "Magelanov oblak" organizovalo je niz predavanja iz astronomije, matematike i informatike u prostorijama Društva, u Gimnaziji i osnovnim školama u Prokuplju. Predavanja su držali profesori sa MF u Beogradu, PMF u Nišu i sa Astronomske opservatorije u Beogradu (AOB). Nada Pejović je bila jedan od aktivnih predavača.

Pomenimo još jednu značajnu aktivnost profesorke Pejović u Prokuplju. Naime, nekoliko godina Nada Pejović je bila rukovodilac jednogodišnjih projekata "Program podsticanja, promocije i popularizacije nauke u Prokuplju", koji su finansirani od strane Ministarstva za nauku Republike Srbije. U okviru tih projekata formirano je fakultativno odeljenje astronomije u Gimnaziji u Prokuplju. Iz ovih projekata je finansiran dolazak predavača iz Beograda, kao i saradnika koji su za fakultativno odeljenje u Gimnaziji osim predavanja obavljali i pripreme učenika za učešće na domaćim takmičenjima i međunarodnim olimpijadama iz astronomije. Za pripreme olimpijaca van Beograda profesorka Pejović je digitalizovala davno rasprodate srednjoškolske udžbenike iz astronomije i postavila ih u Virtuelnu biblioteku MF. Praktične vešbe za olimpijce izvodili su najaktivniji članovi društva "Magelanov oblak", tada studenti geografije Aleksandar Valjarević i Aleksandar Simonović. Posebnu podršku pružio je direktor Gimnazije i professor matematike Dragan Krstić. Sve ove aktivnosti bile su propraćene na lokalnoj televiziji i programima radio stanica u Prokuplju, u lokalnim novinama ali i u dnevnoj štampi (Politika, Večernje novosti, Blic).

Profesorka Nada Pejović je organizovala prvo učešce juniorske grupe đaka (starost 14-15 godina) na X međunarodnoj astronomskoj olimpijadi (MAO) u Pekingu 2005. Na prethodne dve MAO iz Srbije su se takmičili samo seniori (starost 16-17 godina). Prof. Pejović je u oktobru 2005 došla na čelo Nacionalnog astronomskog olimpijskog komiteta (NAOK) za Srbiju. Ovo telo je osnovala 2002. godine prof. Jelena Milogradov Turin koja je bila njegov prvi predsednik. Kada je postala predsednik NAOK, Prof. Pejović je učinila znatan napor u cilju poboljšanja priprema motivisanih učenika. Zahvaljujući njenom trudu bilo je moguće njenim naslednicima na mestu predsednika NAOK dr Slobodanu Ninkoviću i dr Sonji Vidojević da nastave uspešno ovaj važan rad. Godine 2007. pojavilo se još jedno međunarodno takmičenje iz astronomije – Međunarodna olimpijada iz astronomije i astrofizike (MOAA). Tokom tri godine, 2009, 2010 i 2011, takmičari iz Srbije učestvovali su na oba takmičenja – MAO i MOAA. Počev od 2012. godine takmičari iz Srbije učestvuju samo na MOAA. MOAA je predviđena za učenke srednjoškolskog uzrasta (15-20 godina) i ne postoji nikakva pódela takmičara prema uzrastu. Predstavnici Srbije su bili uspešni i na MAO i MOAA, osvojili su mnoge medalje, čak i posebne nagrade.

U Petnici profesorka Pejović je aktivno radila na popularizaciji astronomije. Skoro dve decenije, tokom osamdesetih i devedesetih godina prošlog veka bila je redovni predavač na Zimskom seminaru iz astronomije u Petnici. Znamo da je Istraživačka stanica Petnica jedinstvena institucija u Srbiji koja se bavi naučnim vanškolskim obrazovanjem starijih razreda osnovaca i srednjoškolaca. Kao takva, ova ustanova postala je prepoznatljiva u Evropi i svetu. Profesorka Pejović je sa entuzijazmom promovisala Astronomsku teoriju osunčavanja planeta našeg i svetskog velikana Milutina Milankovića. Promovisala je zanimljivosti i lepote iz astronomskih nauka predavanjima sa temama poput: Sunčev sistem, Sunčani časovnici, Matematičko-dinamički model Zemlje, Kretanje Zemljinih polova, Sistemi vremena, Diferencijalne jednačine kretanja planeta i Najveći teleskopi sveta.

Profesorka Pejović aktivan je član Udruženja "Milutin Milanković", koje je osnovano 2007. s ciljem da promoviše lik i delo našeg i svetskog velikana. Bila je član Upravnog odbora i sekretar ovog udruženja. Organizovala je 2011. saradnju Matematičkog fakulteta i Udruženja Milutin Milanković na izradi Digitalnog legata Milutin Milanković, http://legati.matf.bg.ac.rs/milankovic. Cilj legata je da sakupi, objedini i učini lako dostupnim sav naučni doprinos svetskoj nauci našeg velikog naučnika kao i njegov živototpis. Tamo se nalaze digitalne kopije njegovih knjiga, monografija, univerzitetskih udzbenika, zatim svih naučnih radova koje je objavio, kao i rukopisi njegovih univerzitetskih predavanja. Ovaj portal sadrži mnoge fotografije i druge dokumente koje bliže osvetljavaju njegov život, ali i knjige i radove koje su o Milankoviću napisali drugi autori. Sa Udruženjem obilazila je škole po Srbiji, gde je promovisala izložbe i filmove posvećene Milankoviću i držala predavanja poput: Milankovićevi ciklusi, Digitalni legat Milutina Milankovića, Milutin Milanković - tajne ledenih doba. Više puta je bila gost Osnovne škole u Rabrovu kod Kučeva koja nosi ime našeg velikana. Na Ekonomskom fakultetu i Matematičkom fakultetu u Beogradu i drugim ustanovama održala je više predavanja na temu Milutin Milanković i njegova astronomska teorija klimatskih promena. U prostorijama Udruženja, Pop Lukina 1b, organizovala je u nekoliko navrata Školu mladih talenata iz astronomije, pripreme đaka za učešće na međunarodnim astronomskim olimpijadama, studentske radionice i snimanje priloga iz astronomije za razne TV stanice.

Profesorka Pejović aktivan je član Astronomskog društva Ruđer Bošković (ADRB). Ovo društvo je od 1935. do 1940. izdavalo naučno-popularni časopis *Saturn*, a od 1953. do 2020, i dalje izdaje časopis sličnog profila, *Vasiona*. Časopis nije štampan u periodu od šest i po godina, u intervalu 2011-2017, zbog finansijskih poteškoća. Profesorka Pejović je pokrenula inicijativu i organizovala digitalizaciju nekoliko starih srpskih časopisa iz astronomije. Tako je zajedno sa profesorom Mijajlovićem organizovala da se svi brojevi časopisa *Saturn* (kompletan period 1935-1940) sakupe, digatalizuju i postave u Virtuelnu biblioteku Matematičkog fakulteta. Takođe, u toku je digitalizacija časopisa *Vasiona*. Do sada su digitalizovani svi brojevi za prvih pedeset godina publikovanja *Vasione*, period 1953-2004. Digitalne kopije časopisa postavljene su takođe u Virtuelnu biblioteku MF i tako su postale dostupne širokoj javnosti. U VB postavljene su i elektronske kopije novijih brojeva Vasione, štampane u periodu 2017-2020. Naš kolega Milan Dimitrijević organizuje svake druge godine u okviru AD "Ruđer Bošković" na Narodnoj opservatoriji na Kalemegdanu oko Vaskrsa konferenciju *Razvoj astronomije kod Srba*. Više od decenije profesorka Pejović je redovan učesnik ovih skupova sa saopštenjima uglavnom o digitalizaciji retkih astronomskih knjiga iz XVIII, XIX i prve polovine XX veka u kojima je ispisana istorija astronomske nauke kod Srba. Neke od tih knjiga su već pomenute: *Elementi matematike* Ruđera Boškovića, *Večni kalendar* Zaharija Stefanovića Orfelina, *Zvezdano nebo nezavisne Srbije* Đorđa Stanojevića i *Kanon osunčavanja* Milutina Milankovića.

Akreditacija nastavnih programa je novi termin nastao 2005. prilikom formalnog priznavanja visokoškolskih ustanova u Srbiji. Te godine sprovedena je velika reforma visokoškolske nastave i njenog prilagođavanja Bolonjskoj deklaraciji na Beogradskom univerzitetu i drugim univerzitetima u Srbiji. Profesorka Pejović je ispred svoje katedre od prvog dana bila uključena u Akreditacioni tim za reformu nastave na Matematičkom fakultetu. To je bio težak, zahtevan i iscrpljujući posao koji je iziskivao česta okupljanja, duge i zamorne sastanke. Na primer, trebalo je sve dvosemestralne predmete podeliti na

jednosemestralne, od obaveznih predmeta jednu trećinu prebaciti u izborne, od stručnih trećinu prebaciti u opšteobrazovne predmete, kao i uraditi niz drugih promena. Od školske 2006/2007 Matematički fakultet izvodi reformisanu nastavu i uveden je novi smer Informatika. Imajući u vidu ovu činjenicu i prilagođavajući se novim trendovima i tehnologijama, na predlog i zalaganje profesorke Pejović na Katedri za astronomiju uvedeno je novo usmerenje Astroinformatika. Takođe, na smeru Matematika i smeru Informatika uvedeni su pored postojećeg *Osnovi astronomije* i novi izborni predmeti *Uvod u astronomiju, Uvod u mehaniku, Mehanika, Opšta teorija relativnosti i kosmološki modeli*. Na master studijama na smeru Matematika uveden je novi modul *Astronomija i mehanika*, koji je uključivao niz novih predmeta iz mehanike.

U Srbiji se astronomija izučavala kao poseban predmet u školama već krajem XIX veka pa sve do početka Drugog svetskog rata. Od 1880. na Velikoj školi u Beogradu astronomiju je predavao prof. Milan Nedeljković, osnivač Astronomske opservatorije u Beogradu. Posle njega na Beogradskom univerzitetu (Velika škola je 1905. prerasla u Univerzitet) astronomiju su predavali naš slavni naučnik Milutin Milanković, a potom Vojislav Mišković. Spomenimo da je Mišković osnovao novu Astronomsku opservatoriju na brdu Veliki Vračar, u ono vreme na periferiji Beograda, koje je ubrzo po opservatoriji dobilo ime Zvezdara. U posleratnom periodu astronomija je bila poseban predmet u gimnazijama prirodno-matematičkog smera tokom 25 godina, u periodu 1969-1994. Na žalost, odlukom Prosvetnog saveta Srbije 28. juna 1990. astronomija je ukinuta u gimnazijama i njeni časovi su dodeljeni predmetu fizika. Đaci upisani do ovog datuma ipak su imali astronomiju kao zaseban predmet, sve do 1994. Od tada je prestalo izučavanje astronomije u srednjim školama i od tada kreće mukotrpna borba za njeno vraćanje, što se ispostavilo da je to takoreći sizifoski posao. U tu borbu, u kojoj su učestvovali mnogi profesori Katedre za astronomiju MF i Astronomske opservatorije, od 2000. se uključila i Nada Pejović. Više puta je zajedno sa profesorkom Olgom Atanacković tim povodom pisala dopise i molbe Ministarstvu prosvete i sporta Republike Srbije, Nacionalnom prosvetnom savetu Republike Srbije, Zavodu za unapređenje obrazovanja i vaspitanja, Zavodu za vrednovanje kvaliteta obrazovanja i vaspitanja, Fizičkom fakultetu, pa i dnevnim novinama i TV medijima. Nažalost, ovi napori su ostali bez uspeha. Na tu temu objavljen je članak *Zahtev da astronomija ponovo bude poseban predmet* u dnevnom listu Politika, 23. maja 2013.

U okviru projekta *Digitalizacija naučnog i kulturnog nasleđa* Profesorka Pejović je radila sa timom kolega sa Matematičkog fakulteta na stvaranju Virtuelne biblioteke Matematičkog fakulteta Univerziteta u Beogradu (Virtual library, http://elibrary.matf.bg.ac.rs). Ovo je najveća baza digitalizovanih knjiga u Srbiji sa slobodnim pristupom. U ovom trenutku Virtuelna biblioteka sadrži preko 5000 knjiga. Važne kolekcije u okviru ove biblioteke su zbirka doktorskih disertacija odbranjenih na Matematičkom fakultetu i zbirka retkih knjiga iz XVIII i XIX veka. Većina ovih knjiga su rariteti, neke od njih postoje samo u jednom primerku i praktično su nedostupne javnosti. Tek digitalizacijom postale su lako dostupne đacima, studentima i široj javnosti. Profesorka Pejović je digitalizovala skoro sve srpske stare knjige iz astronomije. Takođe je sa istim timom i u okviru istog projekta uradila i Digitalne legate, http://legati.matf.bg.ac.rs. Ovaj digitalni portal ima za cilj da širokoj javnosti učini dostupnom veliku zaostavštinu najznačajnijih srpskih naučnika iz dalje i bliže prošlosti koji su se bavili matematičkim naukama. Prvi urađen digitalni legat bio je legat Milutin Milanković, urađen 2012. godine. Ovo je prvi digitalni spomenik velikanu srpske i svetske nauke. Pored legata Milutin Milanković urađeni su digitalni legati Bogdan Gavrilović, Anton Bilimovič, Mihailo Petrović Alas, Zaharije Brkić, Đuro Kurepa i Slaviša Prešić. U toku je izrada još tri digitalna legata za tri Jovana, tri profesora Matematičkog fakulteta: Jovana Karamatu, Jovana Simovljevića i Jovana Lazovića. Profesorka Nada Pejović je radila na priređivanju digitalnih legata astronoma Milutina Milankovića i Zaharija Brkića, a sada radi na legatima Jovana Simovljevića i Jovana Lazovića.

Profesorka Pejović je prva žena - profesor astronomije na Katedri za astronomiju Beogradskog univerziteta. Za ovaj podatak se saznalo iz izlaganja prof. Olge Atanacković *Žene na Katedri za astronomiju*, na skupu *Žene u astronomiji* održanom 8. marta 2019 na Matematičkom fakultetu u

Beogradu, povodom obeležavanja Međunarodnog dana žena u nauci (11. februar) i obeležavanja jubileja 100 godina Međunarodne astronomske unije. Profesorka Pejović je bila veoma ponosna na to saznanje, činjenicu da je postala deo istorije Katedre za astronomiju i da je predavala na istoj Katedri na kojoj su pre nje predavali slavni Milutin Milanković, Vojislav Mišković, Milan Nedeljković i drugi profesori.

PRILOZI

Naučni skupovi

Profesorka Pejović učestvovala je na više od 60 domaćih i međunarodnih skupova. Posebno se izdvajaju učešća na sledećim konferencijama koje se održavaju periodično sa intervalima od jedne do četiri godine, a na kojima profesor Pejović učestvuje redovno ili veoma često:

Nacionalna konferencija jugoslovenskih astronoma, učestvuje od 1977. Po finalnom nestanku zajedničke države 2006. godine ovaj periodični skup, drži se svake treće godine, menja naziv i danas je to Konferencija astronoma Srbije. Ipak je zadržan kontitnuitet u rednom broju i poslednja konferencija ove vrste održana je 2020. pod nazivom SAC XIX (19. Serbian Astronomical Conference). Ova konferencija ima međunarodni karakter s obzirom da je skoro polovina učesnika iz inostranstva.

Kongres matematičara Jugoslavije (održava se jednom u četiri godine).
Bulgarian–Serbian Conference on Astronomy and Space science (održava se svake druge godine).
Serbian-Belorussian Symposium on Physics and Diagnostics of Laboratory and Astrophysical Plasmas.
Serbian Conference on Spectral Line Shapes in Astrophysics.
Konferencija: Razvoj astronomije kod Srba (održava se jednom u dve godine).
SEEDI International Conference: Digitization of cultural and scientific heritage (održava se jednom u tri godine).
NCD konferencija Tehnologije i standardi: digitalizacija nacionalne baštine (održava se svake godine).
Profesor Pejović učestvovala je u radu svakog od ovih skupova, od osnivanja 2003. godine.

Profesorka Pejović takođe je učestvovala na velikim međunarodnim skupovima:
International Symposium Geodesy and Physics of the Earth. Potsdam, 1988.
IAU (International Astronomical Union) General Assembly, Buenos Aires, 1991.
Earth Rotation, Reference System in Geodynamics and Solar System, Varšava, 1995.
Tri konferencije *IAU Colloquium*, Beograd 1987 i 2004, Cambridge 2005.
Tri konferencije koje su održane pod pokroviteljstvom *AIP* (American Institute of Physics), Bukurešt 2006, Sinaia 2007, Sremski Karlovci 2007.
Dve konferencije održane pod pokroviteljstvom *Astronomical Society of Pacific*, Side/Antalia 2006. i Sinaia 2006.

Profesorka Pejović takođe je učestvovala na jubilarnim skupovima, gde je učešće bilo po pozivu:

Conference Fifty Years of Romanian Astrophysics, Bukurešt 2006.
International Conference 60 years of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, 2007, Sofia.
Konferencija: Đordje Stanojević- život i delo - povodom 150 godina od rodjenja, 2008, Novi Sad.

Profesorka Pejović održala je nekoliko predavanja po pozivu: „*Project ASV*“ na medjunarodnom skupu The 5th Bulgarian–Serbian Conf. on Astron. and Space science, May 9–12, 2006, Sofia/Bulgaria i takođe „*Teaching of Astroinformatics at the University of Belgrade*“ na VII Bulgarian-Serbian Astronomical Conference: Astroinformatics, 1-4 June 2010, Chepelare-Rozen, Bulgaria.

Profesorka Pejović je često bila direktno angažovana u organizovanju naučnih skupova. Na primer, bila je član Naučnog i organizacionog komiteta (NOK) XI (1996) i XIII (2002) Nacionalne konferencije jugoslovenskih astronoma sa međunarodnim učešćem, u organizaciji Katedre za astronomiju Matematičkog fakulteta u Beogradu, a pod pokroviteljstvom Saveznog sekretarijata za nauku Jugoslavije i Ministarstva za nauku, tehnologiju i razvoj Republike Srbije. Bila je član NOK-a XIV (2005) Nacionalne konferencije u organizaciji Astronomske opservatorije u Beogradu. Bila je kopredsednik zajedno sa kolegom Slobodanom Jankov XVI Nacionalne konferencije astronoma Srbije 2011. Takodje je učestvovala u organizaciji i programskim komitetima međunarodnih i domaćih konferencija: Srpsko-Bugarske konferencije astronoma, NCD i SEEDI konferencija sa temom digitalizacija naučne i kulturne baštine i konferencije Razvoj astronomije kod Srba.

Udžbenici i prevodi.

Profesorka Pejović, sama ili u koautorstvu, napisala je nekoliko udžbenika i uradila prevod jedne knjige.

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Citiranost

Radovi Dr Nadežde Pejović citirani su u domaćim i inostranim naučnim časopisima. U vodećim naučnim časopisima i međunarodnim monografijama ima najmanje 30 citata i to iz svake od četiri oblasti u kojima je radila. Prema Scholar Google ima preko 200 citata. Izdvajamo desetak važnijih citata.

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Učešće na projektima

Tokom svoje akademske karijere profesorka Pejović učestvovala je u više domaćih i međunarodnih projekata. Projekti su bili iz astronomskih nauka, a nekim od njih je rukovodila. Rukovodila je od 1986. do 1990. projektom Kretanje Zemljinih veštačkih sateltita i geodinamika - poredjenje teorije i posmatranja u okviru saradnje Čehoslovačke i Jugoslovenske akademije nauka i umetnosti. Tada je sa kolegom Stevom Šeganom objavila nekoliko zajedničkih radova. U periodu 2006-2009 rukovodila je projektom Program promocije i podsticanja nauke Ministarstva za nauku. Takodje, rukovodila je od 2014 – 2017. potprojektom Projekta III44006 na MF, Digitalizacija naučne i kulturne baštine. Učestvovala je u 10 naučnih i stručnih projekata, 7 domaćih i 3 međunarodna.

1. Projekat iz oblasti Astro-geo nauka Ministarstva za nauku Srbije, 1986-1990.
2. *Dinamika Zemljinih veštačkih satelita i geodinamika – poređenje teorije i posmatranja*, saradnja SANU i Češke akademije nauka, 1986-1990. Rukovodilac projekta: N. Pejović.
3. *Fizika i kretanje nebeskih tela i veštačkih satelita,* proj. 23, podprojekt *Geodinamika i veštački sateliti*, 1990-1995. Projekat je finansiralo Ministarstvo Srbije za nauku i tehnologiju, Rukovodilac: Milan Dimitrijević.
4. *Astrometrijska, astrodinamička i astrofizička istraživanja*, 1996-2000, finansiran od strane Ministarstva Srbije za nauku i tehnologiju. Rukovodilac: M. Dimitrijević.
5. *WG ERHRF – IAU Working Group on Earh Rotation in the Hipparcos Reference Frame* (founded by Baltimor 1998, XX IAU General Assembly). Rukovodilac: Jan Vondrak.
6. *Digitalizacija naučne i kulturne baštine*, proj. 6201, period 2005-2007, finansiran od strane Ministarstva za nauku i tehnologiju. Rukovodilac: Žarko Mijajlović.
7. *Program podsticanja, promocije i popularizacije nauke*, 2006-2009, finansiran je od strane Ministarstva za nauku i tehnologiju Republike Srbije. Ovim projektom je promovisana astronomija u skolama, posebno u Prokuplju, gde je bila u toku izgradnja nove opservatorije. Rukovodilac: Nada Pejović.
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9. *Primena informacionih tehnologija u digitalizaciji naučne i kulturne baštine*, projekt br. 13017, 2007-2010, finansiran od strane Ministarstva za nauku i tehnologiju Republike Srbije. Rukovodilac: Žarko Mijajlović.
10. *Digitalizacija naučne i kulturne baština*, III44006, period 2011-2020, aneksima je produžavan od 2014. do 2020. i sve vreme bio je finansiran od strane Ministarstva za nauku Republike Srbije. Rukovodilac projekta u celom periodu je Zoran Ognjanović, direktor Matematičkog instituta SANU, dok je rukovodilac potprojekta na Matematickom fakultetu bio Žarko Mijajlović. Po njegovom odlasku u penziju 2014, profesorka Pejović je preuzela rukovođenje potprojektom. Po njenom odlasku u penziju 2017, rukovodilac postaje Nenad Mitić, profesor Matematičkog fakulteta.

Administrativne i upravne aktivnosti. Članstva

Skoro neprekidno tokom radnog veka, a i po odlasku u penziju, profesorka Pejović je imala važne funkcije u radu matičnog fakulteta i stručnih udruženja. Tako, bila je sekretar Instituta za astronomiju PMF, član Saveta MF, član Upravnog odbora MF i šef Katedre za astronomiju (1999-2007, 2012-2016) MF. Zatim, bila je član Naučnog veća Astronomske opservatorije u Beogradu i član Komisije MF za izgradnju Astronomske stanice na Vidojevici kod Prokuplja. Član je Astronomskog društva „Ruđer Bošković", zatim Udruženja „Milutin Milanković" i takođe Astronomskog društva „Magelanov oblak" iz Prokuplja. Član je Društva astronoma Srbije (DAS) i Nacionalnog komiteta astronoma Srbije. Član je Evropskog astronomskog društva i takođe Međunarodne astronomske unije (IAU). Bila je predsednik Nacionalnog astronomskog olimpijskog komiteta i potpredsednik Društva astronoma Srbije.

Novinski članci

Profesorka Pejović napisala je nekoliko novinskih članaka posvećenih astronomiji: *U spomen Gagarinu*, dnevne novine Blic, 12. april 2003, str. 24; *Beograđani u Manavgatu u Turskoj - fotografije potpunog pomračenja Sunca*, dnevne novine Politika 31. mart 2006, str 26; *Zahtev da astronomija ponovo bude poseban predmet* dnevne novine Politika 23. maj 2013, str. 8*; Zašto naši najugledniji naučnici tvrde da HAARP ne može da deluje na vremenske prilike*, dnevne novine Politika 25. maj 2014, str.10; *Prof. Jovan Lazović dnevne n*ovine Politika, 18. jul 2020, str. 9. Takođe, pisala je stručne članke i osvrte za naučno-popularni časopis *Vasiona*.

Učešće u TV emisijama

RTS Program za dijasporu, Magazin Srbija na vezi, *Digitalni legati Matematičkog fakulteta.*
16. septembar 2020, https://www.youtube.com/watch?v=1x9MD23uKnE

Studio B Jutro sa Sanjom, *Astronom Jovan Lazović - ko je bio i na čemu je radio?*
5. jun 2020, https://www.youtube.com/watch?v=K-Sdrmml8Mc

Takođe je učestvovala u nekoliko emisija na lokalnoj TV stanici u Prokuplju povodom izgradnje Astronomske stanice na planini Vidojevica kod Prokuplja.

3 **Doktorska disertacija:** *Globalna atmosferska cirkulacija i Zemljina rotacija*
PhD thesis: *Global atmospheric circulation and Earth's rotation*

Nadežda Pejović
Defended at the Faculty of Mathematics in Belgrade
Mentor: Professor Jan Vondrak
102+11 pages, 1989.

Her dissertation, *Global atmospheric circulation and Earth's rotation*, Nada Pejović defended at the Faculty of Mathematics, University of Belgrade, in 1989. The paper contains an extensive study of the influence of global atmospheric circulation on the Earth 's rotation. The topic is very actual, because for the precise control of spacecraft, as well as for reliable geophysical research, it is necessary to monitor the orientation of the Earth in space with the accuracy that is today within the limits of several tens of thousands parts of arc-seconds. A true problem in solving these tasks is the real deviations of the global atmospheric circulation from the known models used in the calculation of the Earth's rotation parameters. Significant progress has been made in overcoming this problem. The functions of the effective angular moments of the atmosphere were calculated at that time using global atmospheric data supplied by at least three world meteorological centers, according to an algorithm provided by Barnes et al. in 1983. Since then, the contribution of atmospheric motion to changes in the parameters of the Earth's rotation has been more precisely determined. However, some issues remained:

- What is the reaction of the ocean to changes in air pressure and is the invert barometer hypothesis sufficient to describe this reaction?
- What is the role of the wind parameter in the polar movement and whether the available data are of the sufficient density and precision for a quality polar motion calculation?
- Is the geophysical model of the Earth used by Barnes and others adequate to calculate the Earth's response to atmospheric excitation?
- Is the atmospheric circulation alone sufficient to excite the observed polar motion or are there other sources of excitation?

In this paper, the author gives answers to these current questions. After a brief presentation of the state of the problem, a brief overview of the theoretical foundations of Earth's rotations, based on Liouville's equations. Atmospheric excitation was estimated taking into account that the Earth is a deformable body that consists of: a liquid core, a viscous-elastic mantle and a dynamic ocean. Special attention is paid to the question of whether the nonlinear reaction of the ocean to polar motion can cause frequency dependence of Chandler nutation from the amplitude of the total motion of the Earth's poles. Comparing the most modern theory, supplemented with new hypotheses, with observations, interesting new results were obtained.

Univerzitet u Beogradu

Matematički fakultet

Nadežda Pejović

GLOBALNA ATMOSFERSKA CIRKULACIJA I ZEMLJINA ROTACIJA

doktorska disertacija

Beograd, 1988

Izražavam svoju iskrenu zahvalnost rukovodiocu, Dr J. Vondraku–naučnom savetniku Astronomskog instituta Čehoslovačke akademije nauka u Pragu, koji je sve vreme pratio i usmeravao ovaj rad velikim brojem sugestija, diskusija i saveta.

Veliku zahvalnost izražavam Dr S. Šeganu– docentu Matematičkog fakulteta u Beogradu, na pruženoj mi pomoći pri izradi programa i štampanju ovog rada u TEX-u.

Autor

Ovaj rad je posvećen
dragim mi roditeljima
Zlati i Raji
Matorčević

Njihova jedina kći

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UVOD

Atmosferska ekscitacija Zemljine rotacije bila je predmet istraživanja već nekoliko decenija. Mnoštvo rezultata i teorijskih razmatranja problema dati su u dve glavne monografije posvećene rotaciji Zemlje i to Munk i MacDonald (1960) i Lambeck (1980). Novije, detaljnije informacije o atmosferi date su mnogim radovima, koji razmatraju pomenuti problem sa različitih gledišta. Neki autori razmatraju, uglavnom, uticaj atmosfere na promene u trajanju dana (na pr. Lambeck i Cazenave, 1973, 1974, 1975; Sidorenkov, 1973, 1979; Okazaki, 1977; Rosen i Salstein, 1983, 1985; Đurović,1983,1986, 1987; Feissel i Nitschelm, 1985), neki se koncentrišu samo na atmosferske efekte u polarnom kretanju (n.pr. Wilson i Haubrich, 1976a, 1976b; Jochman, 1976,1981a,1981b; Lambeck i Hopgood, 1981; Wahr, 1982, 1983; Eubanks i dr., 1987; Brzezinski, 1987) a neki razmatraju oba (napr. Sidorenkov, 1982; Barnes i dr., 1983; Hide, 1984; Dutton i Fallon, 1985; Eubanks i dr., 1985; Vondrak, 1987; Salstein, 1987).Jedan od značajannih radova publikovali su Barnes, Hide, White i Wilson (1983) dajući solidnu osnovu za računanje takozvanih funkcija efektivnog ugaonog momenta (effektive angular momentum functions - EAMF) atmosfere (površinski pritisak i globalni vetar) baziranih na globalnim meteorološkim podacima sakupljenim u Evropskom centru Medium- Range Weather Forecast. Ista globalna analiza je primenjena i na U.S. National Meteorological Center (Salstein, 1985) i Japan Meteorological Agency (Naito, 1985).

Postalo je jasno da postoji saglasnost između kretanja atmosfere i promena parametara Zemljine rotacije (polarno kretanje i promene u trajanju dana). Međutim postoji još mnoštvo nerešenih pitanja. Da bi rešila ova pitanja, formirana je Specijalna radna grupa 5-98 (The Special Study Group 5-98) pod pokroviteljstvom Međunarodne geodetske unije (The International Association of Geodesy) u 1983 godini i njen rad je produžen u 1987 godinu. Nova Međunarodna služba Zemljine rotacije (The International Earth Rotation Service - IERS), koju su osnovali IAU i IUGG aktivno radi počev od jan-

uara 1988, svesna važnosti preciznog poznavanja globalne atmosferske cirkulacije za pitanja Zemljine rotacije, formirala je centar za atmosferske podatke, koji se nalazi u okviru Centralnog biroa IERS-a (The Central Bureau of the IERS).

Ukratko sumirajući ranije dobijene rezultate, možemo reći da kratkoperiodične varijacije u trajanju dana su uzrokovane atmosferskim i plimskim efektima; upoređenja pokazuju odličnu korelaciju između osne komponente atmosferske EAMF i promena u brzini Zemljine rotacije, korigovane za efekte Mesečevih i Sunčevih plima. Osna komponenta EAMF je dominantna u efektima zonalnih vetrova. Najznačajnije promene su sezonske varijacije (sa godišnjim i polu-godišnjim periodama) i kratko-periodične varijacije (sa grupom perioda između 40 i 70 dana). Nalazi se siguran dokaz da postoji bliska korelacija između Sunčeve aktivnosti i globalne atmosferske cirkulacije (Đurović, 1987).

Situacija je mnogo manje očigledna kad se razmatra ekscitacija polarnog kretanja; prenosna funkcija između ekvatorskih komponenata atmosfere EAMF i polarnog kretanja je frekventno-zavisna sa rezonancom na Čendlerovoj frekvenci, stoga je teško rešiti koja bi kombinacija vetar i pritisak člana najbolje predstavljala atmosfersku ekscitaciju polarnog kretanja. Dominantan je član pritiska; efekti vetrova su isčezavajući (Munk i MacDonald, 1960), kasnije je pokazano da nisu nule (Barnes i dr., 1983) ali ponovo se ne preporučuju za korišćenje (Eubanks i dr.,1985, 1987) što je posledica nezadovoljavajućeg kvaliteta podataka i visine posmatračkog šuma. Problematičan je i član pritiska; njegov iznos računat sa i bez korekcije "invertnog barometra" dat je različito i, nalaze se mišljenja da primena njegove korekcije je pogodna samo na severnoj hemisferi (Eubanks i dr., 1987). Mada je godišnji period dominantan i u polarnom kretanju i u ekvatorskim komponentama EAMF, amplitude i faze su različite. Ako su promene u LOD (length-of-day) date, uglavnom, članom vetra, polarno kretanje je uslovljeno, obrnuto, uglavnom članom pritiska (Barnes i dr., 1983). Nameću se sledeća ak-

tuelna pitanja:

1. Koja je reakcija okeana na promene vazdušnog pritiska i da li je hipoteza invertnog barometra dovoljna da opiše ovu reakciju?

2. Šta je uloga vetar člana u polarnom kretanju i da li su prezentirani podaci vetar člana za korišćenje dovoljne gustine i preciznosti za račun polarnog kretanja?

3. Da li je model Zemlje korišćen od Barnesa i dr. adekvatan za računanje reakcije Zemlje na atmosfersku ekscitaciju?

4. Da li je atmosferska cirkulacija sama dovoljna da ekscituje posmatrano polarno kretanje ili ćemo tražiti druge izvore ekscitacije?

U ovom radu potražićemo odgovore na neka od ovih pitanja. Pokušaji korišćenja računatih atmosferskih EAMF za numeričku integraciju polarnog kretanja, napravljeni napr. od Barnes i dr. (1983) ili Dutton i Fallon (1985) uspevaju da fituju posmatrano polarno kretanje samo za vrlo limitirane vremenske intervale (do jednog Čendlerovog perioda); posle kojeg obe krive divergiraju vrlo rapidno. Barnes i dr. (1983) koristili su osno-simetričan model elastične, bezokeanske Zemlje za izvođenje atmosferskih EAMF. Vondrak (1987) koristi realniji model Zemlje: tro-osno telo, fluidno jezgro, visko-elastični omotači statički okean za ocenu Zemljine reakcije na stmosfersku ekscitaciju. Mi ćemo u ovom radu koristiti novi model Zemlje: dvo-osno telo, fluidno jezgro, visko-elastični omotač, dinamički okean (za koji pretpostavljamo da uzrokuje frekventnu modulisanost Čendlero vog kretanja) s ciljem da poboljšamo fitovanje između posmatranog i integraljenog polarnog kretanja na dužim vremenskim intervalima. Na ovakav izbor modela navela nas je Karterova (Carter, 1981) sugestija da model Zemlje sa dinamičkim okeanom može znatno da se razlikuje od modela sa statičkim okeanima i njegov zaključak da dinamička reakcija okeana na polarno kretanje može prouzrokovati promene i u amplitudi i u frekvenci (pogledati tabl.2; Carter, 1981). Prilog ovome su rezultati do kojih je došao Vondrak (1985, 1988), analizirajući posmatrano po-

larno kretanje za dugi vremenskoi interval (1860-1985), i njegov zaključak da je Čendlerova frekvenca nelinearna funkcija amplitude polarnog kretanja. Iz naše ranije spektralne analize ILS (International Latitude Service) koordinata pola (Yumi i Yokoyama, 1980) našli smo da je Čendlerov pik dvostruk (Pejović, 1983) i da je amplituda Čendlerovog kretanja harmonijska funkcija vremena, a da faza Čedlerove nutacije ima dobru stabilnost, izuzimajući iznenadnu i neobjašnjenu njenu rapidnu promenu u periodu 1920-1940 (Pejović,1985). Vondrakov (Vondrak, 1988) postupak smo ponovili na homogenoj seriji ILS i potvrdili njegove rezultate o nelinearnoj zavisnosti Čendlerove frekvence od totalne amplitude polarnog kretanja. Vondrakovi rezultati su potvrđeni i drugom metodom od Durovića (Đurović, 1988) na ILS koordinatama pola. Na osnovu ovoga uvešćemo novu ekscitacionu funkciju dinamičkog okeana s ciljem da poboljšamo modelovanje Zemlje.

Ceo rad je podeljen na 7 glava. U uvodnom delu ukratko je izloženo stanje problema i cilj rada. U glavi 1 diskutovani su dosadašnji rezultati i potvrđena je polazna hipoteza o nelinearnoj zavisnosti Čendlerove frekvence od totalne amplitude polarnog kretanja. Glave 2 i 4 daju kratku osnovu (baziranu na Liuvilovim jednačinama) teorije Zemljine rotacije kao deformabilnog tela. Ekscitacije koje dolaze od različitih slojeva Zemlje (fluidno jezgro, visko-elastični omotač i okean) su procenjene i reakcija ovog realnijeg modela Zemlje na atmosfersku ekscitaciju je izvedena. Takođe je prikazano koliko nelinearna reakcija okeana na polarno kretanje može usloviti slobodno Čendlerovo kretanje pola čija frekvenca je amplitudno zavisna. Treća glava prikazuje tzv. "funkcije efektivnog ugaonog momenta" atmosfere računate iz globalnih meteoroloških podataka i njihove relacije sa ekscitacionom funkcijama prezentiranim u Liuvilovim jednačinama. Podaci koji su korišćeni u radu ukratko su opisani u glavi 5. Glava 6 je glavni deo ovog rada. U njoj su izloženi novi rezultati do kojih se došlo pri istraživanju. Sedma glava sumira sve zaključke ovog rada.

1. POLAZNA HIPOTEZA

Pre nego što pređemo na uvođenje novih ekscitacionih funkcija u Vondrakov (1987) model Zemlje u cilju njegovog poboljšanja, preispitaćemo polaznu hipotezu o nelinearnoj zavisnosti Čendlerove frekvence od totalne amplitude polarnog kretanja.

Promenljivost Čendlerove periode kontraverzna je skoro čitav vek. Ne mali broj autora, uključujući i Čendlera samog, uvodili su hipoteze višestrukosti ili varijabilnosti perioda (Chandler, 1892; Melchior, 1957; Sekiguchi, 1972; Gaposchkin, 1972; Carter, 1981; Dickman, 1981; Pejović, 1983; Vondrak, 1985, 1988).

Okubo (1982) u svojoj opširnoj analizi posvećenoj problemu Čendlerove frekvence pokušava da razreši postojeću kontraverznost. Dajemo Tablicu 1.1, uzetu iz Okubovog rada (Okubo,1982).

Tablica 1.1

Istorijske kontraverzije varijacije Čendlerovog perioda

Promenljivost(višestrukost)	*Nepromenljivost(jedinstvenost)*
Chandler (1892)	Newcomb (1892)
Kimura (1918)	
Hattori (1949)	
Melchior (1957)	Munk i MacDonald (1960)
Colombo i Šhapiro (1968)	
Gaposchkin (1972)	Pedersen i Rochester (1972)
Sekiguchi (1972)	
Sekiguchi (1976)	Ooe (1978)
Carter (1981)	

Analizom amplitude i faze Čendlerovog kretanja (Pejović, 1985) dobijene iz homogene ILS serije koordinata pola za period 1900-1980 (Yumi i Yokoyama, 1980) potvrdili smo rezultate Ginoa (Guinot, 1972, 1982) i Dikmana (Dickman, 1981) da je amplituda Čendlerove nutacije harmonijska funkcija vremena, dok, uprkos tome, faza Čendlerovog kretanja ima dobru stabilnost osim u periodu 1920-1940. Rapidna promena faze u ovom intervalu je jedinstven i neobjašnjen slučaj u do sada raspoloživim serijama koordinata pola. Ovi rezultati do kojih je prvi došao Gino (Guinot, 1972, 1982), potvrđeni su od Vondraka (Vondrak, 1985, 1988) iz druge posmatrane serije polarnog kretanja. Naime, Vondrak (1988) je vršio analizu polarnog kretanja na znatno dužem (50%) vremenskom intervalu (125 godina) od intervala ILS podataka (80 godina). Analizirao je interval (1860-1985) nehomogenih podataka koji se sastoji od (Vondrak, 1988):

a) Polarno kretanje u intervalu 1860-1890 koje je izvela Rykhlova (1970) iz posmatranja apsolutne deklinacije na Pulkovu, Vašingtonu i Griniču.

b) U intervalu 1890-1962 koordinate pola izveo je Fedorov i dr. (1972) iz svih raspoloživih posmatranja latituda .

c) U intervalu 1962-1972 koordinate pola određene u BIH-u (Bureau International de l'Heure) na osnovu optičke astrometrije.

d) U intervalu 1973-1985 koordinate pola određene sa BIH-ovim kombinovanim rešenjem optičke astrometrije i modernih tehnika.

Na ovako dugom vremenskom intervalu, Vondrak (1985, 1988) osim što potvrđuje Ginoove (Guinot, 1972, 1982) rezultate, nalazi, pored rapidne promene Čendlerove faze u intervalu 1920-1940, još jednu u intervalu 1870-1890, jer je analizirao i podatke pre 1900. Ove rapidne promene faze se podudaraju sa minimumima totalne amplitude polarnog kretanja (sl. 7, Vondrak, 1985). Na osnovu Karterove (Carter, 1981) sugestije o postojanju funkcijske zavisnosti Čendlerove frekvence od totalne amplitude polarnog kretanja, Vondrak (1985, 1988) računa koeficijent korelacije između Čendlerove faze i integrala amplitude totalnog polarnog kretanja i dobija za koefi-

cijent korelacije iznos 0.986. Zatim, računajući na uzastopnim nezavisnim šesto-godišnjim intervalima vrednost Čendlerove frekvence iz diferencijala Čendlerove faze, po jednačini (Vondrak, 1988):

$$\sigma = \sigma_o - \frac{\partial \varphi_c}{\partial t}$$

i predstavljajući je sa amplitudom polarnog kretanja na sl. 3 (Vondrak, 1988) nalazi da je Čendlerova frekvenca eksponencijalna funkcija totalne amplitude polarnog kretanja. Ova zavisnost je empirijski dobijena. Karter je našao da funkcijska zavisnost između Čendlerove frekvence i amplitude polarnog kretanja postoji, predpostavljajući da je linearna, matematički je izražava jednačinom 1 (Carter, 1981).

Okubo (1982) u svom radu ne citira poznate radove Ginoa (Guinot, 1972, 1982) niti analizira amplitudu i fazu Čendlerovog kretanja. Koristi kao osnovni metod trenutnu frekventnu analizu (TFA) za traženje vremenski promenljivog spektra za nestacionarnu vremensku seriju (uzimajući takvom seriju polarnog kretanja). U osnovi metode TFA nalazi se metod maksimalne entropije (MME) koji primenjuje na kliznom intervalu od 6.45 godina duž celog intervala od 18 godina dobijajući na taj način trenutnu vrednost amplitude i frekvence Čendlerove nutacije za svakih 0.05 godina. Rezultati su predstavljeni kao trodimenzionalni spektri. Da bi ocenio nivo poverenja metode TFA primenjuje je na IPMS seriju polarnog kretanja (1962-1978, period bez rapidnih promena Čendlerove faze) i nalazi da su fluktuacije Čendlerove frekvence od 1-2% beznačajne za TFA. Zatim, nalazi da odstupanja trenutne od srednje vrednosti Čendlerove frekvence iznose 1.4% za IPMS i BIH podatke (opet interval posle 1962 godine) a ILS podatke (1900-1970, period sa rapidnom promenom Čendlerove faze između 1920-1940) ne podvrgava analizi TFA većih odbacuje kao podatke sa visokim nivoom šuma.

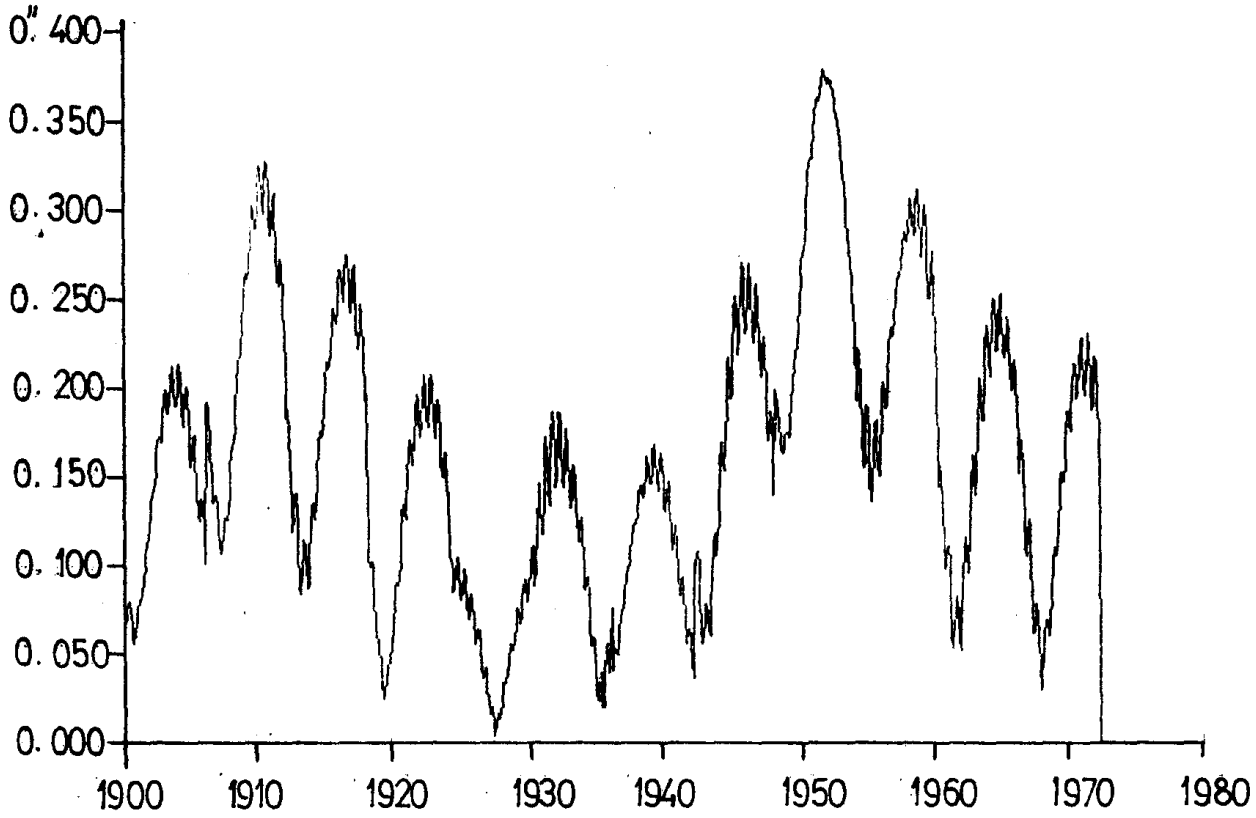
Pošto veći broj autora (Melchior, 1957; Colombo i Shapiro, 1968; Sekiguchi, 1972,1976; Gaposchkin, 1972, Carter 1981) nalazi da je Čendlerov

period ili višestruk ili promenljiv u vremenu a Okubo (1982) odbacuje njihove rezultate upoređujući ih sa svojim rezultatima na sintetizovanim podacima; stoga pogledajmo kako Okubo simulira podatke. Naime, on podatke ne simulira direktno, već prvo simulira spektar sintetizovanih podataka iz kojeg inverznom Furijeovom transformacijom dobija simulirane podatke. I ako Okubo u simulirani spektar uvodi konstantnu Čendlerovu frekvencu, on uvodi i amplitudnu modulaciju Čendlerovog pika, što se vidi iz njegovih jednačina za simulirani spektar (pogledati prilog, Okubo 1982) kao i iz simuliranih podataka, samo Čendlerovog kretanja, predstavljenih na slici A1 (Okubo, 1982). Na slici A2 Okubo daje simulirane ILS podatke polarnog kretanja sa amplitudno modulisanim Čendlerovim kretanjem sa već pomenute slike A1. Jasno je da se ovi podaci razlikuju od simuliranih podataka ILS sa konstantnom Čendlerovom periodom (435 dana) i srednjom amplitudom Čendlera od 0.15" predstavljenih na slikama P1 i P2 za x i y koordinatu redom (u prilogu ovog rada). U prilogu su dati i posmatrani ILS podaci na slikama P3 i P4. Sam Okubo zaključuje da je Čendlerovo kretanje nestacionarno i da je srednja vrednost fluktuirajuće Čendlerove periode 435 dana. Srednja vrednost Čendlerove amplitude je 0.15" a u simulirani spektar je uvodi kao slučajno fluktuirajuću veličinu, odnosno modulisanu. Stoga se nameće zaključak da ono što je za Okuba slučajno fluktuirajuće Čendlerovo kretanje (simulirano kao amplitudno modulisano), za Kartera (Carter, 1981) i druge autore je frekventno modulisano, s tim što Karter traži uzroke ove modulisanosti u ekscitacionim funkcijama, sugerišući da bi uzrok frekventne modulisanosti Čendlerovog kretanja mogao biti dinamički okean. Problem postaje jasniji Vondrakovim rezultatima o funkcijskoj zavisnosti Čendlerove frekvence i amplitude fenomena. Naime, Vondrak (1985, 1988) je dao frekventnu modulisanost Čendlerovog kretanja našavši da je Čendlerova frekvenca eksponencijalna funkcija totalne amplitude polarnog kretanja, računajući trenutnu Čendlerovu frekvencu iz diferencijala Čendlerove faze (što pokazuje da su rapidne promene faze uzete u

obzir).

Mi smo ponovili Vondrakov (Vondrak, 1985, 1988) postupak na homogenoj, metodom Vondraka (Vondrak, 1969, 1977) izravnatoj, ILS seriji koordinata pola i potvrdili njegove rezultate. Totalna amplituda polarnog kretanja predstavljena na slici 1.1 saglasna je Vondrakovim rezultatima na slici 4 (Vondrak, 1985). Trenutnu Čendlerovu frekvencu računatu iz diferencijala Čendlerove faze predstavili smo zajedno sa totalnom amplitudom polarnog kretanja za svaki nezavisni šesto godišnji interval na slici 1.2 i u tablici 1.2. Ova slika potvrđuje Vondrakove rezultate, date na sl. 3 (Vondrak, 1988), da je Čendlerova frekvenca nelinearna funkcija amplitude fenomena. Vondrakove rezultate potvrdio je Đurović(1988) analizirajući ILS koordinate pola drugom metodom. Iz tablice 1.2 i sa slike 1.2 se vidi da je najveća promena faze bila u periodu 1920-1940 što odgovara minimumu totalne amplitude polarnog kretanja.

Slika 1.1. Totalna amplituda polarnog kretanja



Tablica 1.2

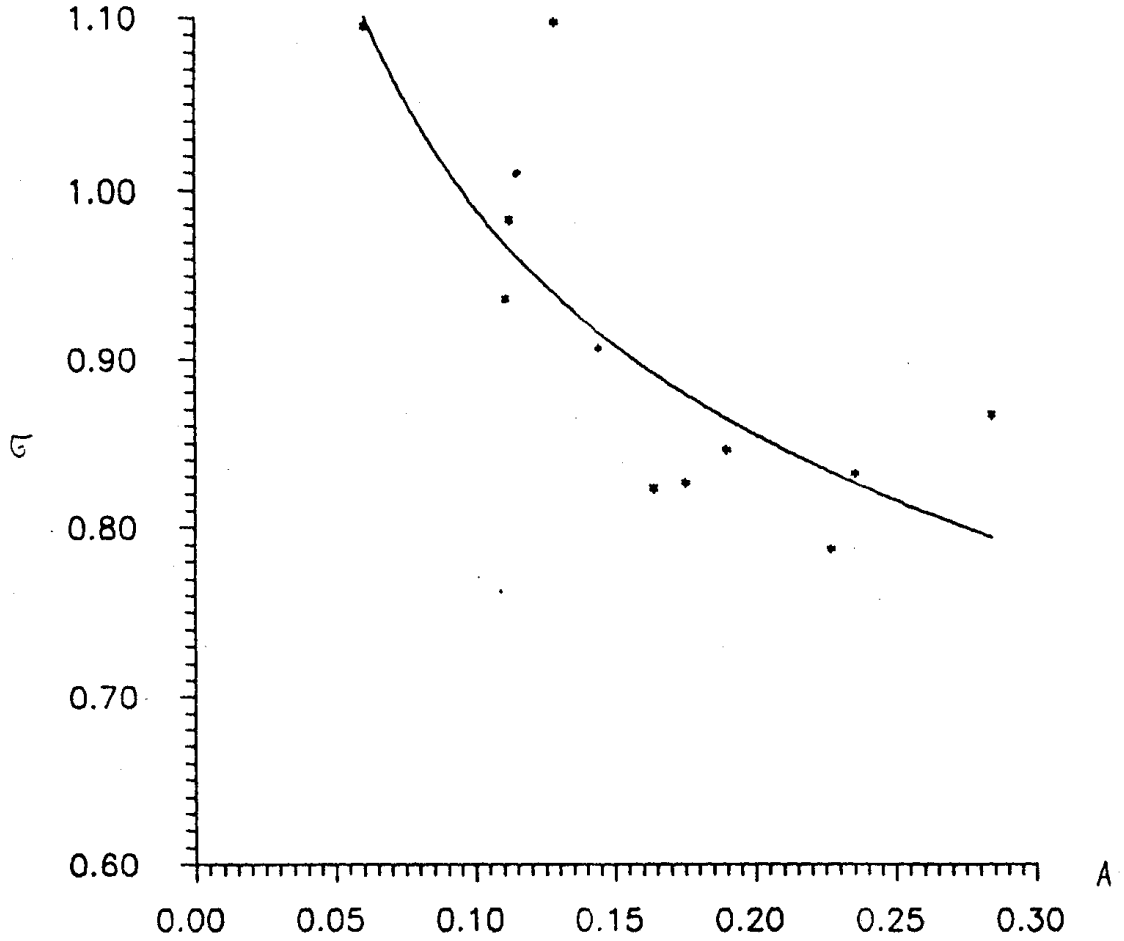
Na uzastopnim šestogodišnjim intervalima ILS podataka računata je amplituda A polarnog kretanja i data je u prvoj koloni a u drugoj koloni data je Čendlerova frekvencija σ računata iz diferencijala Čendlerove faze.

$A\ [''']$	$\sigma\ [rad\ god^{-1}]$
0.1441	0.9065
0.2266	0.7874
0.1898	0.8459
0.1282	1.0974
0.0603	1.0952
0.1120	0.9830
0.1108	0.9358
0.1752	0.8263
0.2839	0.8669
0.2352	0.8319
0.1639	0.8232

Na osnovu naših rezultata ovde izloženih, koji potvrđuju Vondrakove (Vondrak, 1985, 1988) zaključke, smatramo da smo proverili polaznu hipotezu o nelinearnoj zavisnosti Čendlerove frekvence od totalne amplitude polarnog kretanja.

Izvršili smo spektralnu analizu ILS podataka polarnog kretanja, metodom brze Furijeove transformacije (FFT), na dva podintervala i to: 1900-1920 i 1940-1980 izostavljajući interval 1920-1940 u kojem je došlo do rapidne promene Čendlerove faze. Rezultati su predstavljeni na slikama P5, P6 za x, y koordinatu redom za prvi podinterval (1900-1920) i slikama P7, P8 za drugi podinterval (1940-1980) u prilogu. Na slikama P9 i P10 (u prilogu) za x i y koordinatu predstavljen je spektar dobijen metodom FFT na celokupnom intervalu ILS podataka (1900-1980) što predstavlja naše ranije dobijene rezultate u magistarskom radu (Pejović, 1984). Iz upoređenja ovih slika vidimo da se na oba podintervala dobija jedinstven Čendlerov pik a da se na celokupnom intervalu (sl. P9, P10) dobija rascepljen. Odavde sledi da je ras–

Slika 1.2. Nelinearna zavisnost između amplitude polarnog kretanja i Čendlerove frekvence σ



cepljenost posledica uzimanja u obzir, pri spektralnoj analizi, i intervala (1920-1940) sa rapidnom promenom faze, jer promena faze dovodi do promene frekvence. Otuda sledi zaključak da se ne radi o dve, već o jednoj frekventno modulisanoj Čendlerovoj nutaciji.

2. DINAMIKA ZEMLJINE ROTACIJE

2.1 ROTACIJA ZEMLJE

Daćemo kratak pregled osnovnih dinamičkih jednačina koje se mogu naći na pr. kod Munk i MacDonald (1960), Lambeck (1980), Barnes i dr. (1983).

Posmatrajmo koordinatni sistem, za telo fiksiranih, osa $x_i (i = 1,2,3)$, koji rotira ugaonom brzinom ω_i u odnosu na nepokretni sistem u prostoru X_i i u nekom trenutku se poklapa s njim. Ojlerove dinamičke jednačine koje opisuju ovo kretanje su:

$$\frac{dH_i}{dt} + \epsilon_{ijk}\omega_j H_k = L_i. \tag{1}$$

gde su L_i - komponente momenta spoljnih sila,

H_i - komponente momenta količine kretanja,

$\frac{d}{dt}$ - vremenski izvod u okviru rotacije, a

ϵ_{ijk} - tenzor, određen na sledeći način:

$$\epsilon_{ijk} = \begin{cases} 0, & \text{ako su mà koja dva indeksa jednaka;} \\ +1, & \text{ako je } ijk \text{ parna permutacija indeksa 1,2,3;} \\ -1, & \text{ako je } ijk \text{ neparna permutacija indeksa 1,2,3.} \end{cases}$$

Koristi se sumaciona konvencija za ponovljene indekse.

Dobro je podeliti moment količine kretanja na dva dela:

$$H_i(t) = I_{ij}(t)\omega_j(t) + h_i(t), \tag{2}$$

gde je

$$I_{ij} = \int_v \rho(x_k x_k \delta_{ij} - x_i x_i)dv, \tag{3}$$

- promenljivi tenzor inercije definisan zapreminskim integralom, ρ - gustina a δ_{ij} - Kronekerov simbol ($\delta_{ij} = 1$ za $i = j$ i $\delta_{ij} = 0$ za $i \neq j$). Drugi deo jednačine (2) označava relativni moment količine kretanja:

$$h_i = \int_v \rho \epsilon_{ijk} x_j u_k dv, \tag{4}$$

uslovljen kretanjem u_i u odnosu na sistem x_i. Za apsolutno tvrdo telo, koje rotira ugaonom brzinom ω_i, brzina bilo koje čestice u prostoru je $\frac{dX_i}{dt} = \epsilon_{ijk}\omega_j X_k$. Za deformabilno telo možemo iz ω_i izdvijiti veličinu $\bar{\omega}_i$ za koju je minimalan izraz

$$\int_v \rho(\frac{dX_i}{dt} - \epsilon_{ijk}\omega_j X_k)^2 dv. \tag{5}$$

Za ose $\bar{\omega}_i$ možemo pokazati, da za $h_i = 0$, $\bar{\omega}_i$ postaje ugaona brzina srednje ose. Ove ose su nazvane Tiseranovim (Tisserand) srednjim osama.

Stavljajući jednačinu (2) u jednačinu (1) dolazimo do jednačine (6):

$$\frac{d}{dt}(I_{ij}\omega_j + h_i) + \epsilon_{ijk}\omega_j(I_{kl}\omega_l + h_k) = L_i. \tag{6}$$

Ovu jednačinu je prvi izveo Liuvil (Liouville, 1858) i po njemu je nazvana jednačinom Liuvila. Sva sledeća razmatranja različitih neregularnosti Zemljine rotacije vezana su za rešavanje jednačine (6).

Ako mi dalje orijentišemo koordinatni sistem x_i u pravcu glavnih osa inercije Zemlje, možemo izraziti tenzor inercije sa:

$$I_{ij}(t) = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix} + \Delta I_{ij}(t). \tag{7}$$

Na sličan način, pošto rotacija Zemlje odstupa samo malo od rotacije oko polarne ose figure, pišemo vektor ugaone brzine kao:

$$(\omega_1, \omega_2, \omega_3) = (m_1, m_2, 1 + m_3)\Omega, \tag{8}$$

gde je

Ω rotaciona brzina Zemlje, a

$m_1 = \frac{\omega_1}{\Omega}$,

$m_2 = \frac{\omega_2}{\Omega}$,

$m_3 = \frac{\omega_3}{\Omega} - 1$

su male veličine. U jednačini (7) C i A su, redom, polarni i ekvatorski glavni momenti inercije nedeformisane Zemlje. Elementi $\Delta I_{ij}(t)$ predstavljaju odstupanja elastične forme Zemlje od krutog modela, vrlo su mali u poređenju sa glavnim momentima inercije A i C. U ΔI_{ij} su sadržane promene usled prisustva atmosfere,fluidnog jezgra, rotacione deformacije omotača, površinske teretne deformacije, prisustvo okeana. Za Zemlju i atmosferu je:

$$|\Delta I_{ij}| \ll C, |h_i| \ll \Omega C, |m_i| = o(10^{-}7), |\dot{m}_i| \ll \Omega$$

Smenjujući jednačinu (7) u jedačinu (6) i zanemarujući kvadrate i proizvode malih veličina dobijamo jednačine Liuvila u linearnoj formi:

$$A\dot{\omega}_1 + (C - A)\Omega\omega_2 + \Delta\dot{I}_{13}\Omega - \Delta I_{23}\Omega^2 + \dot{h}_1 - \Omega h_2 = L_1$$

$$A\dot{\omega}_2 + (A - C)\Omega\omega_1 + \Delta\dot{I}_{23}\Omega + \Delta I_{13}\Omega^2 + \dot{h}_2 + \Omega h_1 = L_2. \tag{9}$$

$$C\dot{\omega}_3 + \Delta\dot{I}_{23}\Omega + \dot{h}_3 = L_3$$

Ove jednačine mogu biti napisane u jednostavnoj formi koju koristi na pr. Munk i MacDonald (1960), Lambeck (1980), Barnes i dr. (1983):

$$\begin{aligned} m_1 - \frac{\dot{m}_2 A}{\Omega(C-A)} &= \Psi_1 \\ m_2 + \frac{\dot{m}_1 A}{\Omega(C-A)} &= \Psi_2 \\ \dot{m}_3 &= \dot{\Psi}_3 \end{aligned} \tag{10}$$

gde je $\sigma_r = \frac{C-A}{A}\Omega$ Ojlerova frekvenca tj. frekvenca slobodne nutacije krutog tela. Za Zemlju je $\frac{C-A}{A} \approx 300$, a odgovarajuća perioda slobodne

nutacije je oko 10 meseci. Posmatrana Čendlerova frekvenca σ odgovara periodi od oko 14 meseci (videti na pr. Čhandler, 1891a, 1891b). Razmimoilaženje između teorije i posmatranja koje se ispoljilo u tim dvema periodama našlo je ubrzo svoje tumačenje od Njukomba (Newcomb, 1891). Razlika teorijske i posmatrane periode slobodne nutacije je posledica odstupanja elastičnog oblika Zemlje od čvrstog modela.

U jednačinama (10) Ψ_i su poznate kao ekscitacione funkcije i glase:

$$\Psi_1 = \frac{\Omega^2 \Delta I_{13} + \Omega \Delta \dot{I}_{23} + \Omega h_1 + \dot{h}_2 - L_2}{\Omega^2 (C - A)}$$

$$\Psi_2 = \frac{\Omega^2 \Delta I_{23} - \Omega \Delta \dot{I}_{13} + \Omega h_2 - \dot{h}_1 + L_1}{\Omega^2 (C - A)}, \tag{11}$$

$$\Psi_3 = \frac{-\Omega^2 \Delta I_{33} - \Omega h_3 + \Omega \int_0^t L_3 d\tau}{\Omega^2 C}$$

gde $\frac{d}{dt}$ u jednačini (6) zamenjeno sa $(\cdot)$.

Prve dve jednačine u sistemu jednačina (11) možemo da napišemo u konvencionalnoj kompleksnoj notaciji (sa $i = \sqrt{-1}$) kao:

$$\Psi = \Psi_1 + i\Psi_2 = \frac{\Omega^2 \Delta \mathbf{I} - i\Omega \Delta \dot{\mathbf{I}} + \Omega \mathbf{h} - i\dot{\mathbf{h}} + i\mathbf{L}}{\Omega^2 (C - A)}$$

koristeći:

$$\Delta \mathbf{I} \equiv \Delta I_{13} + i \Delta I_{23},$$

$$\mathbf{h} = h_1 + i h_2$$

$$\mathbf{L} = L_1 + i L_2.$$

Stavljajući $\mathbf{m} = m_1 + i m_2$ i $\dot{\mathbf{m}} = \dot{m}_1 + i \dot{m}_2$ prve dve jednačine u sistemu jednačina (10) mogu biti napisane u kompleksnoj notaciji kao:

$$\mathbf{m} + i\dot{\mathbf{m}} \frac{A}{\Omega (C - A)} = \Psi. \tag{12}$$

Poznato je da sistem jednačina (10) može da se reši ako delove ekscitacionih funkcija, sa desnih strana ovih jednačina, koji zavise od m_1, m_2 i m_3 razdvojimo od drugih delova i stavimo u leve strane jednačina (10).

2.2 FLUIDNO JEZGRO

Efekti homogenog fluida jezgra na rotaciju čvrstog omotača dati su na pr. kod Poincare (1910), Melchior (1978), Moritz (1980), Vondrak (1987):

$$A\dot{\omega}_1 + (C-A)\Omega\omega_2 + A_c\dot{V}_1 - A_c\Omega V_2 = L_1$$

$$B\dot{\omega}_2 + (A-C)\Omega\omega_1 + A_c\dot{V}_2 + A_c\Omega V_1 = L_1$$

$$C\dot{\omega}_3 + C_c\dot{V}_3 = L_3 \qquad (13)$$

$$A_c(\dot{\omega}_1 + \dot{V}_1) - C_c\Omega V_2 = 0$$

$$A_c(\dot{\omega}_2 + \dot{V}_2 + C_c\Omega V_1 = 0$$

$$C_c(\dot{\omega}_3 + \dot{V}_3) = 0,$$

gde su A_c, C_c glavni momenti inercije jezgra i V_i komponente vektora rotacije jezgra u odnosu na omotač. Upoređujući jednačine (9), (11) i (13) uzećemo one delove ekscitacionih funkcija koji izražavaju uticaj jezgra:

$$\Delta\Psi_1^c = \frac{A_c\dot{V}_2 + A_C\Omega V_1}{\Omega^2(C-A)} \approx \frac{-A_c}{\Omega(C-A)}\dot{m}_2$$

$$\Delta\Psi_2^c = \frac{A_c\dot{V}_1 - B_c\Omega V_2}{\Omega^2(C-A)} \approx \frac{A_c}{\Omega(C-A)}\dot{m}_1 \tag{14}$$

$$\Delta\dot{\Psi}_3^c = \frac{-C_c\dot{V}_3}{\Omega C} \approx \frac{C_c}{C}\dot{m}_3$$

Prve dve jednačine ovog sistema mogu biti napisane u kompleksnoj notaciji kao:

$$\Delta\Psi^c = i\dot{m}\frac{A_c}{\Omega(C-A)}. \tag{15}$$

2.3 ROTACIONA DEFORMACIJA OMOTAČA

Promene tenzora inercije koje odgovaraju elastičnim rotacionim deformacijama (Vondrak, 1984, 1987) su:

$$\Delta I_{12} = 0,$$

$$\Delta I_{13} = \frac{k\Omega^2 R^5}{3G}m_1$$

$$\Delta I_{23} = \frac{k\Omega^2 R^5}{3G}m_2$$

$$-2\Delta I_{11} = -2\Delta I_{22} = \Delta I_{33} = \frac{4k\Omega^2 R^5}{9G}m_3,$$

gde je k Lovov broj, R srednji poluprečnik Zemlje i G gravitaciona konstanta. Odavde imamo ekscitacione funkcije koje odgovaraju rotacionoj deformaciji:

$$\Delta\Psi_1^{rd} = \frac{\Delta I_{13}\Omega + \Delta\dot{I}_{23}}{\Omega(C-A)} = \frac{k\Omega^2 R^5}{3G(C-A)}\left(m_1 + \frac{\dot{m}_2}{\Omega}\right)$$

$$\Delta\Psi_2^{rd} = \frac{\Delta I_{23}\Omega - \Delta\dot{I}_{13}}{\Omega(C-A)} = \frac{k\Omega^2 R^5}{3G(C-A)}\left(m_2 - \frac{\dot{m}_1}{\Omega}\right)$$

$$\Delta\dot{\Psi}_3^{rd} = \frac{-\Delta\dot{I}_{33}}{C} = \frac{-4k\Omega^2 R^5}{9GC}\dot{m}_3$$

i, ako dalje uvedemo za viskoznost faktor ograničenja Q, kao kod Lambecka (1980, str. 206):

$$\begin{pmatrix}\tilde{m}_1\\ \tilde{m}_2\end{pmatrix} = \begin{pmatrix}\cos\varepsilon & \sin\varepsilon\\ -\sin\varepsilon & \cos\varepsilon\end{pmatrix}\begin{pmatrix}m_1\\ m_2\end{pmatrix}$$

gde za male uglove imamo $\cos\varepsilon \approx 1, \sin\varepsilon \approx Q^{-1}$, dobijamo sledeće izraze:

$$\tilde{m}_1 = m_1 + Q^{-1}m_2$$

$$\tilde{m}_2 = m_2 - Q^{-1}m_1$$

Ako ovo uvrstimo u prethodni sistem jednačina ekscitacionih funkcija , one će glasiti:

$$\Delta\Psi_1^{rd} = \frac{k\Omega^2 R^5}{3G(C-A)}[m_1 + Q^{-1}m_2 + \frac{\dot{m}_2 - Q^{-1}\dot{m}_1}{\Omega}]$$

$$\Delta\Psi_2^{rd} = \frac{k\Omega^2 R^5}{3G(C-A)}[m_2 - Q^{-1}m_1 - \frac{\dot{m}_1 + Q^{-1}\dot{m}_2}{\Omega}]. \qquad (16)$$

$$\Delta\dot{\Psi}_3^{rd} = \frac{-4k\Omega^2 R^5}{9GC}\dot{m}_3$$

Prve dve jednačine ovog sistema mogu biti napisane u kompleksnoj notaciji, koja glasi:

$$\Delta\Psi^{rd} = \Delta\Psi_1^{rd} + i\Delta\Psi_2^{rd} = \frac{k\Omega^2 R^5}{3G(C-A)}(\mathrm{m} - i\frac{\dot{\mathrm{m}}}{\Omega})(1 - iQ^{-1})$$

ili, uvodeći sekularni Lovov broj (Lambeck, 1980)

$$k_s = \frac{3G(C-A)}{r^5\Omega^2}$$

prethodna jednačina postaje:

$$\Delta\Psi^{rd} = \frac{k}{k_s}(\mathrm{m} - i\frac{\dot{\mathrm{m}}}{\Omega})(1 - iQ^{-1}). \qquad (17)$$

2.4 DINAMIČKI OKEAN

Na osnovu Vondrakovih (Vondrak, 1988) i naših (Glava 1) rezultata o nelinearnoj zavisnosti Čendlerove frekvence od totalne amplitude polarnog kretanja i sugestije Kartera (Carter, 1981) da bi uzrok ove zavisnosti mogao biti neravnotežni okean, uvešćemo ekscitacionu funkciju dinamičkog okeana, u kompleksnoj notaciji, u ovom obliku:

$$\Delta\Psi^{o} = \mathrm{m}\xi, \tag{18}$$

gde je ξ faktor koji zavisi od amplitude polarnog kretanja. Jednačinom (18) uvodimo novu okeansku funkciju, jer za statički okean imamo ekscitacione funkcije date kod Lambeka (Lambeck,1980; str. 215):

$$\begin{cases} \Delta\Psi_1 \\ \Delta\Psi_2 \end{cases} = \frac{3}{10}\frac{\rho_w}{\rho}\frac{1}{k_s}(1+k-k') \begin{cases} m_1A_1 + m_2A_2 \\ m_1B_1 + m_2B_2 \end{cases}$$

gde su ρ_w i ρ gustina okeana i Zemlje redom, k_s, k i k' su Lovovi brojevi. A m_1, m_2 su koordinate pola duž nultog meridijana i $90^{o}E$; konstante A_1, A_2, B_1, B_2 zavise od nesimetričnog rasporeda okeana na Zemlji. Koristeći Lambekove formule na str.143 i razvoj okeanske funkcije na str. 53 (Lambeck, 1980), imamo:

$A_1 = 1.724$,

$A_2 = B_1 = -0.025$,

$B_2 = 1.406$

Na osnovu ovoga nalazimo za ravnotežni okean simetrično raspoređen na Zemlji da je, približno, faktor $\xi = 0.064$.

Potrebno je za dinamički okean da nađemo zavisnost faktora ξ od amplitude A polarnog kretanja.

Koristeći rezultate Vondraka (1988), koji je iz analize posmatranog polarnog kretanja, za dugi vremenski interval (1860 - 1985), našao eksponencijalnu zavisnost između Čendlerove frekvence σ i amplitude polarnog kre-

tanja A (grafički predstavljene na slici 3; Vondrak, 1988), dovoljno je za faktor ξ da stavimo:

$$\xi = 0.064(1 - e^{-20A}), \tag{19}^{*}$$

što dopušta da ekscitaciona funkcija (18), za dinamički okean postaje:

$$\Delta\Psi^{o} = 0.064\mathbf{m}(1 - e^{-20A}). \tag{19}$$

Ova ekscitacija (19) je vrlo blizu ekscitacije za statički okean:

$$\dot{\Delta\Psi}^{o}_{s} = 0.064\mathbf{m}, \tag{20}$$

i razlikuju se samo za vrlo male amplitude a jednake su za amplitude veće od 0."2.

Za treću komponentu okeanske ekscitacije $\Delta\Psi^{o}_{3}$, uzetu od Jodera (Yoder i dr., 1981) imamo:

$$\dot{\Delta\Psi}^{o}_{3} = \frac{-4\Omega^2 R^5}{9GC} 0.040\dot{m}_3. \tag{21}$$

2.5 REŠENJE JEDNAČINE LIUVILA U SLUČAJU PROMENLJIVE ČENDLEROVE FREKVENCE

Ako sada zamenimo sve izvedene ekscitacije (15), (17) i (18) u levu stranu jednačine Liuvila (12) dobijamo:

$$\mathbf{m} + i\dot{\mathbf{m}}\frac{A}{\Omega(C-A)} - (\Delta\Psi^c + \Delta\Psi^{rd} + \Delta\Psi^o =$$

$$= \Psi - (\Delta\Psi^c + \Delta\Psi^{rd} + \Delta\Psi^o)$$

Označimo desnu stranu ove jednačine sa Ψ' , tj.

$$\Psi' = \Psi - (\Delta\Psi^c + \Delta\Psi^{rd} + \Delta\Psi^o), \tag{22}$$

i pretpostavimo, za trenutak, da je Ψ' (ekscitacija koja ne zavisi od $\mathbf{m}$ i $\dot{\mathbf{m}}$) jednaka nuli, imaćemo jednačinu:

$$\mathbf{m}[1 - \xi - \frac{k}{k_s}(1 - iQ^{-1})] + i\dot{\mathbf{m}}[\frac{A_m}{\Omega(C-A)} + \frac{k}{k_s\Omega}(1 - iQ^{-1})] = 0, \tag{23}$$

gde je A_m ekvatorski moment inercije omotača, tj. $A_m = A - A_c$.

Potražimo rešenje jednačine (23) u obliku:

$$\mathbf{m} = \mathbf{m}_0 e^{(\alpha+i\sigma)t}, \tag{24}$$

što predstavlja prigušeno kružno kretanje u pozitivnom smeru, jer je

$$\dot{\mathbf{m}} = \mathbf{m}_0(\alpha + i\sigma)e^{(\alpha+i\sigma)t}. \tag{25}$$

Smenom izraza (24) i (25) u jednačinu (23) imamo:

$$1 - \xi - \frac{k}{k_s}(1 - iQ^{-1}) + (i\alpha - \sigma)[\frac{A_m}{\Omega(C-A)} + \frac{k}{k_s\Omega}(1 - iQ^{-1})] = 0$$

Ova jednačina mora posedovati:

realni deo:

$$1-\xi-\frac{k}{k_s}-\frac{\sigma}{\Omega}\left(\frac{A_m}{C-A}+\frac{k}{k_s}\right)+\frac{\alpha Q^{-1}}{\Omega}\frac{k}{k_s}=0$$

imaginarni deo:

$$\frac{k}{k_s}\Omega^{-1}+\frac{\alpha}{\Omega}\left(\frac{A_m}{C-A}+\frac{k}{k_s}\right)+\frac{\sigma}{\Omega}\frac{k}{k_s}Q^{-1}=0$$

Iz ovih dveju jednačina imamo rešenje za σ i α:

$$\sigma\approx\Omega\frac{1-\xi-\frac{k}{k_s}}{\frac{A_m}{C-A}+\frac{k}{k_s}} \tag{26}$$

$$\alpha\approx-\frac{k}{k_s}\frac{\Omega Q^{-1}}{\frac{A_m}{C-A}+\frac{k}{k_s}},$$

gde je $\frac{k}{k_s}\,Q^{-1}$ vrlo malo u poređenju sa $\frac{A_m}{C-A}$.

Koristeći sledeće numeričke vrednosti (Vondrak, 1984):

$$\frac{k}{k_s}=0.3173$$

$$\frac{A_m}{C-A}=269.66$$

$$\Omega=6.3004\ rad\ dan^{-1}$$

dobijamo

$$\sigma=0.01593-0.0233\xi\ rad\ dan^{-1}, \tag{27}$$

$$\alpha=-0.0074Q^{-1}dan^{-1}$$

gde faktor ξ opisuje zavisnost σ od amplitude A . Ako u prvu jednačinu (27) uvrstimo izraz za ξ iz jednačine (19*), dobijamo da je Čendlerova frekvenca jednaka:

$$\sigma=0.01444+0.00149e^{-20A}, \tag{28}$$

što predstavlja odličnu aproksimaciju grafa na slici 3(Vondrak, 1988).

Ako sada nulu na desnoj strani jednačine (23) zamenimo ekscitacijom Ψ' (koja ne zavisi od m i $\dot{m}$) imamo :

$$m + i\frac{\dot{m}}{\sigma - i\alpha} = \frac{\Psi'}{1 - \xi - \frac{k}{k_e}(1 - iQ^{-1})}, \tag{29}$$

gde σ nije konstanta. Rešenje ove diferencijalne jednačine biće:

$$m = e^{\int_0^t (\alpha + i\sigma) d\tau} [m_0 - \frac{(\alpha + i\sigma)}{1 - \xi - \frac{k}{k_e}(1 - iQ^{-1})} \int_0^t \Psi' e^{-\int_0^t (\alpha + i\sigma) d\tau} d\tau]$$

ili

$$m = e^{\int_0^t (\alpha + i\sigma) d\tau} [m_0 - \frac{i\Omega}{\frac{A_m}{C - A} + \frac{k}{k_e}} \int_0^t \Psi' e^{-\int_0^t (\alpha + i\sigma) d\tau} d\tau]. \tag{30}$$

Ovo rešenje je formalno identično sa sličnim rešenjem u kojem je Čendlerova frekvenca σ konstanta, s tim što u eksponentu imamo integral vremenski promenljive Čendlerove frekvence.

Čendlerova frekvenca σ , jednačina (28), ima prvi deo ($\sigma_0 = 0.01444$) što odgovara Čendlerovoj periodi od 435 dana (Okubo, 1982) i drugi deo ($= 0.00149e^{-20A}$) zavistan od totalne amplitude polarnog kretanja A.

Za početni trenutak $t = 0$ je $m(0) = m_0$.

Što se tiče treće jednačine Liuvila iz sistema jednačina (10), njeno rešenje nalazimo ako sve izvedene ekscitacije date jednačinama (14), (16) i (21) zamenimo u jednačinu (10), imamo:

$$\dot{m}_3 [\frac{C_m}{C} + \frac{4\Omega^2 R^5}{9GC}(k + 0.040)] = \dot{\Psi}_3', \tag{31}$$

gde je C_m moment inercije omotača, tj. $C_m = C - C_c$, a

Ψ_3' označava ekscitacionu funkciju bez delova koji zavise od m_3, tj.

$$\Psi_3' = \Psi_3 - (\Delta\Psi_3^c + \Delta\Psi_3^{rd} + \Delta\Psi_3^o)$$

Numeričke vrednosti baziraju na vrednostima korišćenim kod Vondraka (Vondrak, 1984) :

$$C_m = 7.0410^{37}\ kg\ m^2, \qquad C = 8.03610^{37}\ kg\ m^2$$

$$G = 6.67210^{-11}\ kg^{-1}m^2s^{-1}, \qquad R = 6378\ km$$

$$\Omega = 6.3004\ rad\ dan^{-1}$$

Odaberimo onu vrednost Lovovog broja k za koju će posmatrani Čendlerov perid biti 435 dana (tj. $k = 0.2977$) pa jednačina (31) postaje:

$$\dot{m}_3 = 1.126\dot{\Psi}_3'. \tag{32}$$

Rešenje ove diferencijalne jednačine je :

$$-m_3 = \frac{\Delta\lambda}{\lambda_0} = -1.126\Psi_3' + const., \tag{33}$$

gde je

$\frac{\Delta\lambda}{\lambda_0}$ relativna promena trajanja dana u odnosu na srednju vrednost $\lambda_0 = 86400s$. Iz izloženog bi značilo da jednačine (30) i (33) su dobre za ekscitacije koje ne opterećuju Zemlju; ako one to čine , ekscitacione funkcije

$\Psi' = \Psi_1' + i\Psi_2'$ i Ψ_3'

treba množiti sa faktorom $1 + k' = 0.70$, gde je k' teretni Lovov broj (Barnes i dr., 1983). U slučaju izostavljanja ekscitacije

$(\Psi_3 = 0)$

i izostavljanja prigušenja

(Q^{-1})

jednačina (30) izražava Čendlerovo slobodno kretanje, koje je u ovom slučaju kružno u pozitivnom smeru.

2.6 REŠENJE JEDNAČINE LIUVILA U SLUČAJU KONSTANTNE ČENDLEROVE FREKVENCE

Rešenje jednačine Liuvila (12) za visko-elastičnu Zemlju sa statičkim okeanom možemo dobiti, na sličan način na koji smo dobili rešenje (30) za visko-elastičnu Zemlju sa dinamičkim okeanom, unoseći odgovarajuće razlike. Okeanska funkcija za statički okean, data jednačinom (20) je:

$$\Delta\Psi_s^o = 0.064\mathrm{m}$$

jer je faktor $\xi = 0.064 = const.$

Izraz (26) za σ je sada:

$$\sigma_0 = \Omega\frac{1 - 0.064 - \frac{k}{k_s}}{\frac{A_m}{C-A} + \frac{k}{k_s}}$$

i za brojnu vrednost dobijamo:

$$\sigma_0 = 0.01444\ rad\ dan^{-1}$$

Ako ovo uvrstimo u jednačinu (29) za njeno rešenje imamo:

$$\mathrm{m} = e^{(\alpha+i\sigma_0)t}[\mathrm{m}_0 - \frac{i\Omega}{\frac{A_m}{c-A} + \frac{k}{k_s}}\int_0^t \Psi' e^{-(\alpha+i\sigma_0)\tau}d\tau]. \qquad (34)$$

Ovim smo dobili rešenje jednačine Liuvila za model Zemlje sa statičkim okeanom.

3. GLOBALNA ATMOSFERSKA CIRKULACIJA

3.1 FUNKCIJE UGAONOG MOMENTA ATNOSFERE (EAMF)

Umesto ekscitacionih funkcija Barnes i dr. (1983) unose efektivne funkcije ugaonog momenta atmosfere (effective angular momentum functions - EAMF), sada rutinski računate na nekoliko meteoroloških centara. EAMF ne zavise od vremenskih izvoda ΔI_{ij} i h_i i na taj način mogu biti lako računate iz raspoloživih meteoroloških podataka pritiska i vetra. Da bi ovo bilo moguće koristiti u našoj prezentiranoj teoriji,potrebno je da izrazimo ekscitacione funkcije (jednačine 22) Ψ_i' Liuvilovih jednačina u članovima EAMF. Izrazi za EAMF označeni sa χ_i (Barnes i dr., 1983) zamenjuju vrednosti ΔI_{ij} i h_i; njihove relacije su:

$$\chi_1 = \frac{(1+k_2)\Omega\Delta\tilde{I}_{13} + \tilde{h}_1}{\Omega(C_m - A_m)(1 - \frac{k}{k_s})}$$

$$\chi_2 = \frac{(1+k_2)\Omega\Delta\tilde{I}_{23} + \tilde{h}_2}{\Omega(C_m - A_m)(1 - \frac{k}{k_s})} \qquad (*)$$

$$\chi_3 = \frac{(1+k_2)\Omega\Delta\tilde{I}_3 + \tilde{h}_3}{\Omega C_m}$$

gde su A_m, C_m glavni momenti inercije samo omotača, $k \approx 0.30, k' \approx -0.30$ i $k_s = 0.94$ su elastični, teretni i sekularni Lovovi brojevi, redom. Jednačine (*) su bazirane na sledećim pretpo stavkama:

i) Zemlja je rotaciono simetrično i idealno elastično telo,

ii) Fluid jezgra je potpuno odvojen od omotača, i na taj način ne može da prati promene u kretanju omotača,

iii) Zemljina površ je opterećena sa ΔI_{ij} (promenama u tenzoru inercije), dok h_i (relativni ugaoni moment) nema teretni efekat.

Funkcije EAMF (Barnes i dr., 1983) u kompleksnoj notaciji glase:

$$\chi = \chi_1 + i\chi_2 = \frac{1.00R^4}{g(C_m - A_m)} \int_s p_s e^{i\lambda} \sin\phi \cos^2\phi d\lambda d\phi -$$

$$- \frac{1.43R^3}{g\Omega(C_m - A_m)} \int_v (u \sin\phi + iv) e^{i\lambda} cos\phi d\lambda d\phi dp, \qquad (35)$$

$$\chi_3 = \frac{0.70R^4}{gC_m} \int_s p_s \cos^3\phi d\lambda d\phi + \frac{1.00R^3}{g\Omega C_m} \int_v u \cos^2\phi d\lambda d\phi dp.$$

Integrali sa indeksom s su površinski integrali za površinu Zemlje, integrali sa indeksom v su zapreminski integrali atmosfere, ϕ, λ i p_s označavaju geografsku latitudu, longitudu i vazdušni pritisak na Zemljinoj površi, redom; u i v su komponente vetra (u - istočna komponenta, v - severna komponenta). Prvi član jednačina (35) je , često, predstavljen kao član pritiska dok drugi kao član vetra; oni izražavaju uticaj preraspodele vazdušne mase i njen relativni ugaoni moment, redom. Neki meteorološki centri računaju član pritiska sa korekcijom invertnog barometra; tj. površina okeana je predstavljena lokalnom depresijom sa lokalnim porastom atmosferskog pritiska i obrnuto, a sledstveno tome, totalni atmosferski i okeanski efekat je predstavljen konstantnim preko čitave površi svakog okeana.

3.2 VEZA EKSCITACIONIH FUNKCIJA U LIUVILOVIM JEDNAČINAMA SA EAMF

Zbog razlika našeg novog modela Zemlje od Barnesovog (1983) modela, potrebno je da ekscitacione funkcije Ψ_i' date jednačinom (22) izrazimo preko Barnesovih funkcija ugaonog momenta atmosfere (χ_1, χ_2, χ_3).

Izraze (35) možemo napisati u obliku (Vondrak, 1987):

$$\chi = \chi_1 + i\chi_2 = (\chi_1^p + i\chi_2^p) + (\chi_1^w + i\chi_2^w) = E_p \mathbf{I}^p + E_w \mathbf{I}^w \quad \Leftarrow$$

$$\chi_3 = \chi_3^p + \chi_3^w = A_p I_3^p + A_w I_3^w, \qquad (36)$$

gde je

$$\Gamma^p = \int_s p_s e^{i\lambda} \sin\phi \cos^2\phi d\lambda d\phi$$

$$\Gamma^w = \int_v (u \sin\phi + iv) e^{i\lambda} \cos\phi d\lambda d\phi dp$$

$$I_3^p = \int_s p_s \cos^3\phi d\lambda d\phi$$

$$I_3^w = \int_v u \cos^2\phi d\lambda d\phi dp$$

i

$$E_p = \frac{-1.00R^4}{g(C_m - A_m)} = -7.15910^{-10}\ ms^2 kg_{-1}$$

$$E_w = \frac{-1.43R^3}{g\Omega(C_m - A_m)} = -2.20510^{-12}\ s^2 kg^{-1}$$

$$A_p = \frac{0.70R^4}{gC_m} = 1.66910^{-12}\ ms^2 kg{-}1$$

$$A_w = \frac{R^3}{g\Omega C_m} = 5.13410^{-15}\ s^3 kg{-}1$$

Numeričke konstante E i A su računate kod Barnesa (Bar nes i dr., 1983) sa $R = 6370\ km$ srednji Zemljin radijus, $C_m = 7.0410^{37}\ kgm^2$ polarni moment inercije Zemljinog omotača, $\frac{C_m - A_m}{C_m} = 0.00333$ i $g = 9.81\ ms^{-2}$ je srednje gravitaciono ubrzanje na Zemljinoj površi i $\Omega = 7.2910^{-5} s^{-1}$ srednja ugaona brzina Zemlje.

S druge strane, ekscitacione funkcije Ψ_i' definisane jednačinama (11) bolje je računati sa vrednostima:

$R + h_m = 6378\ km$ - Zemljin srednji radijus plus visina atmosfere,

pogledati Johmana (Jochman, 1985)

$C = 8.3610^{37}\ kgm^2$ - Zemljin polarni moment inercije

$\frac{C-A}{C} = 0.003285$

$g = 9.79ms^{-2}$ - gravitaciono ubrzanje na visini iznad Zemljine površine

$\Omega = 7.29210^{-5}s^{-1}$

Zbog različitih brojnih vrednosti za numeričke konstante A i E kod Barnesa i nas, kao i zbog množenja naših ekscitacionih funkcija (11), samo pritisak člana, sa $1 + k_2 = 0.70$ za teretnu deformaciju Zemlje imamo relacije:

$$\Psi' = 0.628(\chi - i\frac{\dot{\chi}_3}{\Omega})$$

$$\Psi_3' = -0.882\chi_3, \tag{37}$$

Ovim smo dobili naše ekscitacione funkcije izražene u funkcijama ugaonih momenata atmosfere (EAMF).

4. MODELI ELASTIČNE ZEMLJE

Pre nego što pređemo na koordinate pola (x, y) daćemo relativne promene trajanja dana $\frac{\Delta\lambda}{\lambda_0}$ u funkciji osnog ugaonog momenta atmosfere χ_3 . To možemo dobiti smenom druge jednačine (37) u jednačinu (33), imamo:

$$\frac{\Delta\lambda}{\lambda_0} = 0.933\ \chi_3 + const \tag{38}$$

Ili

$$-\frac{\Delta LOD}{LOD} = 0.933\chi_3 + const \tag{38*}$$

gde je LOD skraćenica za dužinu dana (length of day), a

$\Delta LOD = LOD - LOD_0$, $LOD_0 = 86400\ sec$.

4.1 NOVI MODEL ZEMLJE SA PROMENLJIVOM ČENDLEROVOM FREKVENCOM

Uvođenjem nove ekscitacione funkcije jednačinom (20) za dinamički okean, koja zavisi od amplitude polarnog kretanja, izvešćemo jednačine novog modela elastične Zemlje sa promenljivom Čendlerovom frekvencom.

Koordinate ekscitacionog pola (x, y) u funkciji ekvatorskih EAMF atmosfere dobijamo smenom prve jednačine (37) u jednačinu (30):

$$\mathbf{m} = e^{\int_0^t(\alpha+i\sigma)d\tau}[\mathbf{m}_0 - \frac{i\Omega}{\frac{A_m}{C-A}+\frac{k}{k_s}}\ 0.628\int_0^t(\chi - i\frac{\dot{\chi}}{\Omega})e^{-\int_0^t(\alpha+i\sigma)d\tau}d\tau] \tag{39}$$

Pošto je σ promenljivo, dato jednačinom (28), za integral u eksponentu jednačine (39) imamo:

$$\int_0^t [\alpha + i\sigma]d\tau = \int_0^t [\alpha + i(\sigma_0 + 0.00149e^{-20A(\tau)})]d\tau$$

odnosno

$$\int_0^t [\alpha + i\sigma]d\tau = \alpha t + i(\sigma_0 t + J)$$

gde je

$$J = 0.00149 \int_0^t e^{-20A(\tau)}d\tau \tag{40}$$

i

$$\sigma = \sigma_0 + 0.00149e^{-20A(t)}$$

ako zbog jednostavnosti pisanja stavimo

$$l = 0.628$$

i

$$L = \frac{A_m}{C - A} + \frac{k}{k_s}$$

jednačina (39) postaje:

$$\mathbf{m} = e^{\alpha t + i(\sigma_0 t + J)}\{\mathbf{m}_0 - \frac{i\Omega l}{L}[\int_0^t \chi e^{-[\alpha\tau + i(\sigma_0\tau + J)]}d\tau -$$

$$-\frac{i}{\Omega}\int_0^t \dot{\chi} e^{-[\alpha\tau + i(\sigma_0\tau + J)]}d\tau]\}. \tag{41}$$

Parcijalnom integracijom drugog integrala jednačine (41) nalazimo da je:

$$\int_0^t \dot{\chi} e^{-[\alpha\tau + i(\sigma_0\tau + J)]}d\tau = (\alpha + i\sigma)\int_0^t \chi e^{-[\alpha\tau + i(\sigma_0\tau + J)]}d\tau +$$

$$+\chi(t)e^{-[\alpha\tau + i(\sigma_0\tau + J)]} - \chi(0)$$

Smenom ovog izraza u jednačinu (41) dobijamo:

$$\mathbf{m} = e^{-[\alpha t+i(\sigma_0\tau+J)]}[\mathbf{m}_0-$$

$$-\frac{l}{L}[\alpha+i(\Omega+\sigma)]\int_0^t \chi e^{-[\alpha\tau+i(\sigma_0\tau+J)]}d\tau]-$$

$$-\frac{l}{L}[\chi(t)-\chi(0)e^{[\alpha t+i(\sigma_0\tau+J)]}]$$

Ovaj izraz u razvijenom obliku glasi:

$$m_1+im_2 = e^{\alpha t}[\cos(\sigma_0 t+J)+i\sin(\sigma_0 t+J)]*$$

$$*\{[m_1(0)+im_2(0)]-\frac{l}{L}[\alpha+i(\Omega+\sigma)]\int_0^t(\chi_1+i\chi_2)e^{-\alpha\tau}[\cos(\sigma_0\tau+J)-i\sin(\sigma_0\tau+J)]d\tau]$$

$$-\frac{l}{L}\{\chi_1(t)+i\chi_2(t)-[\chi_1(0)+i\chi_2(0)]e^{\alpha t}[\cos(\sigma_0 t+J)+i\sin(\sigma_0 t+J)]\}$$

Odavde za koordinate ekscitacionog pola u normalno orijentisanom koordinatnom sistemu, u kojem je x osa usmerena dužnultog meridijana a y osa 90° E, dobijamo:

$$x = m_1(t) = e^{\alpha t}\{x_0\cos(\sigma_0 t+J)-y_0\sin(\sigma_0 t+J)+$$

$$+\frac{l}{L}\Omega(1+\frac{\sigma}{\Omega})[(J_1+J_4)\sin(\sigma_0 t+J)+(J_2-J_3)\cos(\sigma_0 t+J)]\}$$

$$y = m_2(t) = e^{\alpha t}\{x_0\sin(\sigma_0 t+J)+y_0\cos(\sigma_0 t+J)-$$

$$-\frac{l}{L}\Omega(1+\frac{\sigma}{\Omega})[(J_1+J_4)\cos(\sigma_0 t+J)-(J_2-J_3)\sin(\sigma_0 t+J)]\} \qquad (42)$$

Gde smo sa J_1, J_2, J_3 i J_4 označili:

$$J_1 = \int_0^t \chi_1 e^{-\alpha\tau}\cos(\sigma_0\tau+J)d\tau+\frac{1}{\Omega}[\chi_2(t)e^{-\alpha t}\cos(\sigma_0 t+J)-\chi_2(0)]$$

$$J_2 = \int_0^t \chi_2 e^{-\alpha\tau}\cos(\sigma_0\tau+J)d\tau-\frac{1}{\Omega}[\chi_1(t)e^{-\alpha t}\cos(\sigma_0\tau+J)-\chi_1(0)]$$

$$J_3 = \int_0^t \chi_1 e^{-\alpha\tau} \sin(\sigma_0\tau + J)d\tau + \frac{1}{\Omega}\chi_2(t)e^{-\alpha t}\sin(\sigma_0 t + J)$$

$$J_4 = \int_0^t \chi_2 e^{-\alpha\tau} \sin(\sigma_0\tau + J)d\tau - \frac{1}{\Omega}\chi_1(t)e^{-\alpha t}\sin(\sigma_0 t + J) \qquad (43)$$

Jednačine (42) nam predstavljaju koordinate ekscitacionog pola x, y dobijene kao rezultat novog modela Zemlje sa dinamičkim okeanom. Stavljajući u jednačine (42) brojnu vrednost l i izraz za L imamo:

$$x = e^{\alpha t}\{x_0\cos(\sigma_0 t + J) - y_0\sin(\sigma_0 t + J) +$$
$$+\frac{0.628}{\frac{A_m}{C-A}+\frac{k}{k_s}}\Omega(1+\frac{\sigma}{\Omega})[(J_1+J_4)\sin(\sigma_0 t+J)+(J_2-J_3)\cos(\sigma_0 t+J)]\}$$

$$y = e^{\alpha t}\{x_0\sin(\sigma_0 t + J) + y_0\cos(\sigma_0 t + J) -$$
$$-\frac{0.628}{\frac{A_m}{C-A}+\frac{k}{k_s}}\Omega(1+\frac{\sigma}{\Omega})[(J_1+J_4)\cos(\sigma_0 t+J)-(J_2-J_3)\sin(\sigma_0 t+J)]\} \qquad (44)$$

Ovim smo dobili jednačine novog modela a upoređenje sa posmatranjima će slediti .

4.2 VONDRAKOV MODEL ZEMLJE SA KONSTANTNOM ČENDLEROVOM FREKVENCOM

Vondrak (1987) je analitički dao realniji (od Barnesovog, 1983) model Zemlje: tro-osno telo, fluidno jezgro, visko-elastični omotači statički okean (Čendlerova frekvenca $\sigma_0 = const.$), čije jednačine u našoj notaciji glase:

$$x = e^{\alpha t}\{x_0\cos(\sigma_0 t) - (1.006y_0 - 0.001x_0)\sin(\sigma_0 t) +$$
$$\sigma_0(1+\frac{\sigma_0}{\Omega}[(1.014\tilde{J}_2 - 1.019\tilde{J}_3 - 0.002\tilde{J}_4)\cos(\sigma_0 t) +$$

$$+(1.019\tilde{J}_1 + 0.002\tilde{J}_2 + 1.014\tilde{J}_4)\sin(\sigma_0 t)]\}$$

$$y = e^{\alpha t}\{y_0\cos(\sigma_0 t) + (0.994x_0 - 0.001y_0)\sin(\sigma_0 t) -$$

$$-\sigma_0(1 - \frac{\sigma_0}{\Omega})[(1.013\tilde{J}_1 + 0.002\tilde{J}_3 + 1.008\tilde{J}_4)\cos(\sigma_0 t) -$$

$$-(0.002\tilde{J}_1 + 1.008\tilde{J}_2 - 1.013\tilde{J}_3)\sin(\sigma_0 t)]\} \tag{45}$$

gde je

$$\tilde{J}_1(t) = \int_0^t \chi_1 e^{-\alpha\tau}\cos(\sigma_0\tau)d\tau + \frac{1}{\Omega}[\chi_2(t)e^{-\alpha t}\cos(\sigma_0 t) - \chi_2(0)]$$

$$\tilde{J}_2 = \int_0^t \chi_2 e^{-\alpha\tau}\cos(\sigma_0\tau)d\tau - \frac{1}{\Omega}[\chi_1(t)e^{-\alpha t}\cos(\sigma_0 t) - \chi_1(0)]$$

$$\tilde{J}_1(t) = \int_0^t \chi_1 e^{-\alpha\tau}\sin(\sigma_0\tau)d\tau + \frac{1}{\Omega}\chi_2(t)e^{-\alpha t}\sin(\sigma_0 t)$$

$$\tilde{J}_4(t) = \int_0^t \chi_2 e^{-\alpha\tau}\sin(\sigma_0\tau)d\tau - \frac{1}{\Omega}\chi_1(t)e^{-\alpha t}\sin(\sigma_0 t) \tag{46}$$

Vidimo da razlike numeričkih konstanata u jednačinama (45) su različite od jedinice ili nule samo za vrlo male iznose; zato možemo pojednostaviti relacije zanemarivanjem ovih razlika i dobiti Vondrakov model za dvo-osnu Zemlju:

$$x = e^{\alpha t}\{x_0\cos(\sigma_0 t) - y_0\sin(\sigma_0 t) +$$

$$+\sigma_0(1 + \frac{\sigma_0}{\Omega})[(\tilde{J}_2 - \tilde{J}_3)\cos(\sigma_0 t) + (\tilde{J}_1 + \tilde{J}_4)\sin(\sigma_0 t)]\}$$

$$y = e^{\alpha t}\{y_0\cos(\sigma_0 t) + x_0\sin(\sigma_0 t) -$$

$$-\sigma_0(1 - \frac{\sigma_0}{\Omega})[(\tilde{J}_1 + \tilde{J}_4)\cos(\sigma_0 t) - (\tilde{J}_2 - \tilde{J}_3)\sin(\sigma_0 t)]\} \tag{47}$$

Do jednačina (47) mogli smo doći smenom prve jednačine (37) u jednačinu (34), iz koje imamo:

$$\mathrm{m} = \mathrm{e}^{(\alpha+\mathrm{i}\sigma_0)\mathrm{t}}[\mathrm{m}_0 - \frac{\mathrm{i}\Omega}{\frac{\mathrm{A_m}}{\mathrm{C-A}} + \frac{\mathrm{k}}{\mathrm{k_s}}} 0.628 \int_0^{\mathrm{t}} (\chi - \mathrm{i}\frac{\dot{\chi}}{\Omega})\ \mathrm{e}^{(\alpha+\mathrm{i}\sigma_0)\tau} \mathrm{d}\tau]$$

a iz nje dobijamo koordinate pola $\tilde{x}$, $\tilde{y}$:

$$\tilde{x} = e^{\alpha t}\{x_0 \cos(\sigma_0 t) - y_0 \sin(\sigma_0 t) +$$
$$+\frac{l}{L}\Omega(1+\frac{\sigma_0}{\Omega})[(\tilde{J}_1 + \tilde{J}_4)\sin(\sigma_0 t) + (\tilde{J}_2 - \tilde{J}_3)\cos(\sigma_0 t)]\}$$

$$\tilde{y} = e^{\alpha t}\{x_0 \sin(\sigma_0 t) + y_0 \cos(\sigma_0 t) -$$
$$-\frac{l}{L}\Omega(1+\frac{\sigma_0}{\Omega})[(\tilde{J}_1 + \tilde{J}_4)\cos(\sigma_0 t) - (\tilde{J}_2 - \tilde{J}_3)\sin(\sigma_0 t)]\} \qquad (47^*)$$

koje su identične sa Vondrakovim jednačinama (47), jer su razlike numeričkih vrednosti izraza $\sigma_0(1+\frac{\sigma_0}{\Omega})$ kod Vondraka i izraza $\frac{l}{L}\Omega(1+\frac{\sigma_0}{\Omega})$ kod nas zanemarljive.

Jednačine (47) dobijamo kao specijalan slučaj jednačina (44) našeg novog modela sa promenljivom Čendlerovom frekvencom.

Vondrakov model je dat, da bi smo pri upoređenju integraljenog polarnog kretanja po novom modelu sa posmatranim polarnim kretanjem vršili i upoređenje dva modela Zemlje; jednog sa promenljivom Čendlerovom frekvencom a drugog sa konstantnom. Na osnovu dobijenih rezultata slediće zaključci.

5. KORIŠĆENI PODACI

5.1 ATMOSFERSKE FUNKCIJE

EAMF (effective angular momentum functions) su funkcije efektivnih ugaonih momenata atmosfere sada rutinski računate na nekoliko meteoroloških centara po algoritmu koji su dali Barnes, Hide, White i Wilson (1983), datim jednačinama (35) u našem radu. Koristili smo EAMF računate u Američkom nacionalnom meteorološkom centru (U.S. National Meteorologikal Center), privatno dobijenim od Dr J. Vondraka (Prag). Podatke EAMF za period 1976.5 - 1986.0 analizirali smo u pet sledećih kombinacija:

a) $\chi_i^p + \chi_i^w$

b) $\chi_i^{p'} + \chi_i^w$

c) χ_i^p

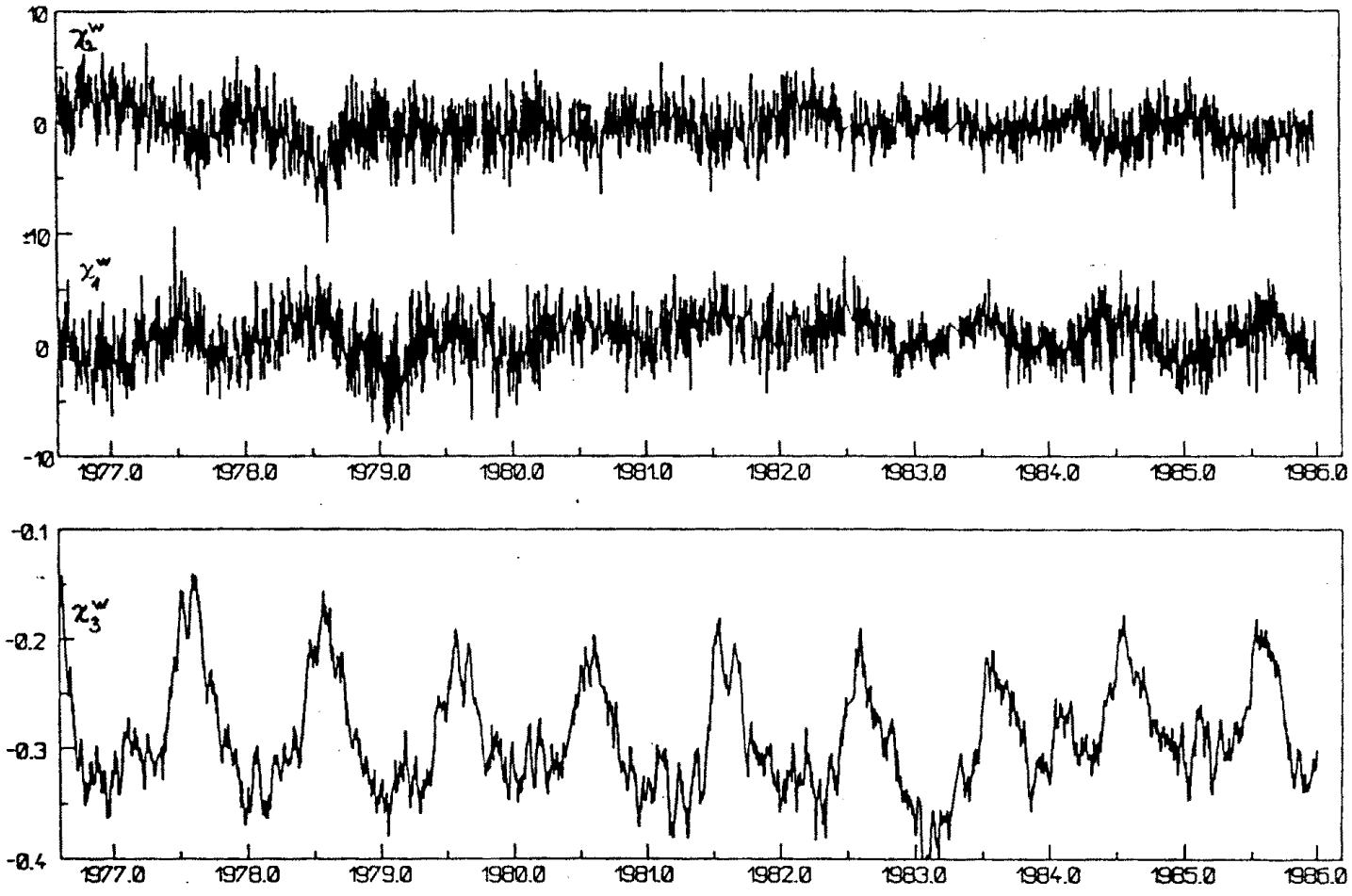
d) $\chi_i^{p'}$

e) χ_i^w

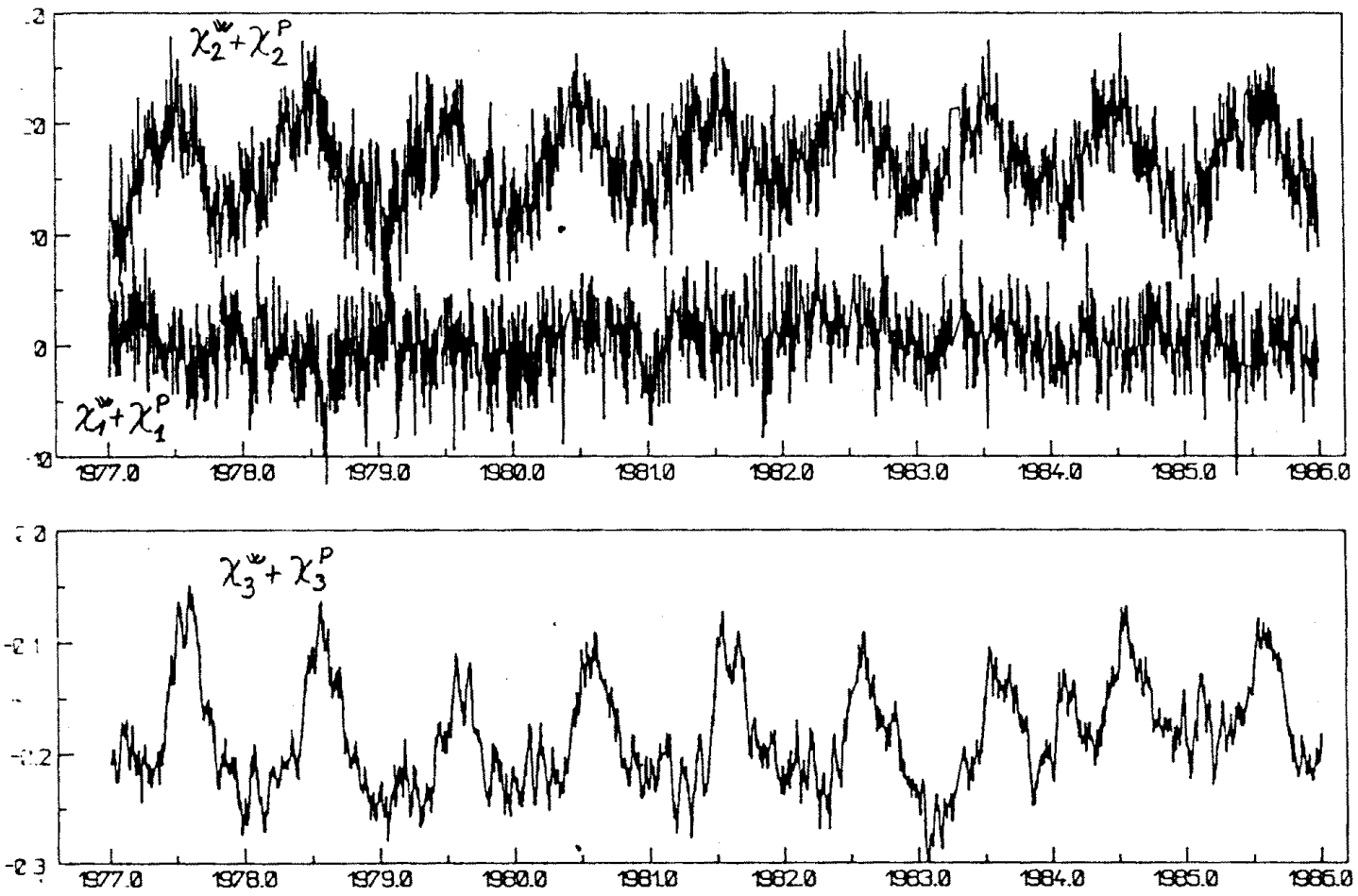
(48*)

gde indeksi p i w označavaju članove pritiska i vetra, redom, a p' označava član pritiska računat sa hipotezom invertnog barometra. Podaci EAMF na intervalu 1976.5 - 1986.0 predstavljeni su na slikama 1 - 3; član vetra na slici 1 i njegov zbir sa članovima pritiska sa i bez korekcije invertnog barometra na slikama 3 i 2 . Sa slika je jasno da su sva tri skupa podataka periodične funkcije vremena, sa izrazitim godišnjim članom. Obe ekvatorske komponente χ_1 i χ_2 su mnogo više šumne nego osna komponenta χ_3 . Član vetra dominira osnom komponentom dok članovi pritiska preovlađuju u ekvatorskim komponentama. Član pritiska u χ_2 ima važan konstantni deo, koji očigledno odgovara nesimetričnoj raspodeli srednjeg pritiska izmedu zapadne i istočne hemisfere. Korekcija invertnog barome–

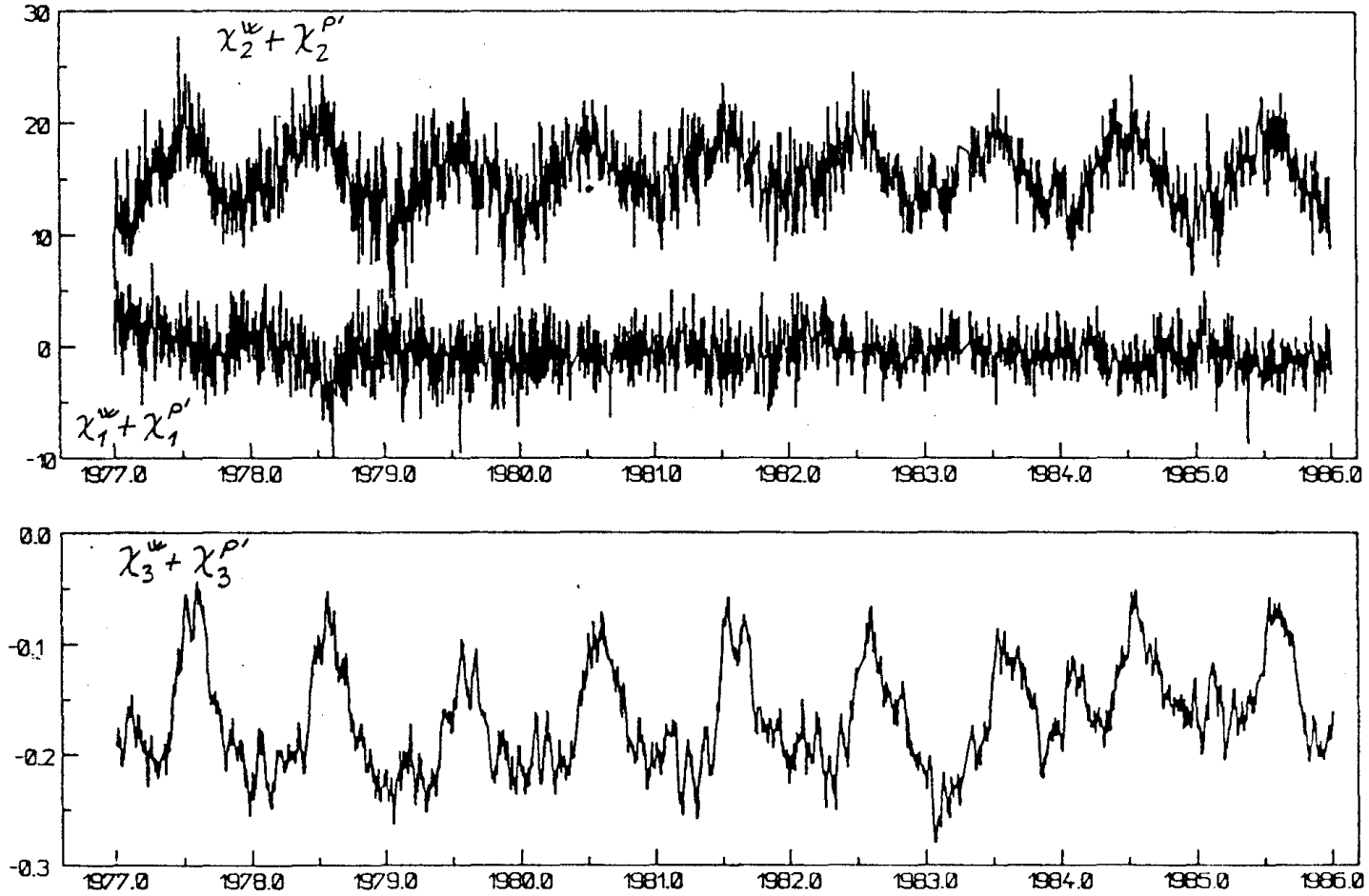
Slika 1. Članovi vetra funkcija efektivnih ugaonih momenata atmosfere kako su raćunati u Nacionalnom meteorološkom centru (NMC). Predstavljeni su samo ponoćni podaci.



Slika 2. Članovi vetra plus pritiska funkcija efektivnih ugaonih momenata atmosfere kako su računati u NMC.



Slika 9. Članovi vetra plus pritiska sa korekcijom invertnog barometra funkcija efektivnih ugaonih momenata atmosfere kako su računati u NMC.

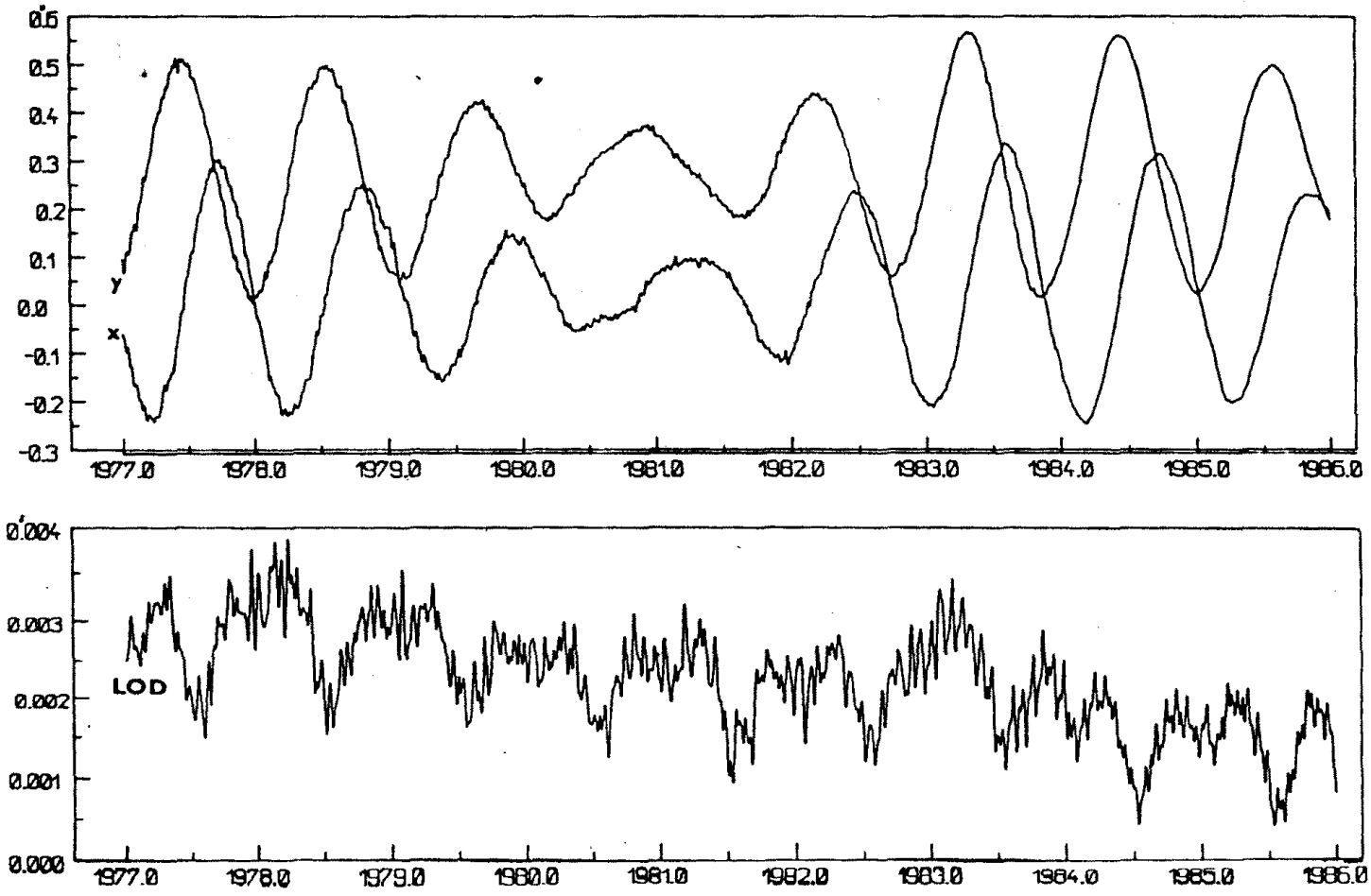


tra utiče na smanjenje fluktuacija u χ_1 i χ_2 dok je to doprinos samo malog konstantnog člana u χ_3 .

5.2 PARAMETRI ZEMLJINE ROTACIJE

Parametri Zemljine rotacije (PZR), tj. polarno kretanje i svetsko vreme su rutinski određeni iz različitih skupova posmatranja (optička astrometrija, Doplerska posmatranja, laserska posmatranja, dugo-bazična interferometrija). Međunarodni Biro za vreme (BIH) u Parizu sakuplja ove podatke, kombinuje ih i razašilje. U ovom radu koristićemo BIH-ova kombinovana rešenja publikovana u BIH Annual Reports (od 1976 do 1986) što je isti vremenski interval kao i za EAMF podatke. Napomenimo da su kombinovana rešenja, u početku , većinom bazirala na optičkoj astrometriji a kasnije sve više i više na posmatranjima modernim tehnikama koje su preciznije naročito u kratko-periodičnom delu spektra (Vondrak,1986). Podaci PZR su prikazani na slici 4; rastuća preciznost podataka u kratkim periodama je predstavljena sa očiglednom činjenicom da posmatrano polarno kretanje je sve izravnatije prema kraju posmatranog intervala. Promene trajanja dana (LOD) su računate iz BIH-ovih publikovanih vrednosti UT1-UTC; njihova složenija struktura je kombinovana od promena uzrokovanih plimama (Yoder et al. 1981) i atmosferskom ekscitacijom.

Slika 4. Polarno kretanje (x, y) i odstupanja u trajanju dana od nominalnih 86 400 sekunada (LOD) određenih iz astronomskih posmatranja u BIH-u. Podaci LOD su bazirani na publikovanim vrednostima UT1-UTC.



6. UPOREĐENJE TEORIJE I POSMATRANJA

6.1 UPOREĐENJE SPEKTRA POLARNOG KRETANJA SA SPEKTRIMA EAMF

Razmotrimo jednačine (47*) uvođenjem veličina za oba skupa podataka: polarno kretanje $\mathbf{X} = \mathbf{x} + \mathbf{iy}$ i ekvatorske komponente EAMF $\chi = \chi_1 + i\chi_2$, imamo jednostavnije relacije u kompleksnoj notaciji:

$$\mathbf{X}(t) = e^{(\alpha+i\sigma_0)t}[\mathbf{X}(0) - i\sigma_0(1+\frac{\sigma_0}{\Omega})\int_0^t \chi(\tau)e^{-(\alpha+i\sigma_0)\tau}d\tau] -$$

$$-\frac{\sigma_0}{\Omega}[\chi(t) - \chi(0)e^{(\alpha+i\sigma_0)t}] \qquad (48)$$

Naravno, ova jednačina može biti lako korišćena, zajedno sa numerički dobijenim vrednostima integrala na desnoj strani, za određivanje trenutne pozicije pola $\mathbf{X}(t)$ za svaki vremenski trenutak t, s tim što je njegov početni položaj $\mathbf{X}(0)$ poznat. Da bi smo našli najbolju kombinaciju EAMF podataka datih jednačinom (48*) izvršićemo kompletnu analizu procesa u domenu frekvencija da bi smo uporedili i amplitude i faze dominantnih članova.

Pretpostavljajući da ekvatorske EAMF (χ) možemo predstaviti, kao zbir periodičnih članova, u obliku:

$$\chi(t) = \Sigma_j a_j e^{i(\omega_j t+\phi_j)} \qquad (49)$$

gde su a_j realne amplitude i ω_j ugaone brzine koje mogu biti pozitivne i negativne (podsetimo se da svako eliptično kretanje može biti izraženo kao zbir dva kružna kretanja sa različitim amplitudama i ugaonim brzinama koje se razlikuju samo u znaku).

Ako smenimo jednačinu (49) u jednačinu (48) i integralimo, dobićemo:

$$\mathbf{X}(t) = e^{(\alpha+i\sigma_0)t}[\mathbf{X}(0) - \Sigma_j a_j F_j e^{i(\phi_j+\Delta\phi_j)}] +$$

$$+\Sigma_j a_j F_j e^{i(\omega_j t+\phi_j+\Delta\phi_j)} \tag{50}$$

gde je

$$F_j^2 = \frac{\sigma_0^2[\alpha^2+(\Omega+\omega_j)^2]}{\Omega^2[\alpha^2+(\sigma_0-\omega_j)^2]}$$

i

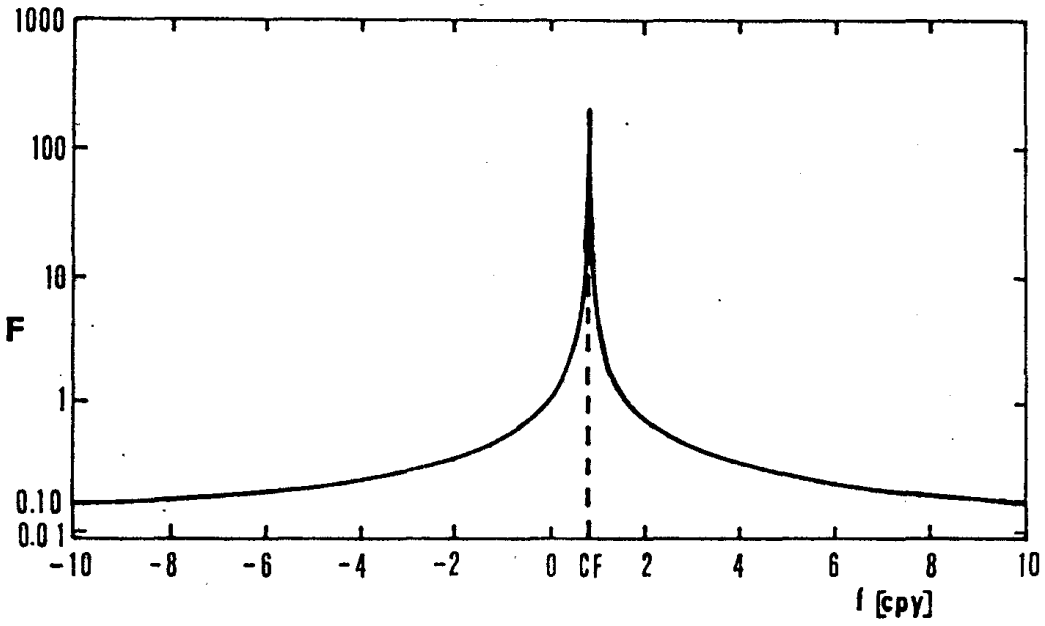
$$\tan\Delta\phi_j = \frac{\alpha(\Omega+\sigma_0)}{(\Omega+\omega_j)(\sigma_0-\omega_j)-\alpha^2}$$

Prvi član jednačine (50) predstavlja prigušeno slobodno kretanje sa ugaonom brzinom σ_0, a drugi član predstavlja zbir prinudnih kretanja sa ugaonom brzinom ω_j. Velične F_j i $\Delta\phi_j$, obe realne funkcije ugaone brzine ω_j, izražavaju odnos amplitude ekscitacionog pola i pola EAMF i promenu faze između ova dva, redom.

Prvo razmotrimo promenu faze $\Delta\phi$. Jasno je da su te promene velike blizu dve singularne tačke, $\omega \approx -\Omega$ i $\omega \approx \sigma_0$, pošto je α vrlo mali negativni faktor. Prva singularna tačka je malo značajna zato što, praktično, ima neatmosfersku ekscitaciju sa ugaonom brzinom blizu $-\Omega$; $\Delta\phi = 0$ za ω u intervalu $(-\Omega, \sigma)$ i $\Delta\phi \approx \pi$ za $\omega > \sigma$. To znači da ekscitacioni pol je u fazi sa ekscitacijom čija je frekvenca niža od Čendlerove frekvence , a u suprotnoj fazi za više frekvence. Prenosna funkcija F (za $Q = 100$) je nacrtana u logaritamskoj skali za $f = \frac{\omega}{2\pi}$ na slici 5. Možemo videti sa slike da ekscitacije sa visokom frekvencom (i pozitivne i negativne) su znatno prigušene, dok ekscitacije blizu Čendlerove frekvence su povećane. Činjenica koja sledi iz posmatranja je ta da je polarno kretanje dobro izravnato. Prenosna funkcija F, koja ide u beskonačnost za $\omega = \sigma_0$ u slučaju apsolutno čvrste Zemlje ili je konačno određena , za elastičnu Zemlju, disipacijom u omotaču Zemlje ($\alpha \approx 0$); za $Q = 100$, $F(\sigma_0) \approx 195$.

Iz jednačine (50) sledi da amplitude spektra od χ i $\mathbf{X}$ su direktno neuporedive; spektar od χ morao bi se pomnožiti sa prenosnom funkcijom F i samo takav upoređivao sa spektrom u $\mathbf{X}$. Spektar S kompleksne funkcije $\mathbf{X}$, uziman na ekvidistantnim vremenskim intervalima Δt, dobijen je kao direktna Furijeova transformacija (FT):

Slika 5. Prenosna funkcija između funkcije efektivnog ugaonog momenta i polarnog kretanja za različite frekvence. CF je Čendlerova frekvenca , korišćen je kvalitativni faktor mantije $Q = 100$.



$$S(\omega) = \frac{1}{N}\Sigma_1^N \mathbf{X}(t_j)e^{-i\omega t_j}, \qquad (51)$$

gde je $t_j = (j-1)\Delta t$. Koristili smo algoritam Cooley-Tukey za brzu Furijeovu transformaciju (FFT) za spektar $S(\omega)$,jednačine (51). Korišćeni podaci su dopunjeni nulama do 2 na ceo broj . Nismo primenjivali spektralne proreze za sužavanje pikova . Pokušaj da primenimo Parzenov prorez na podatke polarnog kretanja dao je veliko smanjenje amplitude i Čendlerovog i godišnjeg člana u odnosu na podatke bez primene spektralnog proreza. Jasno je iz činjenice da primena proreza daje najveću težinu podacima u centru celokupnog intervala gde, specijalno u našem slučaju,amplituda polarnog kretanja je bila ekstremno mala.

Spektri $S(\omega)$ su nacrtani na slikama 6 i 7, zajedno sa sa njihovim likom dobijenim pomoću prenosne funkcije F, već diskutovane. Spektri su dati u intervalu $(-4,4)$ ciklusa po godini , tj. predstavljen je interval sa najizrazitijim pikovima.

Slika 6 prikazuje spektar originalnog polarnog kretanja (na gornjem delu) sa dominantna dva pika, na Čendlerovoj i godišnjoj frekvenciji, i isti spektar podeljen sa F (na donjem delu slike).

Na slikama 7a-7e prikazani su spektri atmosferskih EAMF (na gornjem delu) i isti spektri kao da su pomnoženi sa F (na donjem delu) za pet kombinacija EAMF definisanih u glavi 5, jednačinama (48*). Postoji sličnost svih pet spektara; uvek se nalaze dva dominantna pika na pozitivnoj i negativnoj frekvenci od 1 ciklusa po godini, približno iste amplitude . Pored toga amplitude se razlikuju za različite kombinacije EAMF. Preračunavani spektri na donjem delu slika 7a-7e su namenjeni za upoređenje sa slikom 6. Sličnost je ponovo očigledna ali mi vidimo da samo član vetra (sl. 7c) je dovoljan za ekscitaciju godišnje komponente posmatrane amplitude. S druge strane, samo član pritiska (sl. 7c) je suviše veliki a član pritiska sa korekcijom invertnog barometra (sl. 7d) je mali za ekscitaciju posmatranog polarnog kretanja na ovoj frekvenci. Kombinacije člana ve-

tra sa članom pritiskaa (sa i bez korekcije invertnog barometra) mogu ekscitovati član znatno veće amplitude nego što je posmatrana vrednost. Da bi uporedili i druge parametre godišnje komponente dajemo tablicu 1. Ona prikazuje parametre eliptičnog godišnjeg člana za posmatrano polarno kretanje (BIH) i za svih pet kombinacija od EAMF (a-e).

Tablica 1.

Ovde su dati parametri godišnjeg posmatranog polarnog kretanja (BIH) i ekscitovanog različitim kombinacijama EAMF (48*) na $0^h UT$. A i ϕ označavaju amplitudu i početnu fazu pozitivnog (+) i negativnog (-) kružnog kretanja, a i b su velika i mala poluosa eliptičnog kretanja, φ je pravac velike poluose i ψ je početna faza.

	A^+ [″]	ϕ^+ [°]	A^- [″]	ϕ^- [°]	a [″]	b [″]	φ [°]	ψ
BIH	0.084	115	0.005	266	0.089	0.079	+10	116
w+p	0.222	94	0.017	264	0.239	0.205	-1	95
w+p'	0.181	98	0.011	281	0.192	0.170	+10	98
p	0.123	78	0.014	248	0.137	0.109	-17	79
p'	0.082	78	0.007	261	0.089	0.075	-10	78
w	0.107	113	0.005	310	0.112	0.102	+32	112

Eliptična godišnja komponenta može da se izrazi kao zbir dva kružna kretanja sa suprotnim ugaonim brzinama i različitim amplitudama:

$$\mathbf{X}_a(t) = A^+ e^{i(2\pi t+\phi^-)} + A^- e^{i(-2\pi t+\phi^-)}, \tag{52}$$

gde je t računato u godinama, A^+, A^-, ϕ^+, ϕ^- su amplitude i faze, određene spektralnom analizom $S = Ae^{i\phi}$ na frekvencijama $f = \pm 1 cik/god.$ Iz ove četiri osnovne vrednosti možemo još izvesti veliku i malu poluosu (a i b), pravac velike poluose φ i fazu ψ za trenutak t=0:

$$\begin{aligned} a &= A^+ + A^- \\ b &= |A^+ - A^-| \\ \varphi &= \frac{(\phi^+ + \phi^-)}{2} \end{aligned} \tag{53}$$

$$\psi = \frac{A^+ \sin\phi^+ + A^- \sin\phi^-}{A^+ \cos\phi^+ + A^- \cos\phi^-}$$

Određivanje A^+ i ϕ^+ za godišnju komponentu polarnog kretanja (pogledati sl. 6) je pod uticajem prisustva susednog Čend lerovog člana (pogledati na pr. Jochman, 1986). Ako predpostavimo da se nalaze samo dva člana sa ugaonim brzinama ω_1 i ω_2 i kompleksnim amplitudama a_1 i a_2 imamo:

$$S(\omega) = \frac{1}{T}\int_0^T [a_1 e^{\omega_1 t} + a_2 e^{\omega_2 t}] e^{-i\omega t} dt =$$
$$= a_1 K(\omega_1, \omega) + a_2 K(\omega_2, \omega), \qquad (54)$$

gde je

$$K(\omega_j, \omega) = \frac{e^{i(\omega_j - \omega)T} - 1}{i(\omega_j - \omega)T} =$$
$$= \frac{\sin[T(\omega_j - \omega)] + 2\sin^2\left[\frac{T(\omega_j - \omega)}{2}\right]}{T(\omega_j - \omega)}$$

Spektralni članovi $S_1 = S(\omega_1)$, $S_2 = S(\omega_2)$ koji su procenjeni iz Furijeovih transformacija su:

$$S_1 = a_1 + K(\omega_2, \omega_1) a_2$$

$$S_2 = K(\omega_1, \omega_2) a_1 + a_2 \qquad (55)$$

a odavde dobijamo, stavljajući $K = K(\omega_2, \omega_1), K^* = K(\omega_1, \omega_2)$ (K, K^* su konjugovani kompleksni brojevi),

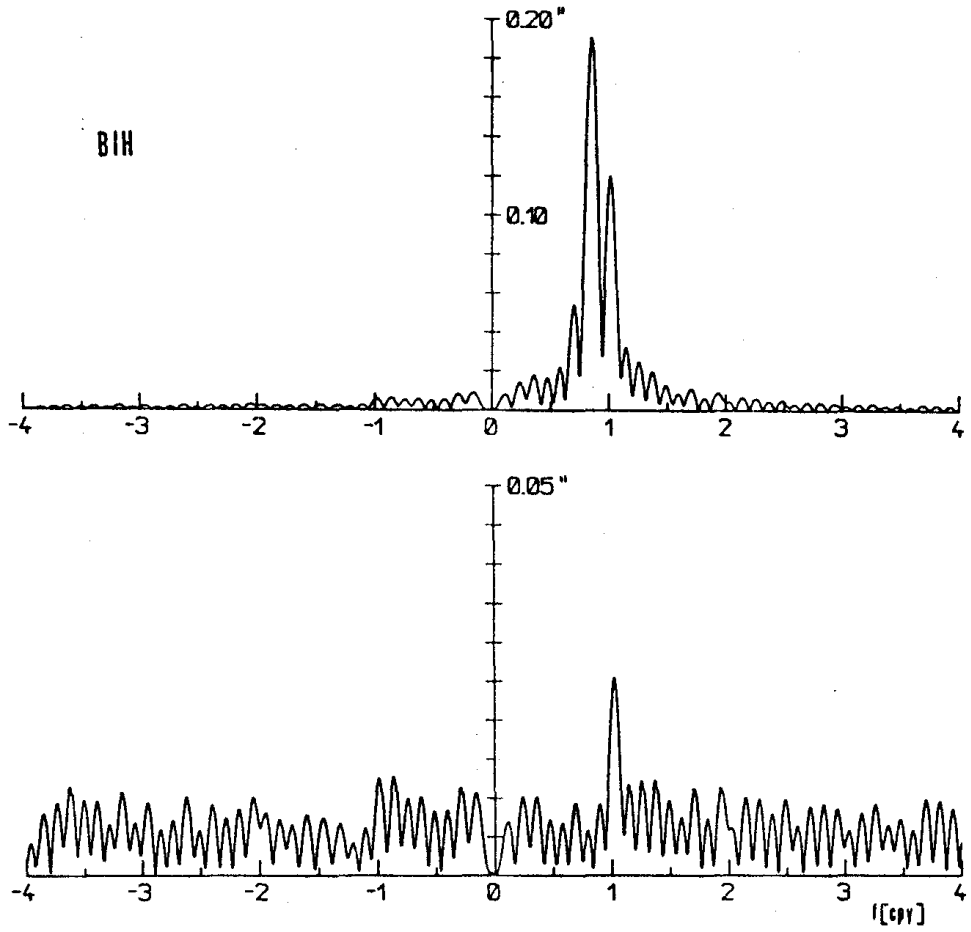
$$a_1 = \frac{S_1 - K S_2}{1 - K K^*}$$

$$a_2 = \frac{S_2 - K^* S_1}{1 - K K^*} \qquad (56)$$

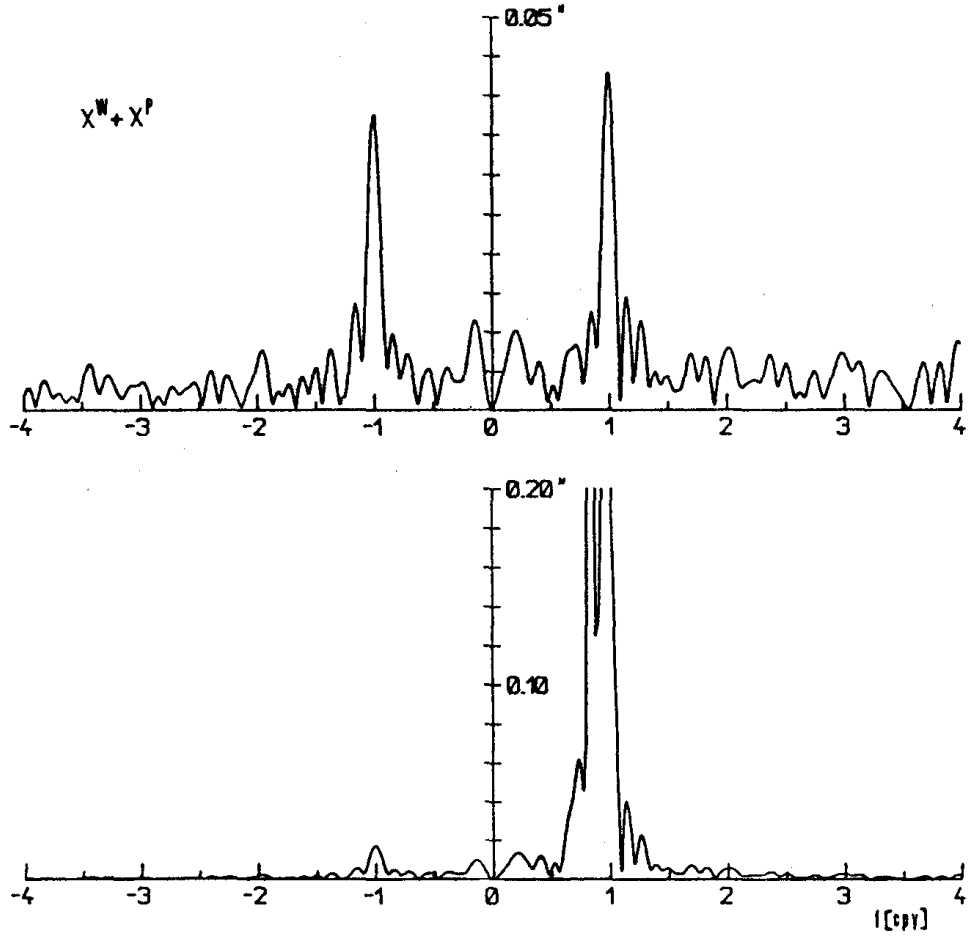
Koristeći vrednosti $\omega_1 = 5.276 rad/god, S_1 = -0".179 + 0".045i$ (Čendlerov član), $\omega_2 = 6.238 rad/god, S_2 = 0."030 + 0".112i$ (godišnji član, T=9 godina, dolazimo na $K = 0.040 + 0.213i, K^* = 0.040 - 0.213i$ i za popravljene vrednosti amplituda dobijamo vrednosti $a_1 = -0".188 + 0".049i, a_2 = -0".036 + 0".076i$. Ustvari ove vrednosti su izložene u tabl. 1 u vrsti označenom sa BIH.

Iz tablice 1 vidimo, relativno, dobro podudaranje parametara posmatrane godišnje komponente sa parametrima računatim iz EAMF u svih pet kombinacija. Amplitude i početne faze su predstavljene grafički na slici 8. Teorijski najbolja saglasnost bi se dobila za varijantu b (tj. član vetra plus član pritiska sa korekcijom invertnog barometra, na sl. 8 označen kao $w+p'$). Tablica i slika prikazuju da samo pravac velike poluose ovog člana je zadovoljavajuće saglasan; razlika u amplitudi i početnoj fazi je velika. Amplituda člana pritiska sa korekcijom invertnog barometra (p') je skoro jednaka posmatranoj amplitudi, ali razlika u fazi ψ je mnogo veća nego za sve druge kombinacije. Najbolja saglasnost se postiže za varijantu e (tj. samo član vetra, označen na sl. 8 sa w). Eubanks i dr. (1987) su dokazali da ekvatorska komponenta vetar člana je suprotne faze na godišnjoj frekvenciji, ako je računata iz podnevnih podataka, stoga oni ne preporučuju korišćenje ekvatorskog vetar člana. Naši računi potvrđuju njihove nalaze; amplitude A^+, A^- godišnjeg vetar člana su manje ako su računate samo iz podnevnih podataka (0".070 i 0".007) ali faze ϕ^+, ϕ^- su skoro suprotne (288° i 75°). Ovo je suprotno vetar članu (varijanta e, jednačinene 48*) ekscitovanom godišnjem kretanju, kao i njegov zbir sa pritisak članovima (varijante a i b, jednačine 48*). Ako se nalaze efekti vetra na polarno kretanje oni su sigurno mnogo manji nego što smo ih prikazali. Na drugoj strani, sam pritisak član sa korekcijom invertnog barometra je dovoljan za posmatrano godišnje kretanje, ali se nalazi velika razlika u fazi. Član vetra bi bilo dobro uzimati usrednjen iz dva skupa podataka, podnevnih i ponoćnih. Rezultat se može videti u tablici 2 (druga vrsta) zajedno sa njegovim zbirom pritisak člana sa korekcijom invertnog barometra (ATM). Po našem mišljenju ovaj član daje najbolju atmosfersku ekscitaciju polarnog kretanja na godišnjoj frekvenci, i u boljoj je saglasnosti sa posmatranim godišnjim članom nego samo pritisak član. Amplituda ovog člana je mnogo veća od posmatrane, orijentacija elipse φ je praktično identična. Razlika u fazi ψ je još veća (31 stepen), ali manja od

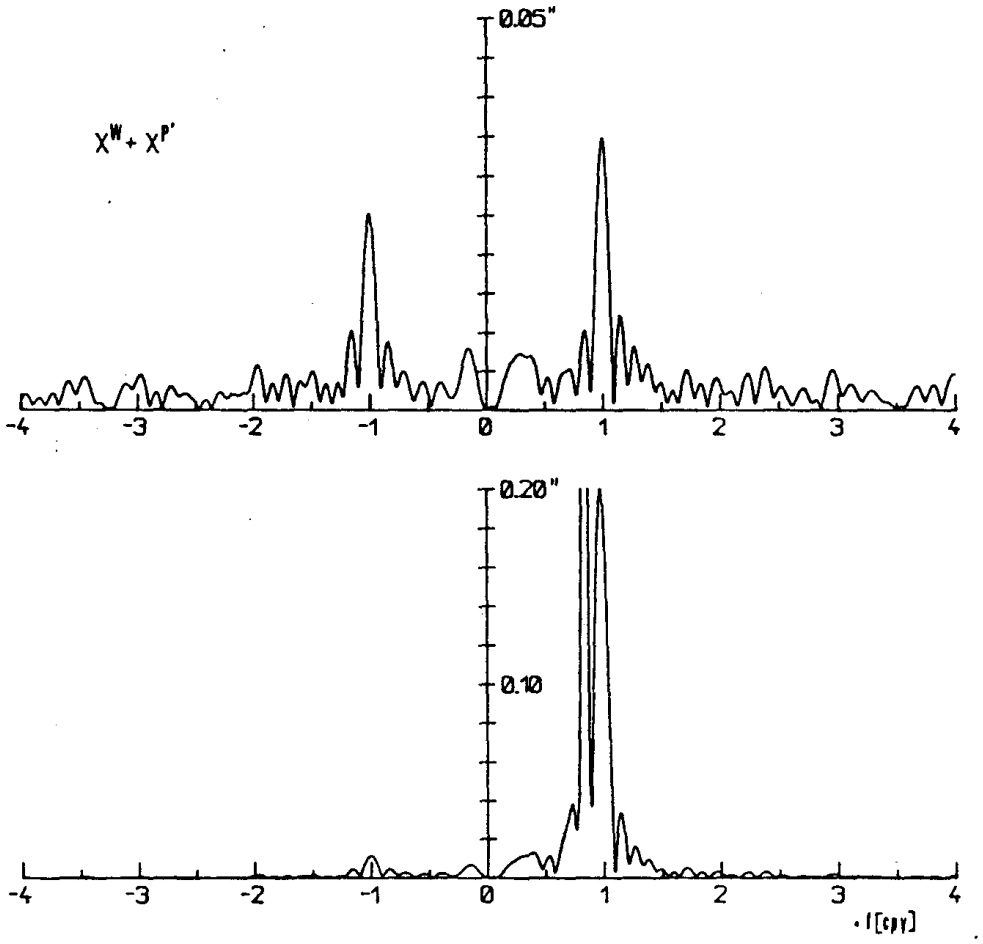
Slika 6. Spektar polarnog kretanja BIH-ovih podataka sa sl. 4 i njegova ponovo računata verzija posle deljenja prenosnom funkcijom F sa sl 5.



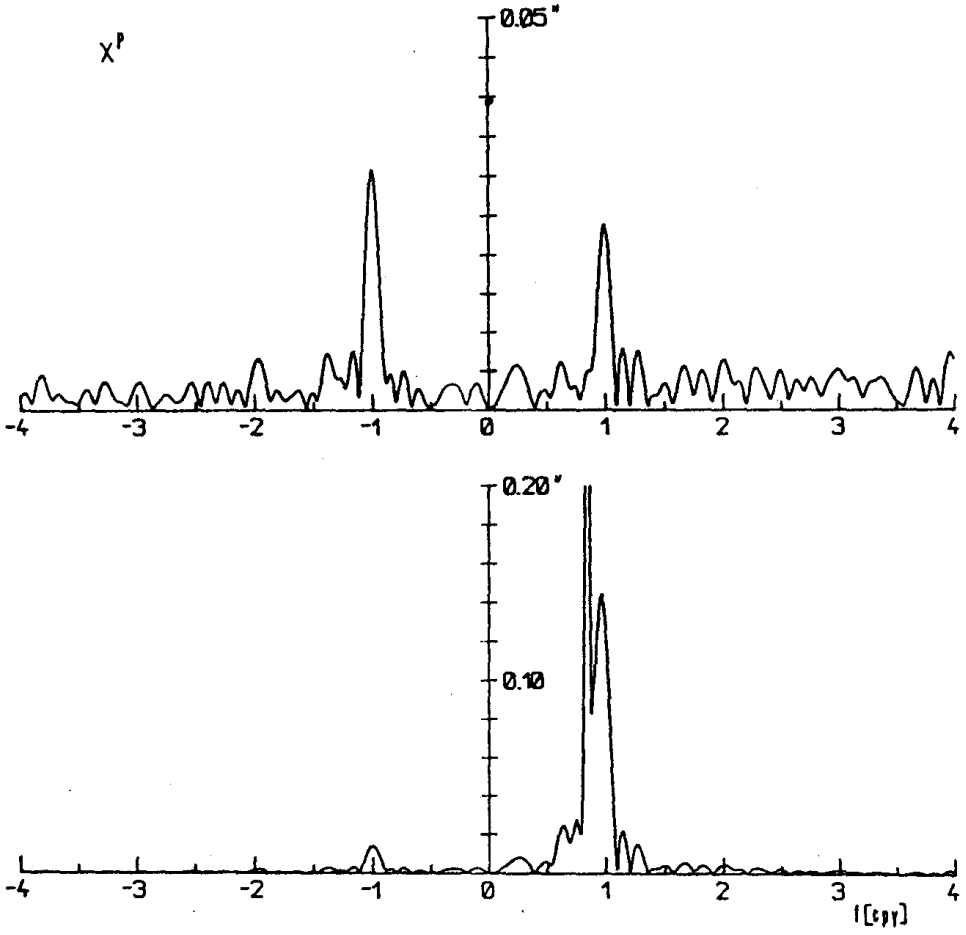
Slika 7a. Spektar ekvatorskih komponenata EAMF (članovi vetra + pritiska) i njegova ponovo računata verzija (tj. pomnožena prenosnom funkcijom F).



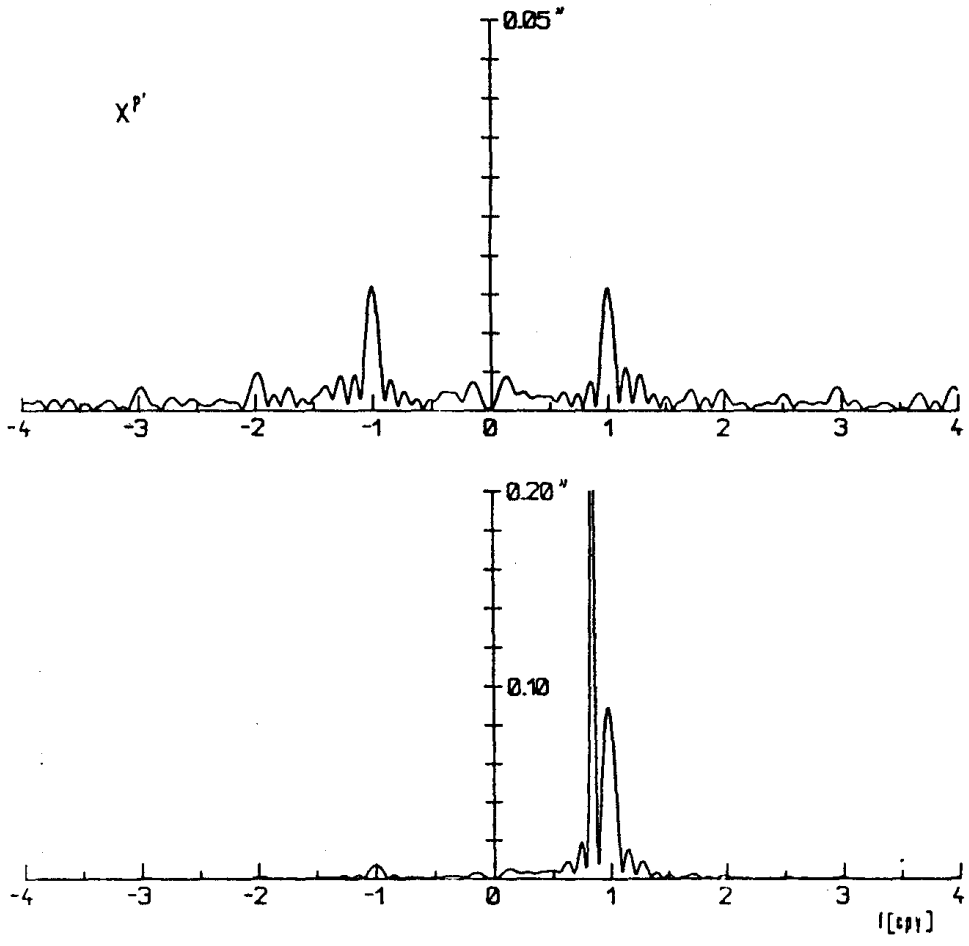
Slika 7b. Spektar ekvatorskih komponenata EAMF (članovi vetra + pritiska sa korekcijom invertnog barometra) i njegova ponovo računata verzija (tj. pomnožena prenosnom funkcijom F).



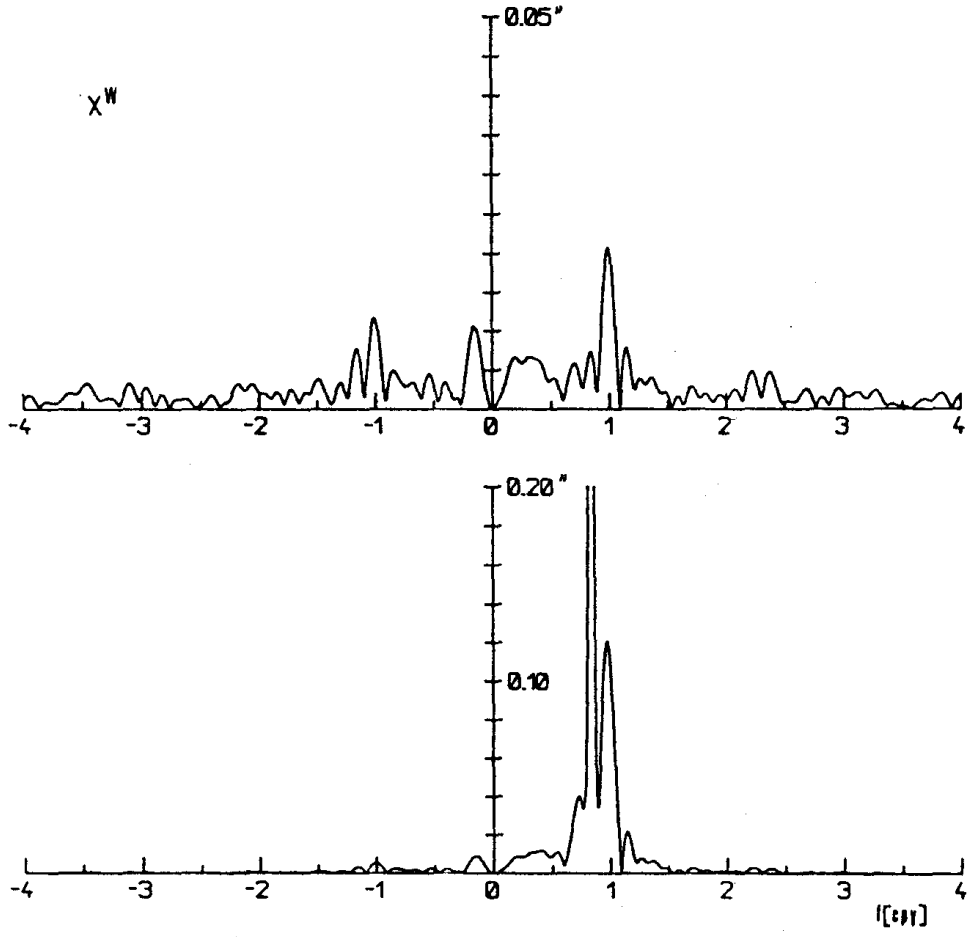
Slika 7c. Spektar ekvatorskih komponenata EAMF (članovi pritiska) i njegova ponovo računata verzija (tj. pomnožena prenosnom funkcijo F).



Slika 7d. Spektar ekvatorskih komponenata EAMF (članovi vetra sa korekcijom invertnog barometra) i njegova ponovo računata verzija (tj. pomnožena prenosnom funkcijom F).

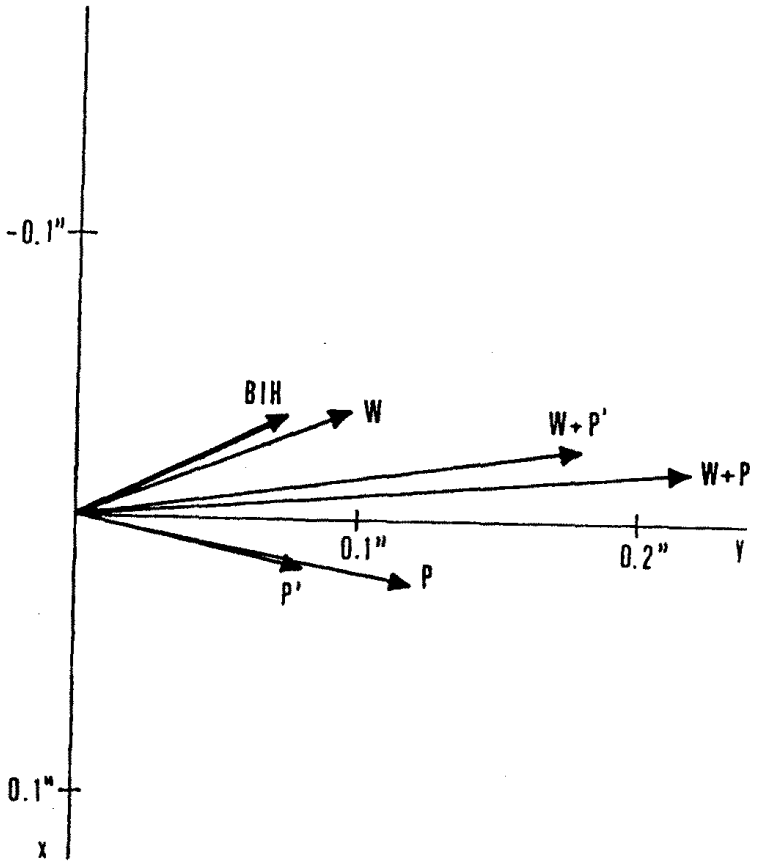


Slika 7e. Spektar ekvatorskih komponenata EAMF (članovi vetra) i njegova ponovo računata verzija (tj. pomnožena prenosnom funkcijom F).



razlike u fazi samo pritisak člana. Neki podaci iz tablice 2 su predstavljeni na sl. 9. Posmatrani godišnji član je označen sa BIH, vetar članovi u ponoć i podne sa W_0 i W_{12} a njihove kombinacije sa pritisak članom sa ATM. Verovatno je da vetar član ima značajnu ulogu u ekscitaciji polarnog kretanja, ali iziskuje dalja istraživanja, zasnovana na pouzdanijim meteorološkim podacima, bar podnevnim i ponoćnim. Takođe su mogući slični efekti, ne veliki, i u pritisak članu.

Slika 8. Godišnji član u posmatranom polarnom kretanju (BIH) i ekscitovanom sa članom vetra, ponoćne vrednosti (W), članom pritiska (P), članom pritiska sa korekcijom invertnog barometra (P') i njihovim kombinacijama (W+P, W+P').



Tablica 2.

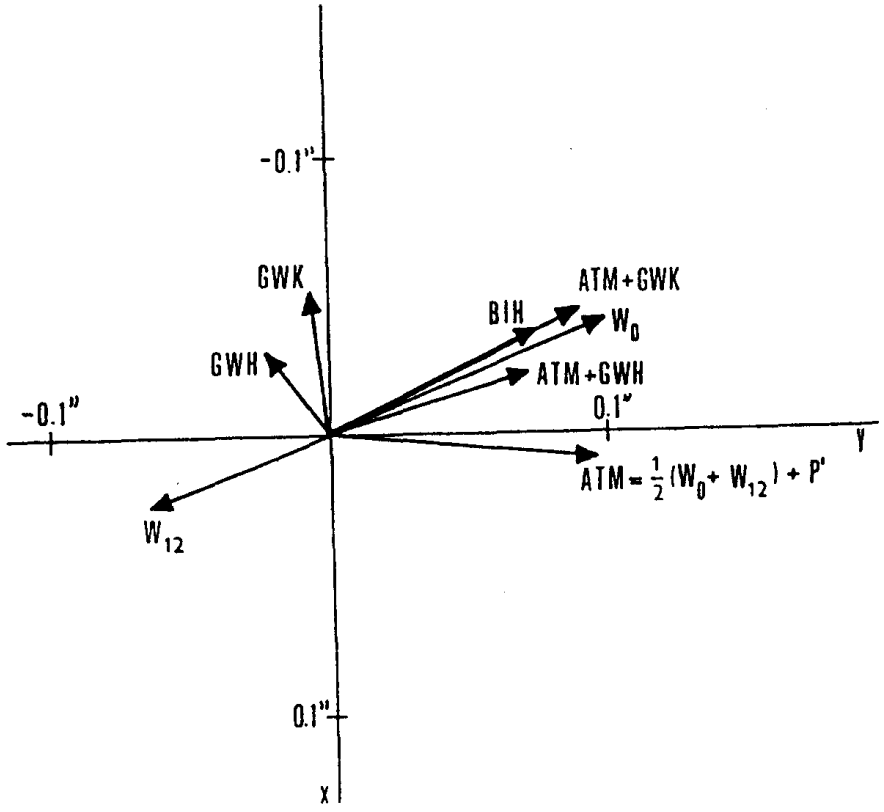
Ovde su dati parametri ekscitovanog godišnjeg kretanja sa članom vetra na 12^h UT (W_{12}), njegova kombinacija sa članom vetra na 0^h UT (W) i zbir ovih kombinacija i člana pritiska sa korekcijom invertnog barometra (ATM). Uticaj podzemnih voda koje daju Van Hylckama (GWH) i Kikuchi (GWK) i njihovi zbirovi sa atmosferskom ekscitacijom su ATM+GWH i ATM+GWK. Ostale oznake su iste kao u tablici 1.

	A^+ [″]	ϕ^+ [°]	A^- [″]	ϕ^- [°]	a [″]	b [″]	φ [°]	Ψ [°]
W_{12}	0.070	288	0.007	75	0.063	0.077	+2	291
W	0.019	122	0.003	30	0.016	0.022	+76	113
ATM	0.097	86	0.006	286	0.091	0.103	+6	85
GWH	0.036	224	0.004	146	0.032	0.040	+5	218
GWK	0.050	190	0.004	146	0.046	0.054	-12	187
ATM+GWH	0.074	105	0.004	256	0.070	0.078	0	107
ATM+GWK	0.101	115	0.004	256	0.097	0.105	+6	116

Pošto nijedna od kombinacija EAMF, koje smo razmatrali , ne može precizno da ekscituje posmatrano polarno kretanje na godišnjoj frekvenci potrebno je uvesti nove izvore ekscitacije različite od atmosfere. Male razlike (i u amplitudi i u fazi) između atmosferski ekscitovanog i posmatranog polarnog kretanja na godišnjoj frekvenci mogu biti uzrokovane globalnim promenama podzemne vode (pogledati na pr. Whar, 1983). Ekscitacionu funkciju koja dolazi od ovog izvora našli su Van Hilkama (Van Hylckama, 1956), Kikuči (Kikuchi, 1977); polarno kretanje ekscitovano sa ova dva, malo različita, godišnja člana prikazano je u tablici 2 i na sl.9 (GWH i GWK),kao i njihovi zbirovi sa atmosferskom ekscitacijom (ATM + GWH i ATM + GWK).

Sa slike 9 se vidi da oba ova rezultata ATM+GWH i ATM+GWK su vrlo blizu posmatranog polarnog kretanja . S toga se, možemo nadati da će u budućnosti biti organizovane službe za praćenje promena podzemne vode kao već postojeći opremljeni meteorološki centri za atmosferu. Dolazimo do zaključka da godišnje polarno kretanje može da se objasni kombinovanim efektima atmosfere i globalnih promena podzemne vode.

Slika 9. Godišnji član u posmatranom polarnom kretanju (BIH) i ekscitovanom sa članom vetra, ponoćne i podnevne vrednosti (W_0 i W_12), njihovim kombinacijama sa članom pritiska sa korekcijom invertnog barometra (ATM), podzemnim vodama prema Van Hilkami i Kikučiju (GWH i GWK) i njihovim kombinacijama (ATM+GWH, ATM+GWK).



6.2 UPOREĐENJE NOVOG MODELA ZEMLJE SA POSMATRANIM POLARNIM KRETANJEM

Spektralna analiza u odeljku 6.1 je pokazala da je najbolja saglasnost između posmatrane i ekscitovane amplitude polarnog kretanja na godišnjoj frekvenci postignuta za član pritiska sa korekcijom invertnog barometra i dodatim samo malim iznosom vetar člana (određenog kao aritmetička sredina ponoćnih i podnevnih podataka). Saglasnost se poboljšava dodavanjem uticaja promena podzemne vode (pogledati predhodni odeljak). Ovde ćemo izvršiti numeričku integraciju polarnog kretanja, korišćenjem ekvatorskih komponenata atmosfere (EAMF) datim jednačinom (35).Pošto je, iz upoređenja podataka sa različitih meteoroloških centara i iz upoređenja ponoćnih i podnevnih podataka (Eubanks i dr., 1985), nađeno da je član vetra prilično šuman i kako ne raspolažemo ponoćnim i podnevnim vrednostima to ćemo ga u ovoj analizi zanemariti. Detaljne informacije o promeni podzemne vode nećemo koristiti, već ćemo njihov uticaj modelovati sa godišnjim članom izvedenim od Van Hilkame (Van Hylckama, 1956) i Kikučija (Kikuchi, 1977).

Reakcija novog modela Zemlje (fluidno jezgro, visko-elastični omotač i dinamički okean) na atmosfersku ekscitaciju izvedena je u odeljku 4.1 (jednačine 44). Za numeričko izračunavanje integrala, neophodnih za određivanje J_i (jednačine 43), ekvatorske komponente atmosfere EAMF mogu se direktno koristiti. Početni položaj integraljene putanje pola, x_0, y_0 može se izabrati potpuno proizvoljno, pošto obe veličine predstavljaju konstante integracije. Numerički ćemo izračunavati integral J (dat jednačinom 40), u kojem je A totalna amplituda polarnog kretanja određena izrazom:

$$A = \sqrt{(x - xo)^2 + (y - yo)^2}$$

gde su xo, yo koordinate srednjeg pola.

Rezultati numeričke integracije na intervalu 9.5 godina (1976.5-1986.0), sa vrednošću za faktor $Q = 100$, predstavljeni su na slikama od 10a do 12b. Tamnije linije (a) prikazuju posmatrano polarno kretanje a svetlije linije (b) predstavljaju putanu pola integraljenu sa početnim položajem (x_0, y_0) , datim u tablici 3, dobijenim iz uslova da je srednjekvadratsko odstupanje između dve krive minimalno. Na slikama 10a,10b predstavljena je integracija samo pritisak člana sa korekcijom invertnog barometra, dok su na slikama 11a,11b i 12a,12b izloženi kombinovani efekti pritisak člana (sa korekcijom invertnog barometra) sa ekscitacijom podzemne vode datom od Van Hilkame (Van Hylckama, 1956) i Kikučija (Kikuchi, 1977), redom. U oba ova slučaja korišćene su samo godišnje ekscitacije, uzrokovane promenom podzemne vode, date jednačinama:

$$\Delta\chi_1 = (A^+ + A^-)\cos 2\pi t + (B^- - B^+)\sin 2\pi t$$

$$\Delta\chi_2 = (B^+ + B^-)\cos 2\pi t + (A^+ - A^-)\sin 2\pi t \qquad (57)$$

sa sledećim numeričkim konstantama:

Tablica 3.

	A^+ ["]	B^+ ["]	A^- ["]	B^- ["]
Van Hylckama	0.0049	0.0048	-0.0060	0.0044
Kikuchi	0.0094	0.0018	-0.0063	0.0041

Vidimo da sve slike prikazuju relativno dobru saglasnost između posmatranih i integraljenih krivih. Doprinos poboljšanju fitovanja od promena podzemne vode najočigledniji je na centru intervala, između 1980 i 1981 godine, kada je amplituda polarnog kretanja bila vrlo mala, naročito za podzemne vode po Kikučiju, slike 12a,12b. Bolja ocena postignutog fitovanja može se dati srednjekvadratskim odstupanjem, δ, između dve krive, koje računamo po formuli:

$$\delta = \sqrt{\frac{\Sigma\nu_1^2(t) + \Sigma\nu_2^2(t)}{2n}} \tag{58}$$

gde su :

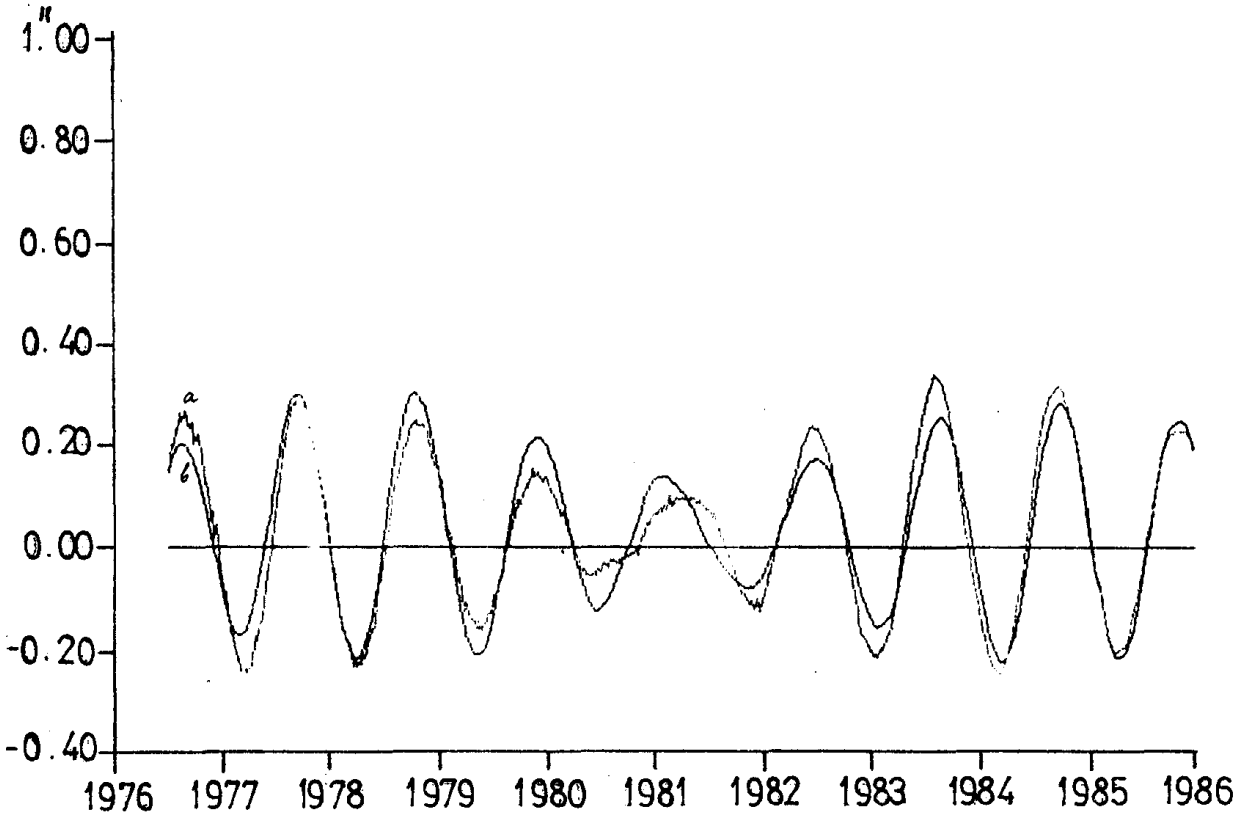
$$\nu_1(t) = x_{BIH}(t) - m_1[\chi(t)]$$

$$\nu_2(t) = y_{BIH}(t) - m_2[\chi(t)]$$

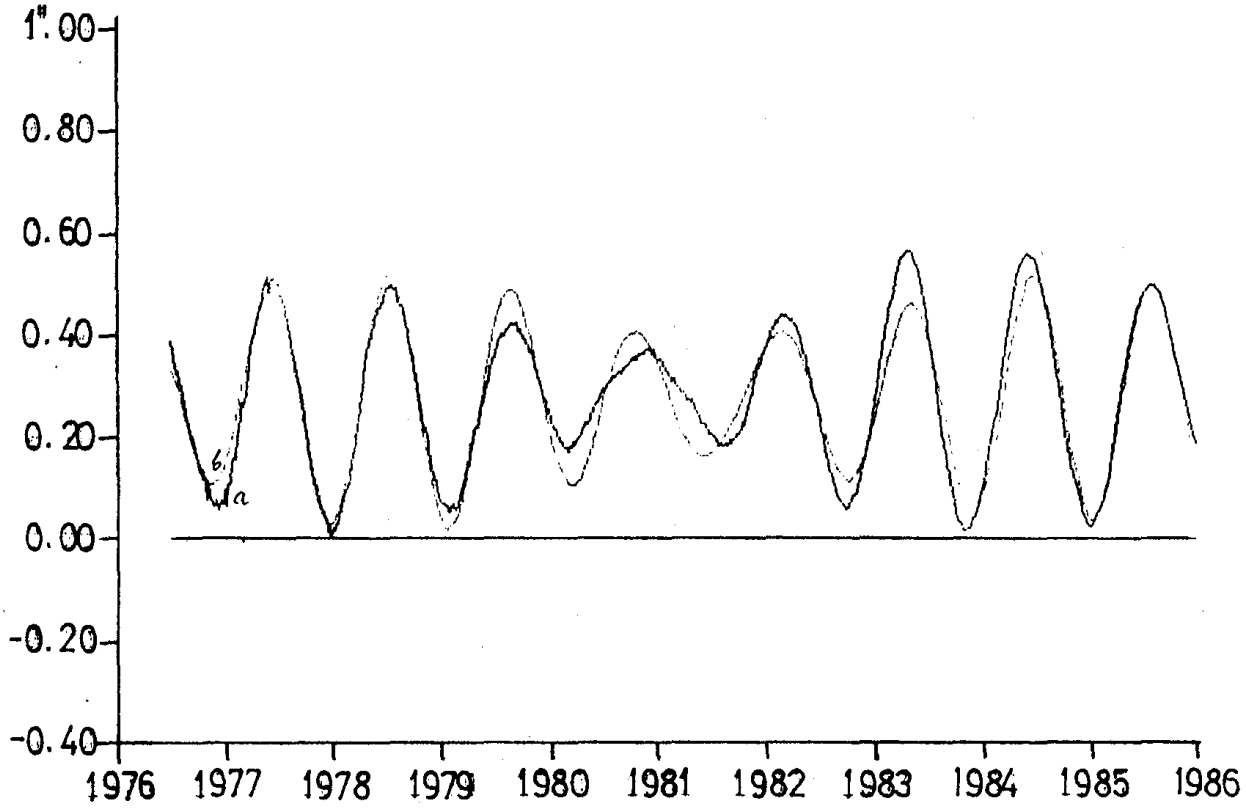
razlike posmatrane i integraljene krive u svakom trenutku vremena t na posmatranom intervalu.

U tablici 4 dato je srednjekvadratsko odstupanje δ^{NM}, gde indeksi NM označavaju novi model, između posmatrane i integraljene krive: $\delta^{NM} = \pm 0".047$ (samo pritisak član); $\delta_{VH}^{NM} = \pm 0".040$ (član pritiska plus Van Hilkama) i $\delta_K^{NM} = \pm 0".037$ (član pritiska plus Kikuči). Dobijeno poboljšanje koje je doprinos dodate ekscitacije podzemne vode, naročito po Kikučiju (Kikuchi, 1977) vidi se iz ovih brojeva. Još jedna karakteristika ovih crteža je sekularni trend u razlici između amplituda posmatranog i integraljenog polarnog kre-

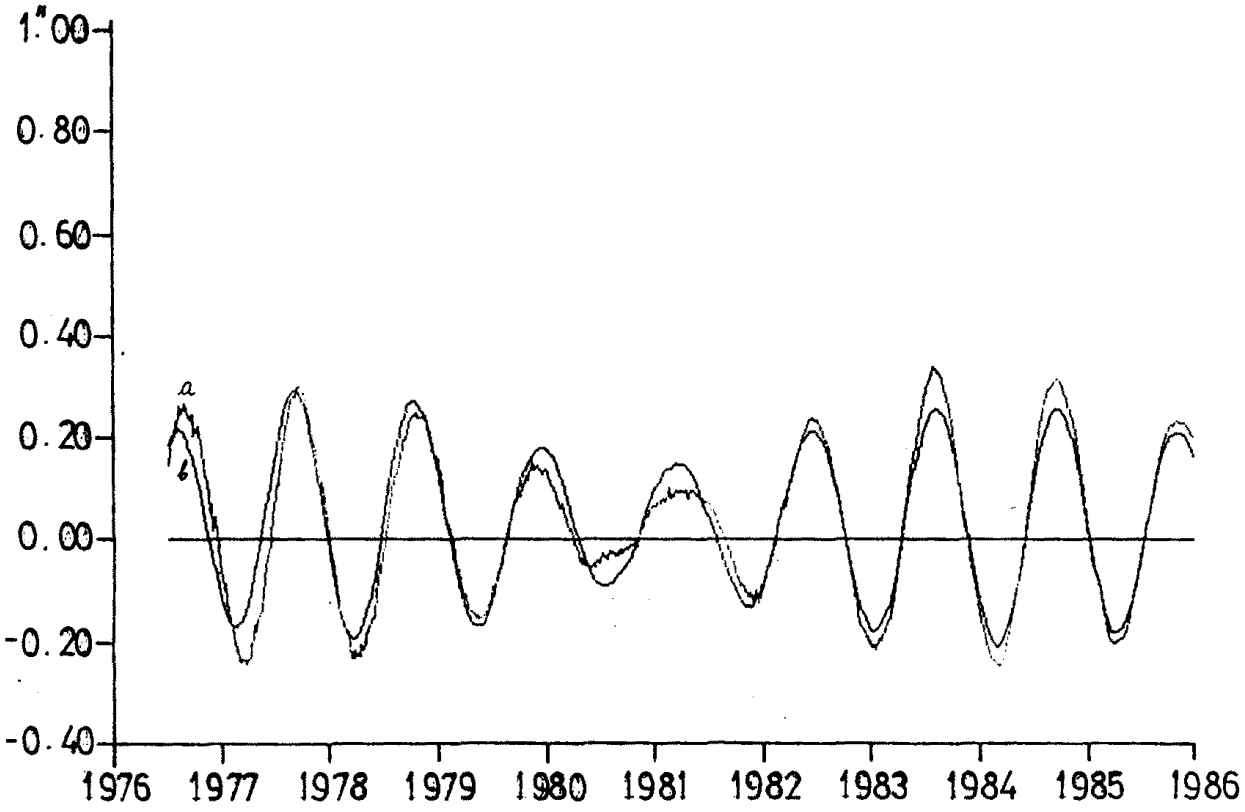
Slika 10a. x komponenta posmatranog (a) i ekscitovanog (b) polarnog kretanja. Za integraciju ekscitovane putanje pola korišćen je atmosferski član pritiska sa korekcijom invertnog barometra i novi model sa amplitudno zavisnom Čendlerovom frekvencom.



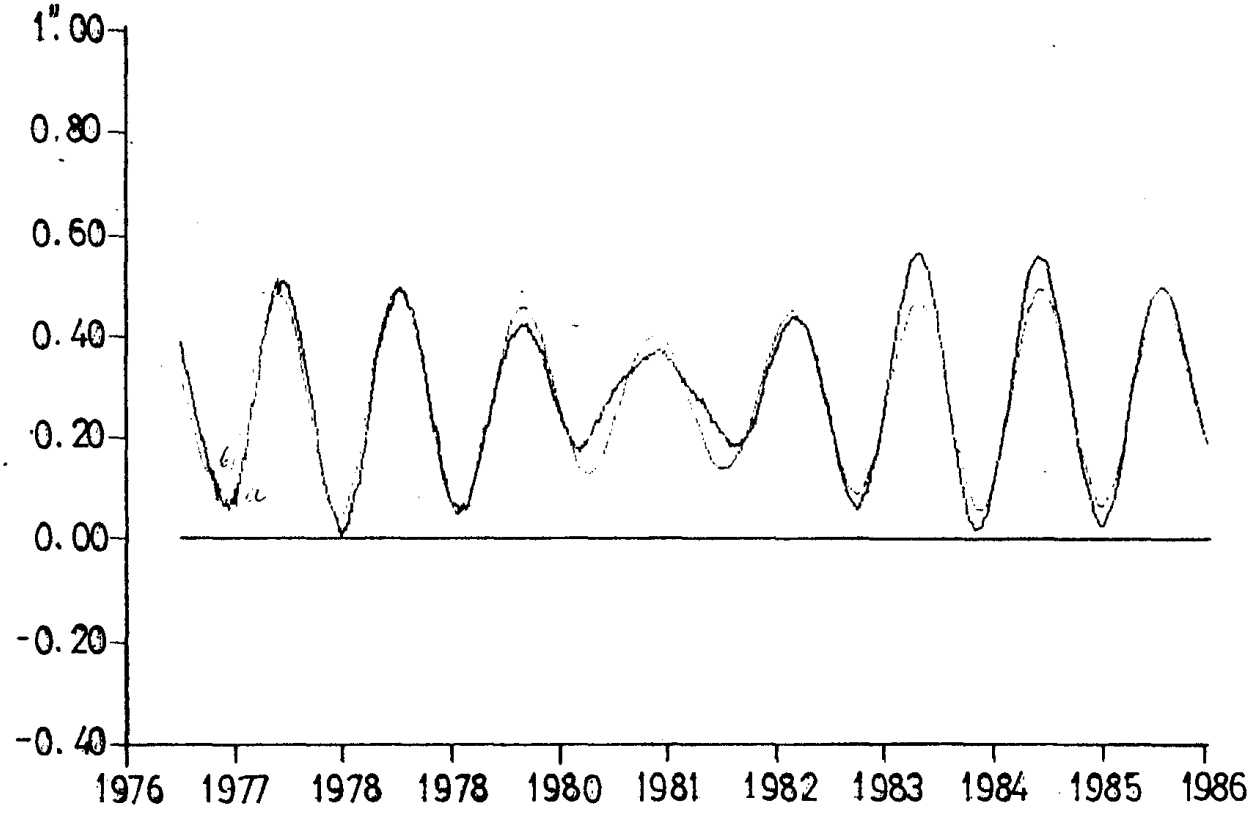
Slika 10b. y komponenta posmatranog (a) i ekscitovanog (b) polarnog kretanja. Za integraciju ekscitovane putanje pola korišćen je atmosferski član pritiska sa korekcijom invertnog barometra i novi model sa amplitudno zavisnom Čendlerovom frekvencom.



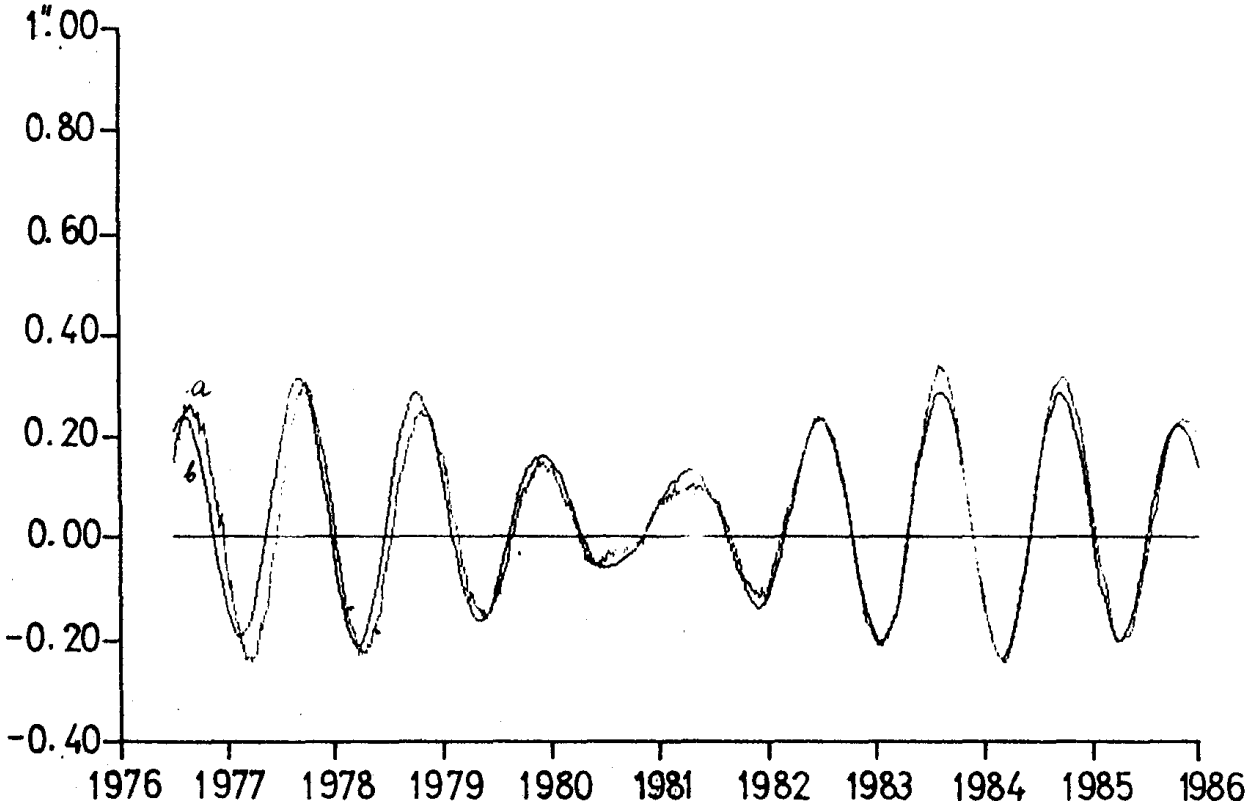
Slika 11a. Isto kao na sl. 10a, s tim što je atmosferskoj ekscitaciji dodata ekscitacija podzemne vode prema Van Hilkami.



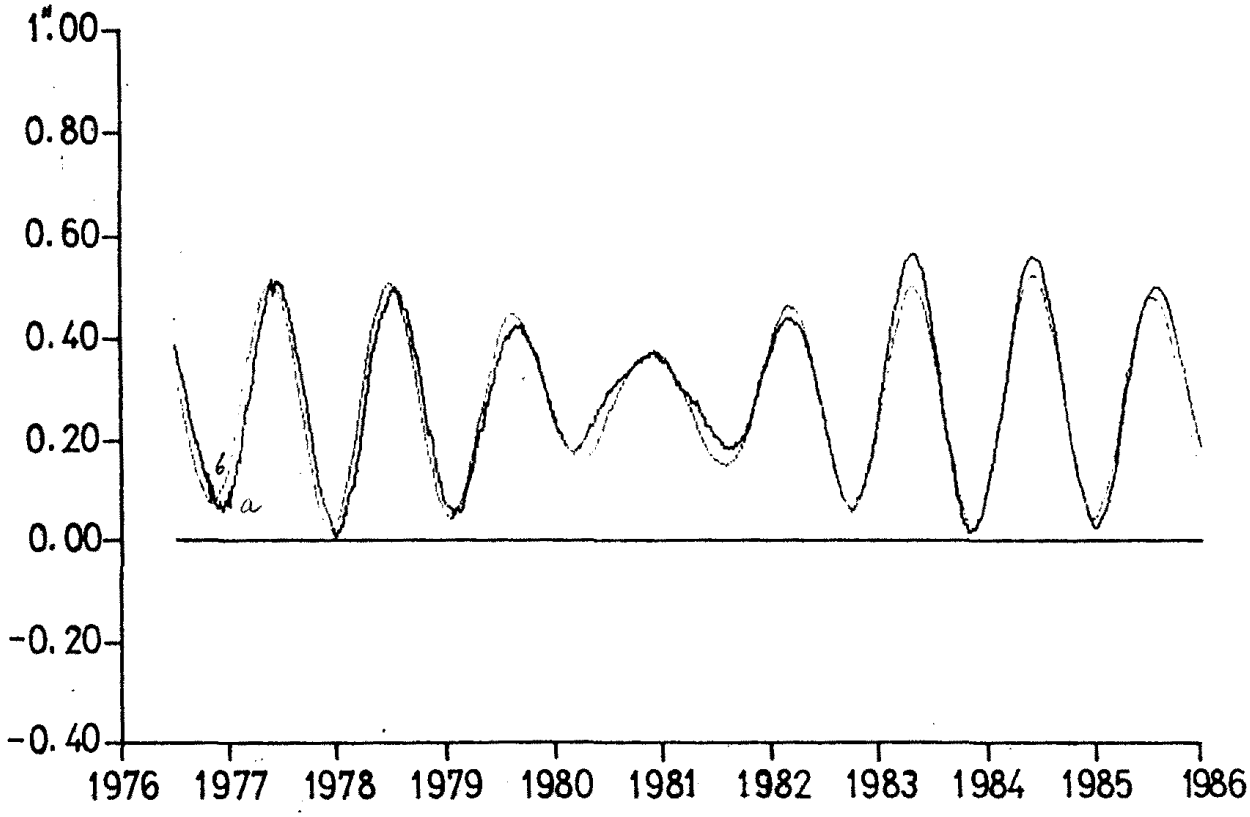
Slika 11b. Isto kao na sl. 10b, s tim što je atmosferskoj ekscitaciji dodata ekscitacija podzemne vode prema Van Hilkami.



Slika 12a. Isto kao na sl. 10a, s tim što je atmosferskoj ekscitaciji dodata ekscitacija podzemne vode prema Kikučiju.



Slika 12b. Isto kao na sl. 10b, s tim što je atmosferskoj ekscitaciji dodata ekscitacija podzemne vode prema Kikučiju.



tanja; posmatrana amplituda kao da raste sa vremenom u odnosu na amplitudu integraljene krive. Ta razlika se malo smanjuje uključivanjem podzemnih voda (vidi se iz upoređenja slika 10a,10b sa slikama 12a,12b) što predstavlja još jedan doprinos poboljšanju fitovanja. Ekscitacija blizu Čendlerove frekvence, koja nije sadržana u EAMF atmosfere, mogla bi objasniti ovaj porast u posmatranoj amplitudi. Kompletniji podaci o podzemnim vodama mogu da imaju takvu ekscitaciju (pogledati Wilson i Hinnov, 1985), zato je neophodna služba za praćenje promena podzemne vode, sličnoj kao za atmosferske EAMF.

6.3 UPOREĐENJE NOVOG MODELA SA VONDRAKOVIM MODELOM ZEMLJE

Da bi ocenili opravdanost uvođenja novog modela sa promenljivom i amplitudno zavisnom Čendlerovom frekvencom uporedićemo ga sa Vondrakovim (Vondrak, 1987) modelom sa konstantnom Čendlerovom frekvencom.

Kao u prethodnom paragrafu, i ovde ćemo izvršiti numeričku integraciju polarnog kretanja po Vondrakovim jednačinama (47), korišćenjem izraza (46) za $\tilde{J}_1, \tilde{J}_2, \tilde{J}_3, \tilde{J}_4$. Dobijeni rezultati predstavljeni su na slikama od 13a do 15b. Tamnije linije (a) prikazuju BIH-ovo posmatrano polarno kretanje, a svetlije linije (b) predstavljaju putanju pola integraljenu sa početnom pozicijom x_0, y_0 (tablica 4) dobijenom iz uslova da je srednjekvadratsko odstupanje δ^{Von} između dve krive minimalno. Na slikama 13a,13b predstavljena je integracija samo pritisak člana sa korekcijom invertnog barometra, dok su na slikama 14a,14b i 15a,15b predstavljeni kombinovani efekti pritisak člana (sa korekcijom invertnog barometra) i člana podzemne vode po Van Hilkami (Van Hylckama, 1956) i Kikučiju (Kikuchi, 1977). Ocenu fitovanja daćemo određivanjem srednjekvadratskog odstupanja δ^{Von} između posmatrane i integraljene krive polarnog kretanja. Ove vrednosti, kao i vrednosti za novi model, date su u tablici 4.

Tablica 4

Srednjekvadratsko odstupanje δ: δ^{NM} (novi model), δ^{Von} (Vondrakov model) i x_0, y_0-početne pozicije pola računate iz uslova $\delta = min$.

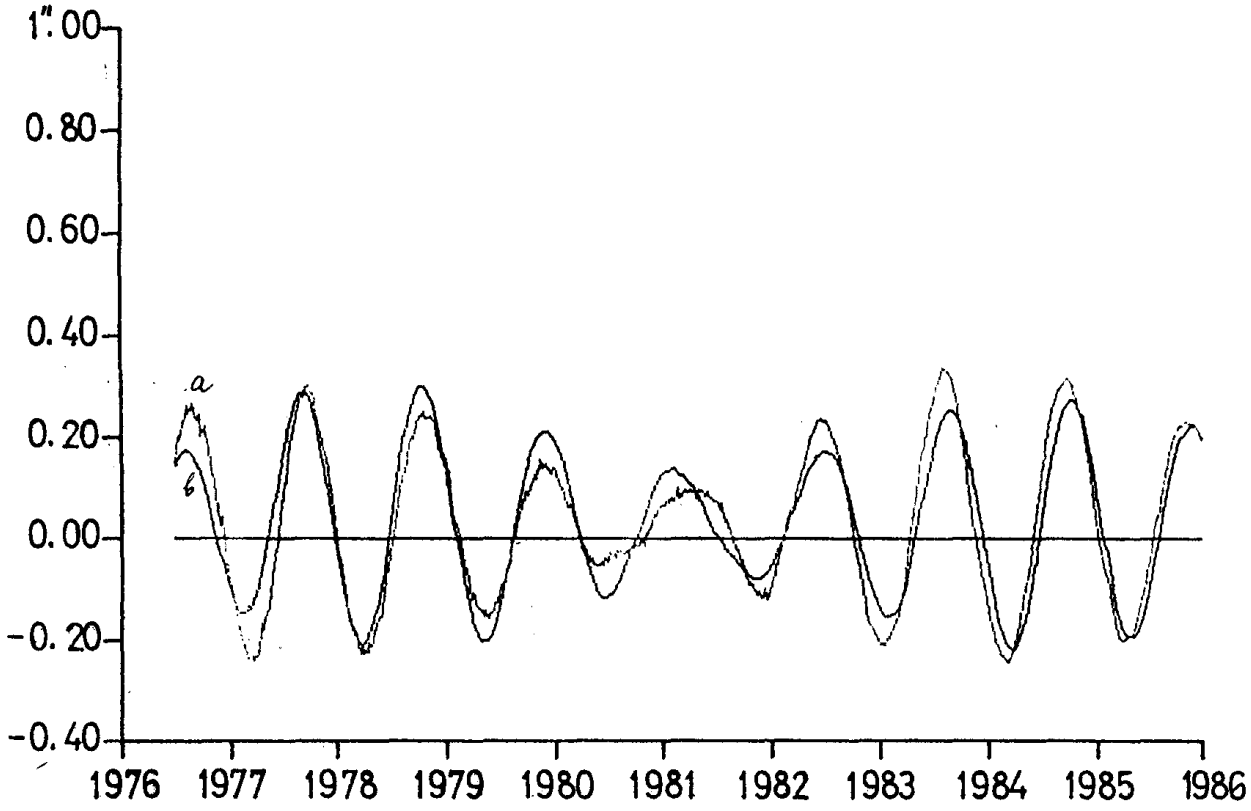
$\sigma_t = \sigma_0 + 0.00149e^{-20A}$ $\qquad$ $\sigma_0 = const$

	δ^{NM} ["]	x_0 ["]	y_0 ["]	δ^{Von} ["]	x_0 ["]	y_0 ["]
$\chi^{p'}$	±0.047	0.112	0.250	±0.057	0.107	0.292
$\chi^{p'} + \Delta\chi_{VH}$	±0.040	0.145	0.265	±0.052	0.142	0.300
$\chi^{p'} + \Delta\chi_K$	±0.037	0.165	0.260	±0.046	0.162	0.282

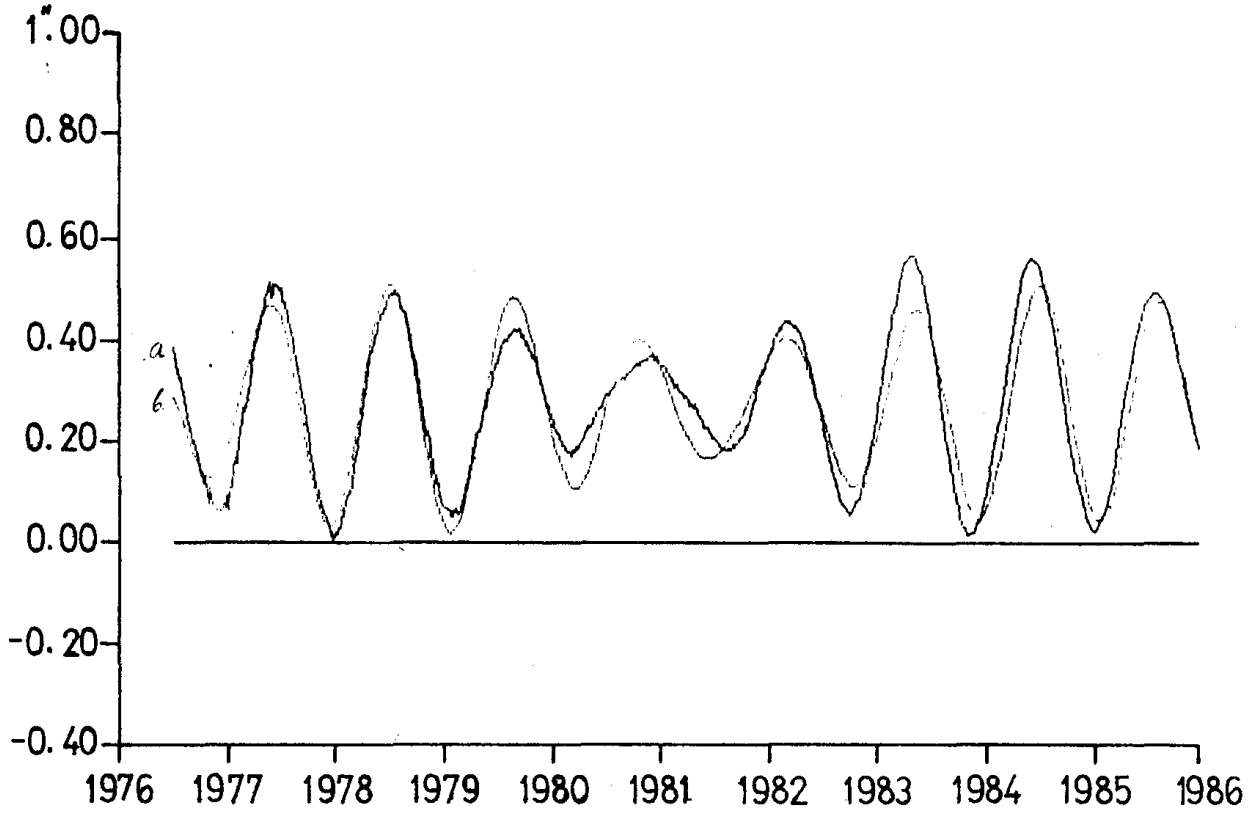
Iz tablice 4 vidimo da su za Vondrakov model razlike između dveju (integraljenog i posmatranog polarnog kretanja) krivih±0".057 a za novi model ±0".047; što predstavlja poboljšanje fitovanja sa novim modelom. Odavde dolazimo do zaključka da je doprinos dinamičkog okeana odgovoran za posmatrano polarno kretanje,kao i nama nepoznate ekscitacije takvog oblika. Ovo opravdava naš pokušaj uvođenja novog modela sa promenljivom i amplitudno zavisnom Čendlerovom frekvencom. Doprinos poboljšanju fitovanja od ekscitacije uzrokovene promenama podzemne vode je, takođe, znatan. Novi model Zemlje sa slobodnom Čendlerovom frekvencom zavisnom od totalne amplitude polarnog kretanja sa podzemnom vodom od Kikučija donosi poboljšanje fitovanja karakterisano srednjekvadratskim odstupanjem između dveju krivih od samo ±0".037.

Odavde imamo konačan zaključak da doprinos atmosferske ekscitacije i ekscitacije podzemne vode je odgovoran za posmatrano polarno kretanje i da naš novi model Zemlje sa promenljivom i amplitudno zavisnom Čendlerovom frekvencom je potvrđen čak za kratak vremenski interval.

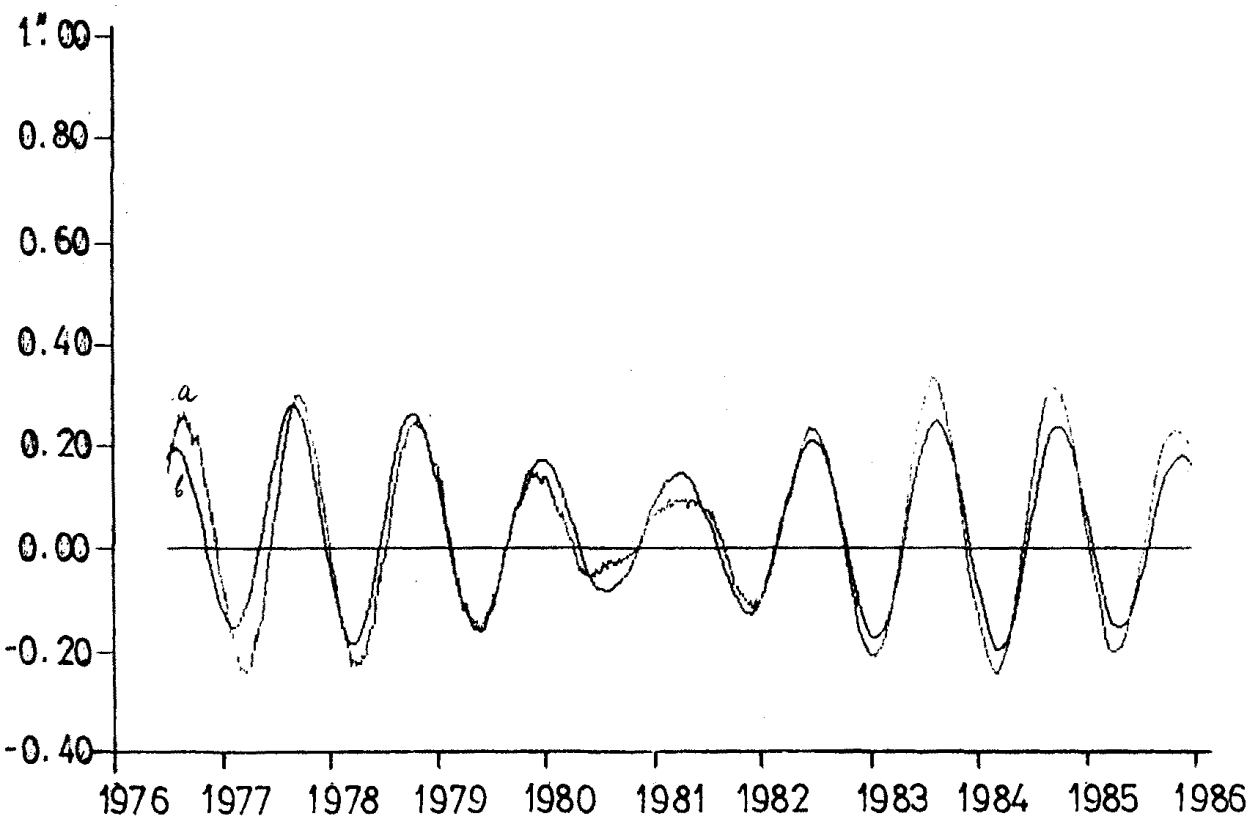
Slika 19a. x komponenta posmatranog (a) i ekscitovanog (b) polarnog kretanja. Za integraciju ekscitovane putanje pola korišćen je atmosferski član pritiska sa korekcijom invertnog barometra i Vondrakov model sa konstantnom Čendlerovom frekvencom.



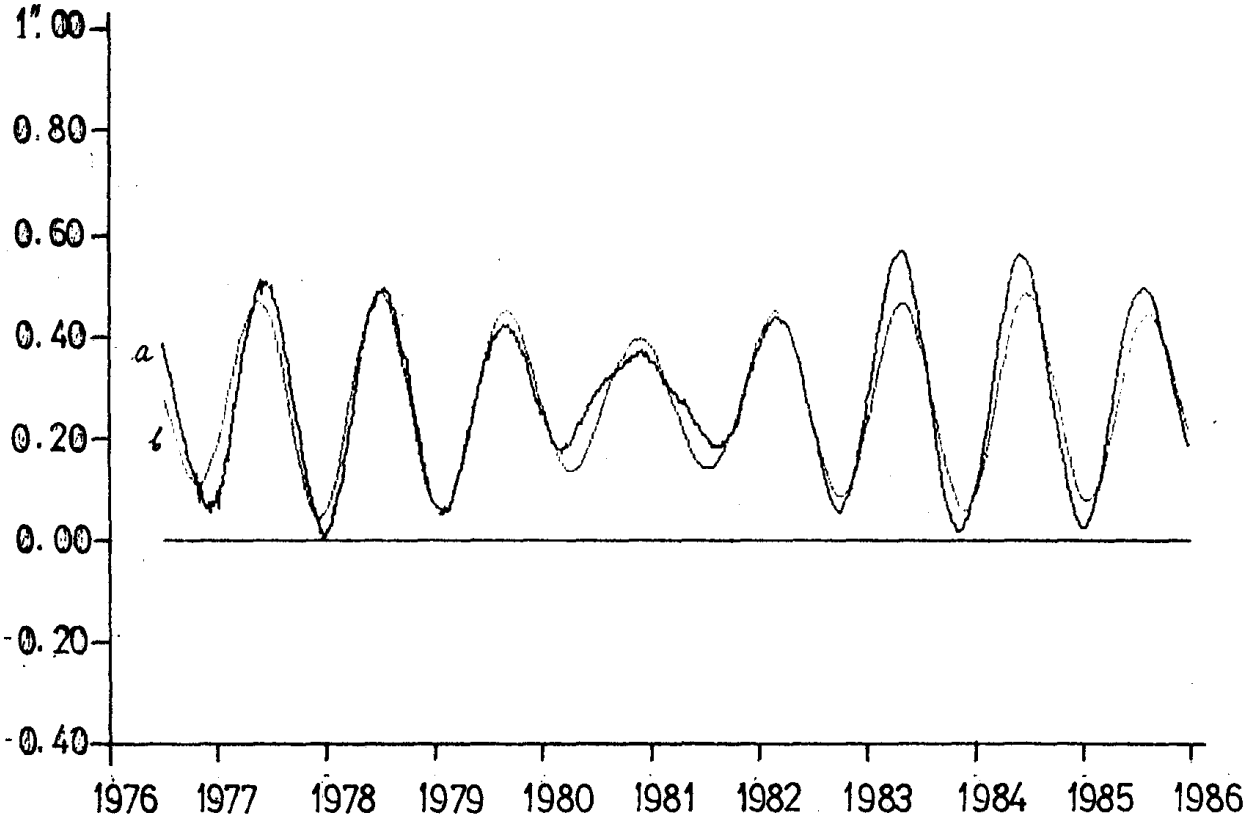
Slika 19b. y komponenta posmatranog (a) i ekscitovanog (b) polarnog kretanja. Za integraciju ekscitovane putanje pola korišćen je atmosferski član pritiska sa korekcijo invertnog barometra i Vondrakov model sa konstantnom Čendlerovom frekvencom.



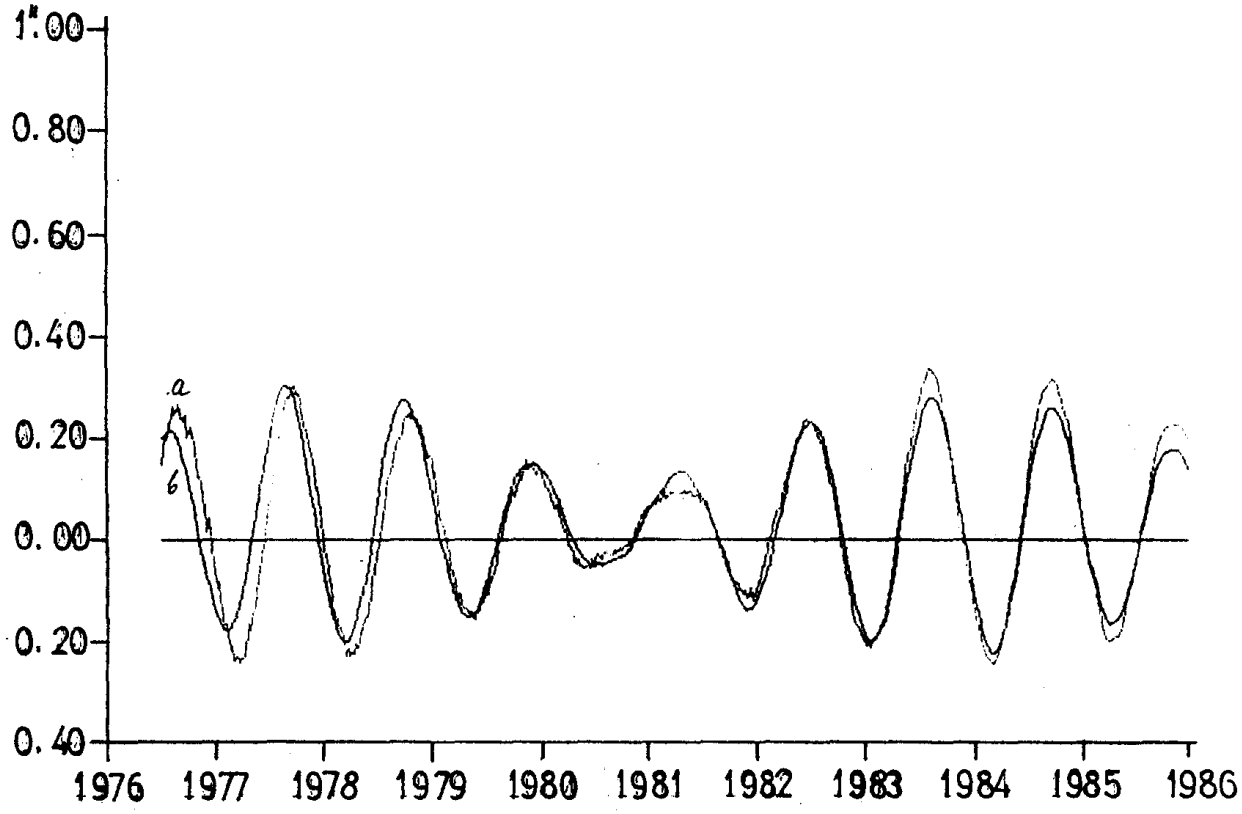
Slika 14a. Isto kao na sl. 13a, s tim što je atmosferskoj ekscitaciji dodata ekscitacija podzemne vode prema Van Hilkami.



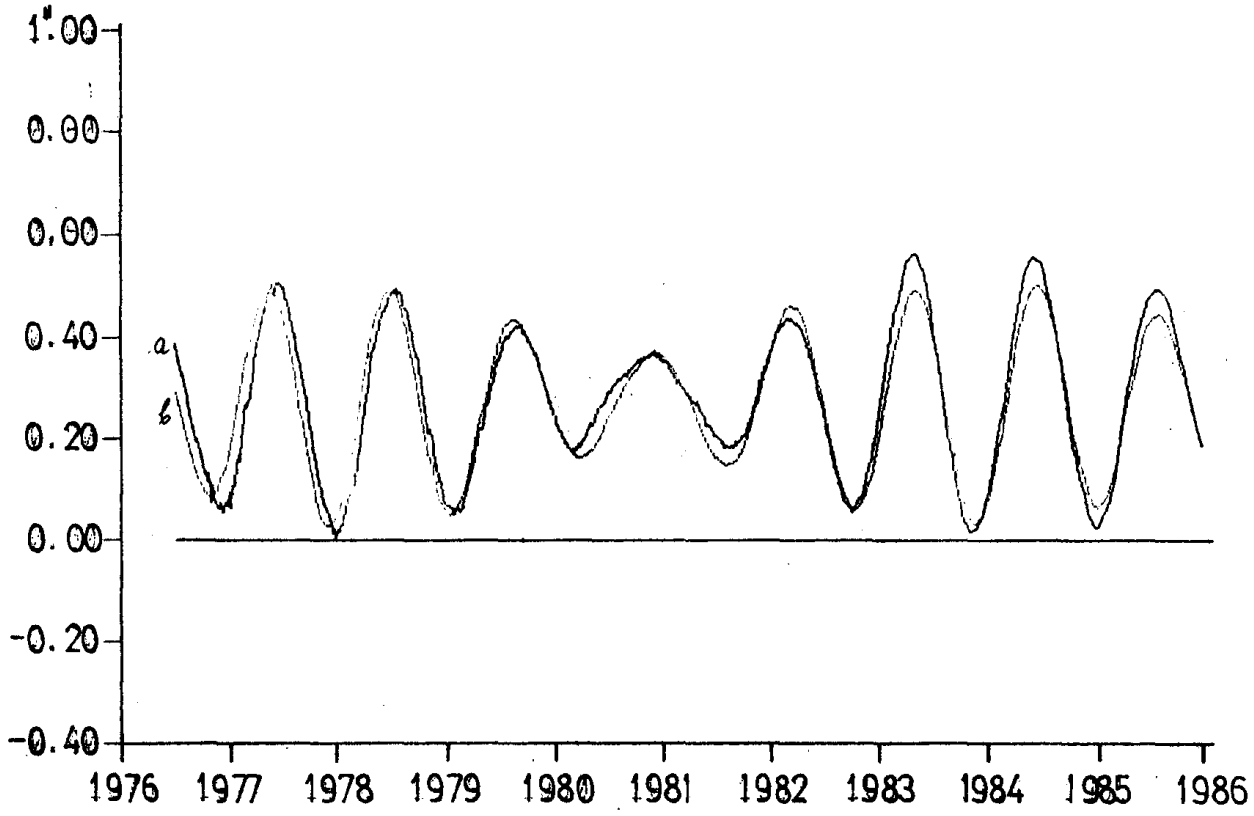
Slika 14b. Isto kao na sl. 13b, s tim što je atmosferskoj ekscitaciji dodata ekscitacija podzemne vode prema Van Hilkami.



Slika 15a. Isto kao na sl. 13a, s tim što je atmosferskoj ekscitaciji dodata ekscitacija podzemne vode prema Kikučiju.



Slika 15b. Isto kao na sl. 13b, s tim što je atmosferskoj ekscitaciji dodata ekscitacija podzemne vode prema Kikučiju.



6.4 UPOREĐENJE SPEKTRA UT1-UTC SA SPEKTROM χ_3

Funkcija osnog ugaonog momenta (axial angular momentum function) atmosfere χ_3 u kombinacujama a), b) i e) (datim varijantama 48*) predstavljena je na donjim delovima slika 1, 2 i 3. Relativne promene trajanja dana (LOD) u odnosu na nominalnih 86400 *sec* računatih iz BIH-ovih publikovanih vrednosti UT1-UTC, predstavljene su na donjem delu slike 4. Relacija između LOD i χ_3 data je jednačinom (38*):

$$0.993\chi_3 + const = -\frac{\Delta LOD}{LOD_0} = \frac{d}{dt}(UT1 - UTC) \tag{59}$$

Na osnovu jednačine (59) možemo napisati UT1-UTC aproksimaciju koja glasi:

$$UT1 - UTC = A + Bt + CT^2 + \int_0^t \chi_3 dt + (CT) \tag{60}$$

u kojoj su sa CT označena 62 plimska člana određena od Jodera (Yoder i dr., 1981), A, B, C su konstante određene metodom najmanjih kvadrata (MNK), njihove vrednosti su date u tablici 5.

Tablica 5.

A, B, C **su konstante aproksimacije UT1-UTC računate MNK metodom za sve tri varijante** χ_3

	A	B	C
χ_3^{w+p}	0.0135324	-0.0011113	0.00000015
$\chi_3^{w+p'}$	0.0192252	-0.0012234	0.00000013
χ_3^{w}	0.0375353	-0.0003767	0.00000019

Ako diferenciramo po vremenu jednačinu (60) dobićemo LOD aproksimaciju koja glasi:

$$\frac{d}{dt}(UT1 - UTC) = -LOD = -(a + bt + \chi_3 + CD), \tag{61}$$

u kojoj su $CD = \frac{d}{dt}(CT)$ plimski članovi (Yoder i dr., 1981), a, b su konstante određene metodom MNK. Dobijene vredosti date su u tablici 6.

Tablica 6.

a, b **koeficijenti određeni metodom MNK za LOD aproksimaciju i** δ **je srednjekvadratno odstupanje.**

	a	b	δ
χ_3^{w+p}	0.0011815	-0.0000003	$\pm 0^s.00028$
$\chi_3^{w+p'}$	0.0012905	-0.0000003	$\pm 0^s.00027$
χ_3^{w}	0.0004083	-0.0000004	$\pm 0^s.00029$

Iz tablica 5 i 6 se vidi da za koeficijente a, b, A, B, C dobijamo približne vrednosti koje slede iz diferenciranja izraza (60) da bi se dobio izraz (61), tj. $-a \approx B$, $-b \approx 2C$.

Na slici 16 (član vetra plus član pritiska), slici 17 (član vetra plus član pritiska sa korekcijom invertnog barometra) i slici 18 (samo član vetra) predstavljene su sve tri kombinacije χ_3 u LOD aproksimaciji (61) i posmatrani LOD. Sa ovih slika vidimo da je saglasnost između dveju krivih izvanredna. Da bi bolje ocenili ovu saglasnost računali smo srednjekvadratno odstupanje δ između dve krive (posmatranog LOD i atmosferskoe funkcije χ_3) po jednačini :

$$\delta = \sqrt{\frac{\Sigma\nu^2(t)}{n-2}}$$

gde je $\nu(t) = LOD - (a + bt + \chi_3 + CD), n$ je broj podataka.

Dobijene vrednosti za δ za sve tri razmatrane varijante χ_3 date su u tablici 6. Na osnovu ove tablice vidimo da je postignuta dobra saglasnost u sva tri sličaja. Iz postignute dobre saglasnosti između krivih LOD i χ_3 zaključujemo da je osna okeanska funkcija $\Delta\Psi_3^o$ data jednačinom (21), koju smo uzeli od Jodera (Yoder i dr., 1981) dobro određena, i da sa njom dobijena treća jednačina modela Zemlje (38) dobro fituje atmosfersku funkciju χ_3 sa posmatranim LOD.

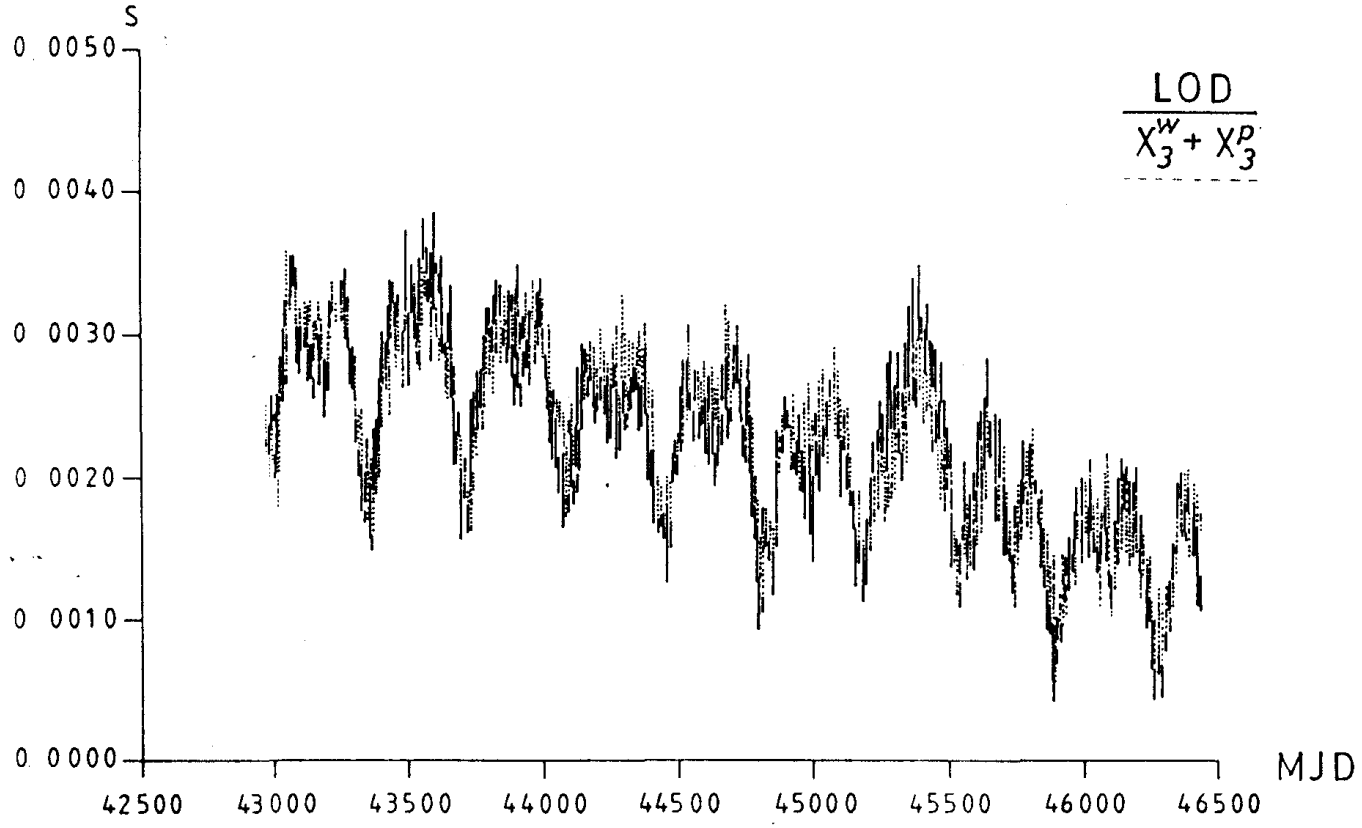
Upoređenje posmatranog LOD sa atmosferskim χ_3 daćemo i upoređenjem njihovih spektara.

Pošto je srednjekvadratno odstupanje između dveju krivih (LOD i χ_3 za sve tri varijante χ_3 (tabl. 6.) oko $\pm 0^s.0003$ to ćemo spektralnu analizu raditi za sve tri varijante.

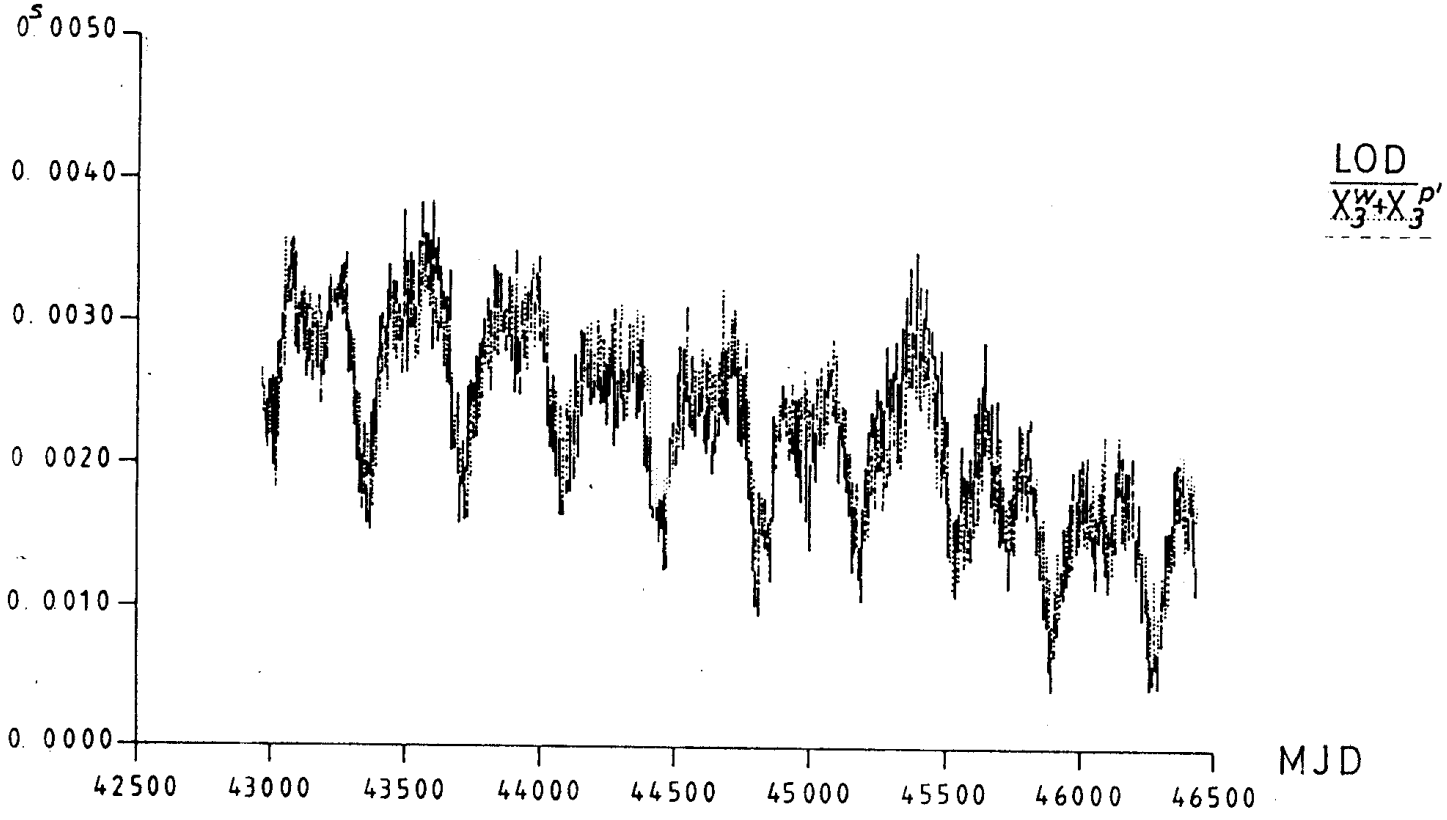
Spektralna analiza LOD i χ_3 dobijena je kao direktna Furijeova transformacija korišćenjem algoritma Cooley-Tukey za brzu Furijeovu transfornaciju (FFT). Rezultati spektralne analize predstavljeni su na slikama od 9-23. Na apscisnoj osi je logaritamska skala periode u godinama a na orinatnoj osi amplitude u vremenim sekundama. Vidimo da LOD i χ_3 u raličitim kombinacijam prikazuju da nema mnogo uticaja član pritiska na tranje dana; ako uporedimo spektar χ^{w+p} (član vetra plus član pritiska) na ici 19 sa spektrom χ_3^w (samo član vetra) na slici 21, vidimo izvarednu

sličnost. Sve do polugodišnjeg pika oni su praktično identični, a za duže periode član pritiska, izgleda, nadoknađuje umerenost amplituda dominantnih članova vetra. Isto je i sa varijantama $\chi_3^{w+p'}$ (član pritiska sa korekcijom invertnog barometra plus vetar član) na slici 20. To je jasno da za osnu komponentu χ_3, član pritiska je tako mali da hipoteza invertnog barometra ne donosi nikakve suštinske promene totalnoj funkciji ugaonog momenta. Uopšteno govoreći kombinacija vetar člana sa svakom od ove dve mogućnosti člana pritiska daje za periode iznad jedne godine bolju saglasnost sa posmatranim LOD, od kojeg su oduzete plime (uporediti spektar LOD i LOD-plime, na sl. 23 i 22), nego sam vetar član. Sličnost spektra LOD (bez plima) i $\chi_3^{w+p}, \chi_3^{w+p'}$ je izvanredna; oni su praktično identični. Postoje neke male razlike u amplitudama oba dominantna člana: godišnjeg i polu- godišnjeg (tablica 7); χ_3 daje manju amplitudu polu-godišnjeg člana a veću amolitudu godišnjeg člana nego posmatrani LOD bez plima, ali razlike se nalaze unutar granice od 10%. Veće razlike su ispoljene za duže periode. Amplitude posmatranog LOD (bez plima), za periode duže od jedne godine, izgledaju bolje saglasne sa samim vetar članom nego sa kombinacijom vetar i pritisak člana. To može značiti da jedan ili drugi član pritiska u njegovom dugo-periodičnom delu nije pouzdan, ali (što je više verovatno), da se nalazi drugi izvor ekscitacije dugo-periodične promene (na pr. elektromagnetska sprega između jezgra i omotača) koja je odgovorna za ove promene rotacione brzine Zemlje.

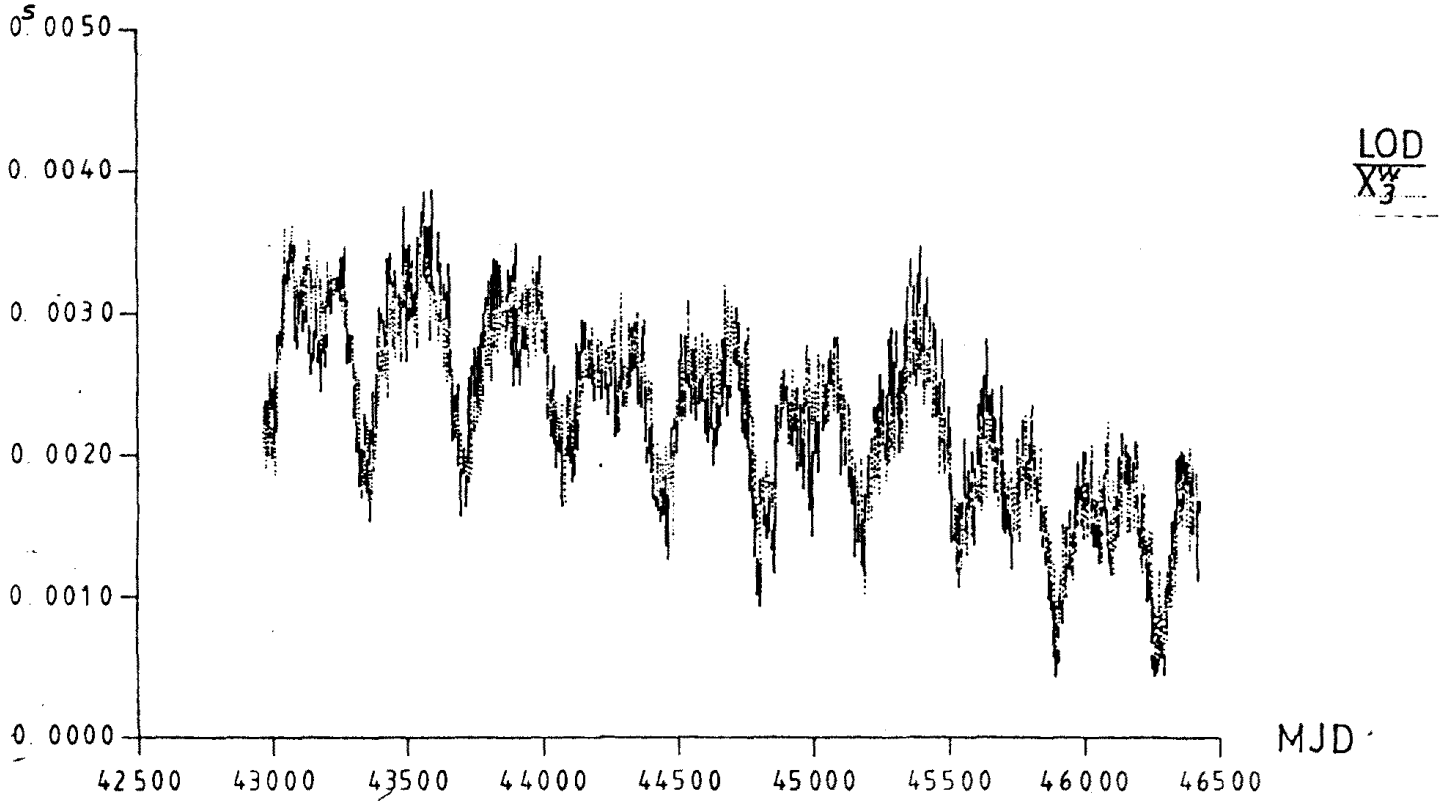
Slika 16. LOD (puna linija) i osna komponenta EAMF (član vetra + član pritiska) računata po jednačini 61 (isprekidana linija).



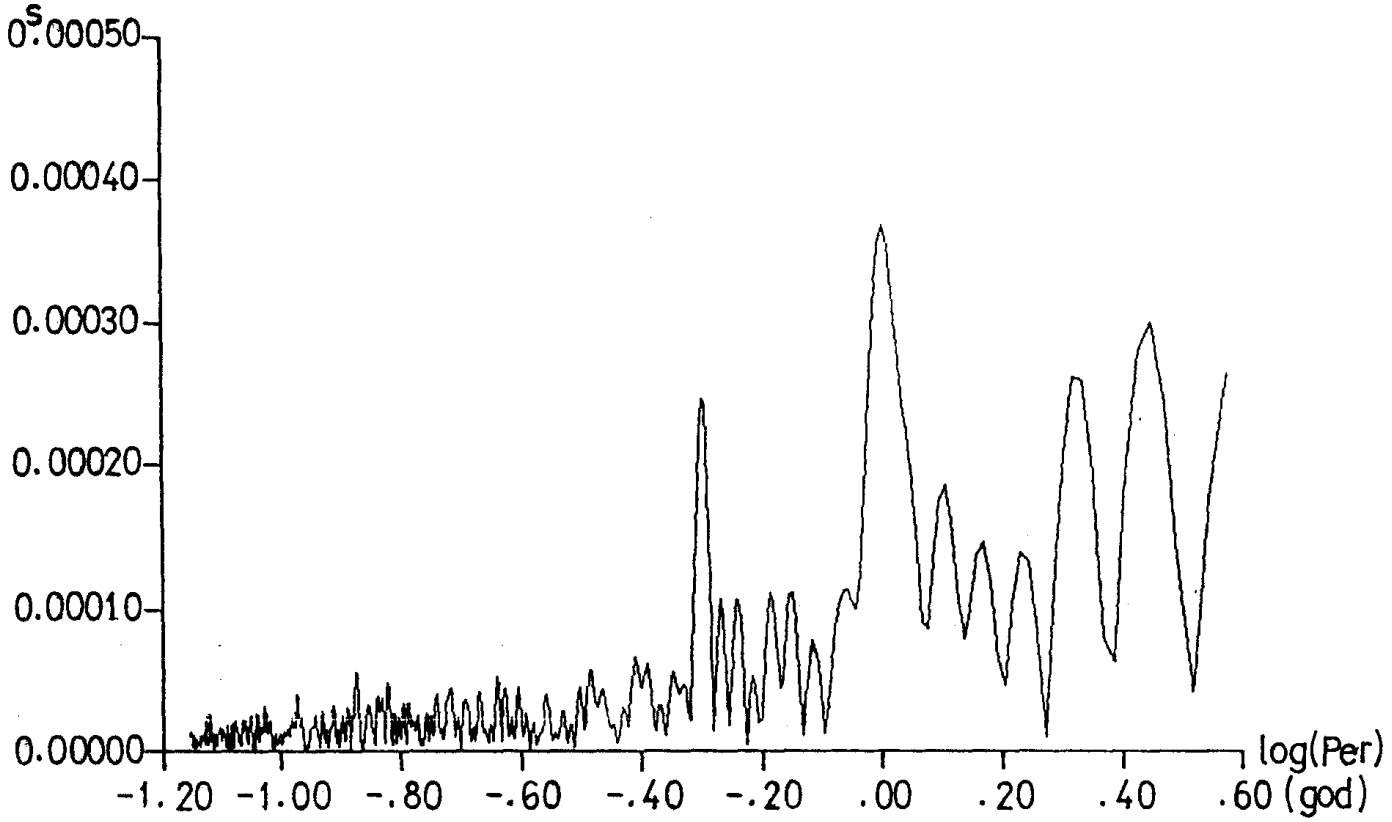
Slika 17. LOD (puna linija) i osna komponenta EAMF (član vetra + član pritiska sa korekcijom invertnog barometra) računata po jednačini 61 (isprekidana linija).



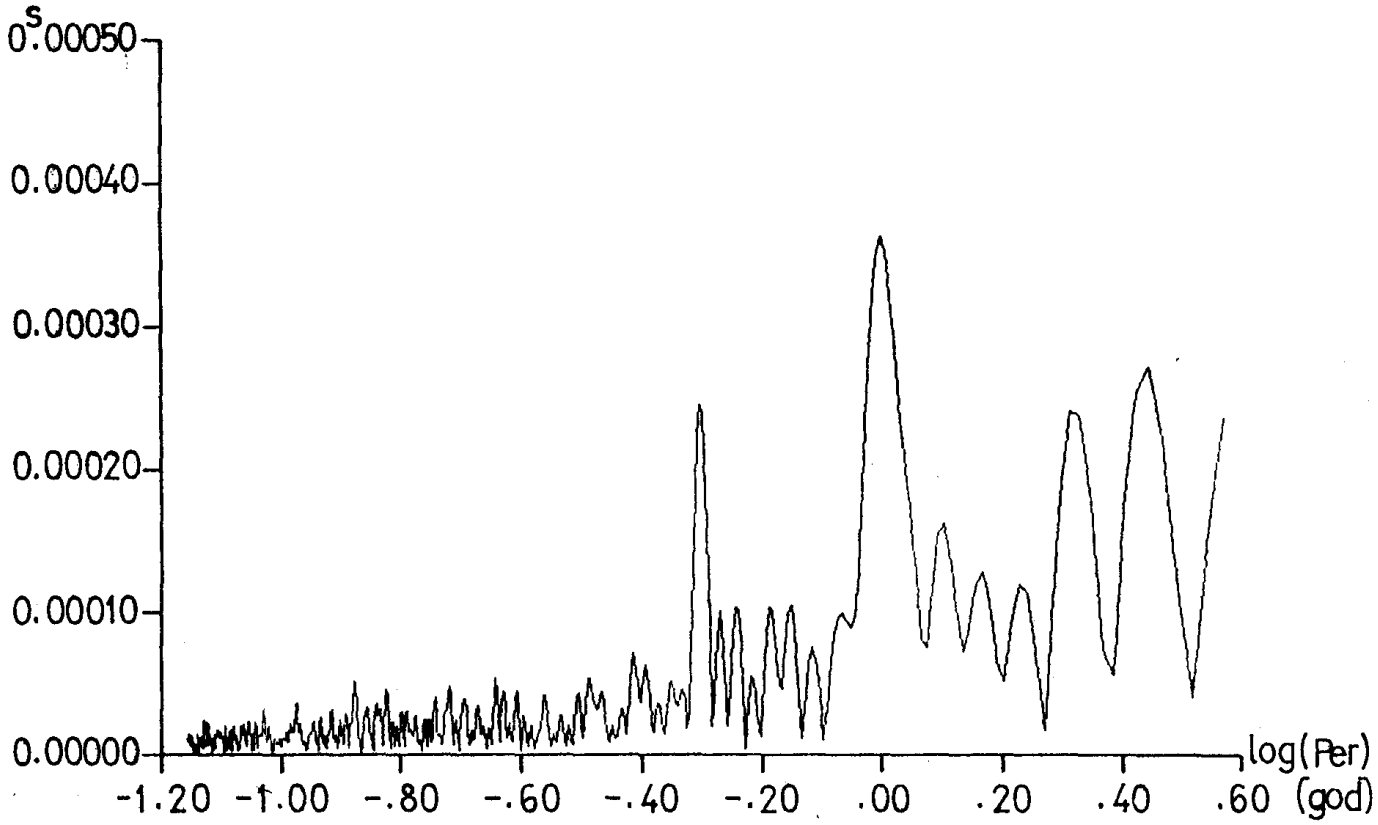
Slika 18. LOD (puna linija) i osna komponenta EAMF (član vetra) računata po jednačini 61 (isprekidana linija).



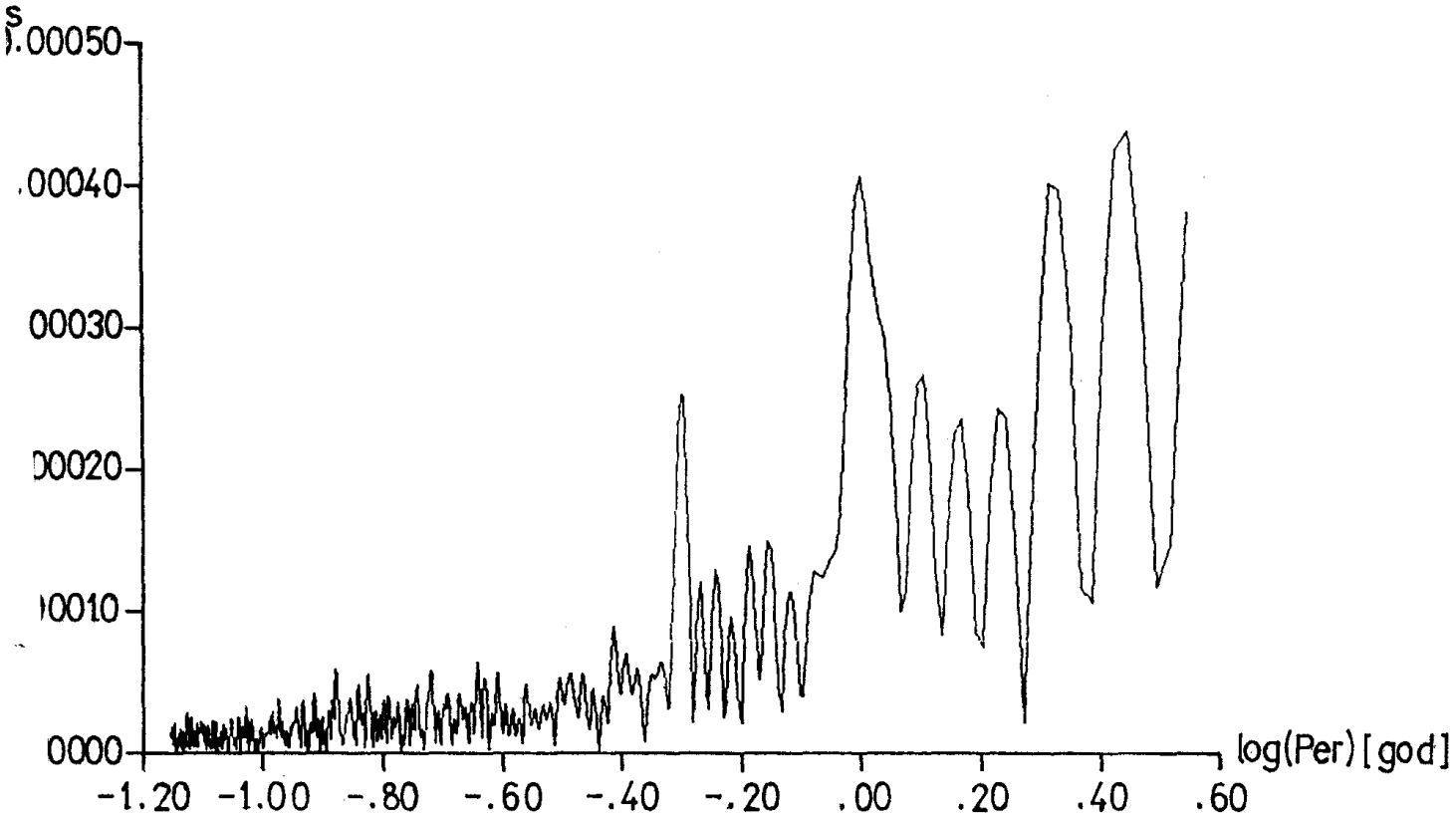
Slika 19. Spektar osne komponente EAMF (član vetra + član pritiska).



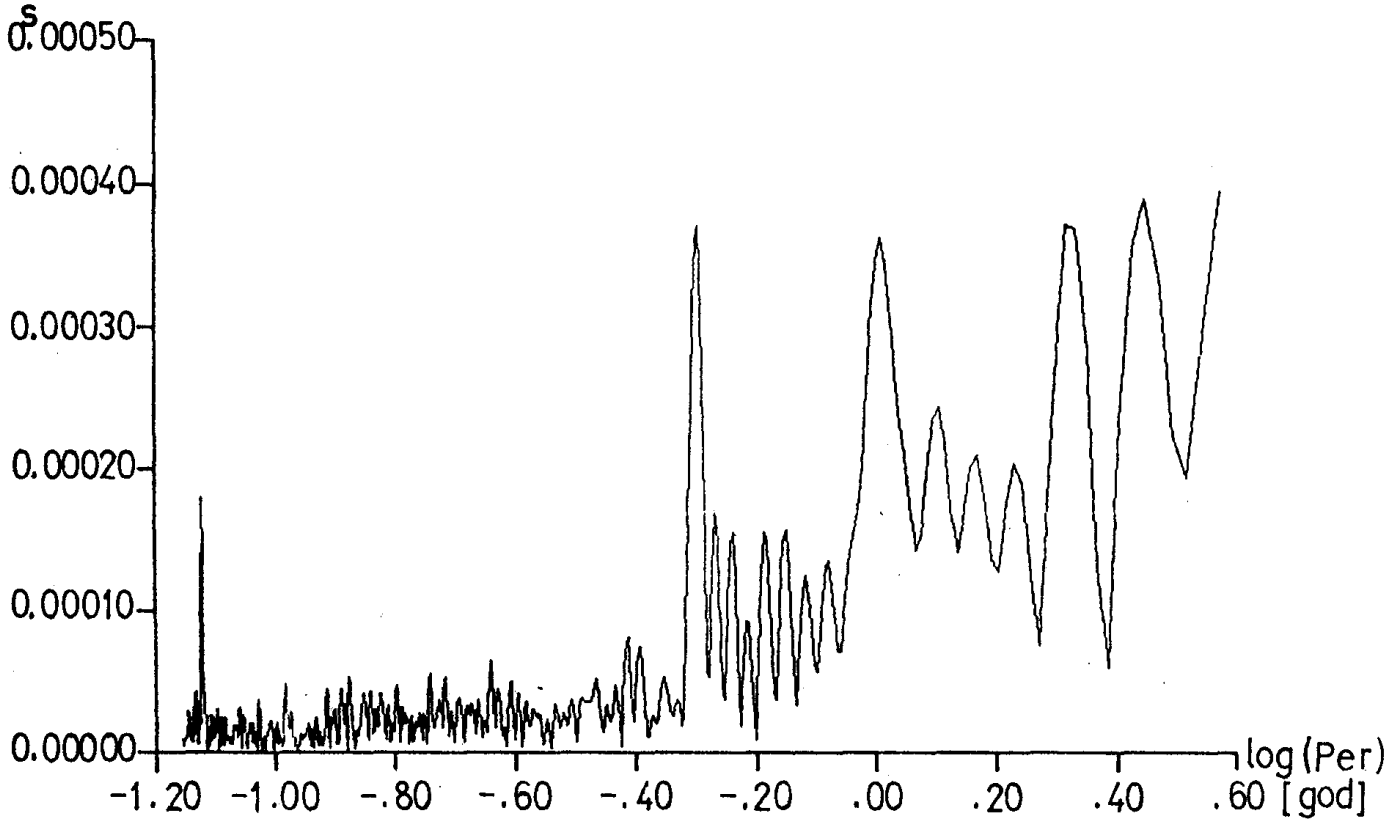
Slika 20. Spektar osne komponente EAMF (član vetra + član pritiska sa korekcijom invertnog barometra).



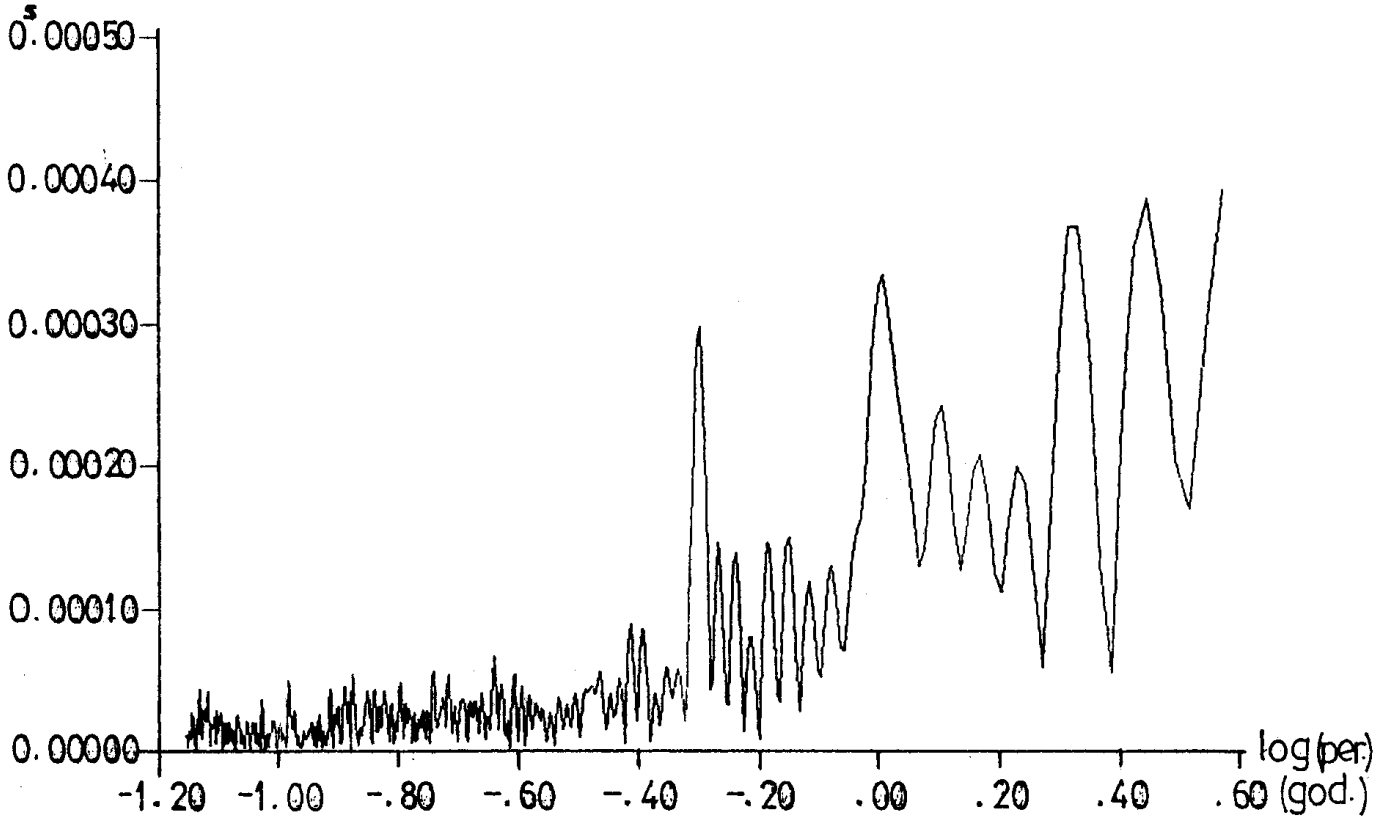
Slika 21. Spektar osne komponente EAMF (član vetra).



Slika 22. Spektar LOD.



Slika 23. Spektar LOD - plime.



Tablica 7.

Date su Periode, Amolitude i Faze dobijene metoddm FFT za sve tri kombinacije χ_3 , LOD i LOD-plime. Faze se odnose na modifikovani Julijanski datum (MJD)=42963.

	Period [*god*]	*Amplituda* [*s*]	*Faza* [^{o}]
χ_3^{w+p}	0.50	0.000246	164.23
	1.00	0.000368	331.74
$\chi_3^{w+p'}$	0.50	0.000246	167.28
	1.00	0.000363	331.84
χ_3^{w}	0.50	0.000253	165.25
	1.00	0.000406	335.74
LOD	0.50	0.000361	159.29
	1.00	0.000354	301.25
LOD-plime	0.50	0.000293	160.92
	1.00	0.000328	327.79

Na drugoj strani, grupa malih pikova koja je praktično identična u oba spektra, za pikove manje od polugodišnjeg potvrđuje nalaze drugih autora (Feissel i Gambis, 1980; Đurović, 1986, 1987) koji su našli fliktuacije sa periodama između 50 i 70 dana u oba, i u LOD i u χ_3. Nalazi se pouzdan dokaz da postoji korelacija između Sunčeve aktivnosti i atmosferske ekscitacije (Đurović, 1987).

Možemo zaključiti da su promene u brzini Zemljine rotacije uglavnom izazvane kombinovanim efektima Mesečevih i Sunčevih plima i atmosferske ekscitacije.

8. ZAKLJUČCI

Reakcija visko-elastične Zemlje na atmosfersku ekscitaciju je izvedena korišćenjem novog, dvo-osnog modela Zemlje sa fluidnim jezgrom i dinamičkim okeanom. Rezultati dobijeni u ovom radu pokazuju sledeće zaključke:

1. Potvrđena je polazna hipoteza o nelinearnoj zavisnosti Čendlerove frekvence od totalne amplitude polarnog kretanja analizom ILS koordinata pola.

2. Pokazano je da se ne radi o dve, već o jednoj frekventno modulisanoj Čendlerovoj nutaciji i da je naša ranije dobijena dvojnost Čendlerovog pika posledica rapidne promene Čendlerove faze u periodu 1920-1940.

3. Član vetra meren samo u ponoć,mnogo je precenjen, posebno što se tiče godišnje komponente. Na drugoj strani, pokazuje se da ga ne trba u buduće zanemarivati, kad podaci sa višom rezolucijom (najmanje dvaput dnevno) postanu rutinski raspoloživi.

4. Hipoteza invertnog barometra reakcije okeana na promene vazdušnog pritiska je podržana; članovi pritiska bez ove korekcije ekscitovali bi polarno kretanje sa amplitudom mnogo većom od posmatrane, dok korekcija invertnog barometra donosi ovu amplitudu vrlo blisku posmatranoj.

5. Razlike između faza atmosferski ekscitovanog i posmatranog godišnjeg kretanja je objašnjena dodatom ekscitacijom uzrokovanom promenama podzemnih voda na godišnjoj frekvenci.

6. Pokušaj numeričke integracije polarnog kretanja na intervalu 9.5 godina bio je uspešan; razlike između dveju (integraljenog i posmatranog polarnog kretanja) krivih su, za Vondrakov model, reda $\pm 0''.06$.

7. Koristeći dodatnu ekscitaciju uzrokovanu promenama podzemnih voda od Kikučija (Kikuchi, 1977) fitovanje je poboljšano.

8. Novi model Zemlje sa slobodnom Čendlerovom frekvencom zavisnom od totalne amplitude polarnog kretanja donosi novo poboljšanje. Fi-

tovanje za ovaj model sa podzemnom vodom od Kikučija karakterisano je sa srednjekvadratnim odstupanjem između dveju krivih od samo ±0".037. Ovako dobro poboljšanje može biti, pored pomenutih, i posledica nepoznatih ekscitacija sličnog oblika.

9. Ima još pitanja na koja treba odgovoriti daljim istraživanjima. Posmatrana amplituda pokazuje spor porast ako je uporedimo sa integraljenim polarnim kretanjem. Očigledno, ovo se može objasniti samo nekom,nepoznatom, ekscitacijom sa frekvencom koja je vrlo bliska Čendlerovoj frekvenci. Zna se da takva ekscitacija nije prisutna u atmosferi, ali je verovatno da bi se mogla naći u promenama podzemnih voda, koje nisu tako precizno poznate kao atmosferske promene. S toga je jasno da, služba slična regularnom izračunavanju funkcija ugaonih momenata atmosfere, trebalo bi da odgovori na ova otvorena pitanja.

10. Iz ovoga možemo izvući konačan zaključak da doprinos atmosferske ekscitacije i ekscitacije podzemne vode je odgovoran za posmatrano polarno kretanje, i da novi model Zemlje sa promenljivom i amplitudno zavisnom Čendlerovom frekvencom je potvrđen čak iz kratkog vremenskog intervala.

11. Što se tiče LOD možemo zaključiti: da promene u brzini Zemljine rotacije uglavnom su izazvane kombinovanim efektima Mesečevih i Sunčevih plima i atmosferskom ekscitacijom.

12. Grupa malih pikova koja je praktično identična u oba spektra (LOD i χ_3) za periode manje od polugodišnjeg potvrđuje nalaze drugih autora da postoje kratko-periodične varijacije sa periodima između 40 i 70 dana i u LOD i u χ_3.

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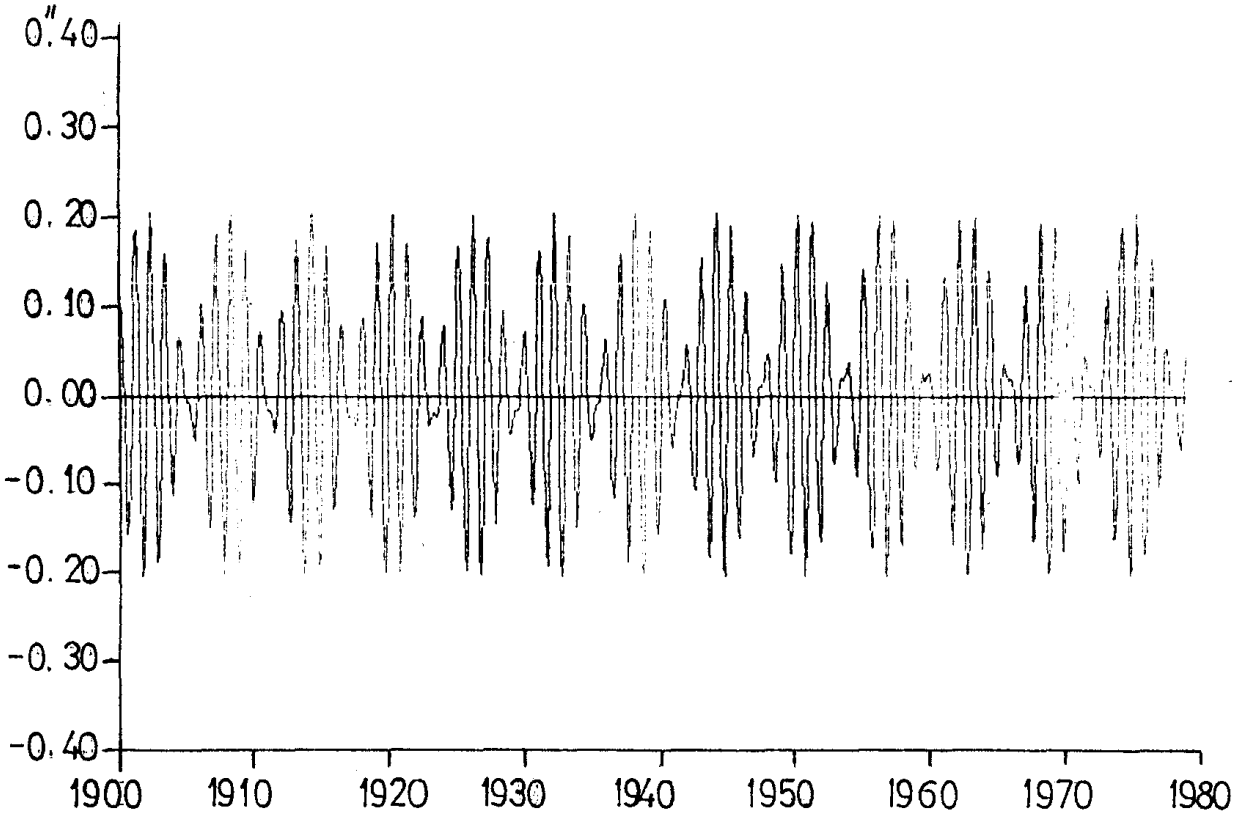
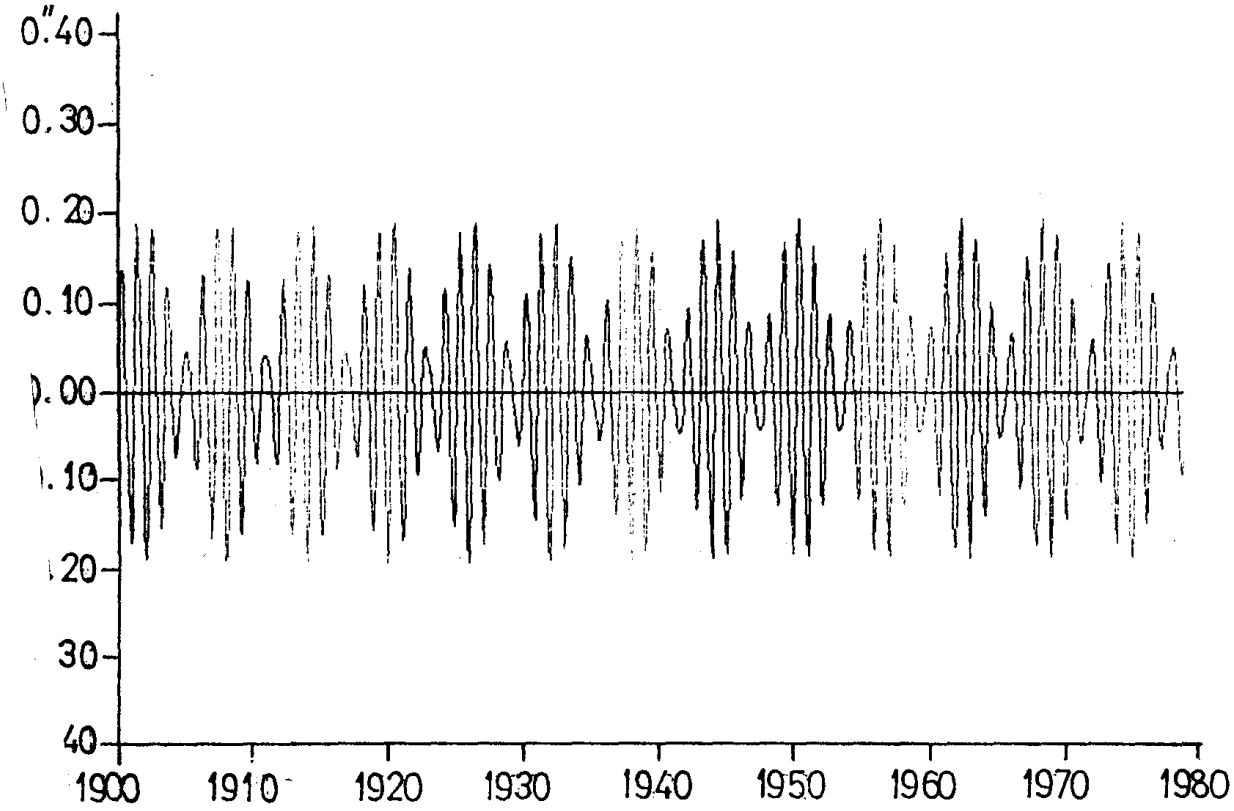
PRILOG

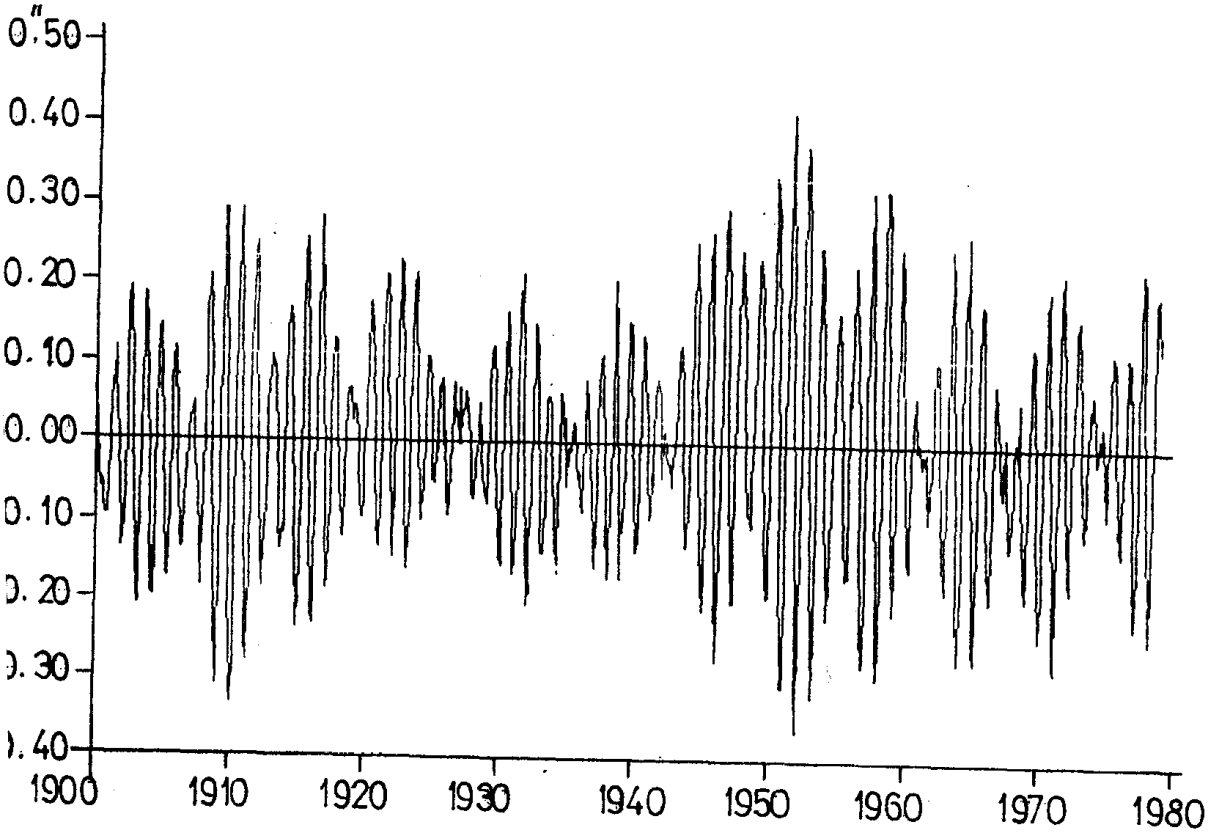
Slika P1. Sintetizovana x komponenta polarnog kretanja sa konstantnom Čendlerovom frekvencom i konstantnom Čendlerovom amplitudom (ILS podaci)



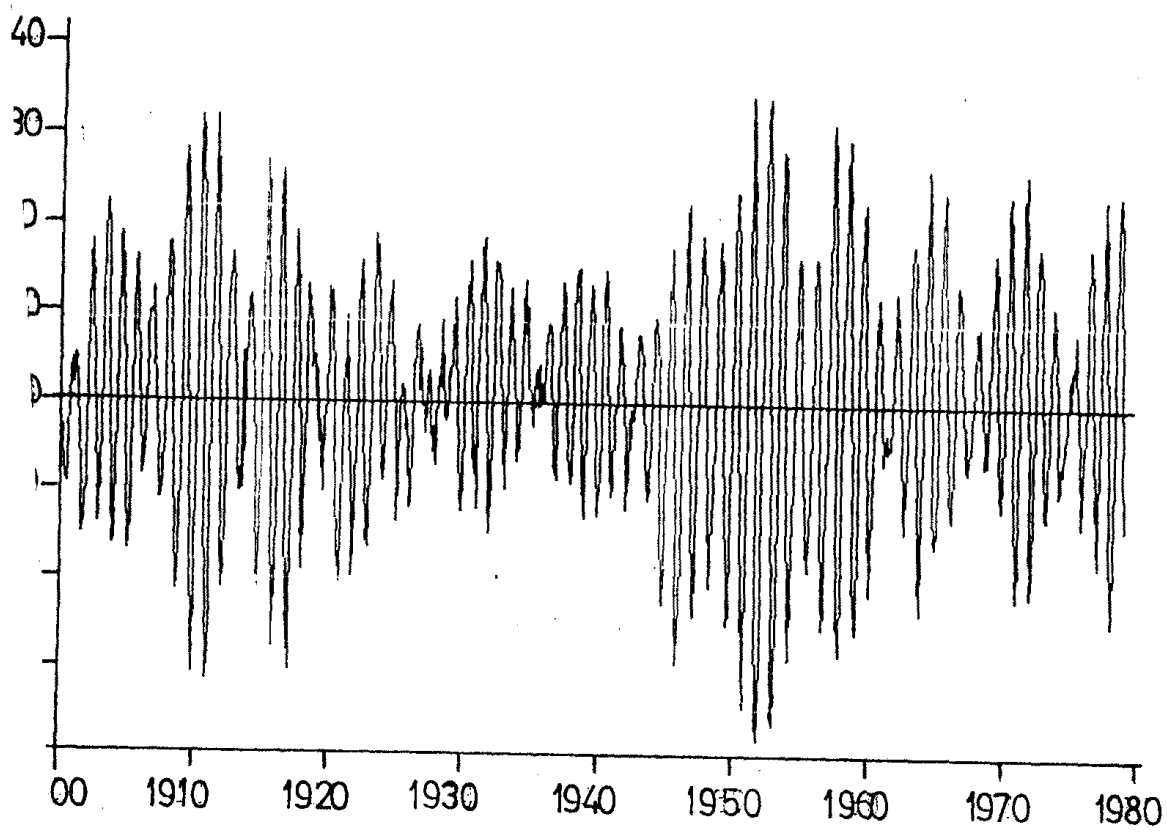
Slika P2. Sintetizovana y komponenta polarnog kretanja sa konstantnom Čendlerovom frekvencom i konstantnom Čendlerovom amplitudom (ILS podaci)



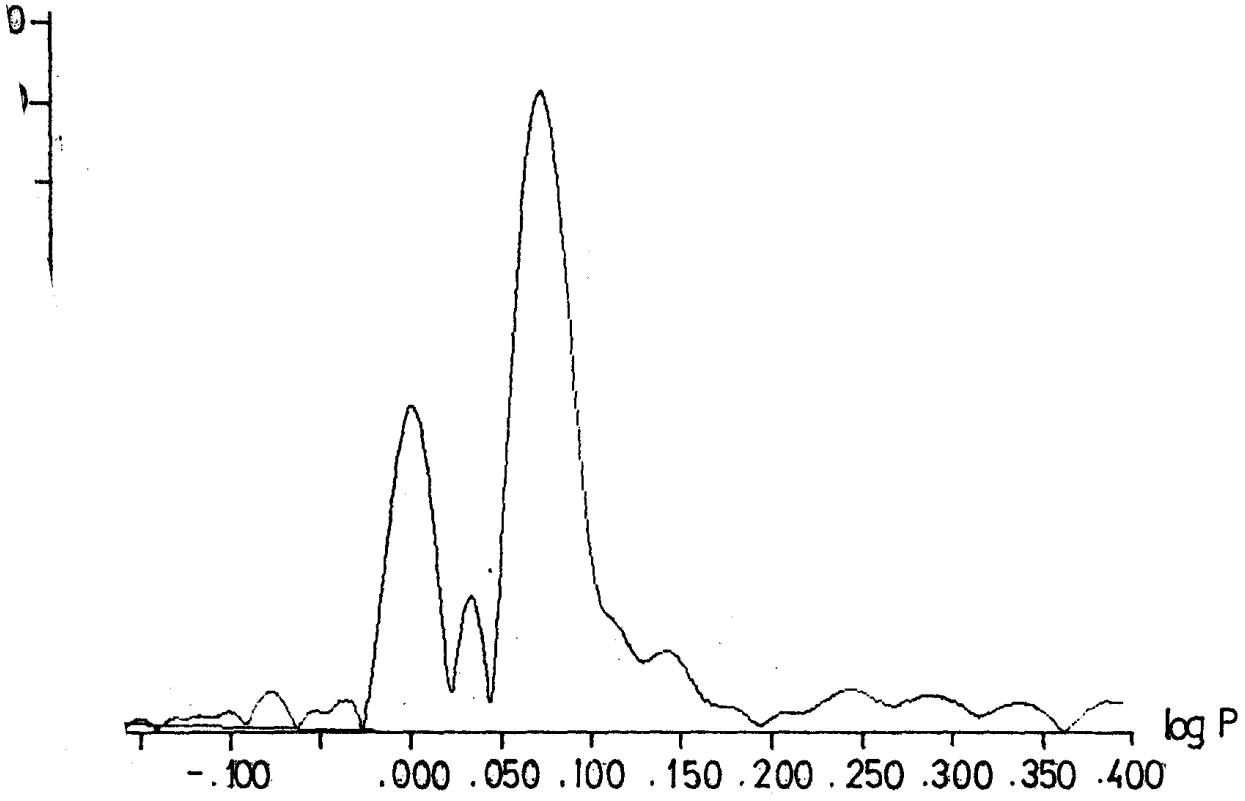
Slika P3. Posmatrani podaci ILS - x komponenta.



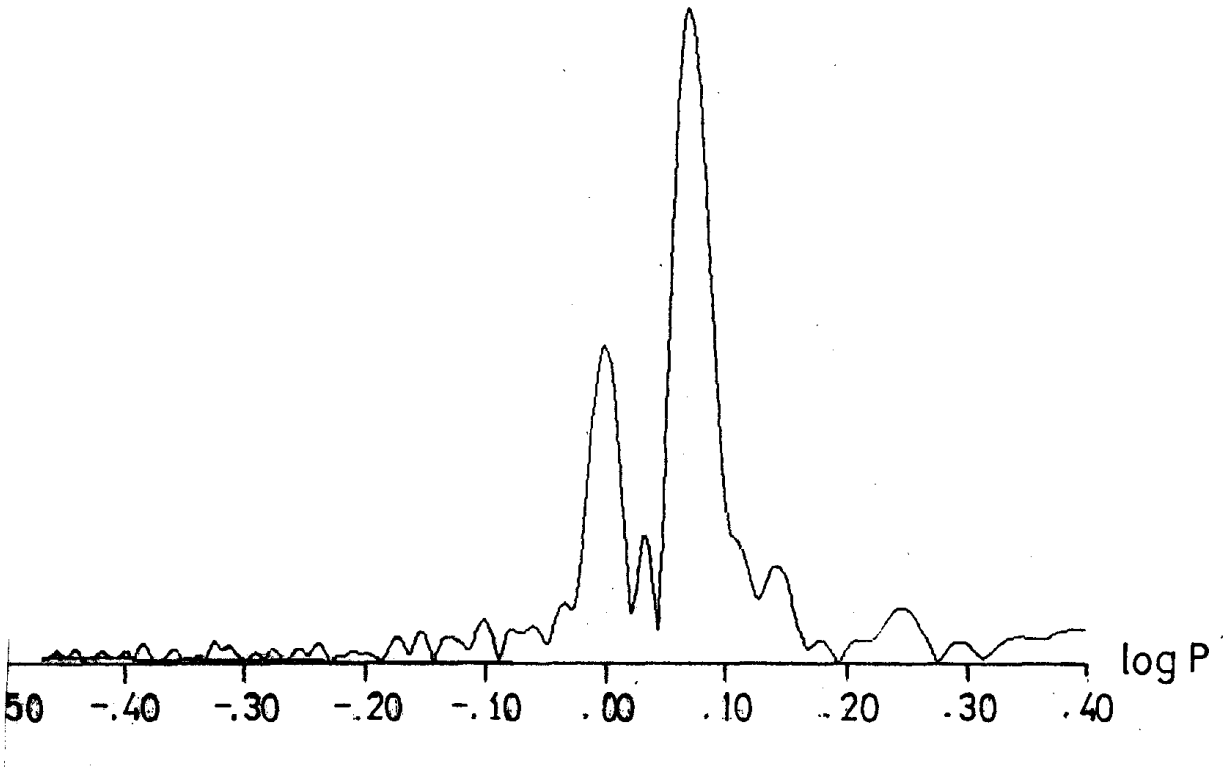
Slika P4. Posmatrani podaci ILS - y komponenta.



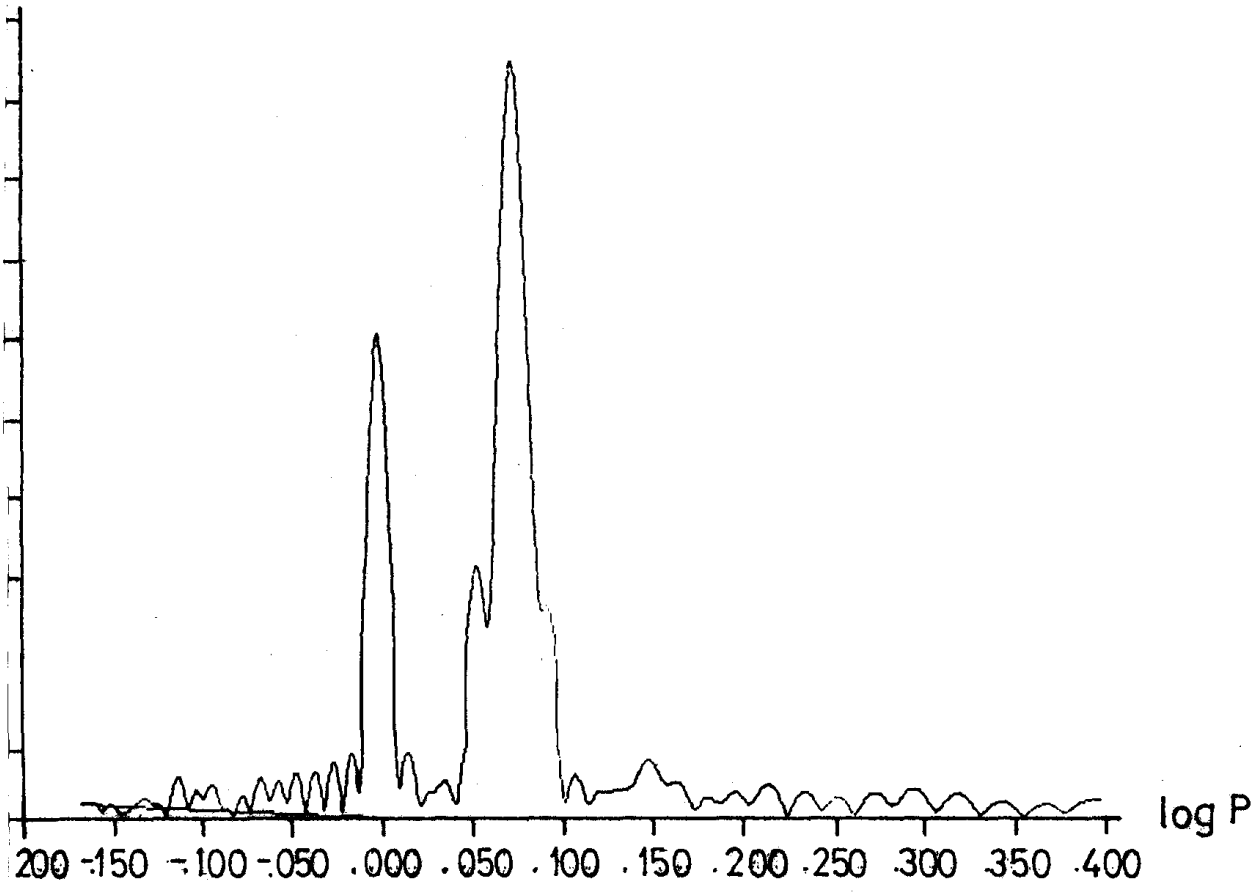
Slika P5. Spektar x komponente ILS podataka za period 1900-1920.



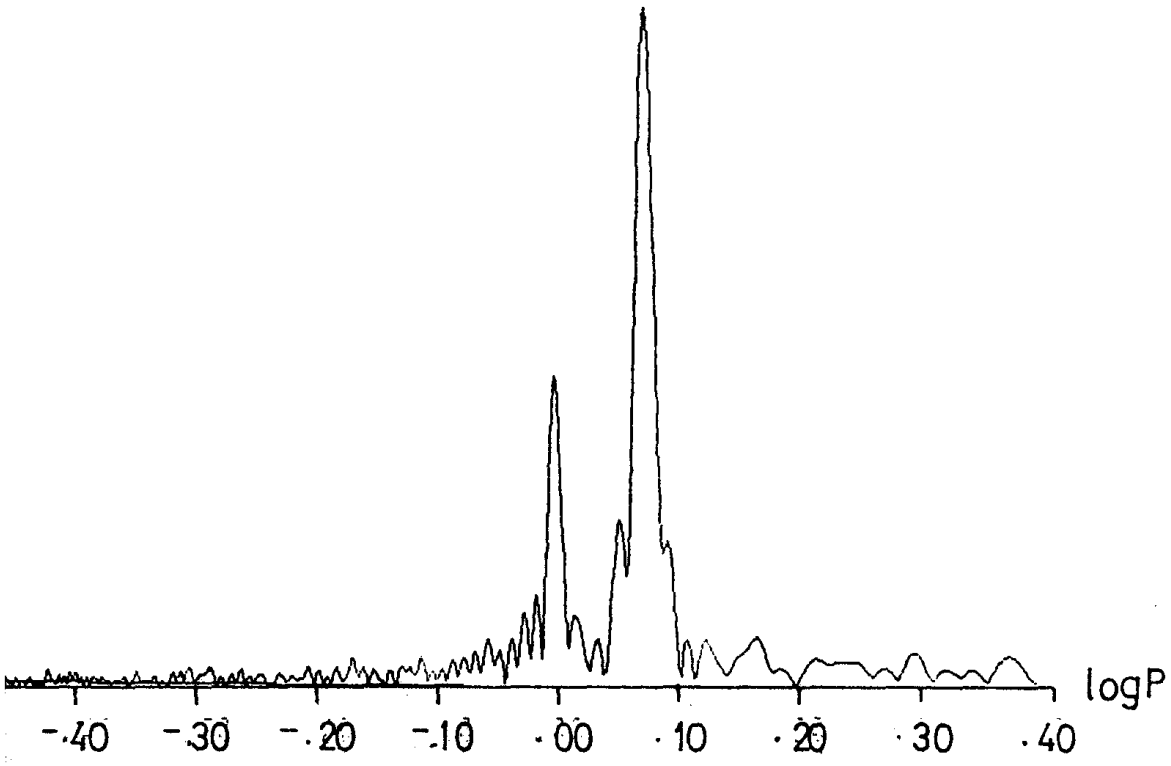
Slika P6. Spektar y komponente ILS podataka za period 1900-1920.



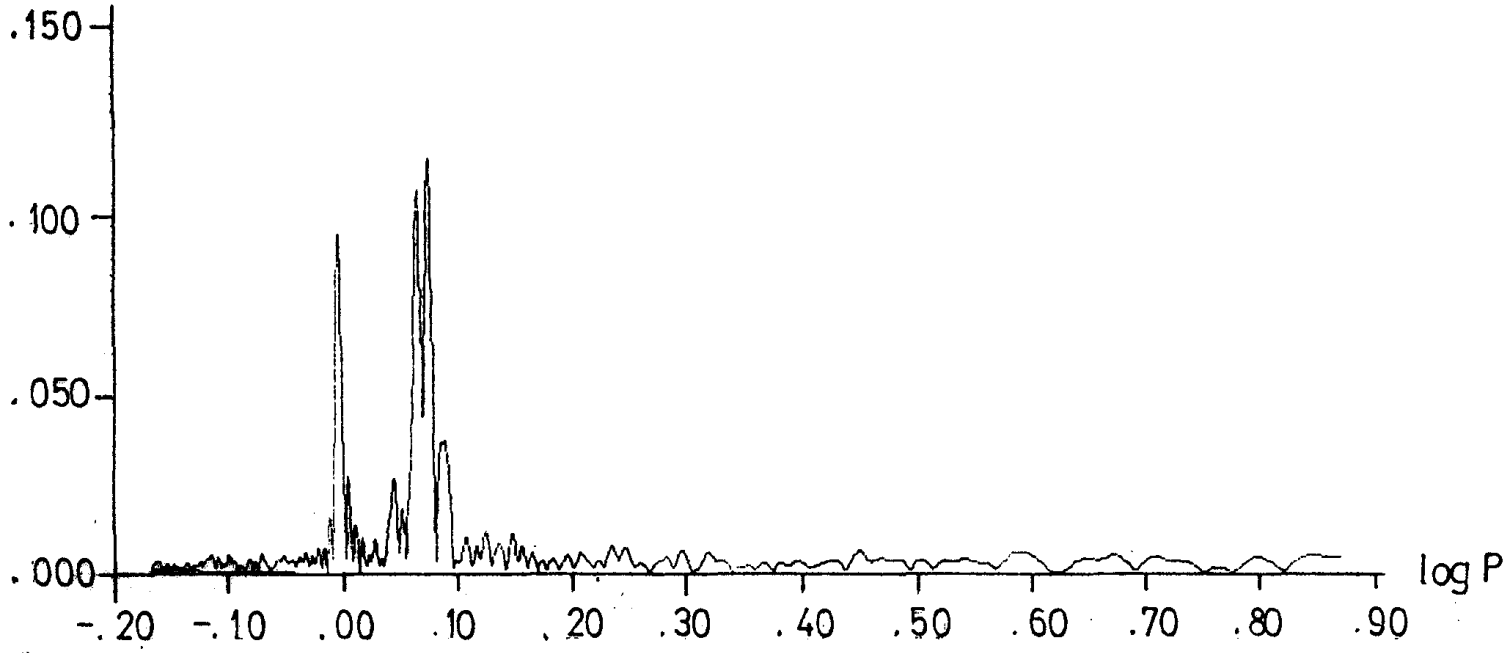
Slika P7. Spektar x komponente ILS podataka za period 1940-1980.



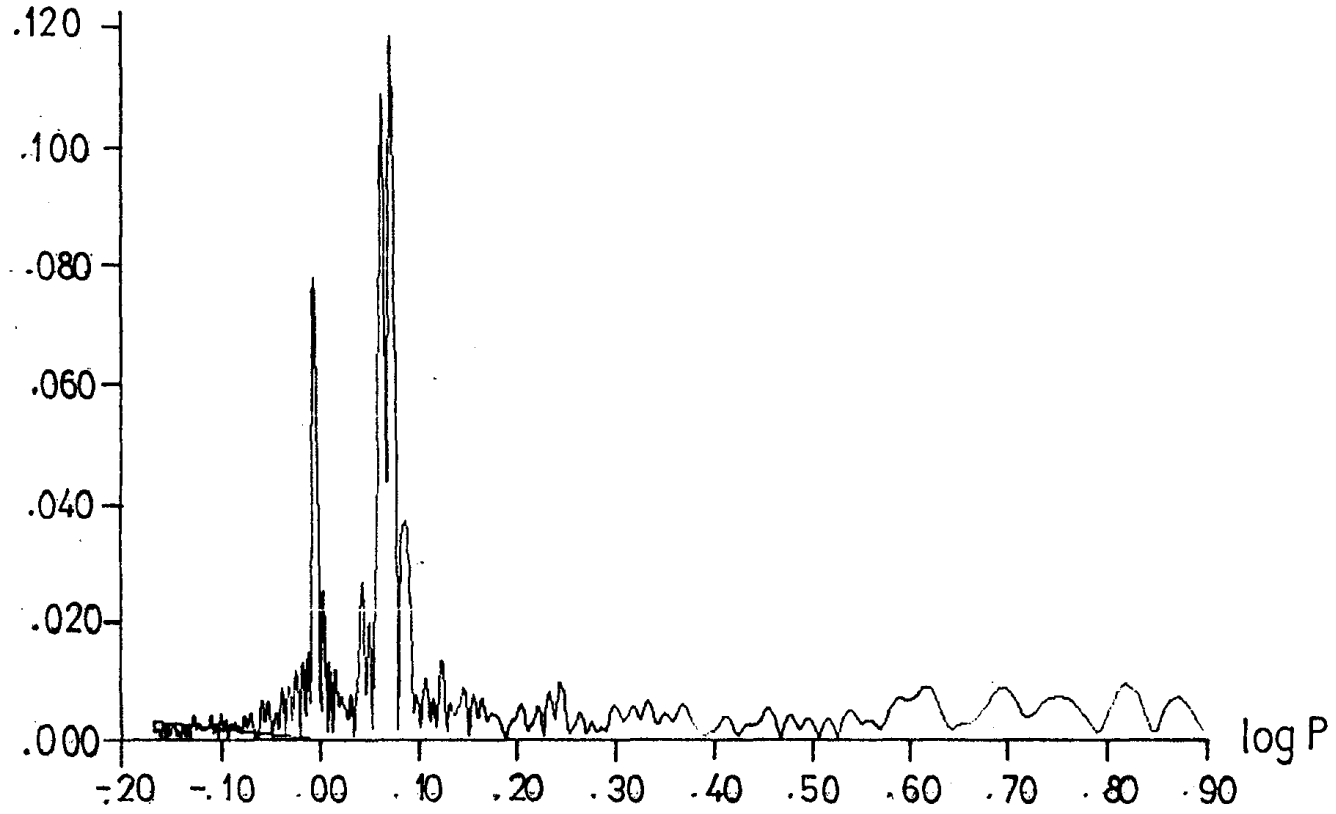
a P8. Spektar y komponente ILS podataka za period 1940-1980.



Slika P9. Spektar x komponente ILS podataka za period 1900-1980.



Slika P10. Spektar y komponente ILS podataka za period 1900-1980.



4 **Magistarski rad:** *Prilog izučavanju kretanja Zemljinih polova*
Master thesis: *Contribution to the study of polar movements*

Nadežda Pejović
Defended at the Faculty of Mathematics in Belgrade
70+24 pages, 1984.
Comission: prof Dragutin Djurović (mentor),
prof. Branislav Ševarlić and prof. Jovan Simovljević.

This paper contains an extensive analysis of the coordinates of the current pole of the Earth's rotation in relation to the International Conventional Pole (OCI), based on data from the International Latitude Service (ILS) for the period 1899.9-1979.0. According to this analysis, Pejović has concluded that there are secular changes in the coordinates of the pole, but it is still uncertain whether it is a consequence of the actual movement of the pole or precession, the proper movements of the stars, etc. For the direction and speed of "movement" of the pole, she got results close to the results of some respectable authors. For example, by spectral analysis of Kimura's Z-member, she found, in addition to the well-known year of cyclic variation, also a semi-annual variation, which represents a completely new discovery.

UNIVERZITET U BEOGRADU

PRIRODNO MATEMATICKI FAKULTET

NADEZDA PEJOVIC

PRILOG IZUCAVANJU KRETANJA ZEMLJINIH POLOVA

MAGISTARSKI RAD

BEOGRAD, 1984.G.

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OVAJ RAD JE POSVECEN MOM JEDINOM,

PRERANO UMRLOM BRATU

DRAGOLJUBU - DRAGI

M A T O R C E V I C U

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IZRAZAVAM SVOJU ISKRENU ZAHVALNOST RUKOVODIOCU, PROFESORU DR D. DJUROVICU, KOJI JE SVE VREME PRATIO I USMERAVAO OVAJ RAD NE MALIM BROJEM KORISNIH SUGESTIJA, PROFESORIMA DR B. SEVARLICU I DR J. SIMOVLJEVICU NA KORISNIM SAVETIMA I KRITICKOM CITANJU RUKOPISA, ASISTENTU MR S. SEGANU NA POMOCI PRI IZRADI PROGRAMA I STAMPANJU OVOG RADA, DIREKTORU MEDJUNARODNE SLUZBE SIRINE DR S. YUMI-JU NA USTUPLJENOM POSMATRACKOM MATERIJALU KAO I OSOBLJU RACUNSKOG CENTRA MATEMATICKOG INSTITUTA SR SRBIJE NA KORISCENJU RACUNARA IBM360 NA KOJEM SU IZVEDENI SVI RACUNI.

AUTOR

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1. UVOD

SKORO DO PRED KRAJ PROSLOGA VEKA MISLILO SE DA ZEMLJINA OSA ROTACIJE NE MENJA SVOJ POLOZAJ U ZEMLIINOM TELU. SMATRALO SE DA JE ZEMLJA PODESILA SVOJ OBLIK PREMA SVOJOJ ROTACIJI TAKO DA SE NJENA GEOMETRIJSKA OSA, TJ. NJENA POLARNA GLAVNA OSA INERCIJE, PO KLAPA SA NJENOM OSOM ROTACIJE. NA PROMENU POLOZAJA ZEMLJINE OSE ROTACIJE U NJENOM TELU PRVI JE TEEORIJSKI UKAZAO JOS U 18. VEKU LEONARD OJLER (EULER, 1736, 1790). PRETPOSTAVLJAJUCI ZEMLJU KAO APSOLUTNO CVRSTU, ON JE NASAO DA NJENA OSA ROTACIJE OCRTA U ZEMLJINOM TELU KONUS POLHODIJE ZA VREMENSKI INTERVAL OD 305 DANA. OD NOSNO, OKO GLAVNE OSE INERCIJE KRECE SE ZEMLJA TAKO DA, RELATIVNO PREMA ZEMLJINOJ POVRSINI, POL ROTACIJE OPISUJE SVOJU USKU KRUZNU PUTANJU OKO GEOMETRIJSKOG POLA ZEMLJE. TO KRETANJE ZEMLJE ZOVE SE NJENA SLOBODNA NUTACIJA. PREDVIDJENU OJLEROM DESETOMESECNU SLOBODNU NUTACIJU ZEMLJINE OSE ROTACIJE POKUSAO JE PRVI EKSPERIMENTALNO DA DOKAZE PULKOVSKI ASTRONOM PETERS (1845). VARIJACIJE LATITUDA KOJE JE OTKRIO OBJASNIO JE SEZONSKIM GRESKAMA POSMATRANJA. ISTRAZIVANJA PETERSA NASTAVILI SU I DRUGI POZNATI ASTRONOMI. KELVIN JE 1876 GODINE OBJAVIO OTKRICE SLOBODNE NUTACIJE U JEDNOJ NJUKOMBOVOJ ANALIZI LATITUDA VASINGTONA U PERIODU 1862-1865 GODINA.KADA SU POSMATRANJA BERLINSKOG ASTRONOMA KISTNERA (KUSTNER, 1888), ZA PERIOD 1884-1885, NESUMNJIVO DOKAZALA REALNOST PROMENA GEOGRAFSKIH SIRINA, ODLUCENO JE DA SE TE PROMENE SISTEMATSKI ISPITAJU.ZATO SU U 1891 I 1892 GODINI VRSENA ISTOVREMENA POSMATRANJA PROMENA GEOGRAFSKIH SIRINA U BERLINU I HONOLULU (PRIBLIZNO SU NA SUPROTNIM MERIDIJANIMA). TA SU POSMATRANJA DOKAZALA DA SE ZEMLJINA OSA ROTACIJE ZAISTA POMERA U ZEMLJINOM TELU.

JEDAN OD NAJVECIH DOGADJAJA U ISTORIJI ISTRAZIVANJA KRETA NJA POLOVA PREDSTAVLJA CHANDLER-OVO OTKRICE 1892 GODINE (CHANDLER, 1892). ON JE ANALIZIRAJUCI VELIKI BROJ POSMATRANJA OTKRIO DA VARI JACIJE LATITUDA IMAJU DVE PERIODICNE KOMPONENTE: JEDNU, KOJA JE DOBILA IME PO SAMOM CENDLERU, SA PERIODOM OD 427 DANA I AMPLITU - DOM 0".12 I DRUGU SA GODISNJOM PERIODOM I AMPLITUDOM U INTERVALU OD 0".04 DO 0".20. RAZLIKU IZMEDJU OJLEROVE I CENDLEROVE PERIODE KAO I GODISNJU KOMPONENTU PRVI JE POKUSAO DA OBJASNI NJUKOMB (NEW KOMB, 1892). PRI DOBIJANJU OJLEROVE PERIODE PRETPOSTAVILO SE DA JE ZEMLJA APSOLUTNO CVRSTA, STO NIJE SLUCAJ. UZME LI SE U OBZIR ZEMLJINA ELASTICNOST, NASTUPA PODUDARNOST IZMEDJU TEORIJE I PRAK- SE. PO NJUKOMBOVOM MISLJENJU, CENDLEROVA PERIODA JE "PRODUZENA" OJLEROVA. STO SE PAK GODISNJEG CLANA TICE, NJUKOMB JE SMATRAO DA SU NJEGOVI UZROCI METEOROLOSKI FAKTORI.

U CILJU STO BOLJEG PRACENJA POLARNOG KRETANJA DOSLO JE 1899 GODINE DO ORGANIZOVANJA MEDJUNARODNE SLUZBE SIRINE ILS (INTE RNATIONAL LATITUDE SERVICE) SA SVOJIH SEST STANICA (MIZUSAVA, CA- RLOFORTE, GAITHERSBURG, UKIAH, CINCINATI, TSCHARDJUI) NA PARALELU GEOGRAFSKE SIRINE φ = +39 08'. OVA MEDJUNARODNA SLUZBA JE OD ORG ANIZOVANJA PA DO DANAS SAKUPILA POSMATRACKI MATERIJAL OD IZUZETNE NAUCNE VREDNOSTI. NA HARMONIJSKOJ ANALIZI OVOG MATERIJALA RADILI SU MNOGI POZNATI ISTRAZIVACI I SVI SU ONI POTVRDILI POSTOJANJE DVE PERIODICNE KOMPONENTE: KRUZNU KOMPONENTU SLOBODNE NUTACIJE, KOJU JE OTKRIO JOS CENDLER I GODISNJU ELIPTICNU KOMPONENTU PRINU- DNE NUTACIJE. UZROK EKSCITACIJE I PRIGUSENJE SLOBODNE NUTACIJE ZE MLJINE OSE ROTACIJE JOS NIJE ODREDJEN (LAMBECK, 1980). PROMENLJI- VOST NJENE AMPLITUDE I FAZE KONTRAVERZNA JE SKORO CITAV VEK. NE MALI BROJ AUTORA, UKLJUCUJUCI I CENDLERA SAMOG, UVODILI SU HIPOTE ZE VISESTRUKOSTI ILI VARIJABILNOSTI PERIODA (CHANDLER,KIMURA, ME-

LCHIOR, SEKIGUCHI, GAPOSCHKIN, CARTER). POSTOJI I DRUGA GRUPA AUTORA (NEWCOMB, OOE, OKUBO) KOJA U SVOJIM RADOVIMA ZASTUPA GLEDISTE DA SLOBODNA NUTACIJA IMA JEDINSTVENU PERIODU. GODISNJA KOMPO - NENTA PRINUDNE NUTACIJE OBJASNJENA JE PRERASPODELOM MASA NA ZEM - LJINOJ POVRSINI, UGLAVNOM VAZDUSNI I VODENI TOKOVI SEZONSKOG KARA KTERA. PRIMENOM SPEKTRALNE ANALIZE NALAZI SE JOS CITAV NIZ PERIODICNIH KOMPONENATA CIJE SE OBJASNJENJE TRAZI U GEOFIZICKOJ GRA - DJI ZEMLJE. RESENJE MNOGIH PROBLEMA ZAHTEVA KOMPLEKSNA ISTRAZIVANJA U VISE NAUCNIH DISCIPLINA: ASTRONOMIJE, GEOFIZIKE, MEHANIKE, GEOLOGIJE, GEODEZIJE I DRUGIH. ZBOG OVAKVE SVOJE SLOZENOSTI OVAJ PROBLEM JOS UVEK NIJE RESEN U KONACNOM OBLIKU, KOJI BI OMOGUCIO DA SE POLOZAJ ZEMLJINOG TRENUTNOG ROTACIONOG POLA MOZE RACUNSKI ODREDITI U SVIM PROSLIM I BUDUCIM VREMENIMA. PROBLEM USLOZNJAVA I POSTOJANJE NEPOLARNIH PROMENA GEOGRAFSKE SIRINE OBUHVACENIH POZNATIM KIMURINIM Z-CLANOM (KIMURA, 1902). ZATO SE ZA PUTANJE ZEMLJINIH POLOVA IZVEDENIH IZ POSMATRACKIH PODATAKA, IZ KOJIH SU ELIMINISANE KOMPONENTE SLOBODNE I PRINUDNE NUTACIJE, DOBIJAJU VRLO ZAMRSENE KRIVE.

RADI UBRZAVANJA PROCESA ODREDJIVANJA KOORDINATA TRENUTNOG POLA ORGANIZOVANA JE BRZA SLUZBA SIRINE IPMS (INTERNATIONAL POLAR MOTION SERVICE) SA MNOGO VECIM BROJEM STANICA. ZAJEDNO SA STANICAMA ILS, STANICE IPMS PORED KLASICNIH METODA POSMATRANJA VRSE I POSMATRANJA NOVIM TEHNIKAMA I METODAMA. ZNACAJAN KORAK U ISTRAZIVANJU KRETANJA POLOVA PREDSTAVLJA PRIMENA METODA ODREDJIVANJA NJIHOVIH KOORDINATA IZ POSMATRANJA VESTACKIH SATELITA. TEHNIKA I TEORIJA POSMATRANJA SATELITA DALE SU DVA METODA: JEDAN, POMOCU DOPLER-UREDJAJA I DRUGI, POMOCU LASERA. OD SAMOG POCETKA POSMATRANJA POMOCU DOPLER-UREDJAJA DALA SU DOBRE REZULTATE. POBOLJSANJE TACNOSTI ASTRONOMSKIH POSMATRANJA, KORISCENJEM PRECIZNIJH INSTRUMENATA

I NOVIH METODA, OMOGUCAVA ASTRONOMIMA SIGURNIJE IZUCAVANJE UZROKA KRETANJA ZEMLJINIH POLOVA.

CILJ OVOGA RADA JE IZUCAVANJE, NA OSNOVU SPEKTRALNE ANALIZE PRAVOUGLIH KOORDINATA POLA, NJEGOVOG KRETANJA. POSEBNA PAZNJA BICE POSVECENA VEKOVNOM (SEKULARNOM) KRETANJU SREDNJEG POLA, GLAVNIM HARMONIJSKIM CLANOVIMA U KRETANJU TRENUTNOG POLA: CENDLEROVOM, GODISNJEM I POLUGODISNJEM I NJIHOVIM VARIJACIJAMA,KAO I NEPOLARNIM PROMENAMA GEOGRAFSKE SIRINE, KIMURINOM Z- CLANU.

2. KORISCENI PODACI

OSNOVNI PODACI KORISCENI U OVOM RADU SU PRAVOUGLE KOORDINATE TRENUTNOG POLA X,Y U ODNOSU NA CIO (CONVENTIONAL INTERNATIONAL ORIGIN) DOBIJENE IZ POSMATRACKOG MATERIJALA ZA PERIOD 1899.9 -1979.0 SAKUPLJENOG NA STANICAMA ILS A PRECISCENOG I OBJAVLJENOG U PUBLIKACIJI MEDJUNARODNE SIRINSKE STANICE MIZUSAWA (YUMI & YOKO YAMA, 1980).

DA BI OBEZBEDILA STO HOMOGENIJI POSMATRACKI MATERIJAL,MEDJUNARODNA SLUZBA SIRINE ILS, SEPTEMBRA 1899 GODINE POSTAVLJA NA SVOJIH SEST STANICA, NA PARALELU SEVERNE GEOGRAFSKE SIRINE φ = 39 08', VIZUELNE ZENIT TELESKOPE (VZT). NA TAJ NACIN POSTIZE DA SVE STANICE POSMATRAJU ISTE ZVEZDE, SA ISTIM INSTRUMENTIMA,POMOCU ISTE TALCOTT-OVE METODE. OVO PREDSTAVLJA NAJVECE PREIMUCSRVO ILS SISTEMA. OD OSNIVANJA PA DO DANAS KONTINUIRANO JE OBEZBEDJIVAN POSMATRACKI MATERIJAL IAKO JE POVREMENO DOLAZILO DO SMETNJI I PREKIDA U POSMATRANJIMA POJEDINIH STANICA. DOLAZILO JE I DO PROMENE POSMATRACKIH PROGRAMA, UGLAVNOM ZBOG PRECESIONOG KRETANJA ZVEZDA KOJE DOVODI DO STVARANJA VECIH RAZLIKA ZENITNIH DALJINA POSMATRANIH PAROVA OD DOZVOLJENE GRANICE (OKO 20') VIDNOG POLJA VZT,PA SU TAKVI PAROVI ZAMENJIVANI NOVIM.

SVAKI POSMATRACKI PROGRAM SADRZI 12 GRUPA, SVAKA GRUPA OBUHVATA 2 CASA REKTASCENZIJE I SADRZI 6 ILI 8 PAROVA ZVEZDA. SVAKE NOCI SE POSMATRAJU 2 ILI 3 GRUPE U TOKU JEDNOG MESECA I SVAKOG SLEDECEG MESECA ISKLJUCUJE SE PRVA GRUPA I ZAMENJUJE NOVOM. U SKLADU SA RAZLIKOM IZMEDJU ZVEZDANOG I SREDNJEG VREMENA (OKO 2 CASA MESECNO) SVIH 12 GRUPA SE POSMATRAJU TOKOM GODINE. U SVAKU GRUPU SU UVEDENI I REFRAKCIJSKI PAROVI ZVEZDA SA ZENITSKIM DALJINAMA PREKO 50 STEPENI.

U PERIODU 1899-1979 POSMATRANO JE 772 395 PAROVA ZVEZDA TOKOM 66 090 POSMATRACKIH NOCI.

VEZU IZMEDJU POSMATRANE PROMENE GEOGRAFSKE SIRINE $\Delta\varphi_i = \varphi_i - \varphi_{ci}$ I PRAVOUGLIH KOORDINATA TRENUTNOG POLA X,Y , ZA SVAKU STANICU DAJE POZNATA JEDNACINA KOSTINSKOG:

$$\varphi_i - \varphi_{oi} = X\cos\lambda_i + Y\sin\lambda_i \qquad (2.1)$$

GDE JE φ_i TRENUTNA SIRINA STANICE i , φ_{ci} NJENA USVOJENA SIRINA (SREDNJA SIRINA ZA PERIOD 1900-1905) I λ_i GEOGRAFSKA DUZINA STANICE i .

U CILJU DOBIJANJA STO TACNIJIH KOORDINATA POLA X,Y KIMURA JE 1902 GODINE PREDLOZIO UVODJENJE JOS JEDNOG KOREKTIVNOG CLANA, NEZAVISNOG OD KOORDINATA POLA I JEDNAKOG ZA SVE STANICE. JEDNACINA (2.1) DOBIJA OBLIK:

$$\varphi_i - \varphi_{ci} = X\cos\lambda_i + Y\sin\lambda_i + Z \qquad (2.2)$$

UVEDENI KOREKTIVNII CLAN NAZIVA SE Z-CLANOM ILI KIMURINIM CLANOM. U NJEMU SU SADRZANE NEPOLARNE PROMENE SIRINE. NJEGOVO SE POSTOJANJE OBJASNJAVA NIZOM UZROKA ALI NI DO DANAS NEMA JEDINSTVENOG MISLJENJA O NJEGOVOM POREKLU (PODOBED & NESTEROV, 1982).

TRENUTNE KOORDINATE POLA X,Y RACUNATE SU IZ JEDNACINE (2.2) U ODNOSU NA MEDJUNARODNI KONVENCIONALNI POL CIO KOJI JE ODREDJEN (IAU, 1967 PRAG) SA USVOJENIM SIRINAMA PET MEDJUNARODNIH STANICA I TO:

MIZUSAWA	+39 08'03".602
KITAB	01 .850
CARLOFORTE	08 .941
GAITHERSBURG	13 .202
UKIAH	12 .096

OVE SREDNJE SIRINE SU DOBIJENE ZA SREDNJU EPOHU POSMATRAN OG PERIODA 1900-1905 GODINA. NA TAJ NACIN I KOORDINATE TRENUTNOG POLA SE RACUNAJU U OVOM SISTEMU.

MESECNE VREDNOSTI NEIZRAVNATIH KOORDINATA POLA DATE SU U TABLICI (PRILOG I) I GRAFICKI PREDSTAVLJENE NA SLICI (PRILOG III). STANDARDNE DEVIJACIJE MESECNIH VREDNOSTI X,Y,Z RACUNATE IZ REZI - DUA USLOVNIH JEDNACINA (2.2), (YUMI & YOKOYAMA, 1980) IZNOSE OD 0".01 DO 0".02 ZA X I Y, A OKO 0".01 ILI MANJE ZA Z.

U TABLICI (PRILOG II) I NA SLICI (PRILOG IV) DATE SU, ZA SVAKIH 0.05 GODINE, METODOM VONDRAKA (1969) IZRAVNATE KOORDINATE POLA X I Y.

NA SLICI (PRILOG VIII) ILUSTROVANA JE PUTANJA POLA IZ SI ROVIH PODATAKA ZA SVAKIH 6 GODINA.VIDIMO DA SE POL KRECE UNUTAR KVADRATA SA STRANICOM DO 1", STO IZRAZENO U METRIMA IZNOSI OKO 30, ODNOSNO TRENUTNI POL ZEMLJINE ROTACIJE OPISUJE SVOJU PUTANJU OKO SREDNJEG POLA NE UDALJAVAJUCI SE OD NJEGA VISE OD 15 METARA .

3.MATEMATICKE METODE KORISCENE U RADU

UKRATKO CEMO SE OSVRNUTI NA TEORIJSKE OSNOVE MATEMATICKIH METODA KORISCENIH PRI OBRADI I FILTRIRANJU POLAZNIH PODATAKA.

METODA IZRAVNANJA WHITAKER-ROBINSON-VONDRAK (WRV) NAM OMOGUCUJE DA OD MESECNIH VREDNOSTI KOORDINATA POLA DOBIJEMO TE PODATKE NA ZELJENIM EKVIDISTANTNIM RAZMACIMA POGODNIM ZA PRIMENU VISESTRUKIH TRANSFORMACIJA ORDINATA I STVARANJU TAKVOG FILTRA KOJI CE PROPUSTITI SAMO KORISNI SIGNAL. PRIMENOM METODE SPEKTRALNE ANALIZE FURIJEOVIH IINTEGRALNIH TRANSFORMACIJA (DFT) MOZEMO ODREDITI AMPLITUDU I FAZU HARMONIJSKIH CLANOVA.

POMENUTE METODE SU KORISCENE U OVOM RADU, NJIHOVE TEORIJSKE OSNOVE DATE SU NIZE.

3.1 IZRAVNANJE POSMATRANJA METODOM VITAKER - ROBINSON - VONDRAK

PRI IZRAVNANJU POSMATRANJA U POSLEDNJE VREME CESTO SE KORISTI METODA VITAKER-ROBINSON-VONDRAK (WHITAKER-ROBINSON, 1946 ; VONDRAK, 1969), KOJU MOZEMO SKRACENO NAZVATI METODOM WRV.

TESKOCE U PRIMENI KLASICNE METODE, METODE NAJMANJIH KVADRATA (MNK), I NJENI NEDOSTACI SU DOBRO POZNATI.MNK METODA MOZE DA SE DIREKTNO PRIMENI AKO JE POSMATRANA FUNKCIJA LINEARNA U ODNOSU NA NEPOZNATE KOJE TREBA ODREDITI. POSTO SE CESTO SUSRECU SLUCAJEVI U KOJIMA TA PRETPOSTAVKKA NE VAZI, PRISTUPA SE LINEARIZACIJI, ODNOSNO, RAZVOJU FUNKCIJE U TEJLOROV RED U KOME SE ZANEMARUJU KVADRATNI I CLANOVI VISEG REDA. DO RESENJA SE DOLAZI METODOM SUKCESIVNIH APROKSIMACIJA. POREDIZVESNIH TESKOCA NUMERICKE PRIRODE, 'LINEARIZACIJA' MOZE DA DOVEDE DO TESKIH PRINCIPIJELNIH PROBLEMA. KADA KORISTIMO METODU NAJMANJIH KVADRATA MI UNAPRED ZADAJEMO ANALI-

TICKI IZRAZ SA KOJIM CEMO PREDSTAVITI POSMATRANJA I ODREDJUJEMO NEPOZNATE POD USLOVOM DA NAM ZBIR KVADRATA REZIDUA BUDE MINIMALAN. TAJ ZBIR JE RELATIVAN I ZAVISI OD IZRAZA KOJIM PREDSTAVLJAMO POSMATRANJA.

AKO RASPODELA VEROVATNOCE GRESKE POSMATRANJA NIJE NORMALNA, ONDA NISMO SIGURNI DA SU RACUNATE NEPOZNATE NAJVEROVATNIJE JEDNAKE NJIHOVIM PRAVIM IZNOSIMA.

U METODI WRV NE ZADAJE SE ANALITICKI IZRAZ KOJIM BI TREBALO PREDSTAVITI POSMATRANJA. U OVOJ CINJENICI JE NJENA SUSTINSKA RAZLIKA I PREDNOST U ODNOSU NA METODU NAJMANJIH KVADRATA.

KORISNIK METODE WRV FIKSIRA NIVO IZRAVNANJA VODECI RACUNA O OSNOVNOM PRINCIPU METODE. DA BI SE SKRATIO POSTUPAK OKO IZNALAZENJA ODGOVARAJUCEG PARAMETRA λ IZ JEDNACINE (3.1) AUTORI METODE PREPORUCUJU DA NIVO IZRAVNANJA BUDE ODREDJEN SLUCAJNIM GRESKAMA. KADA, NAPRIMER, SREDNJA KVADRATSKA GRESKA RACUNATA IZ RAZLIKA POSMATRANIH I IZRAVNATIH VREDNOSTI POSTANE PRIBLIZNO JEDNAKA ONOJ VREDNOSTI KOJA REZULTIRA IZ SLUCAJNIH GRESAKA MERENJA, OSNOVNI PRINCIP METODE JE PRAKTICNO ZADOVOLJEN I TU SE TREBA ZAUSTAVITI.

NEKA SU Y_i I Y'_i POSMATRANA I ODGOVARAJUCA IZRAVNATA VREDNOST FUNKCIJE DATE ZA JEDAN NIZ EKVIDISTANTNIH ARGUMENATA, $\Delta^3 Y'$ RAZLIKE TRECEG REDA FUNKCIJE Y' I λ JEDAN POZITIVAN PARAMETAR OD KOJEG ZAVISI KOLIKO CE BITI Q IZ JEDNACINE (3.1) I, PORED TOGA, KAKAV CE BITI ODNOS DVE KOMPONENTE KOJE DEFINISU Q.

PRINCIP METODE WRV MOZE DA SE ANALITICKI IZRAZI JEDNACINOM:

$$Q = F + \lambda \sum_i (\Delta^3 Y'_i)^2 = MIN \qquad (3.1)$$

ILI

$$Q = F + \lambda S = MIN, \quad \text{GDE JE} \quad F = \sum_i (Y_i - Y'_i)^2$$

DRUGI CLAN U JEDNACINI (3.1) ODREDJUJE HRAPAVOST KRIVE KO

JOM BISMO INTERPOLOVALI Y' (BROJ MINIMUMA I MAKSIMUMA FUNKCIJE NA DATOM INTERVALU ARGUMENTA). OVO JE JASNO DOKAZAO PAKE U SVOM RADU (PAQUET, 1972).

VONDRAK JE RAZRADIO METODU VITAKER-ROBINSON UCINIIVSI JE PRIMENLJIVOM I U SLUCAJEVIMA KADA Y NISU EKVIDISTANTNE I NISU ISTIH TEZINA (VONDRAK, 1969). METODOM WRV NE DOBIJA SE ANALITICKI IZRAZ VEC SE IZ ODREDJENIH JEDNACINA ZA SVAKO Y RACUNA JEDNO Y'_i.

U PROGRAMU ZA RACUNAR IBM 360/44, MATEMATICKOG INSTITUTA SR SRBIJE, NA KOJEM SMO RACUNALI VREDNOSTI PRIKAZANE U OVOM RADU JEDAN TEST FIKSIRA λ U TRENUTKU KADA SREDNJA KVADRATSKA GRESKA E(WRV), RACUNATA IZ RAZLIKA $Y_i - Y'_i$,POSTANE PRIBLIZNO JEDNAKA ODGOVARAJUCEM E.

DA BI ODREDILI VREDNOSTI λ KOJE ZADOVOLJAVAJU JEDNACUNU (3.1) I DA BI VIDELI SA KOJOM BRZINOM $Y'_i \longrightarrow Y_i$ SA POVECANJEM λ , ODDELJIVALI SMO PROMENLJIVOJ λ SLEDECE VREDNOSTI: 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9 I 10^{10} . NASLI SMO DA ZA VREDNOST $\lambda = 10^9$, DOSTIZE SE NAJBOLJE IZRAVNANJE KOORDINATA X I Y.

3.2 FURIJEOVE INTEGRALNE TRANSFORMACIJE

NEKA JE $Y(t)$ STACIONARNA FUNKCIJA DATA ZA 2N+1 EKVIDISTANTNIH ARGUMENATA t. FUNKCIJE :

$$U(\omega) = \frac{2}{2N+1} \sum_{t=-N}^{N} Y(t) \sin \omega t$$
$$V(\omega) = \frac{2}{2N+1} \sum_{t=-N}^{N} Y(t) \cos \omega t \qquad (3.2)$$

SU POZNATE KAO FURIJEOVE INTEGRALNE TRANSFORMACIJE (SEREBRENJIKOV & PERVOZVANSKIJ, 1965). ONE IMAJU EKSTREMUME ZA ONE ω , KOJI SU BLISKI ILI JEDNAKI UGAONIM BRZINAMA HARMONIJSKIH CLANOVA U $Y(t)$. AMPLITUDE $A(\omega)$ I FAZE $F(\omega)$ TIH CLANOVA ODREDJENE SU RELACIJAMA:

$$A^2(\omega) = U^2(\omega) + V^2(\omega)$$

$$tg\,F(\omega) = U(\omega)/V(\omega)\,. \qquad (3.3)$$

SELEKTIVNA SVOJSTVA FURIJEOVIH TRANSFORMACIJA ZAVISE OD DUZINE INTERVALA 2L=2N+1 NA KOJEM SU DATI PODACI. KADA JE POLUINTERVAL L VELIKI, MALOM PROMENOM ω U OKOLINI ω_j POSTIZE SE RELATIVNO VELIKA PROMENA PROIZVODA $(\omega - \omega_j)L$ I NAGLI PAD SELEKTIVNE FUNKCIJE $R(\lambda)=SIN\lambda/\lambda$. ODNOSNO, POSTIZE SE VELIKA OSTRINA PRIMARNOG PIKA, A SAMIM TIM I DOBRA SELEKTIVNOST TRANSFORMACIJE. SA ω JE OZNACENA PROBNA UGAONA BRZINA BLISKA UGAONOJ BRZINI ω_j NEKE OD KOMPONENATA.

KAKO SELEKTIVNA FUNKCIJA FURIJEOVIH TRANSFORMACIJA $R(\lambda)=SIN\lambda/\lambda$ IMA PRVU NULU ZA $\lambda=\pi$, ODNOSNO ZA $(2N+1)(\omega-\omega_j)/2=\pi$, SIRINA PRIMARNOG PIKA JE DATA RELACIJOM:

$$\Delta\epsilon = (2N+1)(\omega - \omega_j). \qquad (3.4)$$

AKO SU SREDISNJE ORDINATE PRIMARNIH PIKOVA, KOJI ODGOVARAJU HARMONIJSKIM CLANOVIMA, UDALJENE ZA $\Delta\epsilon$, UTICAJ JEDNOG OD NJIH NA AMPLITUDU DRUGOG JEDNAK JE NULI.

FUNKCIJA $R(\lambda) = SIN\lambda/\lambda$ IMA PRIMARNI MAKSIMUM ZA $\lambda = 0$ I NIZ SEKUNDARNIH MAKSIMUMA (λ = 4.49, 7.72, 10.91) CIJE AMPLITUDE UNIFORMNO OPADAJU KADA $\lambda \to \infty$. SEKUNDARNI MAKSIMUMI PREDSTAVLJAJU SEKUNDARNE PIKOVE. PRVI SEKUNDARNI PIK IMA AMPLITUDU KOJA IZNOSI 22% OD AMPLITUDE PRIMARNOG PIKA, AMPLITUDA DRUGOG SEKUNDARNOG PIKA JE 13% AMPLITUDE PRIMARNOG PIKA I TAKO REDOM.

AKO SE ZA SVAKU VREDNOST PROBNE PERIODE $P = 2\pi/\omega$ PROMENI DUZINA INTERVALA TAKO DA P PREDSTAVLJA NJEGOV CELOBROJNI DEO, AMPLITUDE PIKOVA BICE JEDNAKE AMPLITUDAMA SINUS I CISINUS KOMPONENTE. ODAVDE SLEDI POZNATO PRAVILO DA SE PRI RACUNU $U(\omega)$ I $V(\omega)$ NE KORISTE SVI PODACI, VEC INTEGRACIJA VRSI NA PROMENLJIVOM INTE-

RVALU KOJI SADRZI MAKSIMALAN BROJ PROBNIH PERIODA 2L=NP. OVO JE POZNATO KAO USLOV SAMERLJIVOSTI.

PRI PRIMENI FURIJEOVIH TRANSFORMACIJA TREBA VODITI RACUNA O NAJKVISTOVOJ FREKVENCIJI. NAIME, AKO JE FUNKCIJA Y(T) ZADATA U OBLIKU NIZA EKVIDISTANTNIH VREDNOSTI SA RAZMAKOM ΔT, TREBA VODITI RACUNA O TOME DA JE SPEKTRALNA GUSTINA S(F) PERIODICNA FUNKCIJA SA PERIODOM $1/\Delta T$ (DJUROVIC, 1979). ZBOG PERIODICNOSTI FUNKCIJE S(F), NJEN DEO NAD INTERVALOM $-F_N \leq F \leq +F_N$, GDE JE $F = 1/2\Delta T$, DAJE MAKSIMUM MOGUCIH INFORMACIJA O SASTAVU POLIHARMONIJSKOG SIGNALA. GRANICNA FREKVENCIJA F_N JE POZNATA POD IMENOM NAJKVISTOVA FREKVENCIJA (NYQUIST FREQUENCY).

3.3 VISESTRUKE TRANSFORMACIJE ORDINATA

OVO SU TRANSFORMACIJE TRANSFORMISANIH NIZOVA KOJE PRUZAJU SIROKE MOGUCNOSTI RESAVANJA PROBLEMA IZOLACIJE JEDNOG DATOG CLANA ILI CLANOVA IZ USKE ZONE FREKVENCIJA. POGODNIM IZBOROM ELEMENTAR - NIH TRANSFORMACIJA MOZE DA SE DOBIJE TAKAV FILTER KOJI CE PROPUSTI TI SAMO KORISNI SIGNAL, A OSTALE SIGNALE I SUM PRIGUSITI.

IZLOZICEMO KORISCENU VISESTRUKU TRANSFORMACIJU ODUZIMANJA:

DAT JE NIZ VREDNOSTI FUNKCIJE (POSMATRACKI PODACI) :

Y1, Y2, Y3,....., YN

ZA EKVIDISTANTNE VREDNOSTI ARGUMENTA:

X1, X2, X3,....., XN

GDE JE

$$X_i - X_{i-1} = \Delta X = h = \text{CONST}$$

POD PRETPOSTAVKOM DA FUNKCIJA Y=F(X) PREDSTAVLJA REZULTAT VISE PERIODICNIH KOMPONENTI, TJ.

$$Y = \sum_{j=1}^{k} \left[a_j \sin(\omega_j X) + b_j \cos(\omega_j X) \right] = \sum_{j=1}^{k} A_j \sin(\omega_j X + \varphi_j) \qquad (3.5)$$

GDE JE k (BROJ KOMPONENTI) PO PRAVILU NEPOZNATO, IZDVAJANJE POJEDINIH (ILI VISE) KOMPONENTI IZ ZAJEDNICKOG SIGNALA Y(X) MOGUCE JE TRANSFORMACIJAMA ODUZIMANJA ORDINATA.

FORMIRAJMO VELICINE Z_i^m NA SLEDECI NACIN:

$$Z_i^m = Y_{i+1} - Y_{i-1} + Y_{i+2} - Y_{i-2} + \ldots + Y_{i+m} - Y_{i-m} \qquad (3.6)$$

GDE JE

$$m>0,\ m\in(1,2,\ldots)\ \ i-m \geq 1,\ \ i \geq m+1,\ \ i+m \leq n \Rightarrow i \leq n-m$$

VELICINA Z_i^m PREDSTAVLJA REZULTAT PRIMENE TRANSFORMACIJA ODUZIMANJA ORDINATA REDA m. USVOJIMO DA JE

$$k=1 \Rightarrow X_{i+\ell} = X + \ell \qquad (3.7)$$

IZ JEDNACINA (3.5), (3.6) I (3.7) IMAMO:

$$\begin{aligned} Y_{i+m} - Y_{i-m} &= \sum_{j=1}^{k} A_j \sin(\omega_j x_i + \varphi_j + m\omega_j) - \sum_{j=1}^{k} A_j \sin(\omega_j x_i - \varphi_j - m\omega_j) \\ &= \sum_{j=1}^{k} 2A_j \sin m\omega_j \cos(\omega_j x + \varphi_j) = \sum_{j=1}^{k} \beta_j^m A_j \cos(\omega_j x_i + \varphi_j) \end{aligned} \qquad (3.8)$$

GDE JE β_j^m AMPLITUDNI MULTIPLIKATOR, A TRANSFORMISANI NIZ VELICINA $Y_{i+m} - Y_{i-m}$ SADRZI ISTE PERIODICNE KOMPONENTE, ALI FAZNO POMERENE ZA $\pi/2$. AMPLITUDNI MULTIPLIKATOR ZA Z_i^m DAT JE IZRAZOM (DJUROVIC, 1979):

$$\xi_j^m = \beta_j^1 + \beta_j^2 + \ldots + \beta_j^m = 2\sin\omega_j + 2\sin 2\omega_j + \ldots + 2\sin m\omega_j$$

ODNOSNO

$$\xi_j^m = 2\,\frac{\sin(m+1)\frac{\pi}{T_j}\ \sin\frac{\pi}{T_j}}{\sin\frac{\pi}{T_j}}$$

UKOLIKO SE NA NOVI NIZ VELICINA Z_i^m PRIMENI TRANSFORMACIJA ODUZIMANJA REDA p (ZATIM q, ... v) KAZEMO DA SE RADI O VISE-

STRUKIM TRANSFORMACIJAMA ODUZIMANJA, KOJE SIMBOLICNO PREDSTAVLJAMO U OBLIKU $\Pi(Z) = Z^m Z^p Z^q Z^r$. AMPLITUDNI MULTIPLIKATOR JE TADA DAT IZRAZOM:

$$\xi_j = \xi_j^m \cdot \xi_j^p \cdot \xi_j^q \cdot \xi_j^r \qquad , r >= \ldots >= q >= p >= m.$$

3.4 METODE ORLOVA I MELKIORA

UKRATKO DA SE OSVRNEMO NA DOSTA KORISCENE METODE ZA DOBIJANJE SREDNJE SIRINE, METOD ORLOVA I METOD MELKIORA. OVE METODE SU, USTVARI, VISESTRUKE KOMBINACIJE SABIRANJA ORDINATA, TIPA (S), POGODNE ZA PRIGUSIVANJE KRATKOPERIODICNIH CLANOVA.

ZA DOBIJANJE SREDNJE SIRINE OSLOBODJENE PERIODICNIH PROMENA ORLOV JE DAO SLEDECU FORMULU:

$$\varrho_m = \frac{1}{40}\,(\varrho_0 + 2\varrho_1 + 2\varrho_2 + \ldots + 4\varrho_8 + \ldots + 2\varrho_{15} + \varrho_{16}) \qquad (3.10)$$

VREDNOSTI ϱ_m SU OSLOBODJENE CENDLEROVOG, GODISNJEG I POLUGODISNJEG CLANA, ODNOSNO FUNKCIJA SELEKTIVNOSTI IMA NULE ZA $n=10$ (GODISNJI PERIOD) I $n=12$ (CENDLEROV PERIOD) JEDINICNIH INTERVALA (0.1 GODINA).

DA BI SE DOBILA JEDNA SREDNJA VREDNOST (JEDNAC. 3.10) POTREBNO JE 17 VREDNOSTI ($\varrho_0, \varrho_1, \ldots, \varrho_{16}$) DATIH SA EKVIDISTANTNIM RAZMAKOM OD 0.1 GODINA.

MELKIOR SMATRA DA FORMULA ORLOVA (3.10) NIJE POGODNA ZA ANALIZU DUZIH POSMATRACKIH SERIJA JER ELIMINISE SAMO GLAVNE HARMONIJSKE CLANOVE A NE ELIMINISE KRATKOPERIODICNE, UKOLIKO POSTOJE. S TOGA, ON PREDLAZE NOVU TRANSFORMACIJU OBLIKA:

$$\varrho_m = \frac{1}{19305}\,(\varrho_{45} + 4\varrho_{44} + 10\varrho_{43} + \ldots + 1038\varrho_{22} + 1047\varrho_{23} + 1030\varrho_{24} + \ldots + 4\varrho_2 + \varrho_1) \qquad (3.11)$$

KOJA STROGO ISKLJCUJE GLAVNE HARMONIJSKE I SVE KRATKOPERIODICNE CLANOVE, JER FUNKCIJA SELEKTIVNOSTI IMA NULE ZA $\eta=9$, $\eta=11$, $\eta=13$ I $\eta=15$ JEDINICNIH INTERVALA (0.1 GODINA). PROIZVOD BROJEVA η JE 19305.

ZA DOBIJANJE JEDNE SREDNJE VREDNOSTI PO FORMULI MELHIORA POTREBNO JE 45 VREDNOSTI ($\varphi_1, \varphi_2, \ldots, \varphi_{45}$).

KOEFICIJENTI JEDNACINA (3.10) I (3.11) DATI SU U TABLICI (3.1), KULIKOV (1962).

TABLICA 3.1

	KOEFIC. ORLOVA	KOEFIC. MELKIOR		KOEFIC. ORLOVA	KOEFIC MELKIOR
0	1	1	12	-	431
1	2	4	13	-	514
2	2	10	14	-	600
3	2	20	15	-	686
4	2	35	16	-	769
5	2	56	17	-	846
6	3	84	18	-	914
7	4	120	19	-	970
8	4	165	20	-	1012
9	-	219	21	-	1038
10	-	282	22	-	1047
11	-	353			

U LITERATURI SE SUSRECU PODELJENA MISLJENJA O PREDNOSTIMA JEDNOG FILTRA U ODNOSU NA DRUGI. S TOGA CEMO U KASNIJOJ ANALIZI KORISTITI OBA.

4. ANALIZA KOORDINATA POLA

SIROVE I IZRAVNATE PRAVOUGLE KOORDINATE TRENUTNOG POLA ZEMLJINE ROTACIJE U ODNOSU NA CIO (PRILOG I I PRILOG II) ANALIZIRANE SU SA CIJLEM DA SE UTVRDI:

A) POSTOJANJE SEKULARNOG KRETANJA POLA

B) POSTOJANJE JEDNE ILI DVE SLOBODNE NUTACIJE ZEMLJINE OSE ROTACIJE (JEDAN ILI DVA CENDLEROVA PERIODA),

C) POSTOJANJE STABILNOSTI AMPLITUDA I FAZA GLAVNIH HARMO-HARMONIJSKIH CLANOVA: CENDLEROVOG, GODISNJEG I POLUGODISNJEG,

D) IDENTIFIKACIJA NUTACIJE MOLODENSKOG (CLANA OD 204 DANA) KAO I DRUGIH NEPOZNATIH ILI MALO POZNATIH KRATKOPERIODICNIH I DUGOPERIODICNIH CLANOVA.

4.1 SEKULARNI CLAN

GEOLOSKA ISPITIVANJA SU POKAZALA DA SU SE POLOVI ZEMLJINE ROTACIJE, U TOKU GEOLOSKE PROSLOSTI, DOSTA DALEKO POMERALI PO POVRSINI ZEMLJE. TO SVEDOCE BOGATE NASLAGE KAMENOG UGLJA PRONADJENE NA SPICBERSKIM OSTRVIMA, KOJE SE NISU MOGLE OBRAZOVATI NA SADASNJOJ GEOGRAFSKOJ SIRINI TIH OSTRVA ZBOG KLIMATSKIH PRILIKA. POLOZAJ ZEMLJINIH POLOVA, U NJENOJ GEOLOSKOJ PROSLOSTI, BIO JE SASVIM DRUGACIJI NO STO JE SADA. OTUDA SE NAMECE PROBLEM POSTOJANJA SEKULARNOG (VEKOVNOG) POMERANJA ZEMLJINIH POLOVA.

NEKA SU X_o ,Y_o PRAVOUGLE KOORDINATE SREDNJEG POLA A X,Y PRAVOUGLE KOORDINATE TRENUTNOG POLA. INDEKSOM u OZNACAVACEMO SIROVE A INDEKSOM v IZRAVNATE KOORDINATE TRENUTNOG POLA.

KAKO JE ZA HARMONIJSKU ANALIZU NEOPHODNO IZ PODATAKA UKLONITI PROGRESIVNI CLAN, U TOM CILJU SMO FORMIRALI SREDNJE SESTOGODISNJE VREDNOSTI X_u, Y_u I X_v, Y_v I IZRACUNALI NJIHOVE RAZLIKE PRVOG, DRUGOG I TRECEG REDA. DOBIJENI REZULTATI DATI SU U TABLICI 4.1 ZA SIROVE I U TABLICI 4.2 ZA IZRAVNATE KOORDINATE POLA.

TABLICA 4.1 (VREDNOSTI SU U 0".001)

U KOLONAMA 1 I 5 DATE SU SREDNJE SESTOGODISNBJE VREDNOSTI KOORDINATA POLA, A U KOLONAMA 2-4 I 6-8 NJIHOVE RAZLIKE PRVOG, DRUGOG I TRECEG REDA.

X	$\Delta^1 X$	$\Delta^2 X$	$\Delta^3 X$	Y	$\Delta^1 Y$	$\Delta^2 Y$	$\Delta^3 Y$
15				2			
	4				28		
19		20		29		- 25	
	24		- 32		2		38
42		- 12		32		13	
	12		- 14		15		7
54		- 25		47		20	
	- 14		30		35		- 10
41		5		82		10	
	- 9		9		45		- 46
32		14		128		- 36	
	5		1		10		8
37		15		137		- 28	
	20		0		- 18		71
56		14		119		43	
	34		- 68		25		- 41
90		- 53		144		2	
	- 20		55		27		8
70		1		172		11	
	- 19		- 3		38		- 17
51		- 2		210		- 7	
	- 21		54		31		- 19
31		52		241		- 26	
	31				6		
61				247			

TABLICA 4.2 (VREDNOSTI U JEDINICAMA 0".001)

U KOLONAMA 1 I 5 DATE SU SREDNJE SESTOBODISNJE VREDNOSTI KOORDINATA POLA, A U KOLONAMA 2-4 I 6-8 NJIHOVE RAZLIKE PRVOG, DRUGOG I TRECEG REDA.

X	$\Delta^1 X$	$\Delta^2 X$	$\Delta^3 X$	Y	$\Delta^1 Y$	$\Delta^2 Y$	$\Delta^3 Y$
53				2			
	27				31		
81		- 42		33		- 40	
	- 15		29		- 9		73
66		- 13		24		33	
	- 29		21		24		- 23
37		8		47		10	
	- 21		37		34		4
17		45		81		14	
	25		- 76		48		- 53
41		- 31		129		- 39	
	- 6		52		9		13
35		21		138		- 26	
	14		- 8		- 17		67
49		13		120		41	
	28		- 51		23		- 36
77		- 38		144		4	
	- 10		33		28		6
66		- 5		172		10	
	- 16		19		38		- 16
51		14		209		- 6	
	- 2		- 26		32		- 21
49		- 13		241		- 27	
	- 15				5		
34				246			

SREDNJE KVADRATSKA ODSTUPANJA U ODNOSU NA SREDNJE VREDNOSTI ODGOVARAJUCIH RAZLIKA I REDA S1, II REDA S2 I III REDA S3 RACUNATA PO IZRAZU:

$$S_K = 1/N \sum_{i=1}^{N} (\Delta^k Y_i)^2$$

U SIROVIM I IZRAVNATIM SERIJAMA DATA SU U TABLICI 4.3.

TABLICA 4.3

SRED.KVADR. ODSTUPANJE	X_u	Y_u	X_v	Y_v
S1	0.0004	0.0005	0.0004	0.0007
S2	0.0007	0.0007	0.0007	0.0007
S3	0.0013	0.0012	0.0016	0.0015

UPOREDJUJUCI SREDNJE KVADRATSKA ODSTUPANJA S1,S2 I S3 NALAZIMO DA JE U SVIM KOORDINATAMA $S1 < S2 < S3$. IZ OVOGA SLEDI DA PRVE RAZLIKE IMAJU NAJMANJE SREDNJE KVADRATSKO ODSTUPANJE,ODNOSNO DA JE LINEARNA APROKSIMACIJA SEKULARNOG CLANA NAJBOLJA.

METODOM NAJMANJIH KVADRATA RACUNALI SMO KOEFICIJENTE REGRESIJE. SLOBODNI CLAN C0 I LINEARNI C1 ZA SIROVE I IZRAVNATE SERIJE DATI SU U TABLICI 4.4.

TABLICA 4.4

KOEFIC. REGRESIJE	X_u	Y_u	X_v	Y_v
C0	0.0179	-0.0155	0.0185	-0.0157
C1	0.0006	0.0035	0.0006	0.0035

SLOBODNI CLANOVI DATI SU SA GRESKOM - 0.0005, A LINEARNI SA GRESKOM $\pm$ 0.0001.

NA OSNOVU TABLICE 4.4 NALAZIMO DA JE SEKULARNI CLAN U SIROVIM KOORDINATAMA POLA PREDSTAVLJEN SA:

$$X = 0".0179 + 0".0006(t - 1900) \qquad (4.1)$$
$$Y = -0".0155 + 0".0035(t - 1900)$$

A U IZRAVNATIM:

$$X = 0".0185 + 0".0006(t - 1900) \qquad (4.2)$$
$$Y = -0".0157 + 0".0035(t - 1900)$$

ODAKLE DIFERENCIRANJEM PO t NALAZIMO DA SE SREDNJI POL KRECE BRZINOM:

$$dX_c/dt = 0".0006/GOD \qquad (4.3)$$
$$dY_c/dt = 0".0035/GOD \ ,$$

REZULTUJUCOM LINEARNOM BRZINOM

$$d\psi/dt = 0^{\circ}.0036/GOD \qquad (4.4)$$

SA TACNOSCU OD $\pm 0^{\circ}.0002/GOD$, U PRAVCU ZAPADNE LONGITUDE 79 .

U RADU MARKOWITZ (1982) NALAZIMO DA JE IZ PODATAKA ILS ZA PERIOD 1900.0-1982.0 NASAO ZA BRZINU SEKULARNOG KRETANJA SREDNJEG POLA IZNOS $d\psi/dt = 0^{\circ}.0035/GGOD$ U PRAVCU ZAPADNE LONGITUDE 75 .

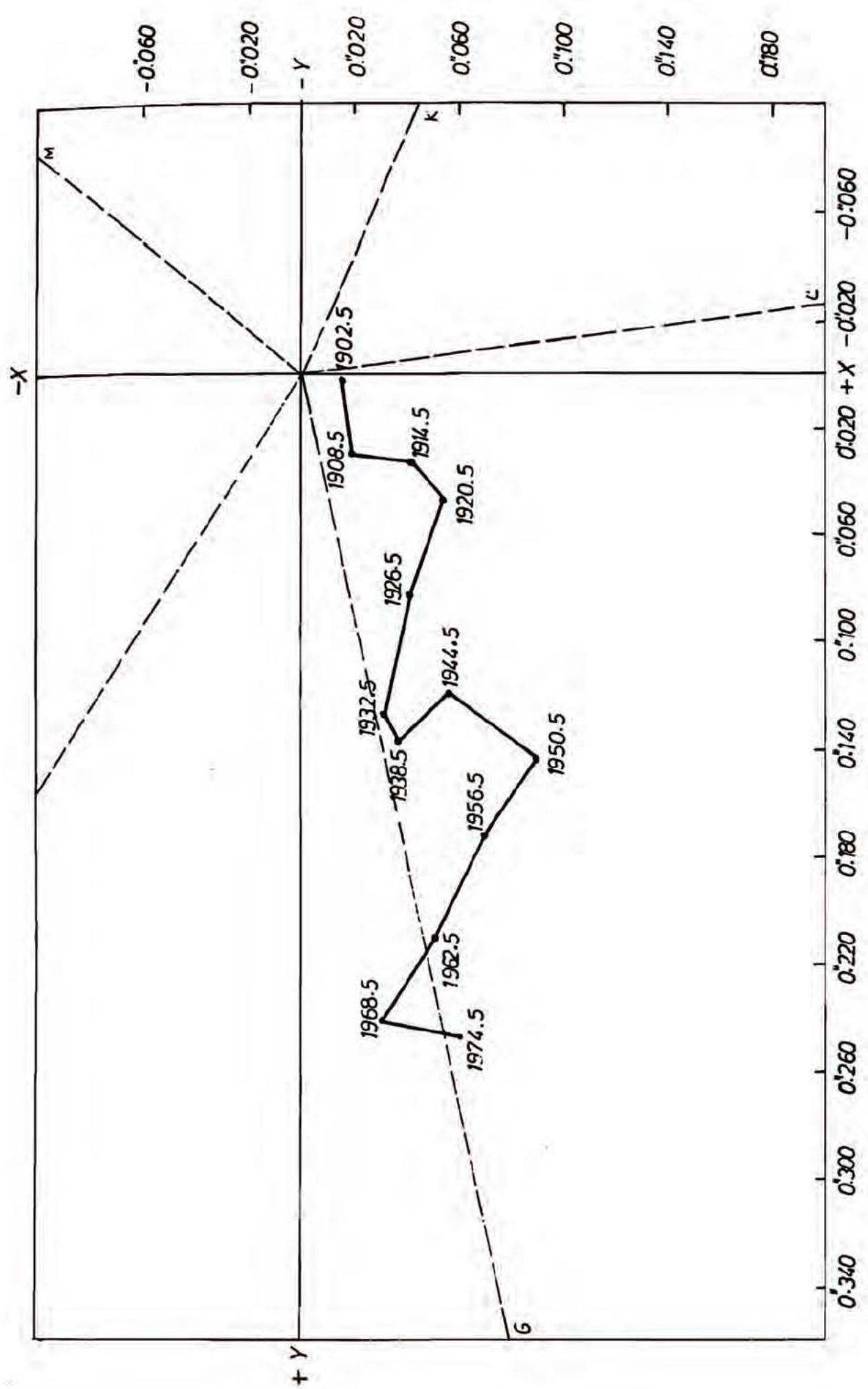
NAVESCEMO I REZULTATE DRUGIH AUTORA DATIH U MONOGRAFIJI LAMBECK (1980):

AUTOR	$d\psi/dt$	PRAVAC (ZAPADNA LONGITUDA)
MARKOWITZ (1970)	$0^{\circ}.0035/GOD$	65°
YUMI & WAKO (1970)	$0^{\circ}.0022/GOD$	77°
STOYKO (1973)	$0^{\circ}.0040/GOD$	73° (PET ILS STANICA)
STOYKO (1973)	$0^{\circ}.0032/GOD$	70° (TRI ILS STANICE)

NA OSNOVU NASIH REZULTATA I REZULTATA DRUGIH AUTORA, KOJI SU U ODLICNOJ SAGLASNOSTI, NALAZIMO DA POSTOJI LINEARNO SEKULARNO KRETANJE SREDNJEG POLA, STO SE JOS UVEK SMATRA NEDOVOLJNO DOKAZANIM.

OD SIROVIH I IZRAVNATIH KOORDINATA POLA ODUZET JE SEKULARNI CLAN I DOBIJENI REZULTATI PREDSTAVLJENI NA SLIKAMA PRILOG V I PRILOG VI.

NA OSNOVU GRAFICKOG PRIKAZA POLHODIJE (SL. 4.1) NA KOJOJ SU PRIKAZANE SREDNJE SESTOGODISNJE VREDNOSTI X , Y UOCAVA SE NJIHOVO SEKULARNO POVECANJE, STO GOVORI O POSTOJANJU VEKOVNOG KRETANJA POLA.



Sl. 4.1 POLHODIJA

NA ISTOJ SLICI OZNACENI SU PRAVCI TANGENATA NA MERIDIJANE OPSERVATORIJA: MIZUSAWA (M), KITAB (K), CARLOFORTE (C), GAITHES - BURG (G) I UKIAH (U).

4.2 GLAVNI HARMONIJSKI CLANOVI

POSLE ELIMINACIJE SEKULARNOG CLANA IZ KORISCENIH PODATAKA DOBILI SMO REZIDUE, GRAFICKI PREDSTAVLJENE U PRILOGU V I PRILOGU VI, NA KOJE SMO PRIMENILI FURIJEOVE INTEGRALNE TRANSFORMACIJE, JEDNACINE (3.2), ZA IDENTIFIKACIJU GLAVVNIH HARMONIJSKIH CLANOVA : CENDLEROVOG, GODISNJEG I POLUGODISNJEG.

PROBNA PERIODA JE VARIRANA U GRANICAMA OD 60 DO 760 DANA SA PRIRASTAJEM OD 0.5 DANA. POCETNA PERIODA PREDSTAVLJA DVOSTRUKU VREDNOST INTERVALA NA KOJEM SU DATI PODACI. PRI IZBORU PROBNIH PERIODA VODILI SMO RACUNA O EFEKTIMA POZNATIM POD NAZIVIMA LEAKAGE I "ALIASING".

DOBIJENE VREDNOSTI ZA PERIODU P_f , AMPLITUDU A_f , I FAZU F POLUGODISNJEG CLANA DATE SU U KOLONAMA 2-4 TABLICE 4.5. U KOLONAMA 5-7, ISTE TABLICE, DATE SU DOBIJENE VREDNOSTI ZA PERIODU P_a , AMPLITUDU A_a I FAZU F_a GODISNJEG CLANA. PERIODE SU DATE U GODINAMA A FAZE F_a , I F_f SE ODNOSE NA 1900.0, JULIJANSKI DATUM D.J.=2415 020.

TABLICA 4.5

KOORD.	P_f	A_f	F_f	P_a	A_a	F_a
X_u	0.501	0".0035	83.3	1.000	0".094	253.9
X_v	0.501	0 .0034	81.9	1.000	0 .094	253.8
Y_u	0.501	0 .0038	348.2	1.000	0 .076	171.7
Y_v	0.501	0 .0037	349.7	1.000	0 .077	170.3

NA ONOM MESTU SPEKTRA GDE SE OCEKUJE CENDLEROV CLAN DOBILI SMO DVOJNI PIK. PERIODE P_c I P'_c , ODGOVARAJUCE AMPLITUDE A_c I A'_c I FAZE F_c I F'_c DATE SU U TABLICI 4.6. PERIODE SU DATE U GOD INAMA A FAZE F_c I F'_c SE ODNOSE NA 1900.0, JULIJANSKI DATUM DJ = 2415 020

TABLICA 4.6

KOORD.	P_c	A_c	F_c	P'_c	A'_c	F'_c
X_u	1.171	0".105	59.4	1.197	0".114	16.4
X_v	1.171	0 .105	68.8	1.199	0 .115	15.4
Y_u	1.171	0 .107	332.2	1.197	0 .118	284.5
Y_v	1.171	0 .107	340.8	1.199	0 .117	285.1

DISPERZIJE PARAMETARSKIH JEDNACINA (4.1) RACUNATE SU PREMA IZRAZU

$$\sigma_u^2 = \sigma_v^2 = \frac{2}{2N+1}\,\sigma_o^2 \qquad (4.5)$$

GDE JE σ_o SREDNJA KVADRATSKA GRESKA POLAZNIH PODATAKA. U NASEM SLUCAJU SREDNJA KVADRATSKA GRESKA KOORDINATA X_u , Y_u , X_v I Y_v IZNOSI σ_o = 0".01. BROJ SIROVIH PODATAKA JE 2N+1=951 A BROJ IZRAVNATIH 2N+1=1583.

IZ IZRAZA (4.5) SLEDI DISPERZIJA AMPLITUDE POJEDINOG CLANA:

$$\sigma_A^2 = (2 - \pi/2)\,\frac{2}{2N+1} \qquad (4.6)$$

ZA SREDNJE KVADRATSKU GRSKU AMPLITUDE σ_A , NA OSNOVU IZRAZA ZA DISPERZIJU AMPLITUDE (4.6), DOBIJAMO ± 0".0003 U NEIZRAVNATIM SERIJAMA I U IZRAVNATIM SERIJAMA ± 0".0002.

POPRAVKA AMPLITUDE ΔA ZA NIVO "BELOG SUMA" ILI NIVO ZA-JEDNICKIH AMPLITUDA JE

$$\Delta A = \sigma_0 \sqrt{\frac{\pi}{2N+1}} \qquad (4.7)$$

ZBOG CEGA JE STVARNA AMOLITUDA POSMATRANOG CLANA UMANJENA ZA A. AMPLIITUDE U TABLICAMA 4.5 I 4.6 POPRAVLJENE SU ZA $\Delta A = 0^{\circ}.001$.

DISPERZIJE RACUNATIH FAZA DOBIJAJU SE RAZVIJANJEM FUNKCIJE $Y=F(U,V)$, TJ. $tg\, F = U/V$ (JEDNACINA 3.3) U OKOLINI MATEMATICKIH OCEKIVANJA $m(U)$ I $m(V)$

$$\sigma_y^2 = \varphi_u'^2 \sigma_u^2 + \varphi_v'^2 \sigma_v^2 + \varphi_{uu}'' \sigma_u^2 + \varphi_{vv}'' \sigma_v'' + \varphi_{uv}'' \sigma_u^2 \sigma_v^2$$

ODAKLE SLEDI

$$\sigma_F^2 = \frac{1}{A^2} + \sigma_u^2 + \frac{1}{A^8}(A^4 + 4u^2v^2)\sigma_u^4 + \dots$$

A ZADRZAVAJUCI SAMO PRVI CLAN REDA

$$\sigma_F^2 \approx \frac{2}{A^2} \frac{\sigma_c^2}{2N+1} \quad [rad]$$

ILI U STEPENIMA

$$\sigma_F^2 \approx (57^{\circ}.296) \frac{\sigma_c^2}{A^2} \frac{2}{2N+1} \qquad (4.8)$$

SREDANJE KVADRATSKA GRESKA FAZA σ_F, RACUNATA PO IZRAZU ZA DISPERZIJU FAZA (4.8) ZA GODISNJI I CENDLEROV CLAN JE U INTERVALU OD $\pm 0^{\circ}.2 - \pm 0^{\circ}.3$, A ZA POLUGODISNJI CLAN IZNOSI $\pm 3^{\circ}.3$.

SPEKTRI DOBIJENI FURIJEOVOM ANALIZOM GRAFICKI SU PREDSTAVLJENI NA SLICI 4.2 ZA SERIJE X_u, Y_u I NA SLICI 4.3 ZA SERIJE X_v, Y_v. NA APSCISNOJ OSI P NALAZE SE PROBNE PERIODE U GODINAMA A NA ORDINATNOJ OSI A AMPLITUDE U LUCNIM SEKUNDAMA.

NA ISTIM SLIKAMA DESET PUTA UVECANO PREDSTAVLJENI SU SPEKTRI NA INTERVALU OD 0.2 DO 0.8 GODINA. NA NJIMA SE NE ZAPAZA POLUGODISNJI CLAN KAO NI PRISUSTVO DRUGIH CLANOVA.

DVOJNOST CENDLEROVOG CLANA ISPITIVALI SMO NA OSNOVU POREDJENJA RACUNATIH I TEORIJSKIH SIRINA PIKOVA.

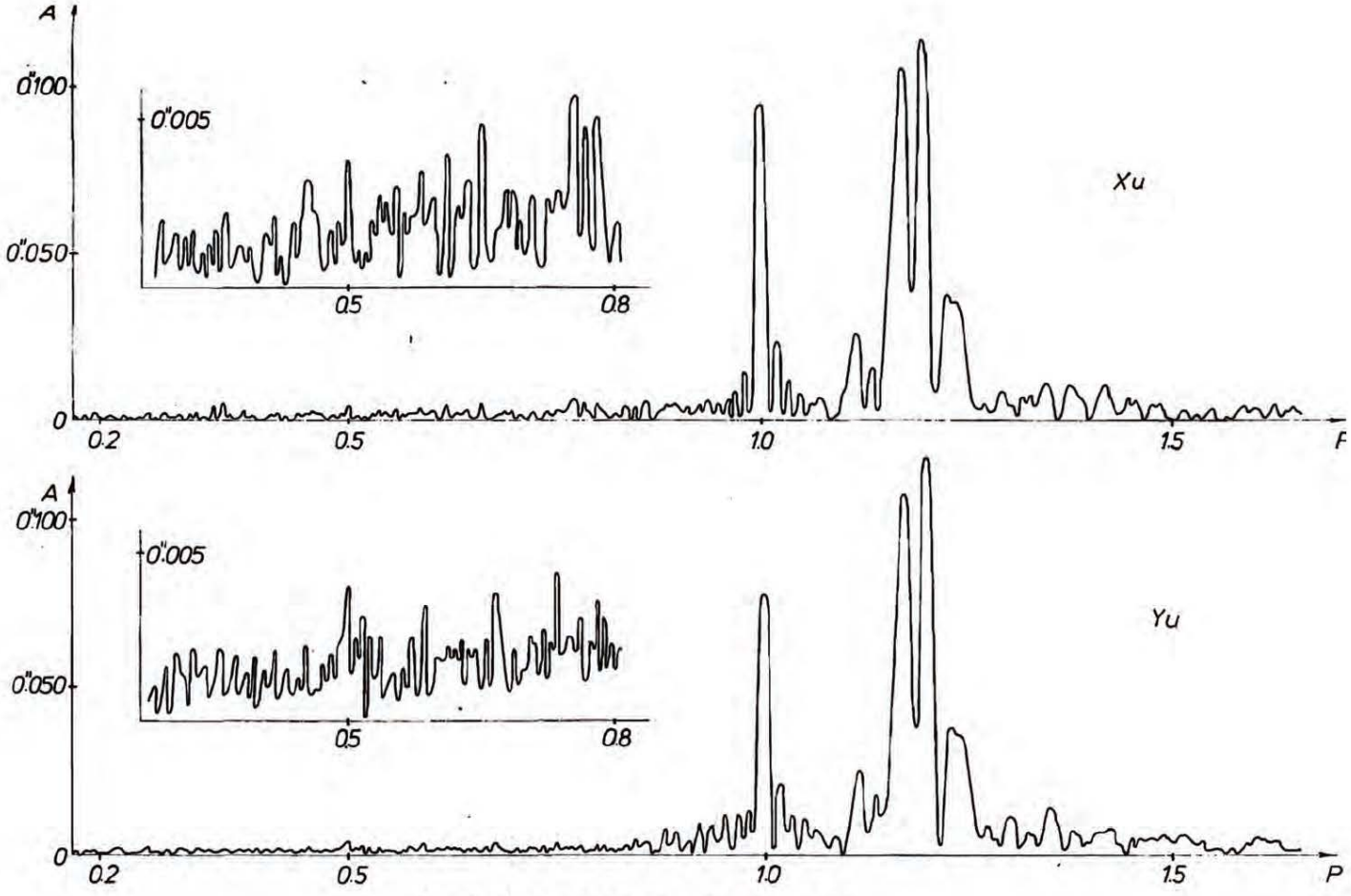
U TABLICI 4.7 DATE SU TEORIJSKE $\Delta\varepsilon$ (JEDNACINA 3.4) I RACUNATE $\Delta\varepsilon'$ SIRINE PIKOVA GLAVNIH HARMONIJSKIH CLANOVA U GODINAMA.

TABLICA 4.7

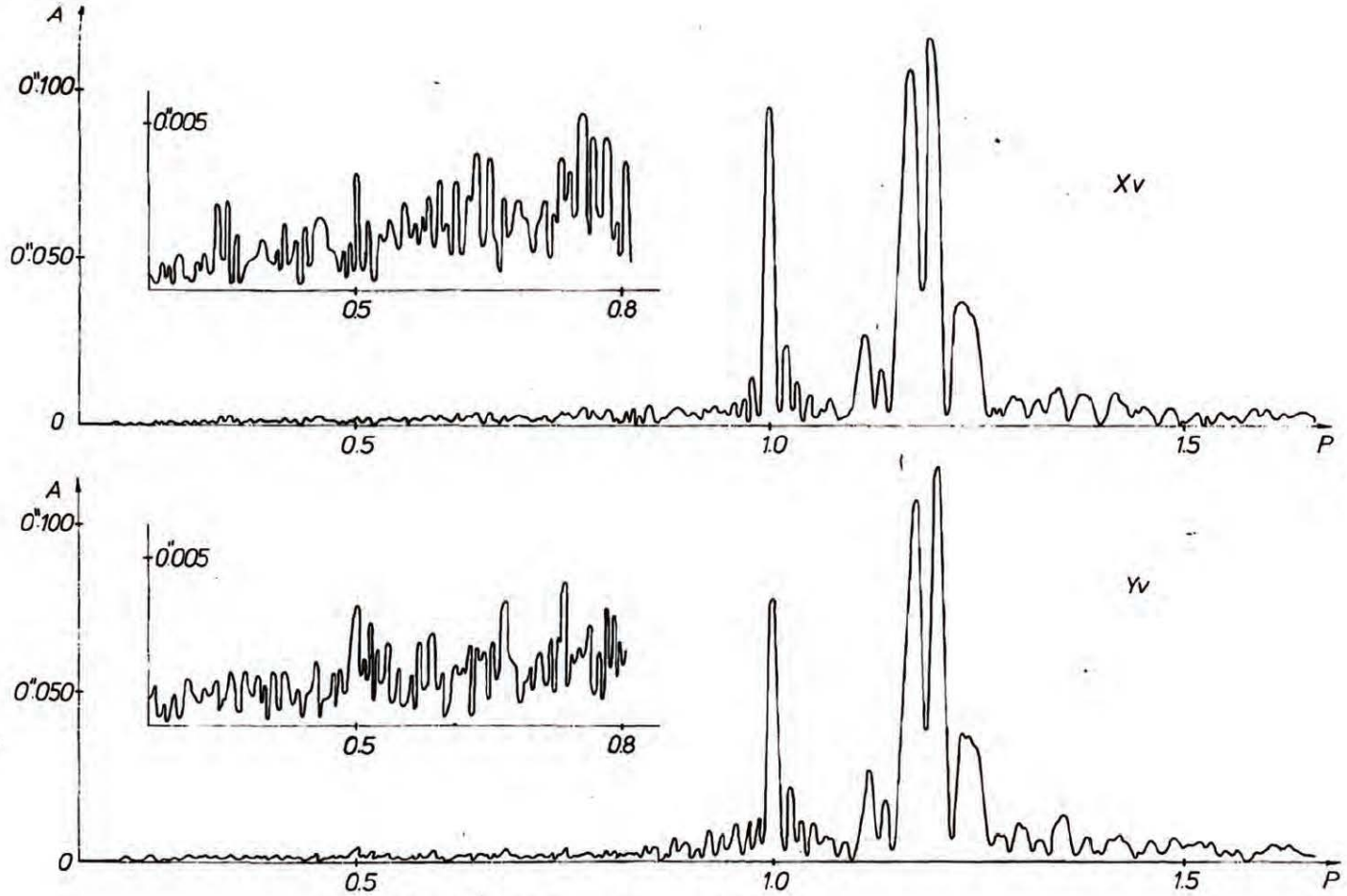
SIRINE PIKOVA	P_a =1.000	P_c =1.171	P_c' =1.199
$\Delta\varepsilon$	0.025	0.035	0.036
$\Delta\varepsilon'$	0.025	0.040	0.031

VIDIMO DA SE TEORIJSKE I RACUNATE SIRINE PIKOVA ZA GODISNJI I OBA CENDLEROVA CLANA DOBRO SLAZU. OVO IDE U PRILOG CINJENICI DA JE CENDLEROV CLAN DVOSTRUK. AKO BI GA POSMATRALI KAO JEDINSTVEN SA SIRINOM $\Delta\varepsilon'$=0.071 BIO BI DVA PUTA SIRI OD TEORIJSKIH VREDNOSTI $\Delta\varepsilon$ ZA BILO KOJU OD DVEJU NJEGOVIH PERIODA (TABLIC.4.7).

DA BI AMPLITUDE PRIMARNIH PIKOVA BILE TACNO ODREDJENE RA-



Sl. 4.2. Spektar sirovih podataka.



Sl. 4.3. Spektar izravnatih podataka.

ZLIKA PERIODA P SUSEDNIH CLANOVA MORA BITI VECA ILI JEDNAKA $\Delta\varepsilon$, STO JE ZADOVOLJENO SAMO KOD GODISNJEG CLANA. POSTO SU DVA CENDLEROVA PIKA UDALJENI ZA 0.028 GODINA, A TO JE MANJE OD $\Delta\varepsilon$, TACNE VREDNNOSTI NJIHOVIH AMPLITUDA (UKOLIKO OBA POSTOJE) NE MOGU SE ODREDITI DATOM METODOM IZ DATOG POSMATRACKOG MATERIJALA.

KAKO SELEKTIVNA FUNKCIJA FURIJEOVIH TRANSFORMACIJA $R(\lambda)=\sin\lambda/\lambda$ IMA EKSTREMUM (PRIMARNI PIK) ZA $\lambda=0$ I NIZ SEKUNDARNIH PIKOVA ZA $\lambda=4.49,\ 7.72,\ 10.91$, CIJE SU AMPLITUDE 22%, 13%, I 9% OD AMPLITUDE PRIMARNOG PIKA, REDOM, TO SMO RACUNALI TEORIJSKE PERIODE SEKUNDARNIH PIKOVA P3,P2,P1 S LEVA OD PRIMARNOG PIKA I P1', P2',P3' S DESNA OD PRIMARNOG PIKA,KAO I NJIMA ODGOVARAJUCE AMPLITUDE A'. DOBIJENI REZULTATI DATI SU U TABLICI 4.8 ZA GODISNJI PIK I TABLICAMA 4.9 I 4.10 ZA CENDLEROVE PIKOVE. U DRUGIM VRSTAMA ISTIH TABLICA, DATE SU AMPLITUDE A SEKUNDARNIH PIKOVA RACUNATE METODOM FURIJEA , GRAFICKI PREDSTAVLJENE NA SLIKAMA 4.2 I 4.3. NA ISTIM SLIKAMA POZICIJE SEKUNDARNIH PIKOVA U DOBROJ SU SAGLASNOSTI SA TEORIJSKIM PERIODAMA SEKUNDARNIH PIKOVA.

TABLICA 4.8

PER.	P3=.958	P2=.970	P1=.982	P=1.000	P1'=1.018	P2'=1.032	P3'=1.046
A	0°.007	0°.009	0°.016	0°.077	0°.024	0°.012	0°.009
A'	0 .007	0 .010	0 .017	0 .077	0 .017	0 .010	0 .007

AKO UPOREDIMO TEORIJSKE A' I RACUNATE A, AMPLITUDE SEKUNDARNIH PIKOVA ZA GODISNJI CLAN DATE U GORNJOJ TABLICI VIDIMO DA SU U ODLICNOJ SAGLASNOSTI.

TABLICA 4.9

PER	P3=1.114	P2=1.130	P1=1.147	P_c=1.171	P1′=1.196	P2′=1.215	P3′=1.235
A	0".027	0".009	0".009	0".107	0".114	0".003	0".035
A′	0 .010	0 .014	0 .023	0 .107	0 .023	0 .014	0 .010

U TABLICAMA 4.9 I 4.10 DATE SU PERIODE PODPIKOVA ZA CENDLEROVE PIKOVE U GODINAMA I ODGOVARAJUCE RACUNATE A I TEORIJSKE A′ AMPLITUDE U LUCNIM SEKUNDAMA.TABLICA 4.9 JE ZA PRVI CENDLEROV PIK A TABLICA 4.10 ZA DRUGI.

TABLICA 4.10

PER	P3=1.139	P2=1.156	P1=1.174	P_c'=1.199	P1′=1.226	P2′=1.246	P3′=1.266
A	0".014	0".036	0"101	0"117	0"037	0".029	0".005
A′	0 .011	0 .015	0.025	0.117	0.025	0 .015	0 .011

IZ TABLICE 4.9 SE VIDI DA PRVI CENDLEROV PIK P_c IMA SVOJ PRVI DESNI SEKUNDARNI PIK P1′ U DRUGOM CENDLEROVOM PIKU P_c' I OBRNUTO DRUGI CENDLEROV PIK IMA SVOJ PRVI LEVI SEKUNDARNI PIK P1 U PRVOM CENDLEROVOM PIKU P_c (TABLICA 4.10). STOGA POSTOJI VELIKI MEDJUSOBNI UTICAJ JEDNOG NA DRUGI.

IZ ISTIH PODATAKA ILS DICKMAN (1981) JE, TAKODJE , DOBIO DVOSTRUKI CENDLEROV CLAN.

MELCHIOR (1957) JE POKUSAO DA OBJASNI PROMENLJIVOST CENDLEROVE PERIODE I AMPLITUDE INTERFERENCIJOM DVE SLLOBODNE NUTACIJE, OD KOJI JEDNA PRIPADA TECNOM ZEMLJINOM JEZGRU, A DRUGA ELIPTICNOJ ZEMLJINOJ KORI.

SHOUXIAN, YINGMIN & SHUHE (1982) DOBILI SU CETVOROSTRUKI CENDLEROV CLAN SA PERIODAMA 1.132, 1,171, 1,198 1,228 GODINA. STO SE TICE NJIHOVE PRVE I CETVRTE PERIODE (1.132 I 1.228 GODINA) U POREDJENJU SA NASIM REZULTATIMA (TABLICE 4.9 I 4.10) VIDIMO DA OD GOVARAJU SEKUNDARNIM PIKOVIMA NASIH IZDVOJENIH CENDLEROVIH PIKOVA.

OKUBO (1982), S DRUGE STRANE, IZ ISTIH PODATAKA METODOM MAKSIMALNE ENTROPIJE DOBIJA CENDLEROV CLAN SA JEDINSTVENOM PERI - ODOM.

VEC SMO SPOMENULI DA POSTOJI NEKI GEOFIZICKI MEHANIZAM KO JI POBUDJUJE (EKSITIRA) CENDLEROVO POLARNO KRETANJE. TAJ MEHANIZAM JOS NIJE OTKRIVEN (LAMBECK, 1980). STO SE TICE DVOJNOSTI CENDLE- ROVOG PIKA MISLJENJA SU KONTRADIKTORNA.

IZ PREZENTIRANIH REZULTATA NASIH ISTRAZIVANJA SLEDI ZAK - LJUCAK DA POSTOJE DVE SLOBODNE NUTACIJE ZEMLJINE OSE. OVAJ ZAKLJU CAK BICE POTKREPLJEN NOVIM (JACIM) ARGUMENTIMA U PARAGRAFU 4.3.

4.2.1 VARIJACIJE GODISNJEG I POLUGODISNJEG CLANA

ZA ANALIZU SMO KORISTILI INTERVALE SAMERLJIVOSTI CENDLER- OVE I GODISNJE PERIODE. U TU SVRHU PODELILI SMO RAZMATRANI PERI - OD 1899.8-1979.0 NA SESTOGODISNJE PODINTERVALE SA PREKLAPANJEM UZ ASTOPNIH PODINTERVALA OD TRI GODINE. POLAZECI OD TOGA DA SU AMPLI TUDE I FAZE GODISNJEG I POLUGODISNJEG CLANA STABILNIJE NEGO U SLU CAJU CENDLEROVOG CLANA, STO CEMO KASNIJE REZULTATIMA ARGUMENTOVA- TI, MNK METODOM ZA SVAKI SESTOGODISNJI PODINTERVAL, RACUNALI SMO AMPLITUDU A_a I FAZU F_a GODISNJEG CLANA I AMPLITUDU A_b I FAZU F_b POLUGODISNJEG CLANA U OBE KOORDINATE. NAMERA NAM JE BILA DA POSMA TRAMO CENDLEROVE PROMENE KOORDINATA POLA U FUNKCIJI VREMENA. DOBI JENI REZULTATI DATI SU U TABLICI 4.11.

TABLICA 4.11

U KOLONAMA 2-3 SU AMPLITUDE GODISNJEG A U KOLONAMA 6-7 AMPLITUDE POLUGODISNJEG CLANA U LUCNIM SEKUNDAMA; U KOLONAMA 4-5 I 8-9 SU FAZE TIH CLANOVA U STEPENIMA.

	X KOORDINATA				Y KOORDINATA			
PERIOD	A_a	F_a	A_b	F_b	A_a	F_a	A_b	F_b
1899.8-1905.8	.080	335.4	.008	282.8	.065	260.7	.001	92.1
1902.8-1908.8	.058	355.5	.005	133.1	.040	279.5	.004	186.2
1905.8-1911.8	.108	329.0	.008	286.0	.093	253.3	.005	263.2
1908.8-1914.8	.104	337.4	.003	224.2	.105	265.5	.005	221.2
1911.8-1917.8	.081	343.5	.005	128.1	.086	265.0	.010	158.7
1914.8-1920.8	.103	340.5	.003	288.4	.105	254.4	.005	132.5
1917.8-1923.8	.090	332.6	.005	191.7	.077	234.9	.005	98.9
1920.8-1926.8	.117	350.8	.005	76.3	.097	263.6	.002	80.7
1923.8-1929.8	.058	360.3	.006	120.3	.045	278.8	.018	121.4
1926.8-1932.8	.104	325.8	.008	308.4	.085	243.0	.005	161.5
1929.8-1935.8	.113	348.8	.006	338.3	.091	267.5	.006	349.4
1932.8-1938.8	.089	345.8	.005	208.0	.062	260.5	.006	115.8
1935.8-1941.8	.107	341.1	.007	300.4	.097	256.4	.003	288.7
1938.8-1944.8	.074	331.7	.006	280.6	.051	251.6	.008	112.7
1941.8-1947.8	.114	340.3	.005	264.5	.088	260.8	.007	184.4
1944.8-1950.8	.070	334.6	.001	289.1	.044	236.8	.011	178.1
1947.8-1953.8	.114	329.5	.009	321.5	.100	238.1	.008	247.1
1950.8-1956.8	.109	354.1	.006	320.0	.100	266.7	.009	328.5
1953.8-1959.8	.085	330.2	.010	301.5	.059	244.2	.006	89.6
1956.8-1962.8	.122	337.7	.010	345.8	.092	252.0	.005	95.0
1959.8-1965.8	.103	336.4	.002	302.1	.069	238.3	.010	120.7
1962.8-1968.8	.106	339.2	.004	306.7	.072	243.3	.005	220.6
1965.8-1971.8	.099	337.4	.006	326.1	.067	254.4	.006	163.6
1968.8-1974.8	.102	349.6	.007	280.0	.095	278.0	.010	123.5
1971.8-1977.8	.078	360.1	.002	306.9	.078	282.7	.016	107.9

NA OSNOVU PODATAKA TABLICE 4.11, UZIMAJUCI SVAKU DRUGU VREDNOST (UZAJAMNO NEZAVISNE) RACUNALI SMO SREDNJU VREDNOST:

$$\bar{A} = \frac{1}{N} \sum_{i=1}^{N-1} A_i \quad , \quad N = 13 \qquad (4.9)$$

GRESKU JEDINICNE TEZINE:

$$\sigma_A = \sqrt{\frac{\sum_{i=1}^{N} (A_i - \bar{A})}{N-1}} \qquad (4.10)$$

I GRESKU SREDNJE VREDNOSTI:

$$\sigma_{\bar{A}} = \frac{1}{\sqrt{N-1}} \sigma_A \qquad (4.11)$$

AMPLITUDA A_a I A_h I FAZA F_a I F_h GODISNJEG I POLUGODISNJEG CLANA. DOBIJENI REZULTATI DATI SU U TABLICI 4.12. FAZE SE ODNOSE NA JULIJANSKI DATUM DJ=2414 948

TABLICA 4.12

KOORD	GODISNJI CLAN		POLUGODISNJI CLAN	
	$\bar{A}_a \pm \sigma_{\bar{A}_a}$	$\bar{F}_a \pm \sigma_{\bar{F}_a}$	$\bar{A}_h \pm \sigma_{\bar{A}_h}$	$\bar{F}_h \pm \sigma_{\bar{F}_h}$
X	$0''.0948 \pm 0''.0050$	$340^\circ \pm 3^\circ$	$0''.0060 \pm 0''.0007$	$266^\circ \pm 21^\circ$
Y	0.0778 ± 0.0046	256 ± 4	0.0077 ± 0.0014	176 ± 25

AKO REZULTATE TABLICE 4.12 UPOREDIMO SA REZULTATIMA DOBIJENIM FURIJEOVIM INTEGRALNIM TRANSFORMACIJAMA (TABLICA 4.5) VIDIMO DA SU AMPLITUDE GODISNJEG CLANA U ODLICNOJ SAGLASNOSTI, DOK SE KOD MALIH IZNOSA AMPLITUDA POLUGODISNJEG CLANA POJAVLJUJU RAZLIKE. FAZE U POMENUTIM TABLICAMA SE ODNOSE NA RAZLICITE TRENUTKE, SVEDENE NA ISTI SAGLASNE SU.

ISPITALI SMO PROMENLJIVOST AMPLITUDE I FAZE GODISNJEG CLANA KORISTECI ABBE-OV KRITERIJUM (DJUROVIC, 1979).

NEKA SU G I GO STATISTIKE DEFINISANE RELACIJAMA:

$$G = \frac{\frac{1}{2N-1}\sum_{i=1}^{N-1}(A_{i+1}-A_i)^2}{\frac{1}{N-1}\sum(A_i-\bar{A})^2}$$

$$GO = 1 + \frac{u_q}{\sqrt{N+0.5(1+u_q^2)}}$$

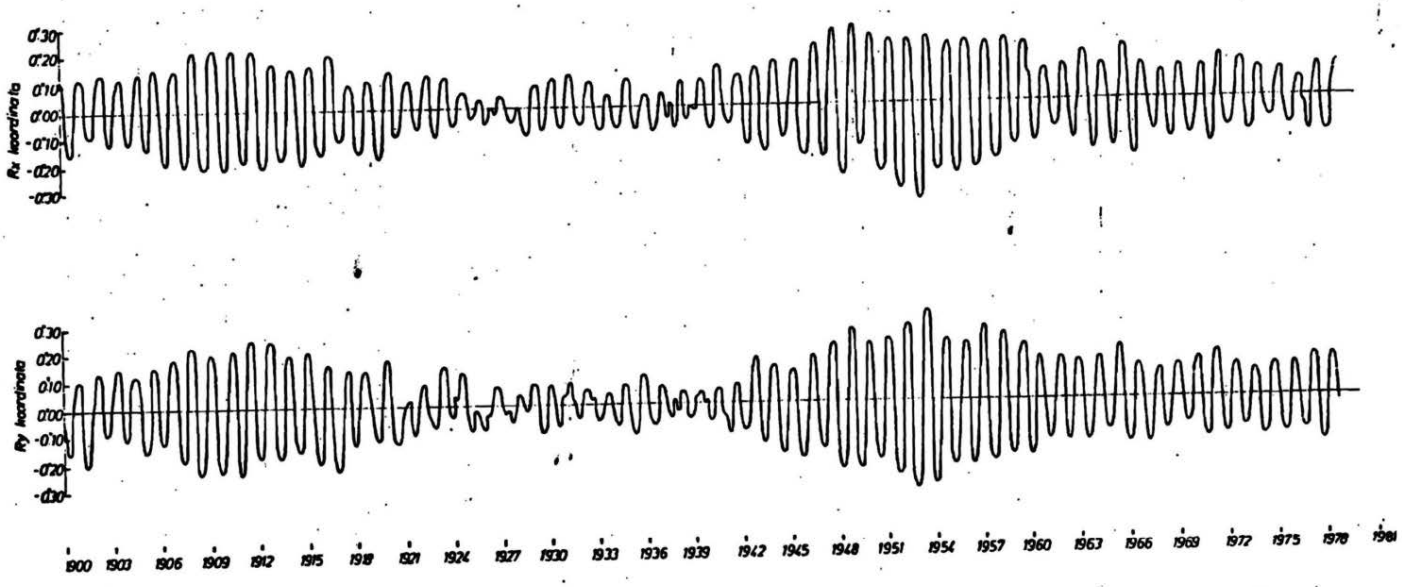
GDE JE u_q = -1.645 KVANTIL REDA 0.05 (u_q JE KVANTIL REDA q AKO JE VEROVATNOCA DA SLUCAJNA PROMENLJIVA X> u_q JEDNAKA q) GAUSOVSKE RASPODELE VEROVATNOCE. STATISTIKA GO RACUNA SE PO NAVEDENOJ JEDNACINI AKO JE N > 20, DOK ZA N<=20, U NASEM SLUCAJU N =13, I ZA q =0.05 UZIMA VREDNOST GO(13)=0.578, DJUROVIC (1979).

REZULTATI PRIMENE NAVEDENE RELACIJE ZA RACUN STATISTIKE G ZA AMPLITUDU I FAZU GODISNJEG CLANA U X I Y KOORDINATI DATI SU U TABLICI 4.13.

TABLICA 4.13

RACUNATA	X KOORDINATA		Y KOORDINATA	
STATIST.	A_a	F_a	A_a	F_a
G	0.973	0.722	0.960	0.872

UPOREDJUJUCI VREDNOSTI G TABLICE 4.13 SA VREDNOSCU GO = =0.578 NALAZIMO DA JE U SVIM SLUCAJEVIMA G> GO. PO ABBE-OVOM KRI-



Sl. 4.4. Rezidui dobijeni oduzimanjem godišnjeg i polugodišnjeg člana od sirovih koordinata pola.

TERIJUMU AKO JE G > GO ODSTUPANJA A - A MOGU SE OBJASNITI SA STANOVISTA SLUCAJNIH GRESAKA.

NAS ZAKLJUCAK JE DA SU FLUKTUACIJE AMPLITUDE I FAZE GODISNJEG CLANA SLUCAJNOG KARAKTERA.

POREMECAJI POLUGODISNJEG CLANA POSTOJE. IMA PERIODA KADA ON SKORO ISCEZAVA, ALI I PERIODA KADA JE AMPLITUDA $A_{\cdot}(X)=0".010$ I $A_{\cdot}(Y)=0".018$ (TABLICA 4.11).

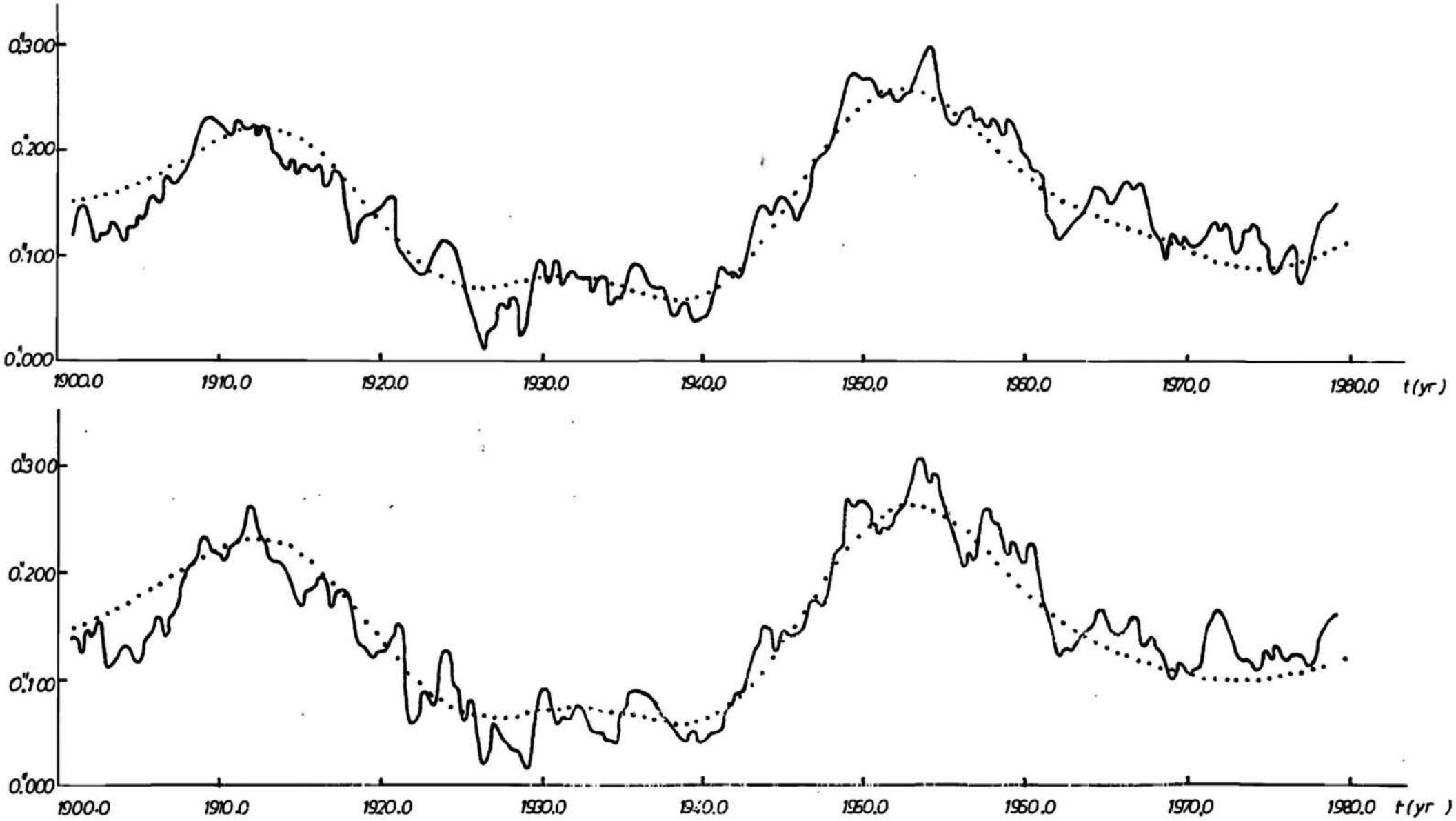
NA OSNOVU PRAVILA "TRI SIGMA" UTVRDILI SMO DA NE POSTOJE GRUBE GRESKE U REZULTATIMA ZA AMPLITUDU $A_{\cdot}$ I FAZU $F_{\cdot}$ POLUGODISNJEG CLANA (TABLICA 4.11). PRIMENOM ABBE-OVOG KRITERIJUMA , KAO KOD GODISNJEG CLANA, NALAZIMO DA SU ODSTUPANJA AMPLITUDE ($A_{\cdot}$ -A) I FAZE ($F_{\cdot}$ -$\bar{F}_{\cdot}$) POLUGODISNJEG CLANA SLUCAJNOG KARAKTERA.

OD SIROVIH PODATAKA KOORDINATA POLA X,Y ODUZELI SMO GODISNJI I POLUGODISNJI CLAN I DOBIJENE REZIDUE $R_{\cdot}$ I $R_{\cdot}$ PREDSTAVILI NA SLICI 4.4.

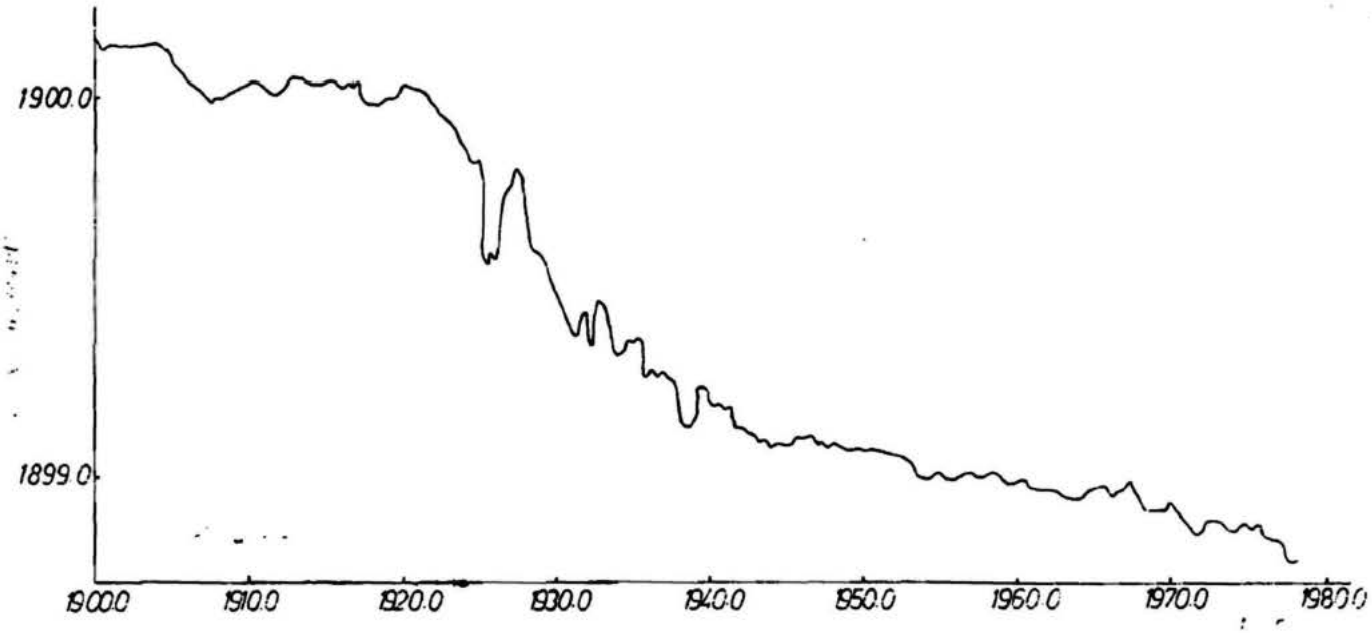
4.2.2 VARIJACIJE CENDLEROVOG CLANA

POSLE ELIMINACIJE GODISNJEG I POLUGODISNJEG CLANA NA DOBIJENE REZIDUE $R_{\cdot}$ I $R_{\cdot}$ (SL. 4.4) PRIMENOM METODE NAJMANJIH KVADRATA RACUNALI SMO AMPLITUDU I FAZU CENDLEROVOG CLANA (SA USVOJENIM PERIODOM OD 1.20 GODINA). RACUN JE VRSEN NA INTERVALIMA OD PO 15 MESECI SA POCECIMA NA RAZMAKU OD GODINU DANA (POCETNI TRENUTAK JE 1899.8, I PREKLAPANJE UZASTOPNIH INTERVALA PO TRI MESECA). DOBIJENE SERIJE VREDNOSTI AMPLITUDE PREDSTAVLJENE SU NA SLICI 4.5 A VREDNOSTI FAZE NA SLICI 4.6. FAZE SE ODNOSE NA POCETNI TRENUTAK JULIJANSKOG DATUMA DJ=2414 948.

REZULTATI PREDSTAVLJENI NA SLLIKAMA 4.5 I 4.6 SU U ODLICNOJ SAGLASNOSTI SA REZULTATIMA GUINOT-A (1972, 1982) I DICKMAN-A



Sl. 4.5. Amplituda Čendlerove nutacije $\Lambda=\Lambda(t)$.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1910.0
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1940.0
1950.0
1960.0
1970.0
1980.0

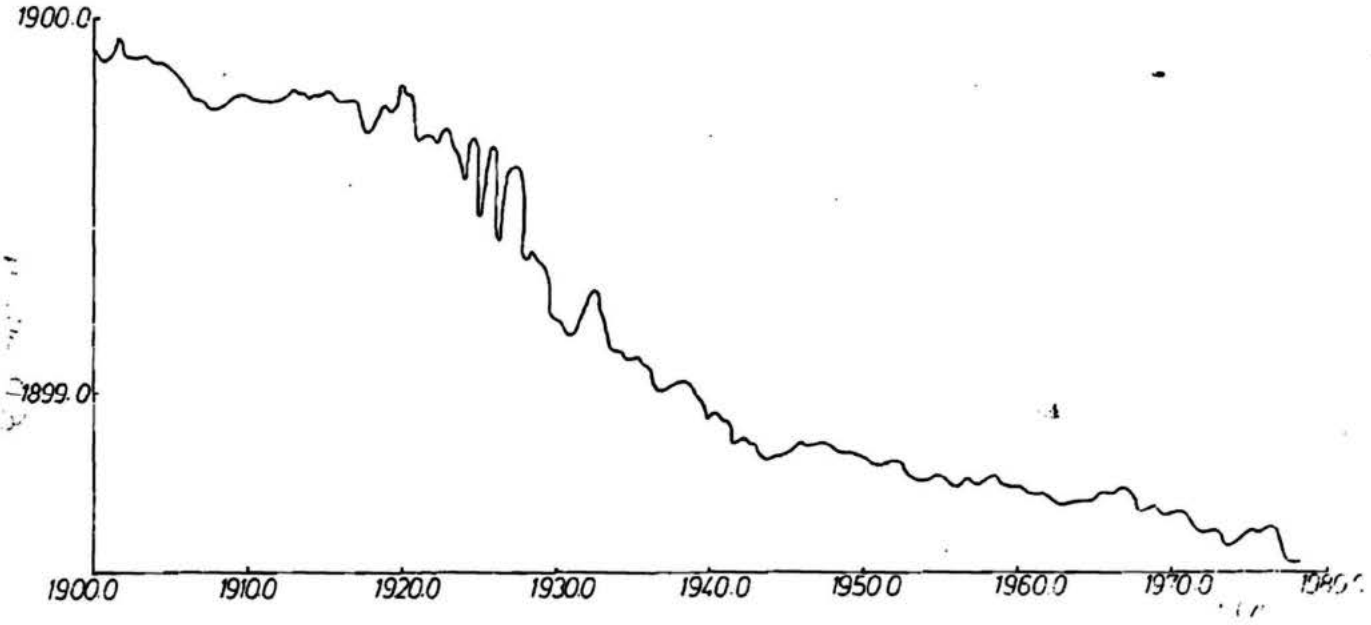

1900.0
1899.0
1900.0
1910.0
1920.0
1930.0
1940.0
1950.0
1960.0
1970.0

Fig 46

(1981). NA SLICI 4.5 SE ZAPAZA PERIODICNA PROMENA CENDLEROVE AMPLITUDE DOK, UPRKOS TOME, FAZA CENDLEROVE NUTACIJE (SL. 4.6) AKO SE IZUZME PERIOD (1925-1940) IMA DOBRU STABILNOST. PROMENA FAZE ZA 180 STEPENI U PERIODU 1925-1940 JE JEDINSTVEN I NEOBJASNJEN SLUCAJ U DOSADA RASPOLOZIVIM SERIJAMA KOORDINATA POLA.

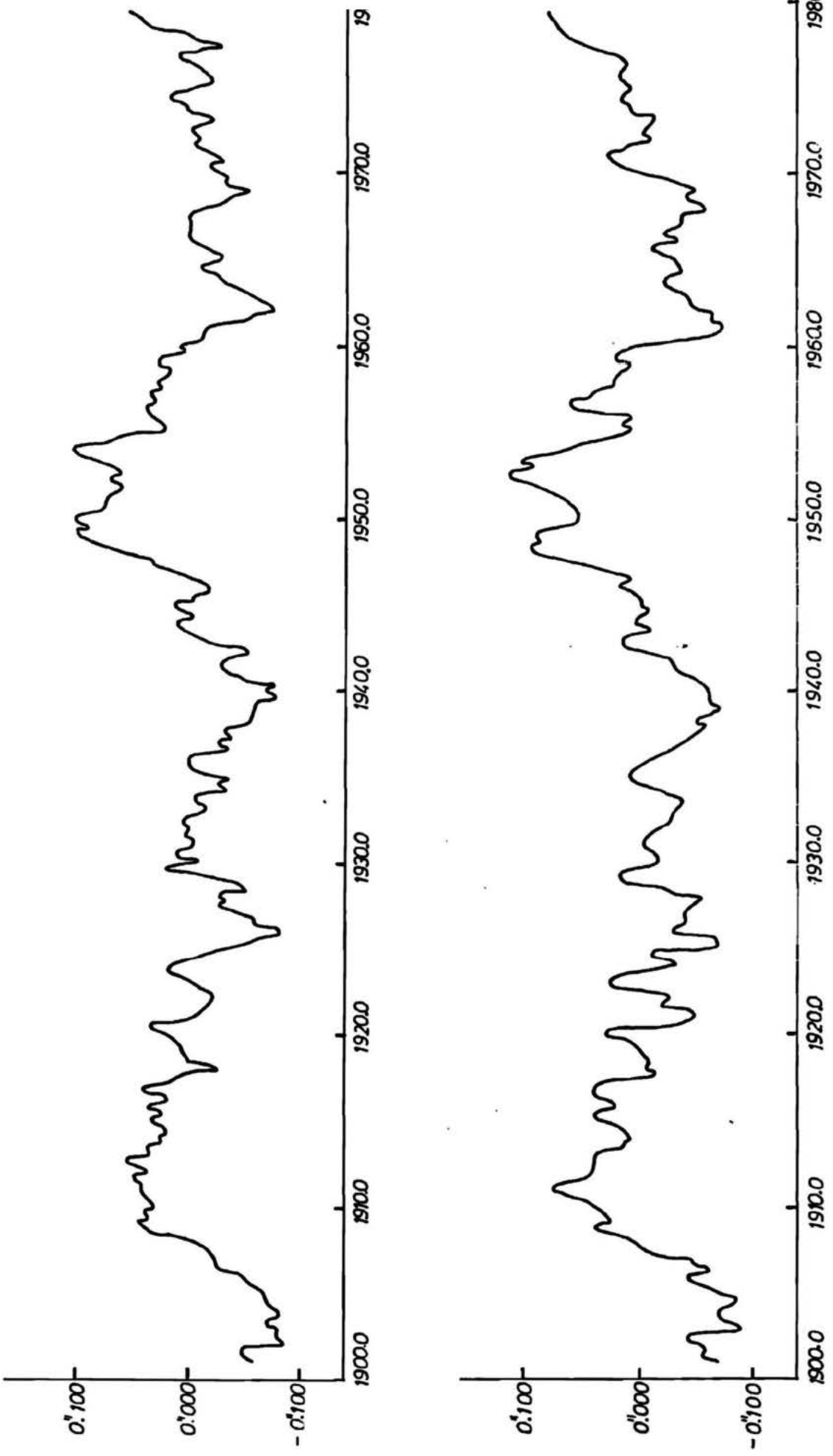
SHOUXIAN, YINGMIN & SHUHE (1982) DOLAZE DO ZAKLJUCKA DA JE FAZA CENDLEROVE NUTACIJE PERIODICNO PROMENLJIVA, STO NIJE U SAGLASNOSTI SA REZULTATIMA NASE ANALIZE. UZROK OVE NESAGLASNOSTI LEZI U RAZLICITOJ INTERPRETACIJI ANALIZIRANIH PODATAKA.

KAKO SE NA SLICI 4.5 ZAPAZA PERIODICNA PROMENA CENDLEROVE AMPLITUDE SA PERIODOM OD OKO 50 GODINA, TO SMO U OKOLINI OVE PERIODE METODOM NAJMANJIH KVADRATA RACUNALI AMPLITUDU I FAZU ZA SVAKU PROBNU PERIODU MENJAJUCI JE OD P0=50 DO P1=55 GODINA SA KORAKOM OD 0.5 GODINA. ODNOSNO, RESAVALI SISTEME JEDNACINA OBLIKA:

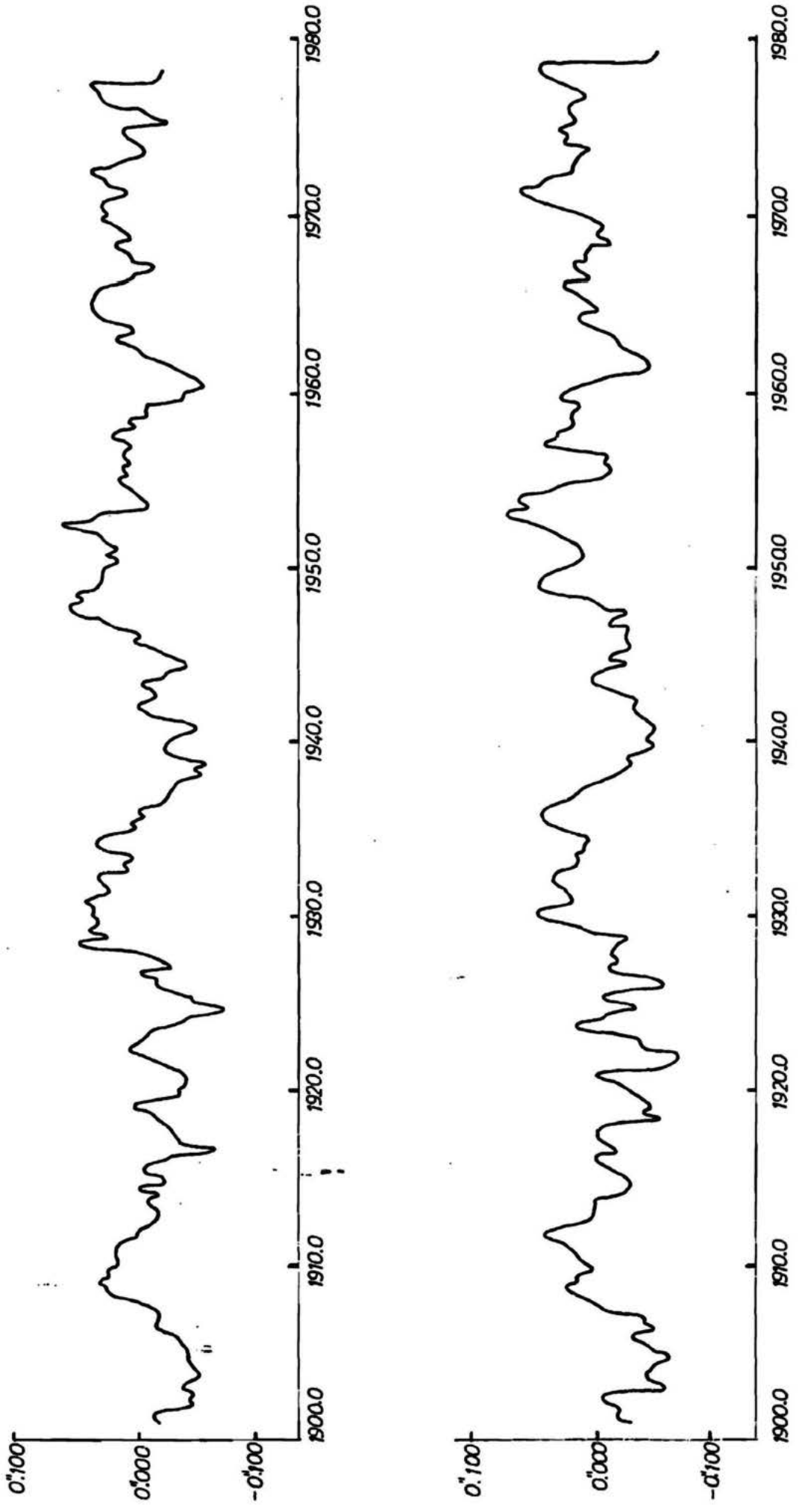
$$X = C0 + C1\cos\omega t + C2\sin\omega t \qquad (4.12)$$
$$Y = C0' + C1'\cos\omega' t + C2'\sin\omega' t$$

GDE JE $\omega = 2\pi/P$, $\omega' = 2\pi/P'$ A P I P' SU PERIODE HARMONIJSKOG CLANA U X I Y KOORDINATI, REDOM. ZA DEFINITIVNE A, F I P USVOJILI SMO ONE KOJI ODGOVARAJU MINIMUMU DISPERZIJE REZIDUA. TAKO SMO IZDVOJILI KOMPONENTU S PERIODOM P=P'=53.0 GODINA I AMPLITUDOM A = $= \sqrt{C1^2 + C2^2} = 0''.0598$ U X I $A' = \sqrt{C1'^2 + C2'^2} = 0''.0632$ U Y KOORDINATI. ZA SLOBODNI CLAN SMO DOBILI VREDNOSTI C0=0".1392 I C0'=0".1396.

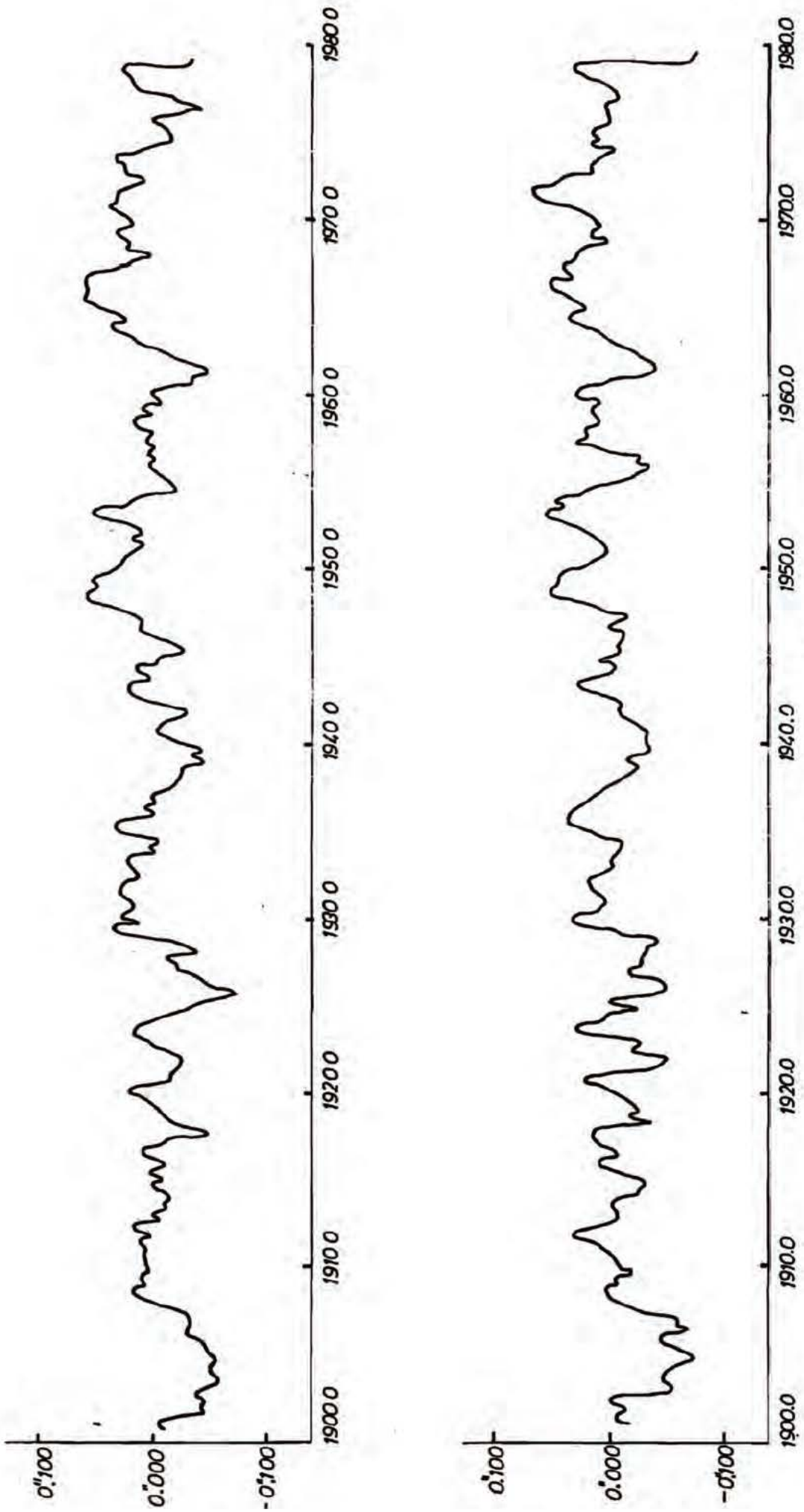
ODUZIMANJEM OVE KOMPONENTE OD VREDNOSTI AMPLITUDA A (SL. 4.5) DOBILI SMO REZIDUE A'_R, PREDSTAVLJENE NA SLICI 4.7. SA SLIKE SE VIDI DA SU REZIDUI A'_R PERIODICNI SA PERIODOM OD OKO 35 GODINA. PONOVIVSI PRETHODNI RACUN, SADA S PROBNIM PERIODAMA U OKOLINI 35 GODINA I ISTIM KORAKOM OD 0.5 GODINA, METODOM MNK IZDVOJILI SMO I



Sl. 4.7. Rezidui I reda A'_R .



Sl. 4.8. Rezidui II reda A''_R .



Sl. 4.9. Rezidui III reda A_R.

DRUGU PERIODICNU KOMPONENTU S PERIODOM OD 34.5 GODINA I AMPLITUDOM 0'.0417 U X I 0".0438 U Y KOORDINATI.

RACUN SMO SPROVELI JOS DVA PUTA IZDVOJIVSI IZ REZIDUA DRUGOG REDA A'_R (SL. 4.8) CLAN SA PERIODOM 23.0 GODINA I IZ REZIDUA TRECEG REDA A''_R (SL. 4.9) CLAN S PERIODOM 17.5 GODINA. DOBIJENI REZULTATI ZA SVA CETIRI IZDVOJENA PERIODICNA CLANA DATI SU U TABLICI 4.14.

TABLICA 4.14

PERIODA	AMPLITUDA	
U GOD.	X KOORDINATA	Y KOORDINATA
53.0	0".0598	0".0632
34.5	0 .0417	0 .0438
23.0	0 .0176	0 .0224
17.5	0 .0205	0 .0167

REZULTATI SU DATI SA GRESKOM - 0".0005

DUGOPERIODICNE VARIJACIJE CENDLEROVE AMPLITUDE IZDVOJILI SMO I NA DRUGI NACIN. NA CELOKUPNOM INTERVALU (A NE U OKOLINAMA POJEDINIH CLANOVA ODREDJENIH SA GRAFIKA, KAO U PRETHODNOM SLUCAJU) OD 15-80 GODINA SA KORAKOM OD 0.5 GODINA VRSENA JE PROMENA PROBNE PERIODE SA KOJOM SE MENJA CENDLEROVA AMPLITUDA. ZA RAZLICITE PROBNE PERIODE PO METODI MNK DOBIJENE SU PERIODICNE PROMENE CENDLEROVE AMPLITUDE,PRIKAZANE NA SLICI 4.10. NA OVOJ SLICI SE IZDVAJAJU DVA DOMINANTNA PIKA S PERIODAMA 53.0 I 34.5 GODINA.VREDNOSTI NJIHOVIH AMPLITUDA KAO I PERIODE I AMPLITUDE DRUGA DVA SLABIJA PIKA DATE SU U TABLICI 4.15.

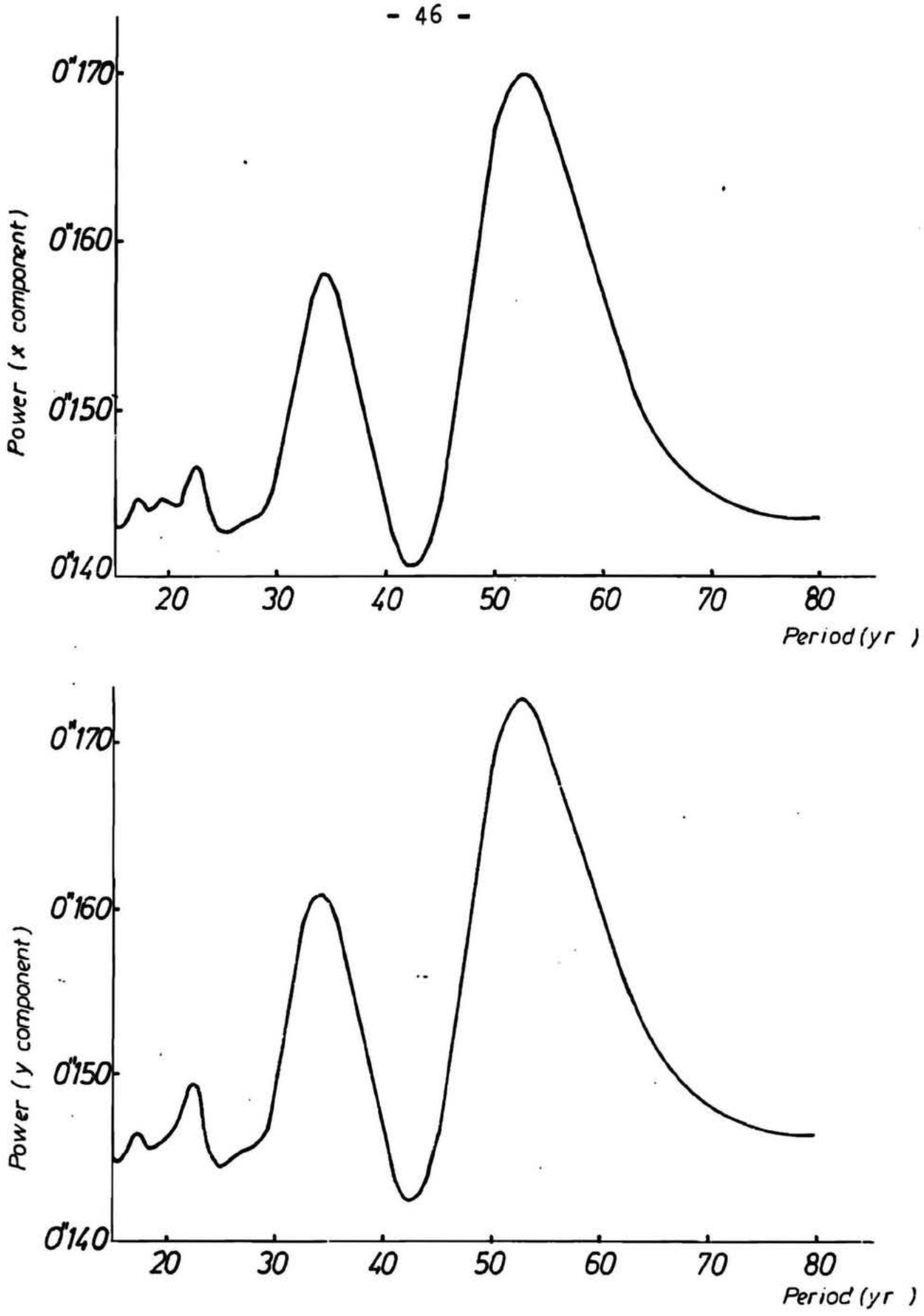


Fig. 4.10 Spektrum of Chandler's amplitude

TABLICA 4.15

PERIODE U GOD.	AMPLITUDE X KOORDINATA	AMPLITUDE Y KOORDINATA
53.0	0".0598	0".0632
34.5	0 .0417	0 .0438
23.0	0 .0188	0 .0234
17.5	0 .0206	0 .0168

REZULTATI SU DATI SA GRESKOM ± 0".005

AKO VREDNOSTI TABLICE 4.15 UPOREDIMO SA VREDNOSTIMA DOBI - JENIM POJEDINACNOM ANALIZOM REZIDUA PRVOG , DRUGOG I TRECEG REDA (TABLICA 4.14) VIDIMO DA SU U ODLICNOJ SAGLASNOSTI. ODNOSNO , DA SU OBA NACINA KORISCENJA METODE MNK ZA IZDVAJANJE DUGOPERIODICNIH CLANOVA PODJEDNAKO DOBRA, S TIM STO DRUGI NACIN IZISKUJE VISE KOM PJUTERSKOG VREMENA.

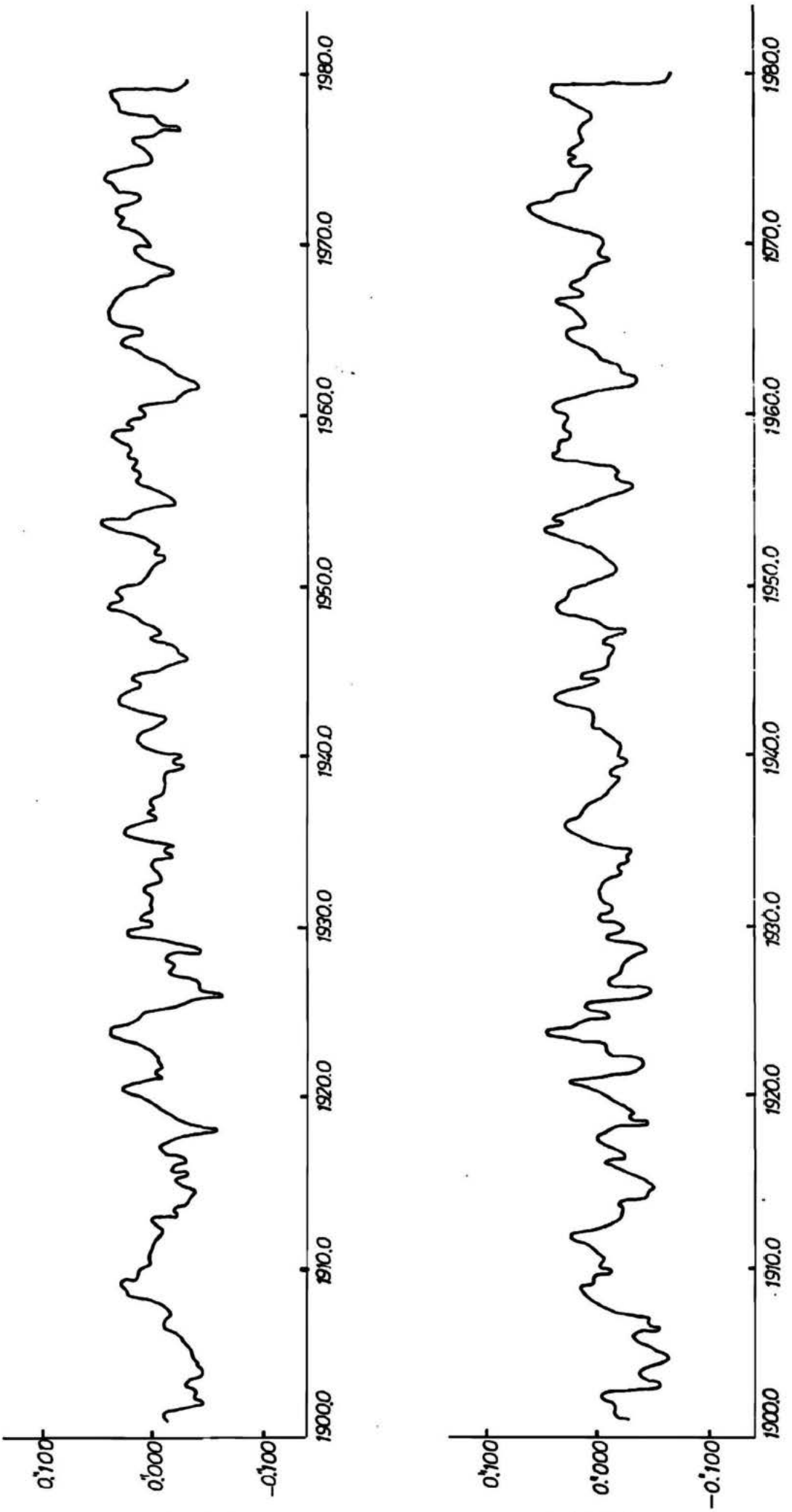
VARIJACIJU CENDLEROVE AMPLITUDE NA OSNOVU TABLICE 4.15 PRE DSTAVILI SMO ZBIROM CETRI PERIODICNE FUNKCIJE:

$$X=C0+C1\cos\omega t+C2\sin\omega t+C3\cos\omega t+C4\sin\omega t+C5\cos\omega t+ +C6\sin\omega t+C7\cos\omega t+C8\sin\omega t \qquad (4.13)$$

$$Y=D0+D1\cos\omega t+D2\sin\omega t+D3\cos\omega t+D4\sin\omega t+D5\cos\omega t+ +D6\sin\omega t+D7\cos\omega t+D8\sin\omega t$$

PRIKAZANE TACKASTOM LINIJOM NA SLICI 4.5. UPOREDJUJUCI KRIVE NA SLICI 4.5 SMATRAMO DA JE APROKSIMACIJA JEDNACINAMA 4.13 DOBRA.

AKO NA SLICI 4.5 OD NEPREKIDNE KRIVE ODUZMEMO TACKASTU KRIVU DOBICEMO REZIDUE IV REDA A_R^{IV} PREDSTAVLJENE NA SLICI 4.11 .



Sl. 4.11. Reziduii IV reda A_R.

ISTE REZIDUE A^{IV}_R DOBIJAMO AKO OD REZIDUA A^{III}_R (SLIKA 4.9) ODUZMEMO PERIODICNI CLAN OD 17.5 GODINA. KAKO SE I KOD REZIDUA A^{IV}_R (SLIKA 4.11) ZAPAZA PRISUSTVO MANJIH PERIODICNIH CLANOVA, TO SMO, RADI DOBIJANJA POTPUNOG SPEKTRA PRIMENILI METODU FURIJEOVIH INTEGRALNIH TRANSFORMACIJA VARIRAJUCI PROBNU PERIODU OD DVA MESECA DO 15 GODINA SA KORAKOM OD 1 MESECA. DOBIJENI SPEKTAR PREDSTAVLJEN JE NA SLICI 4.12. POSTO SE NA OVOJ SLICI ZAPAZA PRISUSTVO SUMA , TO SMO ZA IZDVOJENE PERIODICNE CLANOVE (OD 11.3, 8.2, 7.1 I 6.0 GODINA), METODOM MNK RACUNALI NJIHOVE AMPLITUDE. DOBIJENI REZULTATI DATI U TABLICI 4.16.

TABLICA 4.16

PERIOD	AMPLITUDE	
U GOD.	X KOORDINATA	Y KOORDINATA
[illegible]	[illegible]1	0".008
[illegible]	0 .007	0 .004
7.1	0 .010	0 .009
6.0	0 .008	0 .010

REZULTATI SU DATI SA GRESKOM -0".001

U SISTEMIMA SVETSKOG VREMENA UT1 I UT2 OTKRIVENA JE PERIODICNA VARIJACIJA SA KARAKTERISTICNOM PERIODOM OD 60 GODINA (KALININ & KISELEV, 1980; KISELEV, 1982; EMETZ & KORSUN, 1978; STAJIC, 1983) I AMPLITUDOM OD NEKOLIKO SEKUNADA! NIJE ISKLJUCENO DA 53-GODISNJA VARIJACIJA X I Y IMA ISTI UZROK KAO I VARIJACIJA OD 60 GODINA. I VARIJACIJA OD 35 GODINA JE OTKRIVENA U SISTEMU SVETSKOG VREMENA UT1 (MORISON,1979; STAJIC,1983), STO TAKODJE DAJE OSNOVA ZA RAZMISLJANJE O NJIHOVOM ZAJEDNICKOM UZROCNOM POREKLU.

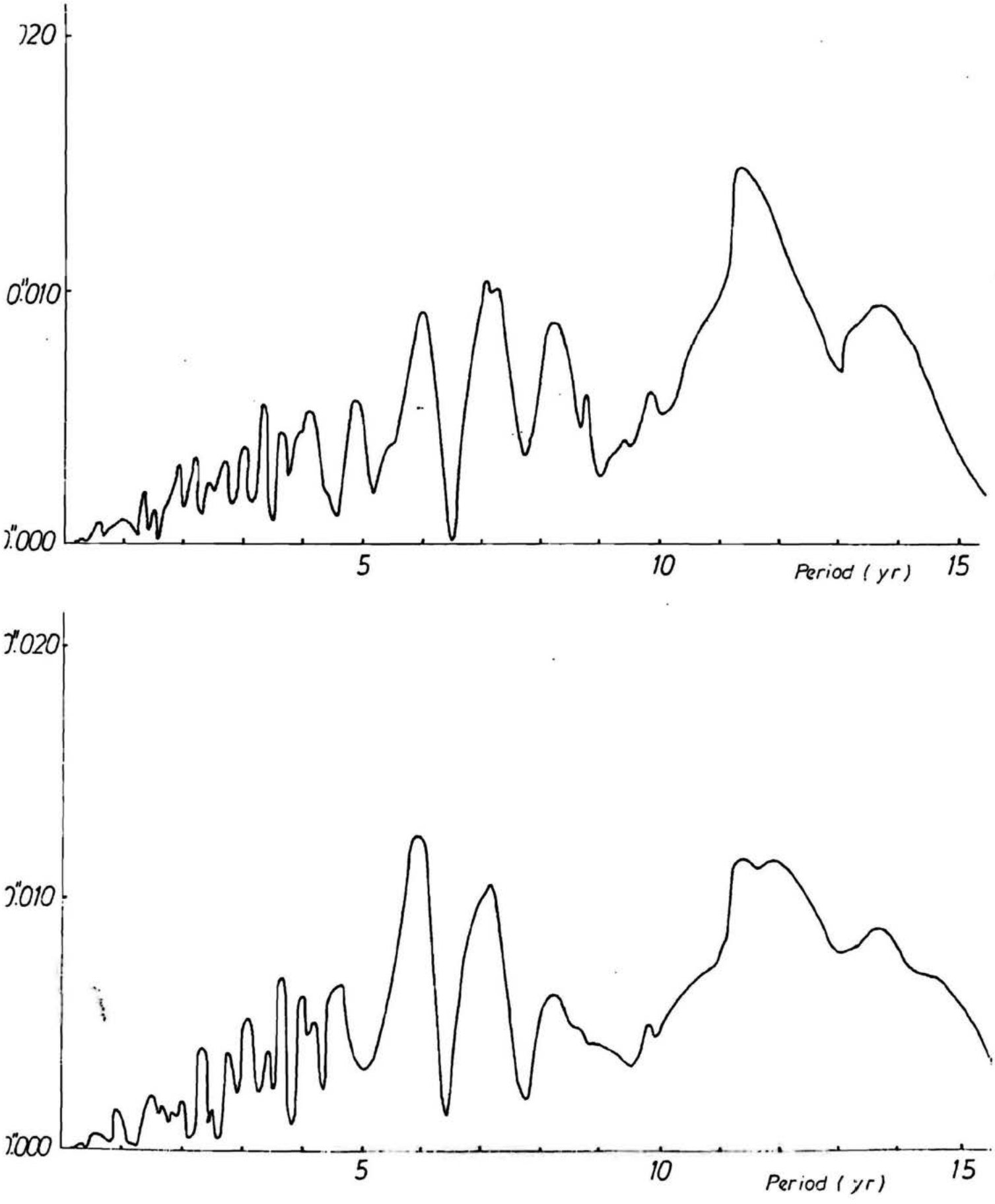


Fig. 4.12 Spectrum of Chandler's amplitude

SLABE VARIJACIJE UT1 OD 22 I 11 GODINA, KOJE PO VELICINI PERIODA ODGOVARAJU GLAVNOM I LAPLASOVOM CIKLUSU SUNCEVE AKTIVNOSTI SU IDENTIFIKOVANE U JEDNOM RADU DJUROVIC I STAJIC (1984).

CLAN SA PERIODOM OD 17.5 GODINA PRIBLIZNO ODGOVARA NUTACIONOM CIKLUSU, ALI NIJE ISKLJUCENO DA SE RADI O PSEUDO POJAVI .

I VARIJACIJE OD PRIBLIZNO 6 I 8 GODINA IZ TABLICE 4.16 SU UOCENE U PROMENAMA UGAONE BRZINE ZEMLJINE ROTACIJE (LAMBECK & CAZENAVE, 1983; DJUROVIC, 1979 ; STAJIC & DJUROVIC, 1984).

4.3 POREDJENJE GENERISANIH I POLAZNIH PODATAKA

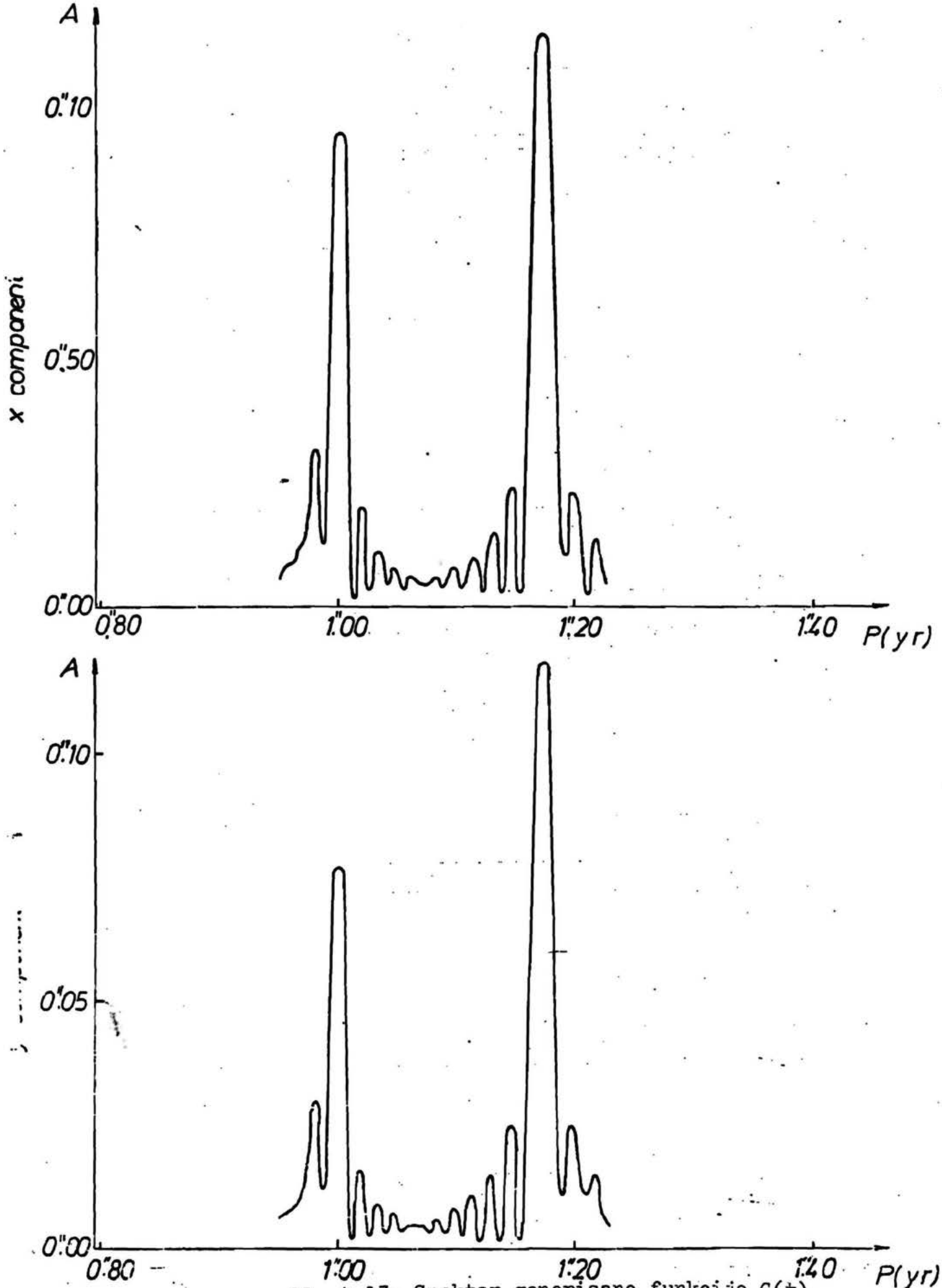
DA BISMO PROVERILI DALI JE DVOSTRUKOST CENDLEROVOG PIKA U SPEKTRIMA KOJE SMO RACUNALI (SLIKE 4.2 I 4.3) PSEUDO POJAVA ILI SE ONA MOZE OBJASNITI POSTOJANJEM DVE BLISKE PERIODICNE VARIJACIJE KOORDINATA POLA, RACUNALI SMO SLEDECE FUNKCIJE:

$$F(t) = A_a \cos(\omega_a t + F_a) + A_c \cos(\omega_c t + F_c) + A_c' \cos(\omega_c' t + F_c')$$
$$G(t) = A_a \cos(\omega_a t + F_a) + A_c' \cos(\omega_c' t + F_c') \qquad (4.14)$$

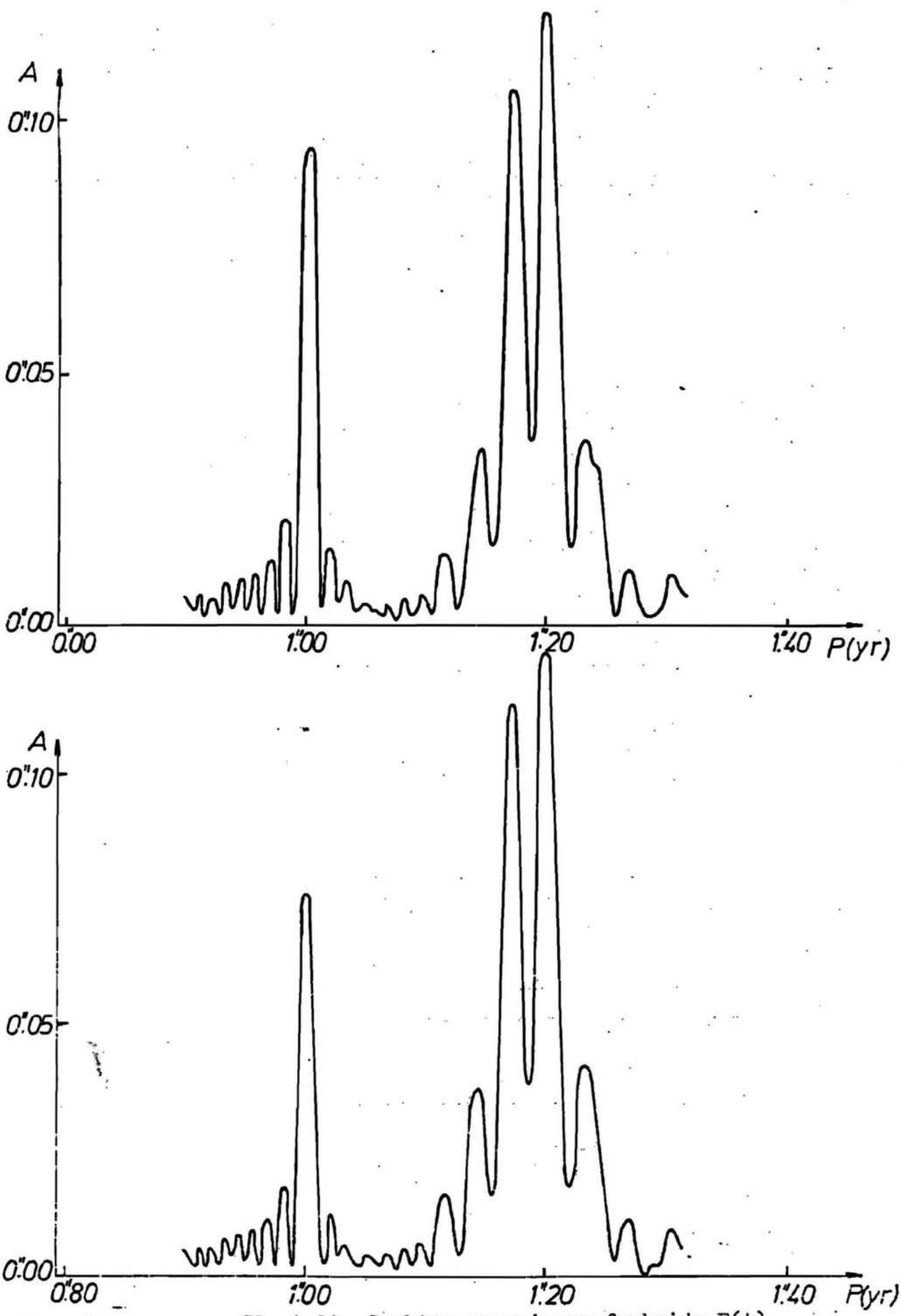
U KOJIMA SU $\omega_a = 2\pi / P_a$, $\omega_c = 2\pi / P_c$ I $\omega_c' = 2\pi / P_c'$ UUGAONE BRZINE JEDNOGODISNJE I DVE CENDLEROVE VARIJACIJE A P_a , P_c , P_c' , A_a, A_c, A_c' , F_a , F_c I F_c' PERIODE, AMPLITUDE I FAZE OVIH CLANOVA (VIDI TABLICE 4.5 I 4.6) DOBIJENE METODOM FURIJEOVIH INTEGRALNIH TRANSFORMACIJA (DFT).AMPLITUDE A_c I A_c' SU PROMENLJIVE (TAB.4.14)

DRUGIM RECIMA FUNKCIJE $G(t)$ I $F(t)$ PREDSTAVLJAJU VARIJACIJE POLA GENERISANE POD PRETPOSTAVKOM DA POSTOJI JEDNA,ODNOSNO DVE SLOBODNE NUTACIJE ZEMLJINE OSE.

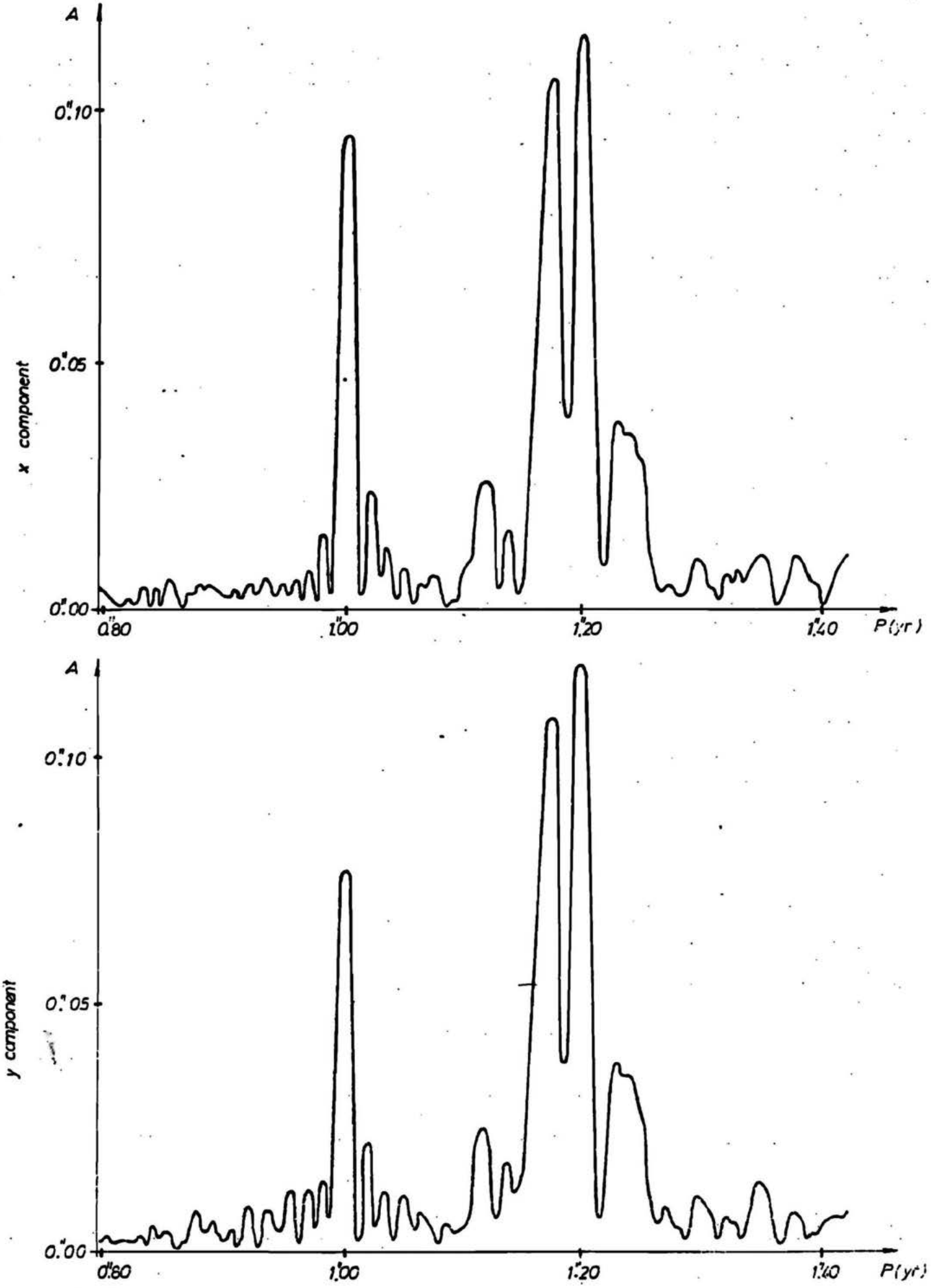
VREDNOSTI GENERISANE FUNKCIJE $F(t)$ PREDSTAVLJENE SU NA SLICI (PRILOG VII). UPOREDJUJUCI OVE VREDNOSTI SA POLAZNIM PODACIMA (SIROVE KOORDINATE POLA) ILUSTROVANIH SLIKOM (PRILOG III) VIDIMO DA SU U DOBROJ SAGLASNOSTI.



Sl. 4.13. Spektar generisane funkcije G(t).



Sl. 4.14. Spektar generisane funkcije F(t).



Sl. 4.15. Spektar polaznih podataka.

METODOM DFT DOBILI SMO SPEKTRE FUNKCIJA $G(\ell)$ I $F(\ell)$ NA SLIKAMA 4.13 I 4.14. KAKO JE FUNKCIJA G(.) GENERISANA POD PRETPOSTAVKOM DA POSTOJI JEDAN CENDLEROV PIK TO SMO I NJEN SPEKTAR DOBILI SAMO SA JEDNIM CENDLEROVIM PIKOM (SL. 4.13) . FUNKCIJA $F(\ell)$ JE GENERISANA POD PRETPOSTAVKOM DA POSTOJE DVE BLISKE SLOBODNE NUTACIJE ZEMLJINE OSE, PA JE I NA NJENOM SPEKTRU CENDLEROV PIK DVOSTRUK (SL. 4.14).

S TOGA MOZEMO ZAKLJUCITI DA METODA SPEKTRALNE ANALIZE DFT NIJE GENERISALA DVA CENDLEROVA PIKA UMESTO JEDNOG.

NA SL.4.15 PREDSTAVLJEN JE U ISTOJ SRAZMERI SPEKTAR POLAZNIH PODATAKA DOBIJEN, TAKODJE, METODOM DFT. IZ UPOREDJENJA SPEKTARA NA SLIKAMA 4.14 I 4.15 EVIDENTNA JE NJIHOVA SAGLASNOST.

OVO IDE U PRILOG CINJENICI DA SE NE RADI O JEDNOJ, VEC O DVE BLISKE SLOBODNE NUTACIJE ZEMLJINE OSE.

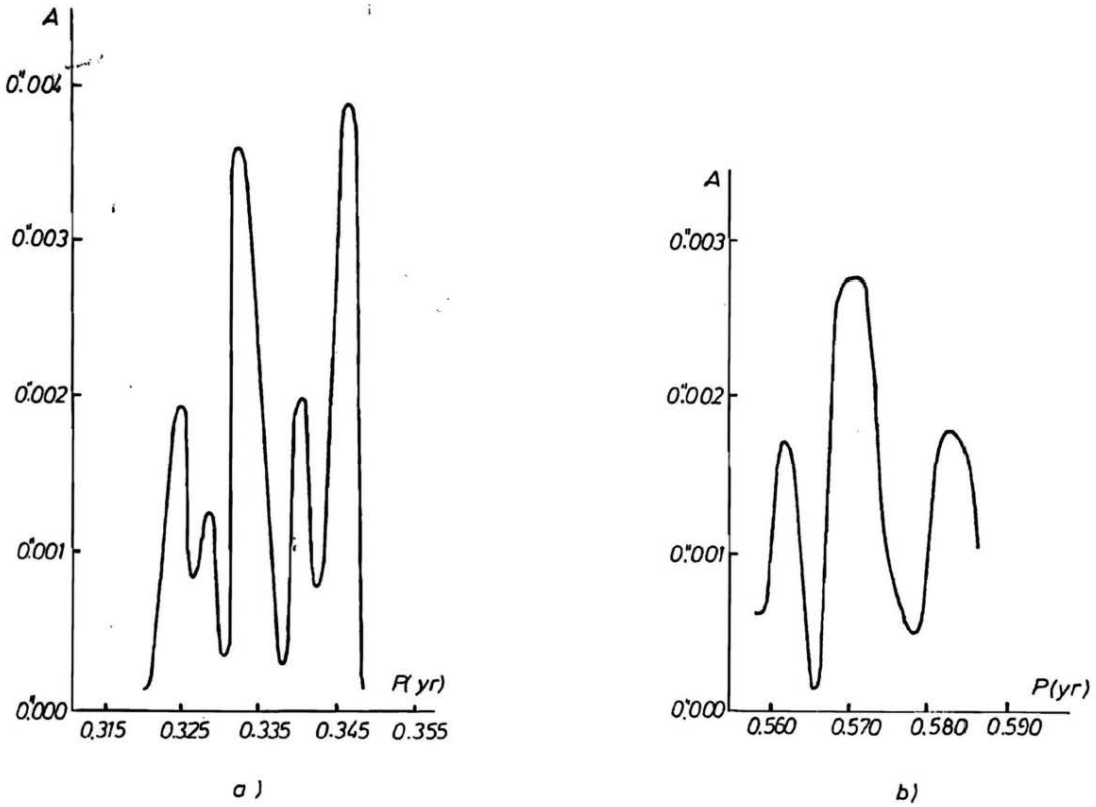
4.4 KRATKOPERIODICNI CLANOVI

ISPITIVANJE POSTOJANJA MOGUCIH KRATKOPERIODICNIH KOMPONENATA VRSILI SMO PRIMENOM VISESTRUKE TRANSFORMACIJE ODUZIMANJA DATE JEDNACINOM (3.6) REDA m, p, q I η, SIMBOLICNO PREDSTAVLJENE SA $\prod(Z) = Z_m Z_p Z_q Z_\eta$ CIJI JE AMPLITUDNI MULTIPLIKATOR OBLIKA:

$$\beta_j = \beta_j^m \beta_j^p \beta_j^q \beta_j^\eta$$

GDE SU VREDNOSTI ZA m, p, q I η VARIRANE U OBLASTI $m = 1,\ldots,7$, $p = 2,3, \ldots,8$, $q = 3,4, \ldots 9$ I $\eta = 4,5, \ldots,10$.
REZULTUJUCI AMPLITUDNI MULTIPLIKATOR IMA IZRAZEN MAKSIMUM ZA 11.7, 15.3, 19.0, 22.9, 26.5, 30.5, 34.5 JEDINICA (RAZMAK ORDINATA η).
ZMAKA ORDINATA).

IZ REZULTUJUCIH NIZOVA IZDVOJENE SU, METODOM FURIJEOVIH INTEGRALNIH TRANSFORMACIJA, HARMONIJSKE KOMPONENTE PRIKAZANE U TA



Sl. 4.16 SPEKTAR KRATKOPERIODIČNIH ČLANOVA
a) ČLANOVI OD 121 i 126 DANA
b) ČLAN OD 205 DANA (NUTACIJA NOLODENSKOG)

BLICI 4.17 I NA SLICI 4.16.

AMPLITUDNI MULTIPLIKATORI ODGOVARAJUCE TRANSFORMACIJE RACUNATI PO FORMULI:

$$\beta_j = 2\sin\left(\frac{2\pi}{n} m\right)\ 2\sin\left(\frac{2\pi}{n} p\right)\ 2\sin\left(\frac{2\pi}{n} q\right)\ 2\sin\left(\frac{2\pi}{n} r\right)$$

GDE JE n PROBNA PERIODA, TAKODJE SU DATI U TABLICI 4.17.

TABLICA 4.17

TRANSFORMACIJA $Z_m\ Z_p\ Z_q\ Z_r$	MULTIPLIKATOR $\beta_m\beta_p\beta_q\beta_r$	JEDINICNI INTERVAL	PERIODA	AMPLITUDA.
$Z_6\ Z_7\ Z_8\ Z_9$	14.35	4 DANA	121 DAN	0".0036
$Z_6\ Z_7\ Z_8\ Z_9$	14.35	4 DANA	126 DANA	0".0039
$Z_7\ Z_8\ Z_9\ Z_{10}$	14.70	6 DANA	205 DANA	0".0027

NA OSNOVU GORNJE TABLICE ZAPAZAMO DA SU IZRAZENI PIKOVI S PERIODAMA 121 I 126 DANA (SL. 4.17) KAO I CLAN OD 205 DANA (SL. 4.17), KOJI ODGOVARA SKORODNEVNOJ NUTACIJI MOLODENSKOG.

POJAVLJIVANJE DVA BLISKA CLANA NA 121 I 126 DANA VEROVATNO JE UZROKOVANO FLUKTUACIJOM PERIODE 122-DNEVNOG CLANA, KOJI JE U BIH-OVIM KOORDINATAMA POLA I U SISTEMU UT1 IDENTIFIKOVAO DJUROVIC (1971, 1979, 1983). S OBZIROM DA JE OVAJ CLAN NEDOVOLJNO POZNAT, NJEGOVA IDENTIFIKACIJA U ILS KOORDINATAMA (NEZAVISNIM OD KOORDINATA BIH-a) IMA IZVESTAN NAUCNI ZNACAJ.

MOLODENSKI JE TEORIJSKI PREDVIDEO NUTACIJU I RACUNAO JE ZA DVA MODELA ZEMLJE. PERIODE KOJE IM ODGOVARAJU RAZLIKUJU SE ZA DVE SEKUNDE.

MNOGI AUTORI SMATRAJU DA SU IDENTIFIKOVALI SKORODNEVNU NUTACIJU MOLODENSKOG, POMENUCEMO NEKE OD NJIH.

POPOV JE IZ POLTAVSKIH POSMATRANJA ZA PERIOD 1939-1969.G. NASAO ZA AMPLITUDU NUTACIJE MOLODENSKOG VREDNOST OD 0".012 .

JATSKIV (YATSKIV) I EMEC (EMETZ) (YATSKIV, 1972) NALAZE U

LATITUDAMA POSMATRANIM U PULKOVU (1905-1941) TRI SPEKTRALNE LINI-JE NA 194, 208 I 219 DANA I TRI LINIJE U VASINGTONSKIM POSMATRA - NJIMA (1916-1940) NA 206, 219 I 234 DANA.ZA AMPLITUDU SKORO DNEV-NE NUTACIJE DOBILI SU VREDNOSTI 0".004 (PULKOVO) I 0".009 (VASIN GTON).

SUGAWA I ODE IZ ANALIZE ILS PODATAKA ZA PERIOD 1955-1967 NALAZE U VARIJACIJI SIRINE CLAN SA PERIODOM 204 DANA I AMPLITUDOM 0".005.

DEBARBA (DEBARBAT, 1970) IZ POSMATRANJA NA ASTROLABU PARI SKE OPSERVATORIJE (1956-1963) DOBIJA ZA AMPLITUDU NUTACIJE MOLO - DENSKOG VREDNOST 0".008.

DJUROVIC (1974) JE IZ ANALIZE BIH-OVIH KOORDINATA POLA (1966-1972) NASAO VARIJACIJU KOJA ODGOVARA NUTACIJI MOLODENSKOG.

UPOREDJUJUCI NASE REZULTATE SA REZULTATIMA POMENUTIH AUTO RA NALAZIMO DA SU SAGLASNI.

STOGA SMATRAMO DA SMO NAŠOM ANALIZOM ILS KOORDINATA DALI JOS JEDAN PRILOG ZA HIPOTEZU O POSTOJANJU SKORO DNEVNE NUTACIJE MOLODENSKOG KAO I KRATKOPERIODICNE VARIJACIJE SA PERIODAMA 121 I 126 DANA (CETVOROMESECNE) KOJE U NAUCNOJ LITERATURI ILI NISU DOVO LJNO POZNATE ILI SE OSPORAVAJU OD STRANE NEKIH AUTORA.

4.5 OSTALI PERIODICNI CLANOVI

DA BI SPEKTRALNOM ANALIZOM U KOORDINATAMA POLA IDENTIFIKO VALI DRUGE NEPOZNATE CLANOVE KORISTILI SMO FILTRE (PO ANALOGIJI NA FILTER ORLOVA I FILTER MELHIORA) ZA PRIGUSIVANJE SVIH KRATKOPE RIODICNIH I GLAVNIH HARMONIJSKIH CLANOVA.

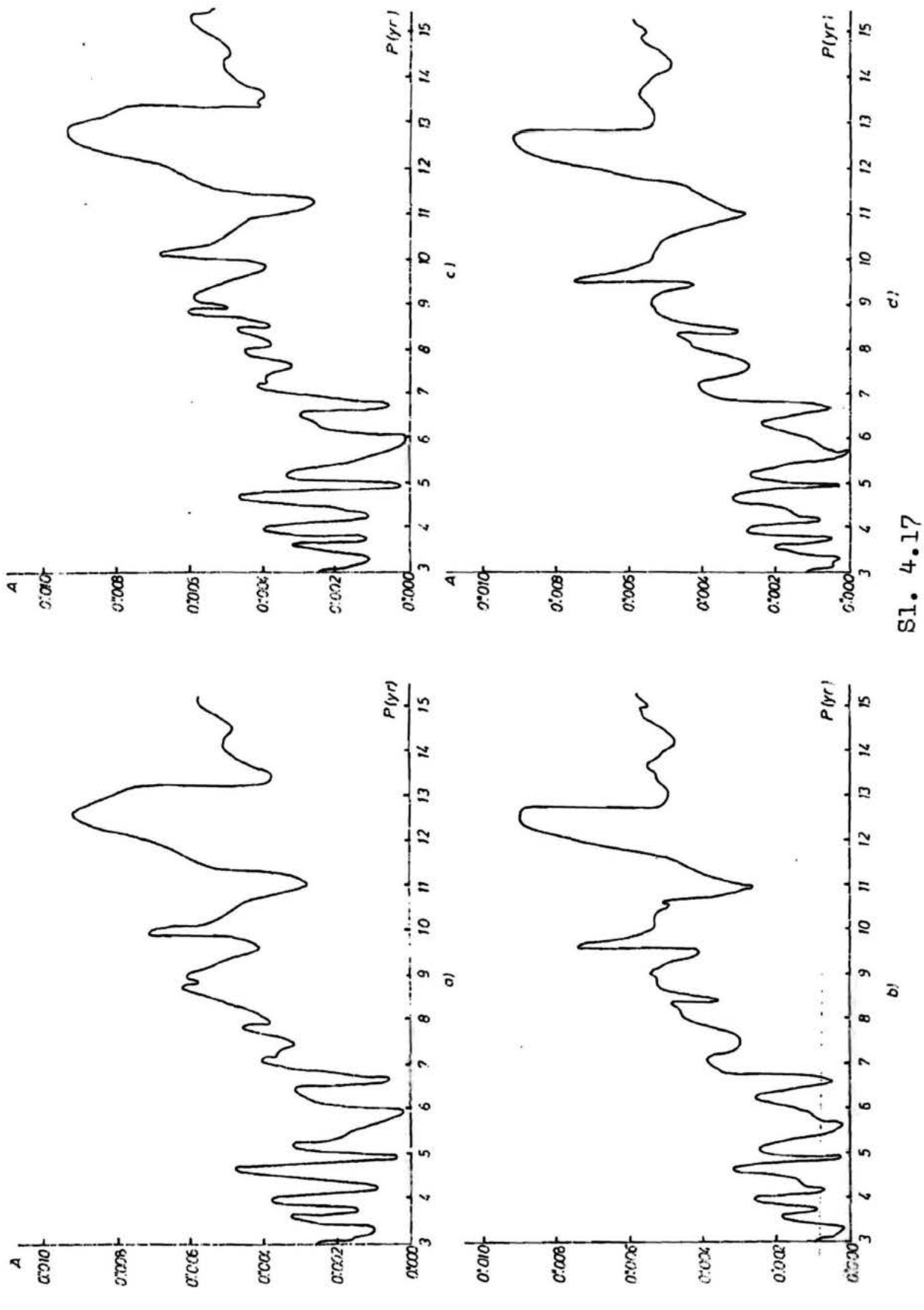
KAKO SU KOEFICIJENTI JEDNACINA (3.10) I (3.11) DATI (TAB LICA 3.1) ZA PODATKE NA EKVIDISTANTNIM RAZMACIMA OD 0.1 GODINA,

TO RASPOLAZUCI PODACIMA NA SVAKIH 1/12 I 1/20 GODINA, NISMO MOGLI NEPOSREDNO DA IH KORISTIMO. S TOGA SMO PO ANALOGIJI NA ORLOVA I MELHIORA RACUNALI KOEFICIJENTE JEDNACINA (3.10) I (3.11), PRAVECI NOVE FILTRE, ZA PODATKE NA EKVIDISTANTNIM RAZMACIMA MESEC DANA I 1/20 GODINE. RACUNATI KOEFICIJJENTI OZNACENI SA "O" I "M" , REDOM DATI SU U TABLICI 4.8. BROJ i JE REDNI BROJ KOEFICIJENTA.

TABLICA 4.8

SIROVI PODACI (1/12 G.)			IZRAVNATI PODACI (1/20 GOD.)			
i	KOEFICIJENTI "O"	KOEFICIJENTI "M"	KOEFICIJENTI "O"	KOEFICIJENTI "M"	i	KOEFICIJENTI "M"
0	1		1		28	3680
1	2	1	2	1	29	3991
2	2	4	2	4	30	4305
3	2	10	2	10	31	4619
4	2	20	2	20	32	4930
5	2	35	2	35	33	5235
6	2	56	2	56	34	5531
7	3	84	2	84	35	5815
8	4	120	2	120	36	6084
9	4	165	2	165	37	6335
10	-	220	2	220	38	6565
11	-	286	3	286	39	6772
12	-	363	4	364	40	6954
13	-	451	4	455	41	7109
14	-	549	4	560	42	7235
15	-	656	4	680	43	7331
16	-	770	-	816	44	7396
17	-	889	-	969	45	7429
18	-	1011	-	1139		
19	-	1133	-	1326		
20	-	1252	-	1530		
21	-	1365	-	1751		
22	-	1469	-	1988		
23	-	1561	-	2240		
24	-	1638	-	2506		
25	-	1698	-	2785		
26	-	1739	-	3075		
27	-	1760	-	3374		

TRANSFORMISANE NIZOVE X_{uo} I X_{vo} (PO ANALOGIJI NA ORLOVA) I X'_{uM} I X'_{vM} (PO ANALOGIJI NA MELHIORA) RACUNALI SMO NOVIM FILTRIMA DATIM IZRAZIMA:



Sl. 4.17

$$X_{UO} = \frac{1}{48}(X_0 + 2X_1 + \ldots + 4X_8 + 4X_9 + 4X_{10} + \ldots + 2X_{18} + X_{19}) \quad (4.16)$$

$$X_{VO} = \frac{1}{80}(X_0 + 2X_1 + \ldots + 4X_{15} + 4X_{16} + \ldots + 2X_{30} + X_{31}) \quad (4.17)$$

$$X'_{UM} = \frac{1}{38\,610}(X_{54} + 4X_{53} + 10X_{52} + \ldots + 1760X_{27} + 1760X_{28} + \ldots + 10X_3 + 4X_2 + X_1) \quad (4.18)$$

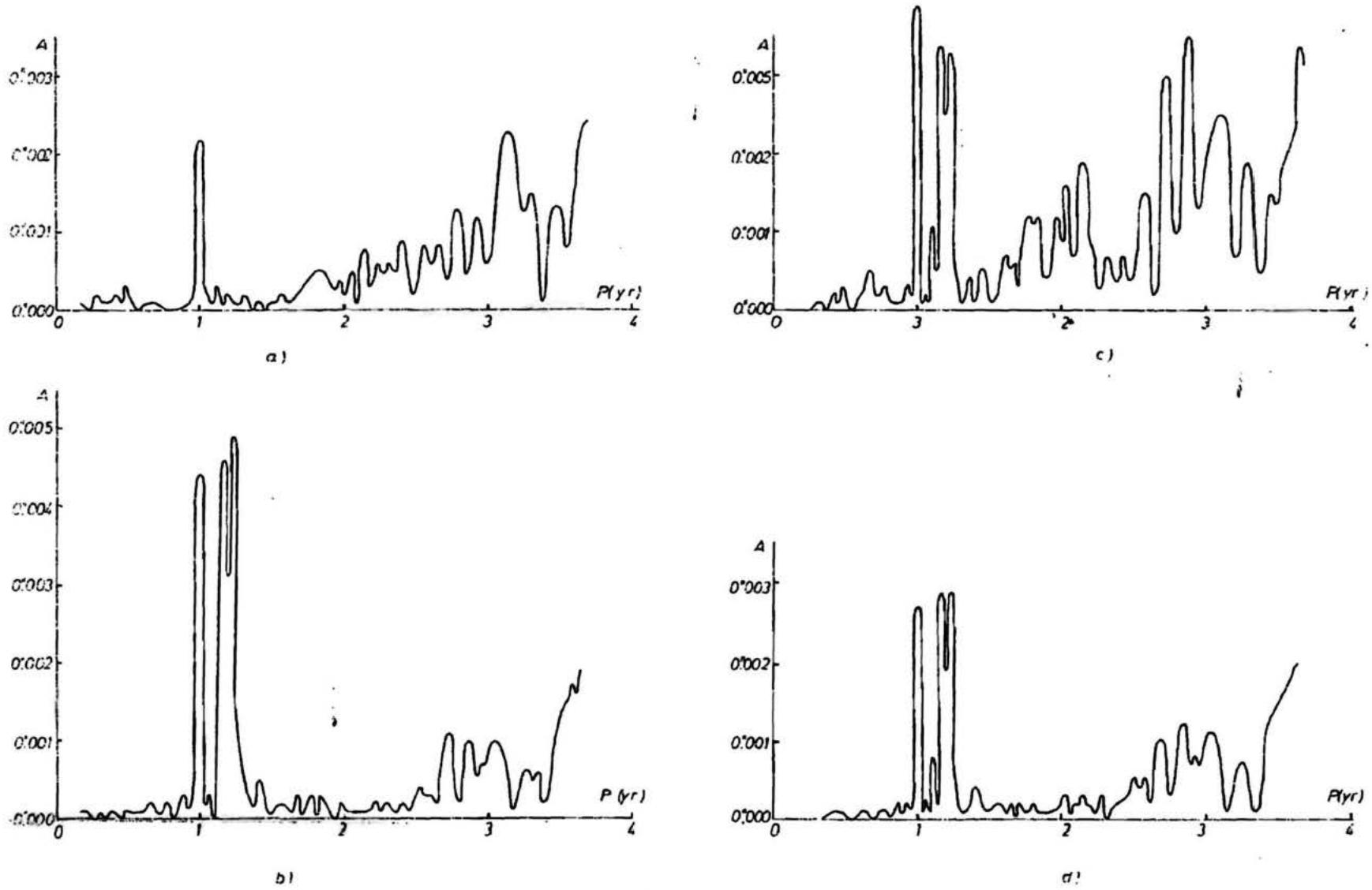
$$X'_{VM} = \frac{1}{267\,750}(X_{90} + 4X_{89} + 10X_{88} + \ldots + 7429X_{45} + 7429X_{46} + \ldots + 10X_3 + 4X_2 + X_1) \quad (4.19)$$

FUNKCIJA SELEKTIVNOSTI (JEDNACINE 4.18) IMA NULE ZA η=18, η=15, η=13 I η=11 JEDINICNIH INTERVALA (MESEC DANA) A JEDNACINE (4.19) ZA η=30, η=25, η=21 I η=17 JEDINICNIH INTERVALA (1/20 GODINE). DRUGIM RECIMA, AMPLITUDNI MULTIPLIKATORI SU JEDNAKI NULI ZA OVE IZNOSE η, ODNOSNO ZA IZNOSE PERIODA: 1.50, 1.25, 1.08, 0.92 I 1.50, 1.25, 1.05, 0.92 GODINA.

IZ NAVEDENIH IZRAZA SE VIDI DA JE ZA SVAKU VREDNOST X_{UO} POTREBNO 20 PODATAKA, ZA X_{VO} 32 PODATKA, ZA X'_{UM} 54 PODATKA I ZA X'_{VM} 90 PODATAKA.

NA OVE REZULTUJUCE NIZOVE PRIMENOM METODE DFT DOBILI SMO SPEKTRE KOORDINATA PRIKAZANE NA SLICI 4.17.NA SLICI SE ZAPAZA NIZ EKSTREMUMA KOJI ODGOVARAJU PERIODAMA OD 3 DO 12 GODINA I AMPLITUDAMA OD 0".002 DO 0".010. NA OSNOVU OVIH REZULTATA NE MOZEMO GOVORITI O IDENTIFIKACIJI ODREDJENIH HARMONIJSKIH VARIJACIJA UKOLIKO ONE POSTOJE.

NA SLICI 4.18 PRIKAZANI SU OSTACI GODISNJEG I CENDLEROVOG CLANA, POSLE NJIHOVOG PRIGUSENJA FILTRIMA DATIM JEDNACINAMA (4.16) (4.17), (4.18) I (4.19). VIDIMO DA SU OSTACI VRLO MALIH AMPLITUDA (OD 0".000 DO 0".005) U ODNOSU NA VREDNOSTI DATE U TABLICAMA 4.5 I 4.6.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P(yr)
d)

Sl. 4.18

5. KIMURIN Z CLAN

OSNOVNA FORMULA ZA IZVODJENJE KOORDINATA POLA X, Y DATA JEDNACINOM (2.2), SADRZI KIMURIN Z-CLAN. POZNATO JE (A. STOJKO , [illegible]; KULIKOV, 1962) DA SE OVAJ CLAN MOZE APROKSIMIRATI SINUSOI - DOM SA GODISNJOM PERIODOM. MEDJUTIM, ANALIZOM ILS PODATAKA POREDGODISNJE, IDENTIFIKOVALI SMO I POLUGODISNJU KOMPONENTU KOJA DO SADA NIJE BILA POZNATA.

DA BI IZ PODATAKA UKLONILI PROGRESIVNI CLAN,FORMIRALI SMO OD MESECNIH VREDNOSTI Z-CLANA (TABLICA PRILOG I, GRFICKI PREDSTAVLJEN PRILOG III) SREDNJE SESTOGODISNJE VREDNOSTI Z I IZRACUNALI NJIHOVE RAZLIKE PRVOG, DRUGOG I TRECEG REDA. DOBIJENI REZULTATI DATI SU U TABLICI 5.1.

TABLICA 5.1

U PRVOJ KOLONI SU SREDNJE TROGODISNJE VREDNOSTI A OD 2-4 NJIHOVE RAZLIKE

Z	$\Delta^1 Z$	$\Delta^2 Z$	$\Delta^3 Z$
0".088			
	-0".018		
.070		0".072	
	.054		-0".160
.124		-.088	
	-.035		.146
.089		.058	
	.023		-.063
.112		-.005	
	.018		-.001
.130		-.007	
	.011		.008
.141		.001	
	.012		-.001
.153		.001	

TABLICA 5.1 (NASTAVAK)

	.013	-.018
.166		-.018
	-.005	.033
.161		.015
	.010	.004
.171		.019
	.029	-.027
.200		-.009
	.020	
.221		

RACUNATA SREDNJE KVADRATSKA ODSTUPANJA U ODNOSU NA SRED - NJE VREDNOSTI ODGOVARAJUCIH RAZLIKA PRVOG REDA S1, DRUGOG REDA S2 I TRECEG REDA S3 SU:

S1 = 0°.0006

S2 = 0 .0015

S3 = 0 .0053

UPOREDJUJUCI SREDNJE KVADRATSKA ODSTUPANJA NALAZIMO DA JE S1< S2< S3. IZ OVOGA SLEDI DA JE LINEARNA APROKSIMACIJA PROGRESIV NOG CLANA NAJBOLJA.

METODOM NAJMANJIH KVADRATA RACUNALI SMO KOEFICIJENTE RE- GRESIJE. SLOBODNI CLAN C0 I LINEARNI C1 SU:

C0 = 0.0084÷0.0005

C1 = 0.0018÷0.0001

NA TAJ NACIN DUGOPERIODICNA VARIJACIJA U NEPOLARNIM PROME NAMA SIRINE PREDSTAVLJENA JE LINEARNO:

$$Z = 0^{s}.0084 + 0^{s}.0018(t - 1900)$$

ODUZIMANJEM OVE DUGOPERIODICNE KOMPONENTE IZ PODATAKA DOB ILI SMO REZIDUE Z-CLANA.

SPEKTAR Z-CLANA (SL. 5.1), KAO I AMPLITUDE, PERIODE I FAZE HAR - MONIJSKIH CLANOVA (TABLICA 5.2) RACUNALI SMO METODOM FURIJEOVUH TRANSFORMACIJA. OVA METODA JE PRIMENJENA NA REZIDUE Z-CLANA.

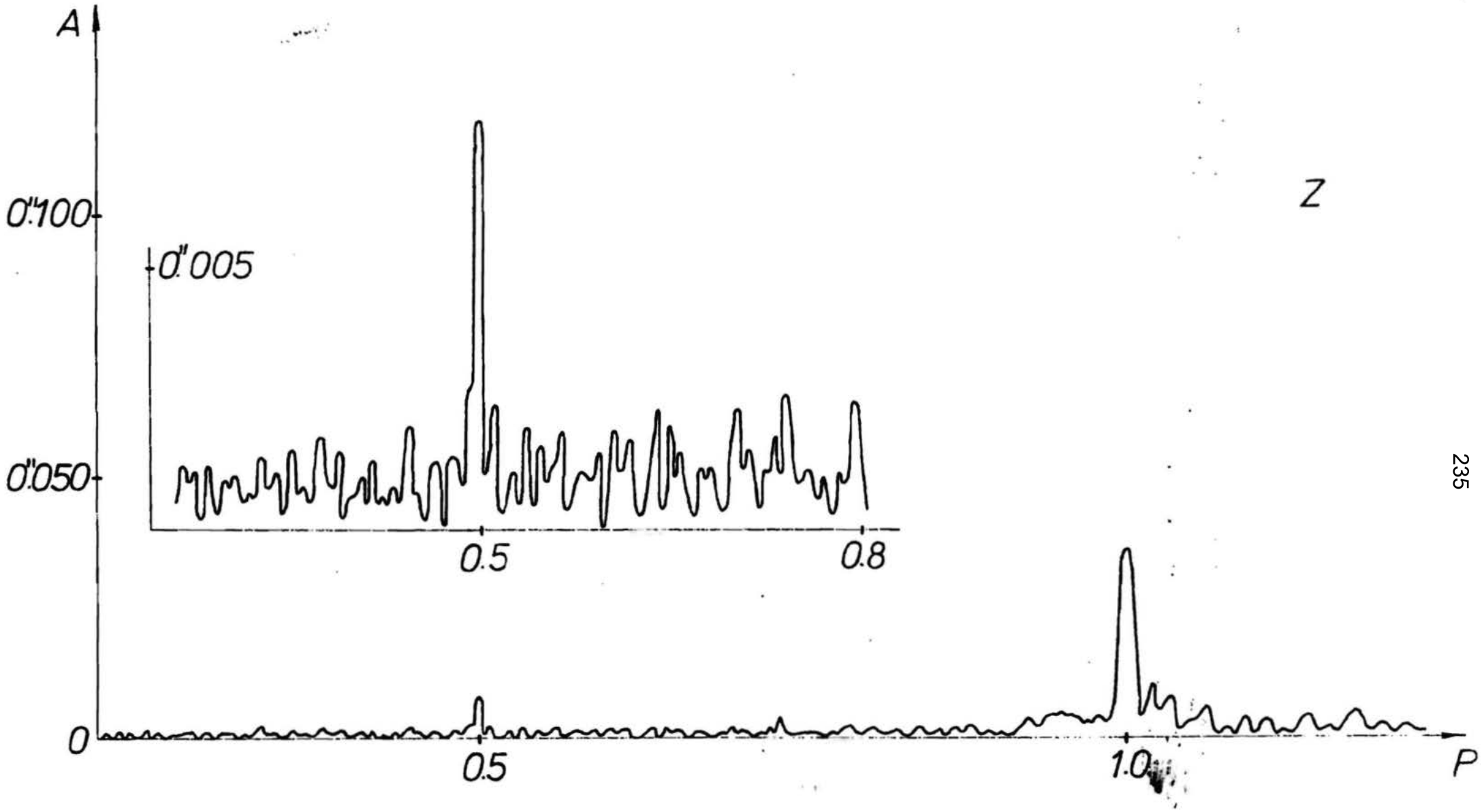
U TABLICI 5.2 SA P , A I F OZNACILI SMO PERIODU, AMPLITU DU I FAZU GODISNJEG CLANA, A SA P', A' I F' PERIODU, AMPLITUDU I FAZU POLUGODISNJEG CLANA. FAZE SE ODNOSE NA JULIJANSKI DATUM DJ= 2415 020, I DATE SU U GODINAMA.

TABLICA 5.2

P	A	F	P'	A'	F'
1.000	$0^{s}.037$	5.8	0.499	$0^{s}.008$	32.3

KAKO SU MESECNE VREDNOSTI Z-CLANA DATE SA SREDNJEKVADRAT- SKOM GRESKOM OD $\pm 0^{s}.01$ TO ZA SREDNJU KVADRATSKU GRESKU AMPLITUDE, RACUNATU PO IZRAZU ZA DISPERZIJU AMPLITUDE (4.6) , DOBIJAMO $\pm$ $0^{s}.0003$, A ZA SREDNJU KVADRATSKU GRESKU FAZE NA OSNOVU JEDNACI- NE ZA DISPERZIJU FAZE (4.8), DOBIJAMO ZA GODISNJI CLAN $\pm 0^{c}.7$ I ZA POLUGODISNJI CLAN $\pm 3^{c}.3$. AMPLITUDE U TABLICI 5.2 POPRAVLJENE SU ZA NIVO "BELOG SUMA", RACUNATOG PO IZRAZU (4.7), KOJI IZNOSI $0^{s}.001$.

APSCISNA OSA P NA SLICI 5.1 JE SA PROBNIM PERIODAMA U GO- DINAMA, A NA ORDINATNOJ OSI A SU AMPLITUDE U LUCNIM SEKUNDAMA. NA



Sl. 5.1. Spektar z-člana.

SLICI SE JASNO VIDE GODISNJI I POLUGODISNJI PIK. POLUGODISNJI PIK JE POSEBNO IZRAZEN NA UVECANOM DELU SPEKTRA (SL.5.1).

S OBZIROM DA JE POLUGODISNJA KOMPONENTA Z-CLANA BILA NEPO ZNATA, A VEOMA JE MALE AMPLITUDE, STATISTICKO ZNACENJE REZULTATA IZ TABLICE 5.2 ISPITIVALI SMO I POSEBNIM KRITERIJUMIMA.

DA BISMO IMALI IZVESNU OCENU STATISTICKOG ZNACENJA AMPLI-TUDE POLUGODISNJEG CLANA A' TESTIRALI SMO NULTU HIPOTEZU HO : DA REZULTAT SPEKTRALNE ANALIZE ZA UCESTANOST $\omega = 2\pi/0.499$ PREDSTAV-LJA BELI SUM, PROTIV ALTERNATIVNE HIPOTEZE H1: PODACI SADRZE HAR-MONIKU UCESTANOSTI ω.

KORISATILI SMO SHUSTER-OV KRITERIJUM (DJUROVIC, 1979)

$$Q\left\{S(\omega) \geq \frac{k\sigma_x^2}{2\pi}\right\} = e^{-k} \qquad (5.1)$$

AKO JE X SLUCAJNA PROMENLJIVA KOJA SE POKORAVA NORAMALNOM GAUSOVOM ZAKONU RASPODELE VEROVATNOCA, SA SREDNJOM VREDNOSCU NULA I DISPERZIJOM σ_x^2, Q PREDSTAVLJA VEROVATNOCU DA SPEKTRALNA GUSTI-NA $S(\omega)$ BUDE VECA ILI JENDAKA GRANICNOJ $SO = k\sigma^2/2\pi$. OVA VERO - VATNOCA ODNOSI SE NA JEDNO SLUCAJNO IZABRANO $S(\omega)$.

SPEKTRALNU GUSTINU $S(\omega)$ RACUNALI SMO U FUNKCIJI AMPPLITU DE:

$$S(\omega) = \frac{2N+1}{8\pi} A'^2(\omega) \qquad (5.2)$$

GDE JE 2N+1 BROJ PODATAKA (U NASEM SLUCAJU 951). DOBILI SMO DA JE $S(\omega)=0.0024$, A $SO=0.00005$ I $SO=0.00007$ ZA NIVOE ZNACAJNOSTI $= e^{-k}=0.05$ I $=0.01$, REDOM.

VEROVATNOCU Q' DA BAR JEDNA OD N SPEKTRALNIH LINIJA $S(\omega_i)$ ($i=1,2,\ldots,N$) DATOG SPEKTRA PREDJE GRANICU $SO = k\sigma_o^2/2\pi$ RACUNALI SMO POMOCU WALKER-OVE FORMULE (DJUROVIC, 1979):

$$Q'\left\{S_1(\omega) \geq \frac{k\sigma_x^2}{2\pi}\right\} = 1-(1-e^{-k})^N \qquad (5.3)$$

GDE JE S ( )=SUP S( ), NAJVECA SPEKTRALNA GUSTINA (U SPEKTRU Z CLANA GODISNJA KOMPONENTA).

U NASEM SLUCAJU IZ IZRAZA (5.2) I (5.3) DOBIJAMO $S_1(\omega)$= =0.0518, A S0=0.00006 I S0=0.00008 ZA NIVOE ZNACAJNOSTI α =1-(1- $-e^{-k})^N$ = 0.05 I α =0.01, REDOM.

NA OSNOVU GORNJIH REZULTATA PRIMENE SHUSTER-OVOG I WAL - KER-OVOG KRITERIJUMA, SA NIVOIMA ZNACAJNOSTI 0.05 I 0.01 DOBILI SMO DA HIPOTEZU HO MOZEMO DA ODBACIMO.

ODNOSNO, SA VEROVATNOCOM 99% NALAZIMO DA S(ω) POLUGODI- SNJEG CLANA NE REZULTIRA IZ REZONANCIJE SLUCAJNIH GRESAKA , VEC DA PODACI SADRZE POLUGODISNJU KOMPONENTU UCESTANOSTI $\omega=2\pi/0.499$

U MONOGRAFIJI KULIKOVA (1962), POSVECENOJ PROBLEMIMA VA- RIJACIJA LONGITUDA I LATITUDA, NALAZIMO DA SU RAZNI AUTORI ZA AM PLITUDE I FAZE Z-CLANA DOBIJALI VEOMA RAZLICITE VREDNOSTI. TO PO TICE OD CINJENICE DA SU KORISTILI RAZLICIT POSMATRACKI MATERIJAL

FIKERA (1971) JE NASAO DA Z-CLAN U GLAVNOM ZAVISI OD KA- TALOGA NA KOJI SE ODNOSE POSMATRACKE LATITUDE. MEDJUTIM , DRUGI AUTORI NAVODE I DRUGA OBJASNJENJA.

SEVARLIC (1961) JE IZ ANALIZE REFRAKCIJSKIH ANOMALIJA BE OGRADSKIH POSMATRANJA LATITUDA NASAO DA VELIKI DEO Z-CLANA MOZE DA SE OBJASNI TERMICKIM GRADIJENTIMA POSMATRACKIH PAVILJONA.

PRIRODA Z-CLANA NI DANAS NIJE POZNATA.

LAMBECK (1980) U SVOJOJ MONOGRAFIJI INTERPRETIRA Z- CLAN KAO GEOFIZICKI FENOMEN. NA OSNOVU RADOVA ISHII-JA, NAITO-A NALA- ZI DA Z-CLAN ZAVISI OD MNOSTVA ASTRONOMSKIH POSMATRACKIH I GEOFI ZICKIH FAKTORA.

6. ZAKLJUCCI

UPRKOS CINJENICI, DA SE PROBLEMOM KRETANJA ZEMLJINUH POLO VA BAVIO VELIKI BROJ ASTRONOMA,ON JE OSTAO NERESEN SVE DO DANAS.TO JE, PRE SVEGA, USLED SLOZENOSTI GEOFIZICKIH MEHANIZAMA, METEOROLO SKIH I DRUGIH UTICAJA KOJI UZROKUJU PERIODICNE I NEPEERIODICNE VA RIJACIJE U KRETANJU POLOVA. S DRUGE STRANE, SISTEMATSKE I SLUCA - JNE GRESKE BILE SU I OSTAJU VELIKA PREPREKA U RESAVANJU POMENUTOG PROBLEMA. POSMATRANJA ZEMLJINIH VESTACKIH SATELITA POMOCU LASERA I DOPLR-UREDJAJA JOS NISU DALA REZULTATE KOJI BI OMOGUCILI DA SE RAZRESE DILEME O KOJIMA SMO DISKUTOVALI U OVOM RADU.

ANALIZOM PRAVOUGLIH KOORDINATA TRENUTNOG POLA ZEMLJINE RO TACIJE U ODNOSU NA CIO ZA PERIOD 1899.9-1979.0 UTVRDILI SMO SLEDE CE:

POSTOJANJE SEKULARNOG KRETANJA SREDNJEG POLA STO SE JOS UVEK SMATRA SPORNIM,

IZ UPOREDJIVANJA TEORIJSKE I RACUNATE SIRINE PIKOVA DOSLI SMO DO ZAKLJUCKA DA JE CENDLEROV CLAN DVOSTRUK,

DA BI SMO PROVERILI DA LI JE DVOSTRUKOST CENDLEROVOG CLA- NA PSEUDO POJAVA ILI SE MOZE OBJASNITI POSTOJANJEM DVE BLISKE VA- RIJACIJE KOORDINATA POLA UPOREDJIVALI SMO POLAZNE I GENERISANE PO DATKE I ZAKLJUCILI DA SE NE RADI O JEDNOJ VEC O DVE BLISKE SLOBO- DNE NUTACIJE ZEMLJINE OSE.

DAKLE, ZA HIPOTEZU O POSTOJANJU DVE BLISKE SLOBODNE NUTA CIJE NASLI SMO NOVE DOKAZE. JEDNA NUTACIJA MOGLA BI DA IMA PERIO- DU P=1.171 GODINA A DRUGA P'=1.197 GODINA.

ZA AMPLITUDU I FAZU GODISNJEG I POLUGODISNJEG CLANA DOBI- LI SMO VREDNOSTI VEOMA BLISKE ONIMA KOJE SU DOBILI DRUGI AUTORI.

STO SE TICE FLUKTUACIJE AMPLITUDE I FAZE GODISNJEG I POLU GODISNJEG CLANA, NASLI SMO DA SU SLUCAJNOG KARAKTERA.

ZA VARIJACIJE AMPLITUDE I FAZE CENDLEROVOG CLANA NASLI SMO DA JE AMPLITUDA PERIODICNO PROMENLJIVA SA DVA NAJIZRAZENIJA HARMONIJSKA DUGOPERIODICNA CLANA (S PERIODAMA 53.0 34.5 GODINA) KAO I NIZOM SLABIJIH PERIODICNIH CLANOVA. UPRKOS TOME, FAZA CENDLEROVE NUTACIJE IMA DOBRU STABILNOST IZUZIMAJUCI IZNENADNU I NEOBJASNJENU NJENU PROMENU ZA 180° U PERIODU 1925-1940.

U SPEKTRIMA KOORDINATA POLA IDENTIFIKOVANE SU KRATKOPERIODICNE VARIJACIJE OD 204 DANA (NUTACIJA MOLODENSKOG) I 122 DANA , KOJE U NAUCNOJ LITERATURI ILI NISU DOVOLJNO POZNATE ILI SE OSPORAVAJU OD STRANE NEKIH AUTORA.

STO SE OSTALIH (KVAZI)PERIODICNIH FLUKTUACIJA (X,Y) TICE, MOZEMO JEDINO RECI DA SE NALAZE ISPOD GRANICE OD 0".010.

SPEKTRALNOM ANALIZOM KIMURINOG Z-CLANA NALAZIMO,POREDPOZNATE GODISNJE CIKLICNE VARIJACIJE,I POLUGODISNJU VARIJACIJU STO PREDSTAVLJA SASVIM NOVO OTKRICE.

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P R I L O Z I

PRILOG I (SIROVE KOORDINATE POLA X, Y I Z KOMPONENTA U 0".001)

	X	Y	Z		X	Y	Z		X	Y	Z		X	Y	Z
1899.055	0	0	0	0.055	6	-31	90	0.055	0	41	118	0.055	104	20	60
0.139	0	0	0	0.139	44	-56	85	0.139	37	18	110	0.139	-80	82	89
0.222	0	0	0	0.222	27	-69	78	0.222	54	39	80	0.222	-39	141	83
0.305	0	0	0	0.305	-14	-86	63	0.305	93	8	72	0.305	42	172	56
0.389	0	0	0	0.389	41	-105	42	0.389	1	-56	70	0.389	13	164	25
0.472	0	0	0	0.472	-32	-110	51	0.472	136	-88	46	0.472	191	111	30
0.555	0	0	0	0.555	-42	-87	83	0.555	102	-152	64	0.555	213	29	60
0.639	0	0	0	0.639	-66	-57	96	0.639	64	-163	96	0.639	185	-40	81
0.722	0	0	0	0.722	74	-19	106	0.722	6	-143	113	0.722	20	-104	97
0.805	4	71	126	0.805	-73	16	128	0.805	-38	-124	115	0.805	38	-147	109
0.889	-42	45	168	0.889	-75	33	156	0.889	-91	-102	116	0.889	-47	-134	103
0.972	-21	35	114	0.972	-25	7	161	0.972	-115	-58	110	0.972	-126	-85	106
1903.055	176	-37	99	0.055	118	-132	117	0.055	7	-169	87	0.055	9	-58	64
0.139	-188	41	107	0.139	-173	-74	118	0.139	-63	-160	98	0.139	-5	-79	33
0.222	-133	109	110	0.222	-118	-8	98	0.222	-140	-128	93	0.222	-45	-69	19
0.305	-75	174	103	0.305	-177	58	89	0.305	-150	-59	70	0.305	-92	-52	39
0.389	7	212	65	0.389	113	158	65	0.389	150	-6	39	0.389	116	-10	26
0.472	84	219	59	0.472	-31	188	56	0.472	-112	87	5	0.472	-110	48	30
0.555	155	185	65	0.555	49	180	67	0.555	-52	150	41	0.555	-97	83	35
0.639	206	104	82	0.639	112	143	86	0.639	30	165	48	0.639	-59	102	51
0.722	3	-4	101	0.722	57	89	92	0.722	1	153	51	0.722	27	108	63
0.805	159	-107	116	0.805	165	26	101	0.805	121	111	66	0.805	-5	105	75
0.889	92	-169	123	0.889	142	-32	122	0.889	137	46	76	0.889	13	108	90
0.972	-12	-163	115	0.972	120	-96	112	0.972	127	-22	82	0.972	28	133	70
1907.055	2	67	37	0.055	35	186	65	0.055	288	58	101	0.055	232	-186	95
0.139	51	25	43	0.139	45	191	77	0.139	-231	178	88	0.139	-290	-38	100
0.222	55	-12	36	0.222	120	166	82	0.222	-155	257	101	0.222	-308	89	105
0.305	70	-46	52	0.305	191	104	68	0.305	13	298	84	0.305	-241	242	84
0.389	7	-74	22	0.389	15	3	49	0.389	60	278	50	0.389	106	304	68
0.472	-3	-102	40	0.472	231	-60	40	0.472	271	203	36	0.472	56	338	50
0.555	-17	-93	35	0.555	183	-142	56	0.555	324	77	43	0.555	202	318	62
0.639	-46	-70	51	0.639	105	-185	67	0.639	308	-25	40	0.639	292	212	77
0.722	111	-31	72	0.722	1	-199	90	0.722	43	-167	76	0.722	21	46	100
0.805	-160	22	79	0.805	-123	-189	116	0.805	105	-253	89	0.805	284	-100	109
0.889	-132	91	65	0.889	-221	-140	118	0.889	-57	-291	97	0.889	197	-200	92
0.972	-89	156	47	0.972	-253	-42	103	0.972	-181	-274	100	0.972	72	-294	115

PRILOG I (NASTAVAK)

1911.055	81	-267	118	0.055	44	-139	111	0.055	7	81	126	0.055	90	147	155
0.139	-174	-225	120	0.139	68	-185	114	0.139	135	60	117	0.139	-8	138	159
0.222	-227	-96	130	0.222	-47	-162	91	0.222	121	-6	65	0.222	89	153	147
0.305	-254	41	103	0.305	-106	-111	50	0.305	124	-43	69	0.305	172	125	92
0.389	193	169	79	0.389	157	-35	43	0.389	19	-71	38	0.389	74	102	69
0.472	-134	295	50	0.472	-140	21	29	0.472	97	-47	35	0.472	194	42	70
0.555	-9	343	35	0.555	-118	126	53	0.555	24	-69	55	0.555	174	-30	70
0.639	113	326	57	0.639	-84	165	89	0.639	-26	86	82	0.639	118	-86	104
0.722	1	263	78	0.722	50	177	113	0.722	78	-58	106	0.722	3	-150	148
0.805	259	153	91	0.805	-11	195	123	0.805	-110	0	128	0.805	-50	-175	173
0.889	278	16	110	0.889	67	194	109	0.889	-91	63	145	0.889	-140	-105	177
0.972	240	-69	136	0.972	99	142	100	0.972	-79	116	155	0.972	-179	-4	150
1915.055	208	29	180	0.055	173	-182	169	0.055	19	-261	199	0.055	6	-144	137
0.139	-168	146	163	0.139	-192	-53	174	0.139	-75	-199	185	0.139	-24	-73	129
0.222	-121	232	173	0.222	-204	101	147	0.222	-157	-106	179	0.222	-50	-5	113
0.305	-2	307	131	0.305	-154	179	125	0.305	-143	14	147	0.305	-68	45	89
0.389	4	260	79	0.389	35	272	94	0.389	87	107	132	0.389	81	107	78
0.472	198	221	89	0.472	60	301	71	0.472	-46	235	127	0.472	-88	177	120
0.555	254	136	133	0.555	166	236	98	0.555	36	236	114	0.555	-23	154	96
0.639	282	47	156	0.639	265	107	110	0.639	119	217	134	0.639	39	150	101
0.722	42	-84	169	0.722	12	4	152	0.722	59	156	174	0.722	1	128	104
0.805	148	-219	181	0.805	287	-80	198	0.805	159	64	197	0.805	82	89	100
0.889	47	-240	188	0.889	210	-197	183	0.889	155	6	184	0.889	95	94	124
0.972	-94	-183	167	0.972	109	-221	203	0.972	150	-72	154	0.972	99	102	132
1919.055	2	80	133	0.055	1	181	59	0.055	97	10	77	0.055	89	35	63
0.139	57	28	123	0.139	38	154	76	0.139	-101	77	70	0.139	-113	55	100
0.222	65	88	107	0.222	52	177	88	0.222	-35	90	45	0.222	-82	151	86
0.305	75	7	51	0.305	144	151	38	0.305	49	123	37	0.305	-23	199	71
0.389	5	30	51	0.389	89	71	24	0.389	50	153	20	0.389	5	218	83
0.472	60	-3	54	0.472	208	16	39	0.472	201	61	41	0.472	146	174	34
0.555	43	-49	55	0.555	198	-46	69	0.555	242	21	41	0.555	226	77	43
0.639	-4	-18	93	0.639	179	-112	72	0.639	239	-20	56	0.639	262	23	71
0.722	42	28	126	0.722	25	-133	75	0.722	91	-93	81	0.722	52	-16	103
0.805	-62	45	135	0.805	56	-136	92	0.805	119	-138	92	0.805	205	-95	131
0.889	-66	49	127	0.889	2	-147	117	0.889	62	-97	73	0.889	138	-65	119
0.972	-37	117	116	0.972	-71	-83	123	0.972	23	-60	81	0.972	12	-102	98

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1923.055	52	-53	100	0.055	9	-20	137	0.055	2	2	111	0.055	0	-46	157
0.139	-100	-13	101	0.139	-29	-3	107	0.139	-13	-64	129	0.139	-27	-33	163
0.222	-127	74	95	0.222	-66	89	108	0.222	-4	-8	93	0.222	-55	30	154
0.305	-77	166	75	0.305	-47	110	78	0.305	-15	63	86	0.305	-33	97	97
0.389	34	242	73	0.389	42	152	61	0.389	4	91	75	0.389	31	107	93
0.472	43	254	69	0.472	-16	181	53	0.472	26	83	58	0.472	-3	142	81
0.555	134	223	68	0.555	43	180	70	0.555	43	94	85	0.555	42	149	69
0.639	230	241	97	0.639	133	205	79	0.639	104	83	96	0.639	74	164	109
0.722	47	142	122	0.722	43	150	103	0.722	0	42	92	0.722	10	130	102
0.805	235	97	123	0.805	142	115	130	0.805	115	54	123	0.805	83	91	141
0.889	197	35	131	0.889	133	102	122	0.889	96	16	139	0.889	67	136	159
0.972	107	-1	133	0.972	106	30	136	0.972	32	22	112	0.972	72	74	155
1927.055	3	79	131	0.055	30	57	168	0.055	26	118	183	0.055	107	64	153
0.139	102	69	119	0.139	-33	88	143	0.139	-39	133	144	0.139	-117	175	126
0.222	34	44	127	0.222	-26	112	112	0.222	-3	159	134	0.222	-63	195	141
0.305	69	75	85	0.305	8	100	93	0.305	73	189	112	0.305	-7	193	141
0.389	0	105	58	0.389	8	173	70	0.389	9	203	88	0.389	7	247	84
0.472	82	116	64	0.472	39	99	62	0.472	150	156	93	0.472	128	210	72
0.555	86	101	69	0.555	53	119	84	0.555	159	88	90	0.555	200	191	80
0.639	99	63	84	0.639	87	69	104	0.639	157	26	104	0.639	203	125	118
0.722	3	23	114	0.722	1	64	111	0.722	6	-15	114	0.722	66	37	132
0.805	57	53	124	0.805	-11	70	139	0.805	25	-13	134	0.805	102	17	155
0.889	42	18	174	0.889	-15	99	173	0.889	-29	-34	165	0.889	18	43	158
0.972	7	12	182	0.972	-27	119	166	0.972	-61	1	170	0.972	-67	-28	168
1931.055	127	70	148	0.055	99	7	178	0.055	30	1	154	0.055	6	37	156
0.139	-105	155	139	0.139	-167	54	124	0.139	-78	37	157	0.139	-50	61	96
0.222	-117	170	128	0.222	-142	150	115	0.222	-101	77	145	0.222	-90	94	80
0.305	-54	257	104	0.305	-69	234	104	0.305	-99	139	112	0.305	-111	117	73
0.389	1	278	79	0.389	77	254	97	0.389	80	188	86	0.389	87	143	57
0.472	122	269	85	0.472	-6	245	78	0.472	-32	227	76	0.472	-43	200	56
0.555	215	234	110	0.555	73	252	109	0.555	14	210	84	0.555	30	240	62
0.639	247	172	136	0.639	153	219	109	0.639	93	190	114	0.639	102	185	113
0.722	53	123	154	0.722	89	150	116	0.722	7	156	135	0.722	5	210	160
0.805	124	-8	191	0.805	179	154	152	0.805	97	130	161	0.805	88	171	169
0.889	52	-12	177	0.889	148	76	202	0.889	84	83	161	0.889	49	122	165
0.972	-8	-54	172	0.972	91	55	202	0.972	53	45	154	0.972	72	119	177

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1935.055	11	114	167	0.055	35	161	187	0.055	97	132	201	0.055	127	40	175
0.139	35	81	191	0.139	-35	177	180	0.139	-114	176	178	0.139	-106	151	176
0.222	21	118	174	0.222	-45	179	167	0.222	-65	209	138	0.222	-95	199	154
0.305	36	117	123	0.305	14	198	133	0.305	0	220	125	0.305	-29	257	117
0.389	2	140	106	0.389	8	195	90	0.389	8	249	91	0.389	5	265	88
0.472	49	145	75	0.472	123	188	79	0.472	117	227	74	0.472	58	268	80
0.555	68	102	123	0.555	116	144	81	0.555	151	175	83	0.555	215	228	132
0.639	60	121	134	0.639	94	91	136	0.639	153	103	100	0.639	249	187	141
0.722	3	151	139	0.722	0	39	138	0.722	4	62	115	0.722	82	118	146
0.805	22	111	163	0.805	20	37	162	0.805	92	51	143	0.805	77	31	158
0.889	-2	119	193	0.889	-24	26	187	0.889	60	50	158	0.889	2	-1	135
0.972	7	110	218	0.972	-14	101	179	0.972	-50	26	180	0.972	-127	-12	97
1939.055	54	28	153	0.055	13	29	153	0.055	4	39	185	0.055	7	10	181
0.139	-59	96	175	0.139	-93	35	177	0.139	15	40	178	0.139	4	44	186
0.222	-68	184	158	0.222	-86	106	161	0.222	-49	108	151	0.222	59	44	157
0.305	-4	205	143	0.305	-88	188	141	0.305	-44	124	149	0.305	62	115	129
0.389	0	247	77	0.389	74	239	95	0.389	0	125	131	0.389	2	96	124
0.472	32	252	86	0.472	17	272	98	0.472	-4	179	127	0.472	37	124	105
0.555	120	247	115	0.555	89	250	105	0.555	54	211	96	0.555	25	114	97
0.639	197	196	129	0.639	164	213	138	0.639	100	216	115	0.639	19	113	119
0.722	94	122	145	0.722	81	177	140	0.722	16	180	122	0.722	4	157	133
0.805	174	84	172	0.805	165	94	156	0.805	127	170	162	0.805	-6	167	138
0.889	138	57	160	0.889	118	48	190	0.889	115	106	193	0.889	16	176	167
0.972	72	-5	150	0.972	104	22	181	0.972	104	104	195	0.972	19	211	157
1943.055	7	210	169	0.055	32	182	157	0.055	128	83	190	0.055	148	-86	195
0.139	57	195	182	0.139	-30	209	177	0.139	-165	147	197	0.139	-208	-18	166
0.222	94	184	175	0.222	4	232	174	0.222	-104	235	172	0.222	-227	80	192
0.305	107	131	155	0.305	86	227	151	0.305	-57	304	145	0.305	-160	209	180
0.389	60	142	131	0.389	99	209	134	0.389	9	317	133	0.389	62	288	133
0.472	171	83	120	0.472	285	178	147	0.472	174	309	108	0.472	6	355	121
0.555	152	40	124	0.555	262	62	129	0.555	286	209	116	0.555	141	369	130
0.639	127	32	122	0.639	301	7	149	0.639	282	114	127	0.639	209	336	144
0.722	3	28	121	0.722	93	-80	143	0.722	16	7	172	0.722	21	217	174
0.805	-35	36	124	0.805	12	-87	137	0.805	265	-88	186	0.805	347	109	163
0.889	-84	93	119	0.889	-47	-52	159	0.889	107	-151	160	0.889	299	11	190
0.972	-70	141	142	0.972	-82	21	173	0.972	-3	-104	140	0.972	241	-58	188

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1947.055	2	-88	171	0.055	59	123	205	0.055	65	297	187	0.055	80	312	227
0.139	-8	-93	194	0.139	219	14	214	0.139	263	266	184	0.139	-5	394	198
0.222	-94	-35	212	0.222	206	-26	199	0.222	276	193	169	0.222	143	391	178
0.305	-151	88	162	0.305	117	-54	181	0.305	284	111	146	0.305	255	321	137
0.389	151	166	131	0.389	0	-14	152	0.389	56	34	114	0.389	35	252	124
0.472	-102	224	125	0.472	-37	22	130	0.472	246	-28	109	0.472	387	140	128
0.555	19	294	115	0.555	-35	45	136	0.555	162	-47	127	0.555	380	28	142
0.639	115	338	145	0.639	-47	129	163	0.639	44	-94	162	0.639	326	-73	172
0.722	92	311	176	0.722	54	185	163	0.722	56	-77	174	0.722	77	-126	202
0.805	264	279	191	0.805	-1	247	182	0.805	-119	9	194	0.805	9	-184	218
0.889	292	239	176	0.889	41	307	180	0.889	-143	150	218	0.889	-129	-153	223
0.972	278	173	216	0.972	91	335	168	0.972	-140	259	217	0.972	-201	6	177
1951.055	258	161	205	0.055	200	-105	214	0.055	5	-196	205	0.055	59	-61	195
0.139	-258	296	184	0.139	-260	96	191	0.139	-93	-154	180	0.139	82	-107	170
0.222	-124	422	162	0.222	-313	243	151	0.222	-221	-59	166	0.222	-49	-116	145
0.305	20	506	140	0.305	-158	385	137	0.305	-243	83	109	0.305	-85	-36	143
0.389	89	455	112	0.389	82	488	95	0.389	267	235	78	0.389	168	10	111
0.472	294	369	92	0.472	86	507	113	0.472	-161	364	94	0.472	-105	113	131
0.555	418	244	117	0.555	248	489	112	0.555	-11	454	135	0.555	-90	209	137
0.639	469	58	142	0.639	383	392	150	0.639	125	445	141	0.639	-2	287	189
0.722	98	-67	162	0.722	29	250	172	0.722	77	430	199	0.722	8	322	189
0.805	266	-193	198	0.805	412	64	180	0.805	303	256	195	0.805	37	338	226
0.889	126	-217	200	0.889	296	-88	197	0.889	303	98	216	0.889	111	332	224
0.972	-29	-185	228	0.972	179	-181	186	0.972	264	51	179	0.972	123	282	224
1955.055	81	197	193	0.055	27	343	225	0.055	215	286	211	0.055	205	95	158
0.139	193	149	197	0.139	107	339	169	0.139	-125	429	179	0.139	-239	234	165
0.222	222	103	178	0.222	224	290	180	0.222	-56	470	194	0.222	-193	333	145
0.305	158	45	150	0.305	232	244	149	0.305	94	497	141	0.305	-85	433	114
0.389	15	4	128	0.389	81	138	115	0.389	4	455	119	0.389	0	482	102
0.472	63	7	130	0.472	243	41	111	0.472	298	357	105	0.472	127	457	102
0.555	-25	-11	120	0.555	210	-35	136	0.555	375	209	122	0.555	295	404	104
0.639	-59	26	162	0.639	165	-75	171	0.639	371	83	144	0.639	379	332	133
0.722	114	80	214	0.722	9	-49	186	0.722	59	-52	165	0.722	78	204	185
0.805	-109	180	199	0.805	-64	15	214	0.805	127	-89	176	0.805	327	96	201
0.889	-110	229	237	0.889	-180	65	217	0.889	9	-99	195	0.889	251	3	184
0.972	-39	289	220	0.972	-225	171	214	0.972	-127	-5	187	0.972	118	-49	183

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1959.055	1	-55	152	0.055	1	2	150	0.055	4	188	200	0.055	8	324	174
0.139	-101	-70	119	0.139	39	-21	175	0.139	65	131	183	0.139	19	278	194
0.222	-139	60	85	0.222	1	35	142	0.222	59	159	145	0.222	87	284	187
0.305	-158	211	106	0.305	-86	75	141	0.305	39	164	134	0.305	118	264	170
0.389	88	325	79	0.389	97	155	104	0.389	5	146	147	0.389	63	227	132
0.472	-14	384	62	0.472	-100	181	104	0.472	33	155	119	0.472	167	154	119
0.555	99	396	94	0.555	-15	253	119	0.555	50	149	121	0.555	164	85	143
0.639	213	416	124	0.639	26	270	121	0.639	40	151	162	0.639	108	56	164
0.722	82	359	143	0.722	0	313	153	0.722	1	182	164	0.722	4	82	179
0.805	306	261	181	0.805	83	278	179	0.805	5	173	191	0.805	-21	94	207
0.889	267	168	163	0.889	120	259	188	0.889	-38	238	196	0.889	-83	145	203
0.972	174	54	182	0.972	115	244	173	0.972	-2	282	184	0.972	-117	214	201
1963.055	127	313	216	0.055	215	164	226	0.055	82	68	221	0.055	3	116	234
0.139	-79	359	214	0.139	-181	287	196	0.139	-178	127	216	0.139	-11	87	220
0.222	-27	374	195	0.222	-150	360	198	0.222	-196	244	190	0.222	-100	83	196
0.305	79	384	169	0.305	-65	472	166	0.305	-215	326	200	0.305	-114	165	183
0.389	82	355	130	0.389	3	448	141	0.389	139	382	160	0.389	135	203	175
0.472	256	280	138	0.472	197	425	112	0.472	-44	450	147	0.472	-98	254	161
0.555	309	172	147	0.555	255	332	136	0.555	96	440	182	0.555	-93	346	193
0.639	269	105	139	0.639	251	236	205	0.639	167	372	199	0.639	-9	349	219
0.722	63	19	174	0.722	76	166	206	0.722	26	301	200	0.722	9	336	219
0.805	53	-32	208	0.805	325	86	213	0.805	241	226	198	0.805	142	316	209
0.889	-42	41	128	0.889	108	56	213	0.889	235	176	201	0.889	137	278	210
0.972	-141	90	188	0.972	32	48	241	0.972	68	104	227	0.972	113	245	200
1967.055	1	220	224	0.055	27	290	241	0.055	73	293	233	0.055	182	204	245
0.139	65	173	189	0.139	6	310	187	0.139	-111	298	258	0.139	-152	299	232
0.222	8	146	153	0.222	43	272	208	0.222	6	371	221	0.222	-135	378	219
0.305	24	185	140	0.305	72	258	201	0.305	46	394	206	0.305	-56	441	181
0.389	2	144	127	0.389	6	230	161	0.389	34	356	158	0.389	0	462	174
0.472	20	160	115	0.472	52	214	154	0.472	162	320	170	0.472	129	414	171
0.555	75	170	155	0.555	119	180	159	0.555	189	257	161	0.555	225	340	179
0.639	21	195	163	0.639	79	153	194	0.639	179	169	165	0.639	261	258	202
0.722	1	199	187	0.722	16	180	195	0.722	0	133	204	0.722	29	150	210
0.805	-36	216	185	0.805	-37	157	180	0.805	49	109	214	0.805	168	102	204
0.889	-67	244	216	0.889	-107	239	205	0.889	-61	113	234	0.889	73	49	225
0.972	-60	304	247	0.972	-131	275	219	0.972	-98	145	252	0.972	-16	9	269

1971.055	132	63	272	0.055	11	18	276	0.055	66	113	241	0.055	37	211	246
0.139	-217	133	264	0.139	-38	39	256	0.139	128	130	220	0.139	125	142	229
0.222	-220	244	241	0.222	-103	131	240	0.222	49	109	210	0.222	85	152	242
0.305	-121	346	215	0.305	-119	246	191	0.305	5	135	200	0.305	49	176	203
0.389	50	437	220	0.389	80	349	177	0.389	36	211	153	0.389	3	172	184
0.472	43	492	163	0.472	12	389	156	0.472	-49	275	185	0.472	55	168	170
0.555	161	465	178	0.555	87	413	151	0.555	-9	311	194	0.555	84	199	187
0.639	261	426	218	0.639	90	403	214	0.639	30	351	207	0.639	68	213	194
0.722	82	297	227	0.722	58	369	235	0.722	4	350	180	0.722	7	221	197
0.805	208	226	209	0.805	216	313	220	0.805	119	315	209	0.805	72	263	221
0.889	164	175	220	0.889	212	279	222	0.889	105	290	221	0.889	38	277	231
0.972	123	90	237	0.972	227	177	227	0.972	128	253	271	0.972	33	246	274
1975.055	18	285	264	0.055	55	215	263	0.055	87	121	297	0.055	61	8	260
0.139	29	284	242	0.139	-67	263	269	0.139	-162	200	290	0.139	-118	98	310
0.222	86	303	237	0.222	-16	352	256	0.222	-132	294	287	0.222	-115	205	300
0.305	101	328	235	0.305	30	426	215	0.305	-71	405	263	0.305	-178	321	262
0.389	38	293	198	0.389	5	423	198	0.389	0	484	220	0.389	101	425	229
0.472	186	254	184	0.472	90	377	194	0.472	75	449	196	0.472	1	493	194
0.555	167	241	193	0.555	183	320	154	0.555	201	440	196	0.555	92	475	188
0.639	178	175	231	0.639	174	277	189	0.639	292	313	192	0.639	253	455	191
0.722	8	132	238	0.722	34	207	196	0.722	71	260	197	0.722	55	400	179
0.805	67	110	246	0.805	180	151	198	0.805	243	137	206	0.805	265	301	157
0.889	8	144	266	0.889	91	96	240	0.889	147	78	175	0.889	238	199	183
0.972	-34	200	263	0.972	-50	69	274	0.972	9	47	254	0.972	194	119	224

PRILOG II (DATE SU IZRAVNATE KOORDINATE X , Y ZA SVAKU GODINU POCEV OD 1900.0 U 0".001)

X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
8	18	-18	20	-114	-33	-147	-72	-49	-157	86	-125	98	-36	37	111
55	-24	0	31	-107	12	-175	-38	-109	-134	29	-160	51	-58	48	77
57	-44	20	29	-95	53	-187	4	-152	-103	-26	-170	16	-72	52	46
43	-57	38	27	-76	92	-181	49	-178	-67	-77	-160	-12	-78	53	19
31	-66	52	34	-51	127	-153	92	-189	-28	-121	-139	-37	-75	57	-3
12	-75	68	29	-13	155	-117	133	-190	13	-146	-107	-63	-66	62	-24
-11	-85	85	9	33	171	-77	170	-177	58	-154	-68	-88	-53	63	-43
-29	-97	94	-23	80	173	-27	198	-147	113	-155	-32	-107	-31	43	-62
-38	-106	104	-56	127	159	24	215	-104	160	-146	10	-116	-2	18	-80
-36	-110	123	-83	172	128	68	221	-54	184	-125	63	-115	32	2	-95
-36	-104	125	-114	203	85	111	213	-4	191	-94	110	-108	61	-7	-101
-42	-90	108	-145	213	37	151	188	44	182	-55	145	-97	82	-16	-95
-54	-73	85	-160	204	-8	186	146	85	164	-8	163	-77	96	-31	-83
-66	-52	58	-161	178	-51	207	90	120	137	38	167	-55	104	-56	-66
-72	-29	29	-151	140	-90	208	25	147	104	76	160	-35	107	-95	-43
-76	-5	-2	-138	94	-123	193	-41	162	69	104	142	-19	106	-130	-15
-76	14	-35	-126	44	-144	165	-101	165	32	121	114	-6	105	-154	19
-75	25	-68	-114	-7	-144	127	-145	158	-4	135	77	5	109	-150	59
-65	27	-95	-97	-59	-129	78	-169	144	-41	139	36	15	116	-130	101
-41	18	-111	-71	-107	-102	16	-170	126	-82	130	-4	25	124	-104	140
-73	169	-272	-11	-201	-251	19	-290	211	-98	104	123	-87	129	-194	3
-38	186	-282	53	-234	-192	-67	-278	156	-137	111	90	-84	143	-204	35
6	193	-265	123	-269	-112	-135	-252	104	-168	122	69	-50	145	-193	91
54	190	-229	189	-295	-25	-185	-206	48	-182	130	46	3	144	-168	155
101	176	-178	240	-308	59	-220	-134	-16	-173	126	12	64	148	-134	214
146	148	-100	277	-293	146	-242	-53	-68	-149	124	-19	120	144	-82	264
184	107	-3	297	-248	227	-249	30	-107	-114	124	-43	162	131	-17	296
208	53	92	295	-176	280	-228	112	-138	-73	123	-59	178	116	44	289
222	-2	178	271	-85	315	-192	191	-153	-31	118	-64	183	93	104	262
229	-46	247	226	13	336	-151	265	-148	10	103	-56	191	59	168	233
219	-91	296	164	108	339	-92	317	-136	60	73	-56	189	19	220	194
189	-135	322	92	192	320	-19	342	-119	114	33	-67	176	-22	254	146
147	-167	324	23	257	269	56	342	-100	149	-2	-79	149	-61	276	93
92	-189	305	-49	300	190	125	322	-81	168	-36	-82	110	-98	278	28
28	-199	265	-130	320	93	181	284	-61	177	-67	-68	65	-137	257	-52
-43	-200	201	199	316	-3	226	229	-38	186	-91	-41	12	-165	216	-135
-115	-191	115	-249	289	-89	257	159	-10	195	-105	-5	-44	-171	159	-206
-178	-167	18	-281	244	-160	276	80	31	197	-101	33	-99	-142	96	-236
-225	-128	-75	-293	182	-220	275	7	69	186	-91	71	-145	-92	23	-234
-252	-72	-150	-285	106	-271	253	-50	91	157	-86	104	-174	-37	-57	-210

PRILOG II (NASTAVAK)

-123	-192	65	-244	122	-103	98	94	-17	139	-82	-53	-19	-27	-9	-85
-167	-174	-4	-254	72	-131	90	77	13	169	-99	3	-75	19	-50	-62
-190	-115	-53	-234	18	-112	73	55	28	171	-105	47	-106	46	-84	-36
-200	-34	-95	-191	-26	-70	60	46	38	166	-92	77	-113	76	-107	0
-205	55	-137	-134	-47	-26	61	63	51	172	-57	90	-97	124	-121	49
-191	128	-155	-68	-59	11	67	55	84	169	-10	106	-67	168	-113	105
-156	181	-145	2	-68	44	74	27	132	151	45	125	-26	199	-86	161
-95	236	-119	67	-78	81	76	22	169	112	103	141	27	216	-58	211
-27	-279	-86	133	-85	120	74	20	194	68	155	134	82	214	-23	244
35	299	-57	201	-87	157	66	3	206	31	192	92	130	188	21	253
98	286	-18	238	-67	170	56	-21	208	-7	220	53	177	143	73	247
160	241	31	243	-30	162	42	-41	201	-44	240	24	220	89	130	235
223	172	82	233	10	155	19	-35	191	-83	245	-2	249	49	188	234
274	96	125	210	43	146	-8	-13	172	-113	234	-33	263	17	231	219
305	32	151	173	64	132	-32	14	141	-129	206	-75	259	-12	248	174
310	-26	161	126	77	113	-51	33	104	-137	168	-111	242	-46	249	131
291	-81	161	75	83	95	-63	42	63	-141	126	-132	212	-79	239	97
252	-144	161	34	90	91	-68	46	26	-146	91	-123	171	-83	218	62
198	-196	159	-8	96	95	-63	62	-11	-137	60	-98	117	-81	184	30
135	-223	152	-57	99	100	-45	99	-52	-103	29	-68	48	-91	133	6
81	-12	96	21	42	-6	57	80	-6	26	-27	118	-80	19	-91	8
34	-21	68	-5	38	-37	49	76	-26	51	-30	119	-104	64	-118	58
-6	-13	26	-38	7	-43	68	71	-34	74	-36	125	-116	124	-121	110
-37	13	-7	-52	-27	-26	79	62	-35	92	-34	136	-110	173	-115	152
-57	59	-13	-27	-47	11	58	51	-29	103	-14	152	-82	192	-114	173
-61	94	-13	14	-50	53	51	54	-14	108	21	171	-48	198	-97	205
-53	113	-8	56	-40	88	61	71	4	113	63	189	-10	202	-64	246
-47	135	10	80	-34	105	67	91	19	138	93	200	31	222	-23	271
-40	156	27	90	-26	116	73	107	29	151	118	197	74	236	28	280
-25	172	28	89	-11	132	79	116	36	127	140	172	115	225	92	275
3	182	33	90	11	145	84	114	45	113	155	136	157	210	155	260
42	187	46	92	37	152	88	102	57	109	161	93	193	191	208	236
92	195	74	88	61	159	94	80	73	90	162	53	207	156	241	205
131	193	99	75	83	157	96	56	75	71	150	19	201	110	256	170
145	167	105	57	100	138	88	37	49	63	121	-4	180	61	252	131
147	141	110	48	101	117	76	35	19	63	80	-16	148	30	210	75
143	121	113	47	88	103	61	43	-6	70	33	-19	107	21	142	13
139	107	104	36	77	113	50	32	-16	85	-4	-28	58	24	88	-15
130	88	84	23	68	117	36	18	-19	101	-33	-29	5	20	44	-29
116	53	56	16	65	95	16	14	-23	113	-56	-13	-47	-5	5	-43

PRILOG II (NASTAVAK)

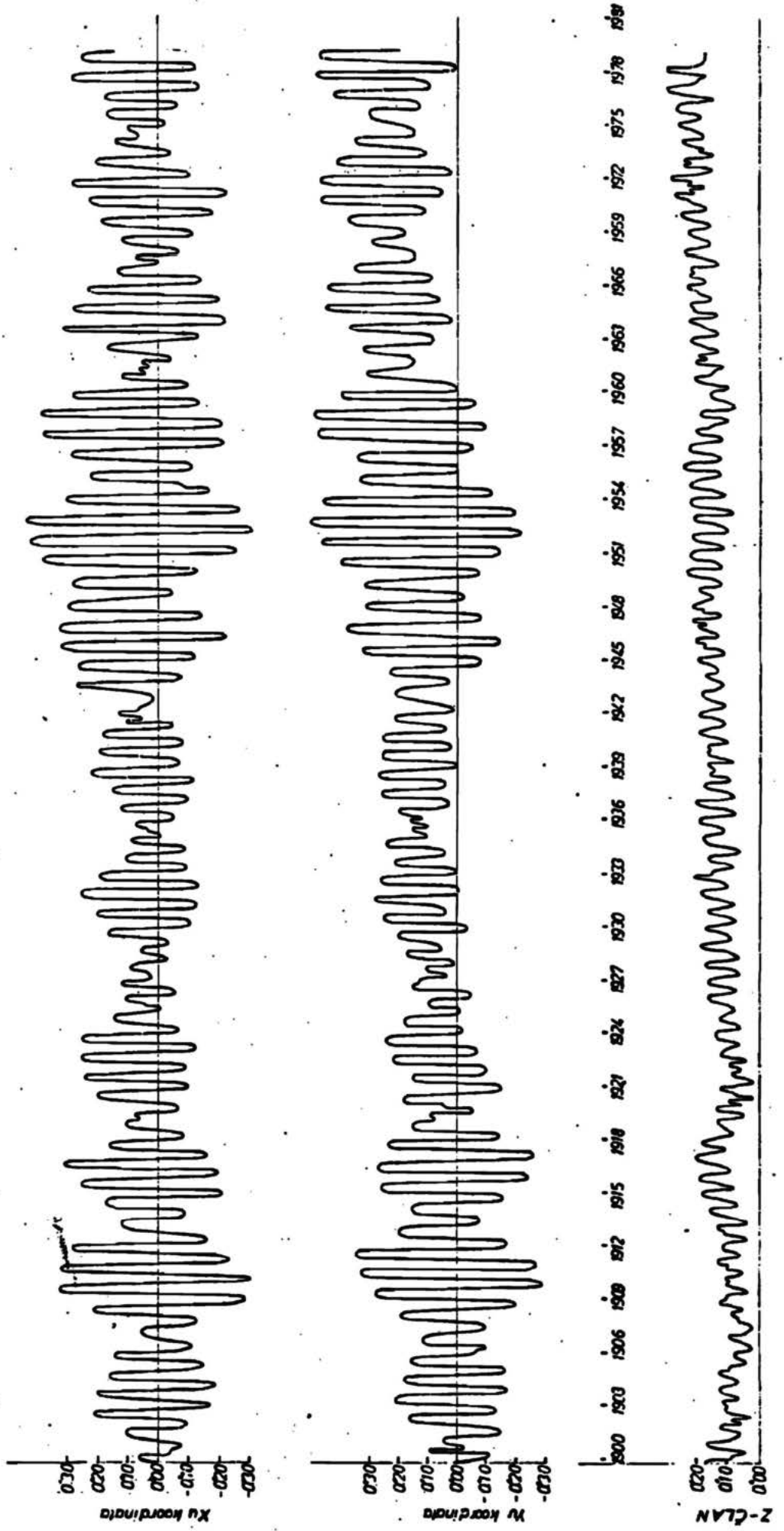
-42	-35	48	31	34	39	39	115	-9	128	-47	108	-78	27	-100	-3
-94	-5	-16	10	0	37	8	110	-27	154	-87	133	-118	45	-72	24
-140	28	-59	16	-30	48	12	97	-37	169	-109	157	-123	95	-62	63
-163	70	-85	38	-57	65	25	91	-41	178	-109	180	-111	151	-60	110
-151	125	-99	65	-82	84	26	104	-42	181	-83	200	-97	190	-61	159
-120	181	-103	98	-100	101	28	115	-27	187	-46	213	-72	224	-45	191
-85	226	-100	134	-109	114	32	121	4	195	-6	223	-40	251	-17	209
-74	248	-91	167	-103	130	33	132	32	198	32	238	-21	265	-5	230
-62	254	-74	196	-84	154	36	141	64	197	68	246	2	269	5	247
-27	252	-48	218	-57	187	45	142	103	191	103	236	44	268	23	254
18	251	-18	223	-18	215	56	129	122	175	132	213	116	256	62	255
68	250	16	214	27	229	65	110	120	148	150	178	197	234	114	247
118	234	58	201	70	215	66	113	108	116	155	136	241	209	163	222
159	208	90	185	97	199	64	126	88	82	144	97	244	177	196	185
183	174	100	166	96	202	62	140	63	53	121	71	208	136	200	143
190	154	101	149	90	194	48	137	39	35	104	57	152	87	193	110
183	144	98	130	83	173	26	120	19	30	94	52	90	40	177	87
168	113	92	104	70	146	10	114	-2	28	77	49	31	10	158	66
143	79	81	77	59	125	2	112	-16	41	41	43	-28	-7	130	43
105	56	62	53	58	119	1	113	-21	76	-20	29	-88	-13	91	14
43	6	91	25	86	66	29	211	-59	156	-101	40	-58	-105	186	-72
-9	18	68	32	57	28	42	211	-39	179	-127	78	-135	-85	101	-90
-58	27	37	38	28	24	52	205	-33	198	-150	117	-187	-53	33	-97
-89	47	2	55	18	35	64	195	-26	214	-154	161	-217	-7	-24	-87
-93	86	-33	88	40	46	81	184	-8	227	-129	212	-227	53	-75	-52
-93	113	-49	111	57	71	96	165	28	232	-96	260	-209	124	-118	5
-91	180	-42	122	62	101	111	143	81	229	-56	298	-166	196	-148	74
-83	219	-20	126	54	107	136	138	147	222	8	318	-113	254	-158	130
-59	247	0	136	44	107	160	128	212	209	80	322	-58	302	-149	180
-12	266	3	163	38	116	170	100	261	185	150	313	-8	341	-118	225
39	267	18	190	32	118	168	69	281	139	217	278	53	366	-63	263
87	254	48	210	26	114	157	45	280	79	270	221	123	372	6	294
133	234	78	216	23	114	143	33	292	32	290	161	179	358	70	321
167	211	102	211	21	122	116	29	282	-13	298	99	233	319	126	333
181	185	115	194	20	143	74	26	223	-59	312	34	293	253	175	322
179	148	122	179	12	158	22	28	133	-86	303	-29	333	183	220	304
165	103	126	165	1	166	-29	38	35	-89	265	-85	346	116	259	282
142	67	123	138	4	173	-66	64	-23	-74	188	-126	331	54	283	258
120	42	117	113	13	184	-83	97	-56	-42	100	-142	299	0	291	229
107	27	108	98	19	201	-76	128	-77	-1	22	-125	256	-42	285	194

PRILOG II (NASTAVAK) 4

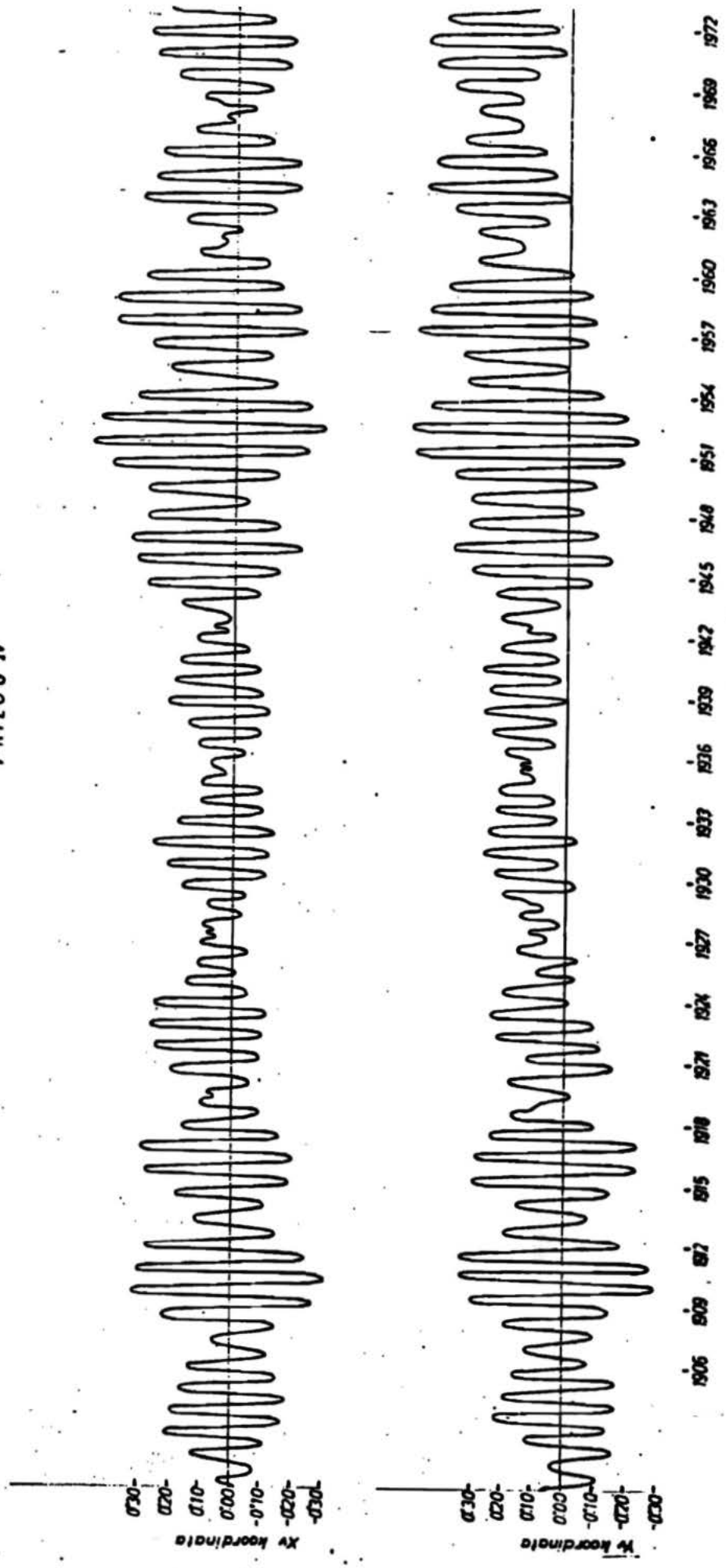
273	159	116	325	-125	279	-229	54	-91	-166	143	-194	231	6	146	254
258	126	163	305	-90	317	-258	149	-182	-104	69	-198	173	-49	172	207
240	64	218	284	-44	361	-266	235	-243	-3	-22	-182	118	-90	190	171
223	12	262	255	18	393	-239	317	-281	109	-112	-144	58	-113	203	142
209	-21	278	214	101	395	-169	395	-299	206	-188	-88	-11	-116	214	114
176	-42	283	167	180	373	-81	459	-263	297	-234	-15	-62	-92	201	82
124	-49	282	117	248	332	13	496	-187	378	-253	72	-95	-50	168	49
65	-35	272	68	303	288	110	489	-124	446	-265	164	-134	-15	139	22
11	-12	261	24	346	235	200	451	-53	492	-251	254	-152	28	108	6
-24	9	250	-11	377	171	272	399	40	510	-195	335	-131	84	75	2
-38	28	220	-35	391	103	344	331	140	510	-117	400	-106	145	32	-3
-39	49	169	-52	385	35	411	249	238	491	-26	445	-82	203	-13	-7
-44	88	102	-76	360	-27	454	147	326	445	64	461	-43	254	-45	5
-50	132	31	-93	306	-79	462	45	392	377	153	455	-11	292	-72	30
-51	170	-31	-87	221	-118	425	-40	426	290	238	436	-4	316	-99	66
-37	207	-81	-54	122	-155	361	-118	433	188	290	371	10	331	-114	114
-8	244	-116	4	21	-181	279	-182	411	78	308	272	36	339	-116	167
19	282	-137	82	-68	-176	195	-214	356	-22	311	174	74	339	-117	206
47	313	-146	163	-139	-131	105	-219	285	-104	301	97	107	326	-100	240
77	329	-143	230	-188	-44	9	-203	214	-163	276	55	124	296	-54	276
18	311	-229	208	-157	23	78	-56	138	33	94	224	1	300	-125	249
95	338	-214	283	-203	91	6	-64	83	1	66	190	13	315	-125	303
120	345	-174	364	-232	168	-58	-70	53	-16	59	157	19	302	-107	340
140	334	-123	431	-235	245	-107	-49	32	-12	61	139	33	285	-77	363
188	309	-73	467	-210	310	-136	19	7	14	59	148	65	282	-42	374
223	279	-4	488	-161	372	-153	107	-30	45	52	158	95	277	8	382
241	244	78	496	-97	427	-154	199	-74	77	42	162	119	267	70	384
262	189	152	482	-33	465	-127	276	-97	117	36	156	144	248	133	373
271	127	219	445	32	480	-84	335	-104	154	33	150	163	217	193	346
257	66	278	387	102	469	-36	372	-100	178	34	151	170	174	242	302
237	12	331	310	188	445	24	392	-72	208	39	151	171	129	282	245
215	-32	371	223	280	411	91	401	-28	242	46	149	163	90	305	185
189	-62	382	140	346	369	161	411	8	265	47	151	137	66	295	135
150	-72	357	59	382	313	223	407	40	282	47	157	102	60	257	88
91	-59	296	-18	386	241	269	378	71	301	48	169	62	70	196	37
19	-31	220	-70	368	168	297	331	87	301	34	175	22	82	128	-4
-57	5	139	-94	335	102	306	272	91	284	7	180	-17	95	62	-22
-130	39	63	-103	291	42	292	211	106	271	-19	206	-56	118	1	0
-188	80	-14	-85	232	-5	255	149	118	259	-30	241	-88	152	-59	39
-220	139	-92	-37	156	-37	199	84	115	247	-15	273	-112	196	-119	74

PRILOG II (NASTAVAK)

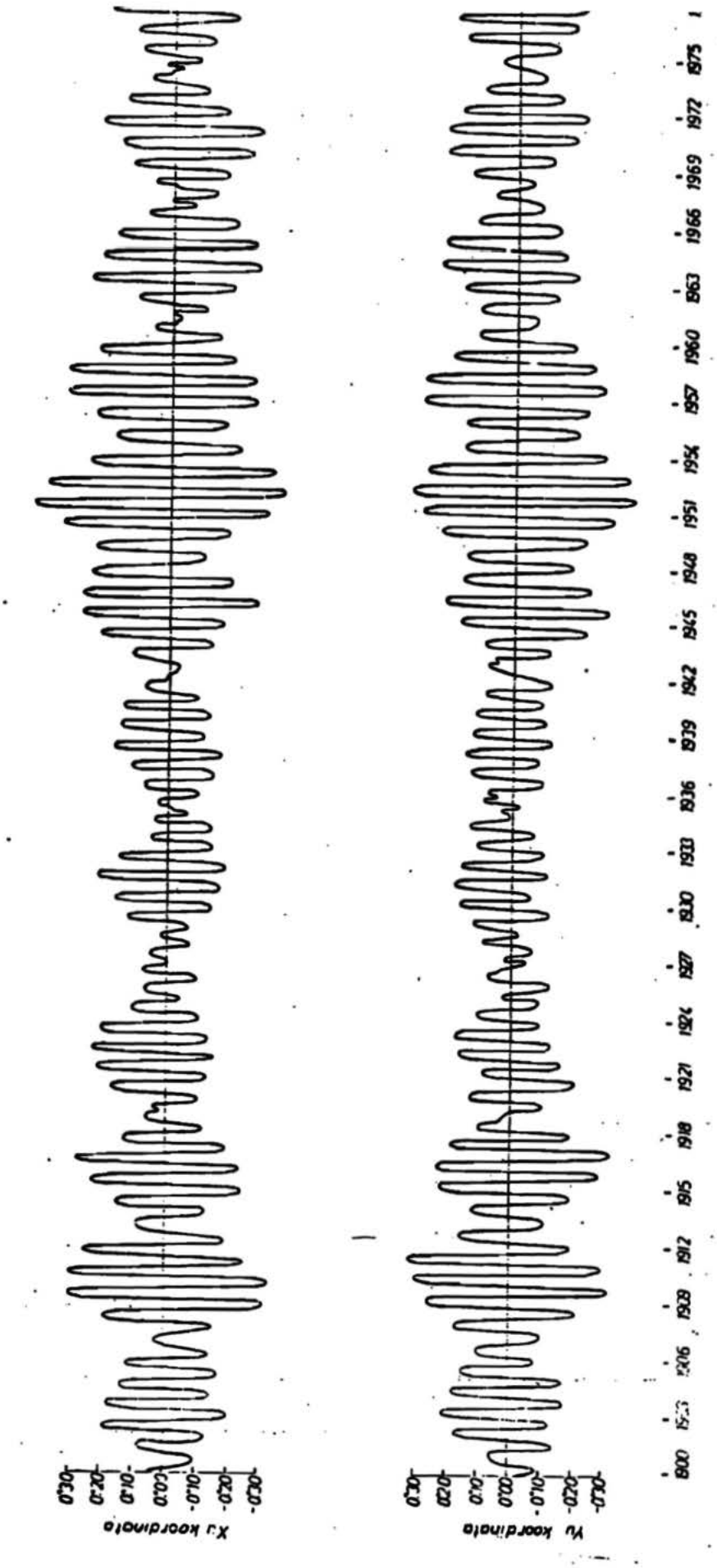
-171	114	-10	50	62	102	103	237	-51	299	-114	282	-133	161	-58	23
-205	162	-75	65	31	108	86	219	-31	299	-93	289	-168	201	-125	54
-204	227	-135	96	5	96	71	194	-10	303	-95	296	-171	253	-184	95
-185	292	-179	145	-28	85	52	170	12	301	-83	313	-158	308	-221	149
-162	347	-199	210	-74	86	27	156	35	285	-33	351	-141	359	-223	213
-125	405	-211	271	-105	110	15	160	55	269	13	380	-110	404	-191	278
-74	456	-210	320	-119	152	15	172	68	257	50	391	-68	439	-136	340
-9	466	-181	360	-128	182	7	163	68	243	95	379	-25	458	-87	399
69	454	-131	397	-127	211	4	152	62	230	134	356	28	457	-37	448
156	434	-69	434	-114	245	17	154	61	218	158	332	97	432	18	480
219	395	5	451	-102	289	41	161	80	201	177	299	164	394	84	489
251	340	81	441	-89	333	62	171	107	182	189	259	219	347	154	474
260	283	137	409	-50	351	50	183	100	166	187	210	251	296	220	449
263	228	179	365	4	351	28	194	67	159	165	166	259	240	266	403
272	181	214	320	63	343	13	198	17	165	121	139	244	180	280	335
265	135	238	275	111	331	-7	204	-20	166	81	121	213	134	261	275
234	94	248	233	139	316	-33	214	-44	168	44	110	172	102	220	232
173	67	241	196	144	296	-54	231	-79	200	-9	108	121	69	189	197
106	52	202	161	136	274	-67	255	-109	239	-60	115	64	40	160	160
50	46	121	126	121	255	-64	285	-122	266	-95	133	7	18	129	110
75	61	207	155	132	239	10	261	-44	202	-63	82	-16	26	0	0
10	24	176	122	137	209	-8	276	-57	216	-96	114	-61	17	0	0
-27	21	147	117	133	172	5	283	-65	239	-134	158	-94	49	0	0
-55	48	114	119	119	147	35	289	-58	276	-155	211	-114	107	0	0
-89	101	71	113	95	147	67	300	-31	328	-145	269	-124	173	0	0
-112	168	34	116	72	158	90	314	-2	380	-117	335	-144	244	0	0
-119	239	6	135	56	171	103	323	23	419	-77	399	-165	314	0	0
-104	304	-21	173	59	174	123	311	36	431	-36	451	-143	380	0	0
-67	354	-40	218	65	172	146	290	52	419	8	477	-95	436	0	0
-13	383	-47	258	62	171	171	268	81	392	56	470	-31	479	0	0
38	402	-37	289	67	179	181	252	125	360	120	454	31	489	0	0
76	412	-14	312	76	194	177	237	169	326	192	431	90	481	0	0
90	411	9	336	76	205	176	206	179	297	253	376	189	466	0	0
106	399	32	351	78	213	164	171	169	266	288	316	249	447	0	0
140	379	52	352	88	221	131	141	154	227	286	270	256	414	0	0
177	352	79	340	88	237	98	120	158	189	271	214	259	366	0	0
208	322	107	320	75	259	68	112	168	153	244	151	262	310	0	0
219	296	113	303	57	270	34	126	134	118	194	105	253	247	0	0
221	259	113	285	40	270	3	152	72	89	126	71	232	188	0	0
223	204	122	264	29	259	-24	182	-5	73	48	47	205	140	0	0



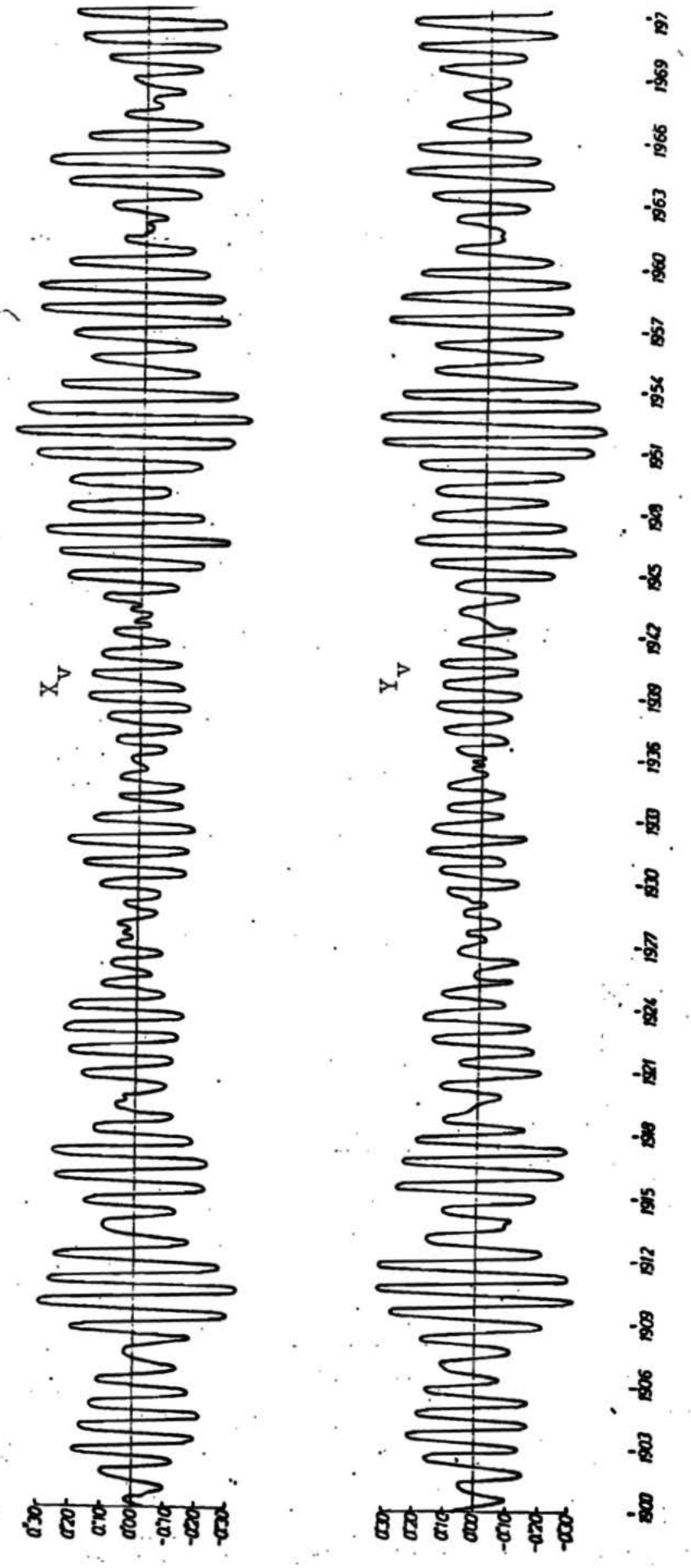
Prilog III. Sirove koordinate pola i z-član.


PRILOG IV
Xy koordinata
Y koordinata
1906 1909 1912 1915 1918 1921 1924 1927 1930 1933 1936 1939 1942 1945 1948 1951 1954 1957 1960 1963 1966 1969 1972
IZRAVNATI PODACI

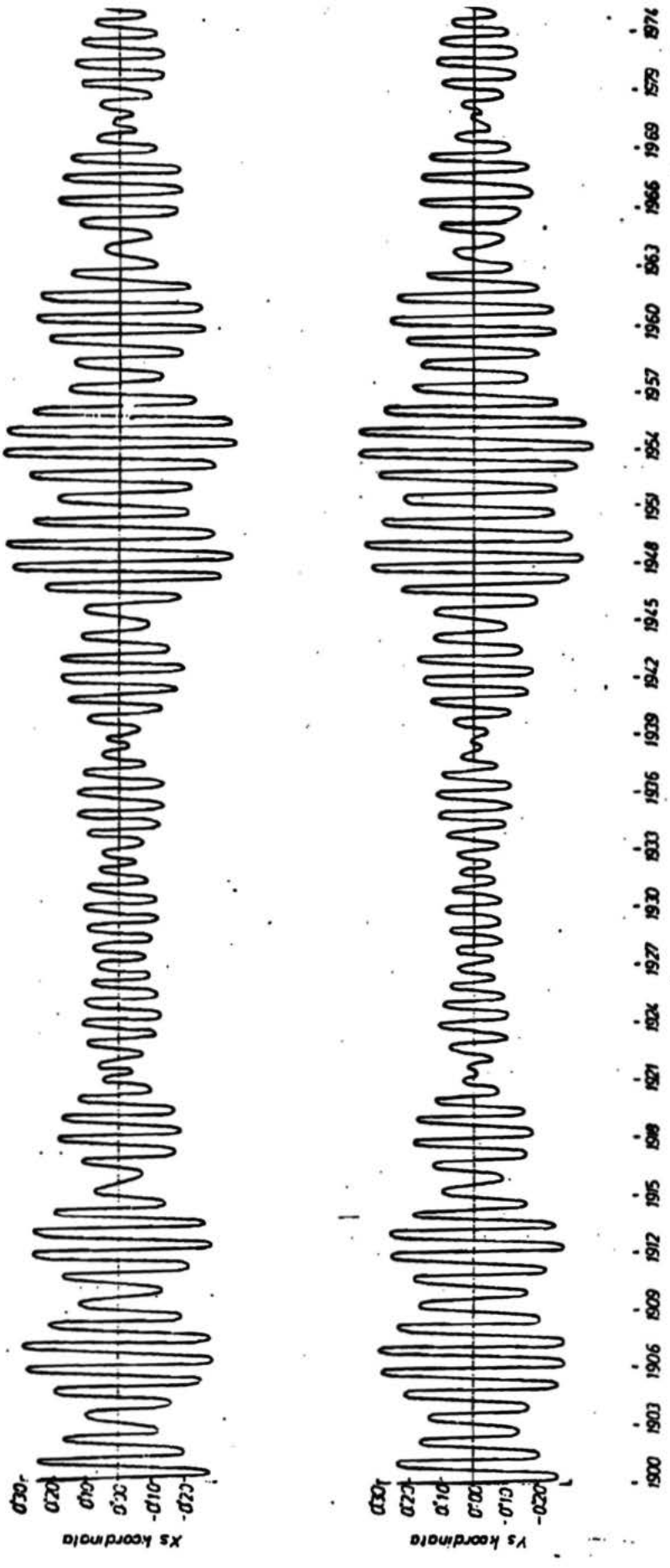
Prilog IV.



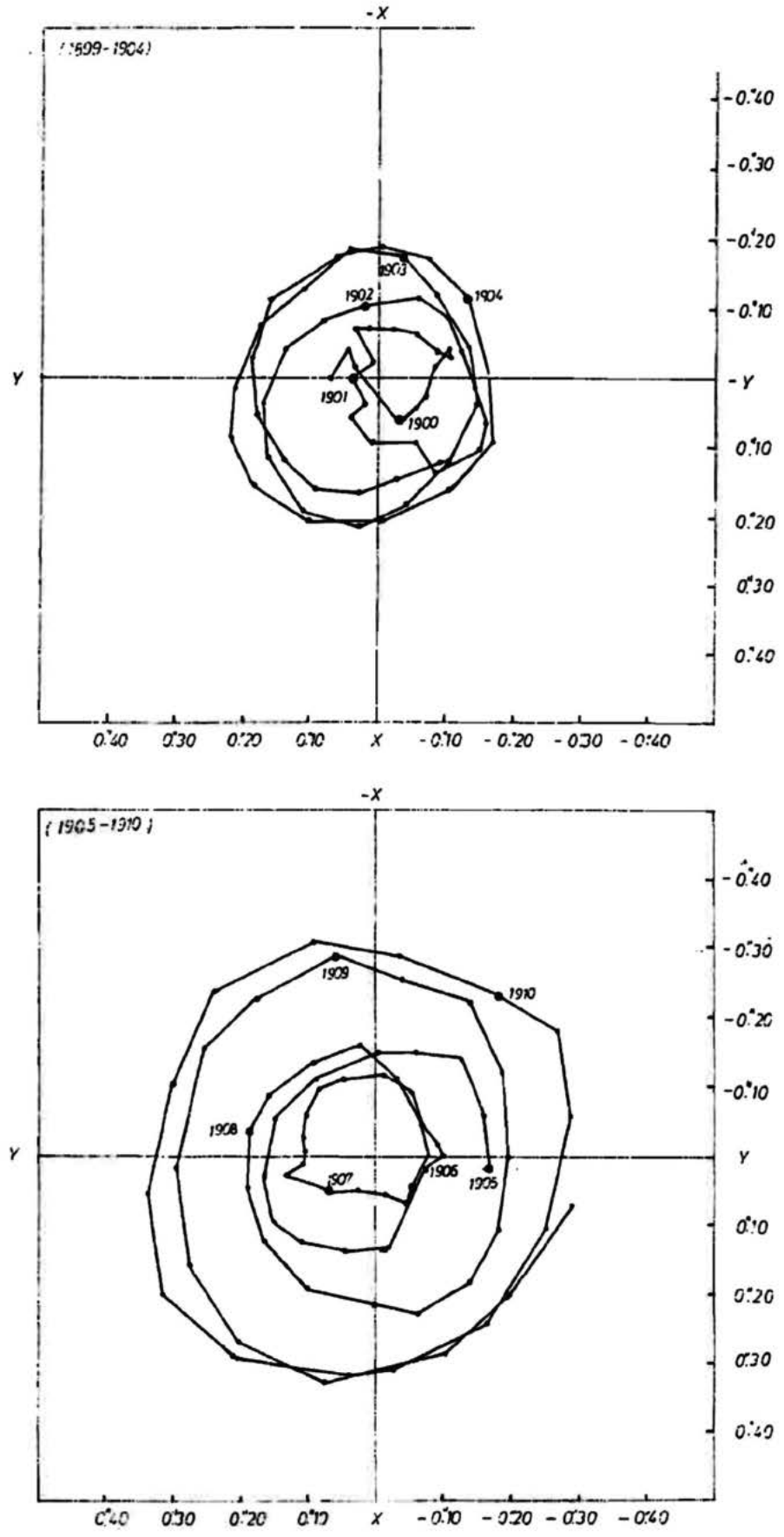
Prilog V. Sirove koordinate pola bez sekularnog člana.



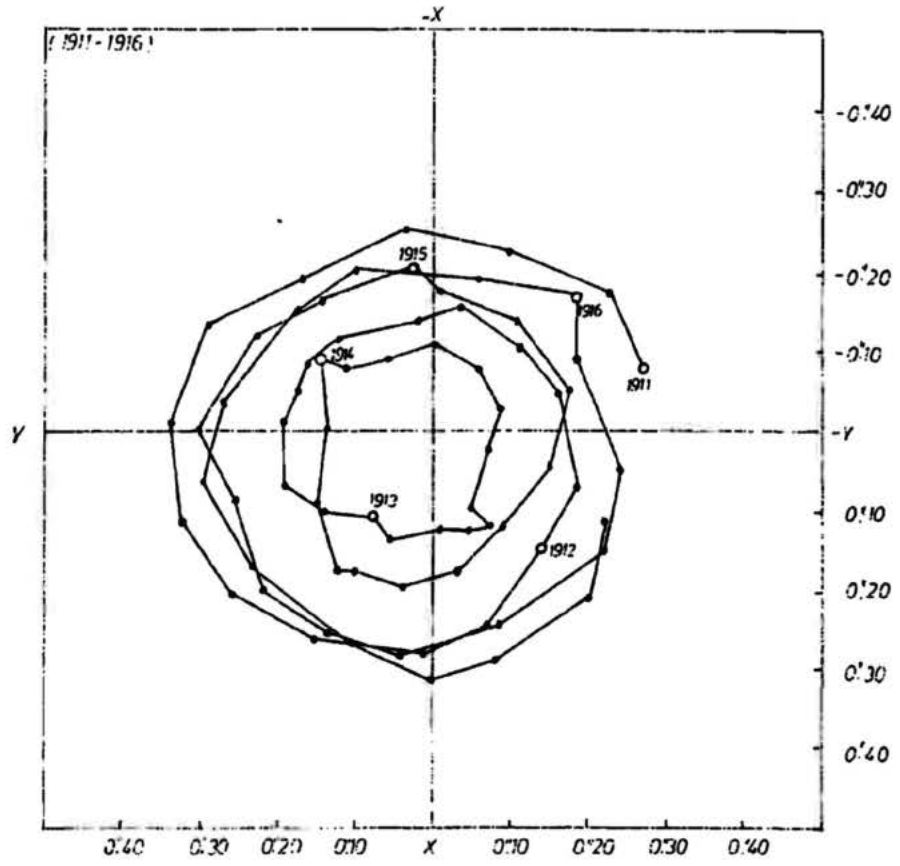
Prilog VI. Izravnate koordinate pola bez sekularnog člana.

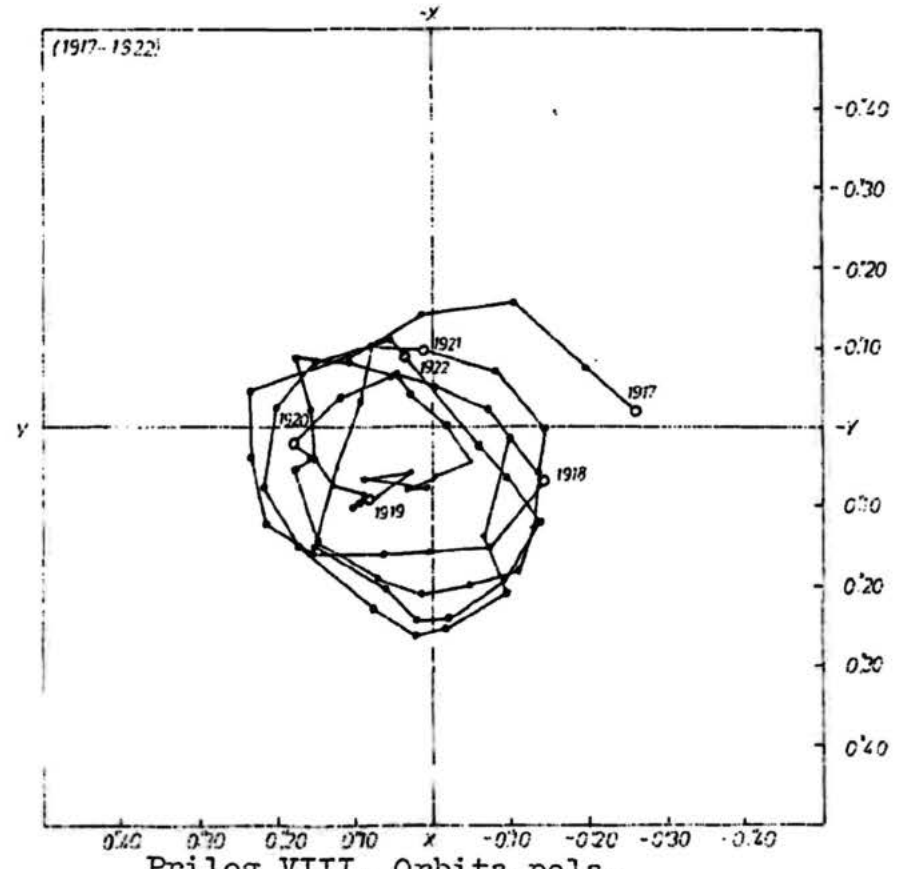


Prilog VII. Generisane koordinate pola.



Prilog VIII. Orbita pola.





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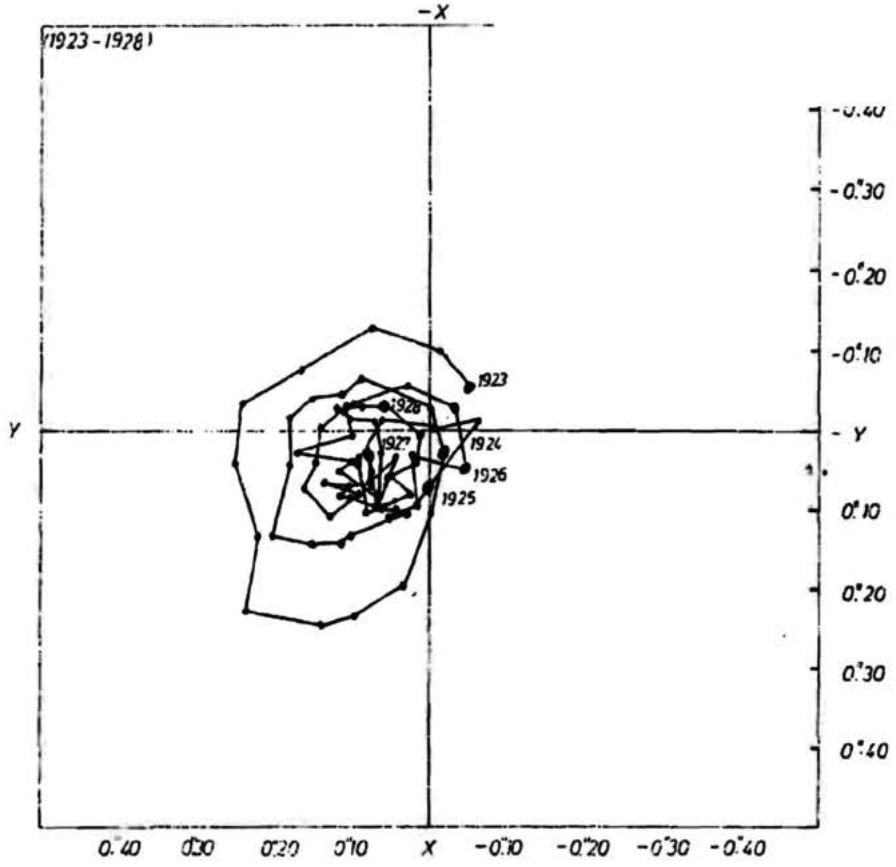

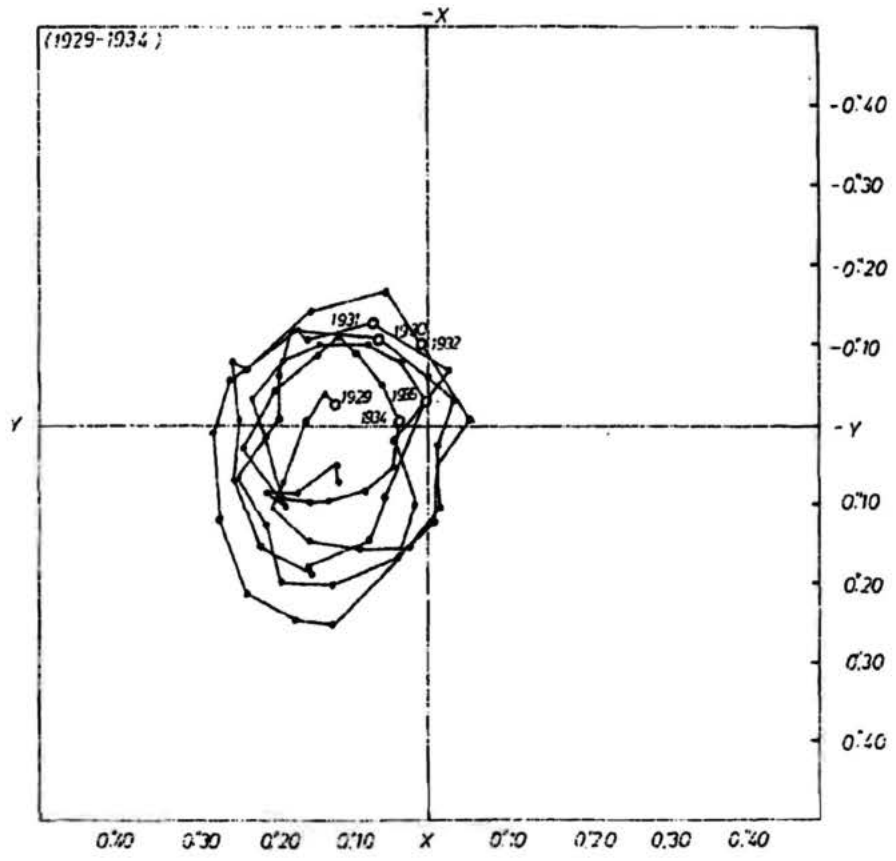

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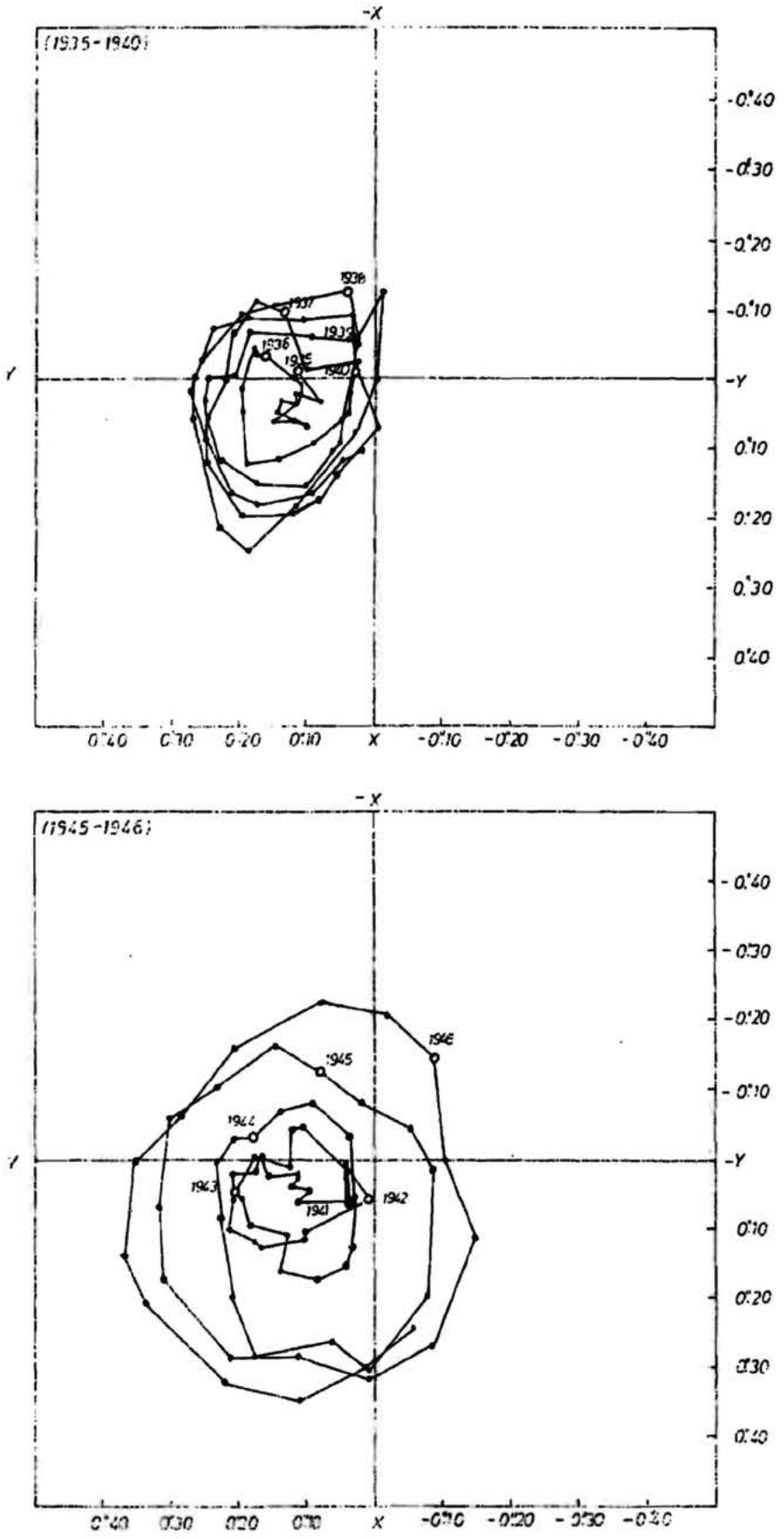

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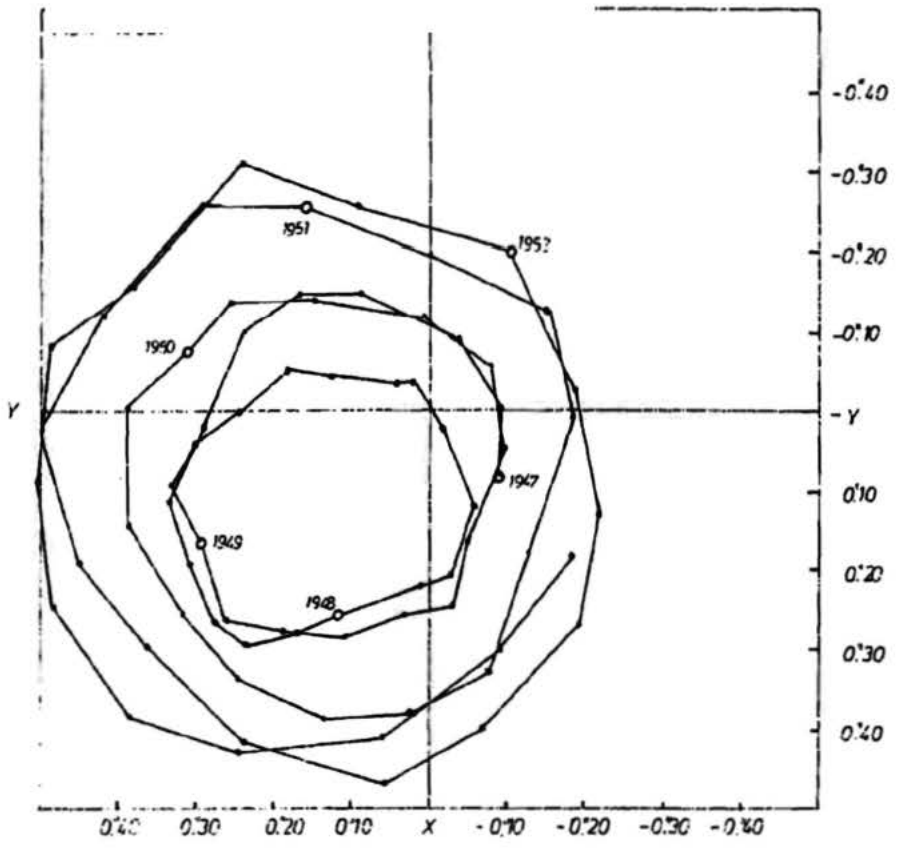

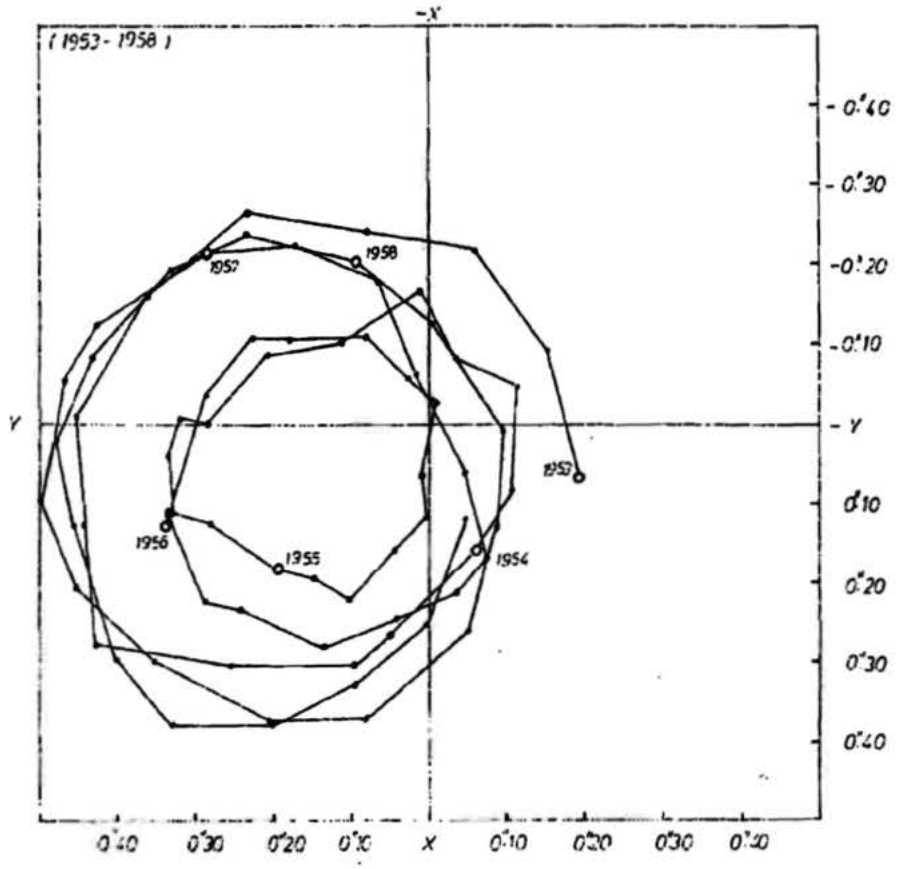

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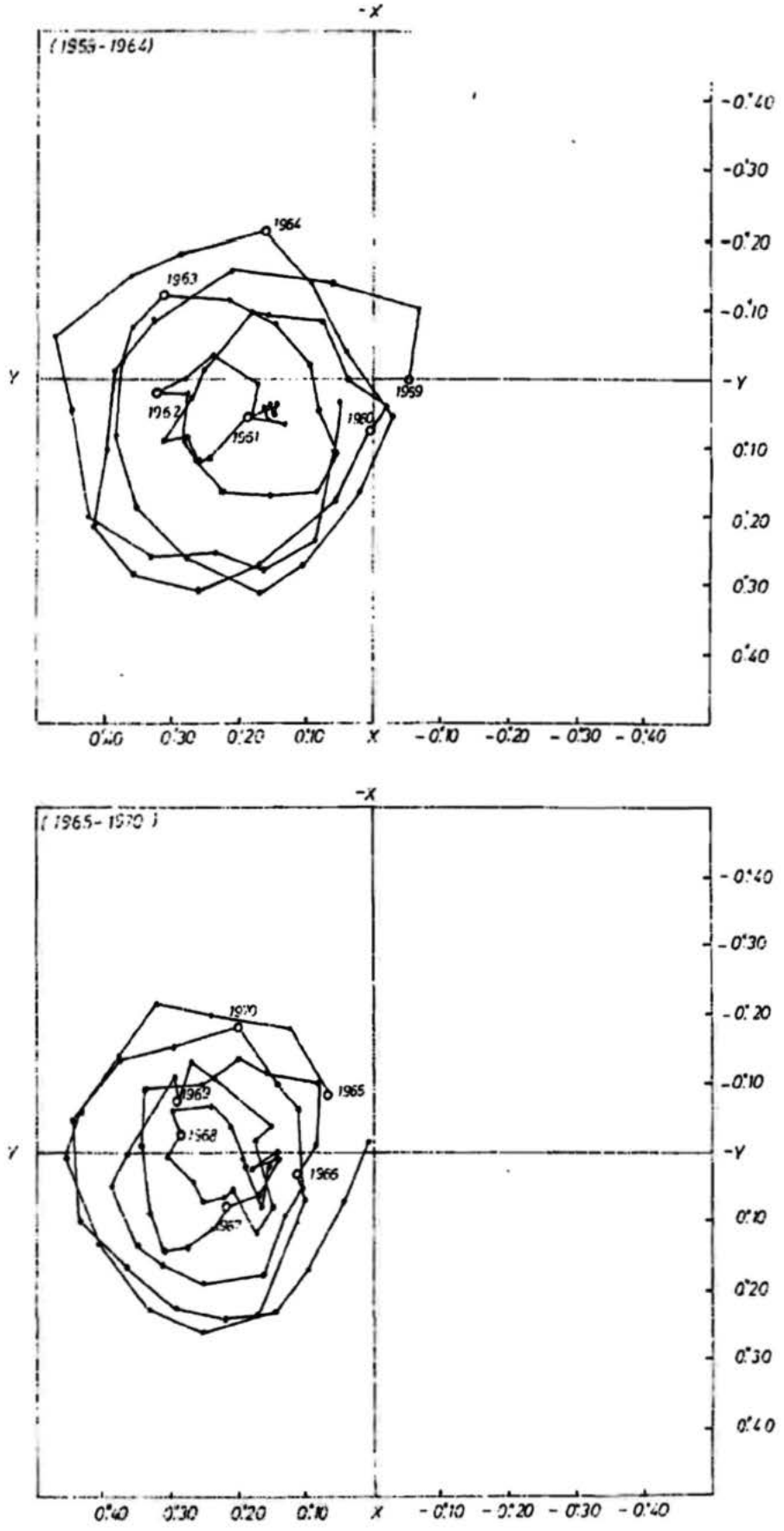

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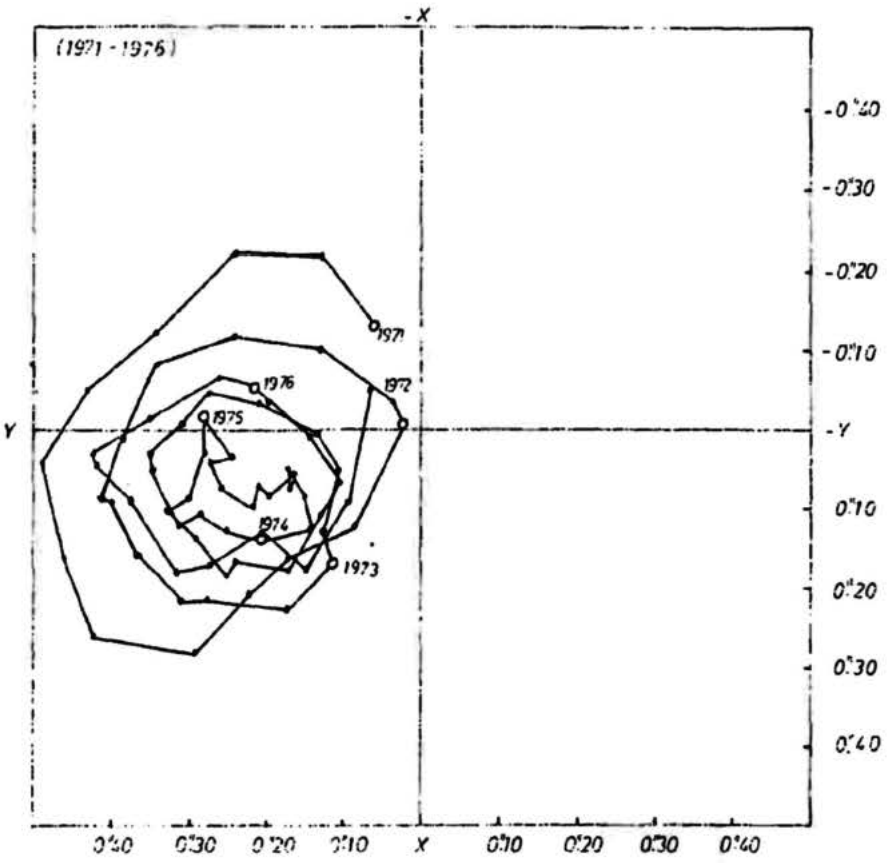

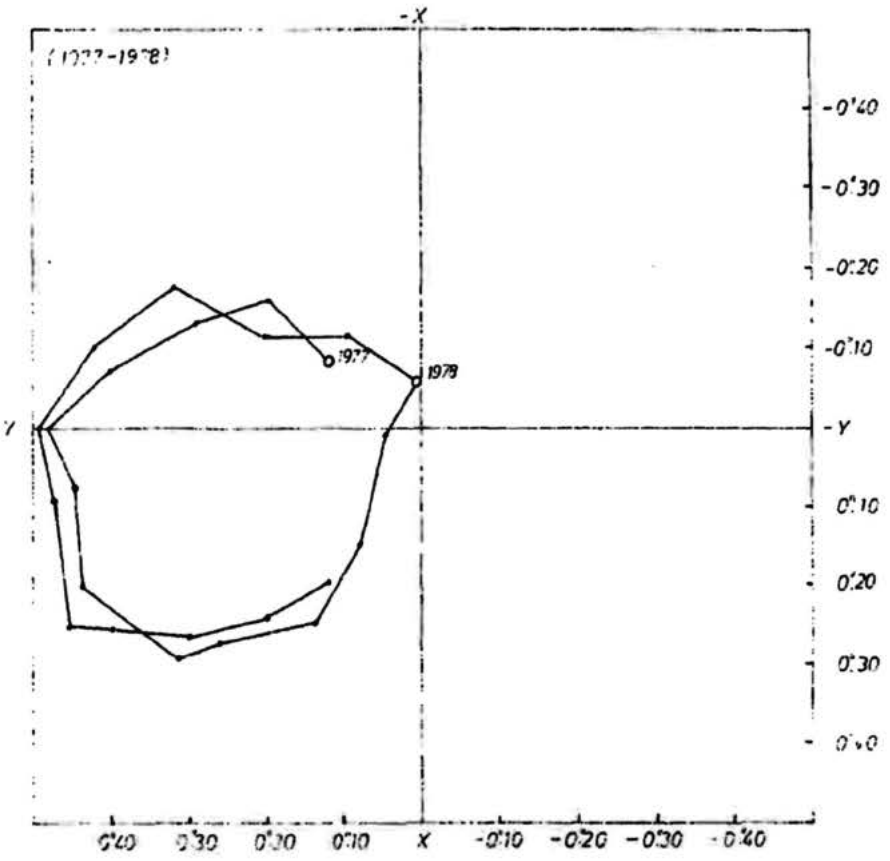

Prilog VIII. Orbita pola.

PART II

Astronomy

Overview

This book is the second part of collected works of Nadežda Pejović, a retired professor of astronomy at the Faculty of mathematics, University of Belgrade. Papers in this collection are related to astronomy.

Professor Pejović is an author or a coauthor of 41 scientific papers. Her interest in astronomy includes several disciplines. The first group of papers which also characterize the first period of her scientific work, are in geodynamics and most of them are related to the change of Earth rotation and polar movement caused by the tidal and atmospheric effects. A particularly notable papers in this group are those she wrote jointly with a leading Czech astronomer Yan Vondrak. The second group of papers are from the middle period of her scientific work and are related to astrometry and astronomical catalogues. Most of these papers she wrote with her doctoral student and later a collaborator Goran Damljanović, an astronomer from the Astronomical Observatory in Belgrade. The third group are from the last period of her scientific work and are related to cosmology. These papers are mainly coauthor works with Žarko Mijajalović, professor of mathematics at the Faculty of Mathematics in Belgrade. She also had excursions to other areas such as artificial satellites and galactic astronomy. There she had coauthor works with professor Stevo Šegan and professor Slobodan Ninković. The contemporary science is characterized by deep specialization, so the scientific work of professor Pejović is decorated with today rare scientific universality and broadness. Particularly if we know that she has a great number of papers on history of astronomy.

5 Preliminary analysis of the secular and main harmonic terms in the polar motion

N. Pejović
Publ. Dept. Astr. Univ. Beograd
No. 12, pp. 41-48, 1983.

Linear secular terms have been identified in both polar coordinates, independently of whether the latter have been derived from raw or smoothed observational data. After elimination, amplitudes and phases of the main harmonic terms in the polar motion have been determined. Our results back up assumption Chandler's cycle being non-unique. Instead of one single peak, two close ones have unequivocally been identified within that cycle in spectra of polar coordinates. Their periods are: 1.171 and 1.198 years, respectively. In the Kimura's z-term, besides the well known annual cycle, a seml-annual one has been identified, whose amplitude is 3 to 4 times weaker.

Publications of the Department of Astronomy — Beograd, № 12, 1983.

UDC 521.93 OSP

PRELIMINARY ANALYSIS OF THE SECULAR AND MAIN HARMONIC TERMS IN THE POLAR MOTION

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Received June 29, 1982

Summury. Linear secular terms have been identified in both polar coordinates, independently of whether the latter have been derived from raw or smoothed observational data. After elimination, amplitudes and phases of the main harmonic terms in the polar motion have been determined. Our results back up assumption Chandler's cycle being non-unique. Instead of one single peak, two close ones have unequivocally been identified within that cycle in spectro of polar coordinates. Their periods are: 1.171 and 1.198 years, respectively. In the Kimura's z-term, besides the well known annual cycle, a semi-annual one has been identified, whose amplitude is 3 to 4 times weaker.

N. Pejović, PRELIMINARNA ANALIZA SEKULARNOG I GLAVNIH HARMONIJSKIH ČLANOVA U KRETANJU POLA — U sirovim i izravnatim koordinatama pola identifikovan je sekularni član u obe koordinate. Posle njegove eliminacije određene su amplitude i faze glavnih harmonijskih članova u kretanju pola. Na osnovu naših rezultata potkrepljena je pretpostavka da poznati Čendlerov ciklus nije jedinstven. Umesto jednog pika, u spektrima koordinata pola pouzdano su utvrđena dva bliska. Njihove periode su: 1.171 i 1.198 godina. U Kimurinom z-članu pored poznatog godišnjeg ciklusa identifikovali smo i polugodišnji ciklus čija je amplituda 3—4 puta manja.

INTRODUCTION

Since the foundation of the ILS (International Latitude Service) in 1899 up to now observational material of an exceptional scientific value has been collected. Many reputed researchers have worked on the harmonic analysis of this material. By all of them the existence has been confirmed of two periodic components: circular component of the free nutation — detected by Chandler — supposed to be excited by the non-coincidence of the Earth's rotational axis with the axis of symetry of its masses (the cause of the non-coincidence of these axes for the time being unknown), and the annual elliptic component of the forced nutation, accounted for by shifts in the masses over the Earth's surface (air and water streams). However, many more periodic components are found by the harmonic analysis, whose source is thought to lie in the specific geophysical composition

*

of the Earth. Due to its complexity this problem has not yet received its final solution, such as would allow the position of the Earth's instantaneous pole of rotation to be determined for all past and future times. This is why the trajectories of the Earth's poles, as deduced from the observations, from which the components of the free and forced nutation are removed, assume a very complex shape.

In the present paper we are going to restrain ourselves to the analysis of the main harmonic terms in the polar motion. A more thorough analysis will be the subject of our future work.

Data used: Observational data, acquired at the ILS stations in the period 1899.9 to 1979.0, cleaned up and published in the Publications of the Mizusawa International Latitude Observatory (Yumi and Yokoyama, 1980). Raw and smoothed (by Vondrak's method, 1969) polar coordinates have in parallel been treated.

SECULAR TERM

Let X_0 and Y_0 be the coordinates of the mean pole and x and y the coordinates of the instantaneous pole with respect to CIO (Conventional International Origin). The raw polar coordinates will be marked by the subscript u and the smoothed ones by the subscript v.

With the purpose of removing the secular term from the data used, as required by the harmonic analysis, mean three-annual values x_u, y_u, x_v and y_v have been formed and their first-, second- and third-order differences calculated.

The mean squares S_1 of the first-, S_2 of the second- and S_3 of the third-order differences in the raw and smoothed series are presented in Table I.

TABLE I

Mean square	x_u	y_u	x_v	y_v
S_1	0.0013	0.0010	0.0006	0.0021
S_2	0.0046	0.0037	0.0016	0.0074
S_3	0.0171	0.0141	0.0051	0.0289

It is evident from the above data that $S_1 < S_2 < S_3$ in all series of coordinates. Hence it follows that the linear approximation of the secular term appears as the best one.

The regression coefficients have been calculated according to the least square method. The free terms C_0 and the linear terms C_1 of the raw and smoothed series are given in Table II.

TABLE II

Regres. coeff.	x_u	y_u	x_v	y_v
C_0	0.01788	—0.01547	0.01846	—0.01567
C_1	0.00065	0.00346	0.00064	0.00348

By using the data from Table II we found that the mean pole moves at the rate $dX_0/dt = +0.''0006$/year and $dY_0/dt = +0.''0035$/year, or by the resulting velocity $dr/dt = 0.''0036$/year, with an accuracy of $\pm 0.''0002$/year.

It is only afterwards that we were aquainted with Markovitz (1982) having obtained results, close to ours own. From our results, as well as from those of Markovitz, there follows that a linear secular motion of the mean pole is present, a matter still considered as questionable.

MAIN HARMONIC TERMS

Upon removing the secular term from the data treated, Fourier integral transformations have been used in the identification of the main harmonic terms: the annual and Chandler's term.

The trial period has been varied within the limits of 60 up to 760 days, with an increment of 0.5 days. The initial period is, in fact, twice the value of the interval for which the data are given. Through such a choice of the trial periods the effects, known as leakage and aliasing, have been reduced.

The obtained values of the period P_a, the amplitude A_a and phase F_a of the annual term are given in the columns 2—4 of Table III. A double peak has come forth at the place where Chandler's term has been expected. The periods P_c, P'_c, the corresponding amplitudes A_c, A'_c and phases F_c, F'_c are listed in the columns 5—10 of the same Table. The periods are given in years. The phases are referred to the Julian date D.J. = 2415020.

TABLE III

Coord.	P_a	A_a	F_a	P_c	A_c	F_c	P'_c	A'_c	F'_c
x_u	1.000	0.''094	253.°9	1.171	0.''105	59.°4	1.197	0.''114	16.°4
x_v	1.000	0.076	171.7	1.171	0.107	332.2	1.197	0.118	284.5
y_u	1.000	0.094	253.8	1.171	0.105	68.8	1.199	0.115	15.4
y_v	1.000	0.077	170.3	1.171	0.107	340.8	1.199	0.117	285.1

The mean square errors $\sigma(x_u)$, $\sigma(y_u)$, $\sigma(x_v)$, $\sigma(y_v)$ of the coordinates x_u, y_u, x_v, y_v are equal $\pm 0.''01$.

By using the expression:

$$\sigma^2_A = (2 - \pi/2)\frac{2}{2n+1}\sigma^2_x$$

the value ± 0.0003 is obtained as the mean square error of the amplitude for the unsmoothed series and the value ± 0.0002 for the smoothed ones.

The mean square errors of the phases, calculated by the formula of

$$\sigma_F \approx 57^9 \cdot 296 \frac{\sigma x}{A}\sqrt{\frac{2}{2n+1}}$$

the annual and Chandler's terms lie in the interval from $\pm 0.^{\circ}2$ to $\pm 0.^{\circ}3$.

The correction to the amplitude ΔA, considering it to represent unbiased estimation of the mathematical expectation, has been calculated according to the formula:

The amplitudes in Table III are corrected for ΔA.

$$\Delta A = \sqrt{\frac{\pi}{2n+1}}\sigma_x.$$

The used expressions for the mean square error of the amplitude and phase and the correction to the amplitude are according to Djurović (1979).

The spectra, obtained by the Fourier analysis, are illustrated in Fig. 1 for the series x_u, y_u and in Fig. 2 for the series x_v, y_v. The abscissae reproduce the trial period P in years. The amplitude is given in seconds of arc.

The spectra in the interval P from 0.2 to 0.8 years, ten times enlarged, are found in the same Figures. By these results one is induced to suspect the presence of the short period terms as well whose amplitude does not exceed 0."006. This point will be the subject of our future study.

The duplicity of the Chandler's term has been put under scrutiny by comparing the theoretical and the calculated peak widths.

In Table IV are presented the theoretical $\Delta\varepsilon$ and the calculated $\Delta\varepsilon'$ peak widths (in years) of the main harmonic terms.

TABLE IV

Peak widths	$P_a = 1.000$	$P_c = 1.171$	$P'_c = 1.199$
$\Delta\varepsilon$	0.025	0.035	0.036
$\Delta\varepsilon'$	0.025	0.040	0.031

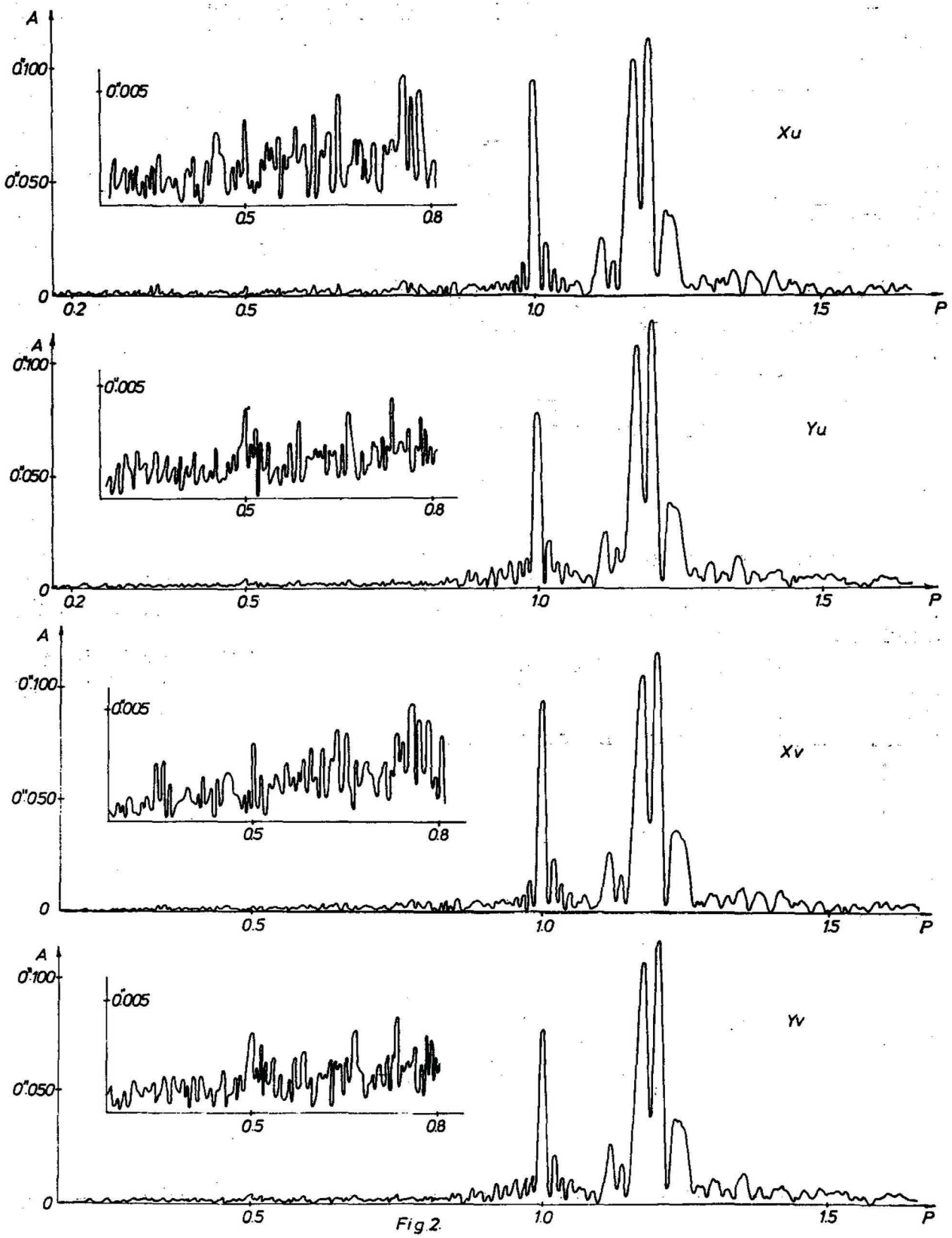

Fig.2.

Evidently, the theoretical and the calculated peak widths of the annual and both Chandler terms are approximately equal. By this fact, the assumption of the Chandler term being twofold, is favoured. If it were considered as a single one, with observed width $\Delta\varepsilon' = 0.071$, it would be two times wider than the corresponding theoretical value $\Delta\varepsilon$ (Table IV).

For the amplitudes of the primary peaks to be accurately determined, the differences of periods ΔP of any adjacent terms have to be greater or equal to $\Delta\varepsilon$, a condition met only with the annual term. However, the two Chandler peaks are 0.028 years apart, i.e. less than $\Delta\varepsilon$. Accordingly, the accurate values of their amplitudes (provided there were two peaks) cannot be determined from the given data by the present method.

From the same data Shouxian, Yingmin and Shuhe (1982) deduced a double Chandler term with the periods 1.171 and 1.198 years respectively. Their amplitudes are in close agreement with our results, arrived at by quite a different method.

With regard to the above results we presume that, what we are faced with, is not one single, but two close free nutations of the Earth's axis.

THE KIMURA'S Z-TERM

It is well known that the Kimura's z-term can be approximated by a sinusoid of an annual period. However, in analysing the ILS data we were able to identify, in addition to the annual, a semi-annual component as well, which heretofore was unknown.

The z-term spectrum (Fig. 3*a*), along with those of the amplitude, the period and the phase of the harmonic terms (Table V), were calculated by the method of Fourier transformations. This method was applied to the z-term residuals after the linear trend has been removed.

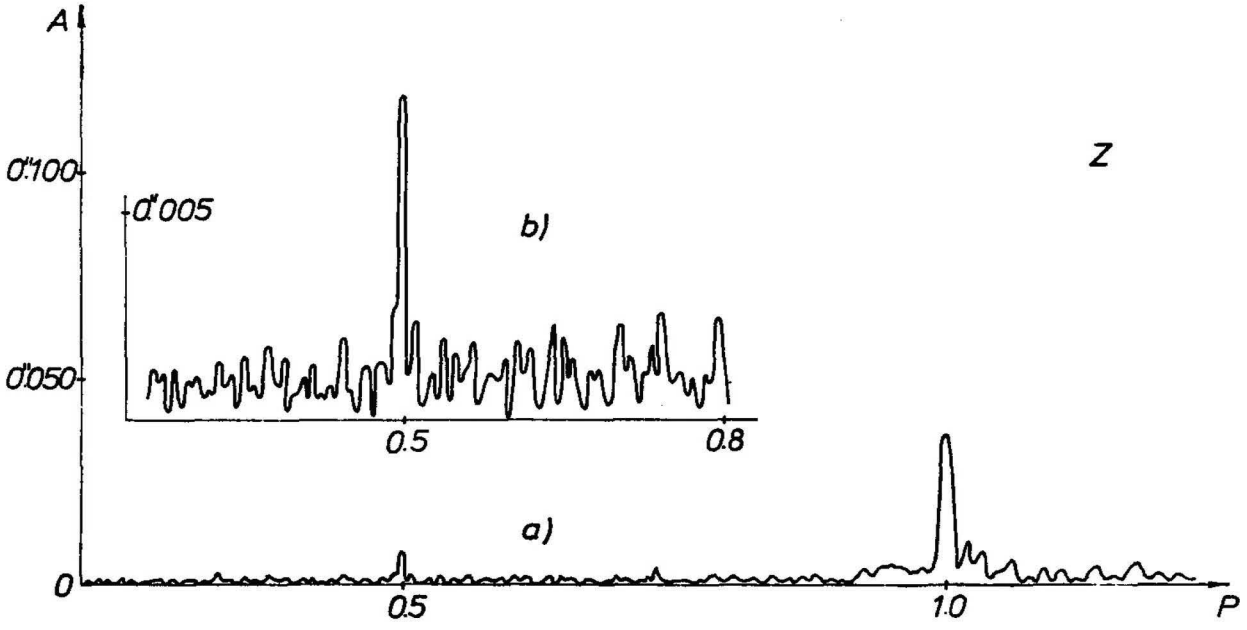

Fig. 3.

The notations in Table V mean, respectively: P, A and F — the period, amplitude and phase of the annual term; P′, A′ and F′ — the period, amplitude and phase of the semi-annual term. The phases are related to the Julian date D.J. = 2415020. The periods are expressed in years.

TABLE V

P	A	F	P′	A′	F′
1.000	0.″037	5.°8	0.499	0.″008	32.°3

The abscissa axis P in Fig. 3 represent the trial period in years, and that of the ordinate the amplitude in second of arc. The annual and the semi-annual peaks are clearly distinguishable (Fig. 3*a*). The semi-annual peak becomes particularly pronounced in the enlarged part of the same Figure (3*b*).

In order to provide an estimate of what is the statistical meaning of the amplitude A′, two alternate hypotheses, H_0 and H_1, have been tested. The first H_0 hypothesis, implied that the results of the spectral analysis, for the frequency $\omega = 2\pi/0.499$, represented a white noise. The implication of the second hypothesis H_1 was: the data embraced a harmonic, with the frequency ω.

It could be established, by employing the Schuster and the Walker criteria, with the significance levels 0.05 and 0.01, respectively that the H_0 hypothesis was to be turned down.

We learn from Kulikov's monography (1962), dedicated to the problems of longitude and latitude variations, that different authors have obtained different values for the amplitude and phase of the zterm. This is explicable by the fact that various authors have used different observational. The origin of the z-term remains unknown even to our days.

Fikera (1971) claims to have found the z-term being dependent on the catalogues used in the latitude observations. Other authors, however, come forward with other interpretations.

The calculations were carried out on the IBM 360/44 of the Computing Centre of the Institute for Mathematics in Beograd.

*

This work is a part of the research project of the Basic Organization of Associated Labour for Mathematics, Mechanics and Astronomy of the Belgrade Faculty of Sciences, funded by the Republic Community of Sciences of Serbia.

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6 The stability the amplitudes and phases of the Chandler's, annual and semi-annual term in the polar motion

N. Pejović
Publ. Dept. Astr. Univ. Beograd
No. 13, pp. 31-39, 1985.

Using the unsmoothed ILS coordinates of the instantaneous pole (Yumi and Yokoyama, 1980) the stability of the amplitudes and phases of the main harmonic terms in the polar motion are investigated. It has been established that the amplitudes and the phases were not subject to systematic fluctuations. As for the amplitude A and the phase P of the Chandler nutation, 1t seems that the amplitude forms a periodic function of time enclosing as the most prominent components with 53.0 and 34.5 year periods, while the phases kept stable over longer time intervals, apart from jump-like change from 1925 to 1940.

Publication of the Department of Astronomy — Beograd, № 13, 1985.

UDC 521.93

THE STABILITY THE AMPLITUDES AND PHASES OF THE CHANDLER'S, ANNUAL AND SEMI-ANNUAL TERM IN THE POLAR MOTION

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Received October 11, 1983

Summary. Using the unsmoothed ILS coordinates of the instantaneous pole (Yumi and Yokoyama, 1980) the stability of the amplitudes and phases of the main harmonic terms in the polar motion are investigated. It has been established that the amplitudes and the phases were not subject to systematic fluctuations. As for the amplitude A and the phase P of the Chandler nutation, it seems that the amplitude forms a periodic function of time enclosing as the most prominent the components with 53.0 and 34.5 year periods, while the phases kept stable over longer time intervals, apart from jump-like change from 1925 to 1940.

N. Pejović, STABILNOST AMPLITUDA I FAZA ČENDLEROVOG, GODIŠNJEG I POLUGODIŠNJEG ČLANA U KRETANJU POLA — Koristeći neizravnate ILS koordinate trenutnog pola (Yumi and Yokoyama, 1980) ispitana je stabilnost amplituda i faza glavnih harmonijskih članova u kretanju pola. Ustanovljeno je da amplitude i faze godišnjeg i polugodišnjeg člana nemaju sistematske fluktuacije. Što se tiče amplitude A i faze P Chandler-ove nutacije, verovatno je da amplituda predstavlja složenu periodičnu funkciju vremena sa najizrazitijim komponentama od 53.0 i 34.5 godina, a da faza ostaje stabilna u dužim vremenskim intervalima, sa mogućom skokovitom promenom kao u periodu 1925—1940.

INTRODUCTION

Following the organising of the ILS in 1899 until the present day the observational data of exceptional scientific value has been collected. Many known researchers have studied this data and all of them confirmed, as is well known, the existence of the circular free nutation component, revealed by Chandler, as well as the existence of the annual elliptical nutation termed the forced nutation. Geophysical causes of exciting of free nutation of the Earth's rotation axis are still unknown (Lambeck, 1980). The variability of its period phase is a matter of controversy for a whole century. In the opinion of some authors, including Chandler himself, there exist two or even more free nutations with close periods to each other or, one single with a variable period (Chandler, Kimura, Melchior, Sekiguchi, Gaposchkin, Carter, Vondrak). There is also another group of authors

(Newcomb, Ooe, Okubo) prooving that there exists one single free nutation having a stable period.

From our previous investigations (Pejović, 1983) we inferred that there exist in the spectra of the polar coordinates two close peaks with periods of 1.171 and 1.197 years, which lends to the free nutations hypothesis. By this hypothesis Melchior (1957) explained the variability of the Chandler period and amplitude. This variability in Melchior's view is due the interference of two free nutations, one of which providing from the Earth's core and the other one from the Earth's elliptic crust. According to Vondrak (1984) the rapid change of the phase in the period 1925—1940 is a consequence of the change of Chandler period.

In the present paper our attention will be directed towards analysing the variation of the amplitude and the phase of the Chandler's, annual and semi-annual terms in the polar motion.

VARIATION OF THE ANNUAL AND THE SEMI-ANNUAL TERMS

For the present analysis the data over six-year subintervals have been treated. According to the assumption that the amplitudes and the phases of the annual and the semi-annual terms are more stable then those of the Chandler term, which was confirmed by the results á posteriori, the amplitude A_a and the phase P_a of the annual term as well as the amplitude A_h and phase P_h of the semi-annual term were calculated and eliminated from both coordinates, using the method of least squares (MLS). Our purpose was to observe the Chandler variation of the polar coordinates as a function of time (Fig. 1). The results A_a, A_h, P_a and P_h are given in Table I. In columns 2 and 6 are listed in seconds of arc the amplitudes of the annual term in X and Y coordinate, and in the columns 4 and 8 those of the semi-annual term. In the columns 3 and 7 and 5 and 9 are presented the phases of these terms in arc degrees.

TABLE I

Period	X coordinate				Y coordinate			
	A_a	F_a	A_h	F_h	A_a	F_a	A_h	F_h
1899.8—1905.8	.080	335.4	.008	282.8	.065	260.7	.001	92.1
1905.8—1911.8	.108	329.0	.008	286.0	.093	253.3	.005	263.2
1911.8—1917.8	.081	343.5	.005	128.1	.086	265.0	.010	158.7
1917.8—1923.8	.090	332.6	.005	191.7	.077	234.9	.005	98.9
1923.8—1929.8	.058	360.3	.006	120.3	.045	278.8	.018	121.4
1929.8—1935.8	.113	348.8	.006	338.3	.091	267.5	.006	349.4
1935.8—1941.8	.107	341.1	.007	300.4	.097	256.4	.003	288.7
1941.8—1947.8	.114	340.3	.005	264.5	.088	260.8	.007	184.4
1947.8—1953.8	.114	329.5	.009	321.5	.100	238.1	.008	247.1
1953.8—1959.8	.085	330.2	.010	301.5	.059	244.2	.006	89.6
1959.8—1965.8	.103	336.4	.002	302.1	.069	238.3	.010	120.7
1965.8—1971.8	.099	337.4	.006	326.1	.067	254.4	.006	163.6
1971.8—1977.8	.078	360.1	.002	306.9	.078	282.7	.016	107.9

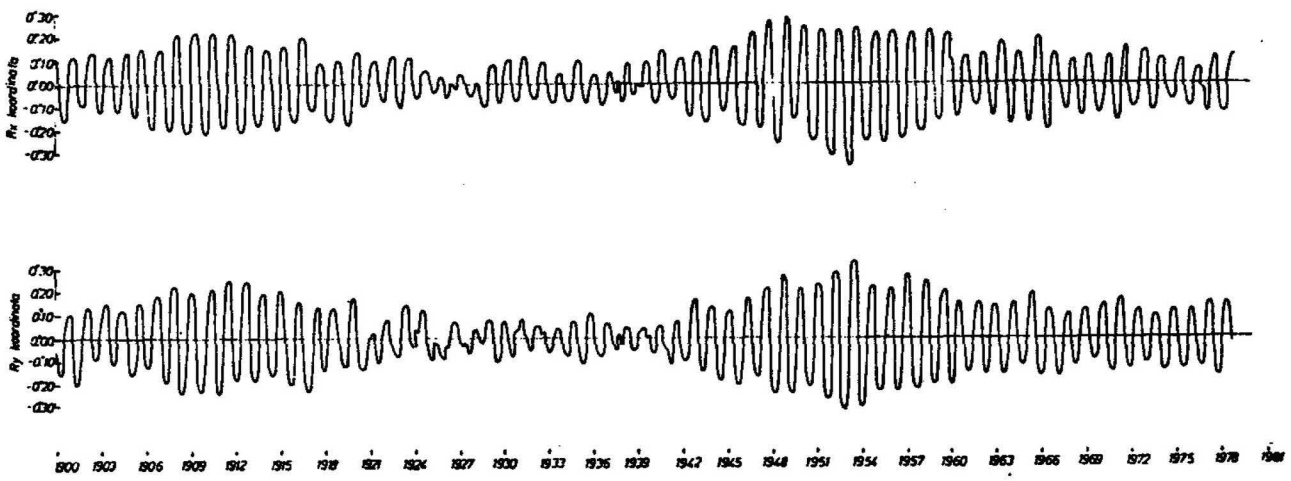

Fig. 1.

From the data of the Table I we computed the mean values of the amplitudes A_a and A_h ($\bar{A}_a$, $\bar{A}_h$), the mean values of the phases P_a and P_h ($\bar{P}_a$, $\bar{P}_h$) and the standard deviations of these variables. The phases are referred to JD = 2414 948. The obtained results are presented in the Table II.

TABLE II

Coord.	Annual term		Semi-annual term	
	$\bar{A}_a \pm \sigma_{\bar{A}_a}$	$\bar{P}_a \pm \sigma_{\bar{P}_a}$	$\bar{A}_h \pm \sigma_{\bar{A}_h}$	$\bar{P}_h \pm \sigma_{\bar{P}_h}$
X	0".0948 ± 0".0050	340° ± 3°	0".0060 ± 0".0007	266° ± 21°
Y	0.0778 ± 0.0046	256 ± 4	0.0077 ± 0.0014	176 ± 25

Practically the same results we have obtained earlier by the method of Fourier transforms (Pejović, 1983).

In order to examine whether the variations of the amplitude and the phase of the annual term were bearing a systematic or accidental feature we used Abbe's criterion (Djurović, 1979).

Let G and G_0 be statistics specified by the relations

$$G = \frac{\frac{1}{2N-1}\sum_{i=1}^{N-1}(A_{i+1}-A_i)^2}{\frac{1}{N-1}\sum_{i=1}^{N-1}(A_i-\bar{A})^2}$$

$$G_0 = 1 + \frac{u_q}{\sqrt{N+0.5\,(1+u_q^2)}}$$

where $u_q = -1.645$ is the quantil of the order 0.05 (u_q is a quantil of the order q provided the probality of the accidentally variable $x > u_q$ is q) of the Gaussian distribution of the probability. The quoted equation is used for calculating the statistics G_0 if $N > 20$, whereas for $N \leqslant 20$, in our case $N = 13$ and for $q = 0.05$ it assumes the value $G(13) = 0.578$ (Djurović, 1979).

The G values relatives to the amplitude and the phase of the annual term in the X and Y coordinates are given in Table III.

The G values in Table III are greater than $G(13) = 0.578$. Thus the $A_i - \overline{A}$ differences might be considered as the accidental errors.

According to Abbe's criterion the variations of the amplitude and phase of the semi-annual term (with respect to the mean values) are also accidental.

TABLE III

Calcul. statist.	X coord. A_a	F_a	Y coord. A_a	F_a
G	0.973	0.722	0.960	0.872

CHANDLER TERM VARIATIONS

The amplitude and the phase of the Chandler term (the adopted period 1.20 years) were computed by MLS for 15 months subintervals by moving their beginings for one year (the zero point is at 1899.8 and the overlapping of the successive subintervals is three months). The amplitudes are plotted in Figure 2 and the phases in Figure 3. The phases are relatives to JD = 2414 948.

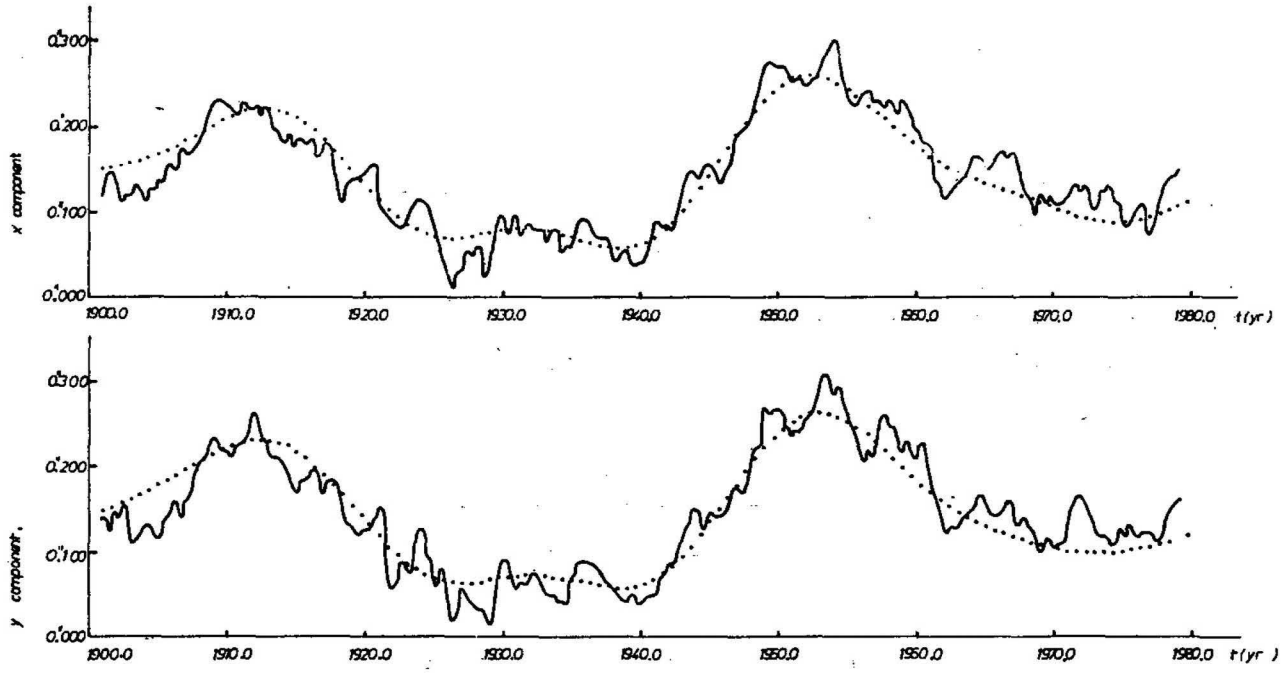

Fig. 2. Periode variation of Chandler's amplitude

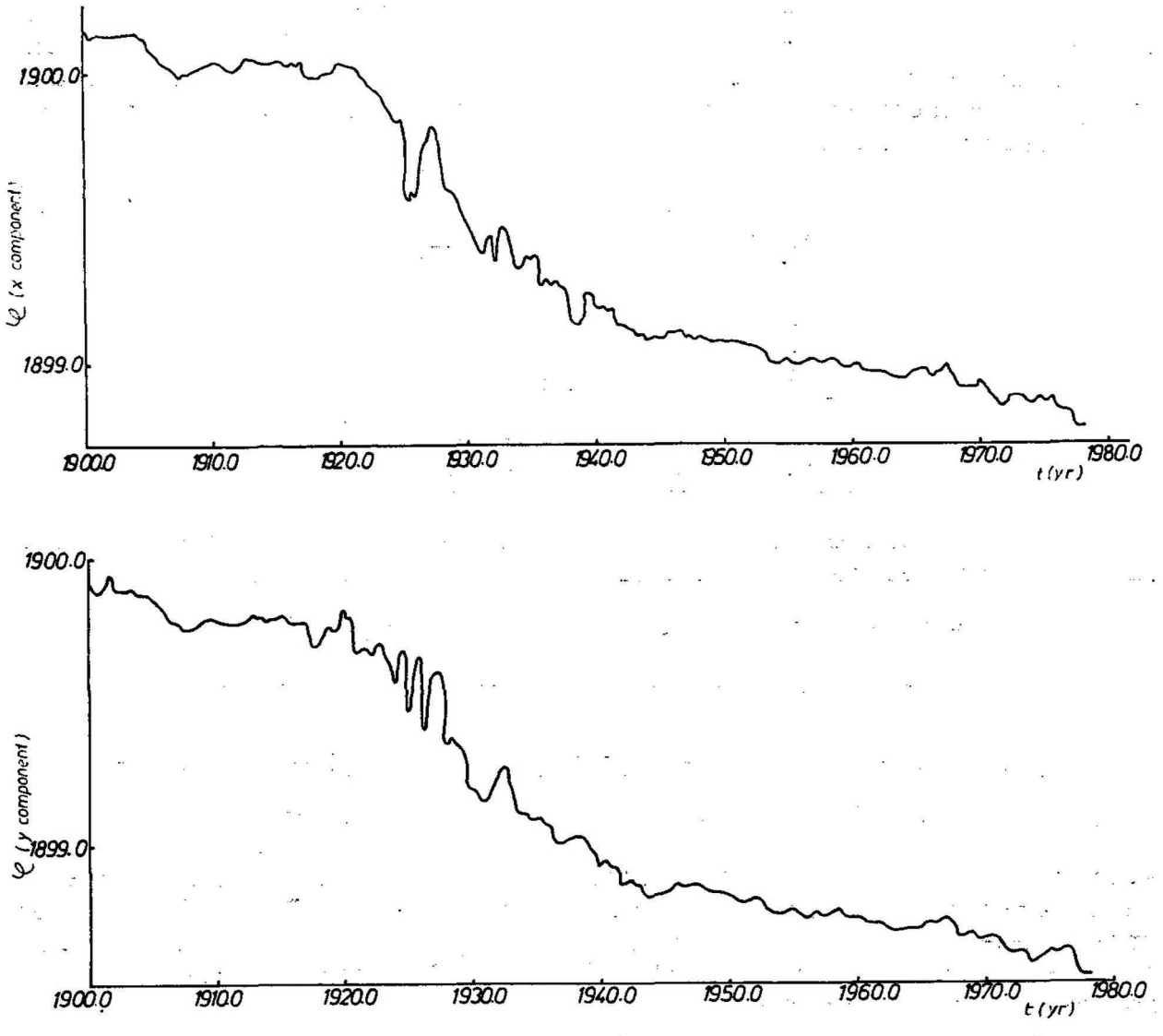

Fig. 3

The results in Figures 2 and 3 are close to those of Guinot (1972, 1973) and Dickman (1981). One gets the impression from Figure 2 that the Chandler amplitude has a periodic variation while the phase, outside of the period 1925—1940, has a relatively good stability. The 180° change of the phase in the period 1925—1940 is a unique and as yet unexplained phenomenon.

Shouxion, Yingmin and Shuhe (1982) have concluded that the phase of the Chandler nutation is periodically variable. This conclusion could not be confirmed from our analysis.

In order to determine the periods of the assumed harmonic variations of the Chandler amplitude we computed by MLS the amplitude m and the phase ν of the sinusoide $A(t) = m\ sin\ (\omega t + \nu)$, varying the trial period

P within a 15 to 80 years range, with the increment $\Delta P = 0.5$ year. In Figure 4 is presented the amplitude m in function of P. Two prominent maxima are evident in Figure 4, corresponding to the periods of 53.0 and 34.5 years, respectively. The amplitudes and the periods of these two and two weaker maxima, are given in Table IV.

TABLE IV

Period (year)	Amplitudes X coord.	Y coord.
53.0	0".0598	0".0632
34.5	0.0417	0.0438
23.0	0.0188	0.0234
17.5	0.0206	0.0168
The standard deviation is	±0".005	

The Chandler amplitude as a function of time is synthetized as a sum of mentioned four periodic functions (in X and Y separately):

$$A_x(t) = C_0 + C_1 \cos\omega_1 t + C_2 \sin\omega_1 t + C_3 \cos\omega_2 t + C_4 \sin\omega_2 t + C_5 \cos\omega_3 t + C_6 \sin\omega_3 t + C_7 \cos\omega_4 t + C_8 \sin\omega_4 t \quad (1)$$

$$Ay(t) = D_0 + D_1 \cos\omega_1 t + D_2 \sin\omega_1 t + D_3 \cos\omega_2 t + D_4 \sin\omega_2 t + D_5 \cos\omega_3 t + D_6 \sin\omega_3 t + D_7 \cos\omega_4 t + D_8 \sin\omega_4 t$$

Adopting for their periods, amplitudes and phases the values of the Table IV we obtained the functions $A_x(t)$ and $A_y(t)$ presented in Figure 2 by dotted lines. These functions (equation 1), including four harmonic components of largest amplitudes (at 53.0, 34.5, 23.0 and 17.5 years), represent a good approximations of Chandler amplitude variation.

Let A_R (Fig. 5) represent the differences between the observed and computed amplitudes $A - A_x(t)$ or $A - A_y(t)$. Since the shape of A_R curves makes one suspect the presence therein of smaller periodical terms, by the Fourier transforms we have computed the spectrum of A_R. The trial period was varied from 2 months up to 15 years, with the increment of 1 month. The periods and the amplitudes of several best pronounced terms are listed in Table V.

TABLE V

Period (year)	Amplitudes X cord.	Y cord.
11.3	0".011	0".008
8.2	0.007	0.004
7.1	0.010	0.009
6.0	0.008	0.010
The standard deviation is	±0".001	

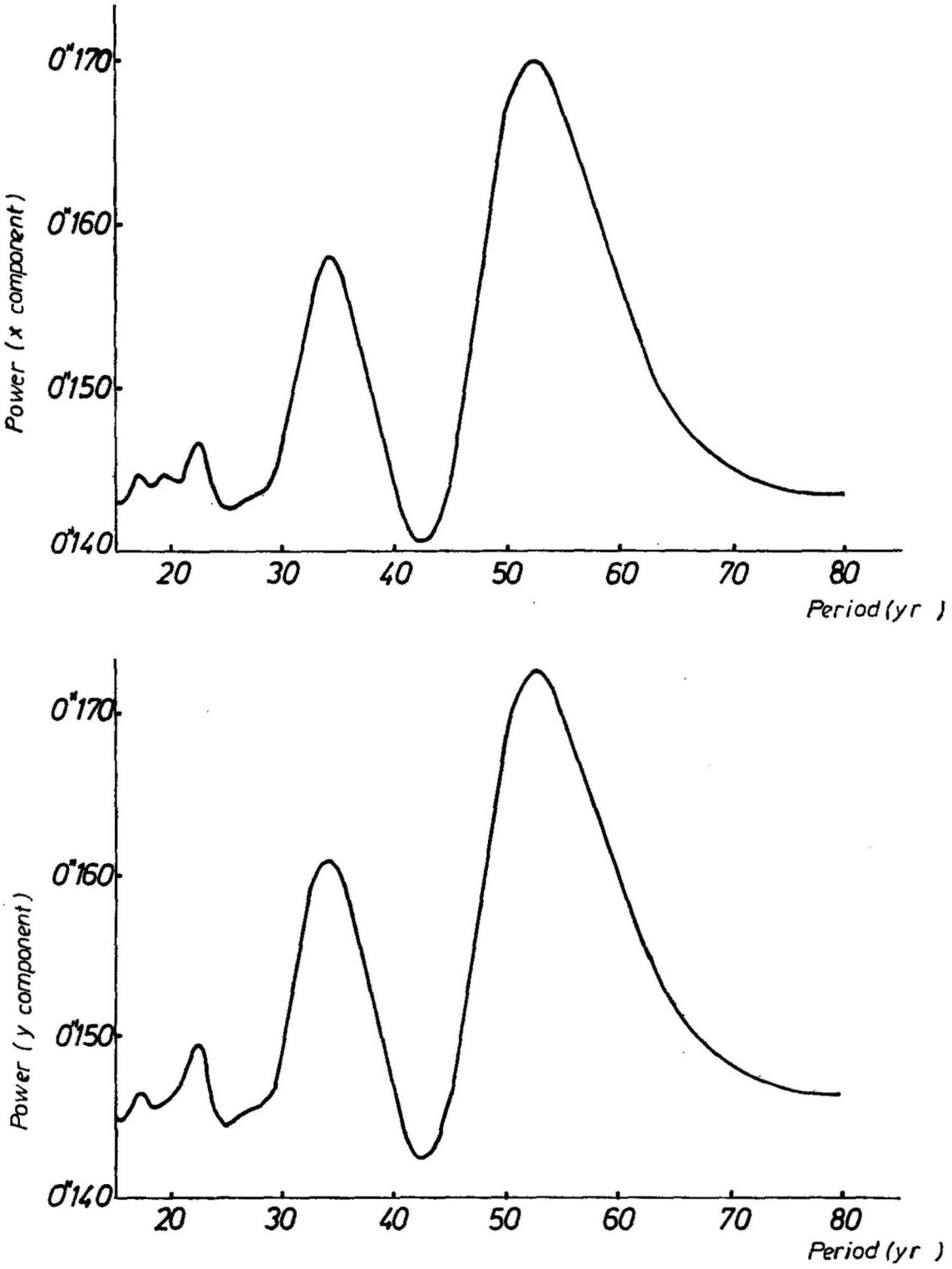

Fig. 4. *Spektrum of Chandler's amplitude*

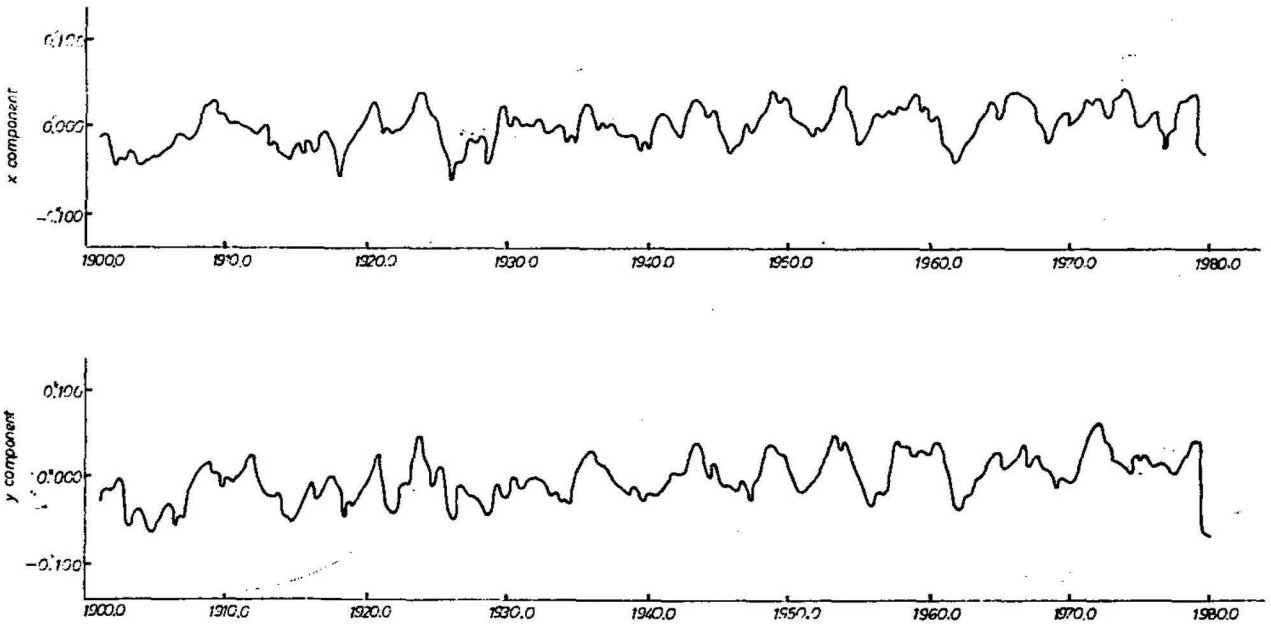

Fig 5

According to the some authors there exists in the universal times UT1 and UT2 a periodic variation with the characteristic period of 60 years (Kalinin and Kiselev, 1980; Kiselev, 1982; Emetz and Korsun, 1978; Stajić, 1983) and an amplitude of few seconds! Should this prove true one should check whether the 53-years variation in X and Y is due to the same cause. A variation of 35 years period has also assumed in UT1 system (Morison, 1979; Stajić, 1983) which in its turn incites one to ponder over their common exciter.

Weak variations of 23 and 11-years have the periods of the known solar activity cycles. They anticipated in the universal time systems (Djurović and Stajić, 1985; Vondrak, 1977). The term of 17.5-year corresponds approximately to the nutation cycle.

The 6- and 8-year variations (Table V) are likewise noted in UT1 or UT2 (Lambeck and Cazenave, 1983; Djurović, 1979; Djurović and Stajić, 1984; Djurović, 1981). Despite the fact that the mentioned periodic or quasiperiodic variations of the polar coordinates and angular velocity of the Earth's rotation (e. g. variations of 60, 35, 22, 11, 18 and 6 years periods), have been assumed conclusive evidence of their existence is still missing. The present paper and the similar papers of other authors may only help us discerning those spectral regions which deserve paying greater attention to.

*

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7 Short-period terms in the motion of the instantaneous pole

N. Pejović
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In the spectrum of ILS coordinates of the instantaneous pole (Yumi and Yokoyama, 1980) short period variations of 204 (corresponding to the quasi-diurnal nutation of Molodensky) and 122 days have been identified. The first one is not accepted by certain authors while the second one is not investigated enough. For other short-period fluctuations it can be only said that they are below the limit of $0.010''$.

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SHORT-PERIOD TERMS IN THE MOTION OF THE INSTANTANEOUS POLE

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Summary. In the spectrum of ILS coordinates of the instantaneous pole (Yumi and Yokoyama, 1980) short period variations of 204 (corresponding to the quasi-diurnal nutation of Molodensky) and 122 days have been identified. The first one is not accepted by certain authors and the second one is not investigated enough. For other short-period fluctuations we can only say that they are below the limit of 0.″010.

N. Pejović, KRATKO-PERIODIČNI ČLANOVI U KRETANJU TRENUTNOG POLA — U spektrima ILS koordinata trenutnog pola (Yumi and Yokoyama, 1980) identifikovane su kratkoperiodične varijacije od 204 (odgovara skoro-dnevnoj nutaciji Molodenskog) i 122 dana, od kojih se prva osporava od strane nekih autora, a druga nije dovoljno poznata u naučnoj literaturi. Što se ostalih kratko-periodičnih fluktuacija (X, Y) tiče, možemo jedino reći da se nalaze ispod granice od 0.″010.

1. INTRODUCTION

In the spectrum of ILS coordinates of the instantaneous pole (Yumi and Yokoyama, 1980) a linear drift has been identified in both coordinates (Pejovic, 1983). After its elimination, the amplitudes and phases of the main harmonic terms have been determined. Our results back up the assumption that the Chandler's cycle is not unique. Instead of a single peak, two close ones have unequivocally been identified with the periods of 1.171 and 1.198 years respectively (Pejovic, 1983).

The stability of the amplitudes and phases of the main harmonic terms has also been investigated. It has been established that the amplitudes and phases were not subject to systematic fluctuations. As for the amplitude A and the phase P of the Chandler nutation, it seems that the amplitude is a periodic function of time containing, as the most prominent ones, the components with 53.0 and

34.5 years periods, while the phase keeps stable over a longer time period, apart from jump-like changes, like the one of 1925—1940 (Pejovic, 1985).

In this paper we continue the analysis of ILS coordinates in order to investigate the existence of short-period terms of 204 days (due to the quasi-diurnal nutation of Molodensky) and 122 days.

Among the authors that recently identified the nutation of Molodensky, e. g.: Debarbat, 1969, Yatskiv, 1972, Đurović, 1975, etc.

As for the 122 days variation, it has first been identified, in the BIH pole coordinates and in the UT1 by D. Đurović (1970, 1979, 1983).

2. SHORT-PERIODIC TERMS

Assuming that function $Y=F(X)$ is the sum of several periodic components, i. e.

$$Y=\sum_{j=1}^{h}[a_j \sin(\omega_j X)+b_j \cos(\omega_j X)]=\sum_{j=1}^{k} A_j \sin(\omega_j X+\varphi_j) \tag{1}$$

where k is the number of unknown components, the separation of one (or more) components from the common signal $Y(X)$ is possible by a transformation of the ordinates.

We form Z_i^m:

$$Z_i=Y_{i+1}-Y_{i-1}+Y_{i+2}-Y_{i-2}+\ldots+Y_{i+m}-Y_{i-m} \tag{2}$$

where $m>0$, $m\in(1,2,\ldots)$, $i\geqslant m+1$, $i\leq n-m$ (see Labrouste et Labrouste, 1943 or Đurović 1979).

Z_i^m results from the subtraction transformation of the ordinates of order m. We note that:

$$X_{i+1}-X_i=h=1. \tag{3}$$

From equations (1), (2) and (3) we obtain

$$Y_{i+m}-Y_{i-m}=\sum_{j=k}^{k} A_j \sin(\omega_j X_i+\varphi_j+m\omega_j)-\sum_{j=1}^{k} A_j \sin(\omega_j X_i-\varphi_j-m\omega_j)=$$

$$=\sum_{j=1}^{k} 2A_j \sin\omega_j \cos(\omega_j X+\varphi_j)=\sum_{j=1}^{k}\beta_j^m A_j \cos(\omega_j X_i+\varphi_j)$$

where β_j is the amplitude multiplier. The transformed sequence of the quantities $Y_{i+m}-Y_{i-m}$ includes the same periodic componerts, but with phase shift $\pi/2$.

The amplitude multiplier Z_i^m is expresed by:

$$\xi_j^m=\beta_j^1+\beta_j^2+\ldots+\beta_j^m=2\sin\omega_j+2\sin 2\omega_j+\ldots+2\sin m\omega_j$$

or

$$\xi_j^m = 2 \frac{\sin (m+1) \frac{\pi}{T_j} \sin m \frac{\pi}{T_j}}{\sin \frac{\pi}{T_j}}.$$

If we apply the substraction transformation of order p $(q, \ldots r)$ to the new sequence of quantities Z_j^m we obtain the multiple transformation of substraction $\Pi(Z) = Z_m Z_p Z_q Z_r$.

Short-period components in the smoothed ILS coordinates (Vondrak, 1969, 1977) have been investigated by means of multiple transformations of orders m, p, q, r, symbolically represented by $\Pi(Z) = Z_m Z_p Z_q Z_r$. The amplitude multiplier of this transformation is:

$$\xi_j = \xi_j^m \xi_j^p \xi_j^q \xi_j^r.$$

The values of m, p, q, r are varied in the region $m = 1,2, \ldots, 7$, $p = 2,3, \ldots 8$, $q = 3,4, \ldots 9$ and $r = 4,5, \ldots 10$.

The amplitude multiplier ξ_j that is finally found has the maxima for 11.7, 15.3, 19.0, 22.9, 26.5, 34.5 units (unit= the elementary time interval between the successive ordinates).

The amplitude multipliers that have been calculated using the expression:

$$\xi_j = 2 \sin \left(\frac{2\pi}{n} m \right) \cdot 2 \sin \left(\frac{2\pi}{n} p \right) \cdot 2 \sin \left(\frac{2\pi}{n} q \right) \cdot 2 \sin \left(\frac{2\pi}{n} r \right)$$

where n is the tested period, are given in Table I.

In this way we have got the results shown in Table I and Fig. 1.

TABLE I

Transform.	Amplitude multiplier	Period	Amplitude
$Z_6Z_7Z_8Z_9$	14.35	121 days	0.''0036
$Z_6Z_7Z_8Z_9$	14.35	126 days	0.''0039
$Z_7Z_8Z_9Z_{10}$	14.70	205 days	0.''0027

Fig. 1. shows explicitly the existence of the peaks with periods of 0.331, 0.345 and 0.561 years, or 121, 126 and 205 days respectively. The last one corresponds to the quasi-diurnal nutation of Molodensky. Two close peaks at 121 and 126 days are probably due to the fluctuation of the period of 122-day term. As the 122-day term is not sufficiently known, its identification in ILS coordi-

nates might be very important for future investigations (the first identification has been done by Đurović (1971, 1979, 1983) in the UT1 system and the BIH pole coordinates).

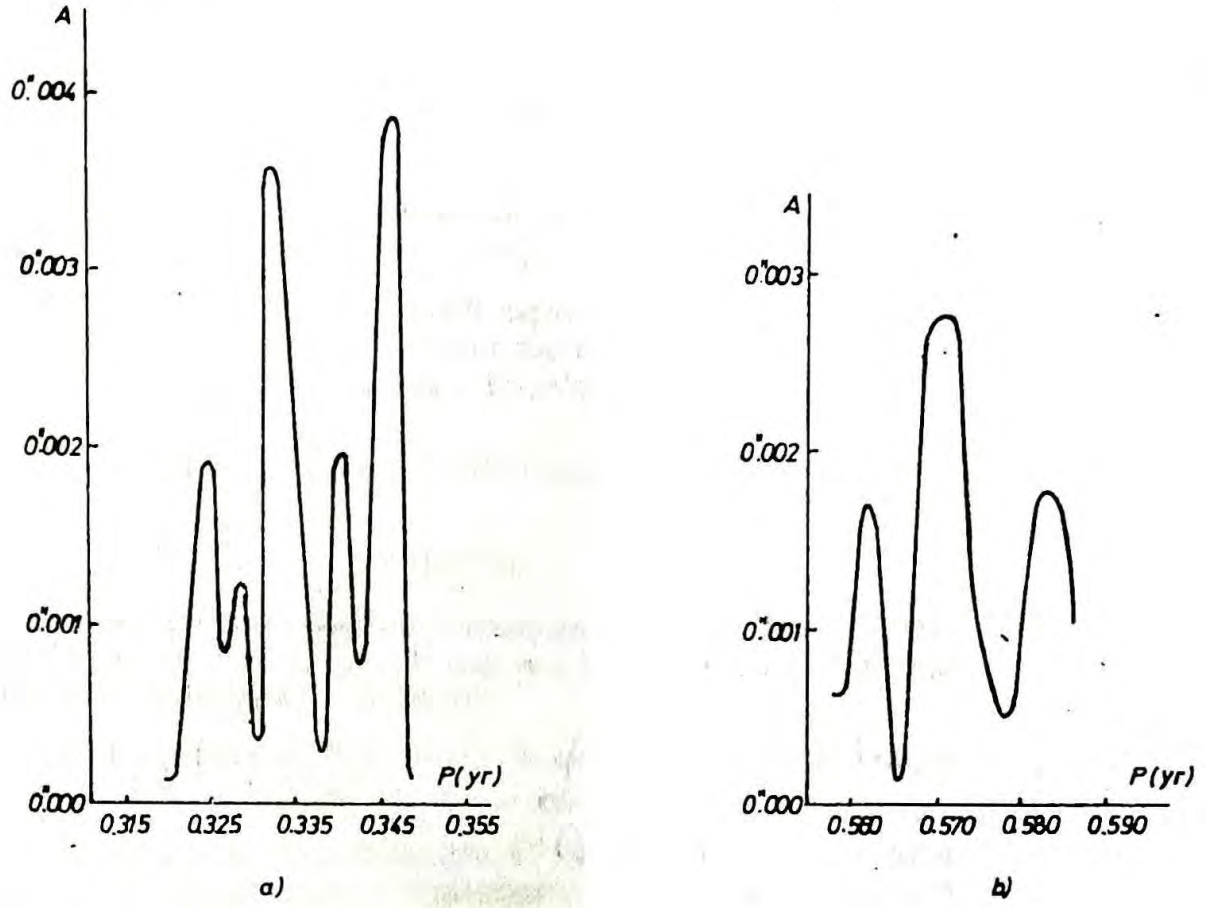

fig.1. THE SPECTRUM OF SHORT PERIODIC TERMS
a) THE TERMS FROM 121 AND 126 DAYS
b) THE TERM FROM 205 DAYS (MOLODENSKY'S NUTATION)

Molodensky has theoretically anticipated and calculated the quasi-diurnal nutation for two models of Earth. Corresponding periods differ for 2 seconds. The verification by observations of this teoretical prediction remains an unresolved problem.

Popov has identified, in Poltava observations for the 1939—1969 period, the nutation of Molodensky and obtained for its amplitude the value 0."012.

Yatskiv (Yatskiv, 1972) has found in the latitudes observed in Pulkovo (1905—1941) three prominent spectral lines at 194, 208 and 219 days. Spectral lines at 205, 219 and 234 days dominate in the spectrum of the Washingthon's observations for the period 1916—1940. For the amplitude of the quasi-diurnal nutation they have obtained 0."004 (Pulkovo) and 0."009 (Washington).

Shugawa and Ooe from the analysis of ILS data for the period 1955—1967 have found in the latitude variations a term with the period of 204 days and the amplitude 0."005.

Débarbat (1969, 1970), from the observations at the Paris observatory (1956—1963) has obtained the value 0."008 for the amplitude of Molodensky's nutation.

From the analysis of BIH pole coordinates, Đurović (1975) has found the variation corresponding to the Molodensky's nutation.

Our results confirm the hypothesis that small variations of pole coordinates of 204 days are due to the quasi-diurnal nutation.

3. OTHER PERIODIC TERMS

For the identification of other periodic fluctuations of (X, Y) we have used the numerical filters, similar to Orlov's and Melchior's filters which are in fact multiple combinations of the addition of ordinates, of type $\Pi(S)$. They are very convenient for a smoothing of the main harmonic terms (semi-annual, annual, Chandler).

In order to obtain the mean latitude, without periodic variations, Orlov has established the formula (Kulikov, 1962):

$$\varphi_m=\frac{1}{40}(\varphi_0+2\varphi_1+\ldots+4\varphi_8+\ldots+2\varphi_{15}+\varphi_{16}). \tag{5}$$

The selectivity function has zeros for $n=10$ (annual period) and $n=12$ (Chandler's period) elementary intervals (0.1 years).

As Orlov's formula eliminates only the main harmonic terms, but not short-periodic terms (if they exist), it is not convenient for the analysis of the longer observational series. Melchior has proposed a new transformation (Kulikov, 1962):

$$\varphi_m=\frac{1}{19305}(\varphi_{45}+4\,\varphi_{44}+10\,\varphi_{43}+\ldots+1038\,\varphi_{22}+1047\,\varphi_{23}+$$
$$+1038\,\varphi_{24}+\ldots+4\varphi_2+\varphi_1) \tag{6}$$

which excludes all the main harmonic and some short-period terms. Its selectivity function has zeros for $n=9$, $n=11$, $n=13$ and $n=15$ elementary intervals (0.1 years). The amplitude multiplier n is 19 305.

To obtain the mean latitude φ_m by Orlov's formula one needs 17 values and by Melchior's formula 45 values of φ_o, φ_1, ... φ_n.

As different opinions about the advantages of two filters exist, we have used both of them for the elimination of semi-annual, annual and Chandler's term.

The coefficients of the equations (5) and (6) are given for equidistant data with intervals of 0.1 years. As we disposed of data for every 1/12 and 1/20 years, we had not the possibility to use them directly. For that reason, by using Orlov's

formula and Melchior's filtres, we calculated the new coefficients of the equations (5) and (6), in order to adapt them to equidistant data with elementary intervals of 1/12 and 1/20 years. The new coefficients are marked by „O" and „M", and are given in Table II, respectively. Index i is the serial number of the coefficients.

The transformed filters corresponding to the methods of Orlov and Melchior are represented by expresions:

$$X_{UO}=\frac{1}{48}(X_0+2X_1+\ldots+4X_8+4X_9+4X_{10}+\ldots+2X_{18}+X_{19}) \tag{7}$$

$$X_{VO}=\frac{1}{80}(X_0+2X_1+\ldots+4X_{15}+4X_{16}+\ldots+2X_{30}+X_{31}) \tag{8}$$

$$X_{UM}=\frac{1}{38610}(X_{54}+4X_{53}+10X_{52}+\ldots+1760X_{27}+1760X_{28}+\ldots$$
$$+10X_3+4X_2+X_1) \tag{9}$$

$$X_{VM}=\frac{1}{267750}(X_{90}+4X_{89}+10X_{88}+\ldots+7429X_{45}+7429X_{46}+\ldots$$
$$+10X_3+4X_2+X_1) \tag{10}$$

where index 0 corresponds to the former, and index M to the latter one. X_U and X_V represent the coordinates given at 1/12 and at 1/20 year, respectively.

The selectivity function of equation (9) has zeros for $n=18$, $n=15$, $n=13$ and $n=11$ elementary intervals (one month). For equation (10), that function has zeros for $n=30$, $n=25$, $n=21$ and $n=17$ elementary intervals (1/20 years). This means, that for such n the amplitude multiplier is zero. The corresponding values of the periods are 1.50, 1.25, 1.08, 0.92 and 1.50, 1.25, 1.05, 0.92 years.

For every filtered value of X_{UO} we need 20 data, for X_{VO}—32 data, for X_{UM}—54 and for X_{VM}—90 data.

The efficiency of these filtres is sufficiently good, what can be seen on Figure 2, where the residuals of the annual and the Chandler's term, after filtration of the pole coordinates, using equation (7)—(10) are represented. The neglected values are small. Their amplitudes are in the range from 0."000 to 0."005. They are negligible with respect to 0."094 obtained for annual, and 0."105 and 0."114, obtained for two close Chandler's peaks (Pejović, 1983).

By the filtres (7)—(10), we can assume that the Chandler's, annual and semi-annual term are eliminated from (X_U, Y_U), (X_V, Y_V). By discrete Fourier transforms (DFT), we have computed the spectra of the mentioned residuals of (X_R, Y_R), represented on the Figure 3. On Fig. 3 the series of the extrema beetwen 3 and 12 years is presented, but their amplitudes are small: from 0."002

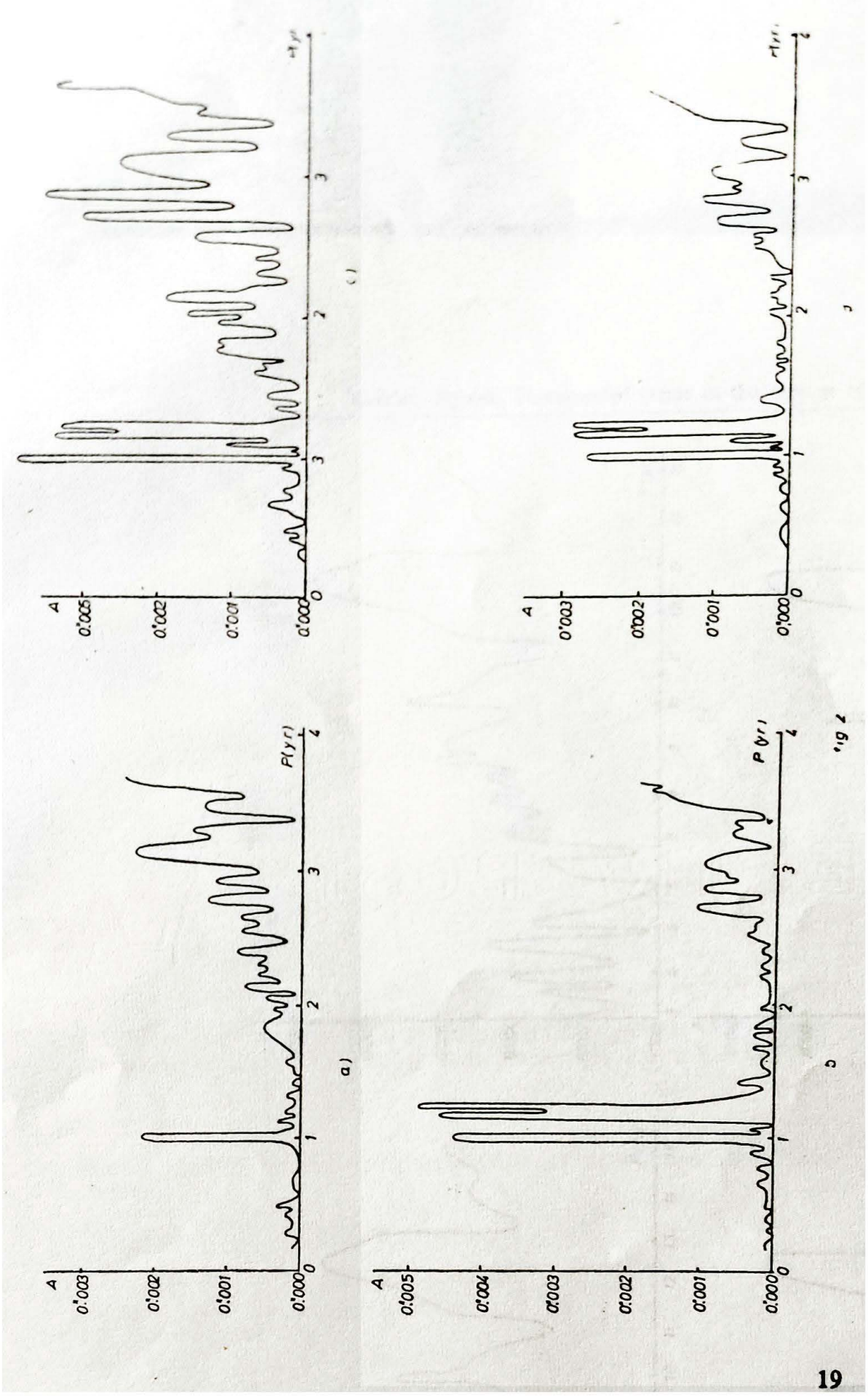

Fig 2

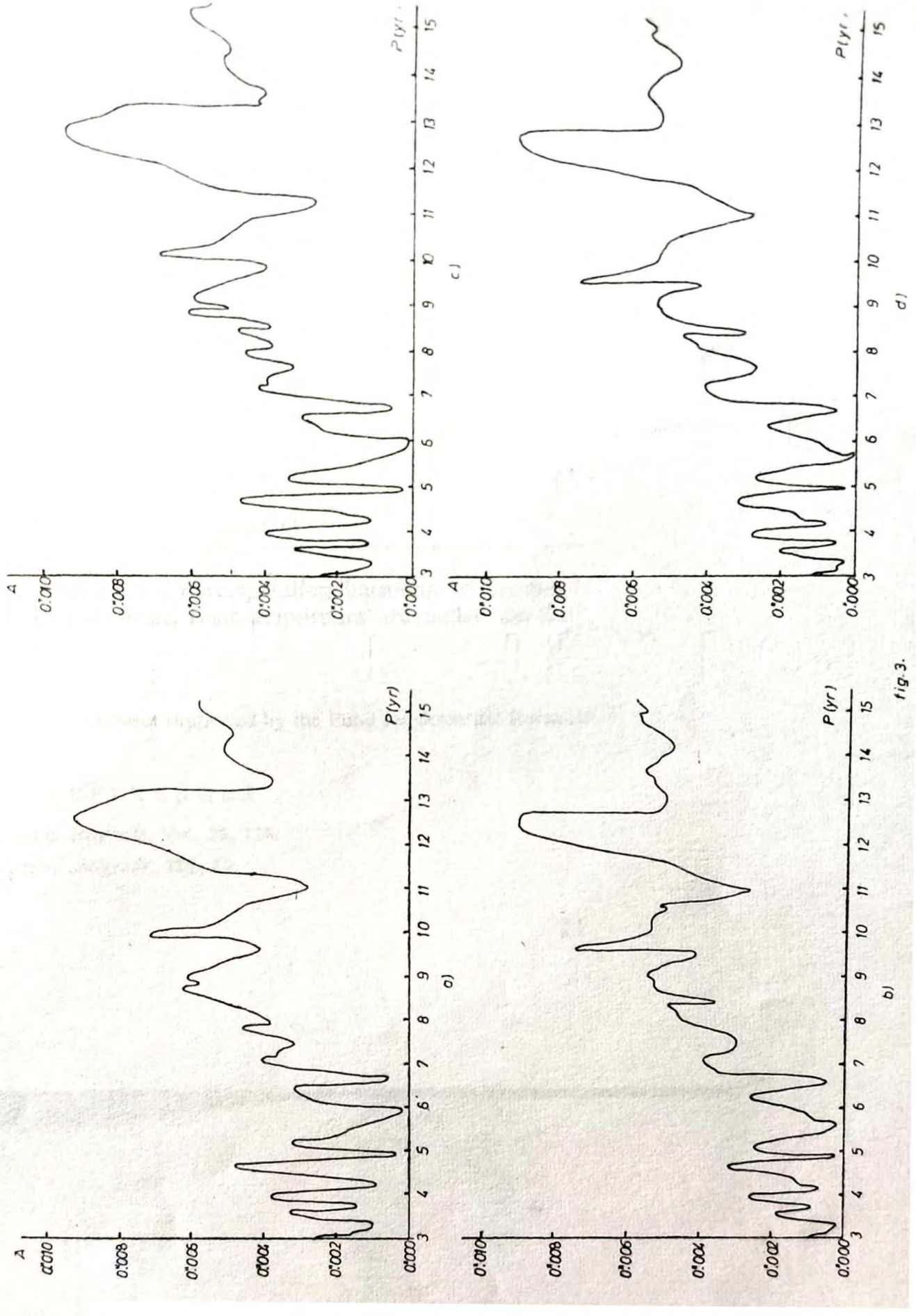

fig. 3.

TABLE II

Elementary Interval (1/12 y.)			Elementary		Interval (1/20 y.)	
	Coefficients		Coefficients		Coefficients	
i	„O"	„M"	„O"	„M"	i	„M"
0	1	—	1	—	28	3680
1	2	1	2	1	29	3991
2	2	4	2	4	30	4305
3	2	10	2	10	31	4619
4	2	20	2	20	32	4930
5	2	35	2	35	33	5235
6	2	56	2	56	34	5531
7	3	84	2	84	35	5815
8	4	120	2	120	36	6084
9	4	165	2	165	37	6335
10	—	220	2	220	38	6565
11	—	286	3	286	39	6772
12	—	363	4	364	40	6954
13	—	451	4	455	41	7109
14	—	549	4	560	42	7235
15	—	656	4	680	43	7331
16	—	770	—	816	44	7396
17	—	889	—	969	45	7429
18	—	1011	—	1139		
19	—	1133	—	1326		
20	—	1252	—	1530		
21	—	1365	—	1751		
22	—	1469	—	1988		
23	—	1561	—	2240		
24	—	1638	—	2506		
25	—	1698	—	2785		
26	—	1739	—	3075		
27	—	1760	—	3374		

to 0."010. Therefore the existence of the corresponding harmonic variations is not proved by these results. If they exist, their amplitudes are under the limit of 0."010.

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8 Atmospheric excitation of the polar motion: comparison of polar motion spectrum with spectra of atmospheric effective angular momentum functions

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The spectra of different combinations of the effective atmospheric angular momentum functions, determined by the USA National Meteorological Center, were compared with the spectrum of polar motion, determined from a combination of all available data by the Bureau International de l'Heure in Paris. The spectra were obtained by use of the fast Fourier transform. To compare the amplitudes and phases, the transfer function, derived for an Earth model with a fluid core, visco-elastic mantle and linearly reacting ocean, was used. It was found, for the dominating annual term, that the influence of the wind component is substantially smaller than would follow from the data sampled at midnight UT, but still significant. The combination of the resting wind term with the pressure term corrected for inverted barometer is, however, not sufficient to explain fully the observed polar motion with annual period. It is shown that the difference in phase can very probably be completely ascribed to the influence of ground water storage changes at this frequency.

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ATMOSPHERIC EXCITATION OF POLAR MOTION: COMPARISON OF THE POLAR MOTION SPECTRUM WITH SPECTRA OF EFFECTIVE ATMOSPHERIC ANGULAR MOMENTUM FUNCTIONS

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АТМОСФЕРНОЕ ВОЗБУЖДЕНИЕ ДВИЖЕНИЯ ПОЛЮСА: СРАВНЕНИЕ СПЕКТРА ДВИЖЕНИЯ ПОЛЮСА СО СПЕКТРАМИ АТМОСФЕРНЫХ ЭФФЕКТИВНЫХ ФУНКЦИЙ МОМЕНТА КОЛИЧЕСТВА ДВИЖЕНИЯ

Выполнено сравнение спектров разных комбинаций эффективных функций моментов количества движения атмосферы, определенных в Национальном метеорологическом центре США со спектром движения полюса по комбинированному решению Международного бюро времени в Париже. Для получения спектров использовался метод быстрого преобразования Фурье. При пересчете амплитуд и фаз использовалась передаточная функция, выведенная для модели Земли с жидким ядром, вязкоупругой мантией и линейно реагирующим океаном. Для важнейшей составляющей с годичным периодом было установлено, что хотя действительное влияние ветра существенно меньше, чем следует из данных отнесенных к гринвичской полуночи, оно все же остается значимым. Комбинация остаточной составляющей ветра с составляющей от влияния изменений атмосферного давления, исправленного за инвертный барометр, является, однако, не совсем достаточной для объяснения наблюдаемого движения полюса с годовым периодом. Показано, что разность фаз можно, по-видимому, полностью объяснить влиянием колебания уровня грунтовых вод с этой частотой.

The spectra of different combinations of the effective atmospheric angular momentum functions, determined by the National Meteorological Center of the USA, were compared with the spectrum of polar motion, determined from a combination of all available data by the Bureau International de l'Heure in Paris. The spectra were obtained by the method of fast Fourier transform. To compare the amplitudes and phases, the transfer function, derived for an Earth model with a fluid core, visco-elastic mantle and linearily reacting ocean, was used. It was found, for the dominating annual term, that the influence of the wind component is substantially smaller than would follow from the data sampled at midnight UT, but still significant. The combination of the resting wind term with the pressure term corrected for inverted barometer is, however, not sufficient to explain fully the observed polar motion with annual period. It is shown that the difference in phase can very probably be completely ascribed to the influence of ground water storage changes at this frequency.

Key words: Earth rotation — polar motion — atmospheric excitation

1. Introduction

Atmospheric excitation of the Earth's rotation has been subject to intensive investigation for a few decades. The earlier results and theoretical considerations of the problem are discussed in two principal monographies, devoted to the rotation of the Earth, by Munk and MacDonald (1960) and Lambeck (1980). Recently, when more detailed information about the atmosphere became available, a large number of papers, devoted to the problem from different points of view, appeared. Some authors mainly studied the influence of the atmosphere on the changes in the length of day (e.g. Lambeck and Cazenave 1973, 1974, 1977, Sidorenkov 1973 and 1979, Okazaki 1977, Rosen and Salstein 1983 and 1985, Feissel and Nitschelm 1985), some concentrated on the atmospheric effects in polar motion only (e.g. Wilson and Haubrich 1976a and 1976b, Jochmann 1976, 1981a and 1981b, Lambeck and Hopgood 1981, Wahr 1982 and 1983, Eubanks et al. 1987, Brzezinski 1987) and others studied both (e.g. Sidorenkov 1982, Barnes et al. 1983, Hide 1984, Dutton and Fallon 1985, Eubanks et al. 1985, Salstein 1987). The paper published by Barnes et al. (1983) provided a solid basis for regular calculations of so-called "effective angular momentum functions" (EAMF)

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of the atmosphere using the surface pressure and wind fields from the "initialized analysis global database" archieved by the European Centre for Medium-Range Weather Forecast. The framework laid out there is now also applied to the global analysis of the U.S. National Meteorological Center (Salstein 1985) and the Japan Meteorological Agency (Naito 1985).

Summarizing in short the results obtained so far, we can say that the short-periodic variations in the length-of-day (i.e. periods shorter than several years) are dominated by the atmospheric and tidal effects; the comparisons show excellent correlation between the axial component of the atmospheric EAMF and Earth rotation rate changes, corrected for the effects of the lunar and solar tides. The axial component of the atmospheric EAMF is, in turn, dominated by the effects of zonal winds. The most important changes are the seasonal variations (with annual and semi-annual periods) and the shorter-period irregular variations (with a cluster of periods between 40 and 70 days).

The situation is much less obvious if the excitation of polar motion is considered; the transfer function between the equatorial components of EAMF and polar motion is frequency-dependent with a resonance at the Chandler frequency, and it is difficult to decide which combination of wind and pressure terms would give the best representation of the atmospheric excitation of polar motion. Dominating is the pressure term; the effects of the winds, originally shown to be vanishing, due to their geostrophic character, by Munk and MacDonald (1960), were later proved to be non-zero (Barnes et al. 1983) but again not recommended to be used (Eubanks et al. 1985 and 1987) due to the unsatisfactory quality and high observational noise of the data. The pressure term itself is also not without problems; its values, calculated with and without "inverted barometer" correction, differ substantially, and there is also an indication that the application of this correction is appropriate only in the Northern Hemisphere (Eubanks et al. 1987). The attempts to use the calculated atmospheric EAMF to integrate polar motion numerically, made, e.g., by Barnes et al. (1983), or Dutton and Fallon (1985), led to results that fit the observed polar motion only for a very limited time interval (typically less than one period of revolution); both curves then diverge very rapidly. Therefore, we shall concentrate our attention on the investigation of atmospheric excitation of polar motion in the frequency domain, in order to find the best fitting combination of EAMF both in amplitude and phase.

2. Response of the Tri-axial, Visco-elastic Earth to Atmospheric Excitation and Observed Earth-rotation Parameters

The response of the non-rigid Earth to any excitation can be described by a set of Liouville equations (see, e.g., Munk and MacDonald 1960, or Lambeck 1980), which can be written, for a tri-axial Earth model, in linearized form (Vondrák 1987) as

$$(1)\qquad \begin{aligned} m_1 - \dot{m}_2 B/\Omega(C-A) &= \psi_1 \\ m_2 + \dot{m}_1 A/\Omega(C-B) &= \psi_2 \\ \dot{m}_3 &= \psi_3 , \end{aligned}$$

where the dot denotes the time derivative and

$$\psi_1 = (\Omega^2 c_{13} + \Omega \dot{c}_{23} + \Omega h_1 + \dot{h}_2 - L_2)/\Omega^2(C-A)$$
$$\psi_2 = (\Omega^2 c_{23} - \Omega \dot{c}_{13} + \Omega h_2 - \dot{h}_1 + L_1)/\Omega^2(C-B)$$
$$\psi_3 = (L_3 - \Omega \dot{c}_{33} - \dot{h}_3)/\Omega C$$

are well-known dimensionless excitation functions. A, B and C are the average values of principal moments of Earth's inertia, Ω the average angular velocity of rotation, m_1 and m_2 denote the instantaneous position of the pole in a right-handed coordinate reference frame pointed towards the directions of the equatorial principal axes of inertia (i.e. m_1 towards 14·9 °W), $m_3 = \omega/\Omega - 1$, where ω is the instantaneous angular velocity of rotation. The excitation functions ψ_i depend on the time-varying components of the Earth's tensor of inertia c_{ij}, of the relative angular momentum vector h_i and of the external torque L_i. Certain parts of the excitation functions ψ_i depend on the varying position of the axis of rotation within the Earth, or on the changes of the angular velocity of the rotation, and can thus be expressed in terms of m_i; this applies namely to the effects of the fluid core, rotational deformation of the mantle and the reaction of the ocean (see, e.g., Wahr 1983). All these changes have to be transferred from the rhs to the lhs of Eqs (1) before solving them. After some lengthy algebra, that can be found in detail in (Vondrák 1987), we arrive at the final solution which reads

$$(2)\qquad x = \exp(\alpha t)\{x_0 \cos \sigma t - (1{\cdot}006 y_0 - 0{\cdot}001 x_0) \sin \sigma t + \sigma(1 + \sigma/\Omega)[(1{\cdot}014 J_2 - 1{\cdot}019 J_3 - 0{\cdot}002 J_4) \cdot \cos \sigma t + (1{\cdot}019 J_1 + 0{\cdot}002 J_2 + 1{\cdot}014 J_4) \sin \sigma t]\}$$

$$y = \exp(\alpha t)\{y_0 \cos \sigma t + (0{\cdot}994 x_0 - 0{\cdot}001 y_0) \cdot \sin \sigma t - \sigma(1 + \sigma/\Omega)[1{\cdot}013 J_1 + 0{\cdot}002 J_3 +$$

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$$+ 1{\cdot}008J_4)\cos\sigma t - (0{\cdot}002J_1 + 1{\cdot}008J_2 - $$
$$- 1{\cdot}013J_3)\sin\sigma t]\}$$

$$\Delta\Lambda/\Lambda_0 = 0{\cdot}993\chi_3 + \text{const.},$$

where

$$J_1(t) = \int_0^t \chi_1 \exp(-\alpha\tau)\cos\sigma\tau\,\mathrm{d}\tau + $$
$$+ \Omega^{-1}[\chi_2(t)\exp(-\alpha t)\cos\sigma t - \chi_2(0)]$$
$$J_2(t) = \int_0^t \chi_2 \exp(-\alpha\tau)\cos\sigma\tau\,\mathrm{d}\tau - $$
$$- \Omega^{-1}[\chi_1(t)\exp(-\alpha t)\cos\sigma t - \chi_1(0)]$$
$$J_3(t) = \int_0^t \chi_1 \exp(-\alpha\tau)\sin\sigma\tau\,\mathrm{d}\tau + $$
$$+ \Omega^{-1}\chi_2(t)\exp(-\alpha t)\sin\sigma t$$
$$J_4(t) = \int_0^t \chi_2 \exp(-\alpha\tau)\sin\sigma\tau\,\mathrm{d}\tau - $$
$$- \Omega^{-1}\chi_1(t)\exp(-\alpha t)\sin\sigma t\,.$$

In Eqs (2) x, y and $\Delta\Lambda/\Lambda_0$ denote the coordinates of the excited pole (oriented towards zero meridian and 90 °E) and the relative change in length-of-day, x_0 and y_0 are coordinates of the pole at the instant $t = 0$ (constants of integration), the damping factor $\alpha = -0{\cdot}0074Q^{-1}$ day^{-1} accounts for the dissipation in the mantle (Q is a dimensionless quality factor), $\sigma \doteq \Omega(0{\cdot}936 - k/k_s)(C - A)/A_m = 0{\cdot}01444$ rad/day is the Chandler frequency and $\Omega \doteq 6{\cdot}300$ rad/day is the average velocity of the Earth's rotation. χ_i are the "effective angular momentum functions" (EAMF) introduced by Barnes et al. (1983) which replace the values c'_{ij} and h'_i (primes denoting that the values are referred to the reference system with the x-axis pointing along zero meridian. Their mutual relations are

(3)

$$\chi_1 = [(1 + k')\Omega c'_{13} + h'_1]/\Omega(C_m - A_m)(1 - k/k_s)$$
$$\chi_2 = [(1 + k')\Omega c'_{23} + h'_2]/\Omega(C_m - A_m)(1 - k/k_s)$$
$$\chi_3 = [(1 + k')\Omega c'_{33} + h'_3]/\Omega C_m\,,$$

where A_m, C_m are the principal moments of inertia of the mantle alone, $k \doteq 0{\cdot}30$, $k' \doteq -0{\cdot}30$ and $k_s \doteq$ $\doteq 0{\cdot}94$ are elastic, load and secular Love numbers. Equations (3) are based on the following assumptions:

i) The Earth is rotationally symmetric and an ideally elastic body.

ii) The fluid core is completely decoupled from the mantle, and thus cannot follow the changes in the motion of the mantle,

iii) The Earth's surface is loaded only by c_{ij} (changes in the tensor of inertia) while h_i (relative angular momentum) has no loading effect.

The small deviations of the numerical constants in Eqs (2) from unity or zero express the fact that the real Earth is equatorially flattened and has an ocean that is non-uniformly spread over the Earth's surface. The main feature of the solution, given by Eqs (2), is the body- and ocean-induced ellipticity of the free wobble, whose semi-major axis points to 7 ° E and ellipticity equals 0·006. In the absence of excitation, the amplitude of the free wobble is slowly damped. The excited polar motion is also elliptic, even in case of circular excitation.

2.1. *Atmospheric Effective Angular Momentum Functions*

The EAMF of the atmosphere are now calculated routinely at several meteorological centers from the formulae proposed by Barnes et al. (1983); they do not depend on the time derivatives of c_{ij}, h_i and thus can readilly be calculated from available meteorological data (pressure, wind):

(4)

$$\chi = \chi_1 + i\chi_2 =$$
$$= -\frac{1{\cdot}00R^4}{g(C_m - A_m)}\int_S p_s \exp(i\lambda)\sin\varphi\cos^2\varphi\,\mathrm{d}\lambda\,\mathrm{d}\varphi - $$
$$- \frac{1{\cdot}43R^3}{g\Omega(C_m - A_m)}\int_V (u\sin\varphi + iv)\exp(i\lambda)\cos\,.$$
$$.\,\varphi\,\mathrm{d}\lambda\,\mathrm{d}\varphi\,\mathrm{d}p$$

$$\chi_3 = \frac{0{\cdot}70R^4}{gC_m}\int_S p_s\cos^3\varphi\,\mathrm{d}\lambda\,\mathrm{d}\varphi + $$
$$+ \frac{1{\cdot}00R^3}{g\Omega C_m}\int_V u\cos^2\varphi\,\mathrm{d}\lambda\,\mathrm{d}\varphi\,\mathrm{d}p\,.$$

The integrals with subscript S are taken over the Earth's surface, with subscript V over the volume of the atmosphere, φ, λ and p_s denote the geographical latitude, longitude and air pressure at the Earth's surface, u and v the eastward and northward components of the velocity of the wind. $R = 6370$ km is the mean Earth's radius, $C_m = 7{\cdot}04 \times 10^{37}$ kg m^2 the axial moment of inertia of the Earth's mantle, $C_m - A_m = 0{\cdot}00333C_m$ and $g = 9{\cdot}81$ m s^{-2} is the average gravitational acceleration at the Earth's surface. The first term is often referred to as the pressure term while the second as the wind term; they express the influence of redistribution of the air mass and its relative angular momentum, respectively. Some of the meteorological centers also calculate the pressure term using the "inverted barometer" hypothesis; the ocean surface is supposed to be locally depressed by a local increase of atmospheric pressure and vice versa and, consequently, the total atmospheric and

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ocean effect is supposed to be constant over the whole surface of each ocean. Therefore, we shall investigate the following five combinations of EAMF:

a) $\chi_i^P + \chi_i^W$

b) $\chi_i^{P'} + \chi_i^W$

c) χ_i^P

d) $\chi_i^{P'}$

e) χ_i^W,

where the superscripts P and W denote the pressure and wind terms, respectively, and P' denotes the pressure term calculated using the inverted barometer hypothesis. The EAMF calculated by the U.S. National Meteorological Center, provided by courtesy of the U.S. National Geodetic Survey (Fallon 1986) will be used throughout the following study. The EAMF (midnight data only) in the interval 1977·0 to 1986·0 are displayed in Figs 1–3; wind terms in Fig. 1 and their sums with pressure terms without and with the inverted barometer correction in Figs 2 and 3, respectively. It is clear that all three data sets displayed are quasi-periodic functions of time, with a pronounced annual term. Both equatorial components χ_1 and χ_2 are much noisier than the axial component χ_3. The wind term dominates the axial component while the pressure term seem to prevail in the equatorial components. The pressure term in χ_2 has a significant constant part, obviously due to the non-symmetric distribution of the average pressure between the Western and Eastern Hemisphere. The inverted barometer correction tends to diminish the magnitude of the fluctuations in χ_1 and χ_2 while it seems to contribute only with a small constant term to χ_3. Notice that χ_3 as given by the NMC has the opposite sign to that defined by Barnes et al. (1983) – compare with (Salstein 1985).

2.2. *Observed Earth Rotation Parameters*

Earth rotation parameters (ERP), i.e. polar motion and Universal time, are routinely determined from various sets of observations (optical astrometry, Doppler tracking of satellites, laser ranging to the Moon and to satellites and very long-base interferometry) at several world centers. The Bureau International de l'Heure (BIH) in Paris concentrates these data, combines and disseminates them. In the present study, we shall use the BIH combined solution as published in (BIH Annual Reports for 1977 through

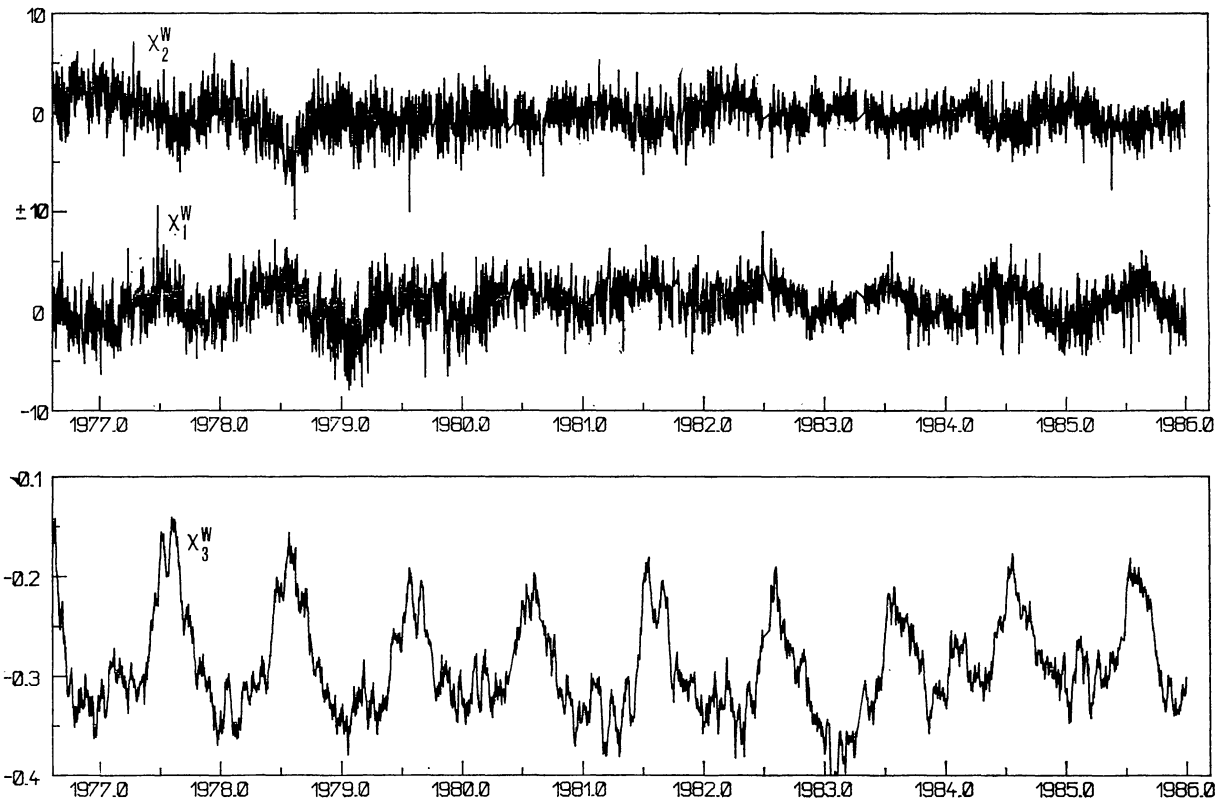

Fig. 1. Wind terms of atmospheric effective angular momentum functions as calculated at NMC. Only the midnight data are displayed.

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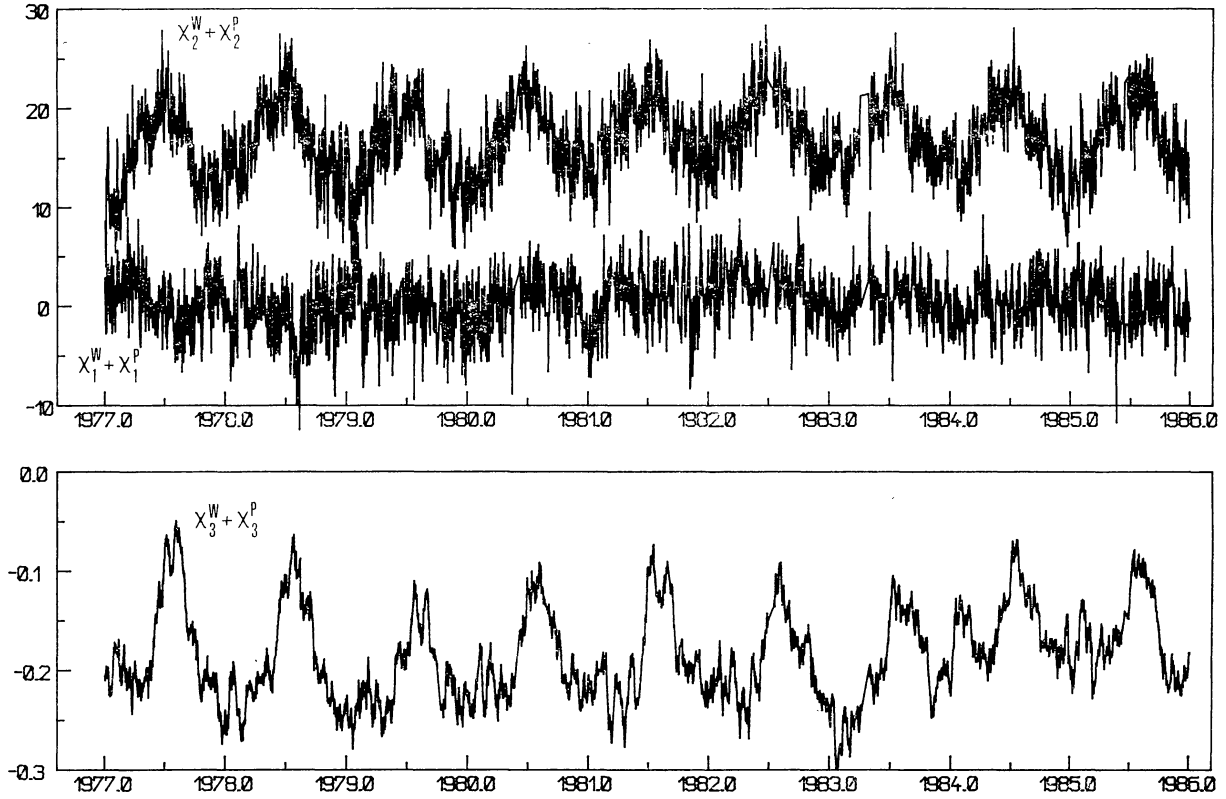

Fig. 2. Wind plus pressure terms of atmospheric effective angular momentum functions as calculated at NMC.

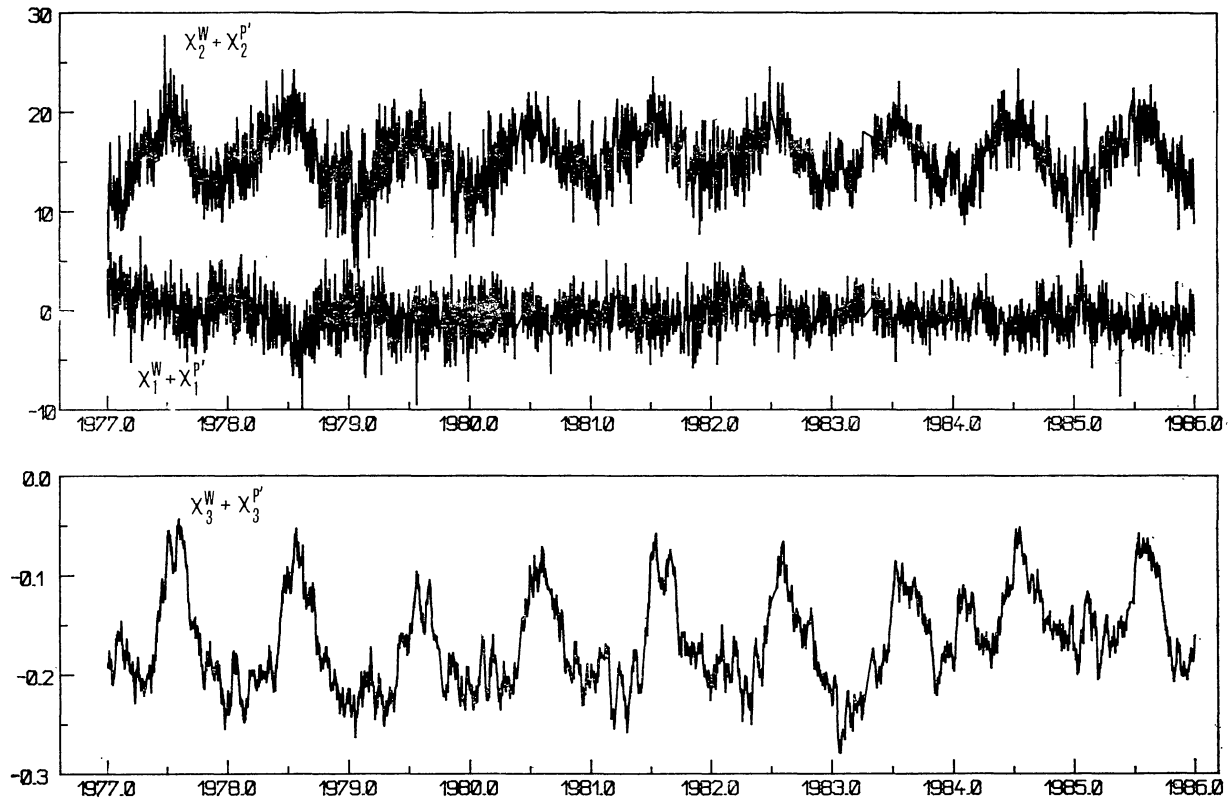

Fig. 3. Wind plus pressure terms with the inverted barometer correction of atmospheric effective angular momentum functions as calculated at NMC.

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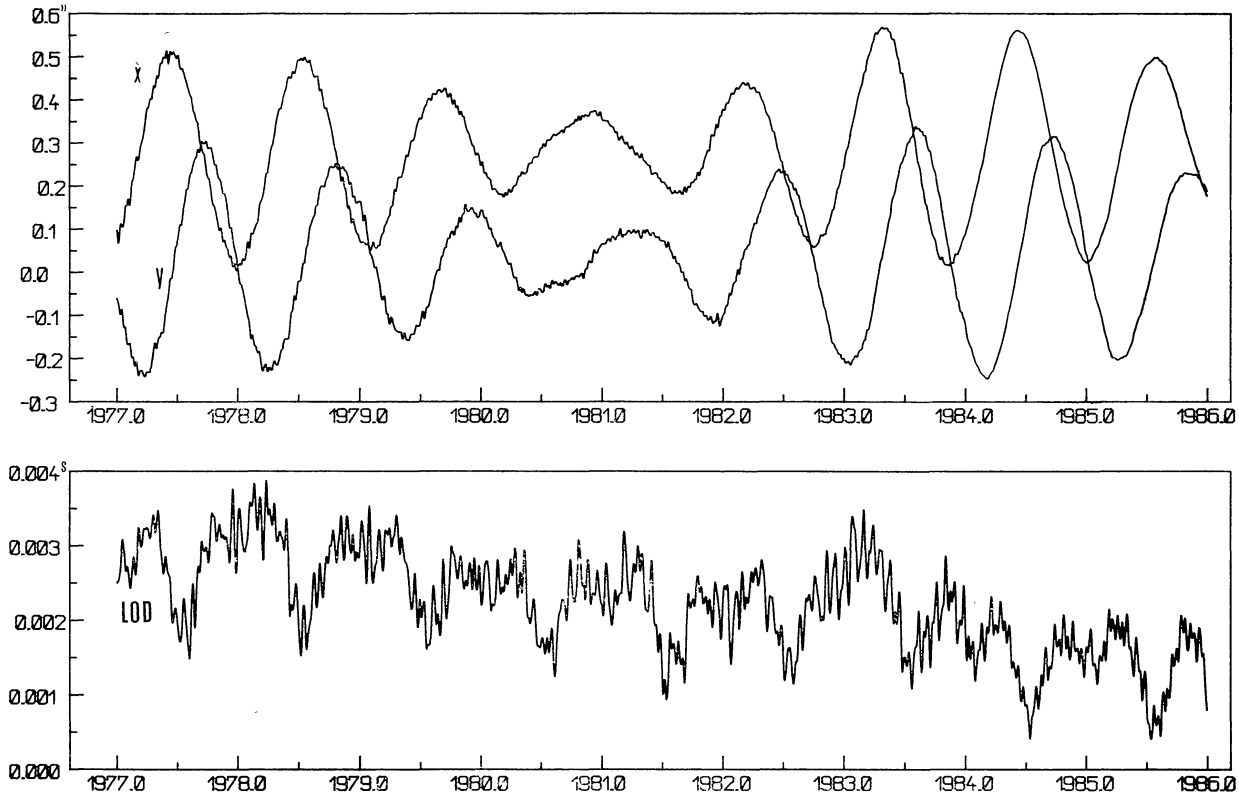

Fig. 4. Polar motion (x, y) and the excess of length-of-day over the nominal 86 400^s (LOD) as determined from astronomical observations at the BIH in Paris. Combined raw data are used, LOD data are based on published UT 1—UTC values.

1985, Table 6) to cover the same time interval as the EAMF. It should be noted that the combined solution, based mostly on optical astrometry at the very beginning, is more and more influenced by modern space techniques of observations which are more precise, especially in the short-period part of the spectrum (Vondrák, 1986). The ERP are displayed in Fig. 4; the ever increasing precision of the data in short periods is demonstrated by the obvious fact that the observed polar motion is smoother and smoother towards the end of the interval studied. The excess of the length-of-day (LOD) over the nominal 86 400 sec has been calculated from the BIH published values UT 1—UTC; its rather complex structure is the combination of the variations caused by zonal tides of the Earth (Yoder et al., 1981) and of atmospheric excitation. We shall not investigate the variations in LOD further, but rather confine our study to polar motion which is more controversial.

3. Comparison of Polar Motion Spectra

First, consider the first two of Eqs (2). We see that the deviations of the numerical constants differ from unity or zero only by very small amounts; we can further simplify the relations by neglecting these differences and introducing complex quantities for both the polar motion data $(X = x + \mathrm{i}y)$ and equatorial components of EAMF $(\chi = \chi_1 + \mathrm{i}\chi_2)$. The equations in complex notation then read:

$$X(t) = \exp[(\alpha + i\sigma)\, t] \,. \tag{5}$$
$$\cdot \{X(0) - i\sigma(1 + \sigma/\Omega) \int_0^t \chi(\tau) \exp[-(\alpha + i\sigma)\, \tau]\, \mathrm{d}\tau\} -$$
$$- \sigma\Omega^{-1}\{\chi(t) - \chi(0) \exp[(\alpha + i\sigma)\, t]\} \,.$$

Of course, this equation can easily be used, together with the numerically obtained values of the integral on the rhs, to calculate the instantaneous pole position $X(t)$ at any time instant t, provided that its initial position $X(0)$ is known. Nevertheless, such an approach leads to unsatisfactory results; our preliminary experiments in this respect show a very rapid divergence of the results, obtained in this way, from the observed polar motion (see also Dutton and Fallon, 1986). Therefore, we decided to have a closer look at both the processes in frequency domain in order to compare both the amplitudes and phases of the dominant terms.

Assume that the equatorial EAMF χ can be developed into a series of periodic terms,

$$\chi(t) = \sum_j a_j \exp[\mathrm{i}(\omega_j t + \Phi_j)] \,, \tag{6}$$

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where a_j are real constants (amplitudes) Φ_j phases and ω_j angular velocities that can be both positive and negative (remember that any elliptic motion can be expressed as a sum of two circular motions with different amplitudes and angular velocities differing only in sign). Then, introducing (6) into (5) and integrating, we arrive at

$$X(t) = \exp[(\alpha + i\sigma)t] \cdot \{X(0) - \sum_j a_j F_j \exp[i(\Phi_j + \Delta\Phi_j)]\} + \sum_j a_j F_j \exp[i(\omega_j t + \Phi_j + \Delta\Phi_j)], \tag{7}$$

where

$$F_j^2 = \sigma^2[\alpha^2 + (\Omega + \omega_j)^2]/\Omega^2[\alpha^2 + (\sigma - \omega_j)^2]$$

and

$$\tan \Delta\Phi_j = \alpha(\Omega + \sigma)/[(\Omega + \omega_j)(\sigma - \omega_j) - \alpha^2].$$

The first term of eq. (7) describes the damped free polar motion with angular velocity σ and the second the sum of forced motions with angular velocities ω_j. The quantities F_j and $\Delta\Phi_j$, that are both real functions of angular velocities ω_j, express the ratio of the amplitude of the excited pole to that of EAMF and the phase shift between these two, respectively.

First consider the phase shift $\Delta\Phi$. It is clear that it changes rapidly near two singular points, $\omega \approx -\Omega$ and $\omega \approx \sigma$, since α is a very small negative factor. The first singular point is less important because there is practically no atmospheric excitation with angular velocities near $-\Omega$; $\Delta\Phi \approx 0$ for ω in the interval $(-\Omega; \sigma)$ and $\Delta\Phi \approx \pi$ for $\omega > \sigma$. This means that the excited pole is in phase with the excitation for frequencies lower than the Chandler frequency (including the negative ones), but in opposite phase for higher frequencies. The transfer function F (for $Q = 100$) is plotted on a logarithmic scale against frequency $f = \omega/2\pi$ in Fig. 5. The graph shows

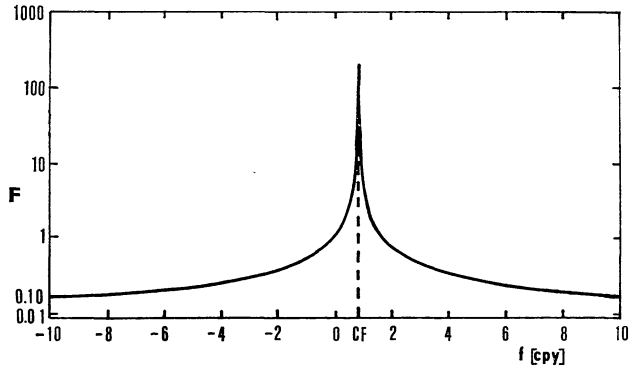

Fig. 5. Transfer function between the effective angular momentum function and polar motion for different frequencies. CF is Chandler wobble frequency, qualitative factor of the mantle $Q = 100$ was used.

that the excitations with high frequencies (both positive and negative) are substantially suppressed, while the excitations near the Chandler frequency are amplified. This explains the observed fact that the polar motion is surprisingly smooth. The transfer function F, which would go to infinity for $\omega = \sigma$ in case of a perfectly rigid or elastic Earth, is finite, due to dissipation in the mantle ($\alpha \neq 0$); for $Q = 100$ $F(\sigma) \doteq 195$.

From eq. (7) it follows that the amplitude spectra of χ and X are not directly comparable; the spectrum of χ must be multiplied by the transfer function F and only then compared with the spectrum of X. The spectrum S of complex function X, sampled at equidistant time intervals Δt, was obtained as a discrete Fourier transform

$$S(\omega) = \frac{1}{N} \sum_1^N X(t_j) \exp(-i\omega t_j), \tag{8}$$

where $t_j = (j - 1)\Delta t$. The Cooley-Tukey algorithm for the fast Fourier transform (*FFT*) was used to calculate the spectra S from eq. (8) in order to save computational time. The data were padded with zeros to complete their number to the integral power of 2 (4096 in the case of polar motion and 16 384 in the case of atmospheric EAMF). No windowing to suppress sidelobes was applied. An experiment with the Parzen window applied to polar motion data showed a severe decrease of amplitudes of both the Chandler and annual term with respect to non-windowed data, evidently because windowing gives the largest weight to the data in the center of the whole interval where, in our special case, the polar motion amplitude was extremely small. The plots of amplitude spectra $|S(\omega)|$ are displayed in Figs 6 and 7, together with their image obtained with the help of the transfer function F discussed above. The spectra are given only in the interval $\langle -4; 4\rangle$ cpy since they are practically null outside this interval.

Figure 6 shows the original polar motion spectrum (in the upper part) with two peaks dominating at the Chandler and annual frequencies and the same spectrum divided by F (in the lower part of the figure), showing the EAMF which would be necessary to excite the observed polar motion. The regular sidelobes, that are obviously only artefacts of the Fourier transform technique, are amplified to such an extent that the hypothetical excitation would have an unrealistically complex character.

Figures 7a–e show the amplitude spectra of atmospheric EAMF (upper parts) and the same spectra as multiplied by F (lower parts) for the five combinations of EAMF defined in Section 2.1. The similarities of all five spectra are evident; there are always two

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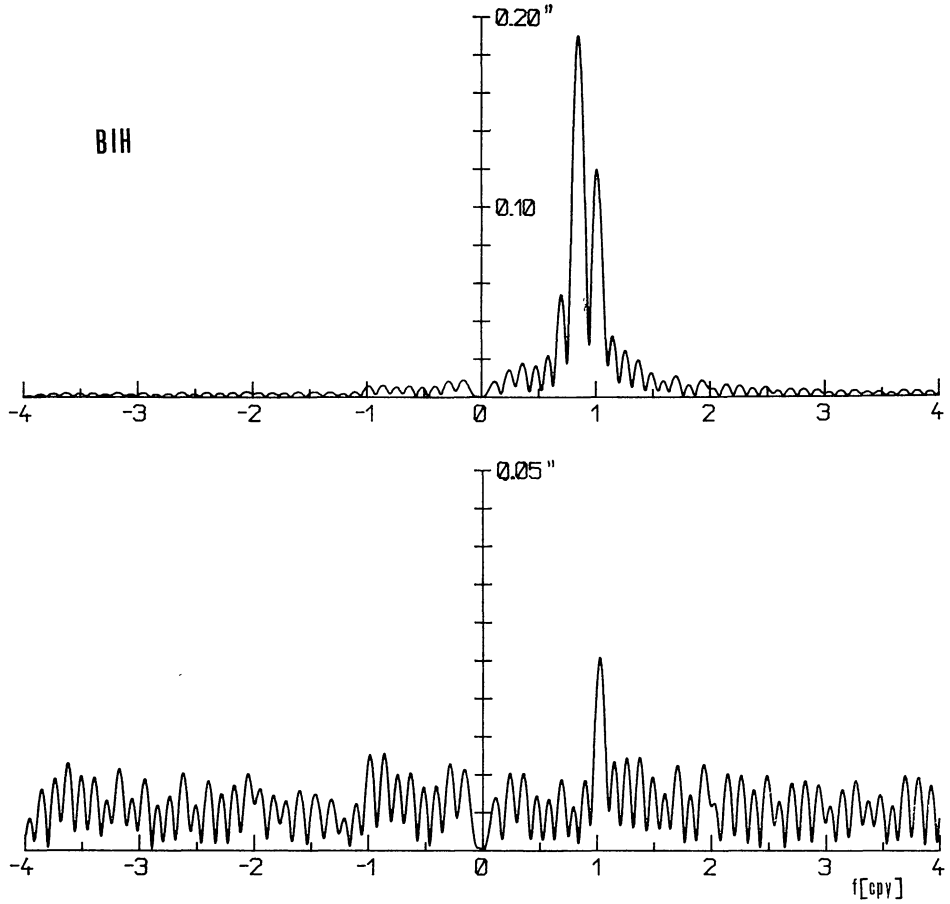

Fig. 6. Polar motion spectrum of BIH data from Fig. 4 and its recalculated version after division by the transfer function F from Fig. 5.

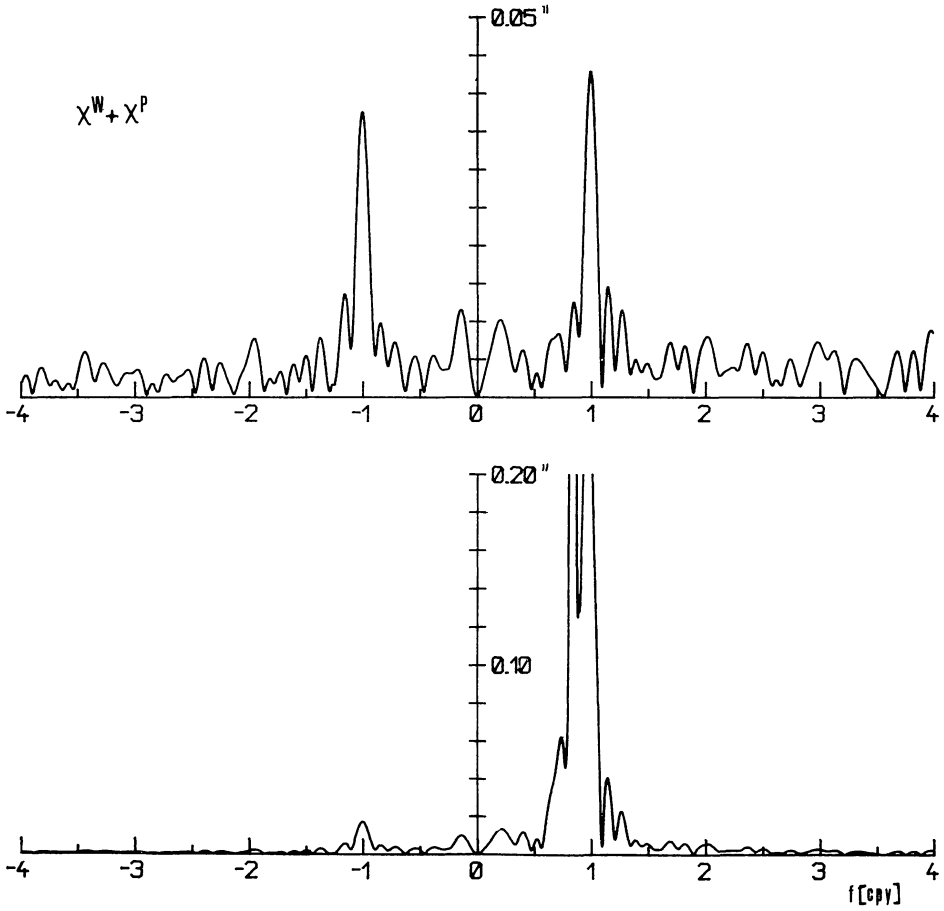

Fig. 7a. Spectrum of equatorial components of EAMF (wind + pressure terms) and its recalculated version (i.e. multiplied by transfer function F).

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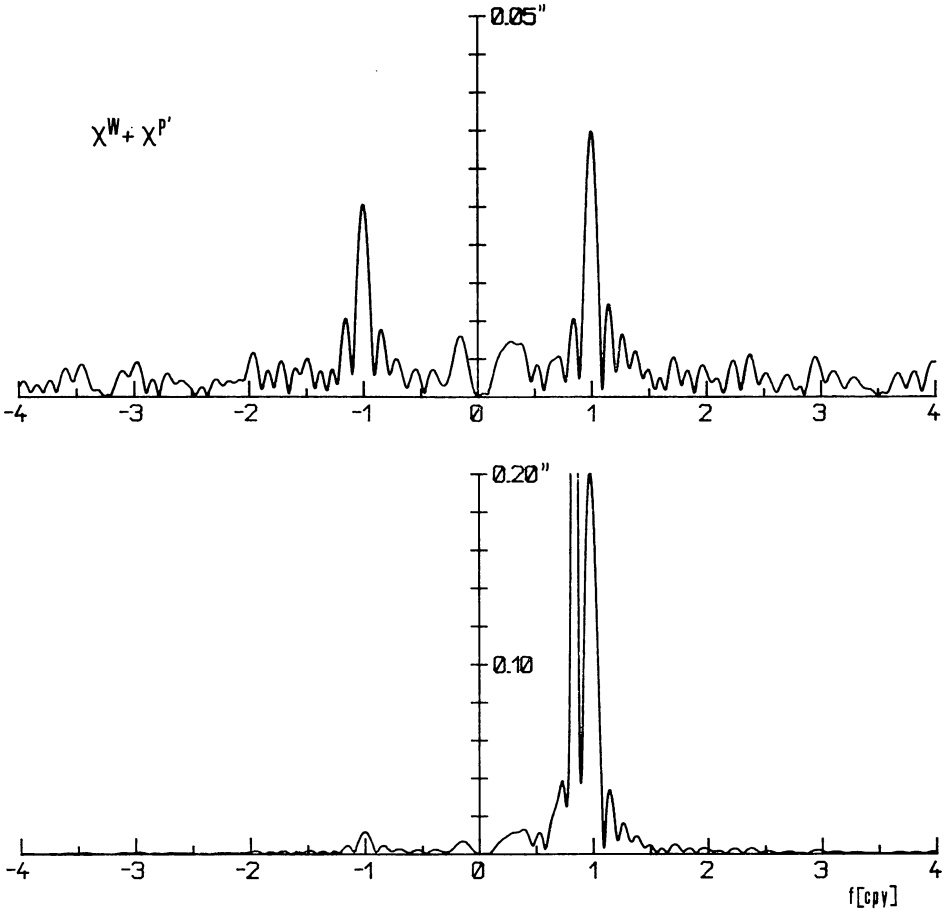

Fig. 7b. Spectrum of equatorial components of EAMF (wind + pressure terms with inverted barometer correction) and its recalculated version (i.e. multiplied by transfer function F).

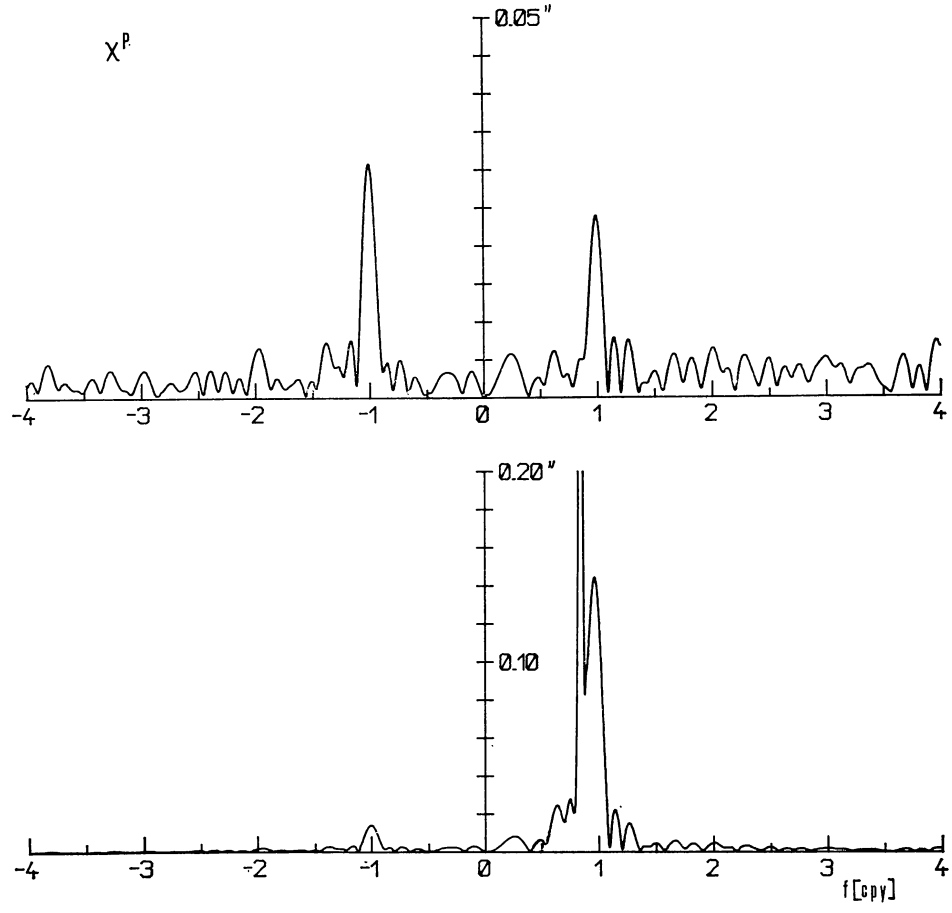

Fig. 7c. Spectrum of equatorial components of EAMF (pressure terms) and its recalculated version (i.e. multiplied by transfer function F).

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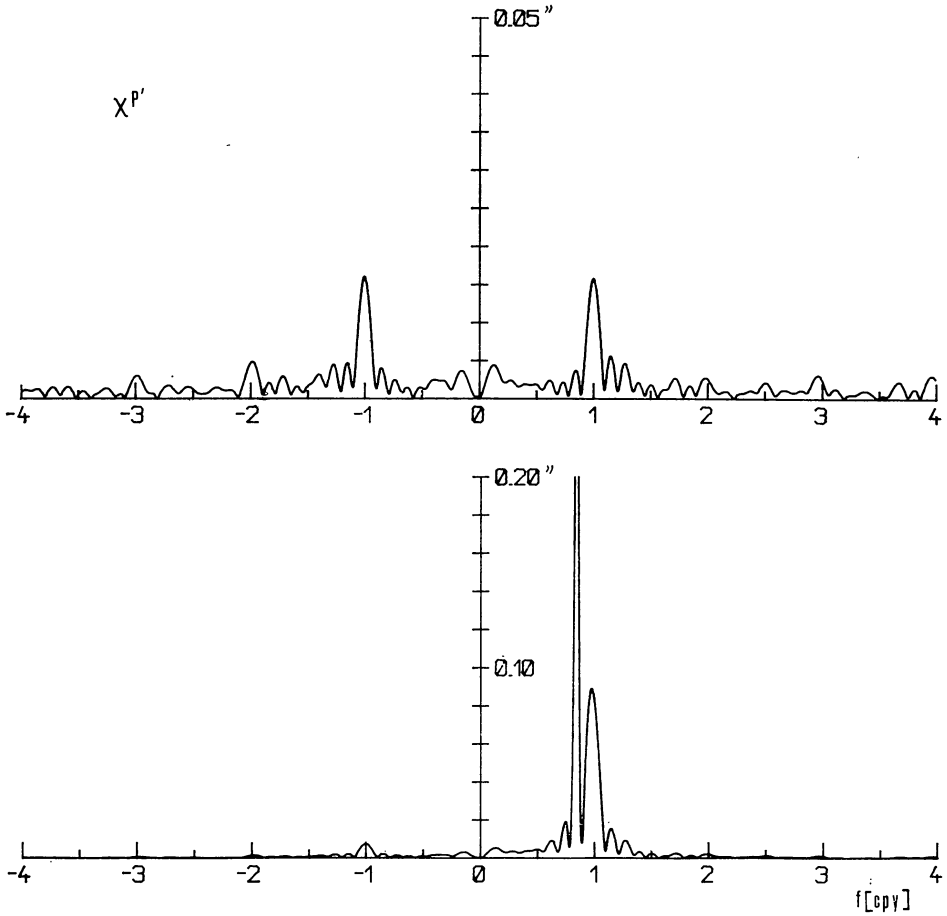

Fig. 7d. Spectrum of equatorial components of EAMF (pressure terms with inverted barometer correction) and its recalculated version (i.e. multiplied by transfer function *F*).

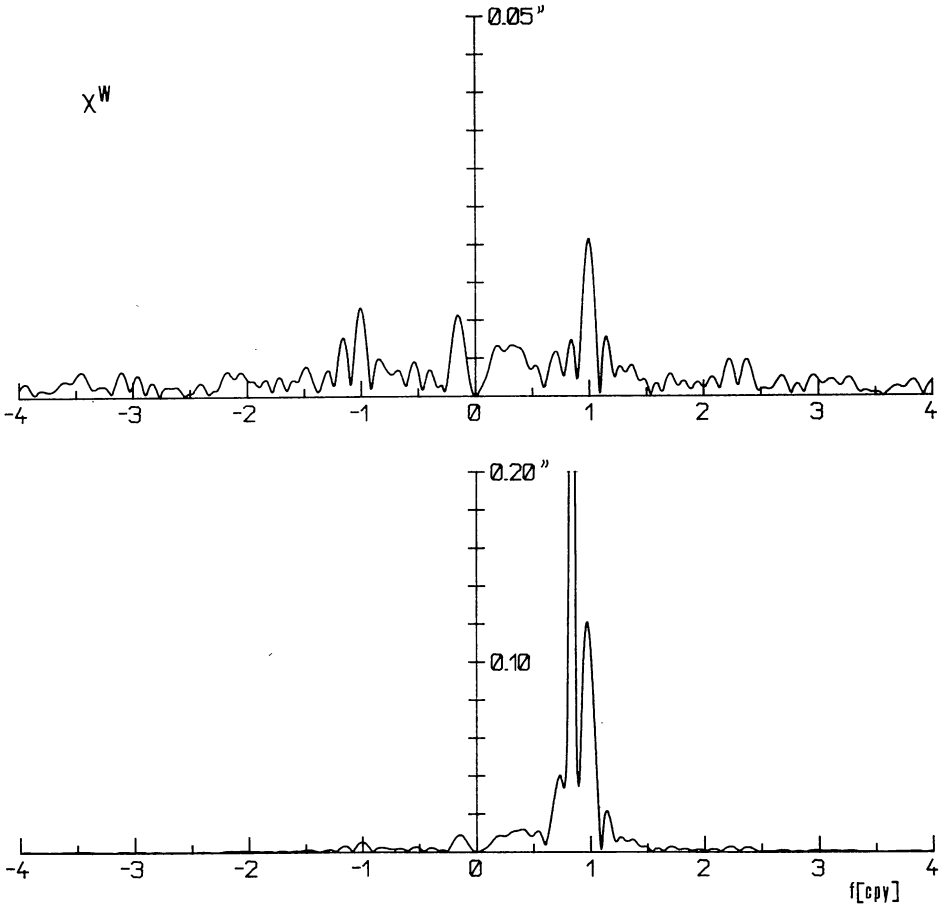

Fig. 7e. Spectrum of equatorial components of EAMF (wind terms) and its recalculated version (i.e. multiplied by transfer function *F*).

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Table 1

Parameters of annual polar motion — observed (BIH) and excited by different combinations of EAMF, sampled at 0^h UT. A, Φ denote the amplitude and initial phase of positive (+) and negative (−) circular motion, a and b are the semi-major and semi-minor axes of the elliptic motion, φ is the direction of the semi-major axis and ψ denotes the initial phase.

	A^+	Φ^+	A^-	Φ^-	a	b	φ	ψ
BIH	0·084″	115°	0·005″	266°	0·089″	0·079″	+10°	116°
a) $W+P$	0·222″	94°	0·017″	264°	0·239″	0·205″	− 1°	95°
b) $W+P'$	0·181″	98°	0·011″	281°	0·192″	0·170″	+10°	98°
c) P	0·123″	78°	0·014″	248°	0·137″	0·109″	−17°	79°
d) P'	0·082″	78°	0·007″	261°	0·089″	0·075″	−10°	78°
e) W	0·107″	113°	0·005″	310°	0·112″	0·102″	+32	112°

dominant peaks at both positive and negative 1 cpy frequency of about the same amplitude (which witness the rather oscillatory character of the excitation at this frequency) and a group of smaller peaks around 0·2, 2, 3 and 4 cpy. The amplitudes, nevertheless, differ for different combinations of the EAMF. The recalculated spectra in the lower parts of Figs 7a–e are meant to be compared with Fig. 6 – the similarity is again obvious but we see that the wind term alone (Fig. 7e) is sufficient to excite the annual wobble of the observed amplitude. On the other hand, the pressure term alone (Fig. 7c) is too big and the pressure term with the inverted barometer correction (Fig. 7d) is too small to excite the observed polar motion at this frequency. The combinations of the pressure and wind terms (both with and without inverted barometer) can excite the wobble of substantially higher amplitudes than the value observed. In order to compare the other parameters of annual wobble as well, Tab. 1 was compiled. It shows the parameters of the elliptic annual motion both for the observed polar motion (BIH) and for all the five combinations of EAMF (a–e). The elliptic annual motion can be expressed as the sum of two circular motions with opposite angular velocities and different amplitudes:

$$(9) \qquad X_a(t) =$$

$$= A^+ \exp\left[i(2\pi t + \Phi^+)\right] + A^- \exp\left[i(-2\pi t + \Phi^-)\right],$$

where t is in years, A^+, A^-, Φ^+, Φ^- are the amplitudes and phases, estimated from the spectral analysis, $S = A \exp(\Phi)$, at the frequencies $f = \pm 1$ cpy. From these four basic values we can further derive both semi-major and semi-minor axes a, b, the direction of the semi-major axis φ and the phase ψ at the instant $t = 0$:

$$(10) \qquad a = A^+ + A^-, \quad b = |A^+ - A^-|,$$

$$\varphi = (\Phi^+ + \Phi^-)/2,$$

$$\tan\psi = (A^+ \sin\Phi^+ + A^- \sin\Phi^-)/(A^+ \cos\Phi^+ + A^- \cos\Phi^-).$$

The estimation of A^+, Φ^+ of the annual term in polar motion (see Fig. 6) is inevitably influenced by the presence of the neighbouring Chandler term; the sidelobes of both peaks, which are present in the Fourier transform, necessarily slightly affect the neighbouring peaks, both in amplitude and phase (see e.g. Jochmann 1986). If we assume that there are only two terms with angular velocities ω_1 and ω_2 and complex amplitudes a_1, a_2, we have

$$(11) \qquad S(\omega) = \frac{1}{T}\int_0^T [a_1 \exp(\omega_1 t) + a_2 \exp(\omega_2 t)] \cdot$$

$$\cdot \exp(-i\omega t)\, dt = a_1 K(\omega_1, \omega) + a_2 K(\omega_2, \omega),$$

where

$$K(\omega_j, \omega) = \{\exp[i(\omega_j - \omega)T] - 1\}/i(\omega_j - \omega)T =$$

$$= [\sin T(\omega_j - \omega) + 2i \sin^2 T(\omega_j - \omega)/2]\,/T(\omega_j - \omega).$$

The spectral images $S_1 = S(\omega_1)$, $S_2 = S(\omega_2)$, from the Fourier transform, are then

$$(12) \qquad S_1 = a_1 + K(\omega_2, \omega_1)\, a_2$$

$$S_2 = K(\omega_1, \omega_2)\, a_1 + a_2$$

and, hence, putting $K = K(\omega_2, \omega_1)$, $K^* = K(\omega_1, \omega_2)$ (notice that K, K^* are conjugate complex numbers),

$$(13) \qquad a_1 = (S_1 - KS_2)/(1 - KK^*)$$

$$a_2 = (S_2 - K^* S_1)/(1 - KK^*).$$

Using the values $\omega_1 = 5{\cdot}276$ rad/year, $S_1 = -0{\cdot}179'' + 0{\cdot}045''$ i (Chandler term), $\omega_2 = 6{\cdot}283$ rad/year, $S_2 = -0{\cdot}030'' + 0{\cdot}112''$ i (annual term), $T = 9$ yerars, we arrive at $K = 0{\cdot}040 + 0{\cdot}213$i, $K^* = 0{\cdot}040 - 0{\cdot}213$i and at the corrected values of the

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amplitudes $a_1 = -0{\cdot}188'' + 0{\cdot}049''\mathrm{i}$, $a_2 = -0{\cdot}036'' + 0{\cdot}076''$ i. These values are displayed in Tab. 1 in the row denoted BIH. From Tab. 1 we can see the relatively good agreement of the observed annual wobble parameters with those calculated from the EAMF in all the five combinations. The amplitudes and initial phases are displayed graphically in Fig. 8. Theoretically, the best agreement should be obtained for variant b (i.e. wind term plus pressure term with inverted barometer correction, denoted as $W + P'$ in Fig. 8). The table and figure show that only the direction of the semi-major axis of this term agrees satisfactorily; the difference in amplitude and initial phase is too large. The amplitude of the pressure

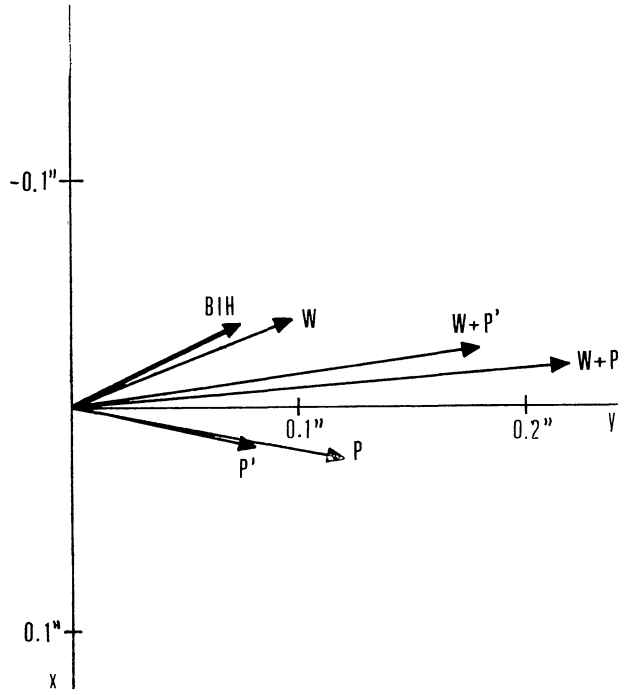

Fig. 8. Annual term in polar motion observed (BIH) and excited by the wind term sampled at midnight (W), pressure term (P), pressure term with the inverted barometer correction (P') and their combinations ($W + P$, $W + P'$).

term with the inverted barometer correction (P') is almost precisely equal to the observed amplitude, but the difference in phase ψ is much larger than for all the other EAMF combinations. Surprisingly, the best agreement has been achieved for variant e (i.e. the wind term alone, denoted in Fig. 8 as W).

4. Discussion and Conclusions

In the preceding section we found that none of the five combinations of the EAMF discussed so far is able to excite precisely the observed polar motion at the annual frequency (see Tab. 1 and Fig. 8). The phases ψ roughly correspond, the largest differences in phase have been found for pressure terms (variants c and d) – almost 40 degrees. The best agreement in phase between the excitation and observation is achieved for the wind term (variant e), though the difference in the directions of the semi-major axes is rather large. The amplitude of the wind term is also very close to the observed value (compare the rows BIH and e). However, the annual component of the wind term, as given in Tab. 1, is calculated from the midnight data only. Eubanks et al. (1987) argued that the equatorial component of the wind term is opposite in phase at the annual frequency, if calculated from the data sampled at noon and, consequently, they did not recommend the use of the equatorial wind term at all. Indeed, our own calculations confirm their findings; the amplitudes A^+, A^- of the annual wind term are smaller if calculated from the noon data only (0·070″ and 0·007″) but the phases Φ^+, Φ^- are nearly opposite (288° and 75°). This seems to rule out the wind term (variant e) as the candidate to excite the annual wobble, as well as its sums with both pressure terms (variants a and b). If there were some effects of the wind on polar motion, they would surely be much smaller than shown in the preceding section. On the other hand, the pressure term with the inverted barometer correction alone is sufficient to generate the whole observed annual wobble, but there is a difference in phase.

Suppose, for a moment, that the wind term has a hypothetical component with a period equal to one sidereal day. Then, due to the sampling interval being equal to one solar day and to the aliasing effect, this component will manifest itself as an annual term, and will combine with any actually existing annual term. If this is the case, we can get rid of this part of the annual term easily by averaging the two annual terms obtained from the two data sets, sampled at daily intervals but mutually shifted by 12 hours. The result can be seen in Tab. 2 (second row), together with its sum with the pressure term, corrected for the inverted barometer (ATM). This very term, in our opinion the best estimation of atmospherically excited polar motion at this frequency, is in a better agreement with the observed annual term than the pressure term alone. The amplitude of this term is slightly greater than necessary, the orientation of the ellipse φ is practically identical. The difference in phase ψ is still rather large (31 degrees), but smaller than the difference in phase of the pressure term alone. Some of the data from Tab. 2 are also displayed graphically in Fig. 9. The observed annual term is denoted as

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Table 2

Parameters of excited annual wobble by the wind term sampled at 12^h UT (W_{12}), its combination with 0^h UT (W) and sum of this combination with the pressure term corrected for the inverted barometer (ATM). The influence of ground water storage after Van Hylckama (GWH) and Kikuchi (GWK) and their sums with atmospheric excitation are also shown. The heading symbols are the same as in Tab. 1.

	A^+	Φ^+	A^-	Φ^-	a	b	φ	ψ
W_{12}	0·070″	288°	0·007″	75°	0·077″	0·063″	+ 2°	291°
W	0·019″	122°	0·003″	30°	0·022″	0·016″	+76°	113°
ATM	0·097″	86°	0·006″	286°	0·103″	0·091″	+ 6°	85°
GWH	0·036″	224°	0·004″	146°	0·040″	0·032″	+ 5°	218°
GWK	0·050″	190°	0·004″	146°	0·054″	0·046″	−12°	187°
ATM + GWH	0·074″	105°	0·004″	256°	0·078″	0·070″	0°	107°
ATM + GWK	0·101″	115°	0·004″	256°	0·105″	0·097″	+ 6°	116°

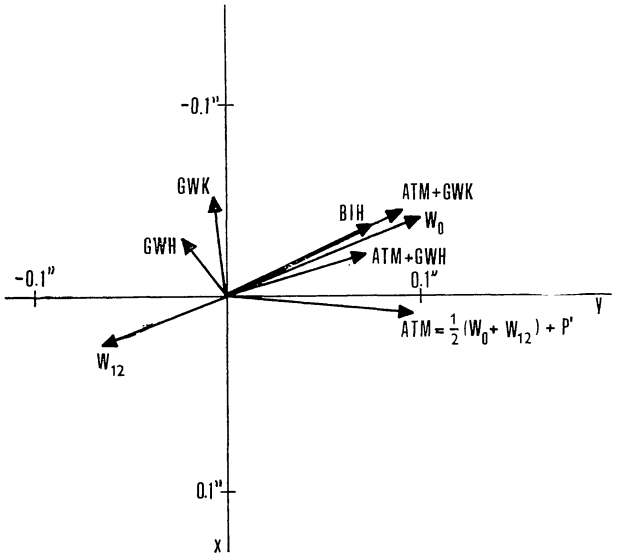

Fig. 9. Annual term in polar motion observed (BIH), excited by the wind term sampled at midnight and noon (W_0 and W_{12}), their combination with the pressure term corrected for the inverted barometer (ATM), ground water storage by Van Hylckama and Kikuchi (GWH and GWK) and their combinations (ATM + GWH, ATM + GWK).

BIH, wind terms at midnight and noon as W_0 and W_{12} and their combination with the pressure term as ATM. It is almost certain that the wind term plays an important role in the excitation of polar motion but it deserves further investigation, based on more reliable meteorological data, sampled at least twice daily. It is also possible that similar effects, not necessarily that large, are present in the pressure term, too.

The small difference (both in amplitude and phase) between the atmospherically excited and observed polar motion at the annual frequency calls for another source of excitation than the atmosphere; global change in ground water storage may be a good candidate to explain this difference (see e.g. Wahr, 1983). The excitation function coming from this source was found, e.g., by Van Hylckama (1956) or Kikuchi (1977); the polar motion, excited by these two slightly different annual terms, is shown in Tab. 2 and Fig. 9 (GWH and GWK), as their sums with the influence of the atmosphere (ATM + GWH, ATM + GWK). It is clear that both the latter results are very close to the observed polar motion. We can only hope that similar regular service as provided by the meteorological centers for the atmosphere will also be organized for the changes in ground water storage in future. In conclusion we can state that the forced polar motion can very probably be almost completely explained by the combined effect of the atmosphere and global change in ground water storage.

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A NON-FRIEDMANNIAN MODEL OF THE UNIVERSE

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НЕ-ФРИДМАНОВСКАЯ МОДЕЛЬ ВСЕЛЕННОЙ

Результаты наблюдений указывают на то, что как поток Хаббла, так и космическое реликтовое излучение являются анизотропными. В стандартных космологических моделях эти эффекты анизотропии интерпретируются как неоднородности гомогенного фридмановского фона, вызванные возмущениями. Однако, если считать неоднородности свойством толмановского фона, то неоднородная нестандартная модель может быть равным способом приемлемой.

Observational results indicate that the Hubble flow as well as the cosmic background radiation tend to be anisotropic. In standard cosmological models such anisotropy effects are interpreted as perturbative inhomogeneities on the homogeneous Friedmannian background. Nevertheless, and inhomogeneous non-standard model can be at least an acceptable alternative, if the inhomogeneities are considered as a property of Tolman's background.

Key words: cosmology — cosmological models

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9 Polar motion: observations and atmospheric excitation

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POLAR MOTION: OBSERVATIONS AND ATMOSPHERIC EXCITATION

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SUMMARY: Observed polar motion in 1976.5-1986.0 is compared with the pole excited by transfer of mass within the atmosphere. It is found that additional excitations are needed to explain the difference between the two data sets. The differences consist mainly in a constant phase shift of the annual term, small fluctuations of the phase of Chandler wobble and a slow secular increase of the total polar motion amplitude. A search for the excitations that could explain these effects is made. The most promising candidates are global groundwater storage changes and non-equilibrium response of the oceans.

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key words: polar motion, atmospheric and groundwater excitation, variable Chandler frequency

1. INTRODUCTION

It has become a well-known fact that the atmosphere has a dominant effect on the changes of the Earth-rotation parameters (ERP), namely length-of-day changes and polar motion. The variations of the length-of-day have been shown by many authors to be caused, for the periods up to several years, mainly by the combined effects of the seasonal changes of zonal winds and tides of the solid Earth. Polar motion exhibits (except for the free Chandlerian nutation with period of about 435 days and a variable amplitude) a dominant annual motion which is known to be caused by the atmosphere. However, this component is shifted in phase with respect to atmospheric excitation by about 30 degrees, or one month (see e.g. [6], [7] or [9]). This phase difference can be almost fully explained by the additional excitation excerted by the groundwater storage changes ([7], [9] or [11]). The combination of both excitations (i.e. atmospheric and groundwater storage, represented by a simple annual term) has recently been used to integrate polar motion on the interval of 9 years ([11]) and found to agree quite well with the astronomically observed path of the pole. For the atmospheric excitation, only the pressure term with inverted barometer correction was used, from the daily values of equatorial components of the effective angular momentum functions (EAMF) of the atmosphere whose conception was introduced in [1]. Three variants of the solution have been studied:

i) atmospheric excitation only,
ii) excitation caused by the atmosphere plus groundwater storage after Van Hylckama [5],
iii) excitation caused by the atmosphere and groundwater storage after Kikuchi [7].

The best agreement with the observed polar motion was achieved for the variant iii). Nevertheless, there still remained some unexplained systematic differences between the integrated and observed polar path, consisting in minor differences in phase and a secular increase in amplitude of the observed polar motion with respect to the amplitude of the integrated polar motion. Tri-axial Earth model with fluid core, visco-elastic mantle and equilibrium ocean was used to calculate the Earth's response to these excitations, as proposed in [8]. The model used gives a constant frequency of the free Chandlerian term. Since there is a strong evidence, based on a long-term study of polar motion, that the frequency of the free nutation is variable and amplitude-dependent (see e.g. [10]), we decided to test this hypothesis also on a short time scale and modern data for both polar motion and atmospheric excitation.

2. THEORETICAL BACKGROUND

For the sake of simplicity, let us confine to a rotationally symmetric Earth model. Then the first two Liouville equations in a linearized form will read (in complex notation)

$$(1) \qquad m + i\,\dot{m}\,A/\Omega(C-A) = \Psi \quad ,$$

where A, C are the principal moments of inertia of the Earth, Ω the mean rotational velocity of the Earth and Ψ the excitation function. The excitations depending on m should be, prior to solving the eq. (1) transferred from the rhs to lhs of the equation. These excitations are caused by

a) the fluid core: $\Delta\Psi_c = i\,\dot{m}\,A_c/\Omega(C-A)$,

where A_c is the moment of inertia of the core,

b) the visco-elastic mantle: $\Delta\Psi_m = k(m - i\,\dot{m}/\Omega)(1 - iQ^{-1})/k_s$,

where k and k_s denote the elastic and secular Love numbers and Q is the quality factor of the mantle,

c) the ocean: $\Delta\Psi_o = \xi\, m$,

where ξ is a factor, dependent on the amplitude of polar motion. For the equilibrium ocean, it is a constant equal to 0.064. Here we shall assume that the non-linear reaction of the ocean is responsible for the dependence of Chandler frequency on the amplitude.

Hence we obtain

$$(2) \qquad m[1 - \xi - k(1 - iQ^{-1})/k_s] + \\ + i\,\dot{m}\,[A_m/(C-A) + k(1 - iQ^{-1})/k_s]/\Omega = \Psi' \quad ,$$

where Ψ' denotes the remaining part of the excitation function. The homogenous solution to this equation for a constant ξ then

reads

$$(3) \quad m = m_0 \exp[(\alpha + i\sigma)t] ,$$

where

$$\alpha \doteq - k\Omega Q^{-1} / k_s [A_m/(C-A) + k/k_s]$$

and

$$\sigma \doteq \Omega(1 - \xi - k/k_s)/[A_m/(C-A) + k/k_s] .$$

If we further consider the numerical values $k/k_s = 0.3173$, $A_m/(C-A) = 269.66$ and $\Omega = 6.3004$ rad/day, we arrive at

$$(4) \quad \alpha \doteq -0.0074\ Q^{-1}\ \text{day}^{-1}$$

$$\sigma \doteq 0.01583 - 0.0233\xi \text{ rad/day} .$$

In order to approximate the dependence of σ on the total amplitude of polar motion M, empirically found in [10], it is sufficient to put

$$(5) \quad \xi = 0.064[1 - \exp(-20M)] ,$$

where M is expressed in arcseconds; this value leads to the expression for the Chandler frequency

$$(6) \quad \sigma = 0.01444 + 0.00148 \exp(-20M),$$

which is an excellent approximation of the empirically obtained graph drawn in Fig. 3 in [10]. The solution (3) holds only in case when the amplitude M is constant. The existence of any excitation Ψ' on the rhs of eq. (2) necessarily causes changes of its value and, consequently, also the variations of σ. Therefore the argument σt has to be replaced by the expression

$$(7) \quad \varphi = \int_0^t \sigma\, d\tau = \sigma_0 t + 0.00148 \int_0^t \exp(-20M)\, d\tau ,$$

in which the integral is to be estimated numerically from the known values of M. If we further use the EAMF of the atmosphere χ instead of the excitation function Ψ' and integrate the equation (2), we arrive at the solution

$$(8) \quad m = \exp(\alpha t + i\varphi)[m_0 - i\sigma_0(1 + \sigma_0/\Omega)\int_0^t \chi \exp(-\alpha\tau - i\varphi)d\tau] -$$

$$-(\sigma_0/\Omega)[\chi - \chi_0 \exp(\alpha t + i\varphi)] ,$$

in which m_0 is the complex constant of integration, defining the initial pole position. It should be noted that m and m_0 are given in a coordinate system whose origin is defined by the mean position of the principal axis of Earth's inertia and its axes are directed along the meridians 0° and 90° E. The mean position of the pole is identical with the mean excitation pole, given by the mean value of χ. On the other hand, the observed pole position is given in a system whose origin is chosen conventionally and whose y-axis is directed to 90° W. Therefore the x and y coordinates to be directly comparable with the observed values should be calculated from the integrated values

$m = m_1 + im_2$ using the formulas

(8) $x = m_1 + \Delta x, \quad y = -m_2 + \Delta y$,

where the values Δx, Δy express the shift between the two origins.

3. THE RESULTS

The daily values of equatorial components of EAMF of the atmosphere χ_1 , χ_2 as calculated by the NMC were used to estimate numerically the integral in eq. (8) needed to calculate the excited pole position. The amplitudes M, necessary to calculate the instantaneous Chandler frequency σ from the formula (6) and its integral from eq. (7) were calculated simultaneously as the distance between the integrated pole position and a chosen mean position of the pole, i.e. from the formula

(10) $M^2 = (x - x_{01})^2 + (y - y_{01})^2$.

The values x_{01}, y_{01} were chosen so that the fit to the observed polar motion would yield the best agreement in a least-squares sense. The constant shifts Δx, Δy were estimated à priori from the mean values of the pole coordinates and EAMF and put equal to $\Delta x = 0.02"$ and $\Delta y = 0.58"$. The initial position of the pole

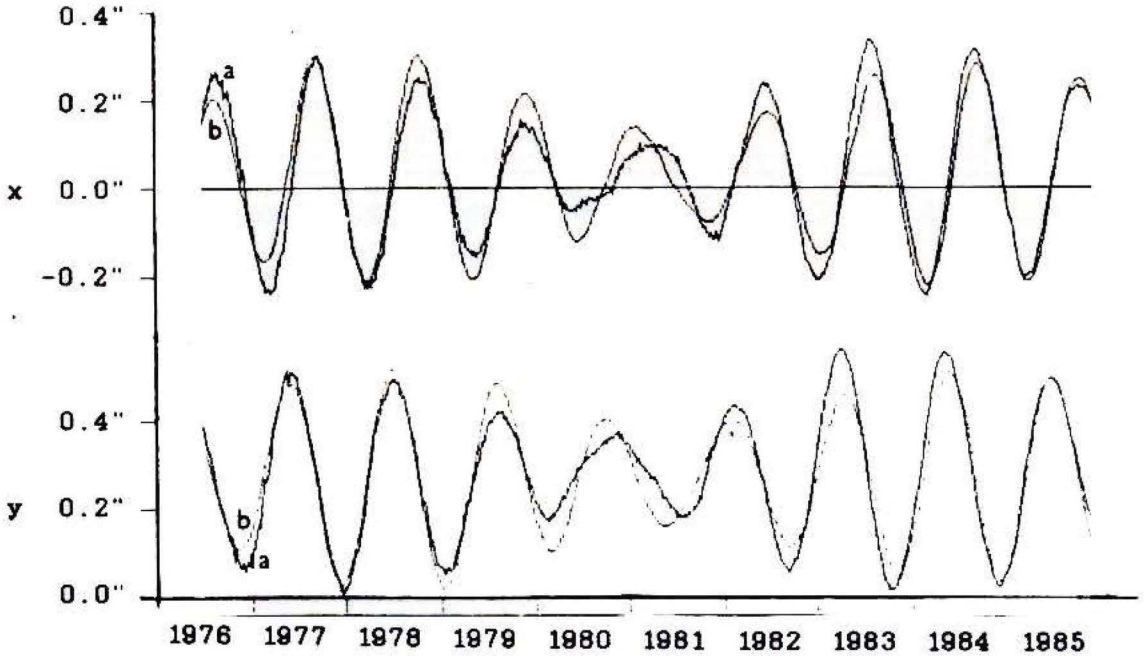

Fig. 1 Observed (a) and excited (b) polar motion. The atmospheric pressure term with inverted barometer correction and amplitude-dependent Chandler frequency have been used to integrate the excited polar path.

at the beginning of the interval studied, i.e. 1976.5, was adjusted by the method of least squares to fit the integrated polar path to the observations. Both EAMF as calculated at the NMC and observed pole positions as determined from the BIH combined solution [2] were used in the interval 1976.5 - 1986.0. Integrated polar motion was calculated in three versions, identical with the study [11]. The resulting curves and their agreement with the observations are shown in Figs 1 - 3. Fig. 1 represents the integration using only the pressure term with inverted barometer correction, while the other two are obtained when combination with annual excitation by groundwater storage changes after Van Hylckama [5] and Kikuchi [7] was used.

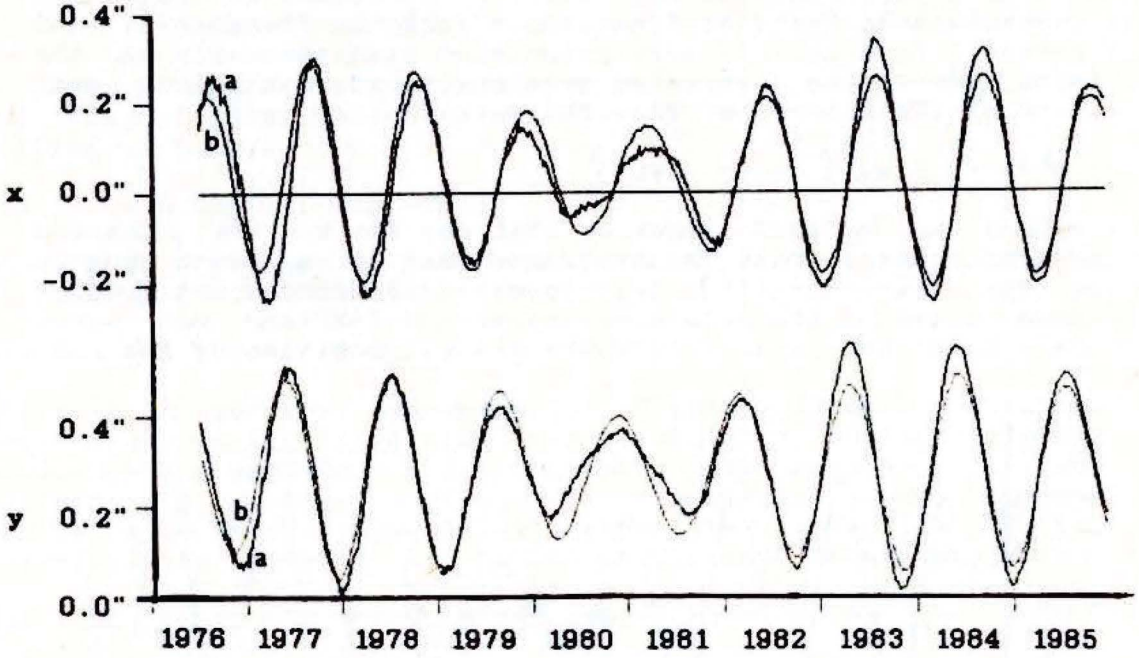

Fig. 2 The same as Fig. 1, but the groundwater storage excitation after Van Hylckama has been added to atmospheric excitation.

In order to estimate the obtained fits numerically, the root-mean-square deviations were calculated from the formula

$$(11) \quad \delta = \pm\sqrt{\Sigma(v_x^2 + v_y^2)/2n} \quad ,$$

where $v_x = x_{BIH} - x$, $v_y = y_{BIH} - y$ and n is the number of data compared. They are given in Tab. 1, together with the other estimated parameters - initial position of the pole x_o , y_o and the mean position of the pole x_{o1} , y_{o1} For comparison, the values obtained with the same data by identical method but assuming the Chandler frequency constant, are also displayed.

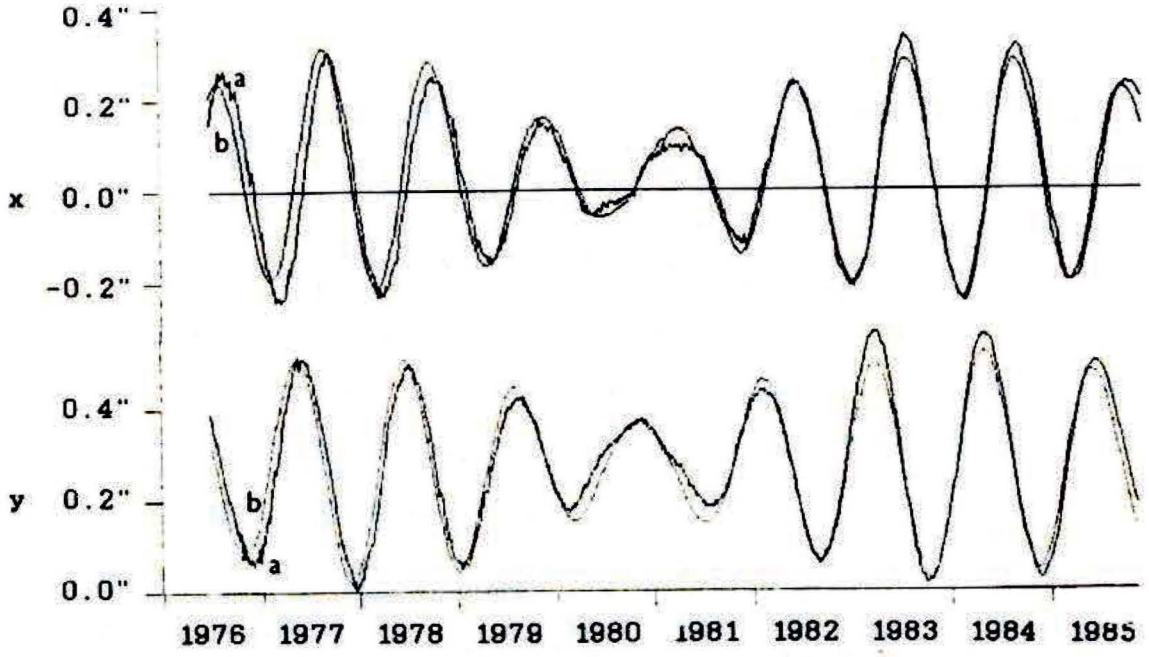

Fig. 3 The same as Fig. 1, but the groundwater storage excitation after Kikuchi has been added to the atmospheric excitation.

Table 1

	σ variable					σ constant		
	δ	x_o	y_o	x_{o1}	y_{o1}	δ	x_o	y_o
χ^P	±0"048	0"112	0"250	0"055	0"285	±0"057	0"108	0"282
$\chi^P + \Delta\chi_{HH}$	40	145	265	28	270	52	142	300
$\chi^P + \Delta\chi_K$	36	170	255	18	280	46	162	283

4.CONCLUSIONS

From Figs. 1 - 3 it can be seen that there is a very good agreement of the integrated polar motion with the observations; the use of additional excitation from the groundwater storage changes, though modelled only by a simple annual term, improves the fit significantly. This is true especially if the groundwater excitation after Kikuchi is used. The substantial improvement in all three variants is obtained when the Earth model with variable Chandler frequency is used, which is demonstrated by the values of δ displayed in Tab. 1 (compare the values in the left and right half of the table). This very fact supports the hypothesis that Chandler frequency is not constant, even on such a short time interval as studied in the present

work. Its dependence on amplitude, found from the long-term changes of polar motion in [10], seems to be valid also in a short-periodic sense. On the other hand, there still exists a secular increase of the amplitude that cannot be explained by any of the excitations in question. It is obvious that there should exist an excitation, yet unknown, with frequency nearly equal to the Chandler frequency. Such an excitation could be present in groundwater storage changes (see e.g. [12]); more detailed knowledge of these changes is necessary for further study in this field.

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Klaus-Harro Tiemann

Zur Entstehungsgeschichte des Internationalen Breitendienstes (1888 - 1899)

Unter dem unscheinbaren Titel "Neue Methode zur Bestimmung der Aberrations-Constante nebst Untersuchungen über die Veränderlichkeit der Polhöhe" berichtete vor 100 Jahren, im Juni 1888, der 31jährige Observator Friedrich Küstner (1856 - 1936) im Heft 3 der Beobachtungsergebnisse der Königlichen Sternwarte zu Berlin mit dreijähriger Verspätung über eine zum Zwecke der praktischen Anwendbarkeit der Horrebow-Talcottschen Meßmethode für die Bestimmung der Aberrationskonstante im Zeitraum 2. April 1884 bis 28. Mai 1885 durchgeführte Beobachtungsreihe, deren Auswertung ihn zu der unbeabsichtigten Schlußfolgerung führte, daß es nicht mehr

> "gestattet sei, die Polhöhe von vornherein als constant anzunehmen, daß im Gegentheil alle Anzeichen dafür sprechen, daß sie in kürzeren Zeitintervallen unregelmäßige Schwankungen bis zu einigen Zehntelsekunden erfahren kann". [1, S. 59]

Die Ursache für dieses Naturphänomen sah er

> "in den gewaltigen, der Energie der Sonne entstammenden Vorgängen in der Atmosphäre und Hydrosphäre der Erde, mit ihrem gesammten Einfluß auf die luftförmigen, flüssigen und festen Theile der Peripherie des Erdballs, durch welchen unablässig Winkelausschläge zwischen der Hauptträgheitsachse und der momentanen Rotationsachse nothwendig hervorgerufen werden müssen". [1, S. 52]

10 Atmospheric excitation of Earth's rotation: comparison of the spectrum of length-of-day and axial component of the angular momentum function of the atmosphere

N. Pejović and J. Vondrak
Bull. Astron. Inst. Czechosl.
Vol 40, pp. 382-393, 1989.

It is shown that the fluctuations observed in length-of-day, for periods from 10 to 180 days, can be fully explained by the combined effect of the redistribution of angular momentum between the Earth and its atmosphere, and tidal deformations of the Earth's body. The tidal effect scaling factor k/C is found to be equal to 0.945 ± 0.021. The small discrepancies between the observed and excited amplitudes of the dominating semi-annual and annual terms can very probably be caused by the influence of zonal stratospheric winds, neglected in the routinely calculated atmospheric angular momentum functions. On the other hand, the observed long-periodic components in the length-of-day (with periods of $2.4y$ and $6.0y$) cannot be explained by any of the two discussed influences.

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ATMOSPHERIC EXCITATION OF EARTH'S ROTATION: COMPARISON OF THE SPECTRUM OF THE LENGTH-OF-DAY AND AXIAL COMPONENT OF THE ANGULAR MOMENTUM FUNCTION OF THE ATMOSPHERE

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АТМОСФЕРНОЕ ВОЗБУЖДЕНИЕ ВРАЩЕНИЯ ЗЕМЛИ: СРАВНЕНИЕ СПЕКТРОВ ДЛИНЫ СУТОК И ОСЕВОЙ СОСТАВЛЯЮЩЕЙ ФУНКЦИИ МОМЕНТА КОЛИЧЕСТВА ДВИЖЕНИЯ АТМОСФЕРЫ

В статье показано, что наблюдаемые изменения длины суток можно для периодов от 10 до 180 суток полностью объяснить комбинированным воздействием перераспределения момента количества движения между Землей и ее атмосферой, и приливных деформаций фигуры Земли. Найдено значение шкального фактора приливных влияний $k/C = 0{,}945 \pm 0{,}021$. Малые несоответствия между наблюдаемыми и возбужденными амплитудами основных полугодовых и годовых членов могут быть, с большой вероятностью, вызваны влиянием зональных ветров в стратосфере, которыми обычно пренебрегают при вычислении моментов количества движения атмосферы. С другой стороны, долгопериодические составляющие в длине суток (с периодами 2,4 года и 6,0 года) не могут быть объяснены ни одим из обоих рассматриваемых влияний.

It is shown that the fluctuations observed in length-of-day, for periods from 10 to 180 days, can be fully explained by the combined effect of the redistribution of angular momentum between the Earth and its atmosphere, and tidal deformations of the Earth's body. The tidal effect scaling factor k/C is found to be equal to $0{\cdot}945 \pm 0{\cdot}021$. The small discrepancies between the observed and excited amplitudes of the dominating semi-annual and annual terms can very probably be caused by the influence of zonal stratospheric winds, neglected in the routinely calculated atmospheric angular momentum functions. On the other hand, the observed long-periodic components in the length-of-day (with periods of 2·4 y and 6·0 y) cannot be explained by any of the two discussed influences.

Key words: Rotation of the Earth — atmospheric excitation — Earth tides

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1. Introduction

It has become a well-known fact that the exchange of angular momentum between the atmosphere and solid Earth plays a dominant role in the changes of Earth rotation parameters (polar motion, length-of-day). The earlier results and theoretical considerations of the problem can be found e.g. in two principal monographs, devoted to the rotation of the Earth, by Munk and MacDonald (1960) and by Lambeck (1980). Many papers have since been devoted to the problem. The seasonal variations (annual, semi-annual) in the length-of-day (LOD) were proved to be caused mainly by the combined effect of the atmosphere and tides – see, e.g., Lambeck and Cazenave (1973, 1974, 1977), Sidorenkov (1973 and 1979) Rosen and Salstein (1983) or Eubanks et al. (1985). Feissel and Nitschelm (1985) proved that the recently found short-peripdic fluctuations (with periods between 50 and 70 days) in LOD are closely correlated to the variations in the axial component of the relative angular momentum of the atmosphere within the same frequency band. Djurović and Pâquet (1988) showed that the origin of these fluctuations lies very probably in physical processes in the Sun. A recent overview of the problem and discussion of the prospects was presented by Dickey and Eubanks (1987). The "effective angular momentum functions" (EAMF) of the atmosphere, proposed by Barnes et al. (1983), have been accepted worldwide and are now routinely calculated by several meteorological centers daily, and after 1984 even twice daily. On the other hand, the measured Earth rotation parameters are known, thanks to modern techniques of observation, with ever increasing precision. This enables these data to be used for detailed analysis in order to estimate to what extent the LOD changes are caused by the atmosphere and which part is left to other influences. In the following, we shall first compare the spectra of the observed LOD changes and atmospheric EAMF, and then try to find a simple functional relation between the observed LOD, on the one hand, and atmospheric EAMF and zonal luni-solar tides, on the other.

2. The Input Data Sets and their Spectral Analysis

The following data sets have been used for the subsequent spectral analysis, in the interval 1976·5 to 1986·0:

a) The observed rate of the Earth's rotation was represented by the series of raw 5-day values of UT1-TAI (i.e. the difference between the rotational and atomic time scale) as derived by the Bureau International de l'Heure in Paris from the combination of all available observations, both from optical astrometry and modern space techniques (BIH 1976–1985, Tab. 6). The numerical differentiation of this time series has further been used to derive the corresponding values of the length-of-day excess over its nominal value (86 400 sec) in seconds of time:

$$\text{LOD} = -\partial(\text{UT1} - \text{TAI})/\partial t\,, \tag{1}$$

where the time derivative on the rhs is reckoned in seconds per day. Since we are mainly interested in periods of less than several years, namely this last data set was used instead of the original UT1 – TAI values (differentiation of a time series amplifies the amplitudes of its short-periodic part while suppressing its long-term changes).

b) The axial component of the effective angular momentum function (EAMF) of the atmosphere, introduced by Barnes et al. (1983), is a dimensionless quantity; its so-called pressure term is

$$\chi_3^p = \frac{0{\cdot}70R^4}{gC_m}\int_S p_s \cos^3\varphi\,\mathrm{d}\varphi\,\mathrm{d}\lambda. \tag{2}$$

It expresses the influence of the redistribution of air mass on the principal moment of inertia of the whole Earth. The axial component of the wind term

$$\chi_3^W = \frac{1{\cdot}00R^3}{g\Omega C_m}\int_V u\cos^2\varphi\,\mathrm{d}\varphi\,\mathrm{d}\lambda\,\mathrm{d}p \tag{3}$$

expresses the relative angular momentum of the atmosphere (with respect to the rotating solid body of the Earth). The subscript S denotes integration over the whole surface of the Earth and V over the volume of the atmosphere, φ, λ and p geographical latitude, longitude and air pressure, respectively. The symbol p_s denotes atmospheric surface pressure (not reduced to mean sea level), u is the eastward component of the wind velocity. $R = 6370$ km is the mean Earth's radius, $C_m = 7{\cdot}04 \times 10^{37}$ kg m^2 is the axial moment of inertia of the Earth's mantle (the core being considered decoupled from the mantle), $g = 9{\cdot}81$ m s^{-2} is the average gravitational acceleration at the Earth's surface and $\Omega = 6{\cdot}3004$ rad/day denotes the mean rotational velocity of the Earth. The pressure term is also calculated with the so-called "inverted barometer" (IB) correction; the reaction of the ocean to the air pressure changes is supposed to compensate these changes in such a manner that the total atmospheric and ocean effect is constant over the whole surface of each ocean. The EAMF as calculated by the U.S. National Meteorological

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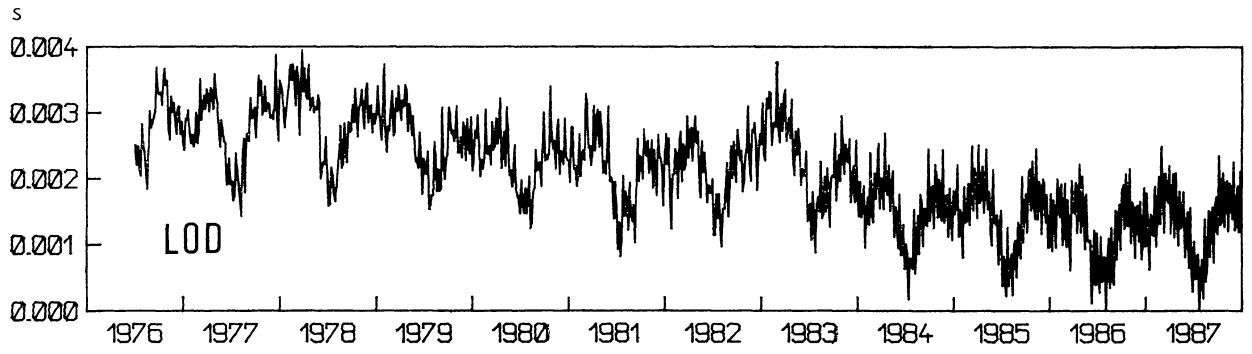

Fig. 1. The excess of length-of-day over the nominal 86 400 sec, calculated from the BIH combined solution. The raw data UT 1 — TAI have been used.

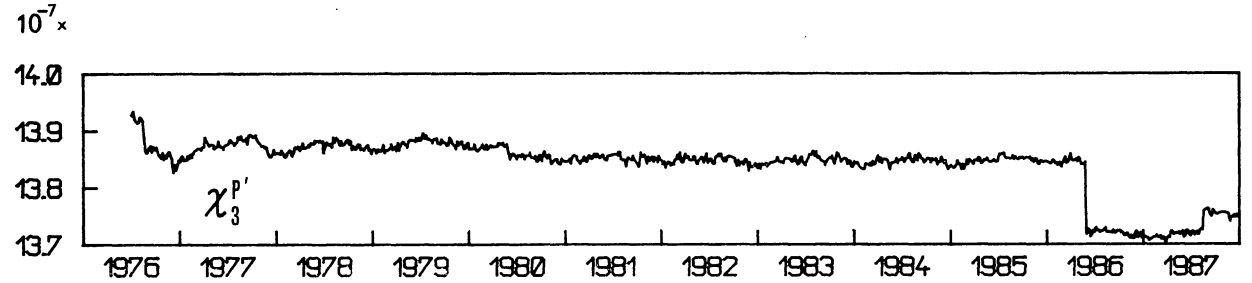

Fig. 2. The axial component of the pressure term of the atmospheric EAMF with the IB correction as determined by the NMC.

Center (NMC) and provided by courtesy of F. W. Fallon (1986) of the National Geodetic Survey and D. Gambis (1988) of the Central Bureau of the IERS has been used.

The data sets are graphically displayed, in the time domain, in Figs 1–4. Figure 1 shows the excess of the LOD over the nominal 86 400 sec; its complex periodic character is mainly due to the combined effects of the atmosphere with the variations caused by the zonal tides of the Earth (Yoder et al. 1981). In order not to suppress the short-periodic tidal variations (with dominating fortnightly and monthly terms), in differentiating the original UT1 – TAI data, the theoretical tides in UT1 with periods shorter than 35 days (i.e. the first 41 terms of Yoder's list) were first removed from UT1 – TAI. These values were then differentiated with respect to time, and only then were the tides in LOD added to obtain the final LOD data set. The scaling factor $k/C = 0{\cdot}94$, recommended by Yoder et al. (1981), has been used to account for the elasticity of the mantle and the effect of the ocean. Since there is no substantial difference (save for a small constant shift) between χ_3^P and $\chi_3^{P'}$ (pressure term without and with the IB correction, respectively), only $\chi_3^{P'}$ is illustrated in Fig. 2. Notice the sudden jump of about $0{\cdot}136 \times 10^{-7}$ in the May 1986 data, due to the change in the forecast model used by the NMC (Salstein 1987). It can be seen that this term does not contribute very much to the total effect, if compared with the wind term in Fig. 3 and the combination of both $(\chi_3^W + \chi_3^{P'})$ in Fig. 4, where the discontinuity mentioned above is less obvious.

The spectral analysis of these data sets (from which the jump in the pressure term has been removed) was carried out using two different methods, in order to estimate which periods are real, and to what extent and at which frequencies the LOD is influenced by the atmosphere. Since the data were sampled at 5-day intervals, only periods longer than 25 days

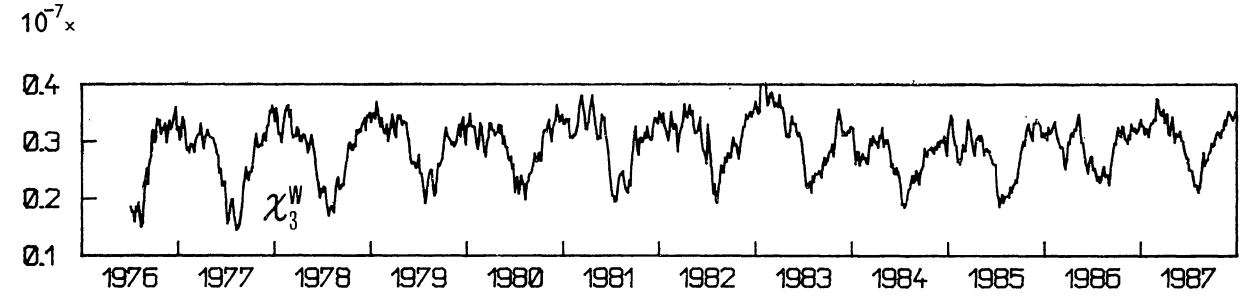

Fig. 3. The axial component of the wind term of the atmospheric EAMF as determined by the NMC.

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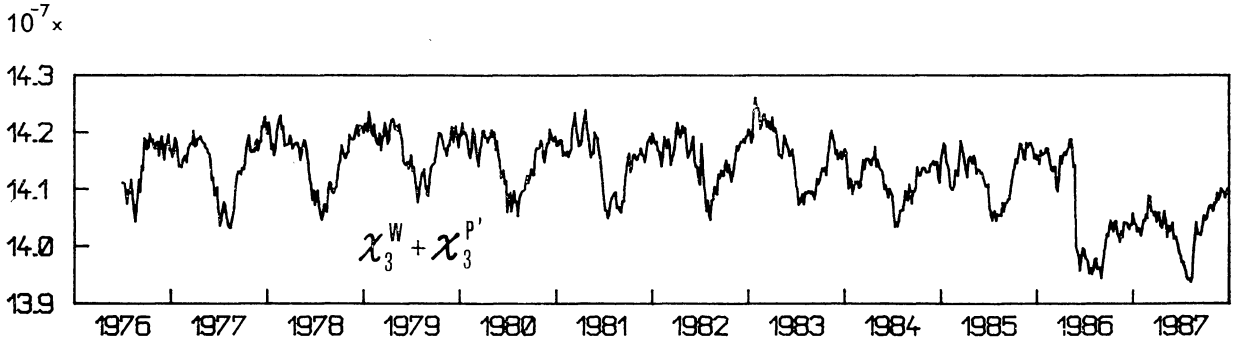

Fig. 4. The axial component of the sum of the pressure and wind terms of the atmospheric EAMF as determined by the NMC.

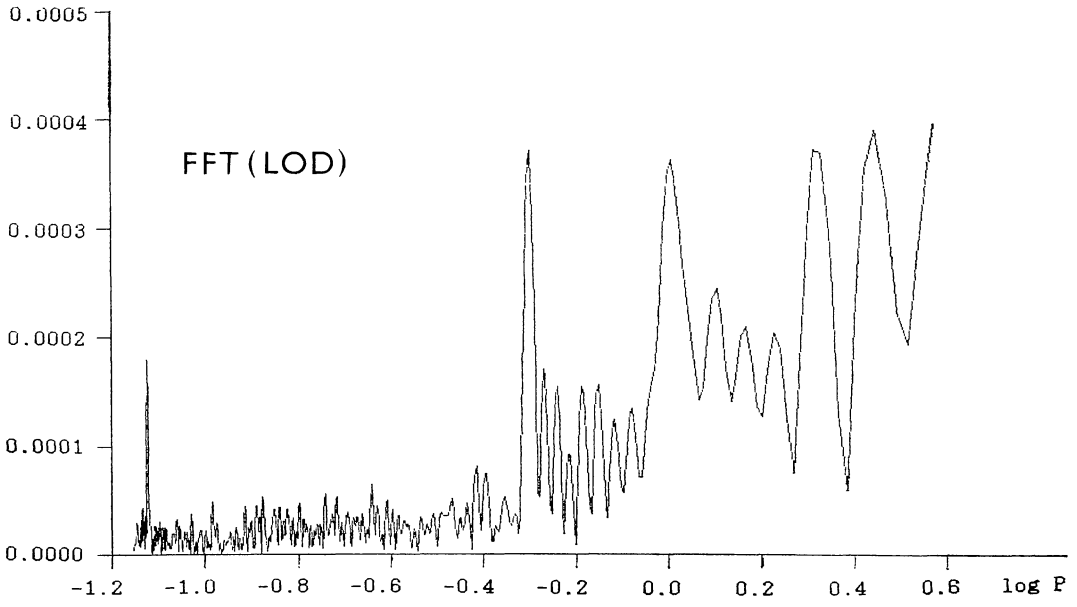

Fig. 5. The spectrum of the observed LOD calculated by the FFT.

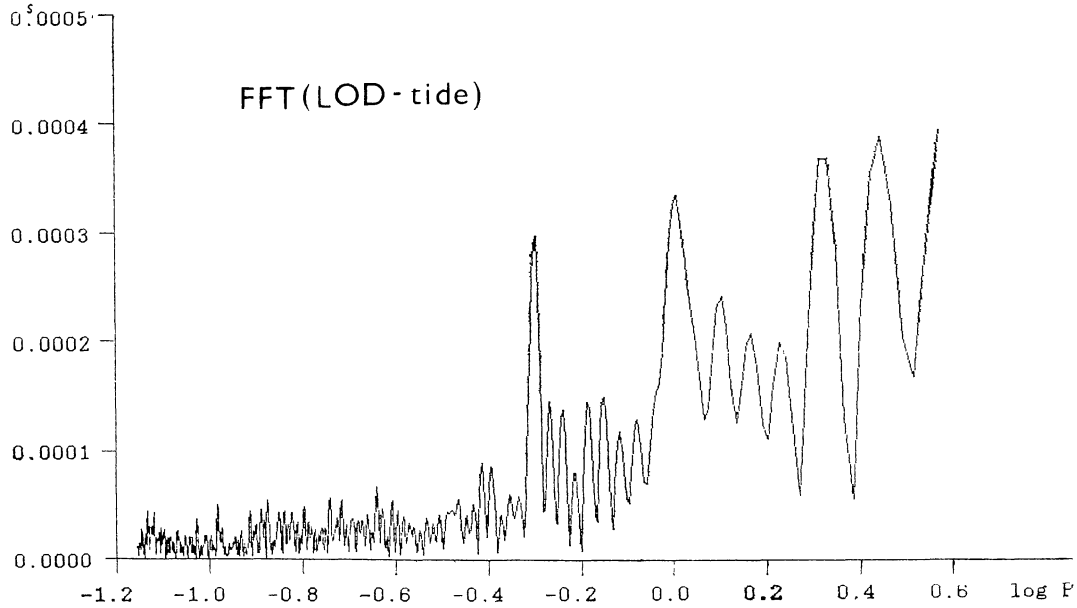

Fig. 6. The spectrum of the observed LOD with tidal variations removed, calculated by the FFT.

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were considered – the spectrum would be distorted for shorter periods:

i) The Fast Fourier transform (FFT), which is known to produce sidelobes, placed symmetrically at both sides of any dominant peak of the spectrum. The sidelobes necessarily have a slight effect on the neighbouring peaks.

ii) The much slower method of least squares (MLS), by which we derived the amplitude and phase of the sinusoidal wave whose period was changed stepwise over a given range of periods. Since a simple relation between LOD and χ_3 should theoretically exist (Barnes et al 1983),

$$\text{LOD} = \chi_3 + \text{constant}, \tag{4}$$

both spectra are directly comparable.

2.1 *FFT Spectra Analysis*

The amplitude spectra obtained with the FFT technique are plotted in Figs 5–8, in which the horizontal scale is logarithmic (period in years) and the vertical in seconds of time. Figure 5 shows the

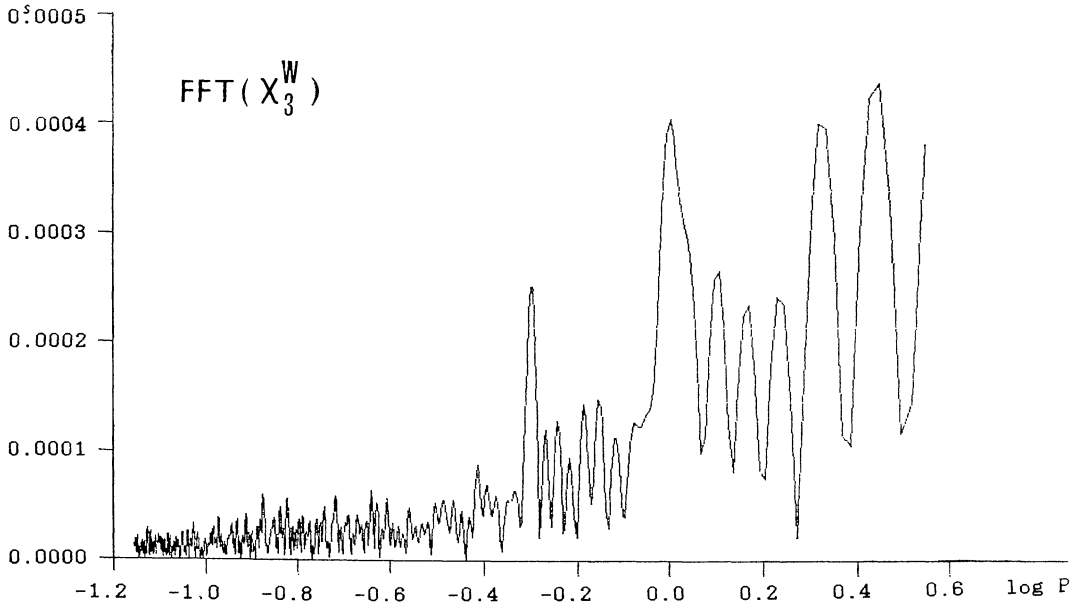

Fig. 7. The spectrum of the axial component of the wind term of the atmospheric EAMF, calculated by the FFT.

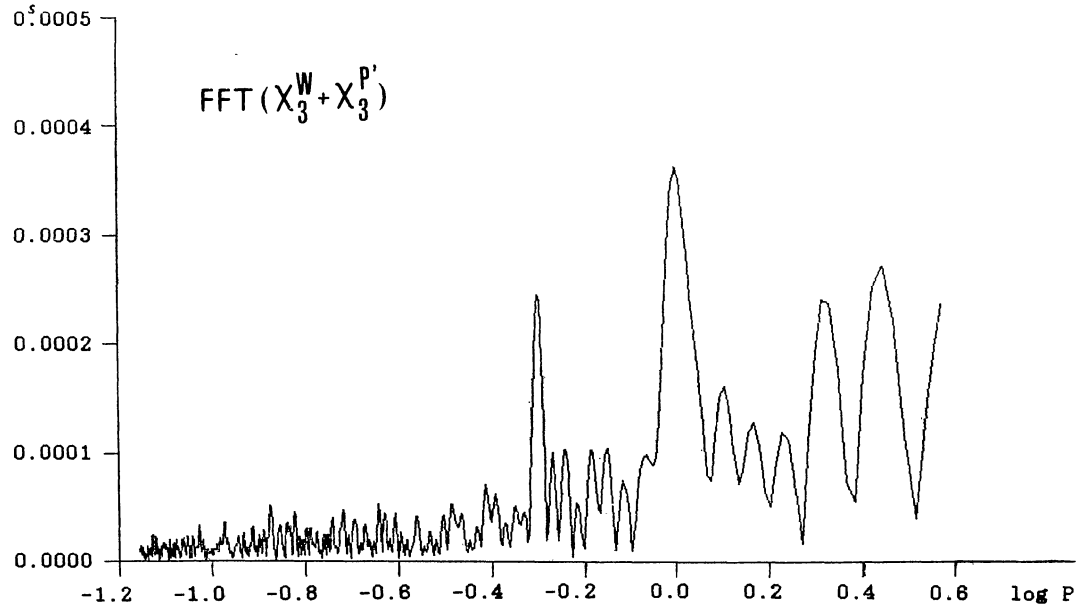

Fig. 8. The spectrum of the axial component of the sum of the pressure and wind terms of the atmospheric EAMF, calculated by the FFT.

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spectrum of the observed LOD changes. Far to the left there is a large peak of the monthly term, evidently of tidal origin. Dominant are semi-annual and annual terms, and also two terms of longer periods, equal to 2 years and 2·7 years. A similar pattern can be seen in Fig. 6, where the theoretical tides with $k/C = 0{\cdot}94$, after Yoder et al. (1981), have been removed. The monthly term has disappeared and the amplitudes of the two dominating terms are slightly reduced. The remaining fine structure of the spectrum is nearly identical with Fig. 5, because there is practically no significant tidal term with a period between one month and half a year, and also between 1 y and 18·6 y. The spectrum of the wind term is displayed in Fig. 7. It is very similar to the spectrum of LOD without the tidal terms (compare Figs 6 and 7), especially for periods shorter than half a year. Nevertheless, there are differences in the two most pronounced terms – the amplitude of the semi-annual term is slightly smaller, and the amplitude of the annual term is larger than the observed value. The pattern improves if the pressure term is added; the amplitude of the semi-annual term remains practically the same, but the amplitude of the annual term is smaller, thus coming closer to the observed value (Fig. 8). Substantially diminished are the long-periodic terms.

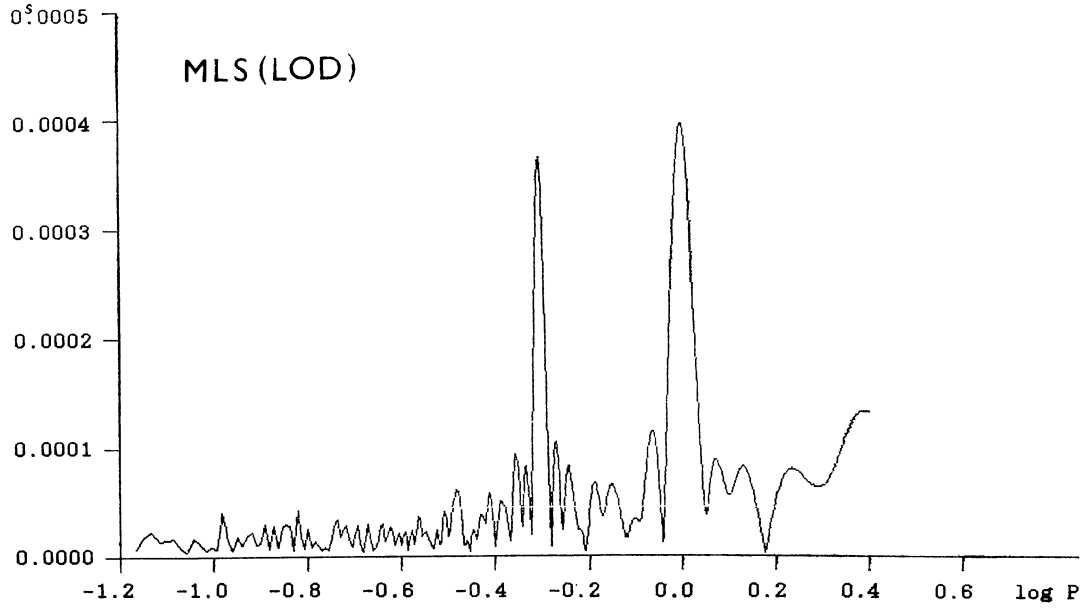

Fig. 9a. The spectrum of the observed LOD, calculated by the method of least squares, with resolution in period of 1d.

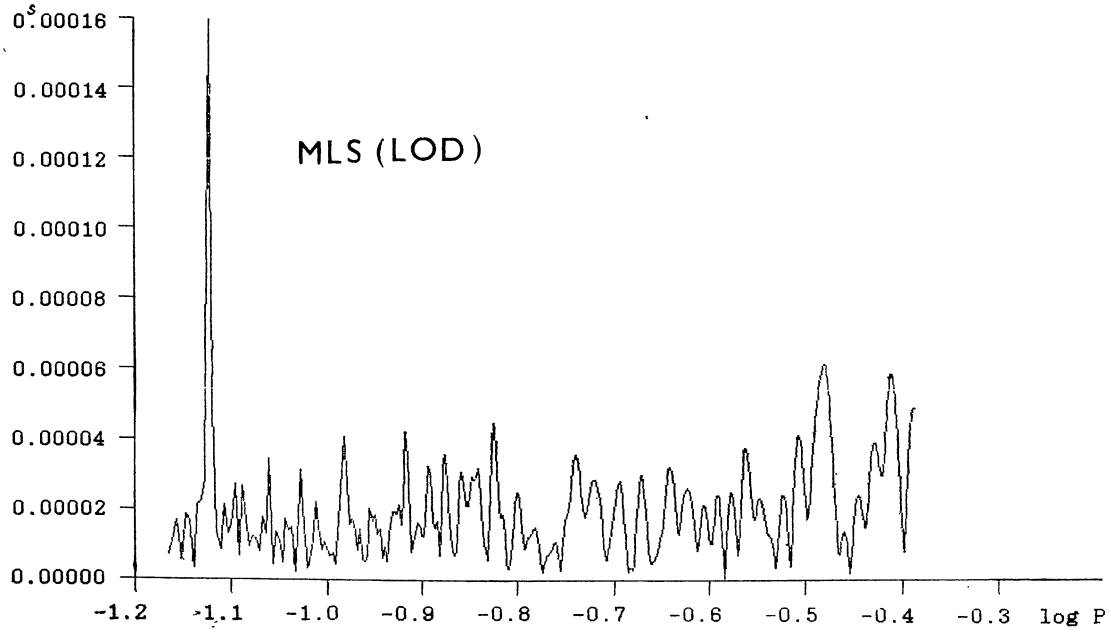

Fig. 9b. Short-periodic part of the LOD spectrum, calculated by the method of least squares, with resolution in period of 0·25 d.

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2.2 *MLS Spectral Analysis*

The results of the spectral analysis, carried out with the least-squares method, are graphically displayed in Figs 9–11. Figures 9a and 9b show the spectrum of the observed LOD; Fig. 9a the spectrum in the whole period range, i.e. from 25 days to 2·5 years, with a period resolution of 1 day, and Fig. 9b the spectrum with a finer resolution (0·25 days) for the shorter period range (from 25 days to 150 days), on an enlarged vertical scale. The spectrum is quite similar to the FFT (LOD) spectrum in Fig. 5, but for the annual peak (which is a little higher here) and for the long-periodic part (that is almost completely missing). Nearly the same situation as in the case of FFT can be seen in Figs 10a and 10b, in which the tides have been removed – the amplitudes of both the semi-annual and annual terms are a little bit smaller and the monthly term has almost completely disappeared. The analysis of the wind term of the EAMF alone is not shown here simply because there is no substantial difference between it and the combination of wind and pressure terms, depicted in Figs 11a and 11b. If these two spectra are compared with those of Figs 10a and 10b, we can see that they are very similar. There are again some differences

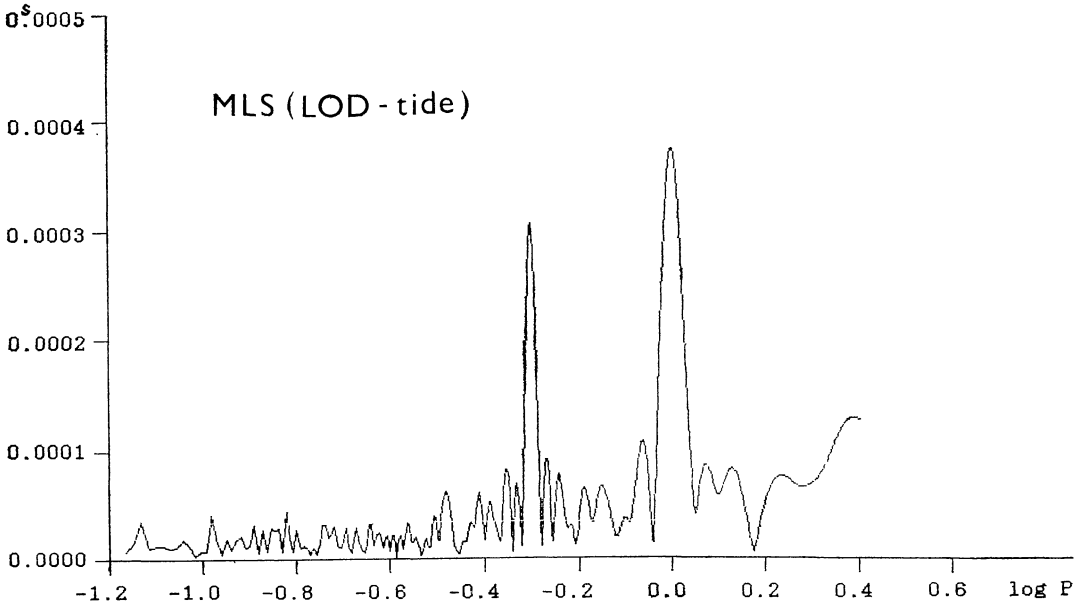

Fig. 10a. The same as Fig. 9a, with tidal variations removed.

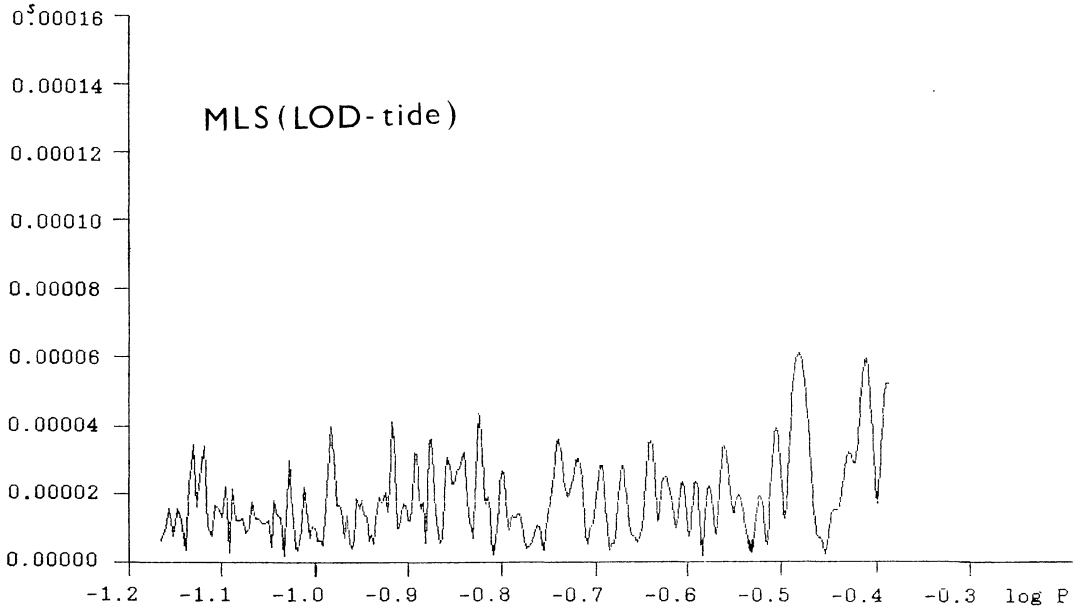

Fig. 10b. The same as Fig. 9b, with tidal variations removed.

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in the amplitudes of semi-annual and annual terms (the observed amplitude of semi-annual term is larger than the amplitude expected due to atmospheric excitation, while the observed amplitude of the annual term is smaller), but the spectra are surprisingly identical for periods shorter than 150 days (compare Figs 10b and 11b).

In comparing the FFT and MLS spectra, one can see that the FFT method highly exagerates the amplitudes of the long-periodic terms with respect to MLS. These terms, if they really exist, are probably much smaller and their atmospheric origin is doubtful. On the other hand, both methods reliably disclose a high number of periodic terms not only in the well-known range between 50 and 70 days, but also for longer periods. An overview of all the periods detected is given in Tab. 1. The periods in days, given in the first column of the table, are average values obtained of all the four analyses, as displayed in Figs 6, 8, 10 and 11. Except for the most significant annual and semi-annual terms, 22 terms have been detected. All of them are present both in the FFT and MLS

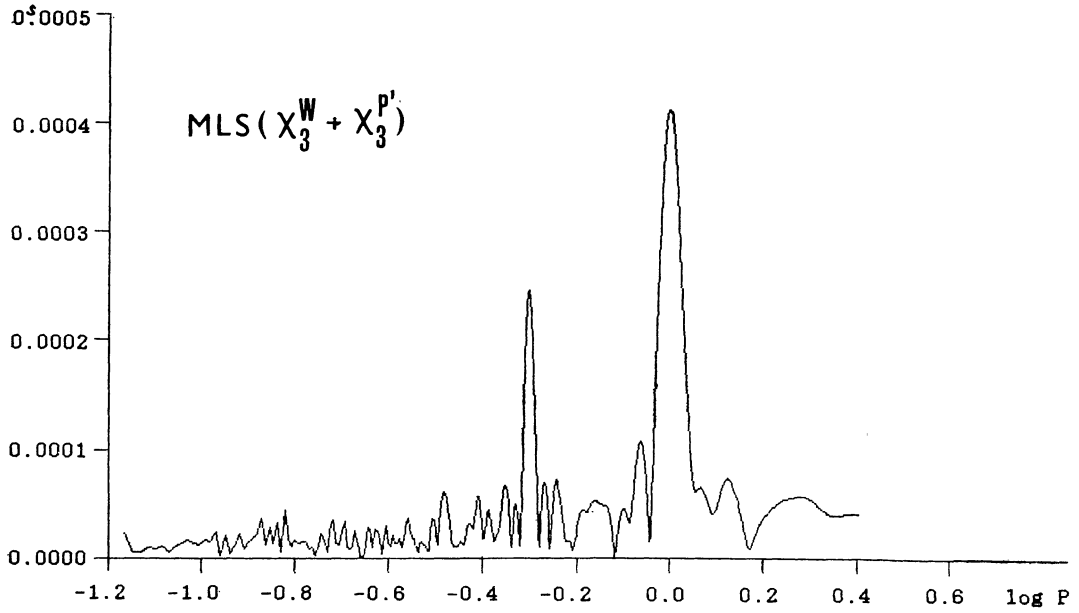

Fig. 11a. The spectrum of the axial component of the atmospheric EAMF, calculated by the method of least squares, with resolution in period of 1d.

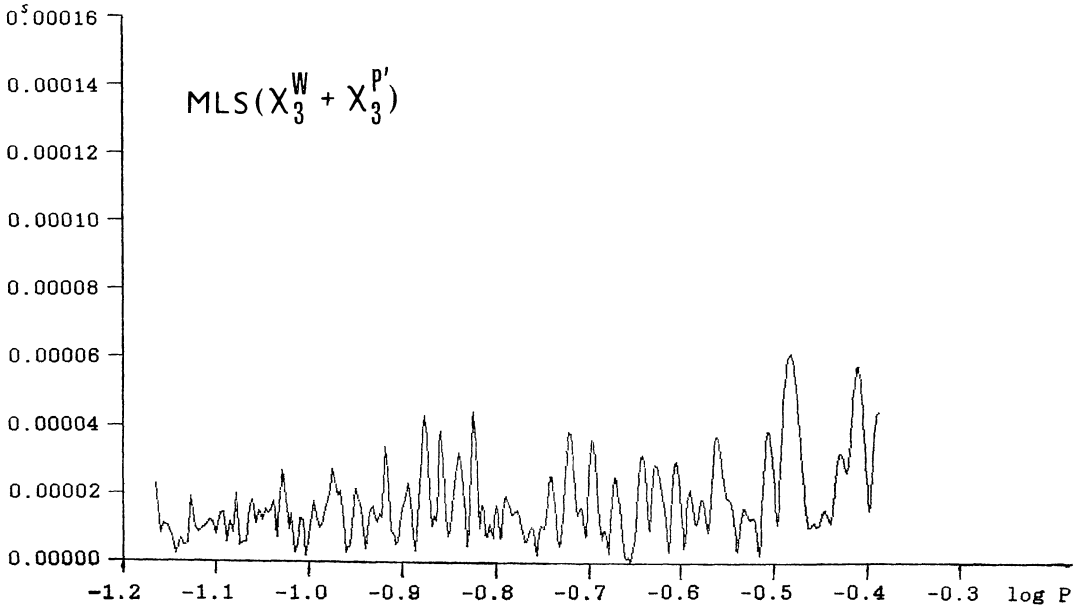

Fig. 11b. The spectrum of the axial component of the atmospheric EAMF, calculated by the method of least squares, with resolution in period of 0·25 d.

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Table 1

Periodic terms up to 1 year found by spectral analysis in LOD and axial atmospheric EAMF

Period in days	Amplitude FFT LOD	FFT EAMF	MLS LOD	MLS EAMF
34·1	0·000038	0·000038	0·000030	0·000027
38·4	50	50	40	27
44·4	43	44	41	34
47·0	45	28	32	23
48·7	54	54	36	43
51·0	43	33	31	39
52·8	44	35	32	32
54·8	43	43	43	44
58·2	48	49	27	17
66·6	57	57	35	26
70·0	54	55	31	39
73·9	36	38	29	37
78·0	36	33	28	26
83·8	67	67	35	32
86·3	47	43	25	29
90·6	54	54	24	30
100·2	34	41	34	37
114·2	41	43	34	38
120·2	45	46	61	61
125·8	55	57	—	—
141·8	90	89	60	58
149·3	87	87	52	44
182·6	293	246	306	246
365·2	328	363	375	412

analyses, save for the term with the period equal to 125·8 days, which is present only in FFT analysis. This term is very probably an artefact due to the leakage of sidelobes from the dominant terms. A surprisingly excellent agreement between the observed amplitudes (headed as LOD) and amplitudes expected due to atmospheric excitation (headed as EAMF) for periods between 30 and 150 days witness that, for this part of the spectrum, all the observed variations in the rotational rate of the Earth can be ascribed to atmospheric excitation. None of this is changed by the obvious fact that the MLS analysis tends to yield systematically smaller amplitudes (both for LOD and EAMF) than FFT. The situation is not that clear with the dominating semi-annual and annual terms. It is very probable that the discrepancies here are mainly due to the higher layers of the atmosphere neglected in the NMC data. The wind term is not in fact calculated for the whole volume of the atmosphere, as it should be, but rather only up to the pressure level of 50 mbars, as discussed by Rosen and Salstein (1985). In analysing the contribution of stratospheric winds on the interval 1980–1981 they found that the annual component in the stratosphere is approximately 180° out of phase with that in the troposphere, while for the semi-annual component they are roughly in phase. Thus the stratosphere acts to decrease the amplitude of the annual component by about 0·060 msec and to increase the amplitude of the semiannual component by about 0·047 msec. These numbers can almost fully explain the difference between atmospheric excitation and the observed LOD changes in Tab. 1. Other possible sources of excitation at these two frequencies could be groundwater storage changes (but these are very small), sea-level changes or ocean currents (which are not sufficiently well known) – for a detailed discussion refer to (Lambeck, 1980).

3. Atmosphere and Tides versus LOD

Now it is clear that the variations in LOD are, at least at high frequencies (i.e. higher than 1 cpy), mostly driven by the combination of tidal deformations of the Earth's body and the redistribution of angular momentum between the solid Earth and its armosphere. The scaling factors for both effects are certain functions of some of the parameters, characterizing the real Earth. To begin with the tides, the scaling factor which is used to multiply theoretical amplitudes after Yoder et al. (1981) is equal to k/C, where k is the elastic Love number of the Earth and C is the dimensionless polar moment of its mantle's inertia. As for the axial component of the atmospheric EAMF, the scaling factor is multiplied by $1 + k'$ for the pressure term and 1 for the wind term, where k' is the load Love number; they both also depend on certain constants preceding the integrals in Eqs (2) and (3). So far we have used the values $k/C = 0{\cdot}94$, $1 + k' = 0{\cdot}70$. In the following, we shall determine new values of the scaling factors, using the time series of LOD and EAMF studied above. Namely, we shall try to fit the LOD, expressed in terms of tidal and atmospheric excitation as

$$(5)\quad \mathrm{LOD}_c = a + bt + ct^2 + d(\chi_3 - \chi_0) + e(tide)$$

to the observed values LOD_0. The unknown coefficients $a - e$ can be derived by the method of least squares to satisfy the condition

$$(6)\qquad (\mathrm{LOD}_c - \mathrm{LOD}_o)^2 = \text{minimum}\,.$$

Coefficients $a - c$ are meant to express the approximation of the decade fluctuations of LOD in the interval studied, while d and e are the scaling factors we are looking for. t denotes the time in Julian years elapsed

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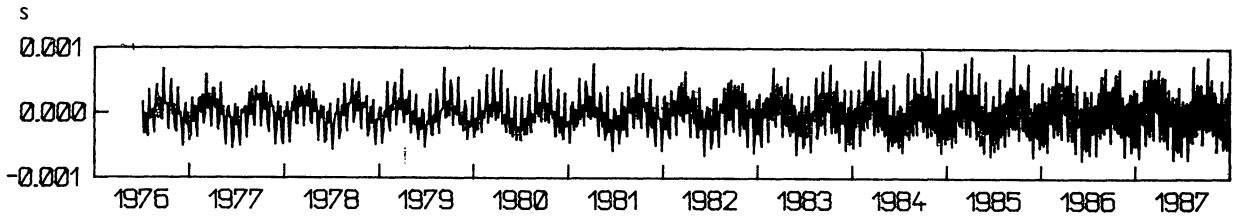

Fig. 12. Tidal variations in LOD with scaling factor $k/C = 0{\cdot}945$. Only the terms with periods less than 8 y have been considered.

since MJD = 4 5061·5, i.e. $t = (\text{MJD} - 45\,061{\cdot}5)\,/\,/\,365{\cdot}25$ and χ_0 is the mean value which is removed from the atmospheric EAMF before solving, i.e. $\chi_0 = 14{\cdot}149 \times 10^{-7}$. In order to convert the EAMF values to LOD changes in seconds of time, they must be multiplied by 86 400. Since the interval studied is too short to enable the analysis of very long-periodic tidal variations, only the first 59 terms of Yoder's list have been used to calculate the *tide* in eq. (5); the last three terms with periods longer than 8 years have been omitted. The tidal variations in LOD derived from the 59 terms are depicted, for the interval in question, in Fig. 12. It can be seen that especially the short-periodic terms towards the end of the interval have very large amplitudes – these are caused by the interference of the two large fortnightly terms with arguments $2F + \Omega$ and $2F + 2\Omega$ (and with close periods of 13·63 and 13·66 days, respectively) which were in phase in November 1987. The observed values of LOD used for this analysis are based on the latest BIH combined solution ERP(BIH) 87 C 02 (Feissel and Guinot 1988), in the interval 1976·5–1988·0. In order to investigate the possible influence of the adjusted tidal scaling factor e on the value k/C actually used to calculate LOD_0 from the observed UT1 – TAI values (see Section 2), the whole solution was repeated several times, with different values of k/C used to derive LOD_0. The results are graphically displayed in Fig. 13, in which the solid line shows the dependence of e on k/C and the dashed line is given by the equation $e = k/C$. The intersection of both lines determines the most probable value of the adjusted coefficient e, i.e. $e = 0{\cdot}945$. Namely this value was then used for k/C in calculating LOD from the observed values UT 1 – TAI in the final solution. The adjusted values of the unknowns $a - e$ are shown, together

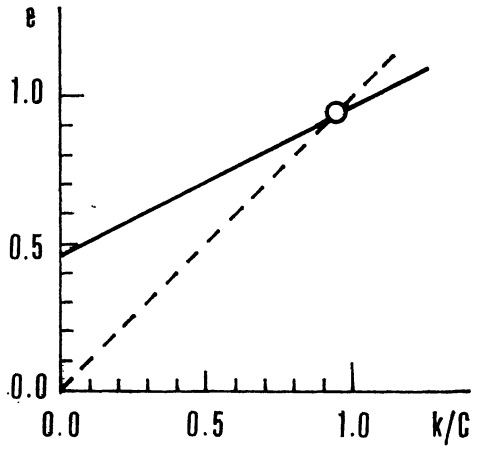

Fig. 13. The dependence of the adjusted tidal scaling factor e on its value used to calculate LOD from the UT 1—TAI values.

with their mean errors, in Tab. 2. The coefficients of correlation between the individual adjusted values, also displayed in the table, are evidence that there is no dangerous correlation in the system of normal equations. This especially holds for the unknowns d and e determined practically independently of one another (the corresponding coefficient of correlation

Table 2

Adjusted values of the unknowns approximating the best fit of LOD changes by the combination of atmospheric exciation and zonal tides after eq. (5)

Unknown	Adj. value	Mean error	Coefficients of correlation				
			a	*b*	*c*	*d*	*e*
a	0·002152	±0·000011	1·00				
b	−0·0001589	±0·0000022	0·00	1·00			
c	−0·00000472	±0·00000072	−0·75	0·00	1·00		
d	0·943	±0·018	0·04	0·06	−0·05	1·00	
e	0·945	±0·021	0·00	0·00	0·00	−0·06	1·00

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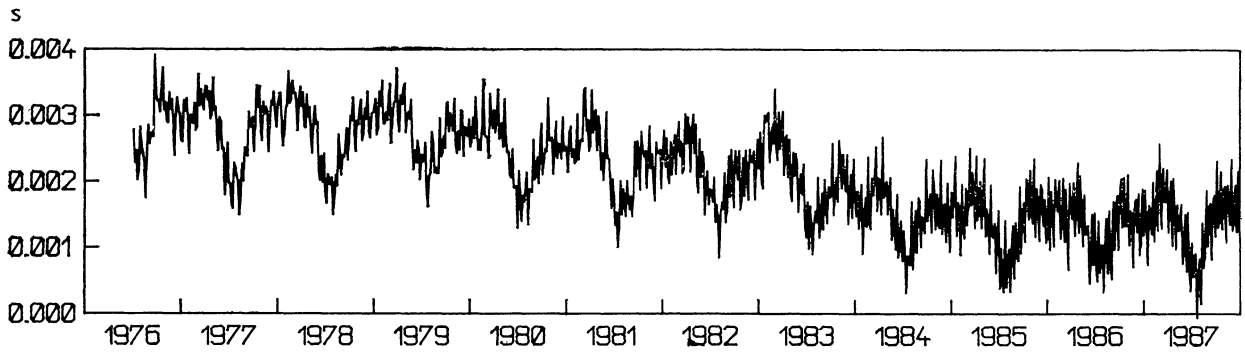

Fig. 14. LOD changes, calculated from the atmospheric excitation and tidal effects using eq. (5).

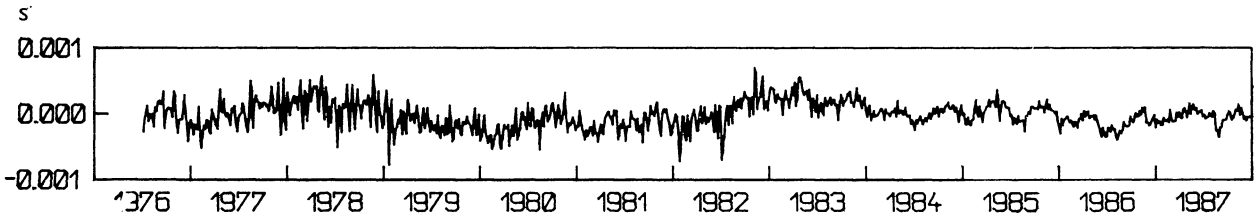

Fig. 15. The differences between the observed (Fig. 1) and calculated (Fig. 14) LOD.

being equal to only −0·06). Fig. 14 shows the resulting LOD_c changes, calculated using formula (5) and the numerical values from Tab. 2. It is obvious that the curve is nearly identical with the curve displaying the observed values LOD_0 (compare with Fig. 1); the difference between LOD_0 and LOD_c is plotted in Fig. 15. It clearly demonstrates the gradual improvement of the fit, especially after 1983, when the new space techniques started to dominate in the observed Earth rotation parameters. The UT 1 component, based until 1978 uniquely on optical astrometry, was also observed by lunar laser ranging after this date, and after 1982 it was VLBI that gradually attained higher and higher relative weight. The average root-mean-square difference between the two LOD representations, calculated over the whole interval studied, is equal to ±0·00021 sec.

The spectral analysis of the residuals from Fig. 15, carried out by the FFT method, is shown in Fig. 16. Clearly visible are the semi-annual and annual peaks most probably caused by the neglected stratospheric winds (see also the discussion at the end of Section 2). The other two dominant peaks at periods of 2·4 y and 6·0 y are evidently of an origin different than the atmos-

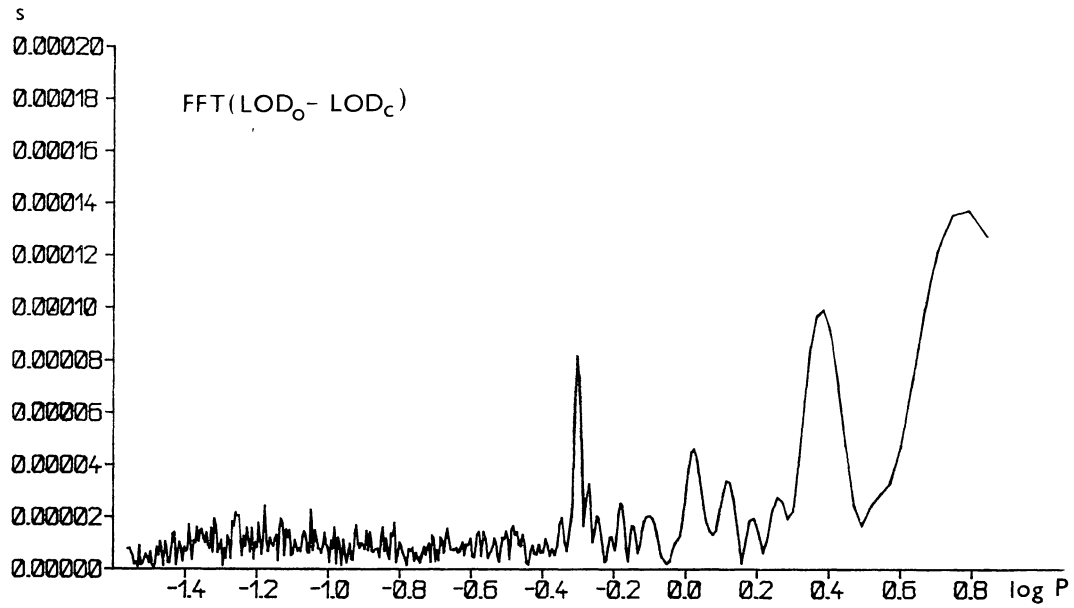

Fig. 16. FFT spectrum of the differences between the observed and calculated LOD.

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pheric or zonal luni-solar tides. The spectrum for periods shorter than 0·5 y clearly demonstrates the obvious fact that the rotation of the Earth at these frequencies can be fully explained by the combined effect of the atmosphere and Earth tides; no additional perturbation wave in the range between 13·5 d and 13·8 d, found by Capitaine and Guinot (1983), can be seen in the spectrum.

4. Concluding Remarks

The present study shows that there is practically no additional excitation to the atmosphere and tides for periods of less than 180 days; the spectrum in Fig. 15 depicts only noise in this frequency range. The small discrepancies in amplitudes of the semi-annual and annual components can very probably be ascribed to stratospheric winds, neglected in the NMC analysis, but this would call for a more detailed investigation, based on data covering an interval longer than the two-year interval used by Rosen and Salstein (1985). On the other hand, there exist variations of longer period in LOD (with periods 2·4 y and 6·0 y and amplitudes 0·10 msec and 0·14 msec respectively) which can be explained neither by the atmosphere, nor by the Earth tides. Here, not unlike in the case of the well-known decade fluctuations, another source of excitation must be present. The scaling factor found for the influence of the atmosphere ($0{\cdot}943 \pm 0{\cdot}018$) seems to show that the constants preceding the integrals in Eqs (2) and (3) should be a little smaller than used by the NMC (compare this number with the scaling factor of 0·993 obtained theoretically by Vondrák 1987). On the other hand, it is certain that the adjusted value of the atmospheric scaling factor depends on the amplitudes of the dominating atmospheric seasonal terms – if the stratospheric winds were included, the adjusted scaling factor would surely change. The obtained tidal scaling factor k/C ($0{\cdot}945 \pm 0{\cdot}021$) fully confirms, and even defines more precisely, the value recommended by Yoder et al. (1981). Better agreement between the observed LOD variations and those calculated from the tides and atmospheric EAMF achieved after 1983 (see Fig. 15) proves that errors of astronomical observations are mainly responsible for the noise of the differences at higher frequencies. The new observing techniques now used will, therefore, lead to a better understanding of the character of the missing excitation in the near future.

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11 Atmospheric excitation and variation in the length of day

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The LOD (length of day) and $\chi 3$ spectra were computed using the maximum entropy method (MEM). The comparisons show excellent correlation between the axial component of the atmospheric EAMF ($\chi 3$) and the changes in the Earth's rotation rate corrected for the effects of the lunar and solar tides. The most important changes are the seasonal variations with annual and semi-annual periods, and a cluster of small peaks with periods between 40 and 70 days i both spectra (LOD and $\chi 3$)

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ATMOSPHERIC EXCITATION AND VARIATIONS IN THE LENGTH OF DAY

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Summary: The LOD (length of day) and χ_3 spectra were computed using the maximum entropy method (MEM). The comparisons show excellent correlation between the axial component of the atmospheric EAMF (χ_3) and the changes in the Earth's rotation rate corrected for the effects of the lunar and solar tides. The most important changes are the seasonal variations with annual and semi-annual periods, and a cluster of small peaks with periods between 40 and 70 days i both spectra (LOD and χ_3).

key words: atmospheric excitation, length of day

N. Pejović: ATMOSFERSKA EKSCITACIJA I PROMENE U TRAJANJU DANA – Spektri LOD i χ_3 računati su korišćenjem metoda maksimalne entropije (MME). Uporedjenja pokazuju odličnu korelaciju izmedju osne komponente atmosferske EAMF (χ_3) i promena u brzini Zemljine rotacije korigovane za efekte Mesečevih i Sunčevih plima. Najznačajnije promene su sezonske varijacije sa godišnjom i polugodišnjom periodom i grupom malih pikova sa periodama izmedju 40 i 70 dana u oba spektra (LOD i χ_3).

1. INTRODUCTION

It is well known that the atmosphere has a dominant influence on the changes of the Earth's rotation parameters (polar motion and variations in the length of day). More recent and more detailed information about the atmosphere has been given in many papers. We should mention authors considering the atmospheric influence on the variations in the day-length variations (e. g. Lambeck and Cazenave, 1973, 1974, 1975; Sidorenkov, 1973, 1979; Okazaki, 1977; Rosen and Salstein, 1983, 1985; Djurović, 1983, 1986, 1987; Vondrak, 1987; Feissel and Nitschelm, 1985). A very important paper providing a solid basis for computations of the so-called effective angular momentum functions (EAMF) of the atmosphere based on global meteorological data collected at the European Centre for Medium-Range Weather Forecasts (ECMWF) has been published by Barnes, Hide, White and Wilson (1983).

Briefly summarizing the results obtained as yet one may say that the short-periodic variations in the day length are caused by the atmospheric and tidal effects; comparisons show an excellent correlation between the axial component of the atmospheric EAMF and the changes in the velocity of the Earth's rotation corrected for the lunar and solar tidal effects. The EAMF axial component is dominant in the zonal wind effects. Seasonal variations (with annual and semi-annual periods) and short-periodic ones (with a group of periods between 40 and 70 days) are the most important. There is a reliable proof that a close correlation between the solar activity and the global atmospheric circulation exists (Djurović, 1987).

2. THEORETICAL BACKGROUND

Vondrak (1987) introduced in the well-known third Liouville equation of the rotation Earth's model

$$\dot{m}_3 = \dot{\psi}_3 \tag{1}$$

excitations depending on m_3 which are due to:

a) fluid core:

$$\Delta\psi_3^c \approx \frac{C_c}{C}\dot{m}_3$$

where C_c is the principal moment of inertia of the core and C is the principal inertia moment of the Earth,

b) rotational deformation of the mantle:

$$\Delta\psi_3^{rd} = \frac{-4k\Omega^2 R^5}{9GC}\dot{m}_3$$

where k is the elastic Love number, Ω is the mean angular velocity of the Earth's rotation, R is the mean terrestrial radius and G is the gravitation constant,

c) ocean:

$$\Delta\psi_3^o = \frac{-4\Omega^2 R^5 0.040}{9GC}\dot{m}_3.$$

Then, by introducing the function of the axial atmosphere angular momentum χ_3 (Barnes et al., 1983) instead of the excitation function

$$\psi_3' = \psi_3 - (\Delta\psi_3^c + \Delta\psi_3^{rd} + \Delta\psi_3^o) \tag{2}$$

and more realistic numerical values of the constants, Vondrak (1987) obtained the equation:

$$m_3 = -\frac{\Delta LOD}{LOD_0} = 0.993\chi_3 + const \tag{3}$$

where $\Delta LOD/LOD_0$ is the relative change in the day length taken with respect to the mean value $LOD = 86400s$. From equation (3) one obtains the calculated value of LOD which should be compared to the observed one obtained from $UT1 - UTC$.

3. USED DATA

The data of the axial atmosphere EAMF calculated on the basis of Barnes' algorithm (Barnes et al., 1983) at the US National Meteorological Center obtained by the courtesy of Vondrak (1986a) are used here. The daily values of χ_3 for the period 1976.5–1986.0 are analysed in the following three combinations:

a) $\chi_3^p + \chi_3^w$

b) $\chi_3^{p'} + \chi_3^w$

c) χ_3^w

where the indices p and w denote the pressure and wind terms, respectively, and p' denotes the pressure term calculated on the inverted barometer hypothesis. The data corresponding to all the three combinations are presented in Fig. 1 and Fig. 2; the wind term is in the lower part of Fig. 1 and its sums with both pressure terms (with and without the inverted barometer correction) are in the lower and upper parts of Fig. 2, respectively. By inspecting the Figures one easily finds out that all the three data sets are periodic functions of time with a prominent annual term. The wind term dominates over the axial component of the atmosphere angular momentum. The second group of the used data are the combined solutions of the UT1-UTC values obtained and published for the period 1976-1986 by BIH (BIH Annual Reports) for the same time interval as for the EAMF data. One should note that the combined solutions in the beginning were mostly based on optical astrometry gradually becoming more and more based on observations performed by using modern technics which are more accurate, especially within the short-periodic part of the spectrum (Vondrak, 1986b). The changes of the day length (LOD) calculated from UT1-UTC are presented in the upper part of Fig. 1; their more complex structure is caused by combination of changes due to tidal effects (Yoder et al., 1981) and to the atmospheric excitation.

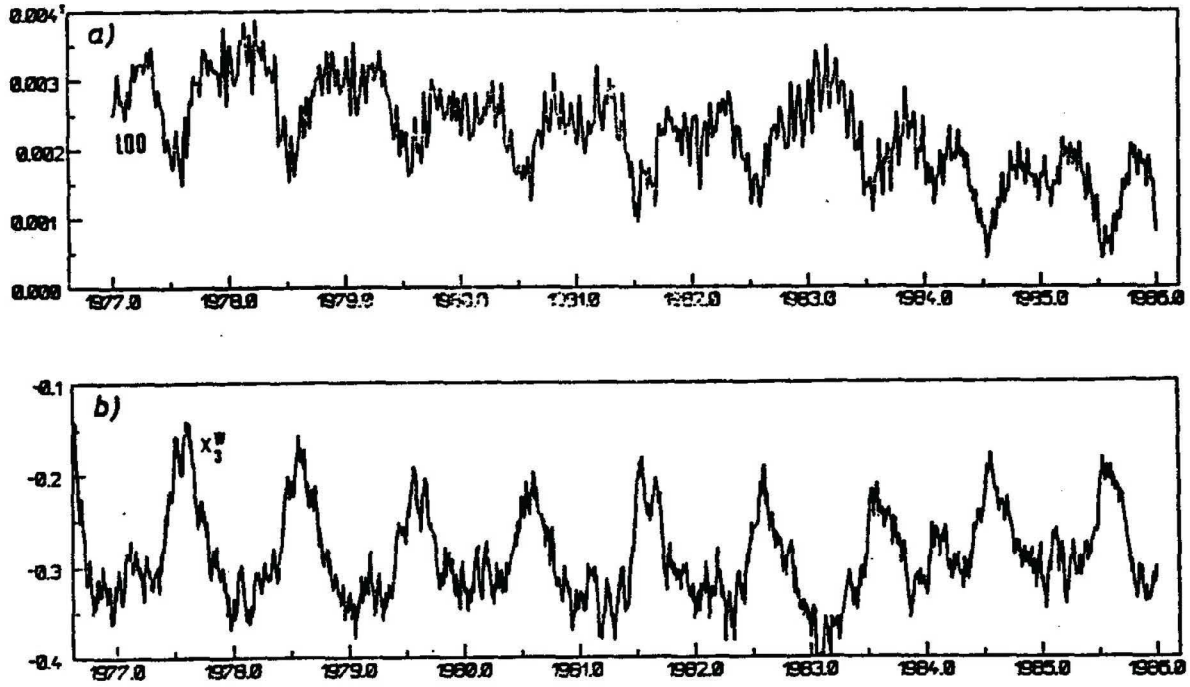

Fig. 1 a) The excess of length of day over the nominal 86400^s (LOD) as determined from astronomikal observations at the BIH in Paris.
b) The axial component of the wind term of the atmospheric $EAMF$ as determined by the NMC.

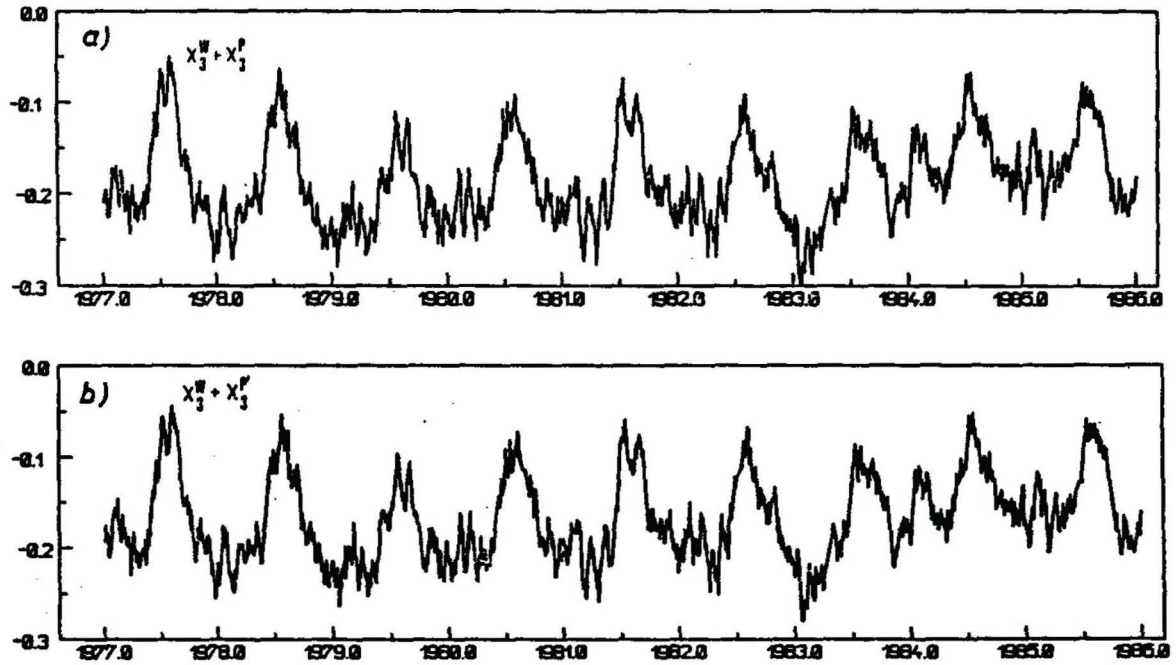

Fig. 2 a) The axial component of the sum of wind and pressure term of the atmospheric $EAMF$ as determined by the NMC.
b) The axial component of the sum of wind and pressure term with the inverted barometer correction of the atmospheric $EAMF$ as determined by the NMC.

4. RESULTS

On the basis of equation (3) one can write the UT1-UTC approximation which is

$$UT1 - UTC = A + Bt + Ct^2 + 0.993 \int_0^t \chi_3 dt + (CT) \tag{4}$$

where with CT are denoted 62 tidal terms determined by Yoder et al. (1981), A, B and C are constants determined by using the least-square-method (LSM). Their values are presented in Table 1.

By obtaining the time derivative of equation (4) one finds the LOD approximation

$$\frac{d}{dt}(UT1 - UTC) = -LOD = -(a + bt + 0.993\chi_3 + CD), \tag{5}$$

TABLE 1.

A, B, C are constants of the UT1–UTC approximation calculated by using LSM for all the three variants of χ_3

	A	B	C
χ_3^{w+p}	0.0135324	−0.0011113	0.00000015
$\chi_3^{w+p'}$	0.0192252	−0.0012234	0.00000013
χ_3^{w}	0.0375353	−0.0003767	0.00000019

where the tidal terms (Yoder et al., 1981) are denoted as $CD = \mathrm{d}(CT)/\mathrm{d}t$, and a, b are constants determined by using LSM. The obtained values are given in Table 2.

It is seen from the Tables that for the coefficients a, b, A, B and C one obtains approximate values following from the derivation of (4) aimed at obtaining expression (5), i.e. $-a \approx B$, $-b \approx 2C$.

TABLE 2.

Coefficients a, b determined by using LSM for LOD approximation, δ is the root-mean-square deviation.

	a	b	δ
χ_3^{w+p}	0.0011815	−0.0000003	$\pm 0^s.00028$
$\chi_3^{w+p'}$	0.0012905	−0.0000003	$\pm 0^s.00027$
χ_3^{w}	0.0004083	−0.0000003	$\pm 0^s.00029$

In Fig. 3 (wind term plus pressure term), Fig. 4 (wind term plus pressure term with invertive barometer correction applied) and Fig. 5 (wind term only) all the three combinations of χ_3 in the LOD approximation (5) and the observed LOD are presented. It is seen from these Figures that the agreement between the two curves is excellent. In order to obtain a better estimate of this agreement one calculates the root-mean-square deviation δ between the two curves (observed LOD and atmosphere function χ_3) by using the following

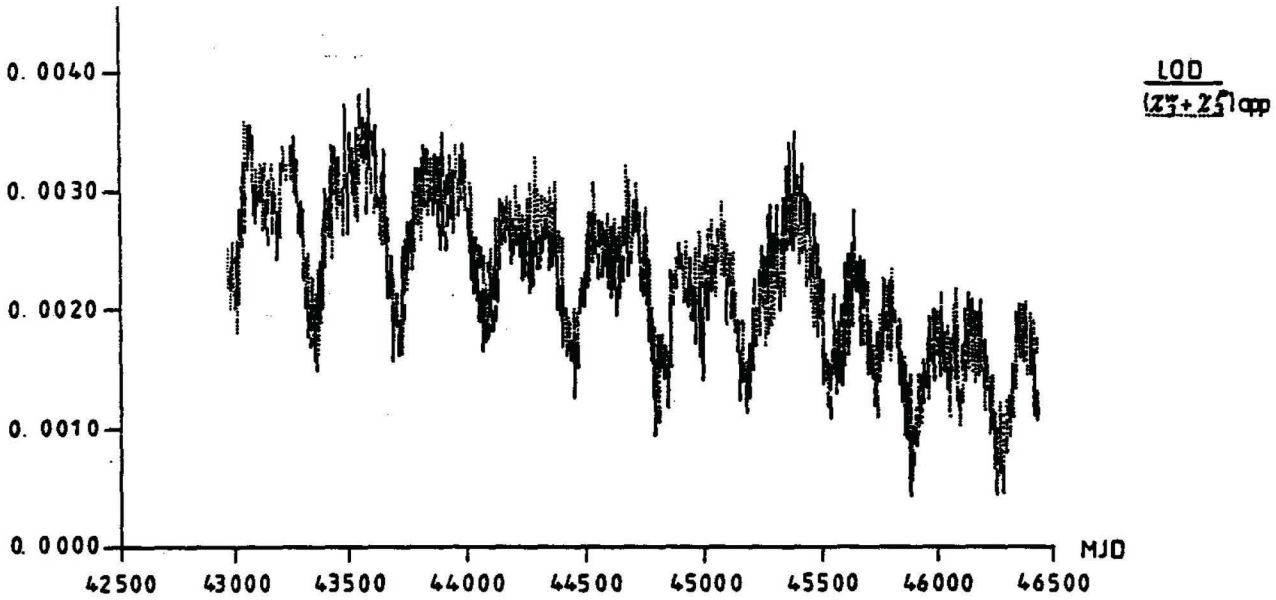

Fig. 3 Comparison of fluctuation in the observed *LOD* with corresponding fluctuations in the axial component of the sum of wind and pressure term of the atmospheric *EAMF*
[see equ. (5); $(\chi_3^w + \chi_3^P)_{app} = a + bt + 0.993(\chi_3^w + \chi_3^P) + CD$].

equation

$$\delta = \sqrt{\frac{\sum \nu^2(t)}{n-2}}$$

where $\nu(t) = LOD - (a + bt + \chi_3 + CD)$, n is the number of data.

The values obtained for δ for all the three considered variants of χ_3 are presented in Table 2. The best agreement is achieved in the case of variant *b*, i. e. for $\chi_3^{w+p'}$ (wind term plus pressure term with invertive barometer correction applied) which is equal to $\pm 0^s,00027$. It follows from this that the LOD and χ_3 data can be directly compared unlike the polar motion data and the equatorial EAMF (Vondrak and Pejović, 1988).

The comparison between the observed LOD and the atmospheric χ_3 will be given also through a comparison of their spectra.

Since the root-mean-square deviation between the two curves (LOD and χ_3) (Table 2) is about $\pm 0^s.0003$, the spectral analysis will be carried out for all the three variants.

The Maximum Entropy Method (MEM)

The parameter analysis of time series and also the spectral analysis as a part of it, is a qualitatively new phase in the theory of stochastic processes and in solving practical problems. The fist steps are due to Yule (1927) who was

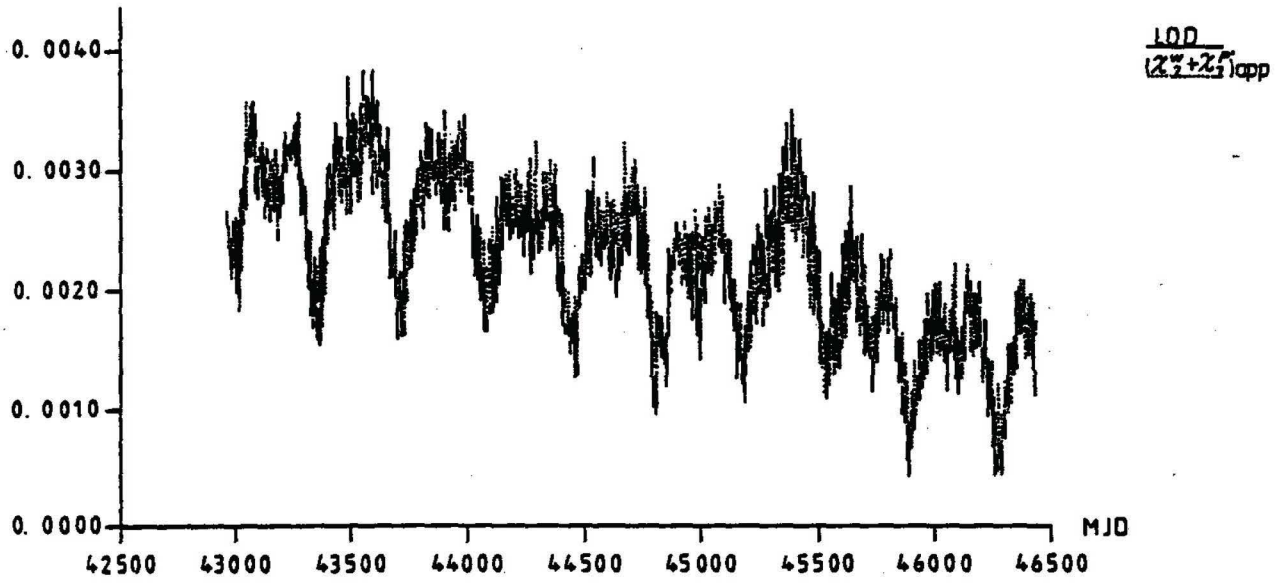

Fig. 4 Comparison of fluctuation in the observed LOD with corresponding fluctuations in the axial component of the sum of wind and pressure term with the inverted barometer correction of the atmospheric $EAMF$
[see equ. (5); $(\chi_3^w + \chi_3^{P'})_{app} = a + bt + 0.993(\chi_3^w + \chi_3^{P'}) + CD$].

the first to construct an autoregressive model of a stochastic process (AR). In 1967 Burg (1967) and independently Parzen (1967) proposed using of the AR model for the purpose of a spectrum estimate, and Burg demonstrated then that such an estimate corresponds to the requirement for a maximal entropy of unknown values of the covariant function. Such a property of the spectral density removes the necessity of making weakly assumptions on a periodical character of a process beyond the interval observed. This is especially important in the case of short time series which are known to be difficult for Fourier analysis method due to leakage and aliasing. We have used the subroutines and the main program written by Privalski (1985) for the autoregressive analysis of an equidistant time series. Parameters of the autoregressive model (M, L...) are estimated by the Burg-Lewinson method (Privalski 1985) and the optimal order of the model ($IBEST$) is chosen by the Parzen's criterium.

The Spectra of the Data Obtained with MEM

The results of the spectral analysis are presented in Figs. 6-10. Along the axis of abscissae is the logarithmic scale of the period expressed in years and along the one of ordinates are logarithmic power density. Computing is done using the parameters of the autoregressive model $M = 60$ and $L = 30$. The optimal order by the Parzen's criterium is $IBEST = 23, 44, 44, 27,$ and 25 for the

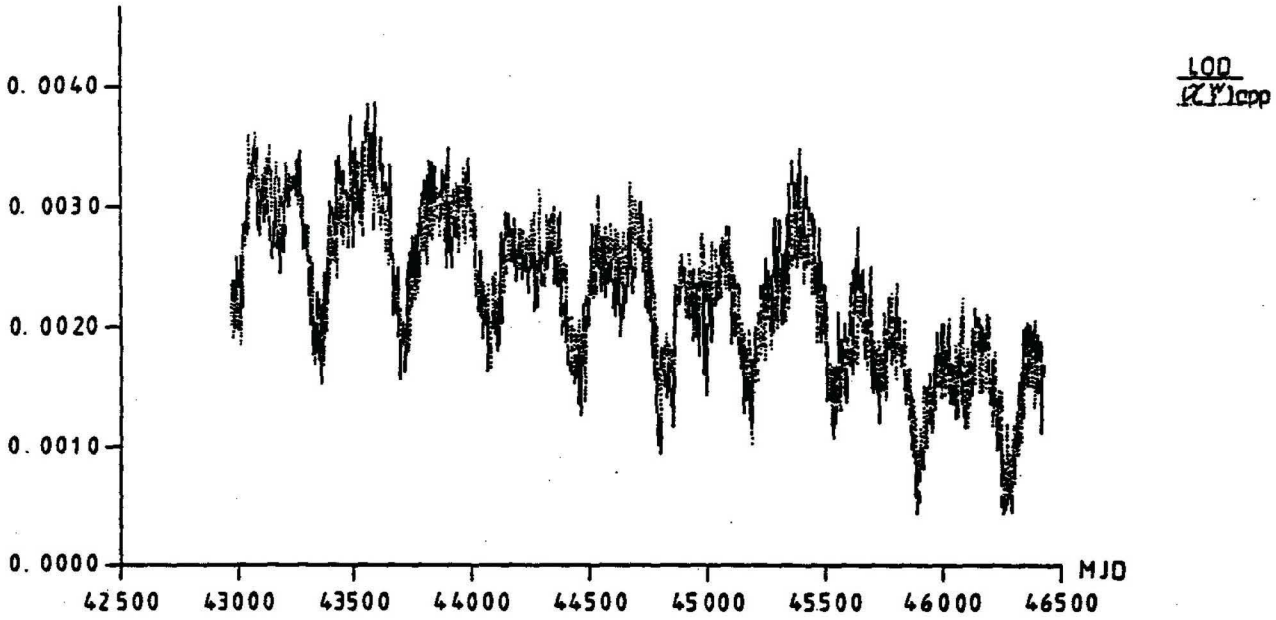

Fig. 5 Comparison of fluctuation in the observed LOD with corresponding fluctuations in the axial component of the wind term of the atmospheric $EAMF$ [see equ. (5); $(\chi_3^w)_{app} = a + bt + 0.993\chi_3^w + CD$].

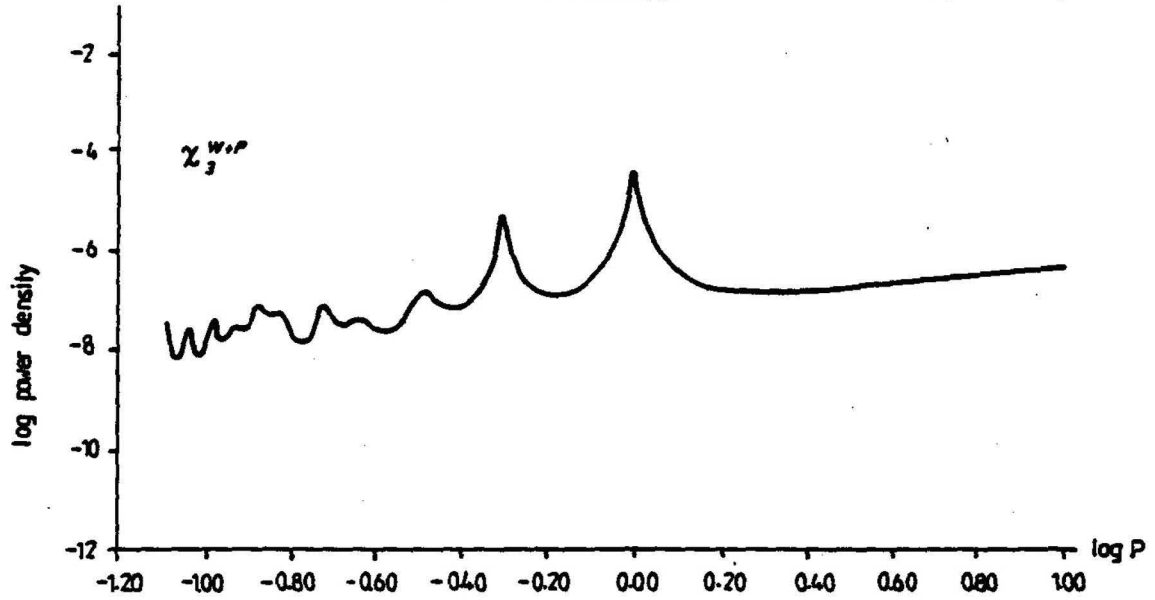

Fig. 6 The spectrum of the axial component of the sum of wind and pressure term of the atmospheric $EAMF$, calculated by the MEM.

spectra on the Figures 6-10,respectively. It is seen from LOD and the various combinations of χ_3 that the pressure term with inverted barometer correction has no important influence on the day duration. If one compares the spectrum

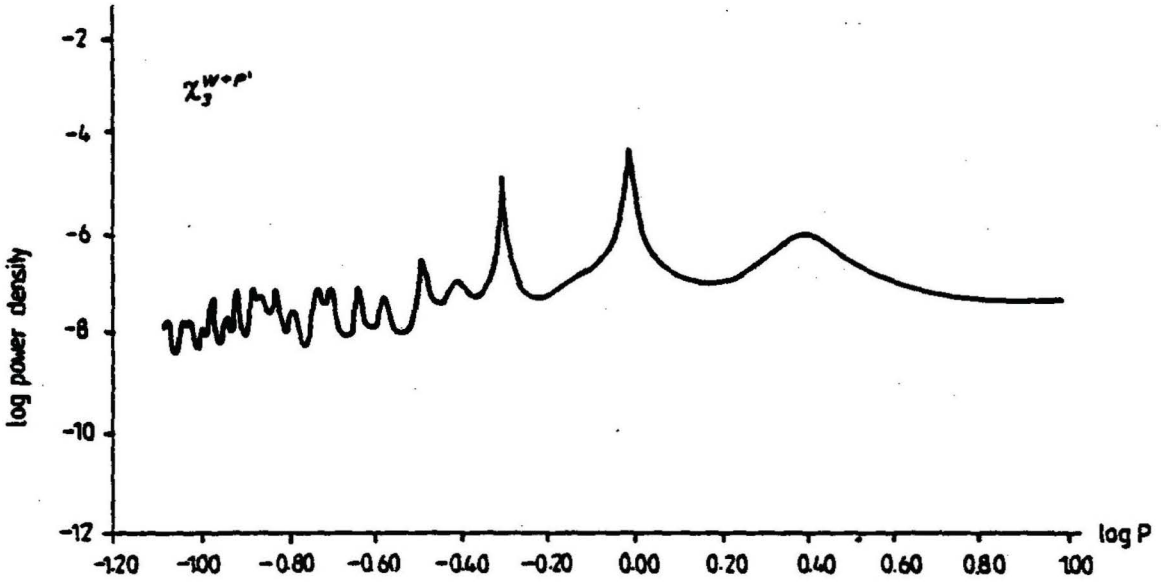

Fig. 7 The spectrum of the axial component of the sum of wind and pressure term with the inverted barometer correction of the atmospheric $EAMF$, calculated by the MEM.

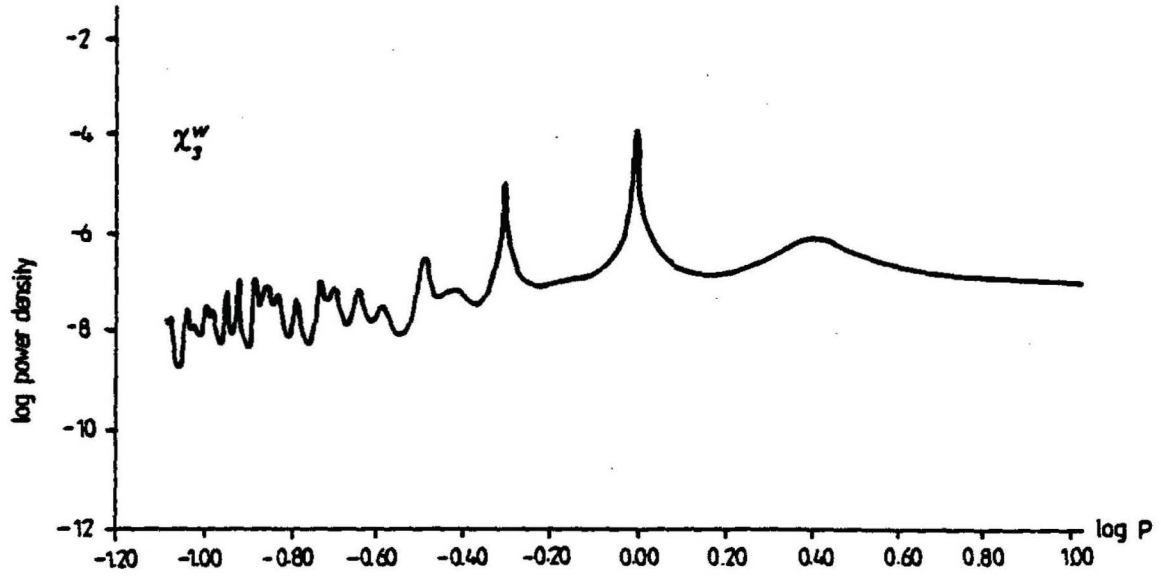

Fig. 8 The spectrum of the axial component of the wind term of the atmospheric $EAMF$, calculated by MEM.

of $\chi_3^{w+p'}$ (wind term plus pressure term with inverted barometer correction) presented in Fig. 7 with the one of χ_3^{w} (wind term only) presented in Fig. 8, one finds a wonderful similarity. They are practically identical. For the

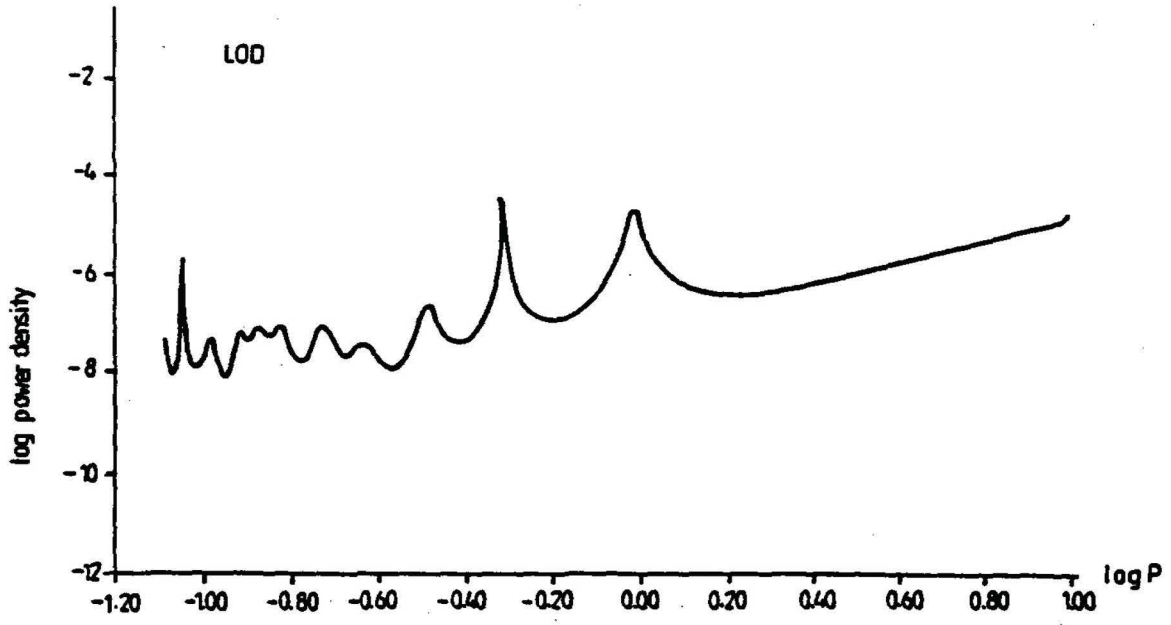

Fig. 9 The spectrum of the observed *LOD*, calculated by the *MEM*.

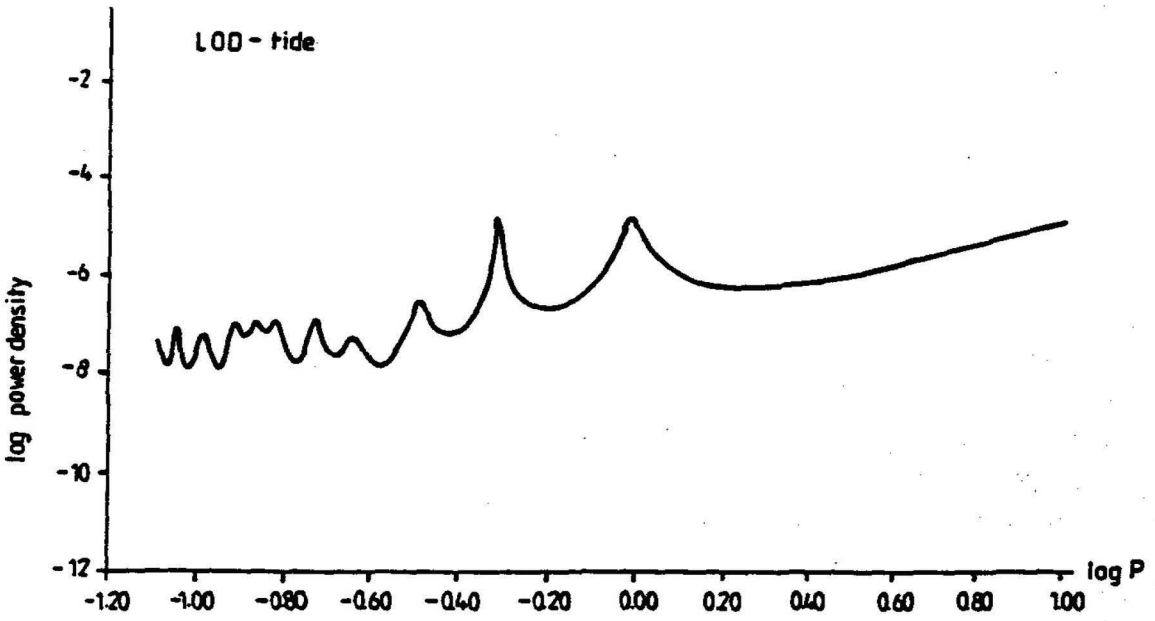

Fig. 10 The spectrum of the observed *LOD* with tidal variations removed, calculated by the *MEM*.

axial component χ_3 the pressure term is so small, that the inverted barometer hypothesis causes no essential changes in the total angular momentum function. Generally speaking by combining the wind term with the pressure term (χ_3^{w+p}) in Fig. 6, one obtains a better agreement with the observed LOD

TABLE 3.

Periods, amplitudes and phases obtained by applying FFT method for all the three combinations χ_3, LOD and LOD-tides are presented. The phases are with respect to modified Julian dates (MJD)=42963.

	Period [*yr*]	*Amplitude* [*s*]	*Phase* [°]
χ_3^{w+p}	0.50	0.000246	164.23
	1.00	0.000368	331.74
$\chi_3^{w+p'}$	0.50	0.000246	167.28
	1.00	0.000363	331.84
χ_3^{w}	0.50	0.000253	165.25
	1.00	0.000406	335.74
LOD	0.50	0.000361	159.29
	1.00	0.000354	301.25
LOD-tides	0.50	0.000293	160.92
	1.00	0.000328	327.79

with tides subtracted (compare the spectra LOD and LOD-tide in Figs. 9 and 10) than by using solely the wind term. The similarity of the LOD spectrum (without tides) with $\chi_3^{w+p}, \chi_3^{w+p'}$ is remarkable; they are practically identical. There are some small differences in amplitudes for both dominant terms: the annual and the semi- annual ones(Table 3);χ_3 yields a smaller semi-annual-term amplitude and a larger annual-term amplitude than the observed LOD without tides, but these differences are within a 10% level. Larger differences appear in the case of longer periods. The amplitudes of the observed LOD (without tides) corresponding to periods exceeding one year seem to be in a better agreement with a combination of the wind term and the pressure term alone than with χ_3^w and $\chi_3^{w+p'}$. This could mean that the pressure term with inverted barometer correction applied in its long-periodic part is not reliable, or (more probably), that another source of long-periodic change excitation responsible for these variations in the angular velocity of the Earth's rotation (e.g. an electromagnetic coupling between the core and the mantle) is present.

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12 Earth's model with variable Chandler's frequency

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The hypothesis of a nonlinear dependence of Chandler's frequency on the total polar motion amplitude is confirmed by analyzing the ILS coordinates of the pole. Assuming that this nonlinear dependence of Chandler's frequency is due to a nonequilibrium ocean, one introduces a new excitation function of the nonequilibrium ocean in Vondrak's (1987) model of the Earth. The response of this more complete Earth's model with a variable and amplitude-dependent Chandler's frequency to atmospheric excitation is derived, as well.

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EARTH'S MODEL WITH VARIABLE CHANDLER'S FREQUENCY

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МОДЕЛЬ ЗЕМЛИ С ПЕРЕМЕННОЙ ЧАНДЛЕРОВОЙ ЧАСТОТОЙ

Подтверждена гипотеза о нелинейной зависимости Чандлеровой частоты от тотальной амплитуды полярного движения из анализа координат полюса МСШ. Предполагая что нелинейная зависимость Чандлеровой частоты вызвана динамическим океаном мы ввели новую возбуждающую функцию динамического океана в модели Земли Вондрака. Выведена реакция этой модели Земли с переменной и амплитудно зависимой Чандлеровой частотой на атмосферное возбуждение.

The hypothesis of a nonlinear dependence of Chandler's frequency on the total polar motion amplitude is confirmed by analysing the ILS coordinates of the pole. Assuming that this nonlinear dependence of Chandler's frequency is due to a nonequilibrium ocean, one introduces a new excitation function of the nonequilibrium ocean in Vondrák's (1987) model of the Earth. The response of this more complete Earth's model with a variable and amplitude-dependent Chandler's frequency to atmospheric excitation is derived, as well.

Key words: Chandler's frequency — polar motion — atmospheric excitation

1. Introduction

One of the most important events in the history of polar motion research is Chandler's discovery of 1892 (Chandler 1892). He found, by analysing a large number of observations, that variations in geographic latitude have two periodic components: one named after Chandler himself, whose period is 427 days, and another with an annual period. Newcomb (1872) was first to attempt an explanation of both the difference between the Eulerian and Chandler's periods and the annual component. In determining the Eulerian period one has assumed that the Earth is an absolute solid body which is not the case; if its elasticity is taken into account, then a satisfactory agreement between theory and observations is reached.

The variability of the Chandler's period has been controversial for almost an entire century. A number of authors, including Chandler himself, have assumed that the Chandler's period is either multi-component, or variable in time (Chandler 1892, Kimura 1918, Hattori 1949, Melchior 1957, Colombo and Shapiro 1978, Gaposchkin 1972, Sekiguchi 1972, 1976, Carter 1981, Dickman 1981, Pejovic 1983, Vondrák 1985, 1988). There are other authors claiming that the Chandler's nutation has a single period (Newcomb 1892, Pedersen and Rochester 1972, Ooe 1978, Okubo 1982).

A variety of results as well as theoretical considerations have been presented in two most important monographs devoted to the Earth's rotation: Munk and MacDonald (1960) and Lambeck (1980).

More recent and more detailed information about the atmosphere has been presented in papers where the influence of atmospheric effects on the polar motion is considered (e.g. Wilson and Haubrich 1976a, 1976b, Jochman 1976, 1981a, 1981b, Lambeck and Hopgood 1981, Wahr 1982, 1983, Eubanks et al. 1977, Brzezinski 1987, Sidorenkov 1982, Barnes et al. 1983, Hide 1984, Dutton and Fallon 1985, Eubanks et al. 1985, Vondrák 1987, 1989, Salstein 1987). Among the important papers is certainly that published by Barnes et al. (1983) where a solid basis for calculating effective angular momentum functions (EAMF) for the atmosphere based on global meteorological data was given.

Attempts of using the computed atmospheric EAMF for the purpose of numerical integration of polar motion, carried out, e.g., by Barnes et al. (1983) or Dutton and Fallon (1985), have been successful in fitting the observed

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polar motion only over very limited time intervals (less than or equal to one Chandler's period); for longer time intervals both curves diverge very rapidly. A more realistic Earth's model has been used by Vondrák (1989) who achieved a better fit between the integrated and observed polar motions over a significantly longer time interval (9 years). The model characterised by triaxial Earth's structure, its fluid core, visco-elastic mantle and equilibrium ocean was proposed by the same author (Vondrák 1987). In order to improve the fit, a modified version of Vondrák (1987) model is proposed in the present paper. The modification is that, instead of Vondrák's equilibrium ocean, a nonequilibrium ocean is introduced. The reason for this modification are Vondrák's (1985, 1988) results achieved by analysing the observed polar motion over a long time interval (1860–1985). Vondrák's basic result is that the Chandler's frequency is a nonlinear function of the polar motion amplitude. On the other hand, Carter (1981) suggested that a nonequilibrium ocean could be the cause of variability of Chandler's frequency. According to an earlier analysis of the present author (Pejović 1983) concerning the ILS pole coordinates (Yumi and Yokoyama 1980) there is no single Chandler's peak.

2. Chandler's Frequency

Prior to introducing a new excitation function of the nonequilibrium ocean in Vondrák's (1987) model of the Earth, the starting hypothesis concerning the nonlinear dependence of the Chandler's frequency on the total polar motion amplitude will be re-examined.

It has already been mentioned above that the variability of Chandler's period has been controversial for almost an entire century. A number of authors, including Chandler himself, have introduced hypotheses about the multiplicity or variability of the period. Okubo (1982) in his extended analysis devoted to Chandler's frequency problem attempted to solve the existing controversy. According to his conclusion Chandler's motion is not stationary and the mean value of the fluctuating Chandler's period is 435 days. It seems that what to Okubo was a randomly fluctuating Chandler's motion, to Carter (1981) and other authors was frequency modulated whereby Carter looked for the causes of this modulation in excitation functions suggesting the non-equilibrium ocean as the possible cause. The problem became more clear with the advent of Vondrák's (1985, 1988) results concerning a functional dependence of Chandler's frequency on the total polar motion amplitude. Namely, Vondrák analysed a long time interval (1860–1985) of inhomogeneous pole coordinate data. Besides confirming the results of other authors (e.g. Guinot 1972, 1982, Dickman 1981, Pejović 1985), Vondrák (1985, 1988) found, in addition to a rapid change in Chandler's phase for the interval 1920–1940, another change for 1870–1890 since his analysis included the interval before 1900. The rapid change of Chandler's phase between 1920 and 1940 cannot be explained by the available series of pole coordinates. However, it is not the only one as seen from the interval 1870–1890. These rapid phase changes coincide with the minima of the total

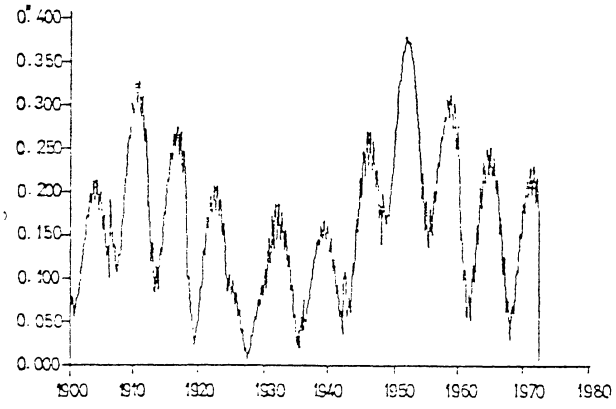

Fig. 1. The total amplitude of polar motion is obtained by analysing the ILS coordinates of the pole.

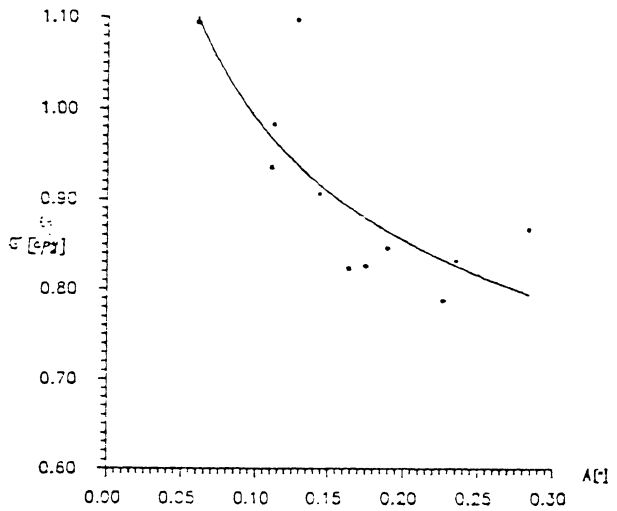

Fig. 2. Non-linear dependence between polar motion amplitude A and Chandler frequency σ.

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polar motion amplitude. The amount of correlation coefficient between Chandler's phase and the integral of the polar motion amplitude was found to be 0·986 (Vondrák 1985, 1988). Calculating the instantaneous Chandler's frequency from Chandler's phase differential, Vondrák also found Chandler's frequency to be an exponential function of the total polar motion amplitude (Vondrák 1988, Fig. 3).

In the present paper Vondrák's (1985, 1988) procedure has been re-applied to another homogeneous ILS pole-coordinate series and his results have been confirmed. The total polar motion amplitude displayed in Fig. 1 agrees well with Vondrák's (1985, Fig. 4) results. The instantaneous Chandler's frequency calculated from Chandler's phase differential is presented in Fig. 2 together with the total polar motion amplitude for every independent six-year interval. This figure confirms Vondrák's (1988, Fig. 3) results according to which Chandler's frequency is a nonlinear function of the polar motion amplitude.

3. Basic Equations

3.1. *Liouville Equations*

The Liouville equations can be written in a simple form used. e.g., by Munk and MacDonald (1960), Lambeck (1980), Barnes et al. (1983), Vondrák (1987)

$$\begin{aligned} m_1 - \frac{\dot{m}_2 A}{\Omega(C-A)} &= \psi_1 \\ m_2 + \frac{\dot{m}_1 A}{\Omega(C-A)} &= \psi_2 , \end{aligned} \tag{1}$$

where $\sigma_r = \Omega(C-A)/A$ is the Eulerian frequency, i.e. the free-nutation frequency of a rigid body, C and A are the polar and equatorial main inertia momenta of the nondeformed Earth, respectively, Ω is the angular velocity of the Earth's rotation, m_1 and m_2 are small quantities. In Eq. (1) ψ_1 and ψ_2 are the known excitation functions,

$$\begin{aligned} \psi_1 &= \frac{\Omega^2 \Delta I_{13} + \Omega \Delta \dot{I}_{23} + \Omega h_1 + \dot{h}_2 - L_2}{\Omega^2(C-A)} \\ \psi_2 &= \frac{\Omega^2 \Delta I_{23} - \Omega \Delta \dot{I}_{13} + \Omega h_2 - \dot{h}_1 + L_1}{\Omega^2(C-A)} , \end{aligned} \tag{2}$$

where $(\cdot)$ denotes the time derivative in the rotating frame. Eq. (2) can be written in a conventional complex notation (Barnes et al. 1983)

$$\psi = \psi_1 + \mathrm{i}\psi_2 = \frac{\Omega^2 \Delta I - \mathrm{i}\Omega \Delta \dot{I} + \Omega h - \mathrm{i}\dot{h} + \mathrm{i}L}{\Omega^2(C-A)} ,$$

where

$$\Delta I \equiv \Delta I_{13} + \mathrm{i}\,\Delta I_{23} ,$$

$$h = h_1 + \mathrm{i}h_2 .$$

$$L = L_1 + \mathrm{i}L_2 .$$

By introducing $m = m_1 + \mathrm{i}m_2$ and $\dot{m} = \dot{m}_1 + \mathrm{i}\dot{m}_2$ equations (1) can be expressed in complex form

$$m + \mathrm{i}\dot{m}\frac{A}{\Omega(C-A)} = \psi . \tag{3}$$

It is known that this equation can be solved if the parts of the excitation function ψ from the right-hand side of (3), which depend on m, are separated from the other parts and moved to the left-hand side of (3).

3.2. *Fluid Core*

The effects of a homogeneous fluid core on the rotation of a solid mantle are given in (Vondrák 1987). The parts of the excitation functions expressing the influence of the core will now be treated:

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$$\Delta\psi_1^c = \frac{A_c\dot{V}_2 + A_c\Omega V_1}{\Omega^2(C-A)} \approx \frac{-A_c}{\Omega(C-A)}\dot{m}_2$$

$$\Delta\psi_2^c = \frac{A_c\Omega V_2 - A_c\dot{V}_1}{\Omega^2(C-A)} \approx \frac{A_c}{\Omega(C-A)}\dot{m}_1\,, \tag{4}$$

where V_1, V_2 are the components of vector of rotation of the core with respect to the mantle.

The equations of the system can be written in complex notation as

$$\Delta\psi^c = i\dot{m}\frac{A_c}{\Omega(C-A)}\,. \tag{5}$$

3.3. *Rotational Mantle Deformation*

The changes of the inertia tensor corresponding to the elastic rotational deformations are given in Vondrák (1984, 1987). The excitation functions corresponding to the elastic rotational deformation are

$$\Delta\psi_1^{rd} = \frac{\Delta I_{13}\Omega + \Delta\dot{I}_{23}}{\Omega(C-A)} = \frac{k\Omega^2R^5}{3G(C-A)}\left(m_1 + \frac{\dot{m}_2}{\Omega}\right)$$

$$\Delta\psi_2^{rd} = \frac{\Delta I_{23}\Omega - \Delta\dot{I}_{13}}{\Omega(C-A)} = \frac{k\Omega^2R^5}{3G(C-A)}\left(m_2 - \frac{\dot{m}_1}{\Omega}\right)$$

and, if one introduces the quality factor Q for the viscosity as Lambeck (1980) did,

$$\begin{pmatrix}\tilde{m}_1\\ \tilde{m}_2\end{pmatrix} = \begin{pmatrix}\cos\varepsilon & \sin\varepsilon\\ -\sin\varepsilon & \cos\varepsilon\end{pmatrix}\begin{pmatrix}m_1\\ m_2\end{pmatrix}$$

where for small angles one has $\cos\varepsilon \approx 1$, $\sin\varepsilon \approx Q^{-1}$, one obtains the following expressions:

$$\tilde{m}_1 = m_1 + Q^{-1}m_2$$

$$\tilde{m}_2 = m_2 - Q^{-1}m_1\,.$$

If the latter expression is inserted into the former system of excitation-function equations, they will transform to

$$\Delta\psi_1^{rd} = \frac{k\Omega^2R^5}{3G(C-A)}\left[m_1 + Q^{-1}m_2 + \frac{\dot{m}_2 - Q^{-1}\dot{m}_1}{\Omega}\right]$$

$$\Delta\psi_2^{rd} = \frac{k\Omega^2R^5}{3G(C-A)}\left[m_2 - Q^{-1}m_1 - \frac{\dot{m}_1 + Q^{-1}\dot{m}_2}{\Omega}\right]. \tag{6}$$

The equations of the system can be written in complex notation as follows:

$$\Delta\psi^{rd} = \Delta\psi_1^{rd} + \mathrm{i}\,\Delta\psi_2^{rd} = \frac{k\Omega^2R^5}{3G(C-A)}\left(m - \mathrm{i}\frac{\dot{m}}{\Omega}\right)(1 - \mathrm{i}Q^{-1})$$

or, by introducing the secular Love number (Lambeck, 1980)

$$k_s = \frac{3G(C-A)}{r^5\Omega^2}\,,$$

the former equation becomes

$$\Delta\psi^{rd} = \frac{k}{k_s}\left(m - \mathrm{i}\frac{\dot{m}}{\Omega}\right)(1 - \mathrm{i}Q^{-1})\,. \tag{7}$$

3.4. *Nonequilibrium Ocean*

Because of Vondrák's (1988) results as well as the present ones concerning the nonlinear dependence of the Chandler frequency on the total amplitude of polar motion and Carter's (1981) suggestion according to which

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the cause of this dependence could be the nonequilibrium ocean, the excitation function of the nonequilibrium ocean in a complex notation is introduced in the following form:

$$\Delta\psi^0 = m\xi\,, \tag{8}$$

where ξ is a factor depending on the polar motion amplitude. With Eq. (8) a new ocean function is introduced because, for the case of an equilibrium ocean, the excitation functions are given in Lambeck (1980):

$$\left.\begin{matrix}\Delta\psi_1\\ \Delta\psi_2\end{matrix}\right\} = \frac{3}{10}\frac{\varrho_\omega}{\varrho}\frac{1}{k_s}(1 + k - k')\begin{Bmatrix} m_1A_1 + m_2A_2 \\ m_1B_1 + m_2B_2 \end{Bmatrix}$$

where ϱ_ω and ϱ are the densities of the ocean and Earth, respectively, k_s, k and k' are the Love numbers; m_1 and m_2 are the coordinates of the pole along the zero meridian and 90E, respectively; the constants A_1, A_2, B_1 and B_2 depend on the asymmetric ocean distribution over the Earth. Using the Lambeck formulae and the ocean function expansion (Lambeck, 1980) one finds: $A_1 = 1{\cdot}724$, $A_2 = B_1 = -0{\cdot}025$, $B_2 = 1{\cdot}406$. In this way one concludes that for the case of an equilibrium ocean symmetrically distributed upon the Earth the factor ξ is approximately equal to 0·064. One should find the dependence of the ξ factor on the polar motion amplitude A for the case of a nonequilibrium ocean. Using the results of Vondrák (1988, Fig. 3), who found an exponential dependence between Chandler's frequency σ and the polar motion amplitude A by analysing the observed polar motion over a long time interval, (1860–1985), it is sufficient to apply

$$\xi = 0{\cdot}064(1 - \mathrm{e}^{-20A})\,. \tag{9}$$

This yields

$$\Delta\psi^0 = 0{\cdot}064m(1 - \mathrm{e}^{-20A}) \tag{10}$$

for the excitation function (8) in the case of the nonequilibrium ocean.

The excitation function (10) is very near the one for the equilibrium ocean:

$$\Delta\psi_s^0 = 0{\cdot}064m\,.$$

They differ only in the case of very small amplitudes and they are equal to each other in the case of amplitudes greater than 0·2″.

4. Solution of the Liouville Equation in the Case of Variable Chandler's Frequency

Substituting all the derived excitations (5), (7) and (8) in the left-hand side of the Liouville equation (3) one obtains

$$m + \mathrm{i}\dot{m}\frac{A}{\Omega(C - A)} - (\Delta\psi^c + \Delta\psi^{rd} + \Delta\psi^0) = \psi'$$

The right-hand side of this equation will be denoted as ψ', i.e.

$$\psi' = \psi - (\Delta\psi^c + \Delta\psi^{rd} + \Delta\psi^0) \tag{12}$$

and it will be assumed, for the moment, that ψ' (excitation independent of m and $\dot{m}$) is equal to zero, so that one obtains the following equation

$$m\left[1 - \xi - \frac{k}{k_s}(1 - \mathrm{i}Q^{-1})\right] + \mathrm{i}\dot{m}\left[\frac{A_m}{\Omega(C - A)} + \frac{k}{k_s\Omega}(1 - \mathrm{i}Q^{-1})\right] = 0\,, \tag{13}$$

where A_m is the equatorial inertia momentum of the mantle, i.e. $A_m = A - A_c$.

The solution of equation (13) will be sought in the following form

$$m = m_0\,\mathrm{e}^{(\alpha + \mathrm{i}\sigma)t}\,, \tag{14}$$

expressing the damped circular motion in the direct sense since

$$\dot{m} = m_0(\alpha + \mathrm{i}\sigma)\,\mathrm{e}^{(\alpha + \mathrm{i}\sigma)t}\,. \tag{15}$$

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By substituting expressions (14) and (15) in Eq. (13) one obtains

$$1 - \xi - \frac{k}{k_s}(1 - iQ^{-1}) + (i\alpha - \sigma)\left[\frac{A_m}{\Omega(C - A)} + \frac{k}{k_s\Omega}(1 - iQ^{-1})\right] = 0 .$$

In this equation both parts should be equal to zero: the real part

$$1 - \xi - \frac{k}{k_s} - \frac{\sigma}{\Omega}\left(\frac{A_m}{C - A} + \frac{k}{k_s}\right) + \frac{\alpha}{\Omega}\frac{k}{k_s}Q^{-1} = 0$$

and the imaginary part

$$\frac{k}{k_s}Q^{-1} + \frac{\alpha}{\Omega}\left(\frac{A_m}{C - A} + \frac{k}{k_s}\right) + \frac{\sigma}{\Omega}\frac{k}{k_s}Q^{-1} = 0 .$$

These two equations yield the solutions for σ and α:

$$\sigma \doteq \Omega\left(1 - \xi - \frac{k}{k_s}\right)\Big/\left(\frac{A_m}{C - A} + \frac{k}{k_s}\right) \tag{16}$$

$$\alpha \doteq -\frac{k}{k_s}\Omega Q^{-1}\Big/\left(\frac{Am}{C - A} + \frac{k}{k_s}\right),$$

since $(k/k_s)\,Q^{-1}$ is very small compared to $A_m/(C - A)$.

By using the following numerical values (Vondrák 1984):

$$\frac{k}{k_s} = 0{\cdot}3173$$

$$\frac{A_m}{(C - A)} = 269{\cdot}66$$

$$\Omega = 6{\cdot}3004 \text{ rad day}^{-1}$$

one obtains

$$\sigma = 0{\cdot}01593 - 0{\cdot}0233\xi \text{ rad day}^{-1} , \tag{17}$$

$$\alpha = -0{\cdot}0074Q^{-1} \text{ day}^{-1}$$

where the factor ξ describes the dependence of σ on the amplitude A. If the expression yielding ξ from equation (9) is substituted in the first Eq. (17), the following value is then obtained for Chandler's frequency:

$$\sigma = 0{\cdot}01444 + 0{\cdot}00149\,e^{-20A} , \tag{18}$$

This is a fine approximation for the plot presented in Fig. 3 (Vondrák 1988).

If instead of the zero on the right-hand side of equation (13) the excitation ψ' (independent of m) is inserted, one finds that

$$m + i\frac{\dot{m}}{(\sigma - i\alpha)} = \frac{\psi'}{1 - \xi - (k/k_s)(1 - iQ^{-1})} , \tag{19}$$

where σ is not constant. This differential equation has a solution:

$$m = \exp\left[\int_0^t (\alpha + i\sigma)\,d\tau\right]\left[m_0 - \frac{(\alpha + i\sigma)}{1 - \xi - (k/k_s)(1 - iQ^{-1})}\int_0^t \psi' \exp\left\{-\int_0^t (\alpha + i\sigma)\,d\tau\right\} d\tau\right]$$

or

$$m = \exp\left[\int_0^t (\alpha + i\sigma)\,d\tau\right]\left[m_0 - \frac{i\Omega}{A_m/(C - A) + k/k_s}\int_0^t \psi' \exp\left\{-\int_0^t (\alpha + i\sigma)\,d\tau\right\} d\tau\right]. \tag{20}$$

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5. Atmospheric Excitation

The equatorial effective angular momentum functions (EAMF) in a complex notation are (Barnes et al. 1983)

$$\chi = \chi_1 + \mathrm{i}\chi_2 = \frac{1{\cdot}00R^4}{g(C_m - A_m)} \int_s p_s \, \mathrm{e}^{\mathrm{i}\lambda} \sin\phi \cos^2\phi \mathrm{d}\lambda \, \mathrm{d}\phi -$$

$$- \frac{1{\cdot}43R^3}{g\Omega(C_m - A_m)} \int_v (u \sin\phi + \mathrm{i}v) \, \mathrm{e}^{\mathrm{i}\lambda} \cos\phi \, \mathrm{d}\lambda \, \mathrm{d}\phi \, \mathrm{d}p \,. \tag{21}$$

The integral with subscript s is a surface integral integrated over the Earth's surface, the one with subscript v is a volume integral integrated over the atmosphere; ϕ, λ and p_s are geographic latitude, longitude and the air pressure on the Earth's surface, respectively; u and v are the wind components (u the eastward one and v the northward). The first term of equation (21) is the pressure term and the second is the wind term; they express the influence of the redistribution of the air mass and its relative angular momentum, respectively. At some meteorological centres the pressure term is calculated with the inverted barometer correction.

Because of differences between the present Earth model and Barnes's (1983) it is necessary to express the excitation functions (Eq. 12) by means of Barnes's equatorial angular momentum functions of the atmosphere (χ_1, χ_2).

Expression (21) can be written in the form (Vondrák 1987):

$$\chi = \chi_1 + \chi_2 = (\chi_1^p + \mathrm{i}\chi_2^p) + (\chi_1^w + \mathrm{i}\chi_2^w) = E_p I^p + E_w I^w \,, \tag{22}$$

where I^p and I^w are a surface and a volume integral, respectively, and E_p and E_ω are numerical constants.

The excitation functions ψ' (Eq. 12) will be calculated using Vondrák's (1987) numerical values.

Because of the different values of the numerical constants E_p and E_w used by Barnes (1983) and by Vondrák (1987), as well as because of multiplying only the pressure term of excitation functions (2) by $1 + k_2 = 0{\cdot}70$ (due to the surface loading deformation)

$$\psi' = 0{\cdot}628 \left(\chi - \mathrm{i} \frac{\dot{\chi}}{\Omega} \right) \tag{23}$$

6. Model of the Earth with Variable Chandler's Frequency

By introducing a new excitation function (11) for the nonequilibrium ocean dependent on the polar motion amplitude, the equations of the elastic Earth model with variable Chandler's frequency are derived.

The coordinates of the excitation pole (x, y) as functions of the atmospheric equatorial EAMF are obtained by substituting the first equation (23) into (20)

$$m = \exp\left[\int_0^t (\alpha + \mathrm{i}\sigma)\,\mathrm{d}\tau\right] \left[m_0 - \frac{\mathrm{i}\Omega}{A_m/(C - A) + k/k_s} \, 0{\cdot}628 \int_0^t (\chi - \mathrm{i}\dot{\chi}/\Omega) \exp\left\{ -\int_0^t (\alpha + \mathrm{i}\sigma)\,\mathrm{d}\tau \right\} \mathrm{d}\tau \right]. \tag{24}$$

Since σ is variable (it is given by Eq. 18), one obtains for the integral in the exponent of (24)

$$\int_0^t [\alpha + \mathrm{i}\,\sigma(\tau)]\,\mathrm{d}\tau = \int_0^t [\alpha + \mathrm{i}(\sigma_0 + 0{\cdot}00149\,\mathrm{e}^{-20A(\tau)})]\,\mathrm{d}\tau \,,$$

i.e.

$$\int_0^t [\alpha + \mathrm{i}\sigma(\tau)]\,\mathrm{d}\tau = \alpha t + \mathrm{i}(\sigma_0 t + J) \,,$$

where

$$J = 0{\cdot}00149 \int_0^t \mathrm{e}^{-20A(\tau)}\,\mathrm{d}\tau \tag{25}$$

and

$$\sigma = \sigma_0 + 0{\cdot}00149\,\mathrm{e}^{-20A(t)} \,.$$

If for reasons of simplicity one writes $l = 0{\cdot}628$ and $L = A_m/(C - A) + k/k_s$, equation (24) becomes

$$m = \mathrm{e}^{\alpha t + \mathrm{i}(\sigma_0 t + J)} \left\{ m_0 - (\mathrm{i}\Omega l/L) \left[\int_0^t \chi \, \mathrm{e}^{-[\alpha\tau + \mathrm{i}(\sigma_0\tau + J)]}\,\mathrm{d}\tau - (\mathrm{i}/\Omega) \int_0^t \dot{\chi} \, \mathrm{e}^{-[\alpha\tau + \mathrm{i}(\sigma_0\tau + J)]}\,\mathrm{d}\tau \right] \right\}. \tag{26}$$

Integrating by parts the second integral in equation (26) one finds that

$$\int_0^t \dot{\chi} \, \mathrm{e}^{-[\alpha\tau + \mathrm{i}(\sigma_0\tau + J)]}\,\mathrm{d}\tau = (\alpha + \mathrm{i}\sigma) \int_0^t \chi \, \mathrm{e}^{-[\alpha\tau + \mathrm{i}(\sigma_0\tau + J)]}\,\mathrm{d}\tau + \chi(t)\,\mathrm{e}^{-[\alpha\tau + \mathrm{i}(\sigma_0\tau + J)]} - \chi(0) \,.$$

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By substituting this expression into equation (26) one obtains

$$m = e^{-[\alpha\tau + i(\sigma_0\tau + J)]}\{m_0 - (l/L)[\alpha + i(\Omega + \sigma)]\int_0^t \chi\, e^{-[\alpha\tau + i(\sigma_0\tau + J)]}\, d\tau\} - (l/L)\{\chi(t) - \chi(0)\, e^{[\alpha\tau + i(\sigma_0\tau + J)]}\}$$

An extended version of this expression is

$$m_1 + im_2 = e^{\alpha t}[\cos(\sigma_0 t + J) + i\sin(\sigma_0 t + J)]$$
$$\{[m_1(0) + im_2(0)] - (l/L)([\alpha + i(\Omega + \sigma)]\int_0^t (\chi_1 + i\chi_2)\, e^{-\alpha\tau}[\cos(\sigma_0\tau + J) - i\sin(\sigma_0\tau + J)]\, d\tau\}$$
$$- (l/L)\{\chi_1(t) + i\chi_2(t) - [\chi_1(0) + i\chi_2(0)]\, e^{\alpha\tau}[\cos(\sigma_0 t + J) + i\sin(\sigma_0 t + J)]\}$$

This yields the coordinates of the excited pole in the normally oriented coordinate system in which the x axis is directed along the zero meridian and the y axis is positive eastwards. We neglected the factor α since it is $4 \sim 5$ orders smaller than Ω.

$$x = m_1(t) = e^{\alpha\tau}\{x_0 \cos(\sigma_0 t + J) - y_0 \sin(\sigma_0 t + J) +$$
$$+ (l/L)\,\Omega(1 + \sigma_t/\Omega)[(J_1 + J_4)\sin(\sigma_0 t + J) + (J_2 - J_3)\cos(\sigma_0 t + J)]\}$$
$$y = m_2(t) = e\{^{\alpha\tau}x_0 \sin(\sigma_0 t + J) + y_0 \cos(\sigma_0 t + J) -$$
$$- (l/L)\,\Omega(1 + \sigma_t/\Omega)[(J_1 + J_4)\cos(\sigma_0 t + J) - (J_2 - J_3)\sin(\sigma_0 t + J)]\}\,. \qquad (27)$$

The quantities designated J_1, J_2, J_3 and and J_4 are:

$$J_1 = \int_0^t \chi_1\, e^{-\alpha\tau}\cos(\sigma_0\tau + J)\, d\tau + \Omega^{-1}[\chi_2(t)\, e^{-\alpha\tau}\cos(\sigma_0 t + J) - \chi_2(0)]$$
$$J_2 = \int_0^t \chi_2\, e^{-\alpha\tau}\cos(\sigma_0\tau + J)\, d\tau - \Omega^{-1}[\chi_1(t)\, e^{-\alpha\tau}\cos(\sigma_0\tau + J) - \chi_1(0)]$$
$$J_3 = \int_0^t \chi_1\, e^{-\alpha\tau}\sin(\sigma_0\tau + J)\, d\tau + \Omega^{-1}\chi_2(t)\, e^{-\alpha\tau}\sin(\sigma_0 t + J)$$
$$J_4 = \int_0^t \chi_2\, e^{-\alpha\tau}\sin(\sigma_0\tau + J)\, d\tau - \Omega^{-1}\chi_1(t)\, e^{-\alpha\tau}\sin(\sigma_0 t + J) \qquad (28)$$

Equation (27) yields the coordinates of the excited pole x, y resulting from the model of the Earth with a nonequilibrium ocean. Substituting the numerical value of l and the expression for L into (27) one has

$$x = e^{\alpha\tau}\{x_0 \cos\sigma_0 t + J) - y_0 \sin(\sigma_0 t + J) +$$
$$+ \frac{0{\cdot}628}{A_m/(C - A) + k/k_s}\,\Omega(1 + \sigma/\Omega)[(J_1 + J_4)\sin(\sigma_0 t + J) + (J_2 - J_3)\cos(\sigma_0 t + J)]\}$$
$$y = e^{\alpha\tau}\{x_0 \sin(\sigma_0 t + J) + y_0 \cos(\sigma_\upsilon t + J) -$$
$$- \frac{0{\cdot}628}{A_m/(C - A) + k/k_s}\,\Omega(1 + \sigma/\Omega)[(J_1 + J_4)\cos(\sigma_0 t + J) - (J_2 - J_3)\sin(\sigma_0 t + J)]\} \qquad (29)$$

In this way the response of the Earth's model with a variable and amplitude-dependent Chandler's frequency to the atmospheric excitation is derived. This model was confirmed on the basis of a comparison with observations (Vondrák and Pejović 1989) even on a short-period scale (1976–1986).

7. Conclusions

The hypothesis of nonlinear dependence of Chandler's frequency on the polar motion total amplitude is confirmed by analysig the ILS pole coordinates. The response of the Earth to atmospheric excitation is derived by using an axially symmetric model of the Earth with a variable and amplitude-dependent Chandler's frequency. This model of the Earth is confirmed by comparing it to observations, even on a short-period scale. In this way the hypothesis of a variable Chandler's frequency due to the influence of the nonequilibrium ocean on polar motion is confirmed.

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13 The Motion of an Artificial Satellite in the Oblate Atmosphere

S. Šegan and N. Pejović
Bull. Astron. Belgrade.
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Under conditions of the TOTAL DENSITY MODEL (Sehnal, 1988) the changes of the satellite orbital elements are concluded (Šegan, 1989) and first and second order theories are constructed. It is assumed that the constants defining thermospheric density model are small (parameters).

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Preliminary report

THE MOTION OF AN ARTIFICIAL SATELLITE IN THE OBLATE ATMOSPHERE

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(Received: December 16, 1991)

SUMMARY: Under conditions of the TOTAL DENSITY MODEL (Sehnal, 1988) the changes of the satellite orbital elements are concluded (Šegan, 1989) and first and second order theories are constructed. We assume that the constants defining thermospheric density model are small (parameters).

1. INTRODUCTION

It is evidently that not exist the general theory which gives the description of the orbital changes for a time interval longer than one revolution. The absence of that is caused by the complexity of the atmospheric density distribution (modeling).

The result of all "aeronomic" models are usable only for the determination of the orbital perturbations merely over one revolution of a satellite around the Earth. The determination of the orbital changes under influence of the atmospheric drag just over one revolution is a common procedure adopted in the present theories.

Using the Sehnal's (1986, 1988a, 1988b) method of approximation of the total thermospheric density distribution and results from our previous paper (Šegan, 1987), we shall develop the theory of second order.

2. THE AIMS

By usage computer algebra we were derived general linear expression for the first order perturbations of the satellite orbital elements under influence of a drag:

$$\frac{d\Theta}{dE} = -f_1(a)\delta f_2(e,E)\rho\binom{sin}{cos}^k i\binom{sin}{cos}^l u, \qquad (1)$$

where

Θ (a, e, i, Ω, ω)

a – semi-major axis

e – eccentricity

E – eccentric anomaly

δ – drag force per unit mass (satellite dependant)

and from Sehnal's total density model

$$\rho = \sum_{k=1}^{K} \rho_s^k e^{\frac{r_{p_0}-r}{kH}} = \sum_{k=1}^{K} \sum_{i=1}^{I} A_i^{(k)} f_i e^{\frac{r_{p_0}-r}{kH}}, K = 3$$

or

$$\rho = k_0 f_0 f_x \sum_{n=1}^{7} h_n g_n, \tag{2}$$

where $A_i^{(k)}$ expresses the geometry while f_i are functions of the physical parameters. By this way we overruned the standard difficulties.

An analytical integration (1) under interval of the one orbital revolution yeald:

$$\Delta\Theta = Nf(a,e), \tag{3}$$

where perturbations and influences of the other orbital elements are neglected.

THE SECOND ORDER PERTURBATIONS

For the prediction of the long time perturbations ("second order") we need

$$\Delta^1\Theta = Nf(a,e), \tag{3a}$$

and expressions for the 2nd order terms

$$\Delta^2\Theta = \frac{1}{2}N^2(f(a,e)\frac{d}{da}(f(a,e))+$$

$$f(a,e)\frac{d}{de}(f(a,e))), \tag{4}$$

and

$$\Delta\Theta = \Delta^1\Theta + \Delta^2\Theta. \tag{5}$$

Since we know the analytical expressions for Δa_0 and Δe_0, we can compute easily their derivatives acc. to a and e (index $_0$ is for perigee distance). Also, we have to take into account the derivatives of the Bessel functions, too.

This is the way to construct the theories of higher orders. As a result the evolution of the elements Θ will be described generally as a polynomial of the desired order.

The results can be checked on the observed satellite orbital changes. We have the observed values of the semi-major axis and eccentricity of the ANS satellite. We see that the first order theory describe the changes as a linear function of time and does not provide reliable information on the evolution of the orbit whereas the second order theory corresponds to the observed values much better.

In much cases we must take into account the influences of the orbital elements changes in their complexity. It will be object for the our next paper.

Acknowledgments – This work has been supported by the Republic Fund for science in Serbia through the project "Physic and Motions of Celestial Bodies".

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КРЕТАЊЕ ВЕШТАЧКИХ САТЕЛИТА У ВИСОКОЈ АТМОСФЕРИ

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УДК 523.4-87:521.322
Предходно саопштење

Под условима *модела тоталне густине* (Sehnal, 1988) изведене су промене путањских елемената сателита (Шеган, 1989) и конструисане су теорије првог и другог реда. Претпостављамо да су параметри који дефинишу модел густине термосфере мали.

14 The Frequency Modulation of the Chandler Wobble

N. Pejović and S. Šegan
Bull. Astron. Belgrade
Vol 145, pp. 53-57, 1992.
UDC 521.93

The spectral analysis of the ILS polar motion data in the interval 1900-1980 are accomplished. The authors discuss obtained results and compare them with the results of the other authors. They conclude that two resonant frequencies in the Chandler wobble do not exist, but only one modulated.

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UDC 521.93
Original scientific paper

FREQUENCY MODULATION OF THE CHANDLER WOBBLE

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(Received: December 5, 1991)

SUMMARY: The spectral analysis of the ILS polar motion data in the interval 1900-1980 are accomplished. The obtained results are discussed and compared with the results of the other authors and with ours, as well. We may conclude that two resonant frequencies in the Chandler wobble do not exist, but only one modulated.

1. INTRODUCTION

The existance of the 14-month period in the polar motion is still an open question. The variability of the Chandler period has been controversial for almost an entire century. The great number of autors, including Chandler himself, have assumed that the Chandler period is either multi-component, or variable in time (Chandler 1892, Kimura 1918, Hattori 1949, Melchior 1957, Colombo and Shapiro 1968, Gaposchkin 1972, Sekiguchi 1972, 1976, Carter 1981, Dickman 1981, Pejovic 1983, 1988, Vondrak 1985, 1988). There are other autors claiming that the Chandler wobble has a single period (Newcomb 1892, Pedersen and Rochester 1972, Ooe 1978, Okubo 1982). In this paper we came to the conclusion that only one phase modulated Chandler frequency exists.

2. COMPARISON OF THE POLAR MOTION SPECTRA

Aalgorithm for the Fourier transform (FT) was used to calculate the spectra of ILS polar motion data (Yumi and Yokoyama 1980). The whole interval (1900 - 1980) was devided into four subintervals, namely: 1900-1920, 1920-1940, 1940- 1960 and 1960-1980. This subdivision was done in such a way to analyse especially the dubious 1920-1940 subinterval. We did not apply windowing to suppress sidelobes. An attempt with the Parsen window to polar motion data showed a severe decrease of amplitudes of both the Chandler and annual term with respect to nonwindowed data. Therefore, it is clear that windowing gives the largest weight to the data in the center of the interval. The amplitude of the polar motion was extremely small especially in the some our subintervals.

The polar motion spectra are shown in Fig. 1, 2 and 3. The spectra of the four analised subintervals for the x component are presented in Fig. 1, and for the y component are presented in Fig. 2. In all of the shown spectra there are two peaks dominating at the Chandler and annual frequencies. It is clear that the amplitude of the Chandler term is distinctly smaller in subinterval 1920-1940 then in other subintervals (Fig. 1b, 2b). In the Table 1 are given parameters of the Chandler wobble with additional subinterval 1940-1980. We can see from this table relatively good agreement of the periods in all of the

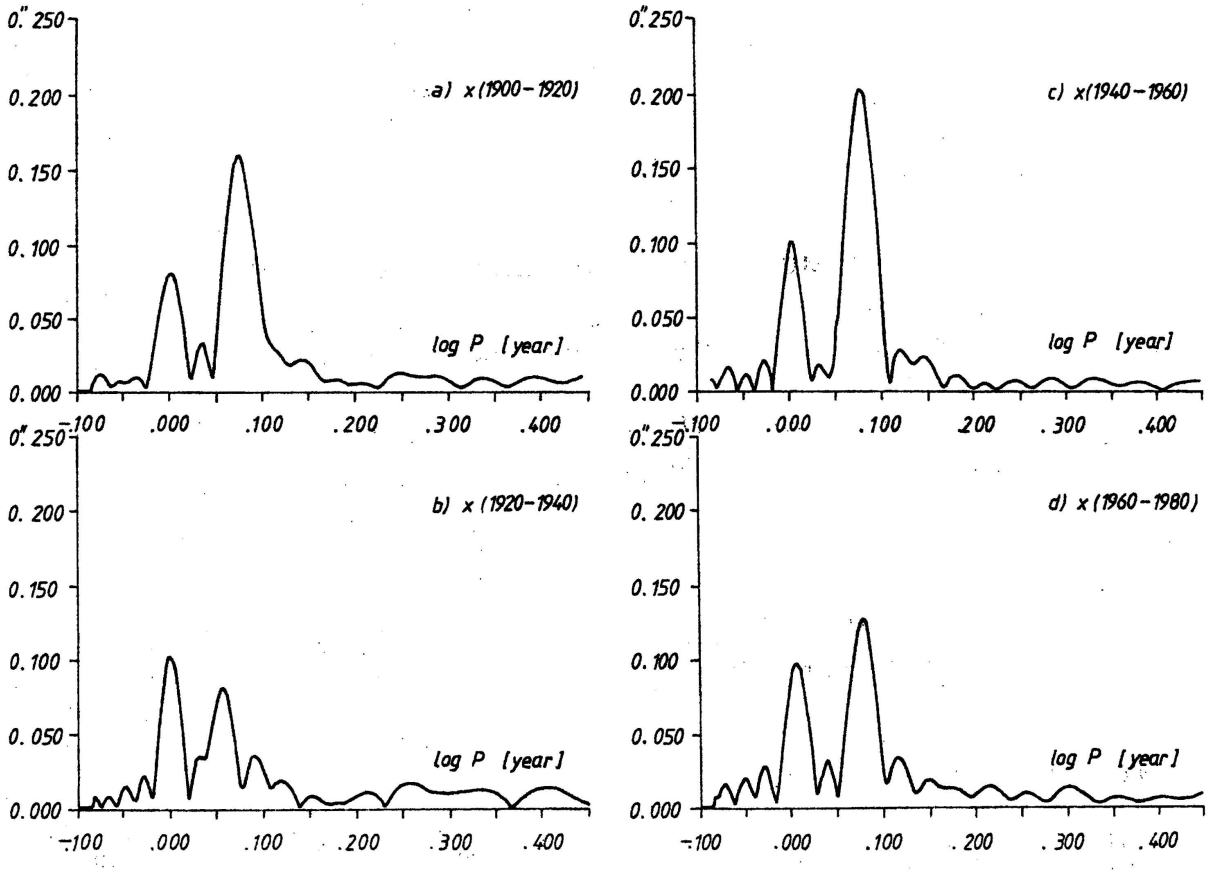

Fig. 1. Spectra of the x component of polar motion for the four subintervals.

Table 1 Parameters of the Chandler wobble - P, A, Φ denote the period, amplitude and phase.

	x component			y component		
Interval	P(day)	A(")	Φ(°)	P(day)	A(")	Φ(°)
1900-1920	434.375	0.162	22.5	434.375	0.165	294.4
1920-1940	415.625	0.081		416.562	0.076	
1920-1940	434.375	0.015	233.6	434.375	0.016	145.2
1940-1960	434.375	0.207	137.1	434.375	0.208	46.5
1960-1980	434.375	0.129	135.9	434.375	0.136	46.1
1940-1980	434.375	0.169	136.4	434.375	0.173	46.3

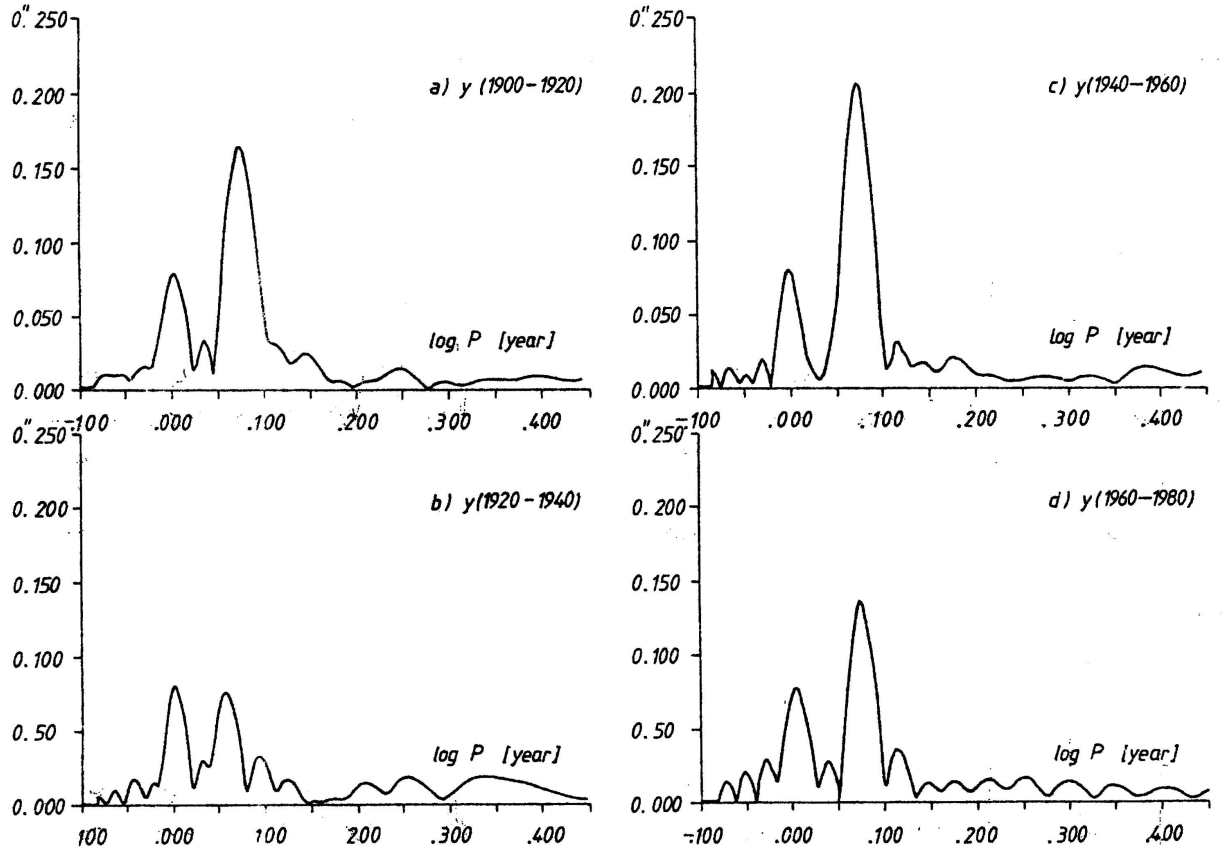

Fig. 2. Spectra of the y component of polar motion for the four subintervals.

Table 2. Parameters of annual polar motion - P, A, Φ denote the period, amplitude and phase.

		x component		y component	
Interval	P(day)	A(")	Φ(°)	A(")	Φ(°)
1900-1920	365.250	0.079	238.8	0.077	162.4
1920-1940	365.250	0.102	238.9	0.081	150.4
1940-1960	365.250	0.103	239.1	0.081	152.4
1960-1980	365.250	0.098	245.0	0.079	162.9
1900-1980	365.250	0.095	240.4	0.078	157.0

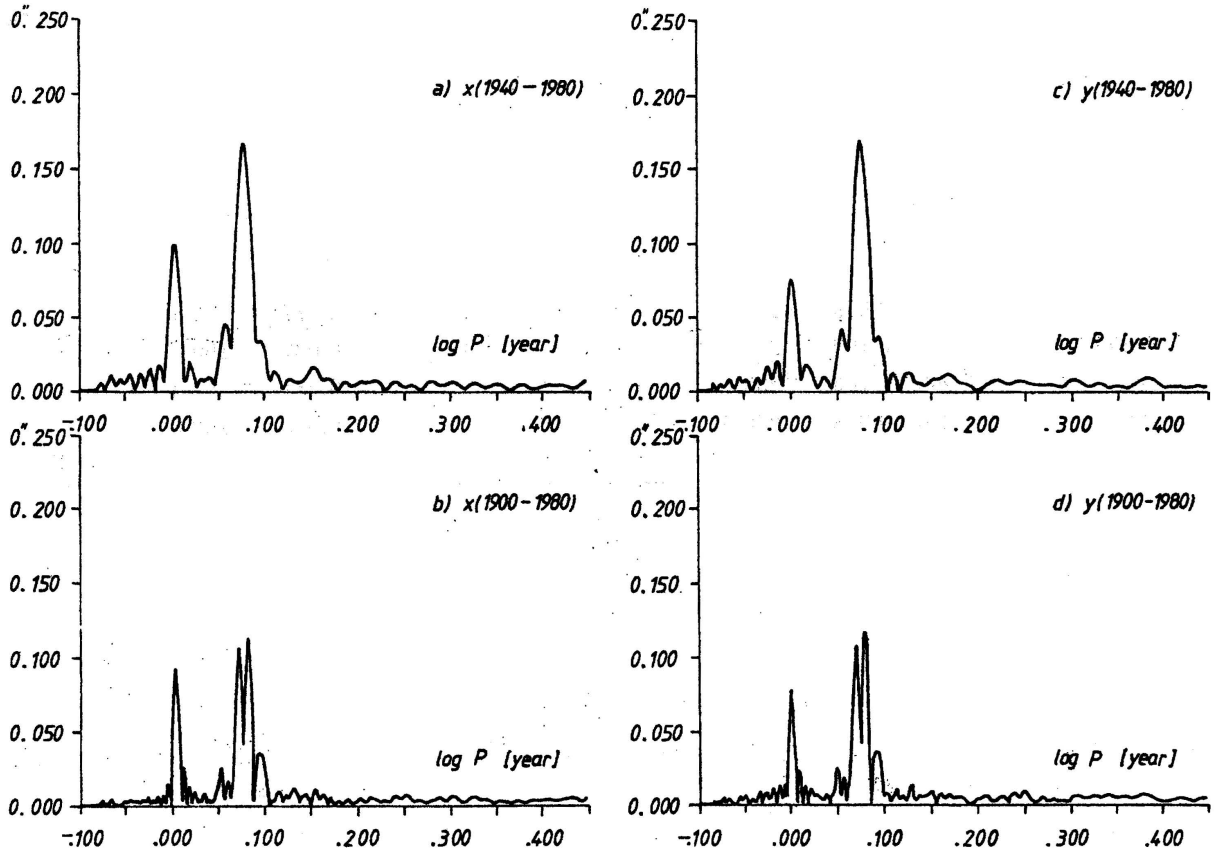

Fig. 3. Spectra of polar motion for subinterval (a, c) and for the whole interval (b, d).

subintervals except in the dubious (1920 – 1940) one, in which the period of Chandler term is considerably smaller, as well as its amplitude. The Table 1 give us a possibility to notice the rapid change of the phase of Chandler term in the dubious subinterval.

The Table 2 ilustrate good agreement of the parameters of annual wobble for x and y coordinates in the all five subintervals as in the Table 1.

The spectra for subinterval 1940-1980 are presented in Fig. 3a and 3c for x and y components, respectively. For the whole interval 1900-1980 the spectra of the polar motion are shovn in Fig. 3b, 3d. This are our previously obtained results (Pejović 1983). Comparing the spectra of the whole interval with the spectra of subintervals it is clear that the Chandler peak is a single in all of the subintervals but it is split into two parts only in the whole interval. We can conclud that the splitting is a result of taking into account the dubious subinterval which has a rapid change of th e phase. Vondrák (1985, 1988) emplasized an important fact that change in phase causes the change in frequency of Chandler wobble. This interrelation was confirmed in our paper (Pejović 1990). Because of that we came to the conclusion that it is not the question of two but of only one frequently modulated Chandler wobble.

3. DISCUSSION

Pedersen and Rochester (1972) showed that the process by wich the polar motion is randomly altered is not a stationary one, in the statistical sense.

The annual peaks are quite consistent with one another, while the spectra in the Chandler region are very dissimilar. The lack of resemblance is strong evidence that the random inputs to the Chandler wobble constitute a highly non-stationary process. Okubo (1982) in this extended analysis attempted to slove the existing problem. According to his conclusion Chandler wobble is not stationary and the mean value of the fluctuating Chandler period is 435 days.

By analysis of the amplitude and phase of Chandler wobble Pejović (1985) confirmed results of Guinot (1972, 1982) and Dickman (1981) that the amplitude of Chandler wobble is the harmonic function of time but on the contrary its phase is stable except during 1920-1940. This rapid change of the Chandler phase was considered by many authors as random until Vondrak (1985) analysing a long time interval (1860-1985) was confirmed this change and descovered a new one 1870-1890. It seems that what for Pedersen and Rochester (1972) and Okubo (1982) was a randomly fluctuating Chandler wobble, for Carter (1981), Vondrak (1985, 1988) and Pejović (1988, 1990) was frequently modulated.

4. CONCLUSION

According to the results received by the spectral analysis of the polar motion it can be concluded that one single Chandler peak is obtained if subintervals are taken into account. But, the Chandler peak is splitted if the whole interval was analysed. It becomes clear that this splitting is a consequence of the inclusion of jast that dubious subinterval. As this subinterval has a rapid change of phase which causes change of the frequency, we conclude that it is not the question of two, but only one, frequently modulated Chandler wobble.

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ФРЕКВЕНТНА МОДУЛИСАНОСТ ЧЕНДЛЕРОВОГ КЛИМАЊА

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УДК 521.93
Оригинални научни рад

Извршена је спектрална анализа ILS података поларног кретања на интервалу 1900 - 1980. Добијени резултати су дискутовани и упоређени, како са резултатима других аутора, тако и са нашим ранијим резултатима. Може се закључити да не постоје две резонантне фреквенције у Чендлеревој нутацији, већ само једна модулисана.

15 Scale Heights for the Aeronomical Models and Perturbations of Satellite Orbits

S. Šegan and N. Pejović
Bull. Astron. Belgrade
Vol 148, pp. 17-20, 1993.
UDC 523.4-87, 521.322

Authors give a general expressions for the scale heights in aeronomic atmospheric density models.

Bull. Astron. Belgrade № 148 (1993), 17 – 20

UDC 523.4-87; 521.322
Preliminary report

SCALE HEIGHTS FOR THE AERONOMICAL MODELS AND PERTURBATIONS OF SATELLITE ORBITS

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(Received: December 12, 1992)

SUMMARY: The general expressions for the scale heights in aeronomic atmospheric density models are given.

1. INTRODUCTION

All "aeronomic" models are usable for the determination of the orbital perturbations merely over one revolution of a satellite around the Earth. The determination of the orbital changes under the influence of the atmospheric drag just over one revolution is a common procedure adopted in the present theories.

The computation of the density scale heights has been applied not only for determining the density at perigee, but also for the determining any orbital element $\{.\} \in (a, e, i, \Omega, \omega)$ and we have (Lagrangian equations in Gaussian form, (King-Hele, 1964)):

$$\frac{d\Theta}{dE} = -f_1(a)\delta f_2(e,E)\rho\binom{sin}{cos}^k i\binom{sin}{cos}^l u, \qquad (1)$$

where
Θ – (a, e, i, Ω, ω)
a – semi-major axis,
e – eccentricity,
... – .....
E – eccentric anomaly,
δ – drag force per unit mass (satellite dependent),
ρ – atmospheric density,
z – altitude,
H – density scale height.

Parameter ρ is unknown. It will be also used for the determination of the effective height, which is actually the perigee height plus $H/2$.

Since the latest trend in the modeling of the upper atmosphere properties (Hedin, 1987) is characterized by the use of spherical harmonic representation of a specific atmospheric parameter

$$\rho = \rho(z, F, F_b, \varphi, ...),$$

and

$$H = -\frac{\rho}{d\rho/dz}$$

where
ρ – atmospheric density
F – solar flux
F_b – mean solar flux
φ – latitude
...

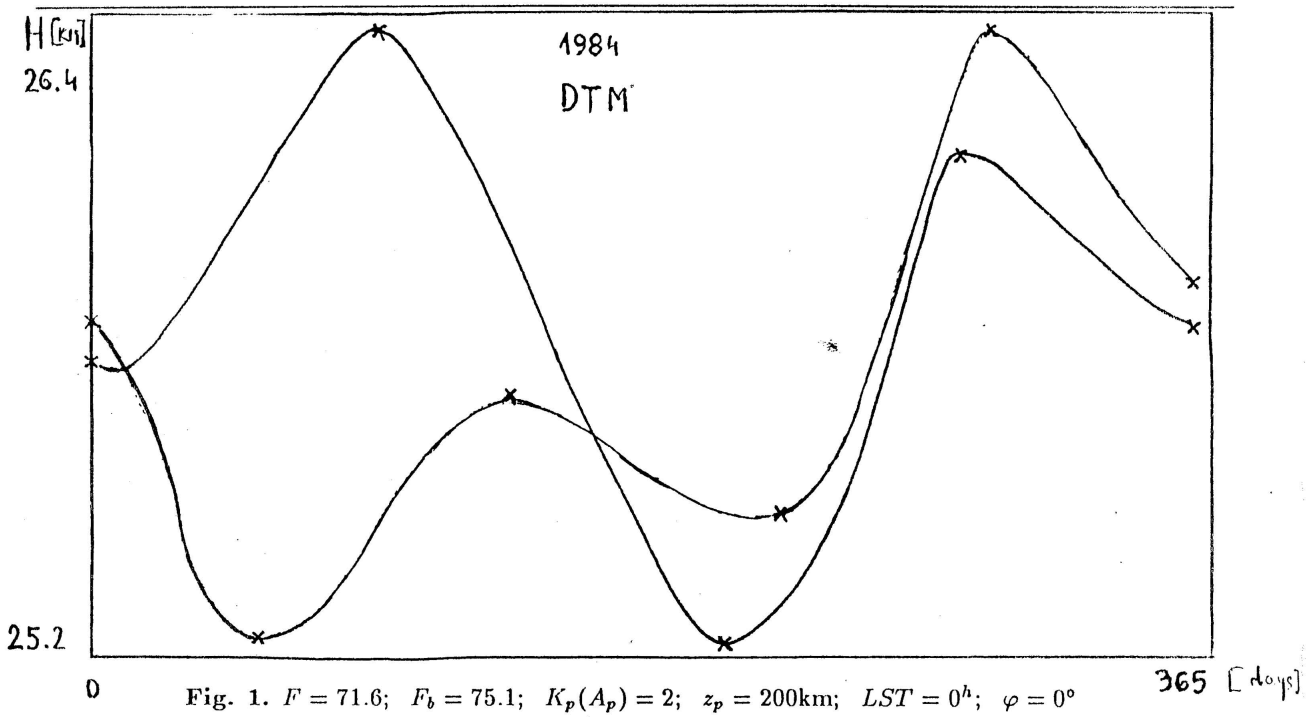

Fig. 1. $F = 71.6$; $F_b = 75.1$; $K_p(A_p) = 2$; $z_p = 200$km; $LST = 0^h$; $\varphi = 0°$

Some of such models were taken into account here. Since the models used not give H explicitly, we shall compute it.

2. COMPUTATION

We have so far assumed that the density scale height H given by

$$H = -\frac{\rho}{d\rho/dz}$$

can be computed from the models itself.

In the aeronomic models the density ρ is given as

$$\rho = \sum_i m_i n_i.$$

For the CIRA72 (Oliver, 1980) it is

$$\rho = \sum_i m_i n_i(z_0) f_i(z'), \qquad (2)$$

where

$$z' = (z - z_0)(R + z_0)/(R + z),$$
$$R = 6356.766 \text{ km},$$
$$z_0 = 125 \text{ km},$$

$$g = 9.80665/(1 + z/R)^2,$$

and

$$n_i(z) =$$

$$= n_i(z_0)((1 - a)/(1 - a^{-sz'}))^{1+\alpha+1/H_i} exp(z'/H_i),$$

where

$$a = (T_\infty - T_{z_0})/T_\infty,$$
$$s = a_1 + a_2/(T_\infty + a_3),$$
$$H_i = kT_\infty/(m_i g_{z_0}).$$

We sum over i constituents with molecular masses m_i and numerical densities n_i.

The only height dependent function is $f_i(z)$ which results from the integration of the diffussive equilbrium distribution with the temperature profile.

Let

$$C_1 = 1 + \alpha + 1/(sH_i),$$

then

$$f_i(z') = ((1 - a)/(1 - a^{-sz'}))^{C_1} e^{-z'/H_i}.$$

By differentiation of the last expression we derive the equation

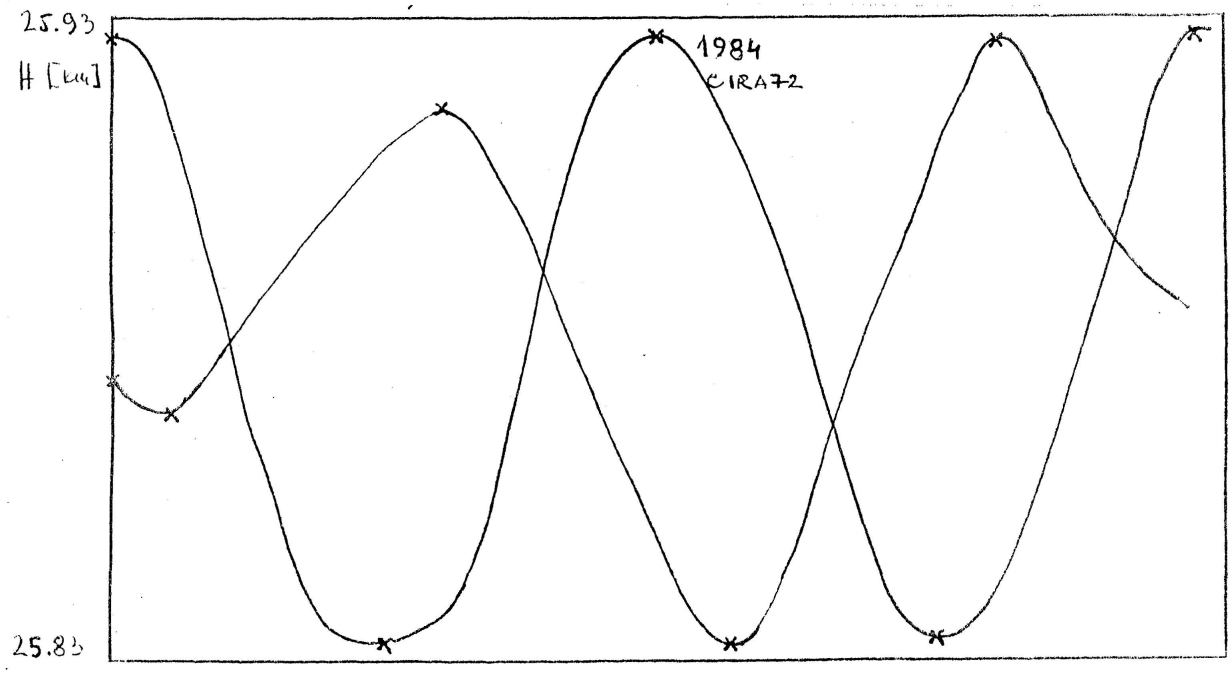

Fig. 2. For the same parameters as on Fig. 1.

$$\frac{d\rho}{dz} =$$

$$= \frac{\sum_i m_i n_i(z_0) f_i(z') \frac{1}{H_i}[-1-(1+\alpha_i)sH_i a e^{-sz'}]}{(1-ae^{-sz'})(R+z)^2(R+z_0)^{-2}}. \quad (3)$$

For different models (MSIS, 1978; DTM, 1977; C, 1977; C1986, 1986; ...) and their constituents i

$$M_i = F(MSIS_i, DTM_i, C_i, ...)$$

$$M_i \simeq CIRA_i exp[G_i(L) - 1], \quad (4)$$

where $G_i(L)$ is a function defining global variation of the densities (Šegan, 1987).

The results can be checked on the observed satellite orbital data concerning atmospheric density. We have the observed values for some satellites. We see (Fig.1 & Fig.2) that they have a very interesting look. The changes are nonlinear functions of the height and do not provide reliable information on the evolution of the density whereas the new models fit to the observed values much better. In many cases we must take into account the influences of the orbital elements changes in their complexity. It's clear that aeronomic models are very complicated for the effective usage and in the next paper we shall try to develop much better theory.

Acknowledgments – This work has been supported by Ministry for Science and Technology of Serbia through the project "Physics and Motions of Celestial Bodies".

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СКАЛЕ ВИСИНЕ ЗА АЕРОНОМСКЕ МОДЕЛЕ И ПОРЕМЕЋАЈЕ САТЕЛИТСКИХ ОРБИТА

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УДК 523.4-87; 521.322
Претходно саопштење

У раду је дат општи израз за скалу висина у аеропомским моделима густина атмосфера.

16 The Hipparcos mission and the Belgrade zenith-telescope observations

G. Damljanović and N. Pejović
Publ. Dept. Astr. Univ. Beograd
Vol 49, pp. 127-130, 1995.

At the beginning of 1990 the authors started started collecting their past observations done with Belgrade ZT and made a new reduction in the FK5 reference frame. After the XXI IAU General Assembly, Buenos Aires 1991, their investigation was in accordance with the task of Commission 19 of the International Astronomical Union, "Rotation of the Earth". which formed the Working Group on Earth Rotation in the HIPPARCOS reference frame - WG ERHRF. The goal of the project was to collect the data and create a central data bank of the optical astrometric observations done in the past.

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THE HIPPARCOS MISSION AND THE BELGRADE ZENITH-TELESCOPE OBSERVATIONS

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Abstract. At the beginning of 1990 we started collecting our past observations made with Belgrade ZT and making a new reduction in the FK5 reference frame. After the XXI IAU General Assembly, Buenos Aires 1991, our investigation was made in accordance with the task of Commission 19 of the International Astronomical Union, "Rotation of the Earth", which formed the Working Group on Earth Rotation in the HIPPARCOS reference frame - WG ERHRF to collect the data and create a central data bank of past optical astrometric observations. We cooperate and some results are presented here.

1. INTRODUCTION

The first value of the precise Belgrade latitude, determined in 1947 from visual zenith-telescope - ZT (Askania-Bamberg No 77241, 110/1287 mm) observations by applying Talcott method, was $\varphi = 44°48'13.''167 \pm 0.''008$ (Djurković, Ševarlić, Brkić, 1951). At the beginning of 1949 started regular ZT observations and are still carried out.

From 1990 we started the preparation of the Belgrade ZT observations in a computer readable form and their re-reduction in the FK5 reference frame (in accordance with Msc Thesis "The analysis of the variation of Belgrade latitude in the period 1949-1985" of Goran Damljanović). The HIPPARCOS catalogue had not been finished yet, and we used the PPM catalogue for our re-reduction.

At the XXI IAU General Assembly, Commission 19 formed the WG ERHRF to collect the data and analyse them in the HIPPARCOS reference frame. The Hipparcos Program (ESA 1989) will give the star coordinates, proper motions and parallaxes at the $0.''002$ level of accuracy (at the epoch of observation) for the reference stars used by most of the astronomical stations. The Belgrade ZT observations are at the WG ERHRF list of the best observations performed in the past (Vondrák, Feissel and Essaifi, 1992).

2. PROCEDURE AND RESULTS

The PPM Star Catalogue (Röser and Bastian, 1991), Vol. I and Vol. II, contains 181731 stars north of $-2^{\circ}.5$ of declination, equinox and epoch J2000.0. It is a representation of the FK5 system at higher star densities and fainter magnitudes. It replaces two older catalogues : AGK3 and the SAO Catalogues. For the stars contained in FK5 Part I (the Basic Fundamental Stars) and in FK5 Part II (the Bright Extension Stars), PPM gives the original FK5 data.

The Old Belgrade Latitude Programme - OP (Djurković, Ševarlić, Brkić, 1951) was observed in the period 1949-1960. The New Belgrade Latitude Programme - NP (Ševarlić and Teleki, 1960) was started in 1960 and the observations are still carried out.

The procedure for the re-reduction of NP has been described and the basic results published (Damljanović, 1994). The re-reduction of OP is finished (it will be published) the basic results being :

- the angular value of the micrometer screw revolution (R) is $40.''1073$ for the period 1949-1960 (it is in good accordance with its NP's value, $40.''1080$ for the period 1960-1963),
- the angular division values of the Talcott's levels (L) were : $1.''2581$ for the upper level and $1.''1547$ for the lower one in the period 1949-1960 (they are also in good accordance with their NP's values, $1.''2684$ for the upper level and $1.''1798$ for the lower one in the period 1960-1968),
- the temperature coefficients : $0.''00606$ for the upper level and $0.''00400$ for the lower one in the period 1949-1960.

The re-reduction is in accordance with MERIT standards (Melbourne et al., 1983). The new IAU(1976) coordinate system of astronomical constants, the IAU(1980) nutation model, and the new dynamical reference system (JPL DE200/LE200 Ephemeris, 1984) are used. The FORTRAN programme for refraction is like that used in forming the "Refraction Tables of Pulkovo Observatory" (Abalakin, 1985). The General Catalogue of Trigonometric Stellar Parallaxes (Jenkins, 1952) and The General Catalogue of Stellar Radial Velocities (Wilson, 1953) are used for the calculation of the apparent places of OP and NP stars.

The numerous systematic errors are taken into account. Under both Belgrade latitude programmes : the effect of the statistical parallaxes for the stars without trigonometric ones, the deviation of the vertical, the wind effect, the E-W effect-the error due to the clamp position of the telescope, the effect of the level bubble length variation, the correction for the curvature of the parallel, the temperature terms of the levels, the systematic errors in declinations and proper motions of Talcott's pairs and (sub)groups of OP and NP. The personal equation was taken into account for the NP only. The temperature term, the progressive and the periodic errors of the micrometer screw revolution are not applied (Milovanović et al., 1981).

We used the Student-Fisher criterion for eliminating the excessive instantaneous latitudes resulting from some Talcott's pairs.

The polar motion was eliminated from the material and the observations were brought in accordance with the mean pole BIH1979. After that we made determina-

tions of the systematic errors in declinations and proper motions of Talcott's pairs and (sub)groups of OP and NP.

3. CONCLUSIONS

We investigated numerous systematic errors and the whole material (OP and NP) is corrected for them. Our basic results are a contribution to Hipparcos program - the mean error (of the instantaneous latitude from one Talcott's pair) being less than before.

From the preliminary ZT observations made in 1947 the mean error was $\pm 0.''255$ (Djurković, Ševarlić, Brkić, 1951). The mean error of the OP (1949-1960) was $\pm 0.''220$ (Ševarlić and Teleki, 1960). After our re-reduction the mean error is $\pm 0.''199$ (1949-1960); $\pm 0.''211$ (1949-1951.5) , $\pm 0.''195$ (1951.5-1953.5) , $\pm 0.''192$ (1953.5-1957) , $\pm 0.''205$ (1958-1960).

The mean error of the NP (from 1960 till now) was $\pm 0.''272$ (1960-1965.5) and $\pm 0.''146$ (1969-1974) (Grujić et al. 1989); after our re-reduction the mean error is $\pm 0.''148$ (1960-1985); $\pm 0.''164$ (1960-1963), $\pm 0.''171$ (1964-1967), $\pm 0.''151$ (1968-1970), $\pm 0.''137$ (1971-1972), $\pm 0.''115$ (1973-1976) (Damljanović, 1994).

With a better catalogue in the future (the Hipparcos Catalogue) we can expect to find systematic errors and obtain results with a better accuracy. The observations made with the classical instruments may attain better accordance with those made with new techniques. They may contribute to more through precession and nutation investigations. We take part in that work with our original ZT observations.

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17 Some results from the re-reduction of Belgrade zenith-telescope observations

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The authors prepared the Belgrade ZT observations done in the period 1949-1985 in a computer-readable form and their new reduction in the FKS reference frame. The mean error of the instantaneous latitude from one Talcott's pair is less than before. Some basic results of the authors re-reduction are presented, too.

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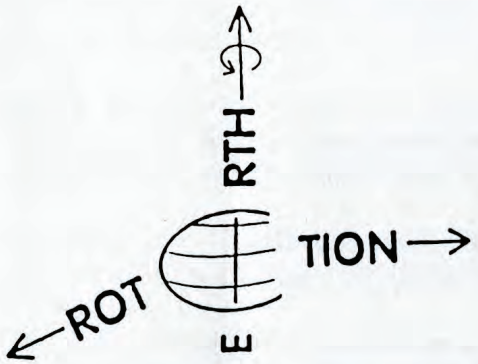

PROCEEDINGS

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SOME RESULTS FROM THE RE–REDUCTION OF BELGRADE ZENITH–TELESCOPE OBSERVATIONS

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Abstract

We prepared the Belgrade ZT observations (made in the period 1949–1985) in a computer–readable form, and their new reduction in the FK5 reference frame. After that, the mean error of the instantaneous latitude from one Talcott's pair is less than before. Some basis results of our re–reduction are presented here.

1 INTRODUCTION

The first value of the precise geographic latitude at the Belgrade Astronomical Observatory was determined in 1947 from ZT (visual zenith–telescope Askania–Bamberg No 77241, 110/1287 mm) observations by applying Talcott's method. At the beginning of 1949 the regular ZT observations started and they are still carried out.

At the XXI IAU General Assembly, Buenos Aires 1991, Commission 19 of the IAU, "Rotation of the Earth", formed the Working Group on Earth Rotation in the HIPPARCOS reference frame–WG ERHRF to collect the past observations and to analyse them in that reference system. Our investigations are in accordance with it. The WG ERHRF has set up a list of the best observations performed in the past. The Belgrade Observatory is on that list (Vondrák, Feissel and Essaifi, 1992) with the observations in the period 1949–1990 obtained with ZT.

The PPM Star Catalogue (Röser & Bastian, 1991) was used. Flag "F" indicates stars that are contained in FK5 (Part I and Part II), flag "H" denotes the stars contained in AGK3R and/or one of the CMC (The Carlsberg Meridian Circle, nos. 1 to 4) catalogues, flag "D" denotes the double star, and flag "P" the problem case.

The Old Belgrade Latitude Programme – OP (Djurković,Ševarlić, Brkić, 1951) was observed in the period 1949–1960. The New Belgrade Latitude Programme – NP (Ševarlić and Teleki, 1960) was started in 1960 and the observations are still carried out.

The numerous stars of OP and NP have got very precise coordinates and proper motions because 81% of stars from OP and 56% of stars from NP are with flag "F" or "H", 10% of stars from OP and 16% of stars from NP (one star is with flag "P") are with flag "D". But "D" stars did make some problems in the observations and the reduction. Their coordinates and proper motions are of lower accuracy than that of single stars and their observations with ZT are difficult.

2 THE PROCEDURE

All the changes in our re–reduction are in accordance with MERIT standards (Melbourne et al., 1983). The new IAU(1976) coordinate system (as defined by FK5) of astronomical constants, the IAU(1980) nutation model, the new dynamical reference system (JPL DE200/LE200 Ephemeris, 1984) and the FORTRAN programme for refraction (Abalakin, 1985) are used.

The trigonometric stellar parallaxes (Jenkins, 1952) and the stellar radial velocities (Wilson, 1953) are used for the calculation of the apparent places of OP and NP stars.

The numerous systematic errors are taken into account. For both Belgrade latitude programmes: the effect of the statistical parallaxes for the stars without trigonometric ones, the deviation of the vertical, the wind effect, the E–W effect–the error due to the clamp position of the telescope, the effect of the level bubble lenght variation, the correction for the curvature of the parallel, the temperature terms of the levels, the systematic errors of declinations and proper motions of Talcott's pairs and (sub)groups of OP and NP. The personal equation was taken into account for the NP only. The temperature term, the progressive and the periodic errors of the micrometer screw revolution are not applied (Milovanović et al., 1981).

We used the Student–Fisher criterion ($\beta = 0.05$) for eliminating the excessive instantaneous latitudes resulting from some Talcott's pairs.

The polar motion was eliminated from the material and the observations were brought in accordance with the mean pole BIH1979. After that we made determinations of the systematic errors of declinations and proper motions of Talcott's pairs and (sub)groups of OP and NP.

We used the new instrument's constants (the angular value of the micrometer screw revolution, the angular division values and the temperature coeficients of the Talcott's levels), see (Damljanović, 1994, 1995).

3 CONCLUSION

The mean error of the instantaneous latitude from one Talcott's pair is less than before. The 12 years period (1949–1960) for OP and 26 years period (1960–1985) for NP were used to derive final systematic errors of declinations and proper motions of the (sub)groups and the Talcott's pairs comprised in OP and NP.

From the preliminary ZT observations made 1947 the mean error was $\pm 0''.255$ (Djurković, Ševarlić, Brkić, 1951). The mean error of the OP (1949–1960) was $\pm 0''.220$ (Ševarlić and Teleki, 1960), and of the NP was $\pm 0''.272$ (1960–1965.5) and $\pm 0''.146$ (1969–1974) (Grujić et al. 1989). After our re-reduction the mean error of the OP is: $\pm 0''.211$ (1949–1951.5), $\pm 0''.195$ (1951.5–1953.5), $\pm 0''.192$ (1953.5–1957), $\pm 0''.205$ (1958–1960). The mean error of the NP is: $\pm 0''.164$ (1960–1963), $\pm 0''.171$ (1964–1967), $\pm 0''.151$ (1968–1970), $\pm 0''.137$ (1971–1972), $\pm 0''.115$ (1973–1976).

The Belgrade latitudes could be used for the final global solution of Earth Orientation Parameters in the Hipparcos reference frame.

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18 The Chandler nutation for the period 1949-1985

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The values of the amplitude, the period and the phase of the Chandler nutation are computed using the homogenized series of Belgrade latitudes for the period 1949-1985. The observations were realized by the Talcott method at ASKANIA zenith - telescope (D= 11 cm, F= 128.7 cm). It was found that the amplitude of the nutation varies with a period of about 38 years; its minimum was around 1971 and the maximum - around 1952. This result was in good agreement with the results of some other authors (Markowitz, 1960; Rykhlova, 1969; McCarthy, 1972; Vicente and Currie, 1976; Wilson and Vicente, 1980; Guinot, 1972, 1982; Nastula et al., 1993; Vondrák, 1985, etc.). The period of the Chandler nutation is also variable. Thus, the results of Carter (1981), Vondrák (1985), Pejović (1985) and some other authors are confirmed.

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THE CHANDLER NUTATION FOR THE PERIOD 1949–1985

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SUMMARY: The values of the amplitude, the period and the phase of the Chandler nutation are computed by using the homogenized series of Belgrade latitudes for the period 1949-1985. The observations were realised by the Talcott method at ASKANIA zenith – telescope (D=11 cm, F=128.7 cm).

The amplitude of the nutation varies with a period of about 38 years; its minimum was around 1971 and the maximum – around 1952. This result is in good agreement with the results of some other authors (Markowitz, 1960; Rykhlova, 1969; McCarthy, 1972; Vicente and Currie, 1976; Wilson and Vicente, 1980; Guinot, 1972, 1982; Nastula et al., 1993; Vondrák, 1985, etc.).

The period of the Chandler nutation is also variable. Thus, the results of Carter (1981), Vondrák (1985), Pejović (1985) and some other authors are confirmed.

1. INTRODUCTION

Different series of classical astrometry data are analysed with the aim to examine the stability in time of the amplitude and the period (or the phase) of Chandler nutation. Besides the 80–year long series of ILS coordinates, the combined series of ILS and pôle coordinates computed from the observations of individual observatories as, for example, Washington, Greenwich, Pulkovo, Kiev, etc. enabled the studies of Chandler nutation to be extended far away in the past. So, for example, the series analysed in Rykhlova (1969) and Nastula et al. (1993) span over the intervals of 119 and 140 years, respectively. In all mentioned series the quasi–periodic instability of the Chandler nutation amplitude whose period lies in the range 30–40 years is detected. In the recent study of Vondrák and Ron (1995), based on the latitudes reduced to HIPPARCOS system of star positions, this result is also confirmed. Having in mind that the HIPPARCOS coordinates are free of known systematic errors and local distortions of classical astrometry coordinates and proper motions, the latter result is particularly interessting.

Despite the above results the observational evidence of the 30–40–yr Chandler nutation amplitude variation is not widely accepted as a real one.This situation could be explained by the lack of the physical explanation of the phenomenon.

Since the homogenized series of Belgrade latitudes provides an opportunity for the studies of long–period variations of latitudes, the present work is focused on examining the Chandler nutation period and amplitude stability and so to give a contribution for a better description of their behavior with time.

The Belgrade latitudes data (in the list of Bureau International de l' Heure (BIH) series denoted by BLZ) for the period 1949 – 1985 are obtained by two observation programs: the old one (Djurković, Ševarlić, Brkić, 1951), until the end of 1960, and the new one (Ševarlić and Teleki, 1960), from the beginning of 1960 and onwards.

All latitudes are reduced to the FK5 reference frame (Damljanović, Pejović, 1995) and apparent star positions are computed following the procedure known as MERIT standards (Melbourne et al., 1983). For this purpose the PPM Star Catalogue (Röser and Bastian, 1991) is used.

The latitudes are analysed for some systematic instrumental, personal, refraction and star position errors. Those of them which are confidently determined are removed from the data (Damljanović, 1994, 1995).

To obtain the equidistant data, the raw latitudes were averaged and after that they were interpolated by the cubic spline method. The "normal points" (the average values of 49–58 Talcott pair latitudes) have approximately equal weights. The interpolated latitudes are computed by the lag of 0.1 yr: their total number being $N = 369$.

2. THE AMPLITUDE, THE PERIOD AND THE PHASE OF THE CHANDLER NUTATION

The Chandler nutation is slightly elliptical or near circular (see, for example, Guinot, 1982). According to Vondrák (1985) and Vondrák and Ron (1995), its ampitude varies from $0\overset{''}{.}07$ to $0\overset{''}{.}28$. Assuming the phase constant, the mentioned authors have found the period of free nutation to be also variable, ranging from 387 days (or 1.06 yr) to 442 days (or 1.21 yr).

The amplitude and phase of Chandler nutation in the present work refere to the Belgrade meridian $\lambda_{BLZ} = 20^o.5E$. This fact has to be kept in mind when one compares the BLZ rezults with the results based on ILS, IPMS, BIH and IERS data.

In the present work the Fourier transforms – DFT of interpolated latitudes are first computed for the whole observation period. As usually, the secular latitude variation was previousely determined (by the least–squares method – LSQ) and removed from the data.

The amplitude periodogram of interpolated latitudes is presented in Fig. 1. The numerical values for the periods, the amplitudes and the phases (for the epoch $1949.^y1$) of three known oscillations are given in Table 1.

The amplitudes are corrected for the white noise contribution.

The standard deviation of residuals (see Fig. 5.), obtained after the removing of the secular term and three mentioned oscillations is $\sigma_0 = 0\overset{''}{.}066$. This value was used to compute the amplitude and the phase standard deviations:

$$\sigma_A = \sigma_0\sqrt{\frac{4-\pi}{N}} = 0\overset{''}{.}003,$$

$$\sigma_F \approx 57.^\circ296\frac{\sigma_0}{A}\sqrt{\frac{2}{N}}.$$

The values of σ_F of the Chandler, the annual and the semiannual terms are: $1.^\circ6$, $4.^\circ0$, $18.^\circ1$ respectively.

The mentioned white noise correction to amplitude is

$$\Delta A = \sigma_0\sqrt{\frac{\pi}{N}} = 0\overset{''}{.}006.$$

The value $A_a = 0\overset{''}{.}063$ is less than the value of A_a from the international data for the same period because of the presence of strong Z–term whose seasonal component has an amplitude $A_Z = 0\overset{''}{.}041$ and a phase $F_Z = 41.^\circ3$ (epoch: $1949.^y1$).

When the results from the Table 1. are used for the removing of annual and semiannual oscillations the residuals presented in Fig. 2. are obtained. These residuals are used to obtain the "instantaneous" amplitude and the phase of Chandler nutation by the LSQ method.

Besides the DFT method, the mean period of Chandler nutation for the time span 1949-1985 was recomputed by the LSQ method. For this purpose P_C is varied to obtain the minimum of standard deviation ($\sigma = 0\overset{''}{.}0836$) of residuals. Thus the value $P_C = 1.185$ yr was obtained. Evidently, this result, as theoretically expected, is very close to the DFT value of P_C.

With constant $P_C = 1.185$ yr the amplitude and the phase (for the epoch $1900.^y0$) of Chandler nutation were recomputed for the independent 1.2 yr subintervals. In this way the results presented in the Table 2. were obtained.

At the bottom of Table 2. the phases relative to the epoch $1949.^y1$ are also given. They may be compared with DFT results.

Evidently, the phases computed by two mentioned methods are close to each other.

In Table 2. are also given the amplitudes and the phases (for the epochs $1949.^y1$ and $1900.^y0$) computed by using $P_C = 1.^y19$. The aim of this computation was to compare our results with those based on international data. For example, with P_C=1.19 yr

Table 1. The period (P), the amplitude (A) and the phase (F, epoch: $1949.^y1$) of the Chandler (c), the annual (a) and the semiannual (sa) terms

P_c	A_c	F_c	P_a	A_a	F_a	P_{sa}	A_{sa}	F_{sa}
$1.^y19$	$0\overset{''}{.}170$	$73.^\circ4$	$1.^y01$	$0\overset{''}{.}063$	$205.^\circ7$	$0.^y49$	$0\overset{''}{.}009$	$168.^\circ1$

Table 2. The values of the amplitude (A_C) and phase (F_C) of the Chandler wobble obtained by LSQ

year (1900+)	No	$A_c('')$	$F_c(^\circ)$ (epoch: $1900.^y0$)
49.7	1	.2365	212.9
50.9	2	.1799	189.8
52.1	3	.3373	209.9
53.3	4	.2202	232.0
54.5	5	.3068	209.5
55.7	6	.2334	208.6
56.9	7	.2100	221.1
58.1	8	.2129	205.8
59.3	9	.2259	224.0
60.5	10	.1958	222.2
61.7	11	.1751	219.5
62.9	12	.1291	206.0
64.1	13	.1960	230.1
65.3	14	.1574	225.0
66.5	15	.1602	233.1
67.7	16	.1507	257.1
68.9	17	.1066	234.6
70.1	18	.1320	248.4
71.3	19	.1071	226.4
72.5	20	.1468	248.2
73.7	21	.1618	239.5
74.9	22	.1601	224.6
76.1	23	.1196	232.0
77.3	24	.1539	229.9
78.5	25	.1686	224.7
79.7	26	.1655	222.2
80.9	27	.1870	202.3
82.1	28	.1598	197.8
83.3	29	.2312	209.0
84.5	30	.2171	213.1
mean		.1848 ±.0096	222.0 ±2.8
period: 1949–1985 ($P_C = 1.^y185$)		.1802	218.6 (epoch: $1900.^y0$)
period: 1949–1985 ($P_C = 1.^y185$)		.1802	62.1 (epoch: $1949.^y1$)
period: 1949–1985 ($P_C = 1.^y19$)		.1749	134.3 (epoch: $1900.^y0$)
period: 1949–1985 ($P_C = 1.^y19$)		.1749	40.5 (epoch: $1949.^y1$)

the mean phase of Chandler nutation from BLZ data at the epoch $1900.^y0$ is $F_c = 134.^o3$. Taking into account the Belgrade longitude, this result is in good agreement with the results of Vondrák (1985, 1988).

The series of independent values of the amplitude and phase (for $P_C = 1.^y185$) are shown in Fig. 3. and Fig. 4. respectively.

The sinusoidal approximation of the amplitude (dotted line in Fig. 3.) is obtained by LSQ varying the period of sinusoid from 35.0 up to 40.0 years with a lag of 0.5 year. The minimum of standard deviation $SD = 0\overset{''}{.}0324$ of residuals (which are shown in Fig. 6.) corresponds to the period $P_{aC} = 38.^y0$ and the amplitude $A_{aC} = 0\overset{''}{.}0597$.

From the results presented in Fig. 4. it seems probable that the Chandler nutation phase in the period of 1964–1978 is systematically above the values outside this period. The corresponding periods are: 447 days (in the interval 1964–1978) and 423 days (outside this interval).

The results obtained from Belgrade data are in relatively good agreement with the results obtained from international data series (Guinot, 1972; Gaposchkin, 1972; Guinot, 1982; Markowitz, 1982; Vondrák, 1985; Vondrák and Ron, 1995). Accordingly the 30 – 40 yr variation of the Chandler nutation amplitude is confirmed. In addition, the phase variation is also shown.

3. CONCLUSION

In the homogenized series of Belgrade latitudes the decade variation of Chandler nutation amplitude is detected. The period of variation is $P_{aC} =$ 38 years, the amplitude is $A_{aC} = 0\overset{''}{.}0597$.

For the period of Chandler nutation two values are found: $P_C = 447$ days, for the period 1964–1978 and $P_C = 423$ days, outside this period.

Assuming P_C constant ($P_C = 1.185$ yr), the corresponding values of the phase are: $F_C = 235.^o7$ (1964–1978) and $F_C = 212.^o8$.

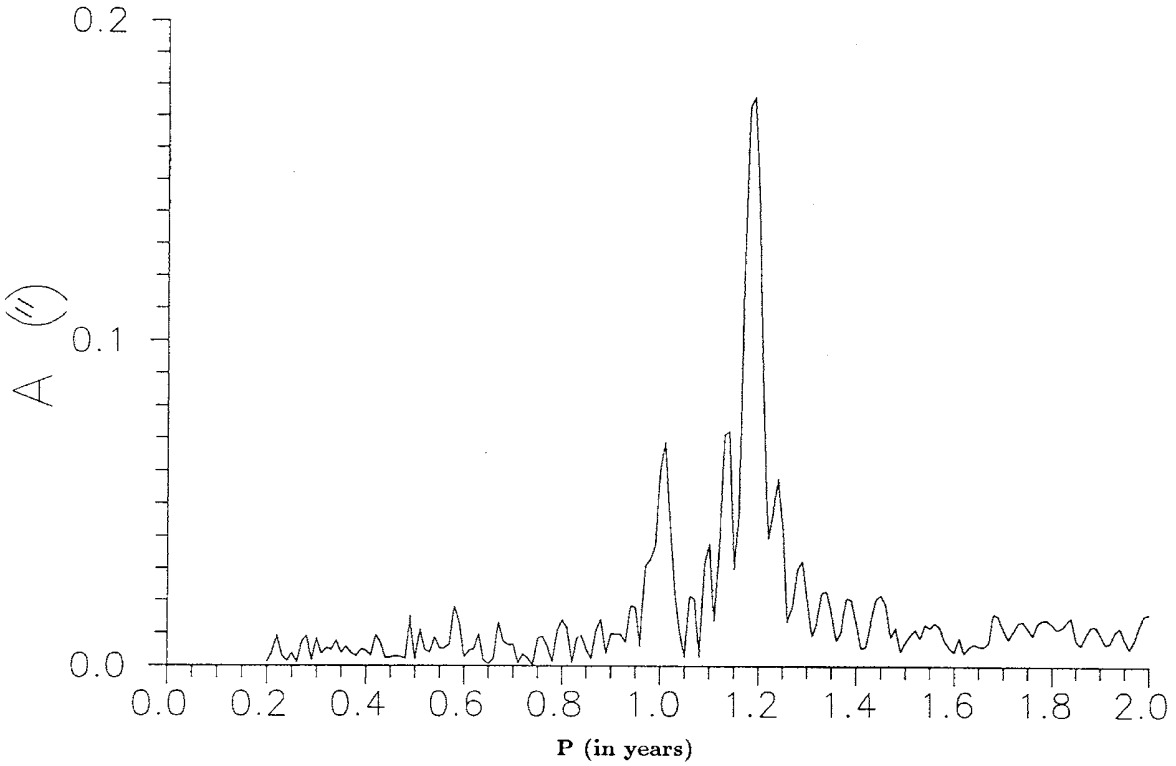

Fig. 1. *Amplitude periodogram of unsmoothed BLZ data.*

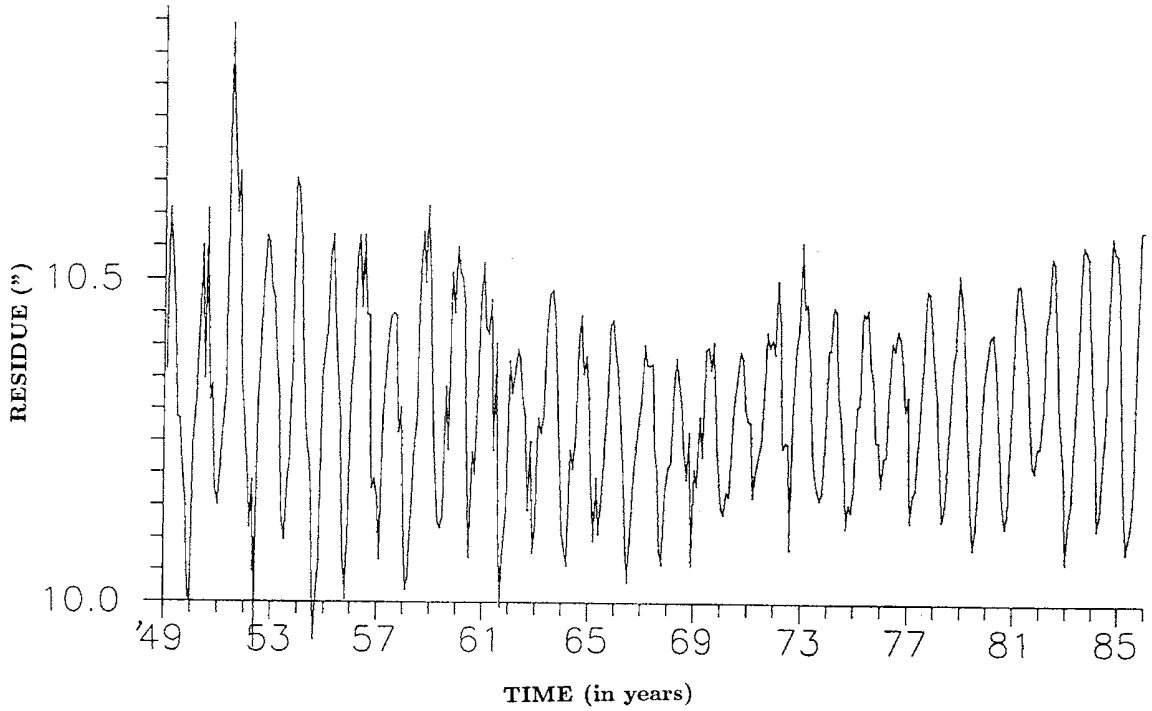

Fig. 2. *Residuals (BLZ data free of the linear, semiannual and annual terms).*

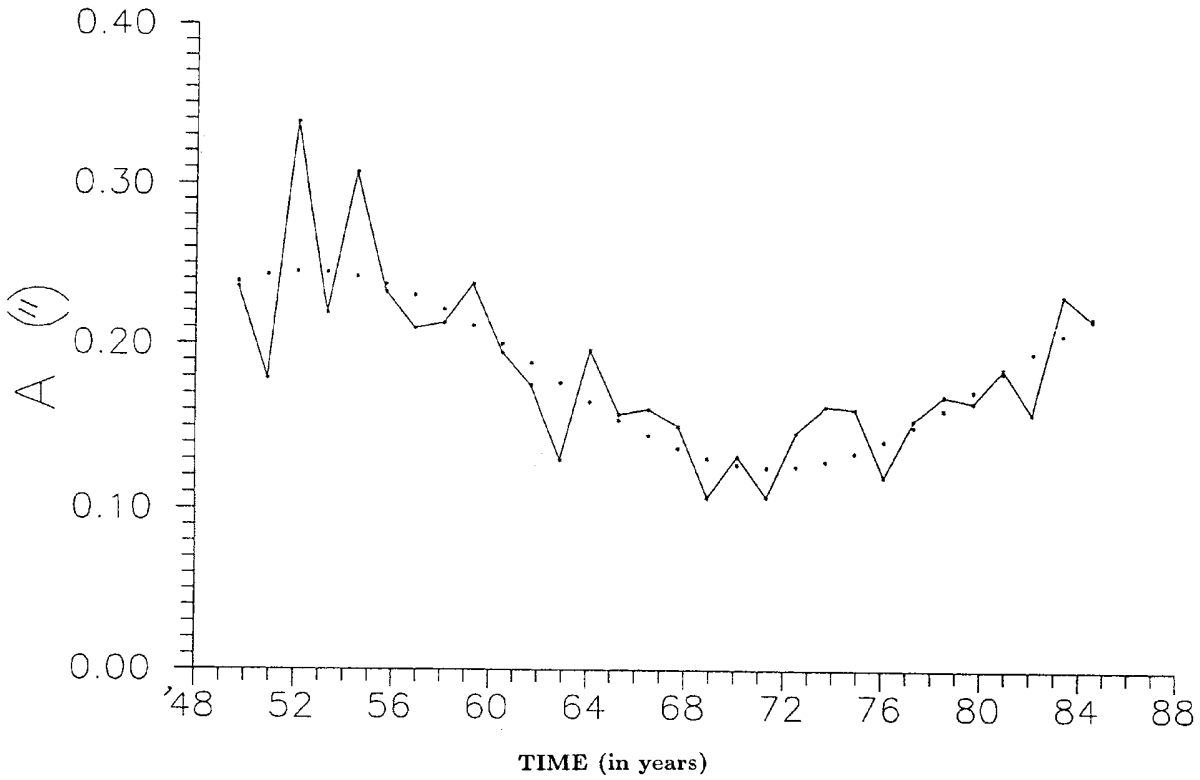

Fig. 3. *The independent values of the Chandler nutation amplitude.*

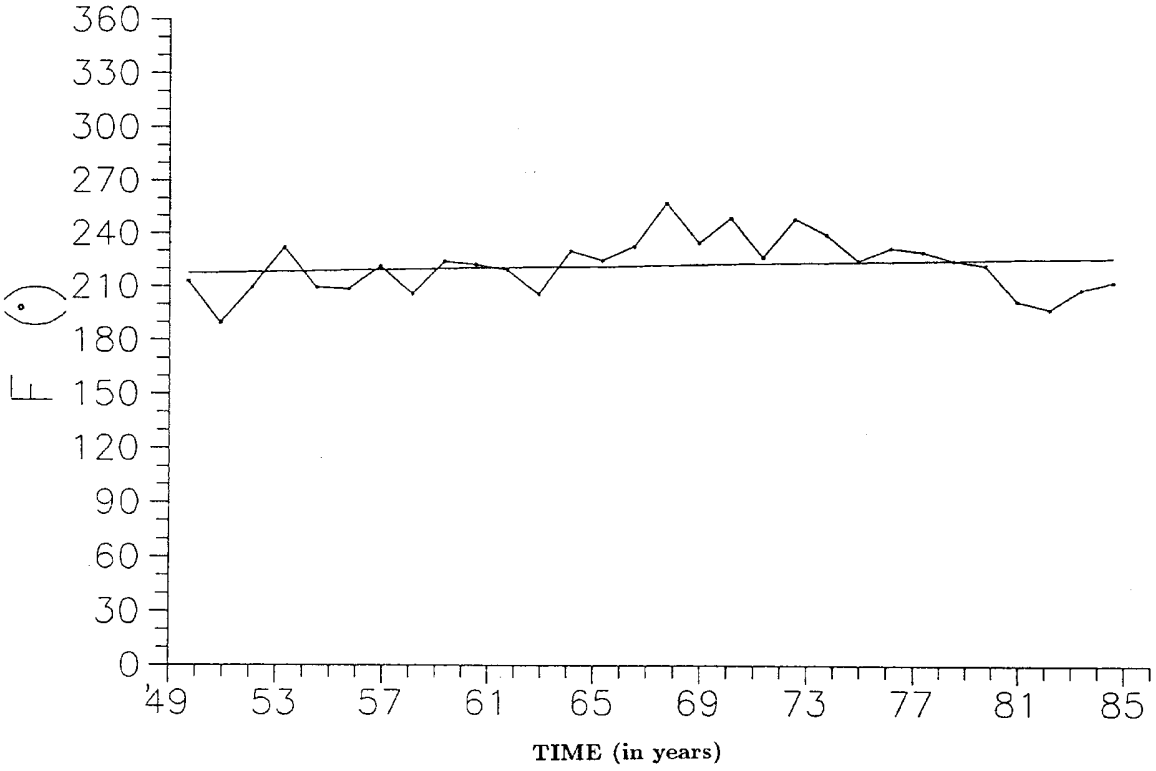

Fig. 4. *The independent values of the Chandler nutation phase.*

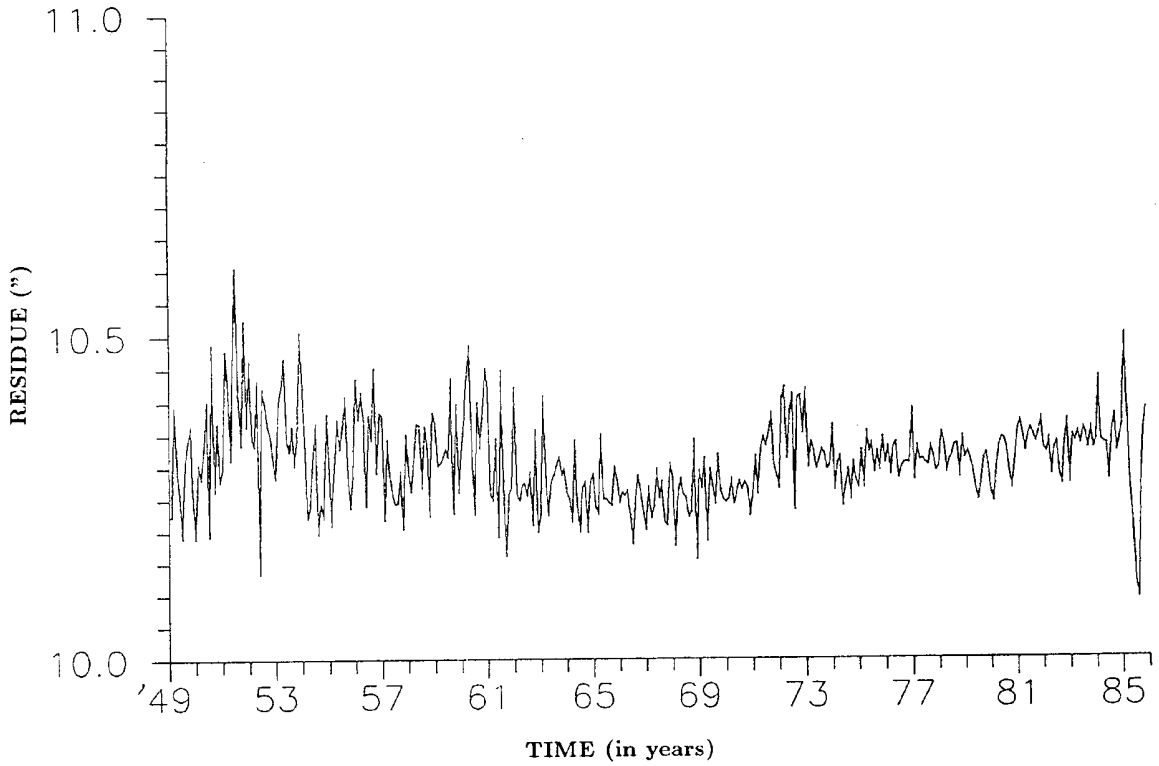

Fig. 5. *Residuals (BLZ data free of the linear, semiannual, annual and Chandler terms).*

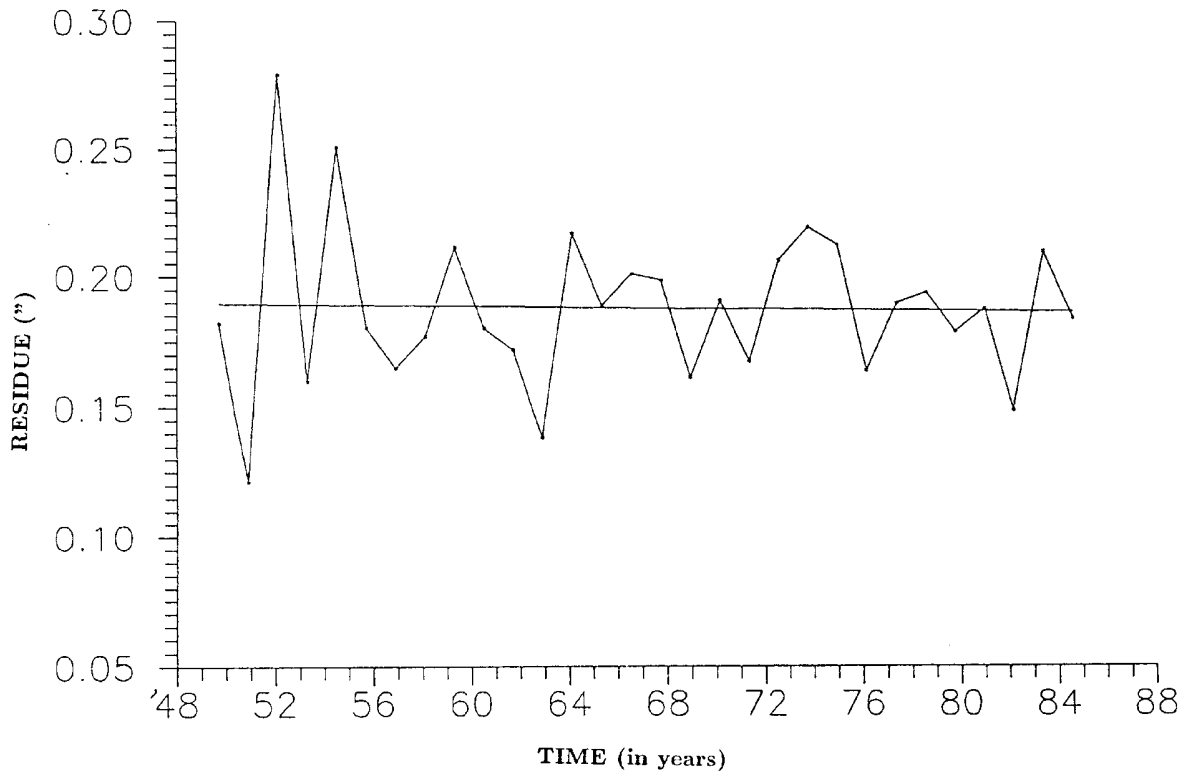

Fig. 6. *Residuals of the Chandler nutation amplitude.*

Table 3. The parameters (A_{aC}, P_{aC}, F_{aC} for the epoch $1900.^y0$) of the harmonic term which was present in the changes of the amplitude of the Chandler nutation, the values of coefficients (C(1), C(2), C(3), with their standard deviations) obtained by LSQ and the standard deviation (SD)

P_{aC}	C(1)	A_{aC}	F_{aC}	C(2)	C(3)	SD
$38.^y0$	$0\,''.\,1878 \pm 0\,''.\,0059$	$0\,''.\,0597$	$137.^\circ7$	$0\,''.\,0402 \pm 0\,''.\,0086$	$-0\,''.\,0442 \pm 0\,''.\,0082$	$\pm 0\,''.\,0324$

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ЧЕНДЛЕРОВА НУТАЦИЈА У ПЕРИОДУ 1949 – 1985

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УДК 520.8;521.93
Оригинални научни рад

Рачунате су вредности амплитуде, периоде и фазе Чендлерове нутације из хомогене серије ширина Београда за период 1949 - 1985. Посматрања су рађена Талкотовом методом на зенит – телескопу ASKANIA (D=11 cm; F=128.7 cm).

Добијене вредности амплитуде Чендлерове нутације варирају са периодом око 38 година; минимум је био око 1971 а максимум око 1952. Овај резултат је у доброј сагласности са резултатима других аутора (Markowitz, 1960; Rykhlova, 1969; McCarthy, 1972; Vicente and Currie, 1976; Wilson and Vicente, 1980; Guinot, 1972, 1982; Nastula et al., 1993; Vondrák, 1985, итд.).

Период Чендлерове нутације је такође променљив. Дакле, потврђени су резултати које су предочили Carter (1981), Vondrák (1985), Пејовић (1985) и други аутори.

19 Wavelet transforms and their application

P. Jovanović and S. Djurović and N. Pejović
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Wavelet transforms with Morlet analyzing function have been discussed. The discussion includes comparisons of wavelet transforms with Fourier transforms and their application for decomposition of a model data time series into time-frequency space. It is also shown that the value of Morlet wavelet function parameter has a great influence on the method's selectivity and statistical significance of obtained spectra. The continuous wavelet transform of a discrete time series was computed as the convolution of the data with scaled and translated version of Morlet wavelet function.

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WAVELET TRANSFORMS AND THEIR APPLICATIONS

P. Jovanović, D. Đurović and N. Pejović

Abstract. Wavelet transforms with Morlet analyzing function have been discussed. The discussion includes comparisons of wavelet transforms with Fourier transforms and their application for decomposition of a model data time series into time-frequency space. It is also shown that the value of Morlet wavelet function parameter has a great influence on the method's selectivity and statistical significance of obtained spectra. The continuous wavelet transform of a discrete time series was computed as the convolution of the data with scaled and translated version of Morlet wavelet function.

1. Introduction

Wavelets are functions that satisfy certain mathematical conditions and they are used in many scientific fields like mathematics (for solving partial differential equations), astronomy, quantum physics, seismic geology, electrical engineering, signal and image processing, magnetic resonance, etc. Wavelet analysis can be assumed as a new branch in harmonic analysis, which contains both the wavelet and the Fourier analysis.

The fundamental feature of the Fourier transforms is their ability to decompose a time series into its frequency content which can then be easily analyzed. Fourier transform of a time series or signal $x(t)$ is defined as:

$$\hat{x}(f) = \int_{-\infty}^{\infty} x(t)\, e^{-i2\pi f t}\, dt, \tag{1}$$

where t is the time and f is the frequency.

Let us assume that $x(t)$ is composed of some sine functions:

$$x(t) = \sum_{k=1}^{N} A_k \sin(\omega_k t + \alpha_k). \tag{2}$$

It can easily be proven that its Fourier transform is the Dirac's comb function. So, instead of the time function $x(t)$, where the recognition of constituents is hardly possible, one can obtain the limited number of "spectral lines" located at the frequencies of constituents.

In practical applications, where $x(t)$ is given over limited time interval $[-T, T]$, instead of the "spectral lines", one obtains the "peaks" whose amplitudes are:

$$B_k = A_k \frac{\sin\lambda}{\lambda}, \tag{3}$$

where $\lambda = \pi(f - f_k)T = \frac{1}{2}(\omega - \omega_k)T$, ω—independent variable angular frequency.

The contribution of the sinusoidal wave at each frequency is represented by the Fourier coefficients of the transformed function, i.e. by the corresponding amplitudes and phases. If the signal properties change in the analysis interval, then the Fourier transforms are not ideally suited because the sines and cosines are not localized in time (they strech out to infinity).

When the sinusoidal wave is replaced by the product of a sinusoid and a window which is localized in time, then the Fourier transform becomes the windowed Fourier transform:

$$\hat{x}(\tau, f) = \int_{-\infty}^{\infty} x(t)\, g(t-\tau)\, e^{-i2\pi f t}\, dt, \tag{4}$$

where $g(t) = g(-t)$ is the symmetric window and τ is the central time. The windowed Fourier transforms can be applied to slowly time-varying signals, but their basis functions (products of a sinusoid wave and a window) are infinitely enduring periodic functions, and such cases are rare in practice. Because of that, a new technique called wavelet transforms, has been advanced to deal with time-varying signals.

The wavelet transform replaces the Fourier transform's sinusoidal wave basis by the basis consisting of a family of functions generated by translations and dilations of a mother wavelet function. The continuos wavelet transform of a continuos signal $x(t)$ is defined as the following convolution [4]:

$$W(a,b) = \int_{-\infty}^{\infty} x(t)\, \frac{1}{\sqrt{|a|}}\, \overline{w}\left(\frac{t-b}{a}\right) dt, \tag{5}$$

where $a \neq 0$ is the dilation (scale) parameter, b is the translation parameter and $\overline{w}(t)$ is the complex conjugate of the wavelet function $w(t)$. The most important dissimilarity between windowed Fourier and wavelet transforms is that the first have a constant time frequency resolution which can be changed only by rescaling the window g. On the other hand, wavelet transforms have a time frequency resolution which depends on the scale a.

Taking into acount that both Fourier and windowed Fourier transforms are invertible, the mathematical conditions which must be satisfied by the wavelet function $w(t)$ are determined to ensure the invertibility of the wavelet transforms, too. This is satisfied if $w(t)$ is integrable, square-integrable and has no zero-frequency component, i.e. $\hat{w}(0) = \int_{-\infty}^{\infty} w(t)\, dt = 0$. Besides theese conditions, wavelet function must be localized both in time and frequency domain. It means that $w(t)$ and $\hat{w}(f)$ must be negligible outside the intervals $[t_{min}, t_{max}]$ and $[f_{min}, f_{max}]$, respectively.

2. Morlet wavelet function

Wavelets comprise an infinite set and some of the most famous families are those named by Daubechies [5], Mexican hat, Paul, DOG (derivative of a Gaussian) and Morlet [6]. The last one is complex-valued analyzing function which is the most widely used. It is defined by the formula:

$$w(t) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}}\, e^{ipt}, \tag{6}$$

where p is the parameter of Morlet wavelet function. Morlet wavelet is nothing else but a plane wave modulated by a Gaussian. It can be easily seen that this wavelet does not satisfy the condition $\int_{-\infty}^{\infty} w(t)\,dt = 0$, but if the parameter p is taken to be $p \geq 5$ then the previous condition is approximately satisfied. For Morlet wavelet function with parameter p, the Fourier period P corresponding to the wavelet scale a is given by the following relationship [6]:

$$P = 1/f = \frac{4\pi a}{p + \sqrt{2 + p^2}}. \tag{7}$$

The choice of the parameter p has a great influence on the selectivity of wavelet transforms and statistical significance of obtained spectra, which will be demonstrated below.

3. Practical application of wavelet transforms

The continuos wavelet transform of a discrete signal x_i is defined as the convolution of x_i with a scaled and translated version of normalized wavelet function $w'(t)$ [6]:

$$W_i(a) = \sum_{j=0}^{N-1} x_j \,\overline{w'}\left(\frac{(j-i)\Delta t}{a}\right), \tag{8}$$

where the overline denotes the complex conjugate. Here, the index i plays the role of translation parameter. Normalization of the wavelet function is done to ensure that the wavelet transforms are comparable to each other at any scale a, and also to the wavelet transforms of other time series, and is performed by the formula:

$$w'\left(\frac{(j-i)\,\Delta t}{a}\right) = \sqrt{\frac{\Delta t}{a}}\, w\left(\frac{(j-i)\,\Delta t}{a}\right). \tag{9}$$

The wavelet transform $W_i(a)$ is in general a complex function of the scaling and translating parameter, so the wavelet power spectrum can be defined as $|W_i(a)|^2$. For comparison reasons, the wavelet power spectrum is also normalized by $|W_i(a)|^2/\sigma^2$.

The global wavelet spectrum is the time-average over all local wavelet spectra:

$$W_g^2(a) = \frac{1}{N} \sum_{i=0}^{N-1} |W_i(a)|^2, \tag{10}$$

and the corresponding normalized value is W_g^2/σ^2.

In order to determine statistical significance of obtained peaks, the global wavelet power spectrum must be compared to an appropriate background spectrum of either white noise or red noise. A simple model of red noise is the first order autoregressive or Markov process:

$$x_n = \alpha x_{n-1} + z_n, \tag{11}$$

where α is the assumed lag-1 autocorrelation, x_0 and z_n are taken from Gaussian white noise [6]. The corresponding background spectrum, after normalization, is:

$$P_k = \frac{1 - \alpha^2}{1 + \alpha^2 - 2\alpha \cos\frac{2\pi k}{N}}, \quad k = 0, 1, \ldots, N/2. \tag{12}$$

For $\alpha = 0$, equation (12) represents a white noise spectrum.

It is shown in [6] that the confidence interval for the true wavelet power $\mathcal{W}_n^2(a)$, assuming a red noise background spectrum, is given by:

$$\frac{2}{\chi_2^2(\beta/2)}|W_n(a)|^2 \leq \mathcal{W}_n^2(a) \leq \frac{2}{\chi_2^2(1-\beta/2)}|W_n(a)|^2, \tag{13}$$

where β is the desired significance.

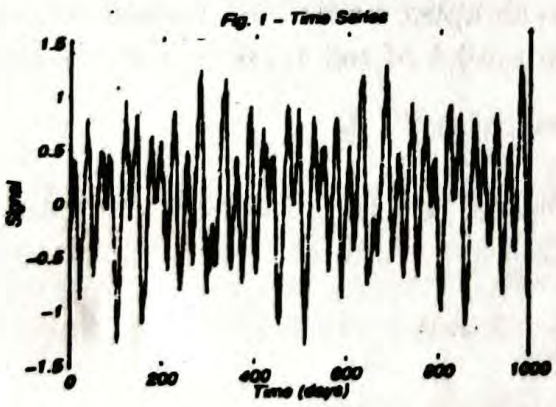
4. Some examples of time series and their transforms

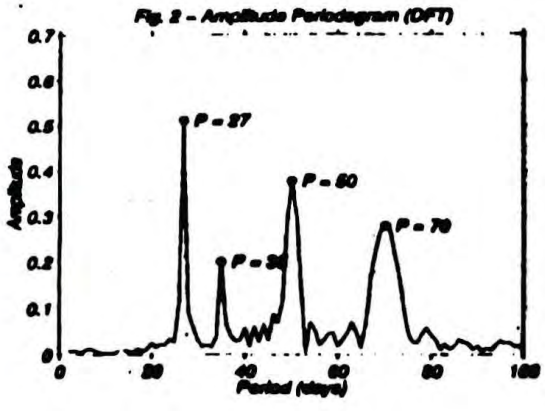
The sum of four harmonics with constant periods, amplitudes and phases which are given in Table 1. is adopted as the model data time series. Pseudo-random generated white noise with standard deviation of $\sigma_n = 0.116$ is added to the previous sum. The sampling time is $\Delta t = 1^d$, the number of data is $N = 1000$ and standard deviation of synthetical signal is $\sigma = 0.530$. Above model is a simulation of the short-periodic oscillations in Earth rotation (i.e. in displacements of Earth's poles and in Earth's rotational period). These oscillations are time-varying while the model does not vary with time. Therefore, the model does not ideally represent the nature of the short-periodic variations in Earth's rotation and is suitable for application of the Fourier, rather than the wavelet transforms. This fact will not affect the main goal of this paper—investigation of the Morlet wavelet parameter influence on the selectivity of the wavelet transforms and statistical significance of the resulting spectra. Time series model is graphically presented in Fig. 1 and its amplitude periodogram obtained by the Discrete Fourier transforms, in Fig. 2. The Discrete Fourier transform (DFT) of x_n is defined by:

i	$P_i(^d)$	A_i	$\Phi_i(^\circ)$
1	27	0.5	225
2	35	0.2	270
3	50	0.4	135
4	70	0.3	45

Table 1. - Periods (P), amplitudes (A) and phases (Φ) of the synthetical signal

$$\hat{x}_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{-2\pi i k n/N}, \qquad k = 0, 1, \ldots, N-1. \tag{14}$$





It can be seen from Fig. 2 that the peaks corresponding to each harmonic in time series are clearly emphasized. Because of dealing with finite-length time series, the width of spectral lines increases with the period. To avoid this problem in DFT, the time-series must be as long as possible.

The results of wavelet transforms applied on the model data series with Morlet wavelet parameters $p = 6$, $p = 7$, $p = 8$ and $p = 23$, are given in Figs. 3a–3d (wavelet power spectra) and Figs. 4a–4d (global wavelet power spectra). The wavelet power spectra are presented in Figs. 3a–3d by the colored contours, while the limits of confidence intervals are indicated by the white contours and they enclose the regions of confidence greater than 95% ($\beta = 0.05$), for a red noise process with $\alpha = 0.72$. The finit length of time series is the cause of errors which occur at the begining and end of the wavelet power spectrum. The region of the wavelet spectrum in which these edge effects become important is called the cone of influence, and is indicated in the same Figs. by the region above two tilted straight lines. A red noise spectrum, calculated from (12) for $\alpha = 0.72$, is shown in Figs. 4a–4d by the dashed curves, while a white noise spectrum is indicated by the horizontal dashed lines.

In Fig. 3a, that corresponds to $p = 6$, there are two regions of confidence greater than 95% (enclosed by the white contours), concetrated at approximately 30 and 45–70 days, respectively. The cone of influence is relatively narrow and affects only the left and right side of the spectrum. Fig. 4a indicates that the maxima of the global power spectrum, located at periods $P \approx 28^d$ and $P \approx 47^d$, exceed the limits of white noise (horizontal dashed line) and red noise (dashed curve). Therefore, these peaks cannot be explained by any kind of noise.

In the case of $p = 7$ (Figs. 3b and 4b), the second 95% confidence region is now devided into two parts, one concetrated at approximately 50^d, and the other at approximately 70^d. This is even more obvious in the global power spectrum, where the broad peak from the Fig. 4a at $P \approx 47^d$ is now devided into two peaks at $P \approx 48^d$ and $P \approx 68^d$, which are clearly above the white and red noise levels. Comparing Fig. 3a and 3b, it can be seen that the cone of influence is wider in the second case, so the edge effects are more emphasized in the case of $p = 7$ than $p = 6$. Figs. 3c and 4c (the case when $p = 8$), are quite similar to those corresponding to $p = 7$, except the 95% confidence regions became more concetrated, the last two global spectrum peaks more separated and their periods almost identical to those obtained by the discrete Fourier transforms. The cone of influence is wider, but still affects only the begining and ending parts of the spectrum.

The cone of influence is dramatically increased for $p = 23$ (Figs. 3d and 4d) and affects not only the spectrum edges, but also a significant part of its central area. On the other hand, three of the 95% confidence regions became very narrow and their maxima close to the real values in the model data (see Fig. 4d). The fourth peak (for $P \approx 27^d$), although it was well emphasized in the previous cases, now is completely under the red noise level.

Fig. 3a – Wavelet Power Spectrum (p = 6)

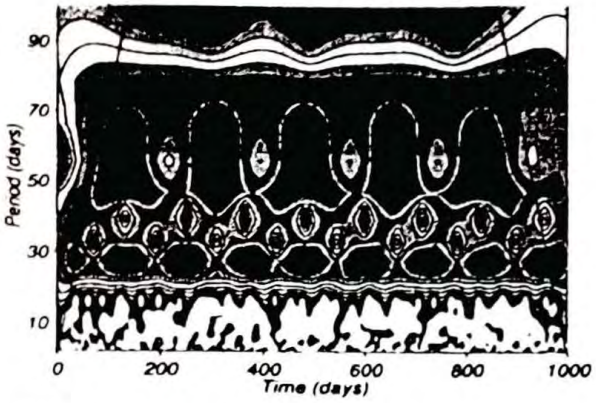


Fig. 3b – Wavelet Power Spectrum (p = 7)

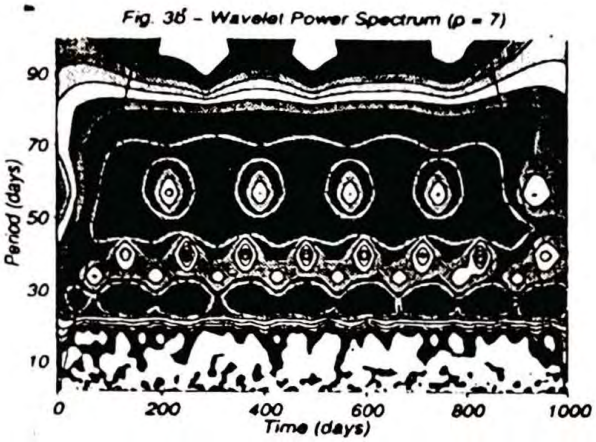


Fig. 3c – Wavelet Power Spectrum (p = 8)

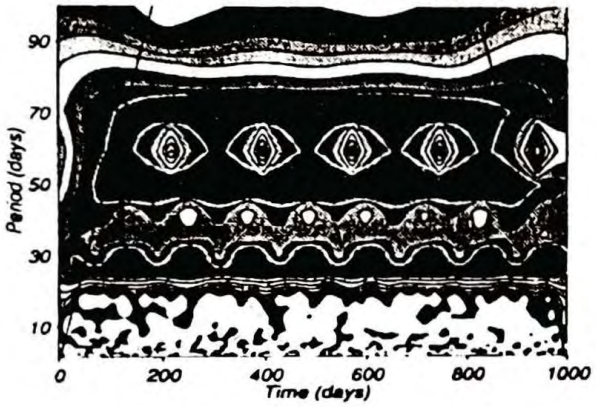


Fig. 3d – Wavelet Power Spectrum (p = 23)

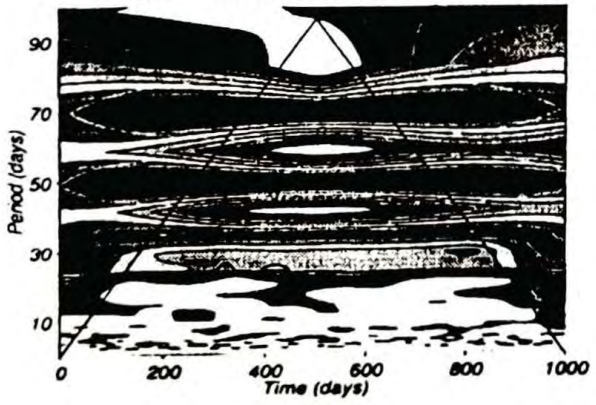


Fig. 4a – Global Wavelet Power Spectrum (p = 6)

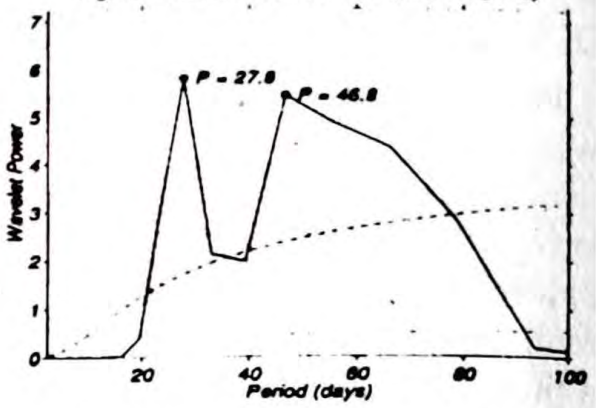


Fig. 4b – Global Wavelet Power Spectrum (p = 7)

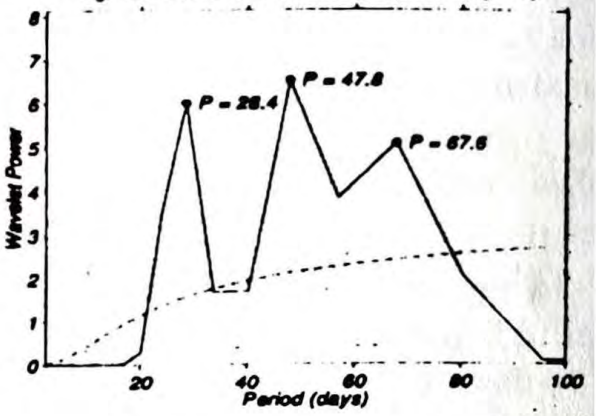


Fig. 4c – Global Wavelet Power Spectrum (p = 8)

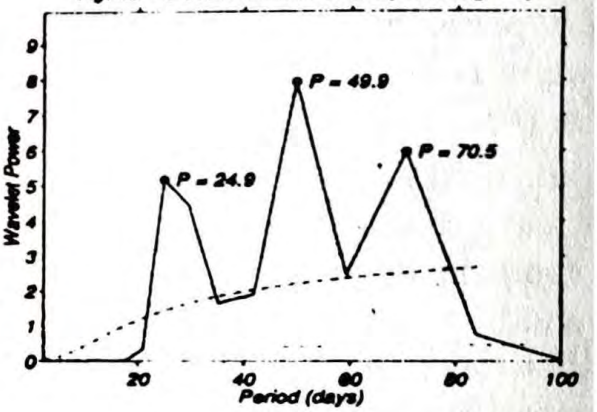
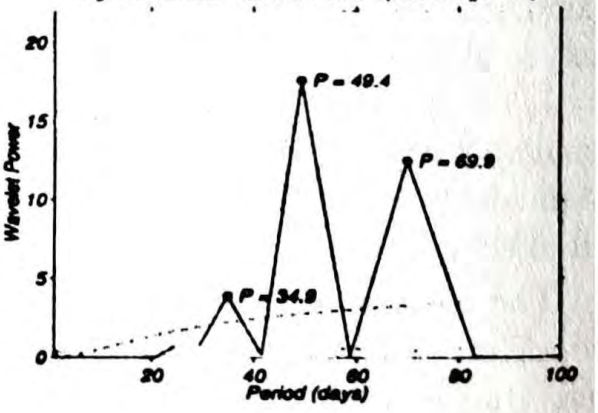


Fig. 4d – Global Wavelet Power Spectrum (p = 23)



All these facts indicate that selectivity of the wavelet transforms and statistical uncertainty of the resulting spectra, increase with the parameter of Morlet wavelet. Therefore, if the statistically significant spectra are wanted, the wavelet transforms must be performed with the relatively modest quantity of Morlet wavelet parameter.

The above model does not vary with time, so the wavelet transforms advantage when dealing with time-varying signals is not quite noticeable. Therefore, these transforms with parameter $p = 2\pi$ were also applied for analysis of the following, real astronomical and geophysical time series:
a) 2800 MHz (10.7 cm) solar flux measures,
b) Earth's pole y - coordinate values and
c) values of χ_1 effective atmospheric angular momentum function for wind + pressure.

Our goal was to detect so called Five Year Oscillation (FYO) in these data. FYO is known as a quasi-periodic oscillation whose period varies between approximately 3 and 8 years. Before spectral analysis, the data were filtered in order to eliminate the stronger oscillations. The amplitude periodograms of the above data, obtained by DFT, are presented in Figs. 5a–7a, and the corresponding wavelet spectra in Figs. 5b–7b, where the darker shades correspond to the greater wavelet moduli and vice versa.

Nearly horizontal dark belt at approximately 5.5 years in Fig. 5b indicates FYO's stability in solar flux time series. In Fig. 5a, corresponding to solar flux DFT amplitude periodogram, there is a strong peak at approximately same period. It shows that in the case of stable time series both methods, DFT and wavelet transforms, give the similar results.

In the case of Earth's pole y-coordinate (Figs. 6a and 6b), two components of FYO are detected. The first one is concetrated between 3 and 4 years, and the second at approximately 6 years. Analyzing only the DFT amplitude periodogram, one could conclude that FYO consists of two oscillations. However, the corresponding wavelet spectrogram shows that FYO is an unique quasi-periodic oscillation whose period varies from 5–6 years (in the first 15 years of the observational interval) to 3–4 years (in the rest of the observational interval).

The presence of several peaks between 3 and 6 years as well as the instability of the DFT amplitude periodogram in the case of χ_1 (Fig. 7a), could be interpreted as an argument for doubting FYO existence in χ_1. However, this characteristic is a consequence of the relatively significant variations in FYO period, making this oscillations quasi-periodic. This statement is obvious from the corresponding wavelet spectrogram (Fig. 7b) where a remarkable fluctuation of the FYO period between 3 and 6 years is recorded. Taking into account that DFT method is not suitable for application in the case of time-varying signals, it could be concluded that the great instability of χ_1 amplitude periodogram is due to this deficiency.

These results clearly show what is the main advantage of the wavelet against the Fourier transorms in the case of time-varying signals.

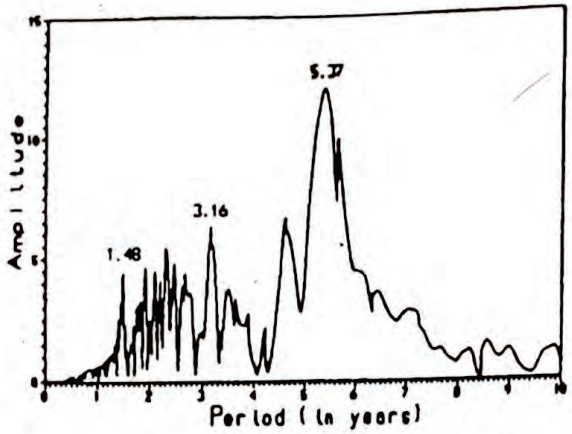


Fig. 5a -*Amplitude periodogram (DFT) of 2800 MHz solar flux*

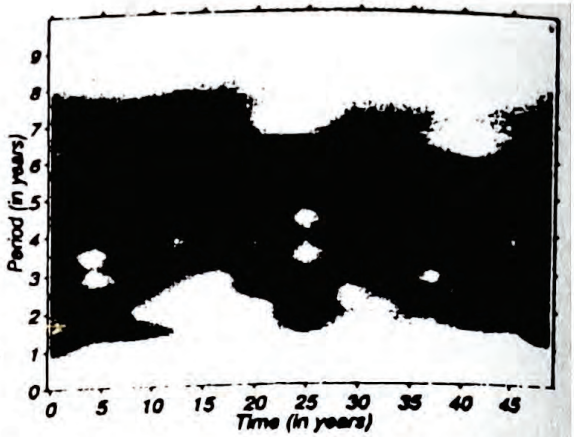


Fig. 5b -*Spectrogram (WT) of 2800 MHz solar flux*

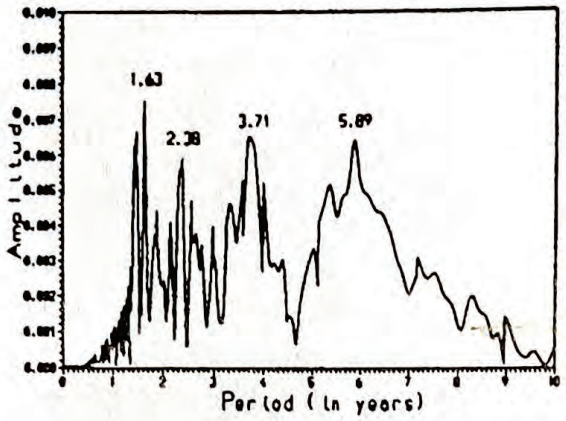


Fig. 6a -*Amplitude periodogram (DFT) of Earth's pole y - coordinate*

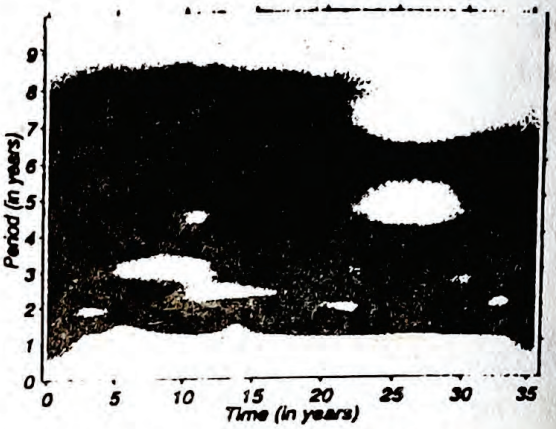


Fig. 6b -*Spectrogram (WT) of Earth's pole y - coordinate*

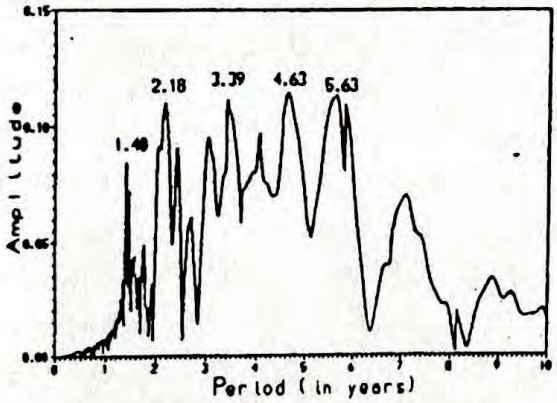


Fig. 7a -*Amplitude periodogram (DFT) of effective atmospheric angular momentum function χ_1 for wind + pressure*

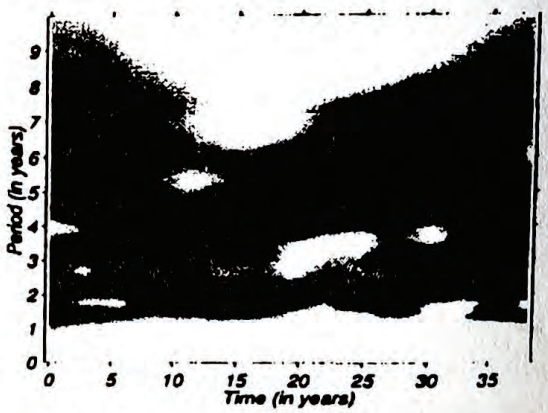


Fig. 7b -*Spectrogram (WT) of effective atmospheric angular momentum function χ_1 for wind + pressure*

5. Conclusions

In this paper, only the continuos wavelet transforms (the transforms that make use of a wavelet function) have been discussed. For a discussion and examples of the discrete wavelet transforms (the transforms that make use of a set of wavelet coefficients), see [5]. As it is shown above, the main advantage of the wavelet transforms is that their windows vary, which is not the case in Fourier analysis. Thus, wavelet analysis can provide an information about time series that can stay hidden with other methods of spectral analysis such as Fourier transforms. The main disadvantage is related to the fact that the wavelet transforms do not have a unique set of basis functions like the Fourier transforms. Even more, some of the wavelet families, like Morlet, depend on parameters whose choice can have a great influence on selectivity of the wavelet transforms and statistical significance of resulting spectra. The last fact is illustrated in this paper on a model data time series. It is shown that selectivity of the wavelet transforms increases as the parameter of Morlet wavelet increases, but quite opposite conclusion can be derived for the resulting spectra significance, which is more important. Thus, the most appropriate values of Morlet wavelet parameter must be relatively modest and vary between $p \approx 6$ and $p \approx 8$.

Nowadays, the wavelet theory is in the refinement stage and it is not completed, yet. This refinement stage includes generalizations and extensions of the theory. Nevertheless, wavelet analysis became a common tool for investigation of localized variations within time series.

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20 CCD Technique for longitude/latitude astronomy

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The authors report about CCD (Charge Coupled Device) experiments with the instruments of astrometry and geodesy for the longitude and latitude determinations. At the Techn. University Vienna (TU Vienna), a mobile zenith camera "G1" was developed, based on CCD MX916 (Starlight Xpress) and F=20 cm photo optic. With Hipparcos/Tycho Catalogue the first results show accuracy up to 0.5″ for latitude /longitude. The PC - guided observations can be completed within 10 minutes. The camera G1 is used for astrogeodesy (geoid, Earth's crust, etc.). At the Belgrade Astronomical Observatory (AOB), the accuracy of (mean value of) latitude/longitude determinations can be a few 0.01″ using zenith stars, Tycho - 2 Catalogue and a ST -8 of SBIG (Santa Barbara Instrument Group) with zenith-telescope BLZ (D=ll em, F=128.7 cm). The same equipment with PIP instrument (D=20 em and F=457.7 cm, Punta Indio PZT, near La Plata) yields a little better accuracy than the BLZ's one. Both instruments, BLZ and PIP, were in the list of Bureau International de l'Heure - BIH. The mentioned instruments have acquired good possibilities for semi or full-automatic observations.

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CCD TECHNIQUE FOR LONGITUDE/LATITUDE ASTRONOMY

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Abstract. We report about CCD (Charge Coupled Device) experiments with the instruments of astrometry and geodesy for the longitude and latitude determinations. At the Techn. University Vienna (TU Vienna), a mobile zenith camera "G1" was developed, based on CCD MX916 (Starlight Xpress) and F=20 cm photo optic. With Hipparcos/Tycho Catalogue the first results show accuracy up to $0.''5$ for latitude /longitude. The PC - guided observations can be completed within 10 minutes. The camera G1 (near 4 kg) is used for astrogeodesy (geoid, Earth's crust, etc.). At the Belgrade Astronomical Observatory (AOB), the accuracy of (mean value of) latitude/ longitude determinations can be a few $0.''01$ using zenith stars, Tycho - 2 Catalogue and a ST-8 of SBIG (Santa Barbara Instrument Group) with zenith-telescope BLZ (D=11 cm, F=128.7 cm). The same equipment with PIP instrument (D=20 cm and F=457.7 cm, Punta Indio PZT, near La Plata) yields a little better accuracy than the BLZ's one. Both instruments, BLZ and PIP, were in the list of Bureau International de l'Heure - BIH. The mentioned instruments have acquired good possibilities for semi or full-automatic observations.

1. INTRODUCTION

Because of better possibilities (accuracy, etc.) of new technic (Lunar Laser Ranging - LLR, Satellite Laser Ranging - SLR, Very Long Baseline Interferometry - VLBI, Global Positioning System - GPS, etc.), after 1988 and the transformation of Bureau International de l'Heure - BIH into International Earth Rotation Service - IERS and Bureau International des Poids et Mesures - BIPM, the observations of latitude and universal time variations by using the classical astronomical instruments declined.

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During the last few years, the CCD technic with computer manipulation and calculation open new exerts and big influence on classical astrometry and geodetic instruments. It means that there are some new possibilities for classical astrometry.

At the AOB there is a zenith-telescope BLZ and at Punta Indio there is a photographic zenith tube (PZT) PIP. Also, at the AOB there are several very good CCD cameras. In this paper we paid attention to the SBIG ST-6 and ST-8. The ST-6 are with 375×242 pixels (1530×1020 for ST-8), 23×27 mikrons pixel size (9×9 for ST-8) and 8.6×6.5 mm array dimension (13.8×9.2 for ST-8). We made the investigations of the accuracy of longitude/latitude determinations by using CCD technic with BLZ and PIP instruments. The experience of AOB and TU Vienna, with CCD technic, was taken under our consideration.

By virtue of the same reasons, the astro-geodesy has got interesting new practicability with the CCDs (good accuracy of geoid determination via astro-geodesy longitude and latitude observations, etc.). At the TU Vienna there is the CCD MX916 with 752×580 pixels count, 11×12 mikrons pixel size and 8.7×6.5 mm active area. The G1 with CCD is under development at the TU Vienna for astro-geodesy objectives.

2. BLZ AND PIP INSTRUMENTS WITH CCD

The instruments presented here, were in the list of BIH: the Belgrade zenith-telescope denoted BLZ (D=11cm, F=128.7cm) and the Punta Indio photographic zenith tube denoted PIP (D=20cm, F=457.7cm).

They participated with their observations (BLZ interval was 1949.0-1986.0 and PIP one 1971.6-1984.5) and took a part in "Earth orientation parameters 1899.7-1992.0 in the ICRS based on the HIPPARCOS reference frame" (Vondrak et al. 1998) with very good weights (among 48 instruments of different types): for BLZ the weight of one observation in latitude was 0.93, and for PIP it was 2.07 and in time 1.96 .

Before the real observations, it is necessary to find a good focus (for each CCD plus optic) and optimal exposure time which depends on observed objects and scientific objectives (Martinez and Klotz 1998).

From the catalogue, we need to take astrometrically "good" stars (to find the min. and max. values of star's magnitude for our CCD and optics, to remove double stars which are not good for precise measurements, to remove the stars with bad position accuracy or without proper motion data, etc.). Nowadays, the Tycho-2 Catalogue (Hog et al. 2000), at ICRS reference system, is very useful for astrometrical observations. The positions are precise to near 60 mas (for all stars), the proper motions are precise to about 2.5 mas/yr, and about 100 000 stars (only 4%) are without proper motion data. The star density is about 150 stars/sq.deg. for $b = 0^o$ (50 stars/sq.deg. for $b = \pm 30^o$, and 25 stars/sq.deg. for $b = \pm 90^o$) and V is near 11.5 mag (with 90% completeness).

During the observations, the computer - CCD connection is of great importance. Also, each astronomical or other instrument needs a good adapter (the connection for mounting CCD on the telescope), and can be the instrument's shutter for better exposure time determination (CCD plus computer exposure time can be with some systematic error). Also, the computer time can be replaced with input time (more precise) for precise time measurement (if it is necessary for reduction).

If we use the CCD with the BLZ's tube in the zenith direction and PZT procedure of observation (four exposures of the same zenith-stars with 180 deg. turns of BLZ,

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between exposures, around its vertical axis) it is possible to determine both coordinates of observed place (longitude and latitude). For PIP with CCD, the process of measurement is like that with the PZT (as it was with photo plate in the past), and it is necessary just to discard the observations by photo plate and to adopt CCD. Both instruments (BLZ and PIP) can be semi-automated or full-automated.

The sky field of BLZ with CCD ST-8 (1."44/px for both, x and y, axis) can be about 37'×25' (about 10'×7' for PIP) and by using Tycho-2 Catalogue we can catch about 20 stars (a few stars for PIP) which is enough to reach the longitude/latitude accuracy of about a few 0."01 (PIP can achieve nearly double accuracy than BLZ because of better optic). We made experiments during 2001 at IGG TU Vienna by using G1 and CCD MX916 (see below, the two CCD figures - observations that can reach the latitude accuracy of 0."89) and if we compare F of BLZ with one of G1 (128.7cm/20cm) or F of PIP with that of G1 (457.7cm/20cm) the longitude/latitude accuracy can be better by using BLZ or PIP with CCD ST-8; also, the ST-8 achieves higher quality for astrometry than the MX916 (the ST-8 has got 1530×1020 pixels, but the MX916 has got 752×580 ones). Another important note: the BLZ levels (for instrument orientation into the zenith direction) are nearly 1" per division (pars), but the G1 ones are more than ten times worse. The standard error of unit weight (for BLZ and PIP longitude/latitude determination) was about 0."15 by using visual (for BLZ) and photo plate (for PIP) observations (Vondrak et al. 1998); the error of the mean value was several times better because of the observations of more stars (group of stars) during about one hour. In the case of BLZ, the error of the mean value reached 0."05. This means, our predicted accuracy of the mean value (of longitude/latitude), for BLZ and PIP with one CCD image procedure (with about 20 Tycho-2 stars for BLZ field and a few stars for PIP) is better than with the visual or photo plate. And the CCD observations can be completed in just a few minutes (see below, the two CCD figures - observations).

3. TRANSPORTABLE GEODETIC ZENITH CAMERA

In geodesy, the rapid determination of the direction of the vertical has 3 additional goals, too:

a) geoid determination by use of Vertical Deflections (VD) along profiles, where VD is the difference between real (astrometric) vertical and ellipsoidal vertical of a point,

b) accurate reduction of terrestrial networks to the reference ellipsoid,

c) inversion of VDs to determine density structures of the Earth's crust.

Geoid projects in flat countries mainly use gravimetry. Observation time is very short, but to get high geoid accuracy 200 - 500 points per 1000 km^2 are necessary, whereas astrogeodesy requires only 10 - 20 VD points (Gerstbach 1997). Therefore in alpine countries an astrogeoid is much more effective than gravimetry.

At the TU Vienna a special research project "CCD IN ASTRO-GEODESY" started in 1999, sponsored by Oesterreichische Nationalbank. The main goal is to develop automatic measurement methods, and if possible to speed them up. During the first 2 years CCD cameras of the photo market (Olympus 800) were used, mounted at the eye-piece of a Zeiss astrolabe (4/20 cm) and a Kern theodolite DKM3a (7/40 cm). The accuracy was ±1" as expected (Gerstbach 2000), but the measuring time not

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shorter than visual. Therefore we changed the project toward the development of a small zenith camera.

Currently we test an astro CCD camera Starlight MX 916. It has 752×580 pixels (with 11×12 mikrons and the sensor 8.7 × 6.5 mm) which can be binned (electronic mix of 2×2 pixels) to increase the sensitivity. A prototype of zenith camera is under construction with a photo tele objective 1:4 / 20 cm. In the vicinity of Vienna exposures of $5^s - 10^s$ show stars up to 12th magnitude.

Four CCD images with the G1 in the meridian direction were taken within a few minutes. The expositions are controlled by a notebook computer. The relation between rotating axis and vertical is measured by two spirit levels mounted on the camera turning plate. The usable field of view is about $1^o30^{'} \times 2^o$. According to first tests the images show 20 - 50 stars, but the faintest stars are not used for measurements. The internal accuracy is about ±1" with 10 Hipparcos (ESA 1997) stars and almost ±0."5 with 20 Tycho stars (see CCD figures below), but the systematic error can be bigger than 1" because of bad levels. Combining four images and with better quality levels we expect real accuracies of ±0."5 (for example, if we replace the present levels with better ones our accuracy is going to be better).

The presented CCD observations (Fig. 1. and Fig. 2.) are made for the illustration. They are made at IGG TU Vienna with G1 and CCD MX916. The date was October 31st, 2001, the temperature was about 16^oC, the air pressure was near 990 mb, $UTC = 17^h36^m1^s.28$ for the Fig. 1. ($UTC = 17^h36^m51^s.45$ for the Fig. 2.), the exposure times were 10^s (because of this, the visible stars appear not like a points but as short lines and the middle of each line is in accordance with the middle of exposure time), the time period between the exposures was about 50^s, the G1 was turned 180° around the vertical axis after each exposure. At IGG TU Vienna, there is a programme for reduction of CCD observations which we used for our investigations. The programme offers necessary options (open for the users) and can process the CCD figures automaticly. After the reduction of presented CCD observations (the Fig. 1. and Fig. 2.), the latitude accuracy was 0."89. During our astro-geodetic CCD observations and investigations at IGG TU Vienna, there were the CCD observations which gave us the results with better and worse accuracy than presented one, but up to 0."5 . The movements of the stars visible in Fig. 1. are up and in Fig. 2. are down (in line with the parallels on the sky), and because of the turning G1 around the vertical axis by 180° between the exposures (the pause between the observations is 50^s) it is possible to see in Fig. 2. very big number of the stars presented in Fig. 1., but just on the other side of the Figure. If some star is visible in Fig. 1. on left side of the figure and down, we can see it in Fig. 2. on the right side and up. For latitude measurements, we used the stars presented on both CCD figures, but at the first figures before the local meridian and at the second one after it. For both coordinates, longitude and latitude measurements, we need four CCD figures, two before and two after the meridian, as it is for the PZT instrument.

4. CONCLUSION

Following the experiences of AOB (with CCD SBIG ST-6/ST-8), and the observations/results and investigations made at IGG TU Vienna (the project "CCD in der Geodatischen Astronomie") by using the G1 with CCD MX916 and the Hipparcos/Tycho Catalogues we can conclude: the level of accuracy of the mean value of

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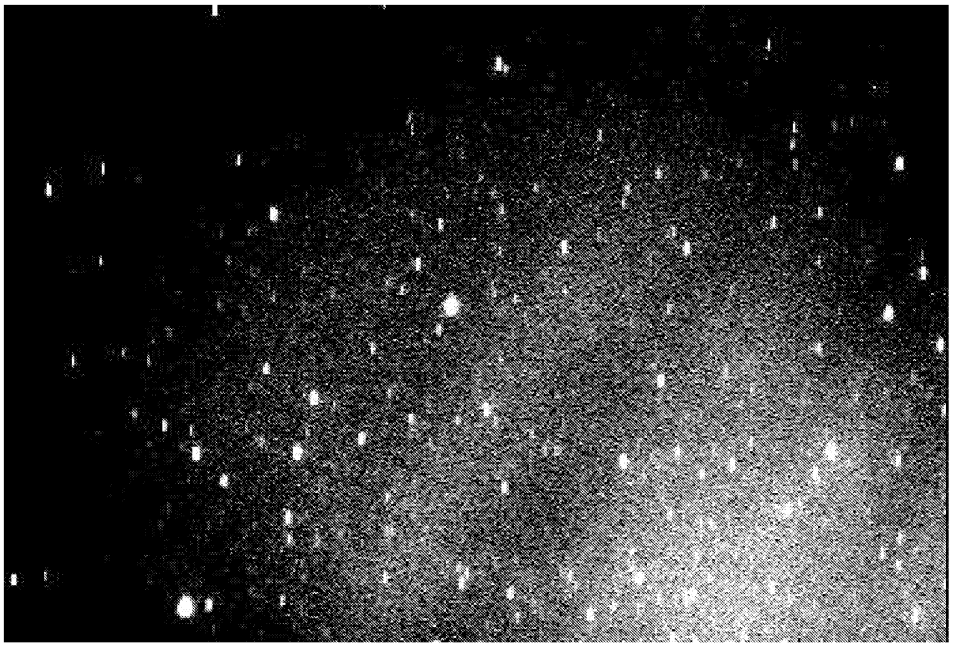

Figure 1: The observations of G1 with CCD MX916.

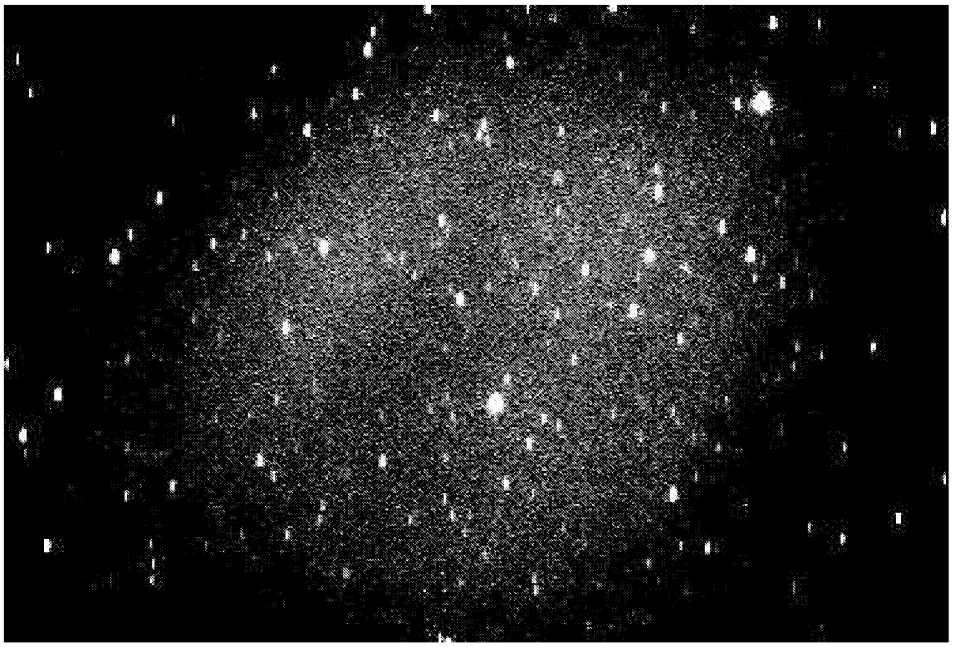

Figure 2: The observations of G1 with CCD MX916.

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longitude/latitude determination by using CCD ST-8 with BLZ/PIP and Tycho-2 can be a few 0.”01 (PIP is with better optic and it can yield us nearly double better accuracy than BLZ one), for G1 (with CCD MX916, Tycho-2 and better levels) the accuracy for astro-geodesy is going to reach 0.”5, the ST-8 has got better resolution than MX916 and can yield us better accuracy, it is possible to adapt BLZ/PIP for CCD technic (also, to adapt BLZ to operate as PZT) and to make semi-automatic or full-automatic observations. Also, BLZ and PIP with CCD can be useful for another kind of observations (to determine the coordinates of the stars, their proper motions, etc.).

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21 Improved proper motions in Declination of some Hipparcos stars via long-term classical observations

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The authors present the new possibilities of classical ground - based observations for the purpose of improving the proper motions of some HIPPARCOS stars. The coordinates of asteroids, comets and other objects are calculated mostly by using the relative method and presented in a relevant reference frame via nearby stars (at the moment of observation) which materialize that reference frame. To improve the proper motions of stars we can use the latitude/ universal time variations of ground - based data. Here, the results of improved proper motions in declination of some HIPPARCOS stars are presented.

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Improved proper motions in declination of some Hipparcos stars via long–term classical observations

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Abstract. In this paper we present the new possibilities of classical ground – based observations for the purpose of improving the proper motions of some HIPPARCOS stars. The coordinates of asteroids, comets and other objects are calculated mostly by using the relative method and presented in a relevant reference frame via nearby stars (at the moment of observation) which materialize that reference frame. To improve the proper motions of stars we can use the latitude/universal time variations of ground – based data. Here, the results of improved proper motions in declination of some HIPPARCOS stars are presented.

Keywords. Stellar catalogues, proper motion

1. Introduction

Two catalogues appear as products of ESA mission, the Hipparcos one and the Tycho one. The former is the primary representation of the ICRS (International Celestial Reference System) in the optical wavelengths (ESA 1997). It contains 118218 stars with precise parallaxes, positions (better than 1 mas), proper motions and photometrical data. The median precision of positions for 1058332 Tycho Catalogue stars is 25 mas. The coordinates of the other objects, (asteroids, comets, etc.) calculated by applying the relative method and using the positions of nearby stars (at the moment of observation) are presented in a catalogue which materializes the reference frame. Thus the quality of the coordinates of the observed solar system objects depends upon the accuracy of the star positions.

During the last years, problems were noted for the proper motion of some Hipparcos stars, because the mission lasted < 4 years and was too short to provide satsisfactory accuracy in proper motion. The proper motions of some Hipparcos stars have comparatively low accuracy, mostly double and multiple stars (Vondrák & Ron 2004).

On the other hand, we can see that ground–based observations with a long history have a good potential to materialize a better reference frame, more stable in time. During last few years some catalogues including ground–based data were made : TYCHO-2, ACT, FK6(I), FK6(III), GC+HIP, TYC2+HIP , ARIHIP and the most recent Earth Orientation Catalogue – EOC (Vondrák & Ron 2003).

There is a large data set of ground–based measurements of latitude/universal time variations. The Hipparcos Catalogue comprises several thousand stars which were observed from the ground for few decades and for which a large number of observations is available (Vondrák *et al.* 1998). We used some of these data to improve the proper

motions in declination for some Hipparcos stars. Our results give a contribution to the investigations of possible improvement of the reference frame.

2. Data

In this paper we present our computational procedure by using the observations of several Photographic Zenith Tubes (PZT):

- PIP data, Punta Indio PZT (Argentina), observation time span 1971.6 – 1984.5,
- MS ones, Mount Stromlo PZT (Australia), 1957.8 – 1985.7,
- OJP ones, Ondřejov PZT (Czech Republic), 1973.1 – 1992.0.

We received the data from J. Vondrák, and decided to apply our procedure star by star, because the data contain star by star latitude observations (the data of each star are independent from each other). The data are referred to the Hipparcos Catalogue, corrected for instrumental constants (micrometer screw, plate scale, etc.), refraction, and errors in proper motions of some Hipparcos stars (but this correction is available for only about 20 % of the stars).

The precession–nutation model IAU 1980 and the MERIT Standards were used to compute the apparent places of the stars. The mean latitudes, corrections of conventional longitudes and tectonic plate motions (from the NUVEL–1 NNR geophysical model) were subtracted from the results in order to bring them all to a homogeneous system.

For each star we prepared a separate file with all observations made with PIP, MS or OJP. There are 165 Hipparcos stars observed with PIP instrument, 184 with MS, and 285 with OJP (157 stars are common to both PIP and MS). We used the polar motion coordinates from the file EOPOA00.dat by Vondrák.

3. Results

The latitude data we received mostly contain time variations due to polar motion changes with time, but we want to investigate the proper motion. Thus, at first it was necessary to calculate and remove the polar motion component.

We took the coordinates of the polar motion (x and y) from the file EOPOA00.dat and adapted x and y by linear interpolation to the moment of interest. Also, we repeated that calculation for each of the mentioned instruments, and removed the calculated polar motion part from the received latitude data for each star. For this purpose, we used the Kostinski's formula (Kulikov 1962) where λ (longitude of the instrument) is calculated from the zero–meridian to the west. The values of λ and φ (latitude) for mentioned stations are from (Vondrák *et al.* 1998).

After removing the polar motion part, for each star we are left with residuals (polar motion value minus mean value of n latitudes of some Hipparcos star observed during the subperiod, approximately one year long). Thus for each star we have got nearly one data point (residual) per year. Each data point contains: MJD, residual, standard error, and number of latitudes n (used in the mean value calculation of the mean value). A few examples are presented in the figures: in Fig. 1 we show the residuals as a function of time (MJD) of the common PIP and MS Hipparcos star H68419 (MS is the longer curve); Fig. 2 shows the same for OJP H80585. The curves are almost straight lines; thus the residuals are well prepared. The calculation of the free term a, of the linear one b and of its standard errors was made for each star by using the Lest-Squares Method (LSM) and linear model

$$Y(i) = a + b * (X(i) - 1991.25),$$

where: $X(i)$ is time in years, $Y(i)$ – mentioned residuals, i is the number of residuals (points). The linear term b is our correction of μ_δ (proper motion in declination).

For the cases presented in the figures, the results are (LSM, linear model, no weights):

- PIP, H68419 , $a = -0.''071 \pm 0.''043$, $b = -0.''0021/year \pm 0.''0032/year$,
- MS, H68419 , $a = -0.''103 \pm 0.''028$, $b = -0.''0042/year \pm 0.''0013/year$,
- OJP, H80585 , $a = -0.''411 \pm 0.''013$, $b = -0.''0103/year \pm 0.''0013/year$.

The results for the star H68419, as obtained from the PIP and from the MS data set, are in good agreement (thus our procedure is correct), but the values for the star H80585 (OJP data) are too high and questionable (the same occurs for the other OJP stars).

To explain the results for the star H80585 it was necessary to identify the trends of all PIP, MS and OJP data. We determined the values of a and b (LSM, the same model, no weights), but for the points of all PIP stars (similar for MS and OJP):

- PIP, $a = -0.''069 \pm 0.''005$, $b = -0.''0018/year \pm 0.''0004/year$,
- MS, $a = -0.''019 \pm 0.''003$, $b = -0.''0008/year \pm 0.''0002/year$,
- OJP, $a = -0.''426 \pm 0.''002$, $b = -0.''0118/year \pm 0.''0002/year$.

Evidently, the results concerning PIP and MS are still in a good agreement (the values of b are small for both instruments), but the results for OJP show the reason why the values for a and b are so large in case of Hipparcos star H80585. Similar results are obtained for most of the other PIP, MS and OJP stars. This means that these data still contain some systematic errors which we need to determine and remove before our final calculations, but the procedure of calculation is correct, which was our purpose to show here. Also, after finishing the calculations for the other instruments (here, we present only three of them) we are going to send the results to Dr. Vondrák, and expect new Vondrák's solutions of the Earth Orientation Parameters (EOP) and full comparison of our results, in line with the long–term Earth rotation studies (Vondrák & Ron 2004). At present, we can conclude that our calculated correction is justified if our procedure for the same star observed at different sites (see Fig. 1) gives us similar results. It means that the procedure can extract the effect of proper motion affecting latitude changes with time.

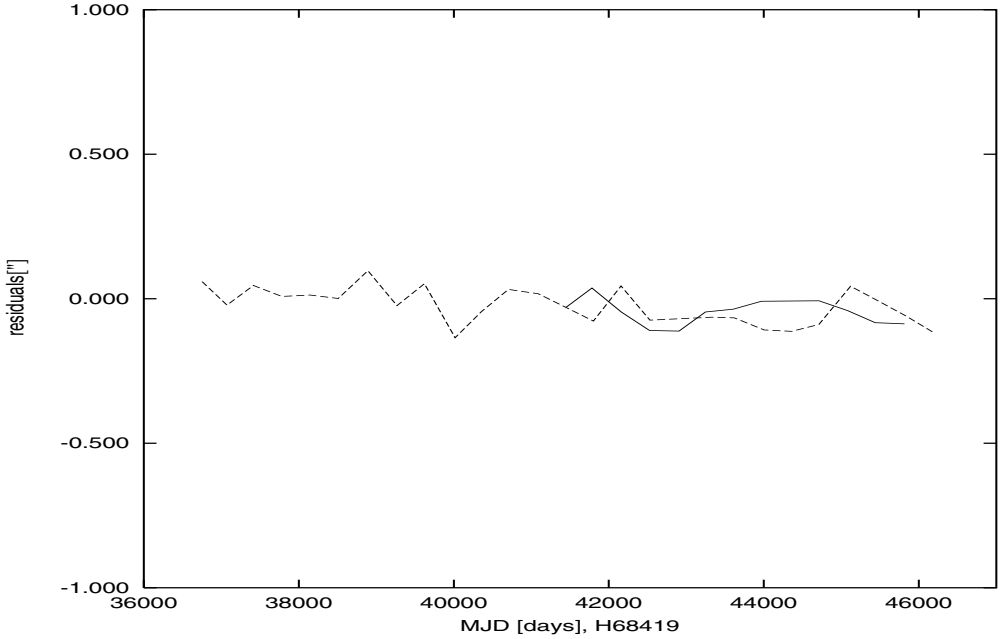

Figure 1. The residuals as a function of time (MJD) of the common PIP (shorter curve) and MS (longer one) Hipparcos star H68419.

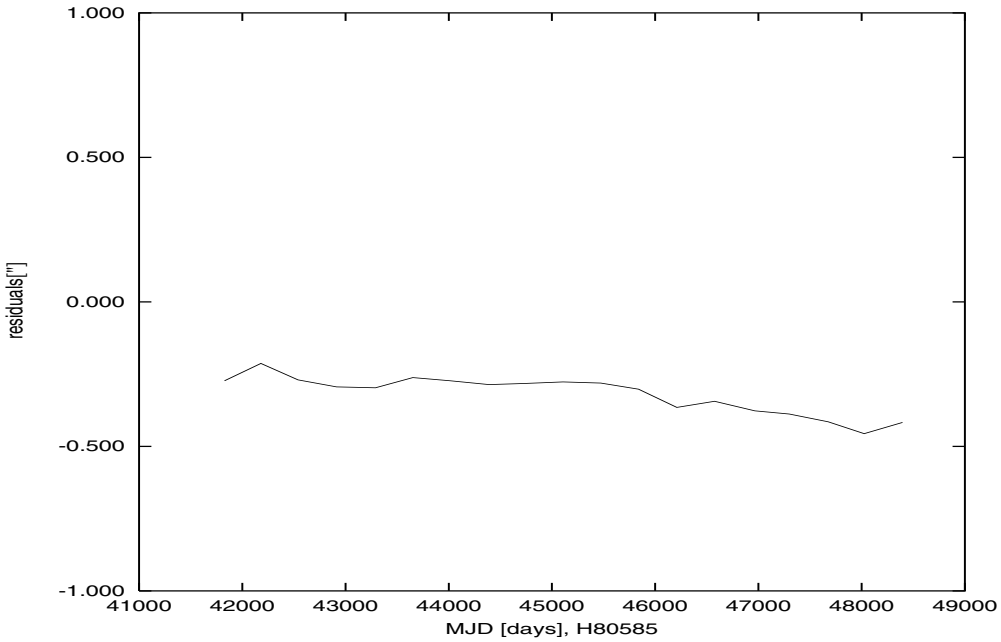

Figure 2. The residuals with MJD of OJP Hipparcos star H80585.

It is necessary to be careful with PZT latitude data because of their level of formal errors (much larger than the Hipparcos ones) and of different kinds of systematic ones, but these data with a long history still can give us useful informations. One possibility is to improve proper motion for some Hipparcos stars as shown here.

4. Conclusions

We used the latitude data of PIP, MS, and OJP PZTs to improve the proper motions in declination for some Hipparcos stars. The main features of the data and our calculation procedure are explained. A few examples (see the Fig. 1 and Fig. 2) are given. This kind of investigation is the way, step by step, to obtain more accurate data than the Hipparcos ones, in a long–term sense, and the combination of ground – based optical observations with space ones can give us a better reference frame.

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22 Improvement of Hipparcos proper motions in declination

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The first astronomical satellite mission HIPPARCOS ESA mission (ESA 1997) was less than 4 years long so that 1991.25 is the epoch of the HIPPARCOS Catalogue. Several projects have checked or improved HIPPARCOS data. Also, a long series of ground - based optical observations of some stars included in HIPPARCOS Catalogue, made with Photographic Zenith Tubes (PZT) are useful for the task of improving the proper motions of these stars. The ARIHIP Catalogue (after ACT, TYCHO-2, FK6, GC+HIP, TYC2+HIP) is a combination of the HIPPARCOS and some ground-based data, and the ARIHIP proper motions are more accurate than the HIPPARCOS ones. The authors presented a new step of their procedure of calculation. Between PZT data they added the HIPPARCOS position with suitable weight - the point with the coordinates (1991.25, $0.0''$). The method was applied to 202 stars observed at Richmond PZTs in the course of a few decades. The result is better proper motions in declination for these HIPPARCOS stars, and a good agreement with ARIHIP proper motions. The authors found 128 common Richmond and ARIHIP stars to check their result. Also, they presented the result for other 74 Richmond stars which are not found in ARIHIP.

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IMPROVEMENT OF HIPPARCOS PROPER MOTIONS IN DECLINATION

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SUMMARY: More than a decade elapsed after the HIPPARCOS ESA mission (ESA 1997) observations have been collected. This first astronomical satellite mission was less than 4 years long so that 1991.25 is the epoch of the HIPPARCOS Catalogue. Many other projects have checked or improved HIPPARCOS data. Also, a long series of ground – based optical observations of some stars included in HIPPARCOS Catalogue, made with Photographic Zenith Tubes (PZT) are useful for the task of improving the proper motions of these stars. The ARIHIP Catalogue (after ACT, TYCHO – 2, FK6, GC+HIP, TYC2+HIP) is a combination of the HIPPARCOS and some ground – based data, and the ARIHIP proper motions are more accurate than the HIPPARCOS ones. Here we present a new step of our procedure of calculation; between PZT data we added the HIPPARCOS position with suitable weight – the point with the coordinates (1991.25, $0\overset{''}{.}0$) in our case. The method was applied to 202 stars observed at Richmond PZTs in the course of a few decades. The result is better proper motions in declination for these HIPPARCOS stars, and a good agreement with ARIHIP proper motions (we found 128 common Richmond and ARIHIP stars to check our result). Also, we present the result for other 74 Richmond stars which are not found in ARIHIP.

Key words. Astrometry – Reference systems

1. INTRODUCTION

The ICRF (International Celestial Reference Frame) materializes the ICRS (International Celestial Reference System) from 1998 via a catalogue of 608 compact radio sources (Ma et al. 1998); the internal accuracy of these sources was between 0.3 and 0.5 mas (milliarcsecond). This list of radio sources has been updated by the ICRF – Ext.1 with 59 new ones (IERS Annual report 1999). The HIPPARCOS Catalogue (ESA 1997) was adopted as the optical counterpart of the ICRF.

The HIPPARCOS Catalogue offers important astrometric data (positions α and δ at 1991.25 – the epoch of the catalogue, proper motions μ_α and μ_δ, parallaxes) for 118218 stars brighter than magnitude 12. It was linked to ICRS with the precision of 0.6 mas in orientation and 0.25 mas/year in rotation (Kovalevsky et al. 1997). The period of HIPPARCOS satellite observations was shorter than four years which was not enough to obtain a satisfactory accuracy of proper motions for some stars. For the purpose of improving these HIPPARCOS data other catalogues have appeared, such as ACT, TYCHO – 2, FK6, GC+HIP, TYC2+HIP, ARIHIP (Wielen et al. 2001), Earth Orientation Catalogue – EOC (Vondrák and Ron 2003), etc.

ARIHIP contains 90842 stars and it represents a compilation of stars with the most accurate data

from the catalogues: FK6(I), FK6(III), GC+HIP, TYC2+HIP, and HIPPARCOS. Their proper motion data are more accurate than the HIPPARCOS ones, and because of this ARIHIP is interesting to us (to compare our results with ARIHIP proper motions in declination). Also, ARIHIP contains other data: for the proper motions and their mean errors there are SI (the single – star), LTP (the long – term prediction), STP (the short – term prediction) and HIP (HIPPARCOS Catalogue) mode (difference = other mode - SI mode), the coefficient K_{ae} with flag for 'astrometrically excellent stars' 3 – astrometrically excellent star with rank ***, 2 – with rank **, 1 – with rank * and blank – star not classified as astrometrically excellent. In ARIHIP, the SI mode is given and the mentioned differences have to be added to the values of the SI mode in order to obtain the values of the other modes.

The values of SI and other modes (in mas/year) refer to the epoch and equinox J2000.0 in the ICRS/HIPPARCOS system. The afore mentioned mean errors (in mas/year) are not the errors of the differences, but those of the full quantities in the other modes.

2. DATA

Even though the accuracy of HIPPARCOS positions is about 1 mas and of proper motions ($\mu_\alpha \cos\delta$, μ_δ) is nearly 1mas/year, the PZTs periods of observation are much longer than the HIPPARCOS one and can give better proper motion data than the HIPPARCOS ones. The PZTs were the part of the different Earth rotation programmes in the interval 1899.7 – 1992.0 (Vondrák et al. 1998).

We use the Richmond (Florida, USA) PZT data (Vondrák 2002). Two instruments were located at the longitude $\lambda_W = 80\overset{\circ}{.}4$ and the latitude $\varphi = +25\overset{\circ}{.}6$, and collected the data in the course of nearly 40 years (1949.8 – 1989.4): RCP operated in the period 1949.8 – 1987.5 and RCQ in 1981.9 – 1989.4 one (during a several years period observations were done simultaneously). The RCP and RCQ data were used for polar motion and universal time UT0 investigations (Vondrák 2004). The code RCP/RCQ is from the monograph (Vondrák et al. 1998).

As input data we use Vondrák's OA00 solution (Ron and Vondrák 2001). The performances of the instruments have been described by Vondrák et al. (1998). The latitude variations φ_i (obtained by RCP and RCQ) around the mean latitude are a part of the OA00 data. We use the RCP and RCQ data because of the long observational period, which provides a good possibility to check our results via the ARIHIP ones. The mean latitude and tectonic plate motion were removed from the RCP and RCQ latitude data for the period 33224 – 47660 MJD (1949.8 – 1989.4) by using $25°36'47\overset{''}{.}116 + 0\overset{''}{.}008/century \cdot (t - t_1)$, where $(t - t_1)$ is in centuries and t_1 is counted from 32000 MJD.

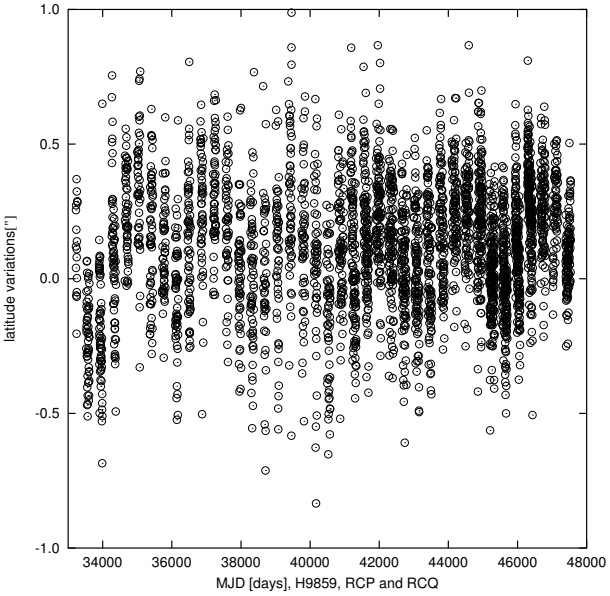

Fig. 1. *RCP and RCQ latitude variations φ_i with time (MJD) for the HIPPARCOS star H9859.*

3. CALCULATIONS

To show the input data, we present in Fig. 1 the latitude variations φ_i with time (MJD) for star H9859 observed for about 40 years with RCP and RCQ. Within each observational year, there are from only a few to a few hundreds of observations of the same star with a standard error of the order of a few tenths of an arcsecond. Also, in Fig. 1, a latitude variation of RCP and RCQ is because of the polar motion and other changes with time clearly seen.

The main steps of the calculation have been already described in Damljanović and Pejović (2005) and Damljanović and Vondrák (2005). Here, we have improved the calculation procedure and instead of the linear approximation for systematic changes with time (local, instrumental, etc.) we take into account more complex variations which are closer to the real situation; some results have been presented by Damljanović (2005). In this paper, we have included in the calculation (for each Richmond star) one new point, the HIPPARCOS position via latitude variation which is with the value $0\overset{''}{.}0$ (in our case here) for 1991.25 (the epoch of HIPPARCOS Catalogue). The weight of this point is 19, defined as the ratio between the HIPPARCOS position error (near 1 mas) and the mean value of errors of points r'_n (see below). The other points are with the weight of 1. With that additional point, we expect to get better results (especially for the stars observed for just a few years). This new point with a large weight is of importance for a better consistency of PZT and HIPPARCOS data, too.

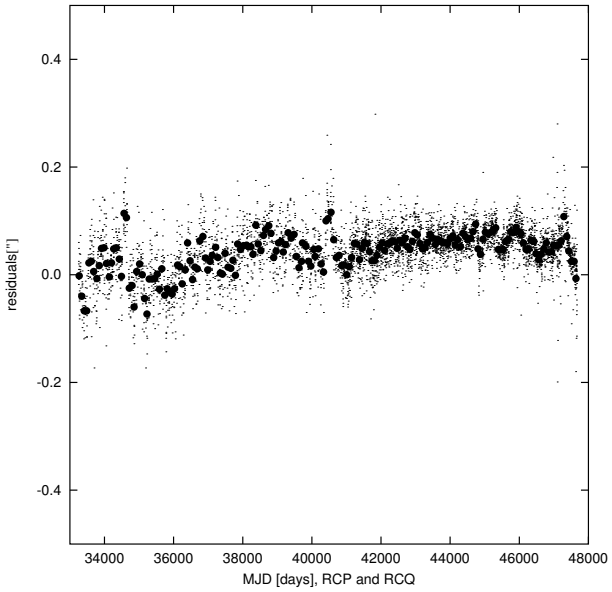

Fig. 2. *Residuals (RCP and RCQ) vs. time (MJD) before elimination of systematic changes: the gray dots r_n are the averaged values of r_i, the black circles are the averaged values of r_n within a* 0.2 *yr interval (for all 202 stars).*

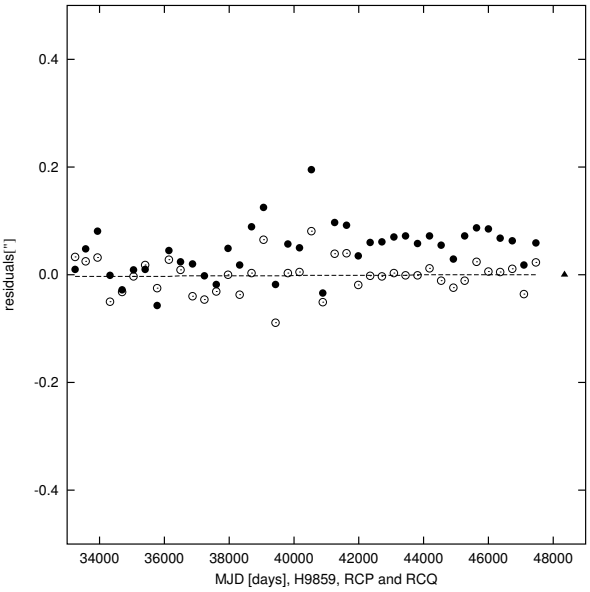

Fig. 4. *Averaged residuals $r_n^{'}$ (white circles, without systematic changes), r_n (black ones, with systematic changes included) vs. time for star H9859; two very similar linear trends of white circles are seen as a single line (one was calculated with and the other without a new point denoted with a triangle).*

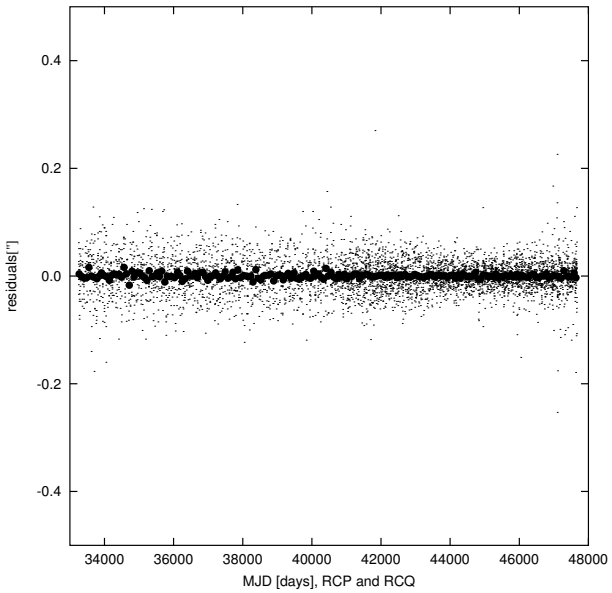

Fig. 3. *The same as in Fig. 2, but after elimination of systematic changes.*

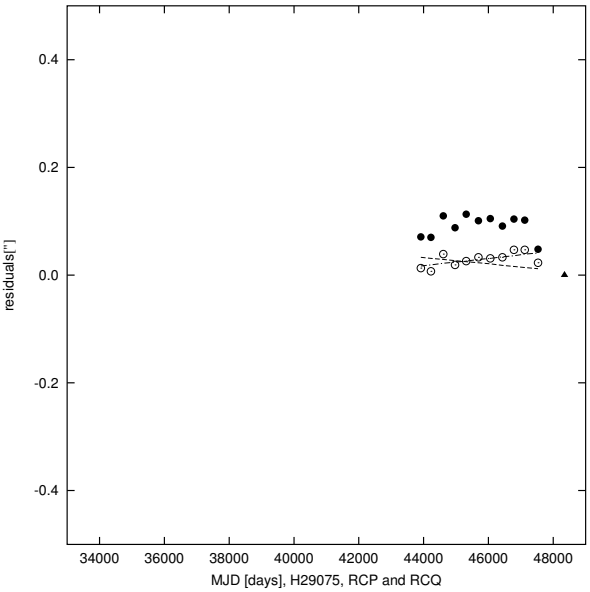

Fig. 5. *The same as in Fig. 4, but for H29075; two different linear trends, calculated with and without new point (triangle), are well separated, here.*

In the first step, the polar motion $\Delta\varphi_i$ was removed from the available φ_i data of the OA00 solution $r_i = -(\varphi_i - \Delta\varphi_i)$ to get the residuals r_i. This was described by Damljanović (2005). The $2.7 \cdot \sigma$ statistical criterion was used to remove the outlier values r_i. It is about one r_n value per year, and r_n are the averaged values of r_i. Then, the systematic changes (local, instrumental, etc.) were removed from the r_n values. These changes were determined by using the observations of all 202 stars.

The calculation procedure for determining and removing the values of the systematic changes was developed by Damljanović (2005). The dots (Fig. 2) are the values r_n which are the averaged values of r_i ones, and the black circles are the averaged values of r_n for all the 202 stars over the subperiod of 0.2 yr which is an optimal one for our case. On one side, it was necessary to find a subperiod, as short as possible, to get real systematic variations with time, and on the other side, with enough points (several tens) r_n in it (several tens of stars observed during this subperiod).

To show the efficiency of the determination of the systematic changes, we present in Fig. 3 the same points as in Fig. 2, but after the elimination of the previously systematic part. The proper motion corrections in declination are applied to each star individually. All steps of our method are valid for any other calculation (not only RCP and RCQ instruments). This is of importance because we want to apply the method described here to other input data.

We use the averaged values over subperiods of 0.2 yr (black circles in Fig. 2 – the systematic variations), and for each star, we remove the systematic changes from r_i by using r'_i. Then, we determine the values of r'_n from the r'_i ones, similarly as the r_n ones were determined from r_i ones; see in Fig. 4 the values r'_n (white circles) for star H9859 as an example.

The difference between the values with (the black circles – r_n values in Fig. 4) and without (the white circles – r'_n ones) systematic changes is evident. Our final residuals, RCP and RCQ data without systematic and polar motion variations, are the values r'_n. For each star, we use r'_n in the calculation of the corrections of the proper motions in declination of the observed HIPPARCOS stars.

4. RESULTS

We applied the Least Squares Method (LSM) to r'_n values for each star (the numerical values of white points, see in Fig. 4), and found that the straight line fits the data. Our corrections of proper motions in declination of the mentioned 202 HIPPARCOS stars are the b values calculated by using LSM according to the equation

$$r'_n = a + b \cdot (t_n - 1991.25), \qquad (1)$$

where t_n is the epoch instant of r'_n. The results for all 202 RCP and RCQ stars are presented in Table 2.

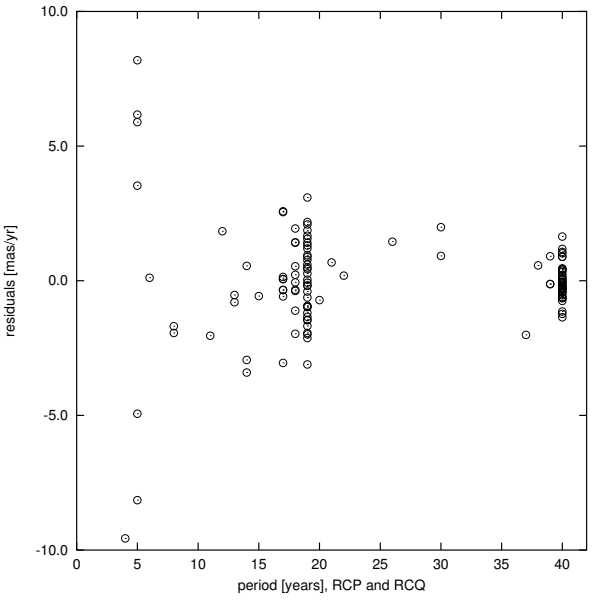

Fig. 6. *Differences r_{HIP} between our and ARIHIP SI mode results (for 125 HIPPARCOS stars observed with Richmond PZTs) vs. observational period n.*

Table 1. Already corrected HIPPARCOS stars.

HIP	bias (mas)	drift (mas/yr)
10053	-23	-3.07
45278	88	0.00
70310	0	2.01
73768	0	-4.21
75350	0	-2.35
81781	-44	-4.07
84821	0	3.76
89847	0	-36.82
97077	0	-6.20
100088	0	-2.52
106872	-81	-12.05

The order of presented stars is in line with the HIPPARCOS one (HIP is the HIPPARCOS number of star, the first column of Table 2), μ_δ is our proper motion in declination (the HIPPARCOS one plus our correction b), ε_b is the standard error of b (and of μ_δ). The value n is the number of input PZT points r'_n (close to the number of years covered by the PZT observations of every star). The LTP, STP and HIP are the ARIHIP modes (see Section 1) with errors (ε_{LT}, ε_{ST} and ε_{HI}, respectively). We put the values of ε_{HI} for all 202 stars from HIPPARCOS Catalogue for the purpose of comparison. The K_{ae} is ARIHIP value. The r_{LTP}, r_{STP} and r_{HIP} are the residuals between our value b and suitable ARIHIP modes (we adapted the differences LTP, STP and HIP in the way that they can be added to the HIPPARCOS

Table 2. The values μ_δ and $b \pm \varepsilon_b$, $LTP \pm \varepsilon_{LT}$, $STP \pm \varepsilon_{\rm ST}$, $HIP \pm \varepsilon_{HI}$ modes, K_{ae}, residuals, and comment C.

HIP	μ_δ ($\frac{\rm mas}{\rm yr}$)	b ($\frac{\rm mas}{\rm yr}$)	ε_b	n (yr)	LTP ($\frac{\rm mas}{\rm yr}$)	ε_{LT}	STP ($\frac{\rm mas}{\rm yr}$)	ε_{ST}	HIP ($\frac{\rm mas}{\rm yr}$)	ε_{HI}	r_{LTP} ($\frac{\rm mas}{\rm yr}$)	r_{STP} ($\frac{\rm mas}{\rm yr}$)	r_{HIP} ($\frac{\rm mas}{\rm yr}$)	K_{ae}	C
42	-6.79	-.28	.80	17	.08	.95	-.04	.55	-.07	.56	-.43	-.31	-.35	2	
2152	-24.39	2.64	.50	17	.04	.80	-.04	.63	-.06	.66	2.54	2.62	2.58	2	
3713	6.16	.34	.51	17	.85	.89	-.14	.56	-.28	.57	-.79	.20	.06	2	
4083	7.56	.57	.44	19	-.29	.93	.08	.49	.20	.50	1.06	.69	.77	1	
4262	-36.67	.75	.29	19	-.10	.77	.31	.56	.66	.63	1.51	1.10	1.41	3	
5777	-60.27	.03	.69	13	1.16	.93	-.40	.60	-.56	.63	-1.69	-.13	-.53		
6830	-9.52	.00	.74	17	-.18	.91	.07	.62	.14	.65	.32	.07	.14	2	
7034	56.56	.28	.31	40	-.90	1.02	.08	.52	.09	.52	1.27	.29	.37	3	
7874	-41.33	-.70	.25	40						.61					
8151	-48.58	-.10	.40	19	.32	1.02	-.06	.64	-.07	.65	-.49	-.11	-.17	2	
8285	-112.35	1.56	.31	39	2.25	1.02	-.56	.66	-.65	.68	-1.34	1.47	.91		
8600	-10.52	.16	.24	40						1.20					
8857	-42.09	-.47	.21	40	-.35	1.00	-.03	.74	-.07	.77	-.19	-.51	-.54	3	
9669	-6.39	1.35	.34	30	.75	.98	-.19	.59	-.43	.63	.17	1.11	.92	3	
9859	-14.45	.09	.30	40						.66					
10378	-67.05	-1.29	.48	17	-.37	1.04	.53	.84	.71	.86	-.21	-1.11	-.58	2	
10540	-66.22	.16	.31	40	.32	.90	-.17	.70	-.20	.72	-.36	.13	-.04	1	
10845	-8.11	-2.09	.34	19	-.40	.85	.43	.93	.75	1.07	-.94	-1.77	-1.34	2	
11265	3.72	.89	.47	19						.92					
11405	-38.39	-.32	.28	40	-.09	1.42	.01	.66	.03	.68	-.20	-.30	-.29		
12524	-5.04	.54	.48	21	-.02	.91	.06	.68	.14	.71	.70	.62	.68		
15273	-30.43	-1.85	.49	17	-.59	1.20	.83	1.23	1.51	1.40	.25	-1.17	-.34	2	
15557	-91.22	2.10	.57	19	1.78	.85	-2.09	.81	-3.08	.88	-2.76	1.11	-.98		
17812	-5.89	-.77	.25	40	.30	.86	-.25	.84	-.59	.98	-1.66	-1.11	-1.36	3	
18194	-23.02	-.94	.55	20	-.39	1.07	.15	.84	.22	.86	-.33	-.87	-.72	2	
18201	-34.97	-.75	.20	40	-.67	1.01	.31	.70	.56	.74	.48	-.50	-.19		
19182	.17	-.08	.15	40	.30	1.07	-.51	.93	-1.15	1.05	-1.53	-.72	-1.23	2	
19341	-2.51	-1.18	.70	19	.19	.88	-.21	.81	-.76	.95	-2.13	-1.73	-1.94	2	
20258	-9.22	.29	.22	40						.86					
20430	-18.32	-.44	.23	40						.81					
20626	-24.07	2.15	.23	40	.26	.78	-.40	.89	-.51	.94	1.38	2.04	1.64	2	
21532	-30.09	-1.71	.20	40	-.53	.74	.39	.69	.57	.75	-.61	-1.53	-1.14	2	
26225	-9.80	1.12	.49	19	-.32	1.08	.25	.69	.75	.73	2.19	1.62	1.87		
26818	-4.29	-.04	.21	40	-.44	.84	.01	.77	-.07	.90	.33	-.12	-.11		
26998	-1.66	1.25	.24	30	-.99	.88	.36	.66	.74	.72	2.98	1.63	1.99	3	
27353	-7.55	-.44	.27	40	-.54	.74	.25	.67	.49	.76	.59	-.20	.05	3	
27850	-2.33	-1.10	.44	13	-.37	.86	.12	.68	.30	.74	-.43	-.92	-.80	2	
27858	-2.95	-.04	.51	19						.68					
28301	-25.60	.28	.27	40	.47	.83	-.25	.49	-.35	.50	-.54	.18	-.07		
29075	-5.34	-2.52	.93	11	-.08	.94	.07	.93	.48	1.18	-1.96	-2.11	-2.04	2	
30142	-46.10	1.84	.45	19	-.16	.72	.09	.61	.25	.67	2.25	2.00	2.09	3	
30452	-1.27	-.13	.71	19	.00	.76	.00	.76	-.83	.88	-.96	-.96	-.96	2	

Table 2. (continued)

HIP	μ_δ ($\frac{\text{mas}}{\text{yr}}$)	b ($\frac{\text{mas}}{\text{yr}}$)	ε_b	n (yr)	LTP ($\frac{\text{mas}}{\text{yr}}$)	ε_{LT}	STP ($\frac{\text{mas}}{\text{yr}}$)	ε_{ST}	HIP ($\frac{\text{mas}}{\text{yr}}$)	ε_{HI}	r_{LTP} ($\frac{\text{mas}}{\text{yr}}$)	r_{STP} ($\frac{\text{mas}}{\text{yr}}$)	r_{HIP} ($\frac{\text{mas}}{\text{yr}}$)	K_{ae}	C
30622	-3.99	-1.17	.25	40	.00	.74	.00	.74	.42	.88	-.75	-.75	-.75	2	
30857	-2.33	.06	.28	39	.03	.75	-.01	.72	-.19	.83	-.16	-.12	-.13	2	
32086	-11.47	2.44	.90	11						.73					
32743	-36.74	-1.13	.34	31						.90					
32964	8.28	1.40	.22	40	1.41	.79	-1.18	.72	-1.73	.78	-1.74	.85	-.33		
33027	-8.96	.04	.58	19	-.45	1.17	.15	.90	.23	.93	.72	.12	.27	2	
33277	24.26	-.88	.22	40	-.56	.68	.64	.58	.74	.59	.42	-.78	-.14		
33855	-1.93	.38	.66	19	-.10	.95	.08	.71	.22	.77	.70	.52	.60	2	
33893	1.09	-.74	.28	40	.37	.90	.07	.51	.10	.52	-1.01	-.71	-.64	3	
34567	-175.78	-.73	.24	40						.67					
34582	-23.53	.52	.57	17						.69					
34861	-3.38	-.94	.30	40	-.16	.75	.19	.79	.31	.85	-.47	-.82	-.63	3	
37704	-22.68	-1.47	.48	19	.54	.65	-.17	.46	-.21	.48	-2.22	-1.51	-1.68	3	
39191	7.41	.92	.32	40	-.08	.38	-.22	.47	-.46	.53	.54	.68	.46		
40023	-347.69	.37	.22	40						.43					
41791	-24.00	.80	.64	14	.79	1.07	-.17	.72	-.25	.73	-.24	.72	.55	2	
42035	-18.00	1.73	.46	24						.81					
44089	-140.05	1.72	.48	19	1.69	1.15	-1.46	1.05	-1.91	1.09	-1.88	1.27	-.19		
44806	-7.52	.57	.44	19	1.68	1.20	-.39	.75	-.64	.79	-1.75	.32	-.07		
47382	16.24	-.05	.23	40	-.93	.99	.18	.53	.26	.55	1.14	.03	.21		
47602	-5.52	-.78	.46	19	-.49	.77	.11	.57	.16	.60	-.13	-.73	-.62		
48511	-42.29	-2.08	.43	19	-.48	.95	.06	.47	.09	.48	-1.51	-2.05	-1.99	3	
50321	-23.26	-1.39	.65	17						.69					
50336	2.78	-.40	.25	40	.42	1.07	-.03	.48	-.05	.49	-.87	-.42	-.45	3	
50915	-56.30	-1.44	.44	19	-.33	.97	.00	.46	-.01	.46	-1.12	-1.45	-1.45	3	
52270	3.71	.60	.47	19	.45	.90	-.08	.57	-.17	.59	-.02	.51	.43	2	
52998	-84.21	-.16	.44	15	1.63	1.15	-.29	.72	-.41	.73	-2.20	-.28	-.57	2	
53355	.93	1.35	.44	19	.98	.93	-.27	.55	-.40	.57	-.03	1.22	.95	1	
53492	-8.14	-.24	.25	40	.40	.95	-.13	.52	-.18	.53	-.82	-.29	-.42	3	
54196	-60.61	1.72	.30	26	.26	.71	-.24	.83	-.27	.90	1.19	1.69	1.45	3	
54906	49.64	1.39	.45	19	.70	1.21	-.15	.68	-.19	.69	.50	1.35	1.20	2	
55166	-29.12	1.15	.22	40	-.23	1.00	.03	.67	.03	.68	1.41	1.15	1.18	2	
56690	19.36	-.59	.26	40	-.51	.93	.44	.90	.63	.96	.55	-.40	.04	2	
57240	9.06	-.69	.34	31						.90					
58135	-9.60	-.58	.21	40	-.36	1.00	.09	.63	.18	.67	-.04	-.49	-.40	3	
58364	-2.63	-.05	.42	18	.64	1.07	-.19	.53	-.29	.54	-.98	-.15	-.34	2	
59165	-54.92	.69	.21	40						.62					
59957	-9.39	.10	.33	22	-.41	1.01	.07	.61	.09	.61	.60	.12	.19	2	
61500	-40.77	.80	.20	40	-.44	.89	.18	.69	.27	.72	1.51	.89	1.07	2	
62102	11.91	1.56	.32	19	-.10	1.20	.00	.61	.00	.61	1.66	1.56	1.56	2	
62778	-1.65	-2.77	.69	13						.86					
64551	-38.66	1.28	.50	19	-1.14	1.14	.02	.42	.03	.42	2.45	1.29	1.31	2	

Table 2. (continued)

HIP	μ_δ ($\frac{\text{mas}}{\text{yr}}$)	b ($\frac{\text{mas}}{\text{yr}}$)	ε_b	n (yr)	LTP ($\frac{\text{mas}}{\text{yr}}$)	ε_{LT}	STP ($\frac{\text{mas}}{\text{yr}}$)	ε_{ST}	HIP ($\frac{\text{mas}}{\text{yr}}$)	ε_{HI}	r_{LTP} ($\frac{\text{mas}}{\text{yr}}$)	r_{STP} ($\frac{\text{mas}}{\text{yr}}$)	r_{HIP} ($\frac{\text{mas}}{\text{yr}}$)	K_{ae}	C
65366	10.84	-.26	.23	40	-.57	.92	.02	.56	.01	.58	.32	-.27	-.25	2	
65495	-3.79	1.79	.49	18	-.18	.82	.10	.72	.15	.77	2.12	1.84	1.94	2	
67239	-57.82	1.64	.62	19						.47					
70259	-17.12	.57	.23	40	-1.75	1.00	.34	.62	.45	.64	2.77	.68	1.02	2	
73178	23.14	.34	.35	40	.23	1.09	.05	.61	.07	.62	.18	.36	.41	2	
74249	24.64	-.76	.46	19	.35	.99	-.25	.88	-.43	.96	-1.54	-.94	-1.19	2	
74678	-2.21	.15	.18	40	-.68	.94	-.09	.69	-.18	.72	.65	.06	-.03	3	
74954	-125.10	.01	.58	14	2.32	1.03	-2.22	.86	-2.95	.91	-5.26	-.72	-2.94		
75233	-9.65	-.96	.68	11						.86					
76639	3.53	-.04	.51	19						.76					
77504	6.68	.63	.81	10						2.31					
77676	-12.89	1.01	.25	38	.47	.94	-.24	.71	-.44	.75	.10	.81	.57	2	
78319	21.07	2.80	.74	19	.00	.68	.00	.68	.29	.79	3.09	3.09	3.09	3	
79225	12.40	.49	.28	40	.10	.94	-.03	.72	-.06	.76	.33	.46	.43	2	
81061	-81.39	-3.36	1.82	14	.06	1.20	-.03	1.00	-.05	1.05	-3.47	-3.38	-3.41	2	
82235	44.33	-.36	.32	40	.26	.93	-.19	.78	-.25	.81	-.87	-.42	-.61	2	
82506	-3.75	-3.33	.51	19	-3.78	1.30	.74	.62	1.20	.64	1.65	-2.87	-2.13		
82780	-19.88	-.46	.23	40						.58					
82987	16.52	.67	1.24	19	.22	.68	-.13	.54	-.18	.57	.27	.62	.49	3	
83367	98.96	-.29	.24	40	-.53	.76	.29	.59	.38	.62	.62	-.20	.09		
83643	5.62	-1.57	.77	19	-.07	.88	.03	.78	.10	.87	-1.40	-1.50	-1.47	2	
84574	4.28	.09	.55	18	-.82	1.12	.61	1.04	1.32	1.15	2.23	.80	1.41	2	
85842	42.83	-3.65	.58	19	-.35	.89	.29	.86	.54	.95	-2.76	-3.40	-3.11	2	
86160	-9.70	.73	.19	40	.46	.86	-.23	.72	-.41	.77	-.14	.55	.32	2	
87001	-43.42	-.19	.27	40	.03	1.35	-.01	.56	-.02	.57	-.24	-.20	-.21	3	
87194	-39.09	-.58	.35	40	-.12	.53	.14	.52	.16	.53	-.30	-.56	-.42	3	
87705	-11.59	.50	.47	19	-.42	.91	.19	.72	.38	.77	1.30	.69	.88	2	
88275	-.73	-.08	.26	40	.43	.89	-.06	.64	-.20	.65	-.71	-.22	-.28	2	
88651	-1.34	-1.38	.55	19	-1.08	1.06	.31	.70	.45	.72	.15	-1.24	-.93	2	
89139	14.12	-.68	.25	40	-.22	1.37	.01	.57	.03	.58	-.43	-.66	-.65	3	
90480	-3.57	-1.86	.61	19						1.45					
90906	15.22	-.01	.47	19	-.09	.88	.02	.61	.05	.63	.13	.02	.04	2	
91168	-21.13	.44	.28	40	.29	.95	-.15	.76	-.26	.80	-.11	.33	.18	2	
91496	-3.41	2.27	.45	17	-.03	1.01	.11	.89	.28	.97	2.58	2.44	2.55	2	
91799	-4.98	2.73	.52	19	.72	.92	-.23	.70	-.55	.74	1.46	2.41	2.18	2	
92127	-7.00	-.30	.25	40	-.05	.97	.01	.63	.02	.64	-.23	-.29	-.28	2	
92896	-4.68	.59	.23	40	-.91	1.02	.18	.68	.30	.69	1.80	.71	.89	2	
93553	-4.19	-2.19	.30	37	-.25	.93	.04	.60	.18	.63	-1.76	-2.05	-2.01	3	
93726	4.78	.12	.50	17	.46	1.10	-.03	.64	-.06	.64	-.40	.09	.06	2	
94485	2.50	.72	.24	40	-.29	.90	.03	.54	.17	.56	1.18	.86	.89	3	
94828	7.82	1.55	.50	19	-.11	.88	.03	.71	.12	.76	1.78	1.64	1.67	2	
95147	-4.88	-.93	.50	19	-.48	.99	.07	.71	.53	.75	.08	-.47	-.40		

Table 2. (continued)

HIP	μ_δ ($\frac{\text{mas}}{\text{yr}}$)	b ($\frac{\text{mas}}{\text{yr}}$)	ε_b	n (yr)	LTP ($\frac{\text{mas}}{\text{yr}}$)	ε_{LT}	STP ($\frac{\text{mas}}{\text{yr}}$)	ε_{ST}	HIP ($\frac{\text{mas}}{\text{yr}}$)	ε_{HI}	r_{LTP} ($\frac{\text{mas}}{\text{yr}}$)	r_{STP} ($\frac{\text{mas}}{\text{yr}}$)	r_{HIP} ($\frac{\text{mas}}{\text{yr}}$)	K_{ae}	C
95476	-5.45	.65	.67	19						1.25					
96081	42.86	-1.87	.30	33						.81					
96225	-20.71	-.05	.25	40	.31	.92	-.10	.81	-.22	.89	-.58	-.17	-.27	3	
96405	.14	.16	.48	18	-.14	.92	.09	.86	.38	.98	.68	.45	.54	2	
97532	-16.77	.16	.56	18	-.09	.80	.01	.64	.06	.65	.31	.21	.22	2	
98188	8.74	-1.96	.43	18	.04	.89	.00	.60	-.01	.61	-2.01	-1.97	-1.97	2	
98392	-17.84	-.99	.38	18	-1.59	1.11	.38	.70	.61	.72	1.21	-.76	-.38	2	
98828	-39.20	-.16	.21	39	-.19	1.04	.03	.57	.04	.57	.07	-.15	-.12	2	
99023	-5.97	-3.03	.37	17	.14	1.02	-.01	.57	-.02	.57	-3.19	-3.04	-3.05		
99353	40.95	1.94	.74	12	.39	.86	-.03	.54	-.10	.56	1.45	1.87	1.84	2	
99824	-4.04	.98	.43	39						.48					
100083	-27.82	1.52	.48	18	.08	1.08	-.05	.69	-.09	.71	1.35	1.48	1.43	2	
100093	-.64	-.05	.37	18	.06	.95	.00	.68	-.01	.69	-.12	-.06	-.06	2	
100832	-6.72	-1.00	.40	18	-.01	1.03	-.06	.67	-.11	.68	-1.10	-1.05	-1.11	2	
101485	-7.28	.87	.92	17						.79					
101641	12.51	.55	.29	39						.52					
102012	-4.24	-.53	.22	39						.63					
102271	-5.87	.22	.19	39						.51					
102728	9.66	1.43	.35	17						.86					
102740	9.22	.61	.23	39						.53					
103252	-1.99	-.43	.45	19						.64					
105247	-8.28	-.52	.48	17						.87					
105484	2.36	.78	.33	19						.65					
105724	-2.36	-.39	.24	39						.50					
106036	-15.99	-1.61	.62	16						.49					
107237	-14.40	-1.59	.36	12						.63					
107462	-5.02	.85	.40	21						.67					
107694	3.98	-.47	.29	39						.94					
108966	-14.02	-2.57	.24	38						1.64					
109276	-36.95	-.04	.46	17						.58					
109696	-145.34	-.62	.38	36						.61					
110152	.88	.52	.25	39						.79					
110733	-8.16	-.43	.27	38						.58					
112453	-9.54	1.07	.34	17						.71					
114025	-3.43	.60	.60	10						.70					
114809	15.21	1.74	.43	19						.54					
115389	-11.35	-.39	.34	30						.48					
115583	-1.63	-1.80	.49	19						.52					
117332	-9.80	1.91	1.02	15						.56					
716	60.67	5.90	1.78	5	.25	.91	-.01	.51	-.01	.51	5.64	5.90	5.89		
5494	-105.68	8.07	2.20	5	-.45	.91	.09	.42	.12	.43	8.64	8.10	8.19	1	
6529	-41.61	-9.69	11.02	4	-.26	1.00	.07	.67	.12	.69	-9.31	-9.64	-9.57	2	

Table 2. (continued)

HIP	μ_δ ($\frac{\text{mas}}{\text{yr}}$)	b ($\frac{\text{mas}}{\text{yr}}$)	ε_b	n (yr)	LTP ($\frac{\text{mas}}{\text{yr}}$)	ε_{LT}	STP ($\frac{\text{mas}}{\text{yr}}$)	ε_{ST}	HIP ($\frac{\text{mas}}{\text{yr}}$)	ε_{HI}	r_{LTP} ($\frac{\text{mas}}{\text{yr}}$)	r_{STP} ($\frac{\text{mas}}{\text{yr}}$)	r_{HIP} ($\frac{\text{mas}}{\text{yr}}$)	K_{ae}	C
9851	-1.59	3.94	1.60	5	.51	.83	-.18	.63	-.41	.69	3.02	3.71	3.53	3	
17583	-39.63	6.77	9.53	5	-.05	.95	-.32	1.01	-.60	1.13	6.22	6.49	6.17	2	
29158	-7.55	-2.32	.98	8	-.04	.87	.04	.85	.63	1.10	-1.65	-1.73	-1.69	3	
39835	-14.95	1.23	.92	8						.62					
40860	-40.02	-4.97	2.64	5	-.63	1.10	.08	.98	.03	1.10	-4.31	-5.02	-4.94	2	
40890	-3.63	10.40	5.13	5	-.06	.80	-.02	.84	-.11	1.04	10.35	10.31	10.29		
54063	3.12	-8.76	1.46	5	-1.56	1.13	.44	.65	.61	.68	-6.59	-8.59	-8.15	3	
54347	-16.78	-2.40	.77	8	-.31	.99	.33	.70	.46	.73	-1.63	-2.27	-1.94	3	
86637	-4.07	.19	4.14	6	.12	.99	-.03	.64	-.08	.66	-.01	.14	.11	2	
92701	-17.67	8.47	3.68	5						.71					
93995	-79.07	.35	1.18	4						.82					
101883	-13.51	-1.16	.28	5						.64					
103901	2.53	-.85	4.15	4						.63					
105000	-34.58	2.80	1.35	7						.59					
110247	-27.77	-.75	.65	7						.75					
113542	.06	.59	1.41	7						.76					
114688	.34	1.22	2.31	5						.63					
115672	19.40	-1.75	1.57	5						.62					
117415	-27.47	-1.04	.83	7						.52					
10053		-.37	.33	38						1.97					12
12744	4.45	.07	.40	39						.89					3
26947	-13.29	6.37	5.95	7						1.42					3
45278	-21.22	-.63	.40	34	-2.12	.91	.39	.58	.61	.60	2.10	-.41	-.02		2
56860	-44.01	.48	.31	39						1.50					3
70310		-.72	.29	30	.11	1.08	-.05	.52	-.05	.53				3	1
73768		-.09	.21	40						1.64					1
75350		-.24	.26	38	-.33	.93	.39	.99	.66	1.12				2	1
75883	4.21	-3.89	1.17	3						1.65					3
81781		-.57	.47	23						1.04					12
84821		-.32	.21	40						.66					1
89847		4.99	7.01	4						2.07					13
97077		-.32	.20	39						.65					1
100088		-.56	.18	39						.59					1
106872		-.31	.82	16						.48					12

ones, because our values b were also calculated in that way). The last column is C – comment: 1 – if our input data were already corrected for the proper motion in declination, 2 – if already corrected for declination, and 3 – for the stars that are contained in HIP, but being erroneously identified, instead other ones were observed.

The corrections (for δ or/and μ_δ) were already applied for 11 RCP and RCQ stars (Vondrák 2002); see Table 1. The first part of Table 2 (165 Richmond stars, 114 ARIHIP ones) exhibits very good results ($n > 10$). The second one (22 Richmond stars, 11 ARIHIP ones) is for the case $n \leq 10$ and with bigger differences between our and ARIHIP results. The third part (15 Richmond stars, 3 ARIHIP ones) is in agreement with the comment C: some μ_δ and other values are not presented because the value b is a correction of the correction for some of these stars.

Both linear trends in Fig. 4, one calculated with the new point ($b = 0.09 \pm 0.30$ mas) and the other one without it ($b = 0.10 \pm 0.50$ mas), are close to each other and seen as a single line. The same situation is also for other stars observed for a few decades. But, a different situation is shown in Fig. 5. Star H29075, as an example ($n = 11$), with two different linear trends clearly separated; with the new point, a quite good value of $b = -2.52 \pm 0.93$ mas ($r_{HIP} = -2.04$ mas) is obtained, and without it, the value is $b = 2.50 \pm 1.00$ mas (which gives $r_{HIP} = 2.98$ mas, bigger than in the previous case). Fig. 6 shows 125 ARIHIP stars (with the third part of Table 2 excluded) by using the values r_{HIP} which are in line with SI mode. To check our values b, we use the data of ARIHIP. It is clear that for the stars observed just for a few years with PZT it is not possible to get good results, and these values of r_{HIP} are large (1 mas $< |r_{HIP}| <$ 10 mas), but if the observational period n is about 20 yrs (central part of Fig. 6) the values r_{HIP} are just a few mas. A very good consistency is evident in the right-hand side of Fig. 6 where some r_{HIP} are of about 1 mas and mostly $|r_{HIP}| \leq 1$ mas. For the stars with a few decades of observations, the results b are good (the values of ε_b are small), and the agreement with ARIHIP data is good. Also, we calculate the average value of the residuals r_{HIP}, r_{LTP} and r_{STP} for 125 stars presented in Fig. 6 (and the standard errors) for both cases $n \leq 10$ (11 stars) and $10 < n \leq 40$ (114 ones), respectively:
0.7 ± 6.7 mas/yr, 0.0 ± 1.2 mas/yr, from r_{HIP},
0.9 ± 6.4 mas/yr, 0.0 ± 1.4 mas/yr, from r_{LTP},
0.7 ± 6.8 mas/yr, 0.0 ± 1.2 mas/yr, from r_{STP}.

For the first and second parts of Table 2. (187 stars), we determined the mean value of our errors ε_b and HIP ones ε_{HI} for $n \leq 10$ (24 stars) and $n > 10$ (163 stars), respectively:
2.5 ± 2.7 mas, 0.4 ± 0.2 mas, from ε_b,
0.8 ± 0.4 mas, 0.7 ± 0.2 mas, from ε_{HI}.

For the case $n > 10$ (163 stars), it is evident that we end up with a better mean accuracy (0.4 mas) than the HIPPARCOS one (0.7 mas), but this is not the case for $n \leq 10$ (24 stars), as we expected. Also, our agreement with the ARIHIP data is much better for the case $n > 10$ (114 stars) than $n \leq 10$ (11 ones). These values are in agreement with our comments regarding Fig. 6. All the three ARIHIP modes are consistent with our results (in line with their errors and ε_b). In Table 2, we present the results for b for all the 202 stars (74 stars are not present in ARIHIP), but the results of the first part of Table 2 are better than the others.

5. CONCLUSIONS

In the paper by Damljanović (2005) we used the data of other PZTs as input, and checked the obtained results in a way different from that used here. The method is slightly changed in this paper, but in both cases the results are good. We compare our results with the ARIHIP ones and for the stars with a long observational history (a few decades) the agreement is good. Because of this, we give the results for all the 202 RCP and RCQ stars. ARIHIP does not include the PZT data, and the 74 stars presented here are not found in ARIHIP. It is possible to get good corrections of proper motions in declination for HIPPARCOS stars observed long enough by using PZT instruments. This means, the long term ground – based observations of the Earth rotation programmes are sufficiently good for the task of improving even the HIPPARCOS μ_δ and the corresponding reference frame.

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ТАЧНИЈА СОПСТВЕНА КРЕТАЊА У ДЕКЛИНАЦИЈАМА НЕКИХ ХИПАРКОС ЗВЕЗДА

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Претходно саопштење

Укратко смо описали процедуру рачунања и ширинске улазне податке две Ричмонд фотографске зенитне тубе (RCP и RCQ за период 1949.8 – 1989.4), а затим презентовали резултате за 202 ХИПАРКОС звезде посматране са RCP и RCQ и упоредили наше са резултатима АРИХИП каталога. Од 202 звезде, у АРИХИП каталогу смо пронашли 128 звезда. Посматрања рађена на RCP и RCQ захватају период од око 40 година, али су многе од 202 посматране звезде са посматрањима од пар деценија а неке и свега неколико година. Због тога смо добили побољшања сопствених кретања у деклинацији посматраних Хипаркос звезда различите тачности, што се уочава и при поређењу са АРИХИП подацима. Наравно, најбоље је слагање наших са АРИХИП резултатима за звезде које су посматране свих 40 година, док је лошије за звезде које су посматране мање од 10 година. Хипаркос сателитска посматрања (ESA 1997) су трајала нешто краће од 4 године. Да бисмо поправили тачност сопствених кретања Хипаркос звезда потребно је више деценија класичних оптичких посматрања тих звезда са Земље. Таква посматрања постоје, јер су током периода 1899.7-1992.0 рађени посматрачки програми звезда у оквиру програма истраживања Земљине ротације. У овом раду, поређењем наших са АРИХИП резултатима, долазимо до доста доброг слагања за звезде које су довољно дуго посматране (више деценија), при чему су наша истраживања и улазни подаци потпуно независни и различити, а 74 RCP/RCQ звезде уопште нису у АРИХИП каталогу. Закључујемо, да се наведена дугогодишња посматрања у оквиру програма истраживања Земљине ротације могу користити за врло захтевне пројекте какав је провера и побољшање сопствених кретања Хипаркос каталога, а тиме и самог референтног система.

23 Ground-based latitude data and the improved reference frame

G. Damljanović and N. Pejović
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This short note deals with the checking and the improvement of data contained in the HIPPARCOS Catalogue.

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GROUND – BASED LATITUDE DATA AND THE IMPROVED REFERENCE FRAME

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Abstract. The HIPPARCOS Catalogue is a product of the HIPPARCOS ESA astronomical satellite mission (ESA 1997). The observations lasted less than 4 years, and the epoch of the catalogue is 1991.25 (there are 118218 stars up to magnitude 12). The precision of positions is 1 mas and of proper motions is 1 $mas/year$. The HIPPARCOS Catalogue is the optical counterpart of the ICRF (International Celestial Reference Frame) and therefore the optical reference frame. Nowadays, it is possible to check and to improve some HIPPARCOS data (and the reference frame) by using other observations, as for example a long series of groundbased optical observations of some HIPPARCOS stars. Here, we used the data of Photographic Zenith Tubes (PZT) as the part of the different Earth rotation programmes made during the last century (Vondrák et al., 1998). These observations lasted much longer than the HIPPARCOS ones, and because of that we can give better proper motions than HIPPARCOS ones. The main steps of calculation (Damljanović and Vondrák, 2005) are presented with the results (the proper motions in declination) for 202 stars observed at Richmond PZTs. Also, we demonstrated a good agreement between our results and the ARIHIP ones, specially for the stars with a few decades of PZT observations.

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24 Corrections of Proper Motions in Declination by Sorting ILS Data

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The authors used data on latitude variations, covering the period 1899.7 - 1979.0, of visual Zenith Telescopes (ZT) of International Latitude Service (ILS), to improve the Hipparcos proper motions in declination for stars observed at seven ILS stations: Carloforte, Cincinnati, Gaithersburg, Kitab, Mizu- sawa, Tschardjui and Ukiah. Their research in this paper is in accordance with the Earth Orientation Catalogue - EOC (Vondrák and Ron 2003) one, based on the Earth rotation programmes ground - based data, but they used different method here. Obtained results yield better proper motions in declination for stars common to ILS and HIPPARCOS, and a good agreement with those from EOC-2.

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Preliminary report

CORRECTIONS OF PROPER MOTIONS IN DECLINATION BY USING ILS DATA

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SUMMARY: There are nowadays numerous astrometric ground–based observations of some stars referred to Hipparcos Catalogue, made at many observatories during the last century. We used the data on latitude variations, covering the period 1899.7 – 1979.0, of visual Zenith Telescopes (ZT) of International Latitude Service (ILS), to improve the Hipparcos proper motions in declination for stars observed at seven ILS stations: Carloforte, Cincinnati, Gaithersburg, Kitab, Mizusawa, Tschardjui and Ukiah. About 15 years elapsed since the HIPPARCOS ESA mission (ESA 1997) observations (1991.25 is the epoch of this catalogue), and with the errors of the Hipparcos proper motions close to 1 mas/yr, the error of apparent places of stars is now more than 15 mas; so that it exceeds the error of the Hipparcos positions by one order of magnitude (which is about 1 mas). Also, for some Hipparcos stars, the errors of proper motions are much larger than the averaged value itself, even not realistic at all (Vondrák et al. 1998); the Hipparcos astronomical satellite mission lasted less than four years, not enough to get a sufficient accuracy of the proper motions. To improve the accuracy of the proper motions for some Hipparcos stars, the ground – based data were used and some new catalogues were published (such as ARIHIP, EOC-2, etc.) during the last decade. Our investigations are in accordance with the Earth Orientation Catalogue – EOC (Vondrák and Ron 2003) one, based on the Earth rotation programmes ground–based data, but we used different method here. Our results yield better proper motions in declination for stars common to ILS and HIPPARCOS, and a good agreement with those from EOC-2.

Key words. Astrometry – Reference systems

1. INTRODUCTION

The HIPPARCOS Catalogue with astrometric data for 118218 stars is the optical counterpart (Kovalevsky et al. 1997) of the International Celestial Reference Frame (ICRF). The standard errors in positions (α and δ) of its stars are around 1 mas at the epoch of the catalogue (1991.25), and the standard errors of its proper motions ($\mu_\alpha \cos\delta$ and μ_δ) are near 1 mas/yr. The stars are brighter than magnitude 12, but most of them are between magnitude 7 and 9. By now several new catalogues have appeared, such as ACT, FK6, GC+HIP, TYC2+HIP, TYCHO-2, ARIHIP, EOC-2, with a better accuracy of the proper motions than in Hipparcos, but as a combination of Hipparcos and/or Tycho with ground–based observations. The main problem for the Hipparcos proper motions accuracy is the period of HIPPARCOS satel-

lite observations which was less than four years; it is not enough to get a good accuracy of proper motions for some stars (such as for double or multiple ones). Also, in some parts of the sky in the Hipparcos Catalogue errors of the proper motion data are larger. Nowadays, 15 years after the epoch of the Hipparcos Catalogue (1991.25), the errors of the Hipparcos proper motions of about 1 mas/yr result in an error of about 15 mas of the calculated apparent positions of the Hipparcos stars; this is close to the ground–based error of the observed positions and an order of magnitude higher than a typical error in a Hipparcos position. Thus, the astrogeodetical observations need a better accuracy of proper motions than that offered by Hipparcos. The aim of some investigations started during the last decade was to improve the Hipparcos proper motions by using ground–based observations covering almost a whole century. As a result, we have several new catalogues of which the ARIHIP (Wielen et al. 2001) and EOC-2 (Vondrák 2004) are of special interest to the present study.

A few years ago, the ground–based astrometric observations of latitude and universal time variations, covering the interval 1899.7 – 1992.0 and made in accordance with the Earth rotation programmes, were included in the investigations aimed at improving the accuracy of the Hipparcos proper motions. Here, we used the data of ZT of ILS and OA00 (Ron and Vondrák 2001) solution of the Earth Orientation Parameters (EOP), but our method was different from that used in EOC-2 (Vondrák 2004). The ILS data cover very long interval (1899.7 – 1979.0) of ZT observations made at seven ILS stations, and by using the ILS latitude variations we determined the corrections of the Hipparcos proper motions in declination with an accuracy close to that of the proper motions given in ARIHIP and EOC-2.

By definition $\mu_\delta = (\delta_1 - \delta_2)/(t_1 - t_2)$, where δ_1 and δ_2 are two declinations (in the same system) of the same star for the epochs t_1 and t_2, respectively. The error of μ_δ is $\epsilon_{\mu_\delta} = (\epsilon_1^2 + \epsilon_2^2)^{1/2}/|t_2 - t_1|$ where ϵ_1 and ϵ_2 are the standard errors of δ_1 and δ_2, respectively. Then, ϵ_{μ_δ} is proportional to $1/t$ (Eichhorn 1974), and with regard to the long ILS interval t we can get a very good accuracy ϵ_{μ_δ} (better than the Hipparcos one) though the Hipparcos data concerning the star positions are better than the ILS ones.

2. DATA AND CALCULATIONS

During each year of the interval 1899.7 – 1979.0, each ILS star pair was observed from a few to a few hundred times. The accuracy of a ZT observation is about $0\overset{''}{.}2$, but the time interval a few decades long and numerous observations for each ILS pair during each year of that interval are decisive in attaining a better accuracy, than the Hipparcos one, of proper motions in declination for stars common to ILS and Hipparcos. Combining the ILS values (observations) with the Hipparcos one (for the epoch 1991.25) can yield better results (corrections of proper motions in declination for some Hipparcos stars) than if only the ILS data are used (Damljanović et al. 2006). The accuracy is about one order of magnitude better than $0\overset{''}{.}2$ for the averaged (over nearly one year observational interval) values of latitude variations.

We used the ZT data (latitude variations φ_i) of 7 ILS stations, at nearly the same latitude of $+39\overset{\circ}{.}1$ (see Table 1).

The value $(t - t_1)$ is in centuries, and t_1 is counted from 32000 MJD. The codes (CA, CI, GT, KZ, MZZ, TS and UK) are from the monograph by Vondrák et al. (1998). The longitude λ_W is west of the zero meridian. The tectonic plate motion and mean latitude were removed from the ILS data (Vondrák 2002). More about the ILS data was published by Yumi and Yokoyama (1980). Here, 256 ILS star pairs (out of total 281 ones) are included in our calculation because some of the ILS stars are not in the Hipparcos Catalogue: the 256 pairs were observed at CA, GT and MZZ, 198 ones at KZ, 117 ones at CI and TS, 228 ones at UK. Some stars are included in more than one star pair and there are 486 common ILS and Hipparcos stars.

The latitude variations were used to improve the Hipparcos proper motions in declination (Damljanović and Pejović 2005), but the ZT data give us these corrections for star pairs, because with a ZT instrument always a star pair is observed. We found the way to solve this problem and to separate them; see Eq. (5) below. From $\varphi_p = \delta_S + z_S$ and $\varphi_p = \delta_N - z_N$ (Yumi and Yokoyama 1980), the latitude obtained from a star pair φ_p (for

Table 1. Some informations of ILS stations.

ILS station	λ_W (^{o})	Observed int.	Break	Mean latitude ($39°8'+$")	Tectonic plate motion("/100yr)	t interval (MJD)
Carloforte(CA)	351.7	1899.8-1979.0	1943.3-1946.5	9.157	$0.061(t-t_1)$	14950-43863
Cincinnati(CI)	84.4	1899.8-1916.0		19.437	$0.002(t-t_1)$	14898-20868
Gaithersburg(GT)	77.2	1899.8-1979.0	1915.0-1932.6	13.309	$0.012(t-t_1)$	14928-43880
Kitab(KZ)	293.1	1930.8-1979.0		2.057	$0.001(t-t_1)$	26304-43878
Mizusawa(MZZ)	218.9	1900.0-1979.0		3.687	$-0.045(t-t_1)$	15002-46010
Tschardjui(TS)	296.5	1899.7-1919.3		11.334	$0.004(t-t_1)$	18512-22090
Ukiah(UK)	123.2	1899.7-1961.0		12.161	$0.025(t-t_1)$	14937-37302

the moment of measurement t) is the main ZT equation (following from the Horrebow – Talcott method):

$$\varphi_p = (\delta_S + \delta_N)/2 + (z_S - z_N)/2, \qquad (1)$$

where δ_S and δ_N are the apparent declinations of stars of the Talcott pair (δ_N for the northern and δ_S for the southern at t, calculated by using Hipparcos Catalogue), $\Delta z = z_S - z_N$ is the measured value of the zenith-distance difference. From the values of the latitude variations, we removed the polar motion component and the systematic variations with time (local, instrumental, etc.), and obtained the residuals with catalogue errors dominant (Damljanović and Vondrák 2005, Damljanović et al. 2006). Thus, one has (Vondrák et al. 1998)

$$\Delta\varphi_p + (d\varphi_p/dt)t \approx (\Delta\delta_S + \Delta\delta_N)/2 +$$

$$+t(\Delta\mu_{\delta_S} + \Delta\mu_{\delta_N})/2\,,$$

where $\Delta\delta_S$ and $\Delta\delta_N$ are corrections of declinations (for S and N star, respectively), $\Delta\mu_{\delta_S}$ and $\Delta\mu_{\delta_N}$ are corrections of proper motions in declination (for S and N stars, respectively), t is time.

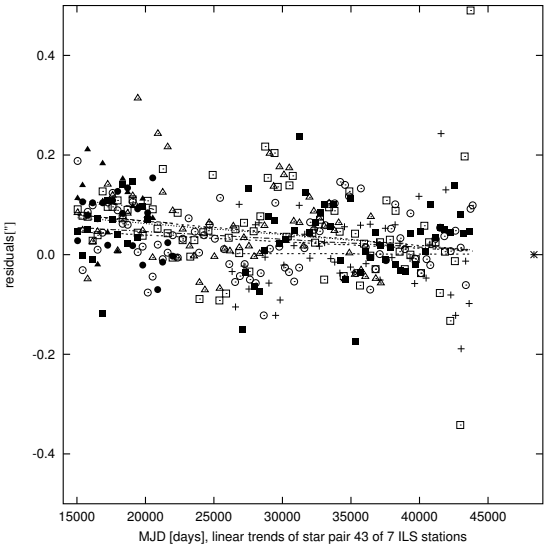

Fig. 1. *Residuals r'_n, linear trend of star pair 43 of each ILS station (CA – open rectangles, GT – solid rectangles, KZ – sign +, MZZ – open circles, TS – solid circles, UK – open triangles, CI – solid triangles), and Hipparcos point (asterisks).*

To calculate the correction of proper motion in declination corresponding to a star pair (of ILS/Hipparcos stars), we use the Least Squares Method (LSM) and linear model (Damljanović 2005, Damljanović and Pejović 2005)

$$r'_n = a + b(t_n - 1991.25), \qquad (2)$$

where r'_n is the star pair residual – about one point per year (residual of latitude variations, without polar motion and the systematic parts, averaged during each year of observational interval), t_n (in years) is the epoch of r'_n, a pertains to $(\Delta\delta_S + \Delta\delta_N)/2$, b pertains to $(\Delta\mu_{\delta_S} + \Delta\mu_{\delta_N})/2$. The values a and b, for each ILS star pair, are in accordance with the epoch of the Hipparcos Catalogue – 1991.25. The points r'_n and the Hipparcos one are taken with appropriate weights (Damljanović et al. 2006). The polar motion and the systematic time changes are calculated and removed from the ILS data in the way presented in an earlier paper (Damljanović 2005). The polar motion part was determined by using the EOP solution OA00 (Ron and Vondrák 2001). The systematic variations of each ILS ZT instrument (local, instrumental and others) were calculated and removed from the ILS data in a similar way as we did in the case of the PZT data (Damljanović et al. 2006).

We have got one equation, $b = (\Delta\mu_{\delta_S} + \Delta\mu_{\delta_N})/2$, but with two unknown quantities ($\Delta\mu_{\delta_S}$ and $\Delta\mu_{\delta_N}$), and we need to introduce another Eq. (3) to solve for the values of $\Delta\mu_{\delta_S}$ and $\Delta\mu_{\delta_N}$,

$$\Delta\mu_{\delta_S} - \Delta\mu_{\delta_N} = (\mu_{\delta_{S1}} - \mu_{\delta_{S2}}) - (\mu_{\delta_{N1}} - \mu_{\delta_{N2}}), \qquad (3)$$

where the values for $\mu_{\delta_{S1}}$ and $\mu_{\delta_{N1}}$ are from the EOC-2 catalogue, and those for $\mu_{\delta_{S2}}$ and $\mu_{\delta_{N2}}$ are from the Hipparcos one. A similar situation is with the errors $\epsilon_{\Delta\mu_{\delta_S}}$ and $\epsilon_{\Delta\mu_{\delta_N}}$ of $\Delta\mu_{\delta_S}$ and $\Delta\mu_{\delta_N}$, respectively. We have got the equation $(\epsilon^2_{\Delta\mu_{\delta_S}} + \epsilon^2_{\Delta\mu_{\delta_N}})/2 = \epsilon^2_b$, but it is necessary to introduce another one, in our case here it is Eq. (4), to solve the system of equations and to calculate the values of $\epsilon_{\Delta\mu_{\delta_S}}$ and $\epsilon_{\Delta\mu_{\delta_N}}$,

$$\epsilon_{\Delta\mu_{\delta_S}}/\epsilon_{\Delta\mu_{\delta_N}} = \epsilon_{\Delta\mu_{\delta_{S1}}}/\epsilon_{\Delta\mu_{\delta_{N1}}}, \qquad (4)$$

where the values of $\epsilon_{\Delta\mu_{\delta_{S1}}}$ and $\epsilon_{\Delta\mu_{\delta_{N1}}}$ (the errors of S and N stars in ILS star pair, respectively) are from EOC-2.

By using Eqs. (3) and (4), it is possible to calculate the values of $\Delta\mu_{\delta_S}$ and $\Delta\mu_{\delta_N}$ from

$$\Delta\mu_{\delta_S} = (2b + (\mu_{\delta_{S1}} - \mu_{\delta_{S2}}) - (\mu_{\delta_{N1}} - \mu_{\delta_{N2}}))/2,$$

$$\Delta\mu_{\delta_N} = 2b - \Delta\mu_{\delta_S}, \qquad (5)$$

and their errors $\epsilon_{\Delta\mu_{\delta_S}}$ and $\epsilon_{\Delta\mu_{\delta_N}}$ from

$$\epsilon_{\Delta\mu_{\delta_S}} = \epsilon_{\Delta\mu_{\delta_N}}\epsilon_{\Delta\mu_{\delta_{S1}}}/\epsilon_{\Delta\mu_{\delta_{N1}}},$$

$$\epsilon_{\Delta\mu_{\delta_N}} = 2^{1/2}\epsilon_b/(\epsilon^2_{\Delta\mu_{\delta_{S1}}}/\epsilon^2_{\Delta\mu_{\delta_{N1}}} + 1)^{1/2}. \qquad (6)$$

Table 2. Our results ($\Delta\mu_\delta$, μ_δ, ϵ_{μ_δ}), Hipparcos ones (μ_{δ_H}, $\epsilon_{\mu_{\delta_H}}$), and value m (number of ILS points r'_n).[1]

HIP	m	$\Delta\mu_\delta$(mas/y)	μ_δ(mas/y)	ϵ_{μ_δ}(mas/y)	μ_{δ_H}(mas/y)	$\epsilon_{\mu_{\delta_H}}$(mas/y)
19	48	.04	-15.03	.39	-15.07	.45
106	151	-.27	-11.46	.15	-11.19	.47
410	126	-.94	-7.51	.10	-6.57	.47

3. RESULTS

As an example, the calculated values $b \pm \epsilon_b$ for ILS star pair No 43, H29246 and H29650 stars, for the ILS stations are (the Hipparcos point was included, all points are with weights):

- CA, 77 points r'_n, -0.87 ± 0.18 mas/yr,
- GT, 62 points r'_n, -0.57 ± 0.22 mas/yr,
- KZ, 49 points r'_n, -0.03 ± 0.30 mas/yr,
- MZZ, 80 points r'_n, -0.49 ± 0.15 mas/yr,
- TS, 20 points r'_n, -0.87 ± 0.15 mas/yr,
- UK, 61 points r'_n, -0.81 ± 0.17 mas/yr,
- CI, 16 points r'_n, -1.20 ± 0.18 mas/yr, and
- all the seven ILS stations, 365 points r'_n, -0.76 ± 0.11 mas/yr.

Star H29246 is of a magnitude of 5.35 and H29650 of 5.71. The values b for star pair No 43 are presented in Fig. 1; all the linear trends are nearly parallel to one another. Only the value of b for the KZ station (-0.03 ± 0.30 mas/yr, the shortest line in Fig. 1) is a little bit further away from that for all ILS stations (and for the other stations individually) because the KZ station started its activity later (at 1930.8 or 26304 MJD) than the other ILS stations (at about 1900 or near 15000 MJD).

By using formulae (5) and (6), for H29246 star (of ILS star pair 43), we get $\Delta\mu_\delta = -0.67 \pm 0.08$ mas/yr, and for the H29650 (the same star pair) $\Delta\mu_\delta = -0.85 \pm 0.14$ mas/yr; our corrections $\Delta\mu_\delta$ can be added to the corresponding Hipparcos values. There are similar results also for the other ILS/Hipparcos stars which are presented in Table 2. Complete Table 2 (408 Hipparcos stars) is given at the website http://saj.matf.bg.ac.yu/173/pdf/Table2.pdf.

Using Table 2, we compared our errors of proper motions in declination with the Hipparcos ones (cases of ILS star pairs observed for more than 10 years): from 204 ILS star pairs (408 Hipparcos stars), the averaged value of the standard errors ϵ_{μ_δ} is 0.21 mas/yr, and it is about 36% of the averaged Hipparcos values $\epsilon_{\mu_{\delta_H}}$ (0.58 mas/yr, that is three times better than Hipparcos values). Thus $\epsilon_{\mu_\delta} \approx \epsilon_{\Delta\mu_\delta}$, because we just added our corrections $\Delta\mu_\delta$ to the corresponding Hipparcos values μ_{δ_H} to get the values μ_δ.

4. CONCLUSIONS

By using the ILS data it is possible to obtain proper motions in declination with a better accuracy than those of the Hipparcos. From 408 Hipparcos stars observed for more than 10 years at the ILS stations, we get an accuracy about three times better than the Hipparcos one. It is evident that the ILS ZT ground–based data are useful and can improve the reference frame (via improvement of Hipparcos proper motions in declination).

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ПОПРАВКЕ СОПСТВЕНИХ КРЕТАЊА У ДЕКЛИНАЦИЈИ ДОБИЈЕНЕ ИЗ ILS ПОДАТАКА

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Претходно саопштење

Постоји обиман астрометријски материјал добијен из посматрања звезда посматраних са Земље током прошлог века на многим опсерваторијама, а сада обрађен коришћењем података из Хипаркос каталога. У овом раду су коришћени подаци ширинских промена у периоду 1899.7 – 1979.0, добијени из посматрања на визуелним зенит телескопима Међународне службе за ширину (*International Latitude Service* – ILS), да би побољшали тачност Хипаркос сопствених кретања у деклинацији за Хипаркос звезде посматране на седам ILS станица (*Carloforte, Cincinnati, Gaithersburg, Kitab, Mizusawa, Tschardjui, Ukiah*). Од посматрања Хипаркос сателитом (ESA 1997), има око 15 година (1991.25 је епоха каталога), што са грешком Хипаркос сопствених кретања од око $0\farcs001$/god уноси грешку у рачун привидних положаја већу од $0\farcs015$, што је за ред величине веће од грешке Хипаркос позиција (око $0\farcs001$). За неке Хипаркос звезде, грешка сопствених кретања је много већа од просечне вредности или није реална (Vondrák et al. 1998), јер су астрометријска посматрања Хипаркос сателитом трајала краће од четири године (недовољно за решавање неких проблема добијања сопствених кретања, што се сада уочава). За добијање тачнијих сопствених кретања неких Хипаркос звезда, користе се класична посматрања са Земље, и током протекле деценије појавили су се нови каталози (као што су ARIHIP, EOC-2 и др.). Наша истраживања и резултати су у складу са резултатима каталога *Earth Orientation Catalogue* – EOC (Vondrák and Ron 2003), који је добијен коришћењем података посматрачких програма рађених са Земље за истраживања Земљине ротације, при чему смо применили другу методу. Приказани резултати су тачнија сопствена кретања Хипаркос звезда које су посматране на ILS станицама, и у складу су са EOC-2 сопственим кретањима.

25 Astronomical station at Vidojevica

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Solar and Stellar Physics Through Eclipses
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The paper is on a project of building a new astronomical station at the mountain of Vidojevica in Serbia (ASV) near Prokuplje as an extension of the Astronomical Observatory in Belgrade (AOB). The project started in 2000 on the joint initiative of AOB and the Faculty of Mathematics of the Belgrade University. The project objective is to improve the research capacities in astronomy and applied mathematics in Serbia and Western Balkan.

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Astronomical Station at Vidojevica

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Abstract. Recently a project was started aimed at building a new astronomical station at the mountain of Vidojevica in Serbia (ASV) as an extension of the Astronomical Observatory in Belgrade. The first phase - ASV1 - is planned to be finished during 2006. ASV1 will consist of one observatory dome, a reflector of 60cm aperture, and a dormitory. In this year, the Faculty of Mathematics and its Department of Astronomy applied for the project of reinforcing and upgrading it to ASV2. The project objective is to improve the research capacities in astronomy and applied mathematics in Serbia and Western Balkan.

1. Astronomical Station ASV

The idea of building an astronomical station at Vidojevica is more than 20 years old. The project was postponed, almost forgotten, due to the circumstances that appeared in the former Yugoslavia during the last two decades. Two institutions, Faculty of Mathematics and Astronomical Observatory in Belgrade (AOB) renewed the project and helped by the Serbian Ministry of Science began raising ASV in 2003. The first stage ASV1, including a 60cm reflector should be finished in 2006. There exists a rather good infrastructure: living building, roads, electricity, water supply, etc. The site was chosen according to good astronomical criteria. The peak of the mountain of Vidojevica was the best observing site even in the former Yugoslavia, with the clearest sky and very good atmospheric conditions. More details can be found at `http://www.asv2.matf.bg.ac.yu`.

1.1. The site

ASV is located at the peak (1155m, N $43^0 08.706'$ E $021^0 33.982'$) of the mountain of Vidojevica in Southern Serbia and it is about 20 km far by road from the town of Prokuplje (30.000 population). The peak is almost flat and occupies about 1000 m^2. The infrastructure at the site is good. The road is in good conditions and it passes about 2.5 km from the station. Also, there is a macadam road connecting ASV and the main road. The site is supplied with electric power and there is a wellspring nearby. There are no significant obstacles on the horizon. The sky is open, particularly to the south. The light pollution is negligible, there are some traces of light pollution in the north, coming from the town of Prokuplje. Astroclimate conditions are also good, in fact best in Serbia. According to meteorological measurements, some of them dating from 1900, most of them since 1950, there are on the average about 120 clear nights

a year. This region is one of the driest areas in Serbia. In Prokuplje the total rainfall is 580mm, while on Vidojevica it amounts only 533mm. Measurements in the last two decades show even decreasing of rainfall (the rainfall was above 600mm until 1980). Other collected data include temperature, rainfall, relative humidity, insolation, haze, cloudiness and number of days with clear sky and all of them show that this site is advantageous for astronomical observations.

1.2. The project

The original project of the whole complex assumes four observatory domes. First, two identical small observatory domes are projected. Telescopes of apertures up to 65cm can be mounted there. According to the project, the third dome can contain a telescope up to 1 m, while the fourth dome is projected for a reflector of 2 m aperture. The living house for the crew is about 200m^2 and it is planned for housing 12 persons. The first stage of the project, ASV1, comprises the building of one small observatory dome and living house, and it should be finished during 2006. A telescope of 60 cm aperture has been already bought with some accessories and one spectrograph. The total cost of the first stage of the project is about 350000EU, and it is financed completely by the Serbian Ministry of Science.

Here are some technical details of the telescope. The reflector is produced by the German company ASTROOPTIK,it has Lomo Sitall optics, primary focal ratio $f/3$ and system focal ratio $f/10$. The servomotor encoders are with $< 0''.08$ resolution, while the maximum slew rate is 3° per second. The software and the controlling system of the telescope are based on a PC WIN XP operating system with standard functions for computerized telescopes. It admits of also remote control. The accessories include a reducer (0.6× for an effective focal length of 2500mm), field corrector, off axis guider and high-accuracy encoders (Heidenhain ERA 780 C Encoders with a very large diameter [473mm] and a very high resolution in both axes [0.2" accuracy per axis]).

However, a group of Serbian astronomers started to think about an alternative as a change of the original project. The alternative would consist of building an array of small identical telescopes (at least three of them, each of 60 cm aperture). They would make a multiple telescope for doing optical and near-infrared interferometry. For this reason, the Faculty od Mathematics applied for the project *Reinforcing research potential in astronomy and applied mathematics* by the European Union under FP6 framework. In fact, the main project objective is to reinforce and improve research capacities in astronomy and applied mathematics in Serbia and Western Balkan (WB). It is planned to do that by upgrading and raising scientific and technological potentials of ASV and exchanging and training scientists and students through networking with other European research centers and neighboring countries. This stage of the project is called ASV2. Our partners in the project would be: **a)** Instituto de Astrofísica de Andalucía, Granada, Spain, key person: Dr Emilio J. Alfaro. **b)** Astronomical Institute, Academy of Sciences of the Czech Republic, Prague, key person: Dr. Jan Vondrak **c)** Oxford University Computing Laboratory, Numerical Analysis Group, United Kingdom, Oxford, key person: Prof. Endre Süli. **d)** School of Informatics, University of Manchester, Manchester, United Kingdom, key person: Dr. Goran Nenadić.

2. ASV2 in Details

The project ASV2 assumes purchasing of a computerized telescope, identical to the already mentioned ASTROOPTIK reflector, then a small supercomputer consisting of a cluster of PC computers and a GPS necessary for a precise time base and the telecommunication equipment. The project consists of the following section parts.

2.1. The multiple telescope

The purpose is to prepare the system to do optical and near-IR interferometry. This is considered as a cheap way to build a big mirror. A further enlargement of the array of telescopes will be possible. The classical principle of interferometry has been little present at optical wavelengths. Interferometry helps to improve the resolving power of an instrument (a telescope). For instance, by synthesizing several small apertures it is possible to achieve the effect of a large aperture. This improvement can be, quantitatively, to an order of magnitude. The base line can attain even 100 m. At the given location such a base line is available to us. The first experiments are planned to be performed with two already mentioned identical telescopes over a base line 70 m long. Once such a telescope already exists at ASV, a multiple telescope of an equivalent aperture of 85 cm will be obtained. The necessary time base line will be provided through a qualitative total station possessing a GPS based on European system Galileo. Mathematical methods will include image and signal processing, frequency analysis and wavelet theory. Additional mathematics includes numerical solutions of ordinary and partial differential equations for solving certain astronomical problems, e.g. in celestial mechanics, then developing algorithms for error correction for applying to the GPS system.

2.2. Robotization and automatic guidance for this system

This will permit a distant guidance and use of each telescope, separately, as well as the system as a whole. Besides, high-tech advanced modern mathematical methods will be used, e.g. the Groebner basis as an important tool in modern computational algebra and analysis. Bearing in mind that a low-budget station is raising, open-source application will be used, where it is appropriate for a professional use. Namely, the revolution in astronomy does not stop at the hardware. Research-grade telescopes at observatories from Spain to Korea are under the control of open-source software and Linux based computers. Under the open-source model, we are free to modify the control software. With a source code freely available, a peer review now occurs not only on the data, but on the gathering as well. In our research, we consider the forefront on this open-source astronomy in *TALON* (can be downloaded at observatory.sourceforge.net). TALON was originally developed by Ellwood Downey as the Observatory Control and Astronomical Analysis Software (OCAAS). As already mentioned, a part of the controlling software for the telescope that is foreseen for purchasing is coming with the telescope, too.

2.3. Data transfer and processing

The objective is an on-line use of the system, transfer of collected data, their storing and manipulating for research purposes. In the previous item we described the intended software platform. Firstly, we plan to develop software for remote control of the multiple telescope and automatic guidance, as of sky patrolling. Secondly, we are planning to develop an application for efficient on-line transfer of so collected data, but off-line, as well. We consider an open-source software for this task, too. For example, a part of this task can be done by software package TALON, as it supports network operations, using X session. However, the quantity and complexity of data that are obtained by astronomical observations may grow rapidly. Therefore, there is an apparent need to provide an efficient data storage, access and processing. In particular, the research objectives would be a sofisticated data mining to discover patterns of any kind, including images and numbers. In particular, we consider a pattern recognition for use in interferometry. Namely, data on light waves collected by telescopes are digitized by DSP processors and so a sequence of numbers is obtained. If two synchronized identical telescopes are used in the same time interval, two nearly equal sequences of numbers will be obtained with a small shift on the time line. The pattern recognition will be used for finding the same base point on the time line for both sequences of light waves. This will enable one to post-process data for the purpose of interferometry. A good time base is needed, of course. We consider this approach as an innovation in interferometry, or generally in astronomy, versus the methods applied elsewhere.

3. Research

The following scientific programs are planned to be performed immediately after finishing the ASV1 stage. Of course, some of them will be possible after finishing the ASV2 stage.

3.1. Time base

As it is well known, the Official Source of Time for the WB region does not exist. It is a very important service for reintegrating the WB region in the European traffic, industry and defence systems, as well as the navigating one. The standard unit for the measurement of intervals of time is the SI second, which is defined by adoption of a fixed value for the frequency of a particular transition of caesium atoms. Caesium frequency standards, hydrogen masers, ion storage devices, and other such devices are able to count seconds and subdivide them very precisely. Thus, such a device can provide a time base and a time-scale whose accuracy is dependent of the measurement precision and the device stability. The results of the measurements of about 200 frequency standards located throughout the world are combined to form a standard time-scale that can be used for identifying uniquely the instants of time at which events occur on the Earth. This standard time-scale is known as International Atomic Time (TAI). It is the basis for all time-scales in the general use. It is distributed by many different means, including radio time signals; navigation systems such as GPS (Global Positioning System), LORAN-C, and OMEGA; communications satellites; and precise time standards. Both the Transit satellite system and the

Global Positioning System (GPS) can be used for one-way time transfer. Although the GPS system is designed for navigation purposes, it also coordinates most of the major time-scales in the world. The GPS system has an accuracy of 10 ns for ten-day averaging. The most accurate time comparison by GPS is based on a common satellite procedure. GPS satellites on a flyby mode make two separate comparisons with respect to the GPS satellite clock. A linear rate for the satellite clock is assumed. This method does not provide the same accuracy as the common view procedure, but the accuracy is enough for the astronomical events registration, among them, for the optical interferometry reduction of the observations. The differential delay effects caused by the Earth's atmosphere and other different sources should not exceed a few nanoseconds. Time has a specific purpose in astronomy. The observational data, with precise time identification by GPS one-way transfer, are then brought together for correlation so that precise values for the differences between the clocks at the individual telescopes can be determined. A base of about 100 m will provide to establish the coordinate accuracy by an order of magnitude better than that obtained with a single telescope. To achieve this accuracy, 400-850 nm bandwidths are required. This process requires distinct measurements based on prearranged experiments involving different locations. The results of the time comparisons are not available unless the two data sets are brought together and correlated. In theory, a sufficiently large data sets are available and sufficiently large bandwidths can be obtained, the synchronization method is mainly limited in accuracy by the difficulty of determining the overall system delay and the atmospheric delay.

3.2. Other astronomical programs

Here we shall mention in short other types of astronomical research at ASV:

1. Monitoring of Main belt asteroids and near - Earth asteroids.
2. Investigation of binary stellar systems.
3. Monitoring of Solar activity.

4. Conclusion

The Project will significantly help the preparation of groups of Serbian scientists in the fields of astronomy and applied mathematics for future Community research. By upgrading and reinforcing ASV by low budget-equipment, it will help technological development policy activities too, including monitoring and assessment activities. In particular, new experiences and connections with European centers will be obtained through partner network activities: conferences, workshops, studies and analyses, working groups and expert groups, operational support and dissemination, information and communication activities. Along this course, the ASV2 Project has a strong support from four leading European scientific institutions and the Serbian leading scientific and high-level educational institution, the Belgrade University.

26 On galactocentric orbits of globular clusters

S. Ninković and N. Pejović,
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A review of galactocentric orbits for many globular clusters is given. It is pointed out that the proper-motion data, necessary to orbit determinations, are still not sufficiently reliable. On the other hand, realistic models of the Milky Way yield very similar galactocentric orbits for the same globular cluster. However, slight changes of both the parameters of a Milky-Way model and initial conditions can result in strong changes of the shapes of these orbits, though the dimensions remain practically unchanged. As a possible interpretation the present authors suggest that the so-called box orbits might be explained by using the two classical integrals (energy and angular momentum) only.

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On Galactocentric Orbits of Globular Clusters

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Abstract. A review of galactocentric orbits for many globular clusters is given. It is pointed out that the proper-motion data, necessary to orbit determinations, are still not sufficiently reliable. On the other hand, realistic models of the Milky Way yield very similar galactocentric orbits for the same globular cluster. However, slight changes of both the parameters of a Milky-Way model and initial conditions can result in strong changes of the shapes of these orbits, though the dimensions remain practically unchanged. As a possible interpretation the present authors suggest that the so-called box orbits might be explained by using the two classical integrals (energy and angular momentum) only.

1 Introduction

Our Galaxy, the Milky Way, is known to contain about 150 globular clusters. With regard that their proper motions are poorly known, or even completely unknown, we cannot be sure that all of them are bound to the Milky Way (MW). For example, Pal 3 has a very high modulus of the heliocentric line-of-sight velocity so that only this velocity component is enough to result in a very large specific (per unit mass) galactocentric kinetic energy that even in the dark-matter presence that cluster can be hardly retained in a finite galactocentric orbit. In the case of, practically, all the other globular clusters (GCs) it might be accepted that they are bound. This circumstance allows us to undertake studies concerning their motion with respect to the MW centre. Due to the proper-motion problem, mentioned above, two different approaches in this matter have appeared. The first of them consists, in fact, of an orbit estimate and, consequently, requires no proper-motion data (*e.g.* Peterson 1974, House & Wiegandt 1977, Ninković 1983). The other one is more rigorous since it requires both the MW potential and the initial conditions to be specified (*e.g.* Brosche *et al.*, 1991, Dauphole *et al.*, 1996).

No matter how we know the potential and the proper motions, the latter approach is more correct and it offers the possibility to get an insight in the real galactocentric motion of GCs. What can be seen from the available results (*e.g.* Ninković *et al.* 1999), is that usually GCs move throughout the Galaxy along real spatial orbits, unlike objects of the galactic disc which almost all the time

remain in the galactic plane, in the vicinity of a given circle. The shapes of these orbits, as well as the influence of the potential parameters, however, deserve further comments.

2 General Comments on Orbit Calculation

The potential of MW is usually presented as stationary and axially symmetric. In such an approach there has been a well-known and long-existing problem. Namely, as an immediate consequence of the steady state appears the integral of energy, whereas the axial symmetry implies the conservation of one component of the angular momentum (that along the axis of symmetry). In other words, we can be sure that there are at least two independent isolating integrals of motion. However, the orbits of MW objects seen in the so-called meridional plane or RZ-plane (R is distance to axis of rotation, modulus of Z is distance to galactic plane) show some properties which are in favour of the possibility that the total number of independent isolating integrals for the two assumptions given above (steady state + axial symmetry) is, nevertheless, three. As can be seen from the figures, the same GC can have rather different orbits caused by a moderate change in only one potential parameter. The differences between, say, two orbits of the same GC are most frequently qualitative, *i.e.* the dimensions in the RZ plane remain, practically, unchanged, whereas the shape of the orbit can be subject of significant changes. These changes of the orbits are not only consequences of the variations in the potential parameters, but also of those in the initial conditions.

3 Results

In the figures given below one can find a number of nice examples for galactocentric orbits of various GCs. These orbits were obtained by applying a particular MW potential, that by Miyamoto *et al.* (1980). Some properties of this potential, of importance to the present work, were described in more details in the paper by Ninković *et al.* (1999). Here it should be pointed out that the pairs of the galactocentric orbits concerning the same GC, seen in the figures, are obtained by varying one of the parameters of the assumed potential, that concerning the flattening of the galactic corona (dark-matter subsystem). In other words, neither the other model parameters nor the initial conditions are subjected to any variations. We have found similar effects also in cases of using alternative potentials (*e.g.* Ninković 1992).

4 Discussion and Conclusion

The present authors find as noticeable that applications of different models (potentials) of MW do not result in significant changes of galactocentric orbits of

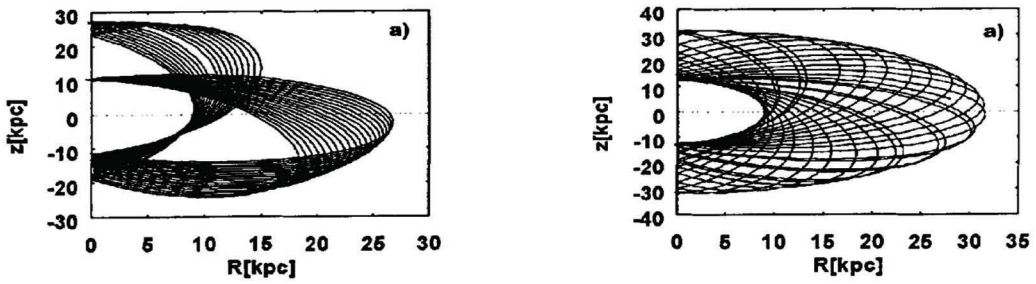

Fig 1. a and b . Orbit of GC 7089 for different values of model parameters

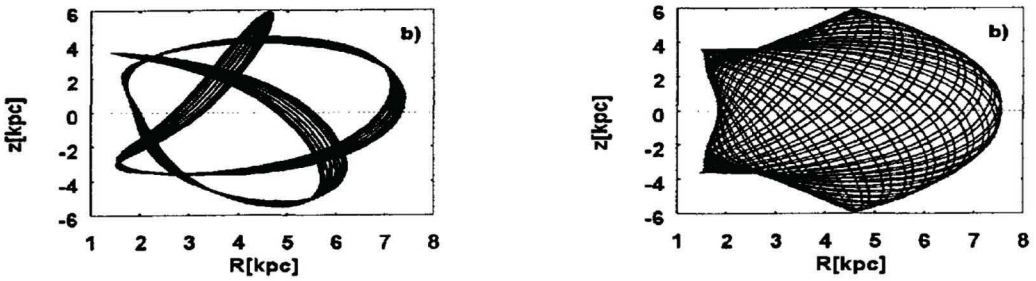

Fig 2. a and b . Orbit of GC 7099 for different values of model parameters

GCs as for their dimensions; of course one here deals with realistic MW models.

However, the uncertainty of the proper-motion data for these clusters is still very large so that the differences in their values between, say, two determinations concerning the same GC usually exceed the error limits indicated by the authors of these determinations. It is quite clear then that the use of two data sets for the same GC based on two different proper-motion sources lead to completely different orbits for the given GC. On the other hand, slight changes of either potential parameters or initial data can result in significant changes of the shape of galactocentric orbits, for instance to convert a box orbit in a tube one and vice versa. Such a change of the shape followed by negligible changes of the orbit dimensions could be due to the existence of a third isolating integral independent of the two classical ones. In the opinion of the present authors the classical integrals (energy and angular momentum) might explain the so-called box orbits alone, without any need for a further isolating integral, independent of them, because they provide natural limits in both R and Z.

Though, at present, no reliable galactocentric orbits for GCs are available for reasons indicated above, it can be, nevertheless, concluded that among GCs groups distincted according to their physical properties, such as, say, chemical composition, clear differences of galactocentric orbits do exist.

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27 A method for determining orbits of small planets and their 3D representation

N. Pejović and M. Marić and Ž. Mijajlović
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The authors propose a method for determining elements of orbits of small planets and comets from data obtained by series of measurements. The particular benefit of the method is seen in the easy and accurate determination of the type of the orbit when the measured points are closely arranged on the trajectory of the body, i.e. the time intervals between observations are small. It is done by computing four quantities related to conics that represent trajectories. Also, software is developed for the 3D graphical representation of trajectories and enveloping surfaces that they make in the course of time in the heliocentric coordinate system.

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A METHOD FOR DETERMINING ORBITS OF SMALL PLANETS AND THEIR 3D REPRESENTATION

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Abstract. We propose a method for determining elements of orbits of small planets and comets from data obtained by series of measurements. The partic ular benefit of the method is seen in the easy and accurate determination of the type of the orbit when the measured points are closely arranged on the trajectory of the body, i.e. the time intervals between observations are small. It is done by computing four quantities related to conics that represent trajectories. Also, software is developed for the 3D graphical representation of trajectories and enveloping surfaces that they make in the course of time in the heliocentric coordinate system.

1. INTRODUCTION

As it is well known, the computing of elements of the orbit of a small celestial body for the given two heliocentric positions r_1 , r_2 in moments t_1 , t_2 is simple if the parameter p of the orbit is known. The problem of finding the parameter p is solved by Gauss (Theoria motus corporum coelestium, 1809) and it is still basic for contemporary computing. In this computation appears a quantity x, the solution of the system of equations:

$$\eta^3 - \eta^2 = mX(x), \quad x = m\eta^{-2} - l.$$

Then the orbit of the body is: elliptic, if x > 0; parabolic, if x = 0; hyperbolic if x < 0.

Now, suppose that we have a series of measurements of positions of a celestial body. The result is the cloud of points, i.e their coordinates that represent the trajectory of the body with certain precision. We propose an algorithm and it's implementation for finding the algebraic equation representing the trajectory. In fact we did more, we proposed the method for finding the equation of the second order surface *S* from a set *SP* of measured points. Then, the trajectory, a second order algebraic curve is obtained simply choosing z = 0 for an appropriately chosen coordinate system.

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The classification of second order surfaces is the key part of the method. We use in the classification two matrices of small order having as elements coefficients of the surface. i.e. it's equation. The surface is determined according two the number of their eigenvalues and their signs. This approach permits simple implementation and good results in recognition of the surface, in particular when the points from the set P are concentrated in the small area on the surface S. The recognition of the surface is performed in the three stages. The first step consists of the finding an approximate equation of the surface, in the second, eigenvalues of matrices are determined, and finally, the type of the surface is concluded. The determination of the equation is done by solving of a system of linear equations, where the method of singular decomposition of matrices is used. Eigenvalues of matrices are computed by the Jacoby method. The algorithm in the details is as follows.

The second order surfaces are presented by the equation:

$$f(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gxz + 2hxy + 2px + 2qy + 2rz + d \quad (1)$$

where $a^2 + b^2 + c^2 + f^2 + g^2 + h^2 \neq 0$

Another form of this equation is:

$$f(x,y,z) = \langle Mr, r\rangle + 2\langle n, r\rangle + d. \quad (2)$$

where $r = [x, y, z]^T$ is the position vector in R^3, u, v denotes the scalar product of vectors u, v, and

$$M = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}, \quad n = \begin{pmatrix} p \\ q \\ r \end{pmatrix}.$$

The advantage of the method we are proposing is that it gives good results when collected data are concentrated in the small area of the supposed surface. This is achieved by introducing the matrix N, usually not considered in other methods, and to which the following theorem refers. This theorem has the crucial role in determination of the type of the surface.

Theorem: Let

$$N = \begin{bmatrix} a & h & g & p \\ h & b & f & q \\ q & f & c & r \\ p & q & r & d \end{bmatrix} \quad (3)$$

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and *r3 = rank M, r4= rank N and δ =det(N)*. Further, *M-sign* and *N-sign* denote the signs of eigenvectors of matrices *M* and *N* respectively. Then the corresponding surface is:

r3	***r4***	***δ***	***M-sign***	***N-sign***	***Surface***
3	4	<0			Ellipsoid
3	4	>0	different		One-sheet hyperboloid
3	4	<0	different		Two-sheet hyperboloid
3	3		different		Conus
2	4	<0	same		Elliptical paraboloid
2	4	>0	different		Hyperbolical paraboloid
2	3		same	different	Elliptical cylinder
2	3		different		Hyperbolical cylinder
2	2		different		Two intersecting planes
1	3				Parabolical cylinder
1	2			different	Two parallel planes
1	1				Two coincident planes

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Imaginary surfaces and degenerate cases (points) are omitted from the table, although they are characterized also by values *r3, r4, δ* and *M-sign* and *N-sign*. The proof of the theorem is long, and we omit it from this presentation.

2. THE ALGORITHM

Suppose that the set P of measurements consists of m points. Instead of determining the equation (1), we are computing the normalized form:

$$k_1x_2 + k_2y_2 + k_3z_2 + 2k_4yz + 2k_5xz + 2k_6xy + 2k_7x + 2k_8y + 2k_9z = -1 \quad (4)$$

having one unknown less. Here $x_2 = x^2$, $y_2 = y^2$ and $z_2 = z^2$. We form the matrix S of measured positions of dimension m × 9,

$$S = \begin{pmatrix} x_2^1 & y_2^1 & z_2^1 & 2y^1z^1 & 2x^1z^1 & 2x^1y^1 & 2x^1 & 2y^1 & 2z^1 \\ x_2^2 & y_2^2 & z_2^2 & 2y^2z^2 & 2x^2z^2 & 2x^2y^2 & 2x^2 & 2y^2 & 2z^2 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ x_2^{10} & y_2^{10} & z_2^{10} & 2y^{10}z^{10} & 2x^{10}z^{10} & 2x^{10}y^{10} & 2x^{10} & 2y^{10} & 2z^{10} \\ . & . & . & . & . & . & . & . & . \\ x_2^m & y_2^m & z_2^m & 2y^mz^m & 2x^mz^m & 2x^my^m & 2x^m & 2y^m & 2z^m \end{pmatrix}.$$

The vector $k = [k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9]^T$ of coefficients k_i is the solution of the system $S \cdot k = P$, where $P = [-1, -1, -1, -1, -1, -1, -1, -1, -1]^T$. The number $m \geq 9$ of measured points might be large, therefore we have more equations (4) then unknowns k_i, so we minimize the function:

$$\Gamma = \sum_{i=1}^{m}\left[\sum_{j=1}^{n} s_{i,j}k_j + 1\right]^2, \qquad S = \|s_{i,j}\|.$$

The minimum of Γ is achieved in the zero of the derivate of Γ along all coefficients k_i, i.e. by solving:

$$\Gamma = \sum_{i=1}^{m}\left[\sum_{j=1}^{n} s_{i,j}k_j + 1\right]^2, \qquad l = 1, .., 9. \quad (5)$$

so k is solution of $Ak = B$, where $A = S^T S$ and $B = S^T P$. This system is solved then by the method of singular decomposition in order to minimize the computing

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error. In fact, $k = VW^{-1} U^T B$ where U, V are orthogonal matrices such that $U^T AV = D$, and D is a diagonal matrix:

$$D = \begin{pmatrix} W & 0 \\ 0 & 0 \end{pmatrix}.$$

The matrix W is also diagonal and it has on the diagonal as elements the eigenvalues of the matrix $A^T A$. The singular decomposition of the symmetric matrix A is given by

$$A = U DV^T.$$

The approximate equation of the surface is not sufficient for determination of the type of the surface. For this we need to find eigenvalues and rank of matrices directly from measured points. Therefore, the next step is to compute the eigenvectors of matrices
M, N. This goal is done using the Jacobi method [1].

This method is iterative, and it reduces the starting matrix $A = \|a_{p,q}\|$ to the matrix in the diagonal form by the sequence of unitary transformations $T_{p,q} = T_{p,q}(\varphi)$, φ is the angle of an appropriate chosen rotation. In each step the non-diagonal element $a_{p,q}$ is annulated, i.e. $a'_{p,q} = 0$ and $a'_{q,p} = 0$ as well by the symmetry of A. As transformations are unitary, they do not change metric properties of the surface. If $A' = T_{p,q}^T A T_{p,q}$ is the iteration step, $A' = \|a'_{p,q}\|$, the angle φ is obtained from:

$$\frac{1}{2}(a_{qq} - a_{pp}) \sin 2\phi + a_{pq} \cos 2\phi = 0$$

i.e. by computing θ from:

$$\theta = \cot 2\phi \frac{\cos^2 \phi - \sin^2 \phi}{2 \sin \phi \cos \phi} = \frac{a_{qq} - a_{pp}}{2a_{pq}}. \qquad (6)$$

Therefore, for $t = \tan\theta = \frac{\sin\theta}{\cos\theta}$, $\theta = (1 - t^2)/2t$, and we take the smaller root $\bar{t}$ of the equation $t^2 + 2t\theta - 1 = 0$. The reason is that to this smaller root $\bar{t}$ value, the rotation angle φ is smaller than 45°, and rotation appears to be more stable. The value t is given by:

$$t = \frac{sign(\theta)}{\sqrt{\theta^2 + 1} + |\theta|}. \qquad (7)$$

Finding the final, diagonal form $\bar{A}$ of the matrix A, we read the eigenvalues on the diagonal of $\bar{A}$, while the number of non-zero elements on the diagonal gives the rank(A). So computed elements are sufficient to determine the type of the surface according to the displayed table.

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A few words about the convergence and program implementation of the algorithm. The convergence is achieved after at most $5n^2$ rotations. Each rotation consists of 4n arithmetical operations, therefore the complexity of the procedure is $20n^3$. The algorithm is first implemented in Matlab. However, it appeared that this package is to slow, so the procedure is implemented again in C++ using OpenGL, the library for graphical presentation. We tested the program on various examples, most of them consisting of several thousands measured points. The obtained results were at least comparable, or better than implementations of other authors.

In astronomical applications we tested procedure for finding distribution characteristics of observations of asteroids Ceres and Pallas. So derived results are quite agreeable with results obtained with other methods.

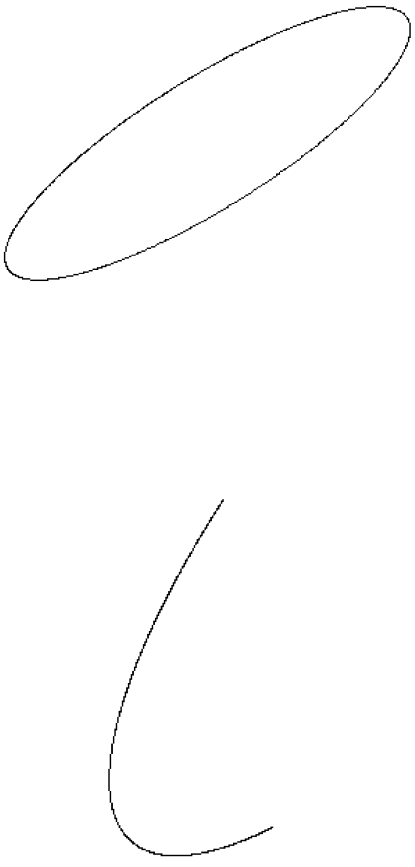

Fig 1. Ellipse and Parabola.

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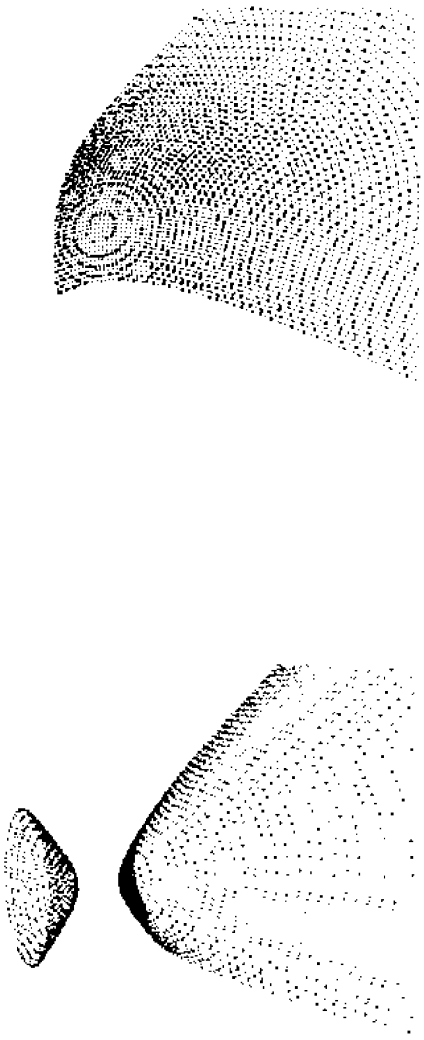

Fig 2. Paraboloid and Hyperboloid

In order to analyze performance of our method, sets of observational data of two largest asteroids in main asteroid belt were downloaded from the public database AstDys (http://hamilton.dm.unipi.it/astdys). We choose as test bodies Ceres and Pallas because of large number of their observations. As a consequence, their

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orbits could be determined with high accuracy. We made several tests with different number of used observations. In almost all cases our method has shown that observations concentrate around an elipse. Our method should be tested on other asteroids, with smaller number of observations. We expact to determine what are the most suitable bodies for appllying our method. Our method could be used in preliminary investigations of distribution characteristics of asteroid observations.

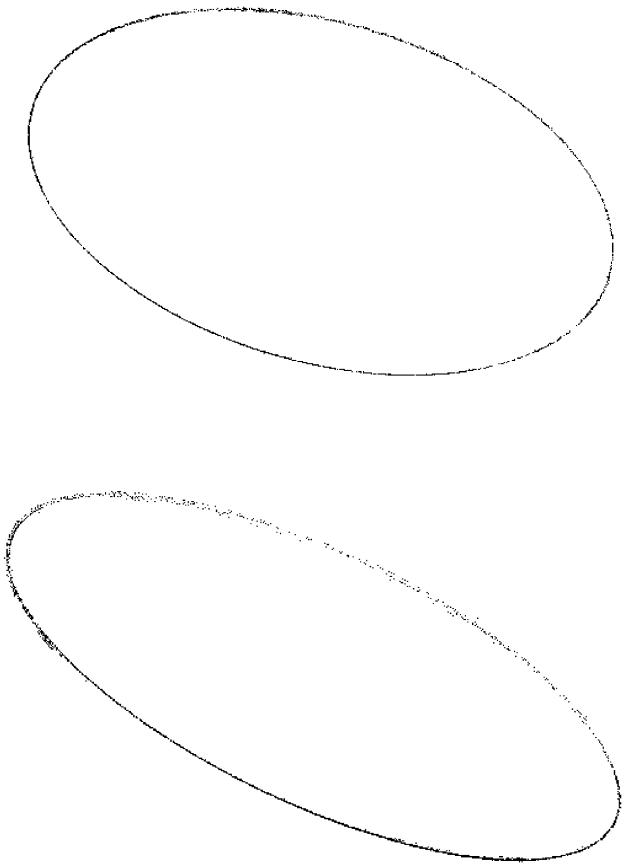

Fig 3. Ceres and Pallas.

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28 Non-Archimedean methods in Cosmology

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The classical study of Standard Cosmological Model (SCM) is based on the mathematics over R, the field of real numbers, or C, the field of complex numbers. Both these structures are archimedean, i.e. they do not admit explicitly infinite quantities. The authors in this paper discuss two nonarchimedean approaches in the analysis of SCM, p-adic analysis and non-standard analysis.

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Non-Archimedean Methods in Cosmology

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Abstract. The classical study of Standard Cosmological Model (SCM) is based on the mathematics over **R**, the field of real numbers, or **C**, the field of complex numbers. Both these structures are archimedean, i.e. they do not admit explicitly infinite quantities. Here we discuss two non-archimedean approaches in the analysis of SCM, p-adic analysis and non-standard analysis.

Keywords: non-standard analysis, p-adic fields, cosmological model
PACS: 02.10.De, 98.80.Jk

INTRODUCTION

p-adic analysis is presented in Cosmology for two decades, in particular in establishing non-archemedean string theory (p-adic strings, adelic cosmology), by many authors: P. G. Freund, E. Witten, A. Yu. Krhennikov, I.V. Volovich, B. Dragovich. The non-standard analysis was less presented, except in explaining certain phenomena in quantum physics (S. Albeverio, J.E. Fenstad, T. Lindstrom, see [**1**]). We remark that infinite quantities reflect certain non-archimedean properties of underlying structure. Our aim is to consider p-adic numbers versus non-standard reals, in particular in regard to the infinitesimal notions, constructions and related transfer techniques. Some applications in the study of SCM will be given. In both cases infinite quantities reflect non-archimedean properties of these fields. However, if $^*\mathbf{R}$ is a nonstandard model of real numbers, then infinitesimals (or infinite numbers) in $^*\mathbf{R}$ arise from non-archimedean property of the order of $^*\mathbf{R}$, while their nature in p-adic fields lies in non-archimedean norm. In either case, some forms of transfer principle hold. It is expressed by Łoś theorem for nonstandard reals. We show that Minkowski-Hasse theorem may be considered as a weak form of transfer principle, while Ax-Kochen-Ershov transfer principle for Hensel fields would be the strong one. It is well known that the ring of p-adic integers $\mathbf{Z}_p$ can be seen as a projective limit. Therefore, we also study which first order properties can be transferred by projective limit to $\mathbf{Z}_p$ and its fraction field $\mathbf{Q}_p$.

INFINITESIMALS

It is generally accepted that Newton and Leibniz independently from each other developed differential calculus. By infinitesimals Leibniz assumed "infinitely small numbers", and he performed usual algebraic operations over them in the same way as he did with real numbers. In particular, each positive infinitesimal ε is lesser than any ordinary (real) positive number, while $1/\varepsilon$ is greater than any ordinary positive number. The central role

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in these considerations plays the **Leibniz principle:**

Every mathematical proposition that is true for finite (real) numbers is also true for the extended system, i.e. the system with infinite numbers, and vice versa.

The major difficulty of Leibniz's approach was a number of paradoxes and a lack of formal framework for consistent foundation of infinitesimal calculus.

In the classical Weierstrass analysis the notion of infinitesimal is replaced by the ε - δ formalism. In particular, zero-sequences, i.e. a sequences that converges to 0, are seen as infinitesimals. However, it is only an auxiliary notion there, and and not all algebraic operations, for example division, are applicable on them.

Abraham Robinson in **Non-Standard Analysis**, Proc. Roy. Acad. Amsterdam, ser. A, 64, 432-440, 1961 solved the 300 years old problem of foundation of infinitesimal calculus. He founded Leibnitz analysis, i.e. introduced actual infinitely small and infinitely large numbers. They admit not only all algebraical operations, but usual mathematical functions, such as sin, cos, exp etc, can be applied over them. Robinson's solution was based on techniques from model theory, a branch of mathematical logic, see [2] and [4].

Here are few facts about nonstandard analysis: by ${}^*\mathbf{R}$ we shall denote an $\aleph_1$-saturated non-archimedean (in the sense of order) elementary extension of the ordered field of reals $\mathbf{R}$. Though $\aleph_1$-saturation provides uniqueness (up to isomorphism) of such structures, there is no canonical representation, such as decimal notation for reals, of nonstandard real numbers.

Theorem (Extension property) Every function $f : \mathbf{R} \longrightarrow \mathbf{R}$ can be extended to the function ${}^*f : {}^*\mathbf{R} \longrightarrow {}^*\mathbf{R}$ which preserves all first order properties of f.

An $a \in {}^*\mathbf{R}$ is *finite* if there is a positive integer n such that $-n < a < n$. By ${}^*\mathbf{R}_{\text{fin}}$ we shall denote the set of all finite elements of ${}^*\mathbf{R}$. The *galaxy* of a is the set $g(a)$ of all nonstandard reals b such that $a - b$ is finite. In particular, ${}^*\mathbf{R}_{\text{fin}} = g(0)$.

It is easy to see that the mapping $\text{st} : {}^*\mathbf{R}_{\text{fin}} \longrightarrow \mathbf{R}$ (*standard part*) defined by

$$\text{st}(x) = \sup_{\mathbf{R}}\{y \mid y \leq x\}$$

is an epimorphism. In particular, ${}^*\mathbf{R}_{\text{fin}}/ker(\text{st}) \cong \mathbf{R}$.

An *infinitesimal* is each finite ε such that $\text{st}(\varepsilon) = \text{st}(0) = 0$. The *monad* of 0 is the set $\mu(0)$ of all infinitesimals. Notice that $\mu(0)$ is closed under addition and multiplication. Further, we say that a and b are infinitely close (denoted by $a \approx b$) if $a - b \in \mu(0)$. In fact, $\mu(0)$ is a kernel of epimorphism st and it is a maximal ideal of ${}^*\mathbf{R}_{\text{fin}}$.

The other monads are got by translations, i.e. $\mu(a) = a + \mu(0)$. By use of homomorphism st one can replace the ε-δ formalism by algebraic identities.

The nonstandard analysis is most useful for proving results in classical analysis, see [5]. The main tools are the Leibnitz principle and the Extension property.

In p-adic analysis: Let p be a prime number and let $\mathbf{Q}_p$ be a field of p-adic numbers. Further, let U be a unit ball containing 0. Then for each $\varepsilon \in U \setminus \{0\}$ and each positive integer n we have that

$$|n\varepsilon|_p < 1.$$

Therefore, we can think of ε as an infinitely small quantity - an infinitesimal.

${}^*\mathbf{R}$ VERSUS $\mathbf{Q}_p$

Here we compare properties of nonstandard reals and p-adic numbers.

${}^*\mathbf{R}$	$\mathbf{Q}_p$
non-archimedean field	non-archimedean norm
infinitesimals: $\varepsilon \approx 0$	infinitesimals: $\lvert\varepsilon\rvert_p < 1$
monads:	*monads*:
$\mu(0)$ reflects	unit ball
certain properties of whole line	
Loeb measure	Haar measure
construction:	*construction*:
ultrapower ${}^*\mathbf{R} = \prod_D \mathbf{R}$	field of fractions of $\mathbf{Z}_p = \varprojlim \mathbf{Z}/p^n\mathbf{Z}$
transfer principle:	transfer principle:
Leibnitz principle	Hasse-Minkowski theorem
$\mathbf{R} \models \varphi$ iff ${}^*\mathbf{R} \models \varphi$	$\mathbf{Q} \models \exists \bar{x}(f(\bar{x}) = 0)$ iff for all prime p
	$\mathbf{Q}_p \models \exists \bar{x}(f(\bar{x}) = 0)$ and $\mathbf{R} \models \exists \bar{x}(f(\bar{x}) = 0)$

Construction. It is well known that $\mathbf{Z}_p$ is the projective limit of rings $\mathbf{Z}/p^n\mathbf{Z}$ ($n \in \omega$), see [3], and that $\mathbf{Q}_p$ is the field of fractions of the ring $\mathbf{Z}_p$. In a certain way the ultrapower construction ${}^*\mathbf{R} = \prod_D \mathbf{R}$ of the nonstandard reals ${}^*\mathbf{R}$ resembles the construction of $\mathbf{Z}_p$, and Łoś theorem is in fact a realization of Leibnitz transfer principle.

Transfer principles. *For each sentence φ of first order language which extends the language of ordered fields it holds*: $\mathbf{R} \models \varphi$ iff ${}^*\mathbf{R} \models \varphi$.

For instance, we can deal with the structure such as $\langle \mathbf{R}, +, \cdot, <, \sin, \cos, \exp, 0, 1 \rangle$ and lift each first order property of it to the adequate ultrapower - nonstandard universe.

In the case of p-adic numbers we do not have such a strong transfer principle. However, Hasse-Minkowsky theorem gives us a kind of downward transfer: if quadratic form $f(\bar{x})$ over $\mathbf{Q}$ has nontrivial zero in each $\mathbf{Q}_p$ (including $\mathbf{Q}_\infty = \mathbf{R}$), then it has a nontrivial zero in $\mathbf{Q}$. Therefore we think of the following theorem as a strong form of transfer:

Theorem [Ax, Kochen, Ershov] Let (K, Z, ord) and (K', Z', ord') be two unramified Henselian fields of characteristic 0, $\mathbf{k}$ and $\mathbf{k}'$ their residue fields. Then the following are equivalent: **a.** $K \equiv K'$ (as valued fields), **b.** $\mathbf{k} \equiv \mathbf{k}'$ (as fields) and $Z \equiv Z'$ (as ordered groups).

${}^*\mathbf{R}$ AND $\mathbf{Q}_p$ IN COSMOLOGY

Nonstandard analysis, based on ${}^*\mathbf{R}$, introduces a specific mathematical method, as well as a way of thinking. It introduces *actual* infinitely small quantities and infinitely large quantities in the mathematical analysis. Therefore, it gives good ground in considering physical systems which in idealized form have infinitely many degrees of freedom. There are a lot of applications of nonstandard analysis in mathematical physics, in particular in quantum mechanics, fluid mechanics, dynamical systems, etc. Definitions

and proofs are more intuitive, and its use is natural and intuitive whenever the considered (idealized) physical system is composed of infinitely many particles. As an example, let us consider Dirac delta function. Let $a(t) = e^{-1/(1-|t|^2)}$ if $|t| < 1$, $a(t) = 0$ otherwise.

This is a simple variation of Cauchy's flat function, and it belongs to the space $\mathscr{E}^\infty$ of infinitely many differentiable functions. Let ε be a positive infinitesimal, and let $b(t) = a(t/\varepsilon)$. Finally, let $k = \int_{-\infty}^{\infty} b(t)dt$ and let $\delta(t) = b(t)/k = a(t/\varepsilon)/k$. Then $\delta(t)$ belongs to ${}^*\mathscr{E}^\infty$, it is positive, and has integral one. In fact, it is what is expected, $\delta(t)$ is a finite compact distribution and it has all properties attributed to the Dirac function.

Further, nonstandard analysis is a consistent and conservative extension of classical analysis. This follows from the ultraproduct construction (consistency), and Łoś theorem (conservativeness). Therefore it cannot produce propositions in the classical mathematical analysis that cannot be proved by means of the classical mathematics. Thus, nonstandard analysis (analysis on ${}^*\mathbf{R}$) is a method of proving, first of all.

However, it introduces new concepts, which do not exist in the classical analysis. For example, ${}^*\mathbf{R}_{fin}$ the finite points of ${}^*\mathbf{R}$ is partitioned into monads:

$$ {}^*\mathbf{R}_{fin} = \bigcup_{r \in R} \mu(r) $$

Remember that $\mu(r)$ is the "cloud" of infinitely closed points to the standard real number r. Thus, if $\mathbf{R}$ is a counterpart of time-line, then it is sound to suppose, at least from the mathematical point of view, that the time is quantized into infinitely small parts (monads), each part having rather rich structure. Similar observation could be asserted for the space $\mathbf{R}^n$, if we are looking into the nonstandard universe ${}^*\mathbf{R}^n_{fin} = \bigcup_{r \in R^n} \mu(r)$, now $\mu(r)$ consisting of space-points infinitely close (as defined in nonstandard analysis) to r. If we could try to connect this mathematical notions to the physical space and time, good candidate would be the Planck distance and Planck time. Nonstandard analysis tells us that it works very well on these structures as well. So, if ${}^*\mathbf{R}$ is accepted as a starting mathematical model for the physical space and time, we can make some conclusions on physics what it would be like below these bounds, even if space-time parts of the Universe below this scale are not observable. If it so, we almost came to the Leibniz principle, that the properties of the World are reflected into monads.

p-adic mathematics is another story. Namely, as we have seen, there is no transfer principle valid for all first-order sentences. Hence, mathematical analysis based on $\mathbf{Q}_p$ differs from the classical mathematical analysis. It does not mean that it is inconsistent, it is simply an alternative to the classical mathematical analysis. Therefore, one should expect that physics based on $\mathbf{Q}_p$ is an alternative to the classical approach to physics.

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29 Spectroscopy at the ASV and Cooperation with Belogradchik Observatory

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The astronomical station at the mountain Vidojevica (ASV) nearby the town of Prokuplje was finished in 2008. A part of the research at the ASV was planned to be dedicated to the stellar spectroscopy. The Astronomical Observatory in Belgrade provided a spectrograph for this purpose. In the town of Belogradchik in Bulgaria, nearby the Bulgarian-Serbian border, a small astronomical observatory of the size as ASV is situated and equipped similarly as ASV. The aim of this paper is to compare the equipment of these observatories and discuss possible cooperation between them, in particular in those areas of astronomy in which the stellar spectroscopy plays an important role.



Spectroscopy at the ASV and Cooperation with Belogradchik Observatory

Nadežda Pejović and Žarko Mijajlović

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Spectroscopy at the ASV and Cooperation with Belogradchik Observatory

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Abstract. The new astronomical station at the mountain Vidojevica (ASV) nearby the town of Prokuplje will be finished till the end of 2007. A part of the research at the ASV will be dedicated to the stellar spectroscopy. The Astronomical Observatory in Belgrade provided a spectrograph for this purpose. In the town of Belogradchik in Bulgaria, nearby the Bulgarian-Serbian border, a small astronomical observatory of the size as ASV is situated and equipped similarly as ASV. The aim of this paper is to compare the equipment of these observatories and discuss possible cooperation between them, in particular in those areas of astronomy in which the stellar spectroscopy plays an important role.

Keywords: astronomical observatory, stellar spectroscopy, photometer, telescope
PACS: 95.75Fg; 95.55Qf

ASTRONOMICAL STATION AT VIDOJEVICA

The idea of building an astronomical station at Vidojevica (ASV) is more than three decades old. The project was almost forgotten due to the circumstances that appeared in the former Yugoslavia during the last two decades. Two institutions, Faculty of Mathematics and Astronomical Observatory in Belgrade (AOB) renewed the project and helped by the Serbian Ministry of Science began raising ASV in 2003. The first stage ASV1, including a 60 cm reflector should be finished in 2007. The site was chosen according to good astronomical conditions. The peak of Vidojevica was the best observing site even in the former Yugoslavia, with the clearest sky and very good atmospheric conditions. ASV is located at the peak (1155 m, N 43^0 08.706', E 21^0 33.982') of Vidojevica in Southern Serbia and it is about 20 km far by road from the town of Prokuplje. The peak is almost flat and occupies about 1000 m^2. The infrastructure at the site is good. The sky is open, particularly to the south. The light pollution is negligible, but there are some traces of light pollution to the north, coming from the town of Prokuplje. Astroclimate conditions are also good, in fact best in Serbia. According to meteorological measurements, some of them dating from 1900, most of them since 1950, there are on the average about 120 clear nights a year. This region is one of the driest areas in Serbia. In Prokuplje the total rainfall is 580 mm, while on Vidojevica it amounts only 533 mm. Measurements in the last two decades show even decreasing of rainfall (the rainfall was above 600 mm until 1980). Other collected data include temperature, rainfall, relative humidity, insolation, haze, cloudiness and number of days with clear sky and all of them show that this site is advantageous for astronomical observations. More details on ASV can be found at http://www.asv2.matf.bg.ac.yu and [1].

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The equipment of ASV

The ASV telescope. The telescope is of the reflector type and it is produced by the German company ASTROOPTIK. It has Lomo Sitall optics, primary focal ratio f/3 and system focal ratio f/10. The servomotor encoders are with < 0".08 resolution, while the maximum slow rate is 3^0 per second. The software and the controlling system of the telescope are based on a PC WIN XP operating system with standard functions for computerized telescopes. It admits also remote control. The accessories include a reducer (0.6x for an effective focal length of 2500 mm), field corrector, of axis guider and high-accuracy encoders (Heidenhain ERA 780 C Encoders with a very large diameter [473 mm] and a very high resolution in both axes [0.2" accuracy per axis]).

Stellar spectrograph SpectraPro 750. This spectrograph is intended for use primarily at ASV telescope. According to [3], the spectrograph has been planed for the recording of low resolution spectra of relatively faint stars and asteroids, medium-resolution spectra of relatively bright stars and studies of the variations in highly broadened spectral line profiles. It can be used also for determination of gas movements in circumstellar disks and radial velocity measurements of close binary stars. In short, the spectrograph is intended for spectral recording and analysis in spectral regions from violet to infrared. The spectrograph is of the Czerny-Turner type and it is produced by the Action Research Company. It has the focal length 750 mm, the aperture ratio f/9.7, resolution 0.023 nm at 435.8 nm, 10 μm slits, dispersion 1.1 nm/mm (nom.) and accuracy $\pm$0.1 nm. The test results with this instrument can be seen in [3] and [4].

BELOGRADCHIK ASTRONOMICAL OBSERVATORY

The Belogradchik Astronomical Observatory (BAO) of the Institute of Astronomy, Bulgarian Academy of Sciences, is situated in the North-Western part of Bulgaria (630 m, N 43^0 37' 35", E 1^h 30' 42"). There are two telescopes: a 60 cm Cassegrain built by Carl Zeiss and a 36.5 cm Schmidt-Cassegrain by Celestron. The 60 cm Cassegrain (600/7500) telescope was equipped with a single-channel electro-photometer without a cooling system and without an offset for guiding. In 1997 the telescope was also equipped with SBIG ST-8 CCD camera. Since 1990 at Belogradchik Observatory has began research of fast brightness variations in chromospherically active stars, cataclysmic, symbiotic stars and asteroids. This kind of research required patrol observations with short integration times of several hours duration. For more than decade, BAO together with NAO-Rozhen has a cooperative program with observatories in Ukraine and Stephanion Observatory in Greece. This project included high-speed photometric observations of short-duration and low-amplitude stellar variability. There is also cooperation with Serbian astronomers, leaded mainly by Milan Dimitrijevic on the Serbian side and Milcho Tsvetkov on the Bulgarian side.

BAO Universal Photometer. The Bulgarian-Ukrainian Universal Photometer was designed in the Laboratory of the Institute of Astronomy, Bulgarian Academy of Sciences and at the Main Astronomical Observatory, National Academy of Sciences of Ukraine. The photometer has a modular structure. It can be used both for single-channel photo-

electric photometry and for CCD photometry. The photoelectric photometer is based on a EMI 9789 QA photomultiplier cooled by a two-cascade Peltier. The main components of the photometer are: photometer head, control and counting module, photon registering module, CCD camera, computer and software. According to [2] the photometer is capable of working with the 60 cm Cassegrain telescope at Belogradchik Observatory and with the 2 meter RCC telescope at the Bulgarian National Astronomical Observatory-Rozhen. It is suitable for observing variable stars, galaxies, stellar clusters, asteroids. A major advantage over the old photometric system is the cooling system, which decreases the dark current level significantly. Another benefit of the instrument is the offset, which allows guiding during observations. This makes possible longer CCD exposures and continuous patrol photoelectric observations that need not be interrupted to center the object in the diaphragm.

THE PLANNED PROJECTS

Shapes of a spectral lines. We shall be concerned particularly with the shapes of spectral lines. The appearance of a spectral line in a star's spectrum is influenced by a number of physical processes occurring in the stellar atmosphere. A line's shape or profile is described as being weak, strong or very strong according to its intensity. The three main processes that affect the shape of a spectral line are known as collisional broadening, Doppler broadening and rotational broadening. There are additional effects, for example the Zeeman effect which can cause splitting of the spectral lines. The aim of the project is to study the shapes and splitting of spectral lines arising from these processes and in connection with this calculation of particular properties of stellar objects e.g. their rotation.

Measuring radial velocities. The aim of the project is the determination of the radial velocities of stellar objects by measuring the Doppler effect, i.e the shifting of spectral lines in their spectrums. Let us remind that that the transverse velocity, i.e. the proper motion of the stelar object, does not affect the wavelengths of the spectral lines and can be disregarded when measuring radial velocities.

Cooperation with BAO Due to similarities in size and equipment between the Astronomical station at Vidojevica and Belogradchick astronomical observatory, we consider that the the cooperation between these two observatories can be very fruitful. The cooperation would consist of exchange and checking collected data, scientific and student training and exchange and organization of local scientific meetings.

CONCLUSION

We believe that the new astronomical station at the mountain Vidojevica and proposed projects will significantly help the preparation of groups of Serbian scientists in the fields of astronomy and applied mathematics for future Community research. By upgrading and reinforcing ASV by low budget-equipment, it will help technological development policy activities too, including contemporary scientific research. In particular, the cooperation with Belogradchik astronomical observatory and other similar observatories in

the neighborhood will strength the partner network activities: exchange and improvement of measured data, conferences, workshops, studies and analyses, working groups and expert groups, operational support and dissemination, information and communication activities. Along this course, the ASV Project just to be finished has a strong support from the Serbian leading scientific and high-level educational institution, the Belgrade University.



FIGURE 1. Positions of Astronomical station at Vidojevica and Belogradchick astronomical observatory. The air distance between ASV and BAO is about 120 km. The distance between ASV and BAO by road is about 200 km.

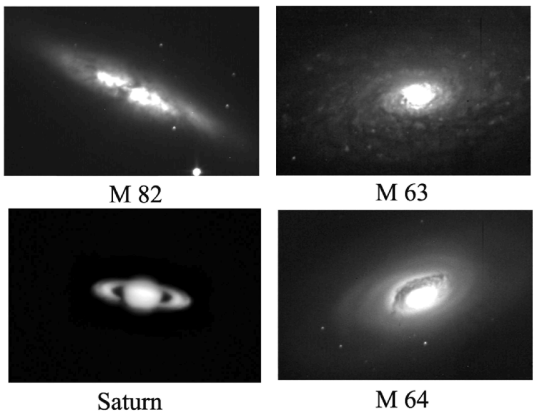

FIGURE 2. Photos of cosmic objects made by BAO telescope during authors visits to Belogradchick, March 2007.

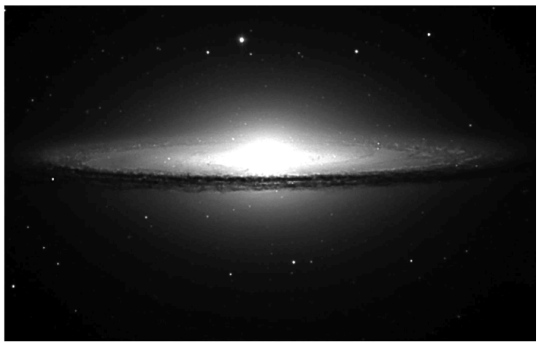

FIGURE 3. Photo of M 104 (Sombrero galaxy) made with the telescope that will be situated at ASV.

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30 The Project ASV

N. Pejović and Ž. Mijajlović
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The initiative for building of an astronomical station at the mountain Vidojevica appeared in 2000, while the construction of the station started in 2004, and the ending of the first stage is expected in the autumn 2006. The aim of the paper is to present the idea of this project, the main technical characteristics of the station, equipment and scientific projects that are planned to be carried out when the station becomes operational.

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The Project ASV

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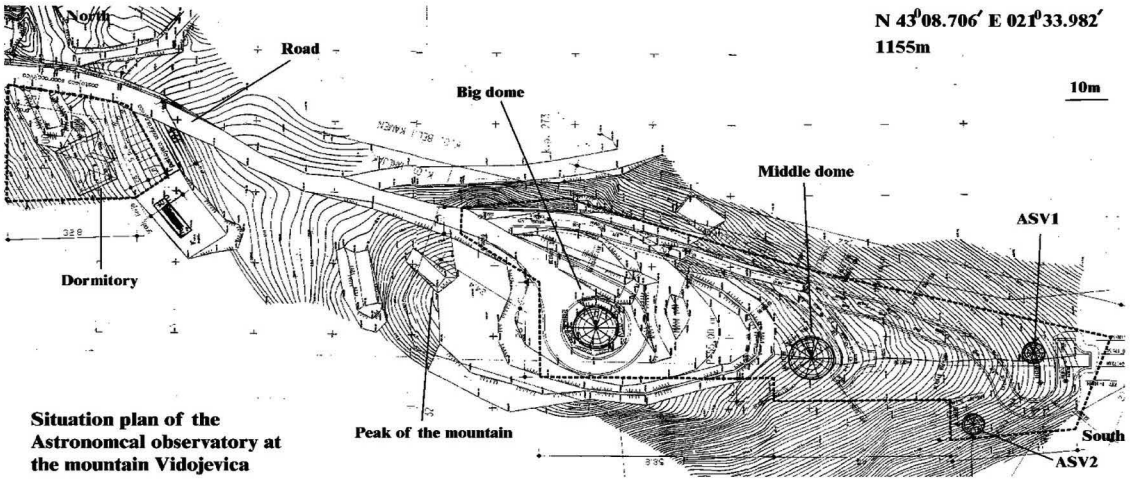
Abstract. The initiative for building of an astronomical station at the mountain Vidojevica appeared in 2000, while the construction of the station started in 2004, and the ending of the first stage is expected in the autumn 2006. The aim of the paper is to present the idea of this project, the main technical characteristics of the station, equipment and scientific projects that are planned to be carried out when the station becomes operational.

Several astronomers from the Faculty of mathematics of the University of Belgrade and the Astronomical Observatory in Belgrade (AOB) started in the year 2000 the joint initiative for building the astronomical station (ASV) at the mountain Vidojevica, near Prokuplje in southern Serbia. In fact, the idea of building an astronomical station at Vidojevica is more than 20 years old. The project was postponed, almost forgotten, due to the circumstances that appeared in the former Yugoslavia during the last two decades. Eventually, the Serbian Ministry of Science decided in 2003 to allocate funds to AOB for building ASV, as well as for equipping it. Now, the first phase of the project is in the final stage, and it is expected that ASV will start operating during the autumn of 2006. The first phase, ASV1, consists of an observatory dome equipped with a reflector of 60cm aperture, and the living house for crew. The planned research themes will include inertial frames, exploration of binary stars, spectroscopy, etc. It will be used also for training students of astronomy. Further plan include robotization and automatic guidance of the system. It is also planed obtaining another identical telescope with the purpose of combining both telescopes into one system, a multiple telescope. The goal is to do optical and near-IR interferometry. This is seen as an inexpensive way to build a big mirror (in this case of 85cm aperture). The CHARA array is the most famous project in the world of this kind. Forming a very precise time base is assumed, while mathematical methods will include image processing and wavelet theory.

1 Site

The whole complex ASV is situated at the peak of the mountain of Vidojevica (1155m, N $43^0$08.706′ E $021^0$33.982′) in Southern Serbia and it is about 20 km far by road to the South from the town of Prokuplje (30.000 population). Other places relatively close to ASV are Niš (about 30 km north-eastward, 250.000 population) and Leskovac (about 30 km south-eastward, 50.000 population). The peak is almost flat and occupies about 1000 m^2. The infrastructure at the site is good. The main road is in good conditions and it passes about 2.5 km from the station. Also, there is a macadam road connecting ASV and the main road. The site is supplied with electric power and there is a wellspring nearby. There are no significant obstacles on the horizon. The sky is open, particularly to the south. The light pollution is negligible, there are some traces of light pollution in the north, coming from the town of Prokuplje.

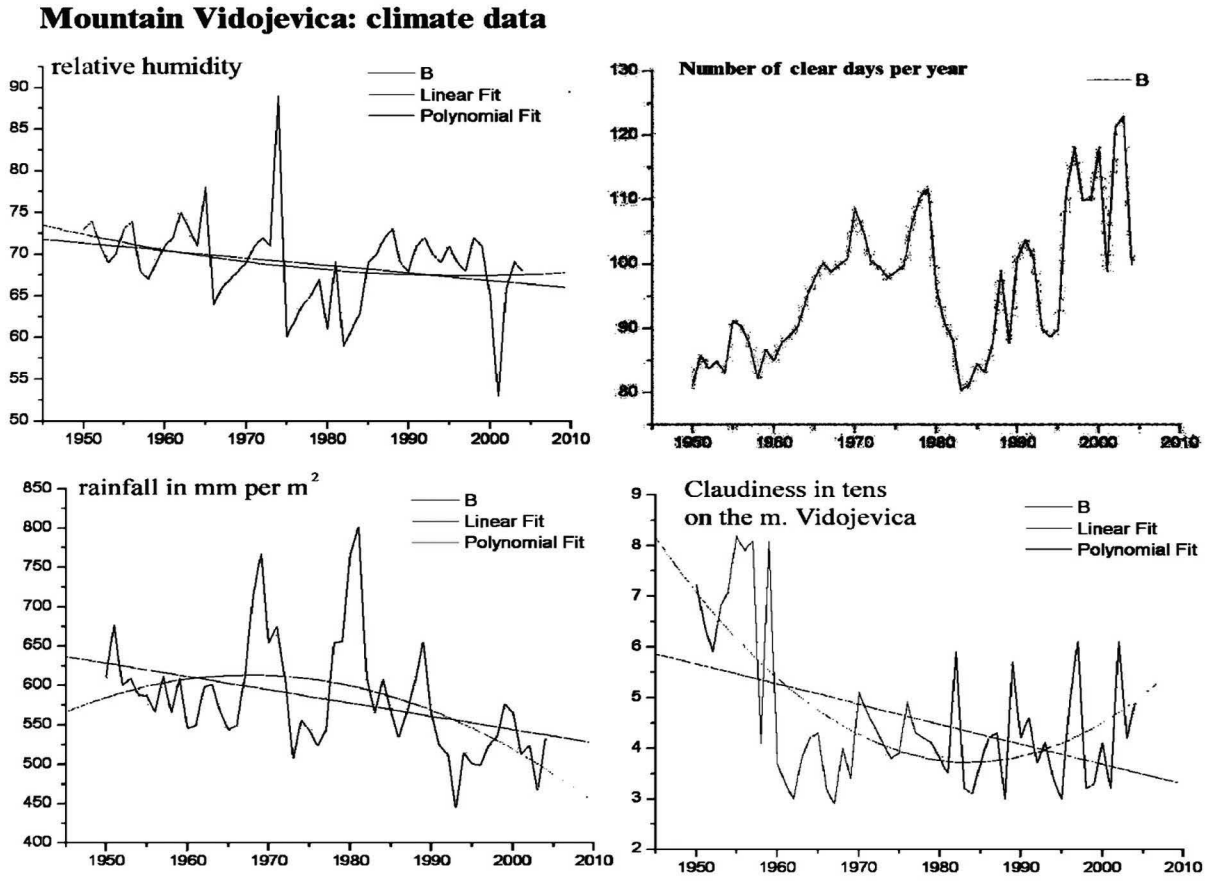


Situation plan of the Astronomcal observatory at the mountain Vidojevica

2 Astroclimate

Astroclimate conditions are also good, in fact best in Serbia. According to meteorological measurements, some of them dating from 1900, most of them since 1950, there are on the average about 120 clear nights a year. This region is one of the driest areas in Serbia. In Prokuplje the total rainfall is 580mm, while on Vidojevica it amounts only 533mm. Measurements in the last two decades show even decreasing of rainfall (the rainfall was above 600mm until 1980). Other collected data include temperature, rainfall, relative humidity, insolation, haze, cloudiness and number of days with clear sky and all of them show that this site is advantageous for astronomical observations.

The microclimate characteristics of the mountain of Vidojevica is rather interesting. Vidojevica is deeply extended into the continental climate, therefore there is the influence of this type of climate in this region. Vidojevica is under snow



1-2.5 months. The temperature on the peak of Vidojevica is lower about 4°C in comparison to the temperature of Toplica valley, in accordance to the temperature gradient principle. However, in wintertime, temperature inversions often appear. It is interesting that the river Toplica which crosses Toplica valley is the coldest hydrological object in the region in summertime. It could be colder up to 7°C than objects on the land. This produces the following interesting phenomena. The colder air along the river keeps under the worm air along the mountain, and on the peak of Vidojevica (nearly flat, area about 4 ha) appears the stream. There it splits into two parts, one directed to the Toplica valley, other goes to the atmosphere. This streaming on the peak can be sensed on the human body, as the temperature difference can amount up to 10°C.

3 The Project ASV

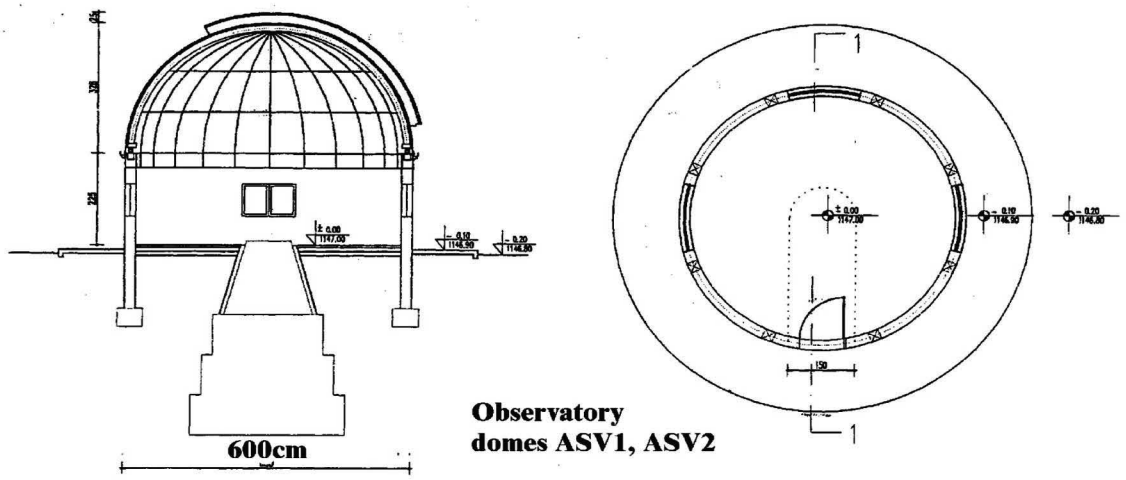
As it was aforementioned, the project is a renewal of an idea, old almost 30 years. Namely, it was planned in the 80's of the last century to build an astronomical station comprising a reflector of 2m aperture at the adjacent mountain Rgaj (1050m). Due to various circumstances, this project was not realized and



it was almost forgotten. It is interesting how this idea appeared again and it should be mentioned here. In mean time, an astronomical society, *Magellan's clouds* appeared in the town of Prokuplje. Members of the society, in particular Aleksandar Valjarević and Aleksandar Simonović, were searching for a good place where to place a small dome for the society's telescope. Finally, they came on the peak of Vidojevica, and introduce senior members of the society, Stevo Šegan and Žarko Mijajlović, professors of the Faculty of mathematics in Belgrade (MATF), with the possibility of building the dome there. They recognized at once the value of the site and recalled the old idea of building of an astronomical station here. The idea was accepted and supported by Milan Dimitrijević, then the director of AOB, and the following director Zoran Knežević who started the realization of the project. Of course, many other people were involved, and they are still working on it: Ištvan Vince (AOB), Nadežda Pejović (MATF), Gojko Djurašević (AOB), Slobodan Ninković (AOB), Vojislava Protić-Benišek (AOB), Dušan Ćirić (Department of mathematics of the University of Niš), many people from the community of Prokuplje and others.

The original project of the whole complex comprises four observatory domes. First, two identical small observatory domes are projected. Telescopes of apertures up to 65cm can be mounted there. According to the project, the third (middle) dome can contain a reflector up to 1 m, while the fourth and the largest dome is projected for a reflector of 2 m aperture. The living house for the crew is about $200m^2$ and it is planned for housing 12 persons. The first stage of the project, ASV1, comprises the building of one small observatory dome and living house, and it should be finished during 2006. A telescope of 60 cm aperture has been already bought with some accessories and one spectrograph.

The total cost of the first stage, ASV1, of the project is about 350000EU, and it is financed completely by the Serbian Ministry of Science.



Observatory domes ASV1, ASV2

Here are some technical details of the telescope. The 60cm-reflector is produced by the German company ASTROOPTIK, it has Lomo Sitall optics, primary focal ratio $f/3$ and system focal ratio $f/10$. The servomotor encoders are with $< 0''.08$ resolution, while the maximum slow rate is 3^0 per second. The software and the controlling system of the telescope are based on a PC WIN XP operating system with standard functions for computerized telescopes. It admits also remote control. The accessories include a reducer ($0.6\times$ for an effective focal length of 2500mm), field corrector, of axis guider and high-accuracy encoders (Heidenhain ERA 780 C Encoders with a very large diameter [473mm] and a very high resolution in both axes [0.2" accuracy per axis]). As can be seen from the following picture taken with the same telescope, stars of the 21st visual magnitude are observable.

However, a group of Serbian astronomers started to think about an alternative as a change of the original project. The alternative would consist of building an array of small identical telescopes (at least three of them, each of 60 cm aperture). They would make a multiple telescope for doing optical and near-infrared interferometry. For this reason, the Faculty od Mathematics applied for the project *Reinforcing research potential in astronomy and applied mathematics* by the European Union under FP6 framework. The project is managed by prof. Nadežda Pejović (MATF). In fact, the main project objective is to reinforce and improve research capacities in astronomy and applied mathematics in Serbia and Western Balkan (WB). It is planned to do that by upgrading and raising scientific and technological potentials of ASV and exchanging and training scientists and students through networking with other European research centers and neighboring countries. This stage of the project is called ASV2. Our partners in the project would be: **a.** Instituto de Astrofísica de Andalucía, Granada, Spain, key person: Dr Emilio J. Alfaro. **b.** Astronomical Institute, Academy of Sciences of the Czech Republic, Prague, key person: Dr. Jan Vondrak **c.** Oxford University



The picture of m104 taken with Astropotik 60cm-reflector.

Computing Laboratory, Numerical Analysis Group, United Kingdom, Oxford, key person: Prof. Endre Süli. **d.** School of Informatics, University of Manchester, Manchester, United Kingdom, key person: Dr. Goran Nenadić.

4 ASV2 in Details

The project ASV2 assumes obtaining of a computerized telescope, identical to the already mentioned ASTROOPTIK reflector, then a small supercomputer consisting of a cluster of PC computers and a GPS equipment necessary for a precise time base and the telecommunication equipment. The project consists of the following section parts.

4.1 The multiple telescope

The purpose is to prepare the system to do optical and near-IR interferometry. This is considered as an inexpensive way to build a big mirror. A further enlargement of the array of telescopes will be possible. The classical principle of interferometry has been little present at optical wavelengths. Interferometry helps to improve the resolving power of an instrument (a telescope). For instance, by synthesizing several small apertures it is possible to achieve the effect of a large aperture. This improvement can be, quantitatively, to an order of magnitude. The base can attain even 100 m. At the given location such a base is available to us. The first experiments are planned to be performed with two already mentioned identical telescopes over a base 70 m long. One such a telescope

is already purchased and it will be settled at ASV during the summer 2006. A multiple telescope of an equivalent aperture of 85 cm will be obtained. The necessary time base will be provided through a qualitative station possessing a GPS which will be based in future on European system Galileo. Mathematical methods will include image and signal processing, frequency analysis and wavelet theory. Additional mathematics includes numerical solutions of ordinary and partial differential equations for solving certain astronomical problems, e.g. in celestial mechanics, then developing algorithms for error correction for applying to the GPS system.

4.2 Robotization and automatic guidance for this system

This will permit a distant guidance and use of each telescope, separately, as well as the system as a whole. Besides, high-tech advanced modern mathematical methods will be used, e.g. the Groebner basis as an important tool in modern computational algebra and analysis. Bearing in mind that a low-budget station is raising, open-source application will be used, where it is appropriate for a professional use. Namely, the revolution in astronomy does not stop at the hardware. Research-grade telescopes at observatories from Spain to Korea are under the control of open-source software and Linux based computers. Under the open-source model, we are free to modify the control software. With a source code freely available, a peer review now occurs not only on the data, but on the gathering as well. In our research, we consider the forefront on this open-source astronomy in *Talon* (can be downloaded at observatory.sourceforge.net). Talon was originally developed by Ellwood Downey as the Observatory Control and Astronomical Analysis Software (OCAAS). As already mentioned, a part of the controlling software for the telescope that is foreseen for purchasing is coming with the telescope, too.

4.3 Data transfer and processing

The objective is an on-line use of the system, transfer of collected data, their storing and manipulating for research purposes. In the previous item we described the intended software platform. Firstly, we plan to develop software for remote control of the multiple telescope and automatic guidance, as of sky patrolling. Secondly, we are planning to develop an application for efficient on-line transfer of so collected data, but off-line, as well. We consider an open-source software for this task, too. For example, a part of this task can be done by software package Talon, as it supports network operations, using X session. However, the quantity and complexity of data that are obtained by astronomical observations may grow rapidly. Therefore, there is an apparent need to provide an efficient data storage, access and processing. In particular, the research objectives would be a sofisticated data mining to discover patterns of any kind, including images and numbers. In particular, we consider a pattern recognition for use in inter-

ferometry. Namely, data on light waves collected by telescopes are digitized by DSP processors and so a sequence of numbers is obtained. If two synchronized identical telescopes are used in the same time interval, two nearly equal sequences of numbers will be obtained with a small shift on the time line. The pattern recognition will be used for finding the same base point on the time line for both sequences of light waves. This will enable one to post-process data for the purpose of interferometry. A good time base is needed, of course. We consider this approach as an innovation in interferometry, or generally in astronomy, versus the methods applied elsewhere.

5 Research

The following scientific programs are planned to be performed immediately after finishing the ASV1 stage. Of course, some of them will be possible only after finishing the ASV2 stage.

5.1 Time base

As it is well known, the Official Source of Time for the WB region does not exist. It is a very important service for reintegrating the WB region in the European traffic, industry and defence systems, as well as the navigating one. The standard unit for the measurement of intervals of time is the SI second, which is defined by adoption of a fixed value for the frequency of a particular transition of caesium atoms. Caesium frequency standards, hydrogen masers, ion storage devices, and other such devices are able to count seconds and subdivide them very precisely. Thus, such a device can provide a time base and a time-scale whose accuracy is dependent of the measurement precision and the device stability. The results of the measurements of about 200 frequency standards located throughout the world are combined to form a standard time-scale that can be used for identifying uniquely the instants of time at which events occur on the Earth. This standard time-scale is known as International Atomic Time (TAI). It is the basis for all time-scales in the general use. It is distributed by many different means, including radio time signals; navigation systems such as GPS (Global Positioning System), LORAN-C, and OMEGA; communications satellites; and precise time standards. Both the Transit satellite system and the Global Positioning System (GPS) can be used for one-way time transfer. Although the GPS system is designed for navigation purposes, it also coordinates most of the major time-scales in the world. The GPS system has an accuracy of 10 ns for ten-day averaging. The most accurate time comparison by GPS is based on a common satellite procedure. GPS satellites on a flyby mode make two separate comparisons with respect to the GPS satellite clock. A linear rate for the satellite clock is assumed. This method does not provide the same accuracy as the common view procedure, but the accuracy is enough for the astronomical

events registration, among them, for the optical interferometry reduction of the observations. The differential delay effects caused by the Earth's atmosphere and other different sources should not exceed a few nanoseconds. Time has a specific purpose in astronomy. The observational data, with precise time identification by GPS one-way transfer, are then brought together for correlation so that precise values for the differences between the clocks at the individual telescopes can be determined. A base of about 100 m will provide to establish the coordinate accuracy by an order of magnitude better than that obtained with a single telescope. To achieve this accuracy, 400-850 nm bandwidths are required. This process requires distinct measurements based on prearranged experiments involving different locations. The results of the time comparisons are not available unless the two data sets are brought together and correlated. In theory, a sufficiently large data sets are available and sufficiently large bandwidths can be obtained, the synchronization method is mainly limited in accuracy by the difficulty of determining the overall system delay and the atmospheric delay.

5.2 Other astronomical programs

Here we shall mention in short other types of astronomical research at ASV:

1. Monitoring of Main belt asteroids and near - Earth asteroids.

2. Planetary transits.

3. Investigation of binary stellar systems, in particular eclipse binary stars and close binary stars.

4. Monitoring of Solar activity.

5. Processing of photometric and spectroscopic data.

6. Synthetical spectra of stars.

7. Writing software for guiding telescopes and domes.

8. Education and training students of astronomy.

6 Conclusion

The Project will significantly help the preparation of groups of Serbian scientists in the fields of astronomy and applied mathematics for future Community research. By upgrading and reinforcing ASV by low budget-equipment, it will help technological development policy activities too, including monitoring and assessment activities. In particular, new experiences and connections with European centers will be obtained through partner network activities: conferences, workshops, studies and analyses, working groups and expert groups, operational support and dissemination, information and communication activities. Along this course, the ASV2 Project has a strong support from four leading European

scientific institutions and the Serbian leading scientific and high-level educational institution, the Belgrade University.

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31 On the motion in the Local group of Galaxies

S. Ninković and N. Pejović
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The authors study the Local Group of Galaxies approximating it as a binary galaxy (Milky Way and Andromeda Nebula). They find that inside 100 kpc from the Milky-Way center the influence of the Andromeda Nebula is very weak and that for the satellites of the Milky Way, even for the outer ones, perigalactic distances of about 30-40 kpc are likely.

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On the Motion in the Local Group of Galaxies

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Abstract. The authors study the Local Group of Galaxies approximating it as a binary galaxy (Milky Way and Andromeda Nebula). They find that inside 100 kpc from the Milky-Way centre the influence of the Andromeda Nebula is very weak and that for the satellites of the Milky Way, even for the outer ones, perigalactic distances of about 30-40 kpc are likely.

Keywords: Local Group of Galaxies, globular cluster, motion, galactic halo, galactocentric distance
PACS: 98.35.Gi, 98.20.Gm, 98.35.Ce

INTRODUCTION

The Local Group of Galaxies consists of a few tens of galaxies. The total number of galaxies in it is still unknown. The problem is that, most likely, a vast majority of them are dwarf ellipticals, galaxies difficult to be observed and, consequently, their detection is also difficult. Besides, within the Local Group (in the further text will be referred to as LGG) several globular clusters, which cannot be identified as members of a particular LGG galaxy, have been found. Therefore, in addition to galaxies LGG also contains globular clusters. It is very likely that the total number of these (common) LGG globular clusters is very large because those where a rather strong evidence exists, have been all found in the part of space dominated by our own Galaxy (Milky Way - MW).

What is rather well known is that in LGG there are only three spiral galaxies, the morphological galaxy type largely known to be connected with giant galaxies; these are MW, the Andromeda Nebula (AN) and M33. Other galaxies are mostly irregular (say NGC 6822 and IC 1613), but, as said above, dwarf ellipticals prevail and also almost all these dwarf ellipticals are satellites of giant galaxies. MW is known to have about ten satellites (not including globular clusters) and, except the two Magellanic Clouds, all of them are dwarf ellipticals. AN is also known to have several satellites (also dwarf ellipticals) and, probably, its total number of satellites is much larger. Thus dwarf elliptical galaxies, members of LGG and having been detected as yet, seem to be largely satellites of giant galaxies. Are there, really, field dwarf elliptical galaxies in LGG? To answer this question one should know the formation mechanism of galaxies in LGG. With regard that a number of theories aimed at throwing more light on it has been developed, the formation mechanism seems not to be quite well known.

The position of the present authors is to study in more details the motion within LGG.

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MILKY-WAY MASS DISTRIBUTION

Globular clusters, as have been mentioned above, are situated, largely in the halo of MW. The majority of them are found at galactocentric distances less than, say, 25 kpc. On the other hand, the space usually associated with MW has been significantly enlarged during last 20-30 years, above all due to the dark-matter hypothesis. Therefore, a MW halo, in its very wide concept, might be thought of as a sphere with radius not less than 100 kpc. Bearing in mind the spatial distribution of its objects one may introduce an inner halo and an outer one. How to put a frontier between them? No doubt, there are many ways, but from the point of view of a study concentrated to LGG it seems reasonable to take into account the galactocentric distance of the closest, but still conserved, galaxy. This is certainly the Large Magellanic Cloud (LMC) situated at 52 kpc from the MW centre (the data about the galaxies from LGG cited here are from [1]- p. 259). Also, at the distance of LMC there is already, practically, no difference between the distance to the MW centre and the heliocentric one (the present authors for the galactocentric distance of the Sun assume the value of 8.5 kpc recommended by IAU). Such distance ratios in the triangle MW centre-Sun-LGG object mean that the angle between the heliocentric and galactocentric directions towards an LGG object is negligibly small, in other words the line-of-sight velocity of any distant object is, as usually, corrected for the galactocentric motion of the Sun only to obtain the radial component of the galactocentric velocity for this distant object. This is, obviously, an advantage; knowing the line-of-sight velocity and the galactocentric motion of the Sun we know immediately one component of the galactocentric velocity (radial) for a given object. Nowadays for close LGG objects, such as the two Magellanic Clouds, we can even know the proper motion [2]. Finally, the reason why the Magellanic Clouds are still conserved, as said above, is very likely that their galactocentric orbits have no points too close to the galactic centre. The debris of other galaxies in the interior parts of MW have been found. Besides, the existence of the so-called Magellanic Stream should be, certainly, mentioned.

Our approach is to try some simulations of motions within LGG. In studying the MW satellites we assume two variants for the MW potential: one following the model proposed by Allen and Martos [3] and another one which follows the model proposed by one of the present authors [4]. A common property of these two cases is that at the distances of galaxies which can be considered as inner MW satellites, such as the Magellanic Clouds and the dwarf galaxies named after the constellation where they are observed, the MW potential may be assumed as spherically symmetric because the two stellar components (bulge and disc) have significantly smaller effective dimensions, whereas the dark-matter subsystem (here referred to as corona) is, as usually, assumed to possess the spherical symmetry. In other words, for these inner MW satellites (closer to MW centre than 100 kpc) the bulge and disc are represented as point masses. In the case of outer MW satellites (more distant than 100 kpc) the potential of MW as a whole can be given by using the point mass formula.

SIMULATIONS

Here one should add that at sufficiently large distances to the MW centre the action of AN is no longer negligible. This effect is also examined here. We find that a MW satellite suffers very small perturbations by AN at distances to the MW centre as large as about 0.45 Mpc. This finding is based on simulations in which AN moves towards the MW centre along a straight line. In such a simulation the potential is no longer stationary (both giant galaxies, as said above, are represented as point masses). Due to the assumption that the approaching between MW and AN takes place along a straight line there is one isolating integral of motion - the component of the angular momentum of the satellite along this straight line remains constant. However, since the potential is not stationary, the energy of the satellite is not constant. This circumstance may yield difficulties, but our interest is in the present situation only, more precisely, when the distance between MW and AN is about 0.7 Mpc (as the present distance of AN usually given 0.69 Mpc - e. g. [1]). Therefore, we concentrate our attention to that rather short time interval about this distance of AN to MW and in such a way the potential becomes quasi-stationary. All the time we assume that the mass ratio AN to MW is 1.5.

In the further simulations (without AN influence) we assume for the MW satellites a distinction: inner and outer ones. The criterion is their present galactocentric distance. For the inner ones we assume that the apogalactic distance does not exceed 100 kpc. This assumption enables us to calculate the modulus of the specific (per unit mass) angular momentum of a satellite by using the energy integral. After this the same formula offers the possibility to determine the perigalactic distance. In this way we find as preliminary results the possible perigalactic distances for the inner satellites. These usually have values of about 30-40 kpc. It is curious to note that for the outer satellites a similar approach, but now assuming a higher apogalactic distance (half the actual distance to AN), yields similar values for the perigalactic distances. The reason is, most likely, in very high values of line-of-sight velocities of these objects (one should remember the case of Leo I).

CONCLUSION

We find that for a majority of the MW satellites the perigalactic distances seem to be not under 30-40 kpc. This may be an explanation why the actual satellites have survived, unlike other ones where the debris can be found at smaller galactocentric distances. We plan to include the tidal effects in our future simulations.

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32 Nonstandard Representations of Processes in Dynamical Systems

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The aim of this paper is to present an alternative description, based on the non-standard analysis, of certain processes in dynamical systems. Some examples of applications will be given, including differential equations and geometry. In particular, bifurcations in perturbed systems are studied which may be applied to the study of a body having a complex magnetic field excited by strong microwave fields. A possible application might include modeling of certain facts of magnetic fields of Sun-like stars. The Karamata notion of a slowly varying function is used for analyzing such processes in infinitesimal neighborhoods (monads). These results, we believe, might give at least new insights into classical problems of dynamical systems.

Nonstandard Representations of Processes in Dynamical Systems

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Abstract. The aim of this paper is to present an alternative description, based on the non-standard analysis, of certain processes in dynamical systems. Some examples of applications will be given, including differential equations and geometry. In particular, bifurcations in perturbed systems are studied which may be applied to the study of a body having a complex magnetic field excited by strong microwave fields. A possible application might include modeling of certain facts of magnetic fields of Sun-like stars. The Karamata notion of a slowly varying function is used for analyzing such processes in infinitesimal neighborhoods (monads). These results, we believe, might give at least new insights into classical problems of dynamical systems.

Keywords: non-standard analysis, dynamical system, bifurcation, slowly varying function
PACS: 02.10.De, 98.80.Jk

INTRODUCTION

Infinite quantities reflect certain non-archimedean properties of underlying structure. If ${}^*\mathbf{R}$ is a non-standard model of real numbers, then infinitesimals (or infinite numbers) in ${}^*\mathbf{R}$ arise from the non-archimedean property of the order of ${}^*\mathbf{R}$. Thus, ${}^*\mathbf{R}$ is the appropriate structure for modelling physical phenomena referring to infinite and infinitesimal notions. The non-standard analysis has been presented in some extent in explaining certain phenomena in quantum physics and statistical physiscs (S. Albeverio, J.E. Fenstad, T. Lindstrom, see [**1**]).

In writing this paper we had in mind several goals:

1. To promote the use of the non-standard (Leibnitz's) analysis in the study of certain physical phenomena, in particular in astronomy.
2. Use of the notion of the Karamata slowly varying functions in thy study of bifurcations in infinitesimal neighborhoods - monads.
3. Formulation of the notions of bifurcations in terms of the non-standard analysis.

NON-STANDARD ANALYSIS

The classical approach to physics is based on the mathematics over $\mathbf{R}$, the field of real numbers, or $\mathbf{C}$, the field of complex numbers. Both these structures are archimedean, i.e. they do not admit explicitly infinite quantities.

In discussing the real line $\mathbf{R}$ we have no way of knowledge what a line in physical space is really like. It might be like the real line $\mathbf{R}$, the hyperreal line ${}^*\mathbf{R}$ or neither. However, in applications of the mathematical analysis it is helpful to imagine a line in physical space as ${}^*\mathbf{R}$. The hyperreal line is, like the real line, a useful mathematical model for a line in physical space. The non-standard analysis is based on properties of ${}^*\mathbf{R}$ and the transfer principle (Łoś theorem) which transfer propositions on elements between ${}^*\mathbf{R}$ and $\mathbf{R}$.

The origin of infinitesimals goes back to Leibnitz who assumed "infinitely small numbers", and he performed usual algebraic operations over them in the same way as he did with real numbers. In particular, each positive infinitesimal ε is lesser than any standard (real) positive number, while $1/\varepsilon$ is greater than any standard positive number. His infinitesimal calculus was one of the most useful achievements in mathematics but it was not founded. Weierstrass founded analysis by introducing the $\varepsilon-\delta$ formalism, but since then the notion of infinitesimal was expelled from mathematics. In fact, the zero-sequences took the role of infinitesimals, however the problem was that even simple arithmetical operations, such as division, could not be performed on them. Abraham Robinson gave the true foundation of the Leibnitz analysis in his seminal paper [2] and this discipline, or better to say this approach to mathematics, since then is called non-standard analysis. We won't go further here into historical details, neither in foundation, but we shall give a short review of some of basic notions of the non-standard analysis.

Let $\mathbf{R}$ be the ordered field of real numbers expanded by the collection S of functions and relations that we intend to study. By ${}^*\mathbf{R}$ we shall denote an $\aleph_1$-saturated non-archimedean elementary extension of $\mathbf{R}$. Then the extension function $*$ assigns to every object $X \in S$ a non-standard counterpart ${}^*X$. All images ${}^*X$ are called internal objects. For example, if N is the set of natural numbers, then ${}^*N$ is the set of non-standard natural numbers and ${}^*N$ is an internal set. The map $*$ adds new elements to every infinite X, i.e. ${}^*X\backslash X$ is nonempty and elements of ${}^*X\backslash X$ are called non-standard. For example, elements of ${}^*N\backslash N$ are called non-standard natural numbers, while elements of ${}^*R\backslash R$ are called non-standard real numbers. The most important is the following transfer principle:

Leibnitz principle: *Every mathematical proposition that is true for finite (real) numbers is also true for the extended system (i.e. system with infinite numbers), and vice versa.*

A consequence of the Compactness Theorem is the fact that the described theory has a model. In particular, each model of that theory contains infinitesimals. We see that this model is in fact a non-archimedean real field but enriched with non-standard counterparts of notions of the mathematical analysis: elementary functions $\sin(x), \ln(x), \ldots$, sets: natural numbers $\mathbf{N}$, integers $\mathbf{Z}$, rational numbers $\mathbf{Q}$, etc. We also see that the following extension principle holds:

Extension property: *Every function* $f : \mathbf{R} \longrightarrow \mathbf{R}$ *can be extended to the function* ${}^*f : {}^*\mathbf{R} \longrightarrow {}^*\mathbf{R}$ *which preserves all first order properties of* f.

For example, if $f(x) = \sin(x)$, $g(x) = \cos(x)$, since

$$sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y),$$

the same identity holds for ${}^*f(x) = {}^*\sin(x)$ and ${}^*g(x) = {}^*\cos(x)$. Something similar is true for analytical continuations of real functions, but only for identities. In the non-

standard analysis all first-order properties are preserved, including monotonicity, the properties of zeros, etc. It is customary that the asterisk * is omitted in the case of elementary functions. So, $\sin(x)$ will denote $^*\sin(x)$ if $x \in {}^*\mathbf{R}$, otherwise it is the standard sine function.

The best of all is that we can do the same with more complex structures. For example, to construct the non-standard enlargement of any infinite structure: complex numbers $\mathbf{C}$, the space of real sequences $\mathbf{R}^{\mathbf{N}}$, the space of real functions $\mathbf{R}^{\mathbf{R}}$, each having the metric according to our choice; then infinite functional, geometrical and topological spaces. This construction simply allows us non-standard but consistent mathematics. The Leibnitz transfer principle enables one to translate theorems from non-standard universe to the standard one. In particular, the Cauchy transfer principle is useful in such translations:

The Cauchy Principle *Let $\varphi(x)$ be an internal formula. Then: If $\varphi(x)$ holds for each infinitesimal x, then there is $r \in \mathbf{R}, r > 0$ so that $\varphi(x)$ holds in $\mathbf{R}$ for all $x \in \mathbf{R}, |x| < r$.*

An element $a \in {}^*\mathbf{R}$ is *finite* if there is a positive integer n such that $-n < a < n$. By ${}^*\mathbf{R}_{\text{fin}}$ we shall denote the set of all finite elements of ${}^*\mathbf{R}$. The *galaxy* of a is the set $g(a)$ of all non-standard reals b such that $a-b$ is finite. In particular, ${}^*\mathbf{R}_{\text{fin}} = g(0)$. The mapping $\mathbf{st} : {}^*\mathbf{R}_{\text{fin}} \longrightarrow \mathbf{R}$ (*standard part*) defined by $\mathbf{st}(x) = \sup\{y \in R \mid y \leq x\}$ is an epimorphism. In particular, ${}^*\mathbf{R}_{\text{fin}}/\ker(\mathbf{st}) \cong \mathbf{R}$.

An *infinitesimal* is each finite ε such that $\text{st}(\varepsilon) = \text{st}(0) = 0$. The *monad* of 0 is the set $\mu(0)$ of all infinitesimals. Notice that $\mu(0)$ is closed under addition and multiplication. Further, we say that a and b are infinitely close (denoted by $a \approx b$) if $a - b \in \mu(0)$. In fact, $\mu(0)$ is a kernel of epimorphism $\mathbf{st}$ and it is a maximal ideal of ${}^*\mathbf{R}_{\text{fin}}$. The other monads are obtained by translations, i.e. $\mu(a) = a + \mu(0)$.

By use of the homomorphism $\mathbf{st}$ one can replace the ε-δ formalism by algebraic identities.

For detailed development of non-standard analysis, one may consult [7]. Notions and tools of model theory (the part of mathematical logic) used in the development of the non-standard analysis one can find, for example, in [6].

Examples

Here we give several examples of basic mathematical notions expressed in non-standard terms and non-standard solutions of some classical problems.

1. Let S be an infinite subset of $\mathbf{R}$. Then ${}^*S \setminus S$ is also infinite.
2. If ε is an infinitesimal, then $\text{st}(a+\varepsilon) = \text{st}(a)$.
3. $f : \mathbf{R} \longrightarrow \mathbf{R}$ is continuous iff for all $a \in {}^*\mathbf{R}_{\text{fin}}$ $\text{st}({}^*f(a)) = f(\text{st}(a))$, i.e. for every infinitesimal ε there is an infinitesimal η such that ${}^*f(a+\varepsilon) = f(a) + \eta$.
4. Let $f : \mathbf{R} \longrightarrow \mathbf{R}$ be a differentiable function and let $\varepsilon \neq 0$ be an infinitesimal. Then $f'(x) = \text{st}\left(\frac{{}^*f(x+\varepsilon)-f(x)}{\varepsilon}\right)$. For example, $(x^2)' = \text{st}\left(\frac{(x+\varepsilon)^2-x^2}{\varepsilon}\right) = \text{st}(2x+\varepsilon) = 2x$.
5. $\int_0^1 f(x)dx = \text{st}(\frac{1}{H}\sum_{i=0}^{H} {}^*f(\frac{i}{H}))$, where $H \in {}^*N \setminus N$ and f is continuous.

6. **Karamata slowly varying and regularly varying functions.** In certain dynamical systems there are slowly varying processes, or it is needed a detailed analysis of the process in the neighborhood of the particular points. Examples of this kind include critical (equilibrium) points of a dynamical system generated by an autonomous system of ordinary differential equations (ODEs). It appears that Karamata slowly varying and regularly varying functions might be useful in these analysis. They are defined as follows:

   Slowly varying functions: $\lim_{x\to+\infty} L(tx)/L(x) = 1, \quad t > 0,$
   Examples : $\ln x,\ \ln(x)^2,\ \ln^{\alpha}(x)\ \ldots$

   Regularly varying functions: $\lim_{x\to+\infty} R(tx)/R(x) = t^{\rho}, \quad t > 0,$
   Examples : $x, x^2,\ x^{\alpha},\ x^{\alpha}\ln^{\beta} x,\ \ldots$

   L and R being continuous and positive on a positive half-axis. The constant ρ is called the index of regularity. This two definitions imply that any regularly varying function has the form $R(x) = x^{\rho}L(x)$. Starting from these two simple definitions, Karamata developed the whole new theory of slowly and regularly varying functions which has included the majority of the most important properties of these functions. The definitions given above appeared in 1930 in the seminal paper [1]. The intent of his paper was to generalize the Tauberian conditions in some inverse theorems of the Tauberian type for the Laplace transform. But it was soon realized that those functions could be successfully applied to many branches of mathematical analysis, probability theory and even in astronomy, [9].
   Jovan Karamata (1902-1967) is one of the most known Serbian mathematician and his discovery of these notions is his most famous result.
   The non-standard description of slowly varying functions is as follows: For all positive infinite $H \in {}^*\mathbf{R}$, $L(tH)/L(H) \approx 1, t > 0$.

7. **Dirac delta function**. Let $a(t) = e^{-1/(1-|t|^2)}$ if $|t| < 1$, $a(t) = 0$ otherwise. This is a simple variation of Cauchy's flat function, and it belongs to the space $\mathscr{E}^{\infty}$ of infinitely many differentiable functions. Let ε be a positive infinitesimal, and let $b(t) = a(t/\varepsilon)$. Finally, let $k = \int_{-\infty}^{\infty} b(t)dt$ and let $\delta(t) = b(t)/k = a(t/\varepsilon)/k$. Then $\delta(t)$ belongs to ${}^*\mathscr{E}^{\infty}$, it is positive, and has integral one. In fact, it is what is expected, $\delta(t)$ is a finite compact distribution and it has all properties attributed to the Dirac function.

8. The next example is from statistical physics. Anderson [4] gave a non-standard description of random walk and Brownian motion: Let $H \in^* N$ be infinite, $\Delta t = H^{-1}$ and $T = \{k\Delta t \mid k \in^* N, k \le H\}$. Then T represents time and $B(\omega,t) = \sum_{s=0}^{t-\Delta t} \omega(s)\sqrt{\Delta t}$, $\omega: T \to \{1,-1\}$, is a hyperfinite random walk, moving by $\sqrt{\Delta t}$ right or left at each infinitesimal interval Δt. Then, $b(\omega, \mathrm{st}(t)) = B(\omega,t)$ is a realization of the Brownian motion on $\mathbf{R}$.

9. The next examples are from geometry, and they could explain certain phenomena in astronomy.
   Tiling the Euclidean plane (patterns, e.g. dominoes): if there is a covering of each square (circle), prove that we can cover the entire plane. One solution goes like this: by the extension principle, we can find the covering of a square with edges (radius) with the length H, $H \in {}^*N \setminus N$. This particular cover induces the covering

of the entire Euclidean plane. Here one can recognize the foundation of one of the basic cosmological principles: homogeneity of the Universe. From the preceding example it follows that from the mathematical point of view at least it is consistent to assume so. This example is connected with famous Hao Wang problem of tiling the plain with dominos (Wang tiling), [3].

Is every parabola ellipse? Let E be an ellipse touching the y-axis and having focuses in $(0,1)$ and $(0,H)$ where $H > 0$ is an infinite real number. Then the set P of all standard points of E, i.e. points laying in $\mathbf{R}$, are points of loci of an "ordinary" (standard) parabola, $y^2 = 4x$. In fact, P is the envelope of the family of all ellipses touching y-axis and having one focus in $(0,1)$, the other one in $(0,b)$, $b > 1$ is a (standard) real number. Here one can recognize the difficulty in determination of the nature of comet orbits with a remote second focus. This example shows that every comet's orbit which we measured (observed) as parabolic, actually is elliptical. But, it's second focus is excessively far away to measure it.

The cell structure of the Universe. The space ${}^*R^n$ is partitioned into monads, ${}^*\mathbf{R}^n_{fin} = \bigcup_{r\in R^n} \mu(r)$. In [10] authors discussed a mathematical model of quantized time and space of the Universe partitioned into cells. In addition, there an interpretation of the Planck distance and Planck time is given, and some conclusions on physics what it would be like below these bounds, as well.

Non-standard methods in dynamical system

The non-standard analysis introduces new concepts, which do not exist in the classical analysis. For example, ${}^*\mathbf{R_{fin}}$, the finite points of ${}^*\mathbf{R}$, is partitioned into monads: ${}^*\mathbf{R_{fin}} = \bigcup_{\mathbf{r}\in\mathbf{R}} \mu(\mathbf{r})$ We can think about $\mu(r)$ as the "cloud" of infinitely closed points to the standard real number r. Thus, if $\mathbf{R}$ is a counterpart of time-line, then it is sound to suppose that the time is quantized into infinitely small parts (monads), each part having rather rich structure. Similar observation could be asserted for the space $\mathbf{R^n}$ (see the last example in the previous section). Accordingly, notions of dynamical systems as described expressed in $\varepsilon - \delta$ formalism have natural and more intuitive description.

We consider the ordinary differential equation:

$$\dot{x} = dx/dt = f(x,\mu), \quad \mu \in \mathbf{R}, \;\; f \in C^2(\mathbf{R}), \tag{1}$$

where μ is a parameter. The variable t represents time, i.e. $t \in R$, and μ is a constant parameter if it does not depend on time t.

A value μ_0 for which the flow of (1) is not structurally stable is called a *bifurcation*. Simply saying, (1) undergoes a bifurcation for μ_0 if the critical point x_c of (1) behavior changes. To remind the reader, the critical points of (1) are the roots of $f(x,\mu_0) = 0$, i.e. $f(x_c,\mu_0) \equiv 0$, so x_c is also the fixed point of (1).

The critical point x_c is *stable* if it captures all paths (trajectories, orbits) in the phase space $(x,\dot{x})$ of (1) crossing close enough to x_c. More formally, if the path $p(t)$ crosses close enough to x_c, i.e. at the distance less than given $\varepsilon > 0$, then for

every neighborhood V_{x_c} of x_c there is t_0 such that for any $t \geq t_0$, Vx_c contains all points of $p(t)$.
If x_c is not stable, than we say x_c is *unstable* and it is the source of bifurcation.
Suppose there is an external influence represented by μ acting on the system modeled by (1) varying in time t. Then $\mu = \mu(t)$, and (1) is not anymore an autonomous system.
We will discuss *delayed dynamic bifurcation* which may occur if $\mu(t)$ is slowly varying function crossing the bifurcation point x_c. The slowly varying function we shall define in non-standard terms.
Dynamical system is *delayed* if there are initial values (t_0, x_0), $x_0 \neq x_c$, and time interval $(0, t_j)$ such that every solution x_j of (1) with $x_j(t_0) = x_0$ is crossing arbitrarily close to x_c.
We consider also delayed dynamic systems with perturbation. Perturbation could be, for example, constant or noise. It appears that if perturbation is small, then delay survives, Walter.
Non-standard formulation of stability. The critical point x_c of (1) in 1.3 is stable if any path $p(t)$ of (1) crossing close to x_c is captured at infinity by the monad $\mu(x_c)$ of x_c. That is, for all infinite time T, $p(T)$ is inside $\mu(x_c)$. Therefore, x_c is unstable if for some infinite time T, the path $p(T)$ is outside of $\mu(x_c)$. This is exactly what we mean by bifurcation. Small changes in the process driving the dynamical system given by (1), produces in certain time t_0 the instability of the system.

From the preceding example we see that it is worth to study path in the infinity and the monad $\mu(x_c)$. For this purpose we introduce the notion of μ-slowly varying function. Let $f: R^2 \longrightarrow R$. Then the function f is μ-slowly varying at point $a \in R$ if for every $x \approx a$, ${}^*f(\varepsilon, t) \in \mu(0)$, $t \in {}^*\mathbf{R}_{fin}$. An example of μ-slowly varying function at x_c is

$$\tau(\varepsilon, t) = \mu_0 + \varepsilon(t - t_0). \tag{2}$$

Other examples of μ-slowly varying functions are $\tau(\varepsilon, t) = \mu_0 + \varepsilon \sin(t)$ and $\tau(\varepsilon, t) = \mu_0 + \sin(\varepsilon t)$. These functions might be useful in dynamical systems with perturbation.

Local time and slowly varying functions

Good source of μ-slowly varying functions are the Karamata slowly varying functions and regularly varying functions. μ-slowly varying functions are easily obtained from Karamata types functions by simple transformations. If $K(t)$ is Karamata slowly varying function and $k(x,t) = (K(x,t) - K(x))/K(x)$, then for all positive infinite H, ${}^*k(H,t) \in \mu(0)$ for all finite t. Thus, $\tau(\varepsilon, t) = k(1/\varepsilon, t)$ is the μ-slowly varying function. Now we take the local time at x_c, $\tau(t) = \mu(t)$, for example taking $\mu(t)$ as in (2). Then (1) becomes:

$$\varepsilon dx/dt = f(x, \tau).$$

It should be noticed that if τ takes positive standard time, t must be infinite, exactly what we want, when the path $p(t)$ might enter $\mu(x_c)$ (and this is the case when x_c is stable).

Non-standard formulation of delayed dynamical system. Consider a system of type (2) with positive infinitesimal ε. For simplicity we shall assume that x_c is stable for $\tau < 0$ and unstable for $\tau > 0$. If there exist initial values (x_0, τ_0), $\tau_0 < 0, \tau_0 \not\approx 0$, and $x_0 \not\approx x_c$, such that the solution of (2) with $x(\tau_0) = x_0$ for some $\tau > 0, \tau \not\approx 0$ is satisfying:

$$x(\tau') \approx x_c\,, \quad \tau' \in (0, \tau)\,. \tag{3}$$

then it is a delayed bifurcation.
The solution of (2) may consist of an infinite family F of functions. Then it is easy to check that F is the set of bounded equicontinuous functions, therefore, according to the Ascoli theorem, there is a continuous function $g(t)$ and a sequence $f_n(t) \in F$ which converges point-wise to $g(t)$. In other words, the family F is glued (or attracted) to $g(t)$. Examples in [8] which arise in the study of high-power magnetic resonance are good illustration of the statement we just noted.

Conclusion

The non-standard analysis could be useful in studying certain problems in astrophysics and physics in general. Definition of many notions are more intuitive, while proofs and inferences are easier. In particular, applications of non-standard analysis in dynamical systems could be fruitful.

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33 The corrections to the Hipparcos proper motions in declination for 807 stars

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The authors used the data on latitude variations obtained from observations with 10 classical photographic zenith tubes (PZT) in order to improve the Hipparcos proper motions in declinations μ_δ for 807 stars. Part of observing programmes, carried out during the XX century for the purpose of studying the Earth rotation, were realized by using PZT instruments. These observations were performed within in the intervals (tens of years) much longer than that of the Hipparcos mission (less than 4 years). Long term examinations of latitude and time variations were used to form the Earth Orientation Catalogue (EOC-2), aimed at a more accurate determination of positions and proper motions for the stars included. The authors method of calculating the corrections of the proper motions in declination from the latitude variations is different from the method used in obtaining the EOC-2 Catalogue. Comparing the results they established a good agreement between their μ_δ and the EOC-2 ones for the star sample used in the present paper.

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CORRECTIONS TO THE HIPPARCOS PROPER MOTIONS IN DECLINATION FOR 807 STARS

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SUMMARY: We used the data on latitude variations obtained from observations with 10 classical photographic zenith tubes (PZT) in order to improve the Hipparcos proper motions in declinations μ_δ for 807 stars. Part of observing programmes, carried out during the last century for the purpose of studying the Earth's rotation, were realized by using PZT instruments. These observations were performed within in the intervals (tens of years) much longer than that of the Hipparcos mission (less than 4 years). In addition, the annual number of observations for every PZT-programme star is several hundreds on the average. Though the accuracy of the star coordinates in the Hipparcos Catalogue is by two orders of magnitude better than that of the star coordinates from the PZT observations, the large number of observations performed a much longer time interval makes it possible to correct the Hipparcos proper motions and to improve their accuracy with respect to the accuracy given in the Hipparcos Catalogue. Long term examinations of latitude and time variations were used to form the Earth Orientation Catalogue (EOC-2), aimed at a more accurate determination of positions and proper motions for the stars included. Our method of calculating the corrections of the proper motions in declination from the latitude variations is different from the method used in obtaining the EOC-2 Catalogue. Comparing the results we have established a good agreement between our μ_δ and the EOC-2 ones for the star sample used in the present paper.

Key words. Astrometry – Reference systems

1. INTRODUCTION

At the IAU General Assembly in Kyoto 1997, the International Celestial Reference Frame (ICRF) was adopted to materialize the International Celestial Reference System (ICRS) from the beginning of 1998, and it was based on a catalogue of 608 compact radio sources (Ma et al. 1998).

The Hipparcos ESA mission (ESA 1997) produced two catalogues in optical wavelength, both linked to the ICRF: Hipparcos (118218 stars with coordinate accuracy close to 1 mas at 1991.25, epoch of the catalogue, 1 mas/yr accuracy of proper motions in $\mu_\alpha \cos\delta$ and μ_δ, and very accurate parallaxes and photometry) and Tycho (1058332 stars with 25 mas accuracy of coordinates). As a result of the IAU resolution from 1997 the Hipparcos Catalogue was

Table 1. The informations for PZT instruments.

observatory	instrument	N	period	MJD	$\lambda(^\circ)$	$\varphi(^\circ)$
Mizusawa	MZP, MZQ	137	02.I 1959-21.XII 1991	36570.6-48611.5	141.1	39.1
Mount Stromlo	MS	184	31.X 1957-27.VIII 1985	36142.5-46304.7	149.0	-35.3
Ondřejov	OJP	285	05.II 1973-14.XII 1991	41718.9-48604.7	14.8	49.9
Punta Indio	PIP	165	03.VIII 1971-29.VI 1984	41166.9-45881.1	-57.3	-35.3
Richmond	RCP, RCQ	202	05.XI 1949-12.V 1989	33225.2-47658.1	-80.4	25.6
Washington	WA, W, WGQ	188	28.X 1915-26.XII 1991	20798.2-48616.0	-77.1	38.9

adopted to be the primary realization of ICRS in the optical domain. It contains stars brighter than $V = 12$ (mostly between $V = 7$ and $V = 9$). The Hipparcos stars positions and proper motions are bases of the optical frame HCRF (Hipparcos Celestial Reference Frame).

It is evident that the combinations of Hipparcos and Tycho catalogues and the ground-based ones can improve the positions and proper motions of observed stars. During the last decade, a number of problems concerning the Hipparcos data appeared: a different accuracy of the data for single Hipparcos stars than for double and multiple ones due to the short duration of the satellite mission, different accuracy of the data for different parts of the celestial sphere, more inaccurate apparent star positions due to the errors of proper motions whose influence is linear in time (from the epoch 1991.25) and by now the errors of apparent position attained about 15 mas (one order of magnitude larger than the average position error in Hipparcos Catalogue), etc. All this is the reason that new star catalogues appeared after the Hipparcos one, such as ARIHIP (Wielen et al. 2001), and the Earth Orientation Catalogue (EOC-2) (Vondrák 2004), with more accurate star positions and proper motions than those from Hipparcos.

2. DATA AND CALCULATIONS

Here we use the results of latitude observations made with PZT instruments over a few decades together with the Hipparcos Catalogue data, in order to obtain proper motions in declination more accurate than the Hipparcos ones for 807 stars.

The reduction procedure for the latitude variations φ_i was described by Vondrák et al. (1998). We investigate these latitude variations φ_i with time around the corresponding mean latitudes. The elimination of systematic effects of interest (polar motion, local and instrumental systematic variations) for this investigation is described in the paper by Damljanović (2005).

Our input data were the latitude variations obtained by using 10 PZT instruments at 6 observatories. Table 1. contains some basic informations about the observatories and PZT instruments. The columns of Table 1. are: observatory – the name of the observatory, instrument – the codes of the instruments, N – the number of observed stars, period – the dates when observations were started and finished, MJD – corresponding Modified Julian Dates ($MJD = JD - 2400000.5$), λ – the longitude of the observatory and φ – the latitude of the observatory. The codes (MZP, MZQ, etc.) are from the monograph by Vondrák et al. (1998).

The first PZT instrument was WA which started with the observations at the end of 1915. The observatories Punta Indio and Mount Stromlo are nearly at the same latitude, their longitudes different, and because of that there are 157 common stars in their observational programmes. The results for these common stars could be used to check our method (Damljanović 2005) and to compare results for the same stars observed by PZTs in different continents. The similar situation is for the observatories Washington and Mizusawa; there are 71 stars in common. The longitude λ_W of PZT instrument is west of the zero meridian and it is useful here to calculate the polar motion component $\Delta\varphi_i$ (dominant in φ_i) by using the Kostinski's formula $\Delta\varphi_i = x_i \cos\lambda_W + y_i \sin\lambda_W$ (Kulikov 1962).

As a first step of our calculations, we remove the polar motion components $\Delta\varphi_i$ from the values of φ_i. The polar motion coordinates, x_i and y_i, are from Vondrák's file EOPOA00.dat. In this way, we get the residuals (for corresponding observational epochs) $r_i = -(\varphi_i - \Delta\varphi_i)$ for every observed star (Damljanović 2005).

As a second step, we calculate the systematic effects se_i (local, instrumental, etc.) for the observational epochs and get new residuals $r'_i = r_i - se_i$, where the values se_i for each instrument are found by using the values r_i for all stars of the observational programme (Damljanović 2005, Damljanović et al. 2006). After eliminating the polar motion changes and the other systematic effects (local, instrumental, etc.) we assume that the differences r'_i are mostly due to the catalogue systematic errors (influence of proper motions in declination). The unknown corrections of the proper motions in declination are contained in r'_i. We process star by star, using the linear fit $r''_i = a + b(t_i - 1991.25)$, where the unknowns a and b are determined by using the Least Squares Method (LSM), the values r''_i are the averaged values of r'_i over subperiods of 1 year for each star, and the values t_i are the times (in years) corresponding to r''_i. Obviously, the value a is the correction of $\Delta\delta$ and b is the correction of $\Delta\mu_\delta$; both are calculated for the epoch of the Hipparcos Catalogue 1991.25. As an input value for the fit, to the points r''_i we add one more point with the coordinates (1991.25, $0\overset{''}{.}0$). All

points are with suitable weights (Damljanović et al. 2006). We add our calculated corrections b to the corresponding Hipparcos proper motions in declination. In this way we obtain the values μ_δ and their errors.

3. RESULTS

The calculated values μ_δ and ϵ_{μ_δ} for 807 stars are presented in Table 2. Also, in Table 2. we list the values of proper motions in declination and their errors from EOC-2 and Hipparcos catalogues for the purpose of comparison of the results.

The columns of Table 2. are: HIP – the Hipparcos number, m – the number of points r_i'', μ_δ – our proper motions in declination, ϵ_{μ_δ} – the standard errors of μ_δ, $\mu_{\delta_{EOC-2}}$ – the EOC-2 proper motions, $\epsilon_{\mu_{\delta EOC-2}}$ – the standard errors of $\mu_{\delta_{EOC-2}}$, $\mu_{\delta_{\rm HIP}}$ – the Hipparcos proper motions, $\epsilon_{\mu_{\delta\rm HIP}}$ – the standard errors of $\mu_{\delta_{\rm HIP}}$, M – Fig. 1 means that the star is common to Washington and Mizusawa, number 2 that the star is common to Mount Stromlo and Punta Indio and empty field means that the star is observed at only one observatory. For each common star observed at two observatories the value m is approximately equal to the sum of observational periods at both observatories expressed in years. For stars observed at only one observatory m is close to observational period expressed in years. The complete Table 2 is given at the URL: `http://saj.matf.bg.ac.yu/177/pdf/Table2.dat`.

Our best results for μ_δ (313 stars observed for almost 20 and for more than 20 years with errors ϵ_{μ_δ} mostly smaller than the Hipparcos ones) are in the first part of Table 2. The results with errors ϵ_{μ_δ} close to the Hipparcos errors (372 stars observed between 10 and 20 years) are in the second part of Table 2. The results with errors ϵ_{μ_δ} mostly exceeding the Hipparcos errors (122 stars observed for less than 10 years) are in the third part of Table 2.

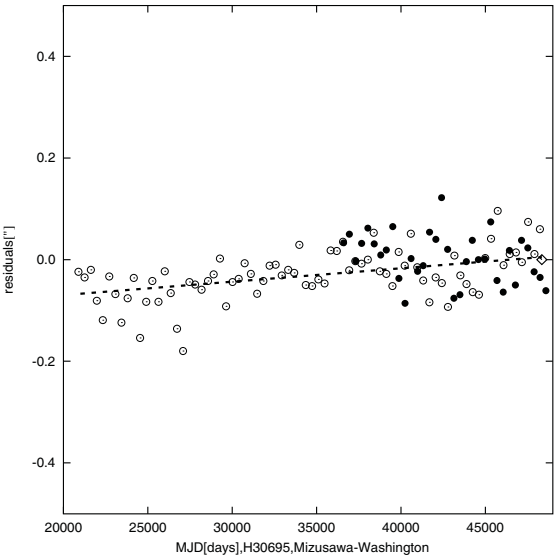

Fig. 1. *Star H30695. Linear trend and residuals r_i'' of Mizusawa (solid circles) and Washington (open circles) observations vs. time (MJD), and the Hipparcos point (open rectangle).*

For example, the calculated value $b \pm \epsilon_b$ for a common Mizusawa and Washington star H30695 (8.33 mag) is $+0.96 \pm 0.15$ mas/yr, with $m = 110$ points r_i'', and we thus got $\mu_\delta = 10.20 \pm 0.15$ mas/yr. The values r_i'' and the linear trend are presented in Fig. 1.

From EOC-2: $\mu_\delta = 10.05 \pm 0.10$ mas/yr.

From Hipparcos: $\mu_\delta = 9.24 \pm 0.76$ mas/yr.

The results with similar errors for the other stars are presented in the first part of Table 2.

4. DISCUSSION

For $m \geq 10$ (685 stars from the first and second parts of Table 2.), the average value of ϵ_{μ_δ} is 0.80 mas/yr (similar to the Hipparcos one for the same stars). For $m \geq 20$ (313 stars of the first part of Table 2.), the average value of ϵ_{μ_δ} is 0.40 mas/yr (0.74 mas/yr is the Hipparcos value for the same stars). This means that, by observing with PZT instruments for 10 or more years, we can get the results μ_δ with similar errors to the Hipparcos ones. If m is greater than 20, our results are much better than the Hipparcos ones. Vice versa, if m is less than 10 (122 stars of the third part of Table 2.), the average value of ϵ_{μ_δ} is 1.87 mas/yr (0.78 mas/yr for the same Hipparcos stars). So, with less than 10 years of PZT observations, it is not possible to get the results μ_δ with errors comparable to the Hipparcos ones.

The further verification of these results is carried out by applying the F-test. The test statistics is

$$F = S_1^2 / S_2^2$$

where S_1^2 is the averaged value obtained by using $\epsilon^2_{\mu_{\delta\rm HIP}}$, and S_2^2 by using $\epsilon^2_{\mu_\delta}$. If $F \geq F_{f_1,f_2;\alpha}$, one concludes that S_2^2 is smaller than S_1^2.

For the case $m \geq 20$ ($n = 313$ stars, $f_1 = f_2 = 312$, $\alpha = 0.05$) we obtain $S_1^2 = 0.61$ and $S_2^2 = 0.21$, so that

$$F = 2.9 > 1.2 = F_{312,312;0.05}.$$

The hypothesis $H_0(\epsilon^2_{\mu_\delta} = \epsilon^2_{\mu_{\delta\rm HIP}})$ can be rejected. This means that the values $\epsilon^2_{\mu_\delta}$ are smaller than the $\epsilon^2_{\mu_{\delta\rm HIP}}$ ones, and our results on μ_δ are better than the Hipparcos ones.

For the case $m \geq 10$ ($n = 685$ stars, $f_1 = f_2 = 684$, $\alpha = 0.05$) we obtain $S_1^2 = 0.81$ and $S_2^2 = 1.09$, so that

$$F = 0.7 < 1.1 = F_{684,684;0.05}.$$

Table 2. The values μ_δ and ϵ_{μ_δ} for 807 stars: ours, EOC-2 and Hipparcos ones, M corresponds to the observatory.[1]

HIP	m	μ_δ (mas/yr)	ϵ_{μ_δ} (mas/yr)	$\mu_{\delta_{\rm EOC-2}}$ (mas/yr)	$\epsilon_{\mu_{\delta\rm EOC-2}}$ (mas/yr)	$\mu_{\delta_{\rm HIP}}$ (mas/yr)	$\epsilon_{\mu_{\delta\rm HIP}}$ (mas/yr)	M
167	60	-3.30	.18	-3.41	.14	-4.37	.61	
630	40	-57.69	.25	-56.55	.20	-56.82	.64	
732	37	-10.82	.39	-11.37	.29	-12.15	.53	2

This means that the values $\epsilon^2_{\mu_\delta}$ are not better than, but rather similar to the Hipparcos ones.

The similar investigations (for $m \geq 10$), but for the stars of each observatory separately, gave us the following average values of ϵ_{μ_δ}:
- Richmond (165 stars), 0.39 mas/yr (0.74 mas/yr for the same Hipparcos stars),
- Washington/Mizusawa (46 stars), 0.41 mas/yr (0.71 mas/yr),
- Punta Indio/Mount Stromlo (144 stars), 0.57 mas/yr (0.82 mas/yr),
- Washington (84 stars), 0.66 mas/yr (0.67 mas/yr),
- Mizusawa (58 stars), 1.14 mas/yr (0.77 mas/yr),
- Ondřejov (187 stars), 1.48 mas/yr (0.92 mas/yr).

The best results are those of the Richmond PZTs. The averaged value of ϵ_{μ_δ} is about 53% of the value from the Hipparcos Catalogue. The PZT observations made at the Richmond Observatory cover about 40 years. The results for the Mizusawa Observatory are mostly with higher values of ϵ_{μ_δ} than the Hipparcos ones even though the observed period is longer than 30 years. In combination with the Washington observations (for the common Washington/Mizusawa stars) the results are much better. The errors of the results of the Ondřejov data are similar to the Mizusawa ones. The reason for poor results in the case of Mizusawa is that this is a very active seismic region, but for the Ondřejov ones it is the observational programme with a lot of double and multiple stars (the average value of $\epsilon_{\mu_{\delta\rm HIP}}$ is 0.92 mas/yr which exceeds the values of the other observational programmes). The high values of the Ondřejov systematic errors (local, instrumental, etc.) are well known (Vondrák et al. 1998).

We also compare our results with those from EOC-2. We calculate the average values of the modulus of the differences $(\mu_\delta - \mu_{\delta\rm EOC-2})$ to find:

0.82 mas/yr for $m \geq 10$ (685 stars),
0.35 mas/yr for $m \geq 20$ (313 stars),
2.16 mas/yr for $m < 10$ (122 stars).

This means that our results are in good agreement with the EOC-2 ones, especially for $m \geq 10$ and $m \geq 20$.

5. CONCLUSIONS

We have calculated the proper motions in declination for 807 stars observed with 10 PZT instruments.

Our results are in good agreement with the EOC-2 ones although different methods are used.

In general, if $m \geq 10$ (stars are observed 10 or more than 10 years) our results are close to the Hipparcos ones, if $m \geq 20$ (stars are observed 20 or more than 20 years) our results are better than the Hipparcos ones. The F-test leads to the same conclusion. To obtain good results it is necessary to have data covering a long time interval of PZT observations.

It is evident that PZT data are useful and can improve the reference frame (via improvement of proper motions in declination of Hipparcos stars).

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[1]The complete Table 2 is given at the URL: `http://saj.matf.bg.ac.yu/177/pdf/Table2.dat`.

ПОПРАВКЕ HIPPARCOS СОПСТВЕНИХ КРЕТАЊА ПО ДЕКЛИНАЦИЈИ ЗА 807 ЗВЕЗДА

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Оригинални научни рад

Користили смо промене ширине добијене из посматрања са 10 класичних фотографских зенитних туба (PZT) да бисмо израчунали поправке Hipparcos сопствених кретања по деклинацији μ_δ за 807 звезда. Део посматрачких програма рађених због изучавања Земљине ротације током прошлог века реализован је помоћу PZT инструмената. Ова посматрања рађена су у много дужим временским интервалима (десетине година) него што је временски интервал Hipparcos мисије (краћи од 4 године). Осим тога, годишњи број посматрања сваке звезде PZT програма је неколико стотина. Иако је тачност координата звезда у Hipparcos каталогу за два реда величине боља од тачности координата звезда добијених из посматрања PZT инструментима, велики број посматрања урађен у много дужем интервалу времена омогућава да се добију поправке Hipparcos сопствених кретања и да се побољша њихова тачност у односу на тачност дату у Hipparcos каталогу. Дугогодишња испитивања промена ширине и времена послужила су за израду Earth Orientation Catalogue (EOC-2) чији је циљ било тачније одређивање положаја и сопствених кретања посматраних звезда. Наш метод рачунања поправки сопствених кретања по деклинацији из промена ширине разликује се од метода коришћеног при добијању EOC-2 каталога. Из поређења резултата установили смо добру сагласност наших и EOC-2 μ_δ за коришћени узорак звезда.

34 Envelopes of cometary orbits

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The authors discuss cometary orbits from the standpoint of Nonstandard (Leibnitz) analysis, a relatively new branch of mathematics. In particular, they consider parabolic cometary paths. It appears that, in a sense, every parabola is an ellipse with one focus at infinity.

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ENVELOPES OF COMETARY ORBITS

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SUMMARY: We discuss cometary orbits from the standpoint of Nonstandard (Leibnitz) analysis, a relatively new branch of mathematics. In particular, we consider parabolic cometary paths. It appears that, in a sense, every parabola is an ellipse.

Key words. Comets: general – Methods: analytical

1. INTRODUCTION

The standard approach to physics is based on mathematics over **R**, the field of real numbers, or **C**, the field of complex numbers. Both these structures are Archimedean, i.e. they do not admit explicitly infinite quantities. In discussing the real line **R** we have no way of knowing what a line in physical space is really like. It might be like the real line R, the hyperreal line ${}^*R$ which contains infinitesimals and infinite numbers, or neither. However, in applications of the mathematical analysis it is helpful to imagine a line in a physical space as ${}^*R$. The hyperreal line is, like the real line, a useful mathematical model for a line in the physical space. One of the aims of this paper is to advocate the use of methods from nonstandard analysis (also known as Leibnitz analysis, non-Archimedean analysis or Robinson's analysis) in studies of certain phenomena in astronomy. Here we shall discuss trajectories of comets.

2. NONSTANDARD ANALYSIS

It is generally accepted that Newton and Leibnitz, independently from each other, developed differential calculus. By infinitesimals Leibnitz assumed "infinitely small numbers", and he performed the usual algebraic operations over them in the same way as he did with real numbers. In particular, each positive infinitesimal ε in this contemplation was lesser than any ordinary real (standard) positive number, while $1/\varepsilon$ was greater than any standard positive number. The following rule was implicitly supposed:

Leibnitz principle: *Every mathematical proposition that is true for finite (real) numbers is also true for the extended system (i.e. system with infinite numbers), and vice versa.*

The major difficulty of Leibnitz's approach was a number of paradoxes and a lack of formal framework for consistent foundation of infinitesimal calculus. Introducing Weierstrass analysis the infinite quantities are expelled, for example the notion of the infinitesimal is replaced by the ε - δ formalism. In particular, zero-sequences (i.e. sequences that converge to 0) are seen as infinitesimals. However, this is only an auxiliary notion there, and they lack the use of all algebraic operations (such as division) over them.

Abraham Robinson (1961) solved the 300 years old problem of foundation of infinitesimal calculus. He founded Leibnitz analysis, i.e. introduced actual infinitely small and infinitely large numbers. They admit not only all algebraical operations, but also an application of usual functions from analysis (such as sin, cos, exp etc) on them. Robinson's solution was based on certain constructions and techniques from mathematical logic, such as the ultraproducts, the Compactness theorem and the saturated models. The reader can find details about these notions in Chang and Keisler (1990).

The nonstandard analysis is based on properties of ${}^*R$ and the transfer principle (Łoś theorem), the counterpart of the Leibnitz principle, which exchange propositions between ${}^*R$ and $\mathbf{R}$. The nonstandard analysis has been used since then in explaining certain phenomena in physics, in particular in statistical physics and quantum mechanics (e.g. S. Albeverio, J. E. Fenstad, T. Lindstrom, see Anderson 1976, Albeverio, et al. 1986).

Mathematical models of nonstandard analysis are non-Archimedean real fields enriched with nonstandard counterparts of notions of the mathematical analysis: elementary functions $\sin(x), \ln(x), \ldots$, sets: natural numbers N, integers Z, rational numbers Q, etc. As they are non-Archimedean, they contain infinitesimals and infinite quantities. The best of all is that we can do the same with more complex structures. For example, to construct the nonstandard enlargement of any infinite structure: complex numbers C, the space of real sequences R^N, the space of real functions R^R, each having the metric of our choice; then infinite functional, geometrical and topological spaces. This construction simply allows us to do nonstandard but consistent mathematics. Leibnitz transfer principle enables one to translate theorems expressed by special, so called internal formulas from nonstandard universe to the standard one. In particular, the Cauchy transfer principle is useful in such translations:

The Cauchy Principle Let $\varphi(x)$ be an internal formula. Then: *If $\varphi(x)$ holds for each infinitesimal x, then there is $r \in R, r > 0$, such that $\varphi(x)$ holds in R for all $x \in R, |x| < r$.*

Let us mention few facts about nonstandard analysis: by ${}^*R$ we shall denote some $\aleph_1$-saturated non-Archimedean (in the sense of order) elementary extension of the ordered field of reals R. Though $\aleph_1$-saturation provides uniqueness (up to isomorphism) at the given cardinal number of such structures, we do not have the canonical representation (such as decimal notation for reals) of nonstandard real numbers. Another useful property is expressed by the following theorem

Theorem (Extension property) *Every function $f\colon R \longrightarrow R$ can be extended to ${}^*f\colon {}^*R \longrightarrow {}^*R$ which preserves all first order properties of f.*

For example, if $f(x) = \sin(x)$, $g(x) = \cos(x)$, since

$$sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y),$$

the same identity holds for ${}^*f(x) = {}^*\sin(x)$ and ${}^*g(x) = {}^*\cos(x)$. Something similar is true for analytical continuations of real functions, but only for identities. In nonstandard analysis all the first order properties are preserved, including monotonicity, properties of zeros, etc. It is customary that the asterisk * is omitted in the case of elementary functions. Thus, $\sin(x)$ will denote ${}^*\sin(x)$ if $x \in {}^*R$, otherwise it is the "ordinary" sinus function.

Another useful notion in nonstandard analysis are monads. An element $a \in {}^*R$ is *finite* if there is a positive integer n such that $-n < a < n$. By ${}^*R_{fin}$ we shall denote the set of all finite elements of ${}^*R$. The *galaxy* of a is the set $g(a)$ of all nonstandard reals b such that $a - b$ is finite. In particular, ${}^*R_{fin} = g(0)$. It is easy to see that the mapping $st\colon {}^*R_{fin} \longrightarrow R$ (*standard part*) defined by

$$st(x) = \sup_R\{y \mid y \leqslant x\}$$

is an epimorphism. In particular, ${}^*R_{fin}/ker(st) \cong R$. An *infinitesimal* is each finite ε such that $st(\varepsilon) = st(0) = 0$. The *monad* of 0 is the set $\mu(0)$ of all infinitesimals. Notice that $\mu(0)$ is closed under addition and multiplication. Further, we say that a and b are *infinitely close*, denoted by $a \approx b$, if $a - b \in \mu(0)$. In fact, $\mu(0)$ is a kernel of epimorphism st and it is a maximal ideal of the ring ${}^*R_{fin}$. We get the other monads by translations, i.e. $\mu(a) = a + \mu(0)$.

By use of homomorphism st one can replace the ε-δ formalism by algebraic identities. Let us illustrate this with several examples:

Examples from mathematics

1. If ε is an infinitesimal, then $st(a+\varepsilon) = st(a)$.
2. $f\colon R \longrightarrow R$ is continuous iff (if and only if) for all $a \in {}^*R_{fin}$, $st({}^*f(a)) = f(st(a))$.
3. $f\colon R \longrightarrow R$ is uniformly continuous iff for all $a, b \in {}^*R$, $a \approx b \to {}^*f(a) \approx {}^*f(b)$.
4. Let $f\colon R \longrightarrow R$ be a differentiable function and let $\varepsilon \neq 0$ be an infinitesimal. Then $f'(x) = st\left(\frac{{}^*f(x+\varepsilon)-f(x)}{\varepsilon}\right)$. For example,

$$(x^2)' = st\left(\frac{x^2+2x\varepsilon+\varepsilon^2-x^2}{\varepsilon}\right) = st(2x+\varepsilon) = 2x.$$

5. Every subset S of R has the nonstandard enlargement ${}^*S$. If S is finite then ${}^*S = S$, but if S is infinite then ${}^*S \setminus S$ is also infinite.
6. If f is continuous, then the Riemann integral may be defined, for example $\int_0^1 f(x)dx = \frac{1}{H}\sum_{i=0}^{H} {}^*f(i/H)$, where H is an infinite number, i.e. $H \in {}^*N \setminus N$.

One can find detailed development of nonstandard analysis in Stroyan and Luxemburg (1976).

Examples in Geometry and Astronomy. Nonstandard analysis, based on ${}^*R$, introduces a specific mathematical method, as well as a way of thinking. As we saw, it introduces *actual* infinitely small quantities and infinitely large quantities. Therefore, it gives good ground in considering physical systems which in idealized form have infinitely many degrees of freedom. Definitions and proofs are more intuitive, and its use is natural and intuitive whenever the considered (idealized) physical system is composed of infinitely many particles. There are a lot of applications of nonstandard analysis based on this assumption in mathematical physics, in particular in quantum mechanics, fluid mechanics, dynamical systems, etc. As an example, let us first consider Dirac delta function.

1. Dirac δ function Let $a(t) = e^{-1/(1-|t|^2)}$ if $|t| < 1$, $a(t) = 0$ otherwise. This is a simple variation of Cauchy's flat function, and it belongs to the space $\mathcal{E}^\infty$ of infinitely many differentiable functions. Let ε be a positive infinitesimal, and let $b(t) = a(t/\varepsilon)$. Finally, let $k = \int_{-\infty}^{\infty} b(t)dt$ and let $\delta(t) = b(t)/k = a(t/\varepsilon)/k$. Then $\delta(t)$ belongs to ${}^*\mathcal{E}^\infty$, it is positive, and has integral one. In fact, this is what is expected, $\delta(t)$ is a finite compact distribution and it has all properties attributed to the Dirac function.

2. Tiling the Euclidean plane, Hao-Wang dominoes problem: if there is a covering by the certain pattern of the finite type τ of each bounded domain in the plain such as squares and circles, prove that there is a cover of the type τ of the entire plane. One solution goes like this: by the extension principle, we can find the covering $\mathcal{C}$ of the type ${}^*\tau$ of a square with edges having the infinite length H, i.e. $H \in {}^*N \setminus N$. Since τ is finite, we have ${}^*\tau = \tau$. Therefore, this particular nonstandard cover induces the covering of the entire Euclidean plane by restricting $\mathcal{C}$ to the standard (finite) part of ${}^*R \times {}^*R$.

In this example we have seen how to extend certain local property to the global one. We can try to interpret this covering property to the foundation of fundamental cosmological principles. Namely, all observations from the Earth are local, even on the large scale. But observations on the large scale show that the Universe is homogeneous and isotropic. Identifying observations with tiling, we see at once that we may assume two basic cosmological principles: homogeneity and isotropy of the Universe. Therefore, from the mathematical point of view at least it is consistent to assume so.

Consistency and conservativeness of nonstandard analysis. Nonstandard analysis is a consistent and conservative extension of classical analysis. This follows from the ultraproduct construction (consistency), and Łoś theorem (conservativeness). Nonstandard analysis cannot produce propositions in the classical mathematical analysis that are not possible to prove by means of classical mathematics. Thus, nonstandard analysis is a method of proving, first of all. However, we should mention that some problems, such as Bernstein-Robinson theorem on polynomial operators, were first solved by means of nonstandard analysis.

There are other mathematical non-Archimedean methods that are used in physics. Particularly popular in last two decades became p-adic physics, which is based on the so called p-adic mathematics. It admits counterparts of all basic notions of classical analysis, but they are not true extensions of standard functions of classical analysis. Also, it lacks the transfer principles such as the Leibnitz transfer principle, or they are much weaker, such as the Hasse-Minkowski theorem for Henselian fields. Without any intention to doubt the trustiness on p-adic physics, it certainly gives interpretation of physical phenomena that differs from those in the main-stream physics. Detailed discussion on this topic one can find in Mijajlović et al. (2006), Mijajlović and Pejović (2007), Mijajlović et al. (2007).

3. ELLIPSE IN THE NONSTANDARD UNIVERSE

Let E be an ellipse having foci at the points $(p,0)$ and $(q,0)$ where $p > 0$ is a fixed positive real number and $q > 0$ is an infinite real number. Then all standard points of E, i.e. the points lying in the real plane R^2, are the points of loci of an "ordinary" parabola P having the focus at $(p,0)$. We show that P is actually the envelope of the family of all (standard) ellipses having one focus in $(p,0)$, the other one in $(b,0)$, $b > p$ is a positive real number.

From the stated assumptions on the ellipse E, see Fig. 1, we infer the following formulas:

$$\begin{cases} l_1 + l_2 &= d \\ l_1^2 &= (x-p)^2 + y^2, \\ l_2^2 &= (x-q)^2 + y^2. \end{cases} \qquad (1)$$

From these equations it follows

$$\begin{aligned} l_1 &= ((q-p)x + pd)/d, \\ l_2 &= ((p-q)x + qd)/d. \end{aligned}$$

By eliminating l_1 and l_2 from the set of formulas (1), we obtain finally the equation of the ellipse E:

$$y^2 = 4px - 4p\frac{p(p+q)x + qx^2}{(p+q)^2} \tag{2}$$

We can interpret the formula (2) in the following two ways.

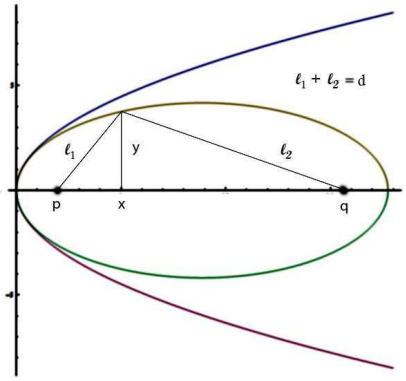

Fig. 1. *The ellipse having the foci respectively in* $(p,0)$ *and* $(q,0)$ *with the vertex at the coordinate origin and* $l_1 + l_2 = d$, $d = p + q$, *where* l_1 *and* l_2 *are distances of a point on the ellipse from the foci.*

1. Ellipse in the nonstandard plane. Assume that $p \in R$ and that $x \in {}^*R$ is finite and $q \in {}^*R$ is infinite. Then the term

$$4p\frac{p(p+q)x + qx^2}{(p+q)^2} \tag{3}$$

is an infinitesimal, while $4px$ is finite. Hence y is also finite and $y^2 \approx 4px$. Thus, $\mathrm{st}(y)^2 = 4p\mathrm{st}(x)$, so by replacing $\mathrm{st}(x)$ by x and $\mathrm{st}(y)$ by y we obtain the equation $y^2 = 4px$ of a parabola. Therefore, the standard part of the ellipse E (see Fig. 2) in the nonstandard plane with the finite focus $(p,0)$ and the infinite focus $(q,0)$ is the confocal parabola P determined by the equation $y^2 = 4px$. Observe that P does not depend on the choice of the infinite focus $(q,0)$.

All geometric and differential properties of the parabola P can be derived from the properties of the ellipse E. For example, the optical property that if a ray of light travels parallel to the symmetry axis of a parabola and strikes the concave side of the parabola, then it will be reflected to the focus follows immediately from the corresponding optical property of the ellipse E. Just note that if a ray r' is coming from

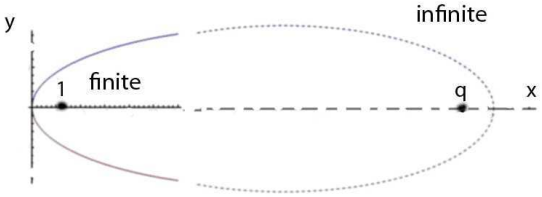

Fig. 2. *The ellipse* E *in the nonstandard plane.*

the infinite focus $(q,0)$ it reflects from the ellipse to the focus $(p,0)$. Then the standard part $r = st(r') = \{(st(x), st(y)) : (x,y) \in r', x, y \in {}^*R_{fin}\}$ is a (standard) ray parallel to the x-axis and as r is infinitely close to r' it also enters into the focus.

2. Family of confocal ellipses. We may take (2) as the equation of the family of (standard) ellipses sharing the fixed focus $(p,0)$, while the second focus $(q,0)$ runs over the x-axis. Observe that from the astrodynamics point of view this family of ellipses may be regarded as Hohmann-Vetchinkin transfer orbits connecting co-planar circular orbits.

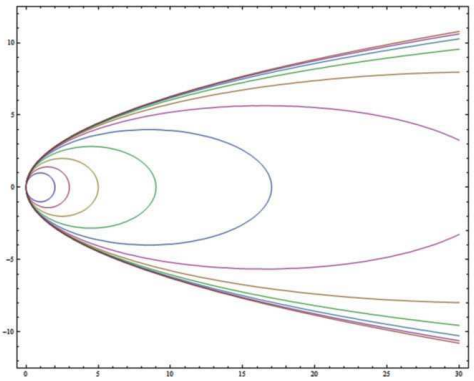

Fig. 3. *Family of confocal ellipses.*

The parabola P is the limit curve enveloping ellipses (see Fig. 3) from this family. In the classical approach in mathematical analysis, the existence of the limit curve is guaranteed by the Arzelà-Ascoli theorem applied in the L^2 space.

However, it should be mentioned that P is not the envelope of the family of ellipses given by the Eq. (2) as it is defined in mathematical analysis. Namely, if a family of plane curves are given by a formula $F(x,y,q) = 0$, q is a parameter, then the envelope of this family is a curve osculating each member of the family. The equation of the envelope is obtained by elimination of q from the system of equations

$$F(x,y,q) = 0, \quad \partial F(x,y,q)/\partial q = 0.$$

In our case,

$$F(x,y,q) = y^2 - 4px + 4p(p(p+q)x + qx^2)/(p+q)^2,$$

and it is easily found that the envelope is in fact the critical point $x = 0, y = 0$, the perihelion of q-ellipses.

4. COMETARY TRAJECTORIES

First studies of cometary orbits serve as a historical introduction to astrodynamics. Namely, their trajectories are also influenced by non-gravitational forces, for example by the acceleration resulting from ejection of a jet of a material from the comet. Most cometary orbits are very elongated. Many physical quantities related to the very elongated cometary orbits change by several orders of magnitude. Every cometary orbit which is observed as parabolic actually is elliptical as further calculations usually show. If this is not the case, this is due to the fact that the second focus is too remote to measure it. Therefore, nonstandard analysis could be the appropriate mathematical tool in the study of cometary trajectories. In the rest of this article we shall use the terminology of nonstandard analysis and the words *standard* and *infinitesimal* will have meaning as explained in the previous sections. For example, if the value of the velocity at the perihelion is assumed to be standard, then the velocity at aphelion may be taken as an infinitesimal. In particular, we shall discuss parabolic paths. By our consideration in the previous section we may assume that every parabolic trajectory is an ellipse. Our discussion is relied on available cometary data, so we shall first shortly review them.

The number of observed comets is rapidly growing due to the development of space technology. For example The ESA/NASA SOHO spacecraft, `http://sohowww.nascom.nasa.gov`, discovered exactly 1500 comets since 1995, the last one 27. June 2008 when this paper was submitted. About 2300 comets are catalogued, even if it is believed that there are more than 10^9 of them. As very few comets have periods less than 12 years, their trajectories are good illustration for very elongated or nearly parabolic ellipses. Here is the short history on comet discoveries in the last four decades.

According to Baker and Makemson (1960), in 1960, of the 1000 comets for which orbits have been computed, fewer than 100 had periods of revolution less than 100 years. Some 40 or 50 had periods between 100 and 1000 years, and the periods of the rest were very uncertain. Fewer than 30 comets were known to have been observed on two or more returns to the Sun. About 50 comets had periods less than 12 years (Jupiter family).

The Catalog of Cometary Orbits, compiled by Marsden, 1989 edition, lists 1292 computed orbits from 239 BC to AD 1989; only 91 of them were computed using the rare accurate historical data from before the 17th century. More than 1200 are, therefore, derived from cometary passages during the last three centuries. Sets of orbital elements in Marsden's catalog involve only 810 individual comets; the remainder represents the repeated returns of periodic comets. Four of these comets had been definitely lost, and three more were probably lost, presumably because of their decay due to the solar heat. Of the 155 short-period comets, 93 have been observed at two or more perihelion passages.

The 16th edition of the Catalogue of Cometary Orbits of Smithsonian Astrophysical Observatory issued in 2005 contains 3031 sets of orbital elements (in the J2000.0 system) for 2991 cometary apparitions of 2221 different comets through mid-August 2005. There is a special tabulation giving osculating elements for the 170 designated periodic comets, excluding seven deemed to be lost.

According to the list of periodic comets on the Planetary Data System Small Bodies Node, NASA, last update on 10 April 2008, `http://pdssbn.astro.umd.edu/comet_data`, there are 420 designated periodic comets. According to Seiichi Yoshida's Comet Catalog, `http://www.aerith.net/`, there are 243 non-designated periodic comets (the last discovered C/2008 L3, 13. Jun 2008) and 200 designated periodic comets.

In discussion of very elongated cometary orbits we shall rely on Marsden Catalog of Cometary Orbits. Of the 655 comets of long period contained in the Catalog, 192 have osculating elliptic orbits, and 122 have osculating orbits that are very slightly hyperbolic. Finally, 341 are listed as having parabolic orbits, but this is false because either it has not been possible to detect unequivocal deviations from a parabola on the usually very short arc over which the comets have been observed or the final calculations have never been made. However, the parabola is always assumed first in the computation of the preliminary orbit. If the osculating orbit is computed backwards to when the comet was still far beyond the orbit of Neptune and if the orbit is then referred to the centre of mass of the solar system, the original orbits almost always prove to be elliptic.

These data show that the methods of nonstandard analysis are suitable in studying of cometary trajectories. For example, in the computation of the preliminary orbits, the value of the parameter p is computed. Simply, the second term (3) in Eq. (2) may be omitted as we may consider it as an infinitesimal. It also shows that the formula (2) could be very appropriate in calculation of cometary orbits in the sense that it represents better starting point for the method of differential corrections than the simple parabola.

Let us consider very-long-period comets and comets having orbits not significantly different from a parabola. It is believed that these comets originate in the Oort cloud which is located 10000 and 100000 AU from the Sun. By our previous discussion it is appropriate to use here methods of nonstandard analysis. So let us assume that a hypothetical comet C is moving along an ellipse E in the nonstandard plane having the second focus at $(q, 0)$ where q is an infinite number. Therefore, the aphelion of E is at infinity, and by the second Kepler's law the velocity v of the comet near the aphelion (i.e. at the finite distance from aphelion in terms of nonstandard analysis) is an infinitesimal. Otherwise, the surface swept by the comet for the finite time $\triangle t$ would be infinite due to the infinite distance of the comet from the Sun, and that would contradict the Second Kepler's law. In reality, a simple calculation shows that the velocity v of the comet C near aphelion would be around 100 m/sec, negligibly small comparing to the velocity at the perihelion. Therefore, the momentum $p = mv$ of the comet C is an infinitesimal too;

we could say that the comet C floats in the Oort cloud instead of travelling around the Sun. Hence the trajectory of the comet C is subject to a very small perturbation. We show that an infinitely small force, or impulse, would change significantly its trajectory. Simply saying, parabolic orbits at large distances are very unstable. We can see that using the following formula from astrodynamics (see Andjelić 1983, pages 132-134):

$$v_{a_t} = \sqrt{\frac{2\mu}{\rho_{a_2}} \frac{\rho_{p_1}}{\rho_{p_1} + \rho_{a_2}}} \tag{4}$$

which determines the velocity v_{a_t} needed for the body C to continue travelling along the other orbit which is coplanar and confocal to E.

So let O_2 be the orbit E with the apoapsis ρ_{a_2} along which the body C travels so that C is at the aphelion (i.e. in the Oort cloud) with the infinitely small velocity v_C. Further, let O_1 be the orbit, coplanar and confocal to O_2, with the periapsis ρ_{p_1} to which the body C will be transferred under an action $\mathcal{A}$. Observe that ρ_{a_2} is infinite, while ρ_{p_1} is finite. According to the Maupertuis' principle of the least action, we may assume that the transition path from O_2 to O_1 will be the Hohmann transfer orbit H touching the orbit O_1 at the aphelion, so the equation (4) can be applied. Here, μ is the gravitational parameter and v_{a_t} is the velocity resulting from the action $\mathcal{A}$. Under these assumptions, we have

$$v_{a_t} = \frac{\sqrt{2\mu\rho_{p_1}}}{\rho_{a_2}}(1 - \frac{\rho_{p_1}}{2\rho_{a_2}} + \frac{\varepsilon}{\rho_{a_2}})$$

where ε is an infinitesimal. Simplifying the previous formula, we find $v_{a_t} = \sqrt{2\mu\rho_{p_1}}/\rho_{a_2} - \kappa/\rho_{a_2}^2$, where κ is a standard finite value. Observe that v_{a_t} is an infinitesimal of the order $1/\rho_{a_2}$.

Let $v_{a_2} = v_C$ be the velocity of the body C at the aphelia on the orbit O_2. Then $v_\delta = v_{a_2} - v_{a_t}$ is the velocity needed for transition from the orbit O_2 to the transfer ellipse which would carry the comet C to the orbit O_1. Since v_{a_2} and v_{a_t} are infinitesimals, it follows that v_δ is an infinitesimal, too. The transfer ellipse H with the second focus at infinity will be seen from the near neighborhood of the Sun as a parabola.

There are astronomical evidences that support our discussion. Namely, according to Delsemme (2008) among the very-long-period comets, there is a particular class of comets that Oort showed as having never passed through the planetary system before, notwithstanding the fact that their original orbits were elliptic, which implies repeated passages. This paradox vanishes when it is understood that their perihelia were outside of the planetary system before their first appearance but that their orbits have been perturbed while they resided near aphelia, either by stellar or dark interstellar-cloud passages or by galactic tides, in such a way that their perihelia were lowered into the planetary system.

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ОБВОЈНИЦЕ КОМЕТСКИХ ОРБИТА

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Оригинални научни рад

У раду се разматрају кометске орбите са становишта нестандардне анализе, релативно нове области математике. Посебно се изучавају параболичне кометске орбите. Показује се да је, у одређеном смислу, свака парабола заправо елипса.

35 Cometary Trajectories in the nonstandard plane

Ž. Mijajlović, N. Pejović, G. Damljanović
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Nonstandard (Leibnitz) analysis, based on nonstandard real numbers *R, introduces a specific mathematical method, as well as a way of thinking. It introduces actual infinitely small quantities and infinitely large quantities. Therefore, it gives a good ground in considering physical systems which in idealized form have infinitely many degrees of freedom. Definitions and proofs are more intuitive, and its use is natural and intuitive whenever the considered physical system is composed of infinitely many particles. There are a lot of applications of nonstandard analysis based on this assumption in mathematical physics, in particular in quantum mechanics, fluid mechanics, dynamical systems, etc. Here, the authors discuss cometary trajectories, in particular the parabolic one, from the standpoint of nonstandard analysis. It appears that in a sense every parabola is an ellipse. Or from the standard point of view, every parabola P is a limit curve of a family F of confocal and coplanar ellipses. In a sense, the parabola P is also the envelope of the family F. Based on the observed data, this approach gives also a good mathematical model of

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COMETARY TRAJECTORIES IN THE NONSTANDARD PLANE

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Abstract. Nonstandard (Leibnitz) analysis, based on nonstandard real numbers ${}^*\mathbf{R}$, introduces a specific mathematical method, as well as a way of thinking. It introduces *actual* infinitely small quantities and infinitely large quantities. Therefore, it gives good ground in considering physical systems which in idealized form have infinitely many degrees of freedom. Definitions and proofs are more intuitive, and its use is natural and intuitive whenever the considered physical system is composed of infinitely many particles. There are a lot of applications of nonstandard analysis based on this assumption in mathematical physics, in particular in quantum mechanics, fluid mechanics, dynamical systems, etc. Here, we discuss cometary trajectories, in particular the parabolic one, from the standpoint of nonstandard analysis. It appears that in a sense every parabola is an ellipse. Or from the standard point of view, every parabola P is a limit curve of a family *F* of confocal and coplanar ellipses. In a sense, the parabola P is also the envelope of the family *F*. Based on the observed data, this approach gives also a good mathematical model of paths of comets. In addition, this paper is written in order to promote the use of methods of nonstandard analysis in astronomy.

1. INTRODUCTION

The standard approach to physics is based on mathematics over **R**, the field of real numbers. This structure is archimedean, i.e. it does not admit explicitely infinite quantities. We have no way of knowledge what a line in physical space is really like. It might be like the real line **R**, the hyper-real line ${}^*\mathbf{R}$ which contains infinitesimals and infinite numbers, or neither. However, in applications of the mathematical analysis it is helpful to imagine a line in a physical space as ${}^*\mathbf{R}$. The hyper-real line is, like the real line, a useful mathematical model for a line in the physical space. One of the aims of this paper is to popularize the use of methods

from nonstandard analysis (also known as Leibnitz analysis, non-archimedean analysis or Robinson's analysis) in studies of certain phenomena in astronomy. Here we shall discuss trajectories of comets from the stand point of Nonstandard analysis.

2. NONSTANDARD ANALYSIS

First, let us review the basic notions of nonstandard analysis. Newton and Leibnitz independently from each other developed differential calculus. By infinitesimals Leibnitz assumed "infinitely small numbers", and he performed the usual algebraic operations over them exactly in the same way as he did with real numbers. In particular, each positive infinitesimal ε in this contemplation was lesser than any ordinary real (standard) positive number, while $1/\varepsilon$ was greater than any standard positive number, i.e. $1/\varepsilon$ is an infinite number. The following rule was implicitly supposed:

Leibnitz principle: *Every mathematical proposition that is true for finite (real) numbers is also true for the extended system (i.e. system with infinite numbers), and vice versa.*

The major difficulty of Leibnitz's approach was a number of paradoxes and a lack of formal framework for consistent foundation of infinitesimal calculus. Introducing Weierstrass analysis the infinite quantities are expelled, for example the notion of the infinitesimal is replaced by the ε - δ formalism. In particular, zero-sequences (i.e. sequences are seen as infinitesimals. However, this is only an auxiliary notion there, and they lack the use of all algebraic operation (such as division) over them.

Abraham Robinson in Robinson (1961) solved the 300 years old problem of foundation of infinitesimal calculus. He founded Leibnitz analysis, i.e. introduced actual infinitely small and infinitely large numbers. They admit not only all algebraic operations, but also an application of usual functions from analysis (such as sin, cos, exp etc) over them. Robinson's solution was based on certain constructions and techniques from mathematical logic, such as the ultraproducts, the Compactness theorem and saturated models. The reader can find details about these notions in Chang and Keisler (1990).

The nonstandard analysis is based on properties of ${}^*\mathbf{R}$ and the transfer principle (Łoš theorem), the counterpart of the Leibnitz principle, which exchange propositions between $^*\mathbf{R}$ and $\mathbf{R}$. The nonstandard analysis has been used since then in explaining certain phenomena in physics, in particular in statistical physics and quantum mechanics (e.g. Anderson, 1976; Albeverio et al., 1986).

Mathematical models of nonstandard analysis are non-archimedean real fields enriched with nonstandard counterparts of notions of the mathematical analysis: elementary functions sin(x), ln(x), …, sets: natural numbers $\mathbf{N}$, integers $\mathbf{Z}$, rational numbers $\mathbf{Q}$, etc. As they are non-archimedean, they contain infinitesimals and

infinite quantities. We can do the same constructions with more complex structures. We can build the nonstandard enlargement of any infinite structure: complex numbers $\mathbf{C}$, the space of real sequences $\mathbf{R}^N$, the space of real functions $\mathbf{R}^R$, each having the metric on our choice; then infinite functional, geometrical and topological spaces. This construction simply allows us to do nonstandard but consistent mathematics. Leibnitz transfer principle enables one to translate theorems expressed by special, so called internal formulas from nonstandard universe to the standard one. Another useful property is expressed by the following theorem

Theorem (Extension property). Every function $f: \mathbf{R} \to \mathbf{R}$ can be extended to $f: {}^*\mathbf{R} \to {}^*\mathbf{R}$ which preserves all first order properties of f.

For example, if $f(x)= \sin(x)$, $g(x)= \cos(x)$, since $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$, the same identity holds for ${}^*f(x) = {}^*\sin(x)$ and ${}^*g(x) = {}^*\cos(x)$. Something similar is true for analytical continuations of real functions, but only for identities. In nonstandard analysis all first-order properties are preserved, including monotonicity, properties of zeros, etc. It is customary that the asterisk * is omitted in the case of elementary functions. So, sin(x) will denote ${}^*\sin(x)$ in ${}^*\mathbf{R}$ as well.

Another useful notion in nonstandard analysis is monads. An element $a \in {}^*\mathbf{R}$ is *finite* if there is positive integer n such that $-n < a < n$. By ${}^*\mathbf{R}_{fin}$ we shall denote the set of all finite elements of ${}^*\mathbf{R}$. The *galaxy* of a is the set $\gamma(a)$ of nonstandard real numbers b such that $a - b$ is finite. In particular, ${}^*\mathbf{R}_{fin} = \gamma(0)$. The mapping st: ${}^*\mathbf{R}_{fin} \to \mathbf{R}$ (*standard part*) is defined by $st(x)= \sup_{\mathbf{R}}\{y: y < x\}$. An *infinitesimal* is each finite ε such that $st(\varepsilon) = st(0) = 0$. The *monad* of 0 is the set $\mu(0)$ of all infinitesimals. Note that $\mu(0)$ is closed under addition and multiplication. Further, we say that numbers a and b are *infinitely close*, denoted by $a \approx b$, if $a - b \in \mu(0)$. In fact, $\mu(0)$ is the kernel of epimorphism st and it is a maximal ideal of the ring ${}^*\mathbf{R}_{fin}$ The other monads we get by translations, i.e. $\mu(a)= a + \mu(0)$.

By use of homomorphism st one can replace the ε - δ formalism by algebraic identities. Let us illustrate this with several examples:

1. $f: \mathbf{R} \to \mathbf{R}$ is continuous iff (if and only if) for all $a \in {}^*\mathbf{R}_{fin}$, $st({}^*f(a))={}^*f(st(a))$.

2. Let $f: \mathbf{R} \to \mathbf{R}$ be a differentiable function and let $\varepsilon \neq 0$ be an infinitesimal. Then $f(x)' = st({}^*f(x + \varepsilon) - f(x)/ \varepsilon)$. For example, $(x^2)'= st(x^2 + 2x\varepsilon + \varepsilon^2-x^2)/ \varepsilon = st(2x + \varepsilon) = 2x$.

3. If f is a continuous function, then the Riemann integral of f on the interval $[0,1]$ may be defined as $1/H \; \Sigma_{i=0}^{H} \; {}^*f(i/H)$, where H is an infinite number, i.e. $H \in {}^*\mathbf{N} \setminus \mathbf{N}$.

Detailed development of nonstandard analysis one can find in Stroyan and Luxemburg (1976).

Examples in Geometry and Astronomy.

As we saw, nonstandard analysis introduces *actual* infinitely small quantities and infinitely large quantities. Therefore, it gives good ground in considering physical systems which in idealized form have infinitely many degrees of freedom. Definitions and proofs are more intuitive, and it's use is natural and intuitive whenever the considered (idealized) physical system is composed of infinitely many particles. As an example, let us first consider Dirac delta function.

1. Dirac δ – function. Let $a(t)= \exp(-1/(1-|t|^2))$ if $|t| < 1$, $a(t)=0$ otherwise. This is a simple variation of Cauchy's flat function, and it belongs to the space E^∞ of infinitely many differentiable functions. Let ε be a positive infinitesimal, and let $b(t)= a(t/\varepsilon)$. Finally, let $k = \int_{-\infty}^{\infty} b(t)dt$ and let $\delta(t)= b(t)/k = a(t/\varepsilon)/k$. Then δ(t) belongs to ${}^*E^\infty$, it is positive, and has integral one. In fact, it is what is expected; δ(t) is a finite compact distribution and it has all properties attributed to the Dirac function.

2. **Tiling the Euclidean plane, Hao-Wang dominoes problem.** If there is a covering by the certain pattern of the finite type τ of each bounded domain in the plain such as squares and circles, prove that there is a cover of the type τ of the entire plane. One solution goes like this: by the Extension principle, we can find the covering C of the type $*\tau$ of a square with edges having the infinite length H, i.e. $H \in {}^*\mathbf{N} \setminus \mathbf{N}$. Since τ is finite, we have $*\tau = \tau$. Therefore, this particular nonstandard cover induces the covering of the entire Euclidean plane by restricting *C* to the standard (finite) part of ${}^*\mathbf{R} \times {}^*\mathbf{R}$.

In this example we have seen how to extend certain local property to the global one. We can try to interpret this covering property to the foundation of fundamental cosmological principles. Namely, all observations from the Earth are local, even in the large scale. But observations in the large scale show that the Universe is homogeneous and isotropic. Identifying observations with tiling, we see at once that we may assume two basic cosmological principles: homogeneity and isotropy of the Universe. Therefore, from the mathematical point of view at least it is consistent to assume so.

3. ELLIPSE IN THE NONSTANDARD PLANE

Let E be an ellipse having foci at the points (p,0) and (q,0) where p>0 is a positive real number and q>0 is an infinite real number. Then all standard points of E, i.e. the points lying in the real plane R^2, are the points of loci of an "ordinary" parabola P having the focus at (p,0). We show that P is the envelope of the family of all (standard) ellipses having one focus in (p,0) and the other one in (b,0), b is a positive real number.

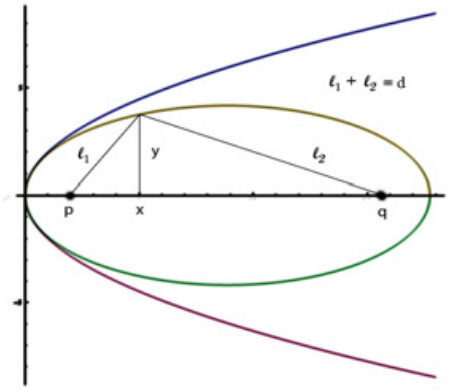


Figure 1: The ellipse having the foci in (p,0) and (q,0) with the vertex at the coordinate origin and $\ell_1+\ell_2= d$, d=p+q, where ℓ_1 and ℓ_2 are distances of a point on the ellipse from the foci.

From the stated assumptions on the ellipse E, see Figure 1, we infer the following equations:

$$\begin{aligned} \ell_1 + \ell_2 &= d, \\ \ell_1^2 &= (x-p)^2 + y^2, \\ \ell_2^2 &= (x-q)^2 + y^2. \end{aligned} \tag{1}$$

By eliminating ℓ_1 and ℓ_2 from the set of formulas (1), we obtain the equation of the ellipse E:

$$y^2 = 4px - 4p(p(p + q)x + qx^2)/(p + q)^2 \tag{2}$$

We can interpret the formula (2) in the following two ways.

1. Ellipse in the nonstandard plane. Assume that $p \in R$ and that $x \in {}^*\mathbf{R}$ is finite and $q \in {}^*\mathbf{R}$ is infinite. Then the term

$$4p(p(p + q)x + qx^2)/(p + q)^2 \tag{3}$$

is an infinitesimal, while 4px is finite. So y is also finite and $y^2 \approx 4px$. Hence $st(y)^2 = 4p\ st(x)$, so by replacing st(x) by x and st(y) by y we obtain the equation $y^2 = 4px$ of parabola. Therefore, the standard part of the ellipse E in the nonstandard plane with the finite focus (p,0) and the infinite focus (q,0) is the parabola P determined by the equation $y^2 = 4px$. Observe that P does not depend on the choice of the infinite focus (q,0).

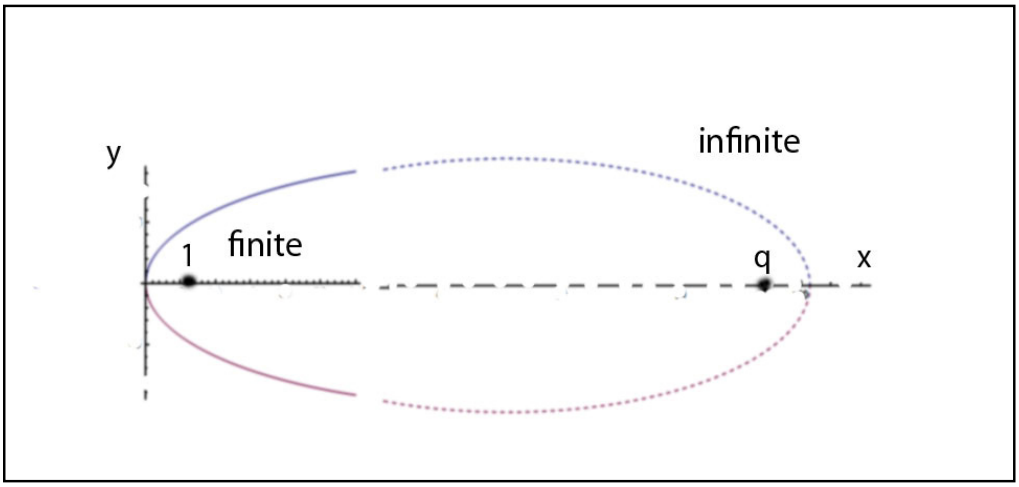


Figure 2: The ellipse E in the nonstandard plane.

It should be mentioned that all geometric and differential properties of the parabola P can be derived from the properties of the ellipse E. For example, the optical property that if a ray of light travels parallel to the symmetry axis of a parabola and strikes the concave side of the parabola, then it will be reflected to the focus follows immediately from the corresponding optical property of the ellipse E. Just note that if a ray r is coming from the (infinite) focus (q,0) it reflects from the ellipse to the focus (p,0) and that the standard part of r is a line parallel to the x-axis.

2. Family of confocal ellipses.

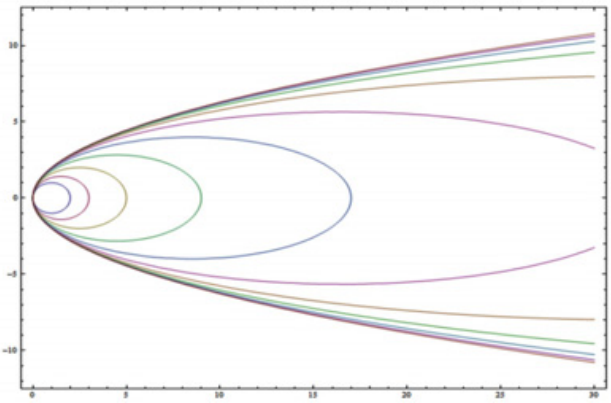


Figure 3: Family of confocal ellipses.

We may take (2) as the equation of the family of (standard) ellipses sharing the fixed focus (p,0), while the second focus (q,0) runs over the x-axis. Observe that from the astrodynamics point of view this family of ellipses may be regarded as Hohmann-Vetchinkin transfer orbits connecting co-planar circular orbits. We see that the parabola P is the limit curve enveloping ellipses from this family. However, it should be mentioned that P is not the envelope of the family of ellipses given by the equation (2) as it is defined in mathematical analysis. Namely, if a family of plane curves are given by a formula F(x,y,q) = 0, q is a parameter, then the mathematical envelope of this family is a curve touching each

member of the family. The equation of the envelope is obtained by elimination of q from the system of equations $F(x,y,q)= 0$, $\partial F(x,y,q)/ \partial x = 0$. In our case, $F(x,y,q) = y^2 - 4px + 4p(p(p + q)x + qx^2)/(p + q)^2$, and it is easily found that the envelope is in fact the critical point $x = 0$, $y = 0$, the aphelia of q-ellipses.

4. COMETARY ORBITS

Most cometary orbits are very elongated. Namely, every cometary trajectory which is observed as parabolic actually is elliptical as further calculations show. But, the second focus is too remote to measure it. Many physical quantities related to the very elongated cometary orbits change for several orders of magnitude. For example, if the value of the velocity at the perihelion is assumed to be standard, then the velocity at aphelion may be taken as an infinitesimal. Therefore, we shall consider cometary trajectories assuming nonstandard analysis. By our consideration in the previous section we may assume that every parabolic trajectory is an ellipse. Our discussion is relied on available cometary data, so we shall first shortly review them.

The number of observed comets is rapidly growing due to the development of space technology. For example the ESA/NASA SOHO spacecraft, http://www.nascom.nasa.gov, discovered exactly 1500 comets since 1995, the last one on 27. June 2008. About 2300 are catalogued, even if it is believed that there are more than 10^9 of them. As very few comets have periods of 12 years, their trajectories are good illustration for very elongated or nearly parabolic ellipses. Here is the short history on recent comet discoveries.

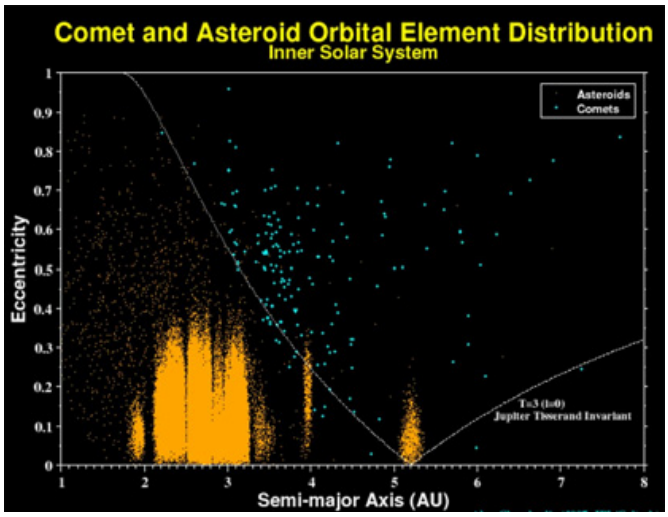
The Catalog of Cometary Orbits, compiled by Marsden, 1989 edition, lists 1292 computed orbits from 239 BC to AD 1989; only 91 of them were computed using the rare accurate historical datafrom before the 17th century. More than 1200 are therefore derived from cometary passages during the last three centuries. Sets of orbital elements in Marsden's catalog involve only 810 individual comets; the remainder represents the repeated returns of periodic comets. Four of these comets had been definitely lost, and three more were probably lost, presumably because of their decay in the solar heat. Of the 155 short-period comets, 93 have been observed at two or more perihelion passages.

The 16th edition of the Catalogue of Cometary Orbits of Smithsonian Astrophysical Observatory issued in 2005 contains 3031 sets of orbital elements (in the J2000.0 system) for 2991 cometary emersions of 2221 different comets through mid-August 2005. There is a special tabulation giving osculating elements for the 170 numbered periodic comets, excluding seven deemed to be lost.

According to the list of periodic comets on the Planetary Data System Small Bodies Node, NASA, last update on 10 April 2008, http://pdssbn.astro.umd.edu/comet\data, there are 420 designated periodic comets. According to Seiichi Yoshida's Comet Catalog, www.aerith.net, there are 243 non-numbered periodic comets (the last discovered C/2008 L3, 13. Jun 2008) and 200 numbered periodic comets.

In next discussion we shall rely on Marsden Catalog of Cometary Orbits. Of the 655 comets of long period contained in the Catalog, 192 have osculating elliptic orbits, and 122 have osculating orbits that are very slightly hyperbolic. Finally, 341 are listed as having parabolic orbits, but this is rather false because either it has not been possible to detect unequivocal deviations from a parabola on the usually very short arc along which the comets have been observed or, more simply, the final calculations have never been made. However, the parabola is always assumed first in the preliminary computation. If the osculating orbit is computed backward to when the comet was still far beyond the orbit of Neptune and if the orbit is then referred to the centre of mass of the solar system, the original orbits almost always prove to be elliptic.

These data exactly validates methods of nonstandard analysis in studying cometary trajectories. For example, in the preliminary computation, the value of the parameter p is computed. Simply, the second term (3) in the formula (2) may be omitted as we may consider it as an infinitesimal. It also shows that the formula (2) could be very appropriate in calculation of cometary orbits.



Let us consider very-long-period comets and comets having orbits not significantly different from a parabola. It is believed that these comets originate in the Oort cloud which is distant around 100000 AU from the Sun. By our previous discussion it is appropriate to use here methods of nonstandard analysis. So let us assume that a hypothetical comet C is moving along an ellipse E in the nonstandard plane having the second focus at $(q,0)$ where q is an infinite number. Therefore, the aphelion of E is at infinity, and by the second Kepler's law the velocity v of the comet near the aphelia (i.e. at the finite distance from aphelia in terms of nonstandard analysis) is an infinitesimal. Otherwise, the surface swept by the comet for the finite time Δt would be infinite due to the infinite distance of the comet from the Sun, and that would contradict the Second Kepler's law. In reality, a simple calculation shows that the velocity v of the comet C near aphelia would be around 100 m/sec, negligible small comparing to the velocity at the perihelia. Therefore, the momentum $p = mv$ of the comet C is an infinitesimal too; we would rather say that the comet C floats in the Oort cloud instead of it travels around the Sun. Hence the trajectory of the comet C is subject to any small perturbation, i.e. any infinitely small force, or impulse, would change significantly its trajectory. Simply saying, *parabolic orbits at far distances are very unstable*. This follows from the fact that the velocity Δv needed for transition from an orbit O_2 to the transfer ellipse which would carry the comet C to the other orbit O_1 is an infinite-

simal. The transfer ellipse will be seen from the near neighborhood of the Sun as a parabola with the second focus at infinity. The graph above (A. Chamberlin, JPL/Caltech, 2007) illustrates the instability of cometary orbits. The dotted line represents the Jupiter Tisserand invariant (T) evaluated at T=3 and zero inclination. This boundary *very roughly* separates small-bodies which are dynamically bound to Jupiter from those which are not. The region above this curve represents objects with T<3 (i.e. bound to Jupiter). Notice that *most* comets as well as the Trojan asteroids appear in that upper region (T<3, bound to Jupiter) while *nearly* all asteroids are contained in the region below the curve where T>3 (i.e. not bound to Jupiter). Therefore most of the comets having now elliptical orbits were captured once in the past by Jupiter, the dominant planet of the Solar system.

There are other astronomical evidences that support our discussion. Namely, according to Delsemme (2008) among the very-long-period comets, there is a particular class of comets that Oort showed as having never passed through the planetary system before, notwithstanding the fact that their original orbits were elliptic, which implies repeated passages. This paradox vanishes when it is understood that their perihelia were outside of the planetary system before their first appearance but that their orbits have been perturbed near aphelia by interstellar-cloud passages or by galactic tides, in such a way that their perihelia were lowered into the planetary system.

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36 Classical observations of latitude and the improved reference frame

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It was decided at the General Assembly of IAU in 1997 at Kyoto that the International Celestial Reference Frame (ICRF) materializes the International Celestial Reference System (ICRS) from the beginning of 1998, and the HIPPARCOS Catalogue was accepted as the primary representation of the ICRS in optical wavelengths. HIPPARCOS is one of the two catalogues (another one is Tycho) of ESA mission, and it gives for each of 118218 stars: very precise positions, proper motions, parallaxes, etc. However, nowadays we can see that the proper motions of some stars (mostly double or multiple) have problematic values because the mission was too short, less than four years. To improve these proper motions, it is possible to use also the ground-based long history optical observations of latitude/universal time variations (near 4.4 million observations of more than four thousand stars were collected), and the reference frame can be more stable. The goal is the Earth Orientation Catalogue (EOC). In this paper, the authors present some results of proper motions in declination of HIPPARCOS stars observed with Photographic Zenith Tubes (PZT) throughout the 20th century.

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CLASSICAL OBSERVATIONS OF LATITUDE AND THE IMPROVED REFERENCE FRAME

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Abstract. It was decided at the General Assembly of IAU in 1997 at Kyoto that the International Celestial Reference Frame (ICRF) materializes the International Celestial Reference System (ICRS) from the beginning of 1998, and the HIPPARCOS Catalogue was accepted as the primary representation of the ICRS in optical wavelengths. HIPPARCOS is one of the two catalogues (another one is Tycho) of ESA mission, and it gives for each of 118218 stars: very precise positions, proper motions, parallaxes, etc. However, nowadays we can see that the proper motions of some stars (mostly double or multiple) have problematic values because the mission was too short, less than four years. To improve these proper motions, it is possible to use also the ground-based long history optical observations of latitude/universal time variations (near 4.4 million observations of more than four thousand stars were collected), and the reference frame can be more stable. The goal is the Earth Orientation Catalogue (EOC). In this paper, we present some results of proper motions in declination of HIPPARCOS stars observed with Photographic Zenith Tubes (PZT) throughout the 20th century.

1. INTRODUCTION

The standard errors of Hipparcos stars positions are near 1 mas at the epoch of the catalogue (1991.25), and the st. errors of its proper motions are near 1 mas/yr. Nowadays, a few new catalogues appeared with better accuracy of proper motions than the Hipparcos one. The main problem of Hipparcos proper motions accuracy is the period of Hipparcos satellite observations which was shorter than four years; it is not enough to get a good accuracy of proper motions for some stars (as double or multiple ones). Also, some regions of Hipparcos Catalogue are with big errors of proper motions data. Nowadays, there are near 15 years after the epoch of Hipparcos Catalogue (1991.25), and the errors of Hipparcos proper motions (near 1 mas/yr) introduce the error of calculated apparent positions of Hipparcos stars of

about 15 mas, which is near the ground-based error of observed positions and one order bigger than the errors of Hipparcos positions. So, the astro-geodesy observations need better accuracy of proper motions than Hipparcos ones. Some investigations started during last decade by using useful ground-based observations (to get improved proper motions better than Hipparcos ones) and produced new catalogues: the ARIHIP (Wielen et al. 2001), EOC-2 (Vondrak 2004), etc.

The ground-based astrometric observations of latitude and universal time variations, covering the interval 1899.7-1992.0 and made in accordance with the Earth rotation programmes, were included into the investigations to improve the accuracy of proper motions, also. Here, we use the latitude variations of 10 PZT plus the OA00 (Ron and Vondrak 2001) solution of the EOP. Our method is different than that used for the EOC-2. The PZT data are: 3 at Washington, 2 at Mizusawa and Richmond, 1 at Ondrejov, Punta Indio and Mount Stromlo.

We determined the corrections of Hipparcos proper motions in declination with accuracy similar to accuracy of proper motions of ARIHIP and EOC-2.

2. DATA, CALCULATIONS AND CONCLUSIONS

The proper motion of the star (tangential part on the sphere) can be separated into two parts, along the coordinate right ascension and along the declination (the radial velocity is normal on both, and all together give the motion of the star in the space). Here, we are using PZT data, the linear model and the Least Squares Method (LSM) to calculate the corrections to Hipparcos proper motions in declination. To determine the proper motion in declination, it is necessary to have at least two positions (declinations, in the same system) of the same star for the epoch t1 and t2, respectively. The error of proper motion in declination is in line with the standard errors of mentioned declinations, but proportional to 1/t (Eichhorn 1974). It means, with long observations interval t, we can get good accuracy of proper motion in declination (better than the Hipparcos one) even Hipparcos accuracy of star positions is better than ground-based one. It is of importance for this work.

During each year of the observed interval, each PZT star was observed from a few until few hundred times. The latitude accuracy from one PZT star observation is mostly between 0.1 and 0.2 arcsec, but a few decades long time interval and a lot of observations of each star during each year of that interval are useful for calculations of better accuracy (than Hipparcos one) of proper motions in declination of common PZT and Hipparcos stars. The combination of latitude observations with Hipparcos one (for the moment 1991.25) can give us better results than only PZT latitude data. The accuracy of PZT data is about one order better than 0.2 arcsec for the averaged (over near one year observational interval) values of latitude variations.

In the case of the Punta Indio – Mount Stromlo stations (common stars), we used the PZT data (latitude variations) of both stations, with nearly the same latitude (-35.3 arc degrees). Similar situation is for the PZT stations Washington - Mizusawa.

We used the latitude PZT variations to improve Hipparcos proper motions in declination (Damljanović, 2005; Damljanović and Pejović, 2005), and the main PZT formula from the paper of McCarthy (1970). The polar motion part was calculated and removed from our input data (PZT latitudes) by using the Kostinski formula (Kulikov 1962). The polar motion coordinates x and y are from the EOP solution OA00 (Ron and Vondrak 2001). Also, the systematic variations (local, instrumental, etc.) for each instrument were calculated and removed from our input data. So, without the polar motion and systematic variations (local, instrumental, etc.), our input data (for LSM) are mostly with catalogue errors. It was our intention, in line with the connection (Vondrak et al. 1998) between the correction of proper motion in declination and the latitude data (without polar motion part and mentioned systematic variations). The main steps of our calculations, by using LSM with the linear model, are presented in few published papers (Damljanović, 2005; Damljanović and Pejović, 2005). Our calculated corrections to the Hipparcos proper motions in declination of common PZT/Hipparcos stars are in line with the epoch of Hipparcos Catalogue (1991.25). Input latitude data and the Hipparcos one are with suitable weights (Damljanović et al., 2006).

For example, the calculated correction to common PZT RCP/RCQ (Richmond) and Hipparcos star H9859 (5.99 mag) is +0.12 mas/yr with st. error 0.26 mas/yr. There are 40 input PZT points (about 40 observed years) plus the Hipparcos one, and the points are with weights. So, our proper motion in declination for H9859 is -14.42 mas/yr with st.error 0.26 mas/yr.

From EOC-2, it is -13.98 mas/yr with st. error 0.20 mas/yr.

From HIPPARCOS, it is -14.54 mas/yr with st. error 0.66 mas/yr.

The similar results are for the other PZT/Hipparcos stars. Our calculated corrections can be added to the Hipparcos values and get better values of proper motions in declination than the Hipparcos ones.

We calculated the proper motions in declination of 807 PZT/Hipparcos stars observed with 10 PZT instruments. Our results are in good agreement with the EOC-2 ones. The differences between our solutions and EOC-2 ones with number of observed years are mostly less than 1 mas/yr. In general, if the PZT stars are observed more than 10 years our results are better than the Hipparcos ones. For good results, it is necessary to have the input data with long interval of observations, as we remarked at the beginning of this paper.

It is evident that PZT data are useful and can improve the reference frame (via improvement of proper motions in declination of Hipparcos stars).

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37 Astroclimatic characteristics of Vidojevica mountain

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The Vidojevica Mountain belongs to the mountain group of the Southern Serbia that elongates the Rodop mountain system. Other mountains of the group are: Arbanaska, Sokolovica, Pasjača and Rgaja. The Vidojevica Mountain is near to the town of Prokuplje and directly faces the town from the South. In fact, it extends into the direction from the North to the South West. The average height of the Vidojevica Mountain is 780 m and it is the natural barrier of the Southern side of the Toplica valley. It has one dominant peak Bandera (1154m), and two minor ones, Beli kamen (1072 m) and the peak Perina Livada (1072 m). The new astronomical station (ASV) of the Astronomical Observatory in Belgrade (AOB) is situated at the peak Bandera, therefore the astroclimatic properties at the mountain will have an important role in the future plans for the utilization of ASV. We present in this paper the comparative investigations of the microclimatic and macroclimatic properties of the mountain Vidojevica and the vicinity and some of the astroclimatic conditions, as well.

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ASTROCLIMATIC CHARACTERISTICS OF VIDOJEVICA MOUNTAIN

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Abstract. The Vidojevica Mountain belongs to the mountain group of the Southern Serbia that elongates the Rodop mountain system. Other mountains of the group are: Arbanaska, Sokolovica, Pasjača and Rgaja. The Vidojevica Mountain is near to the town of Prokuplje and directly faces the town from the South. In fact, it extends into the direction from the North to the South West. The average height of the Vidojevica Mountain is 780 m and it is the natural barrier of the Southern side of the Toplica valley. It has one dominant peak Bandera (1154m), and two minor ones, Beli kamen (1072 m) and the peak Perina Livada (1072 m). The new astronomical station (ASV) of the Astronomical Observatory in Belgrade (AOB) is situated at the peak Bandera, therefore the astroclimatic properties at the mountain will have an important role in the future plans for the utilization of ASV. We present in this paper the comparative investigations of the microclimatic and macroclimatic properties of the mountain Vidojevica and the vicinity and some of the astroclimatic conditions, as well.

1. MACROCLIMATIC CHARACTERISTICS OF THE TOPLICA VALLEY

We investigate the temperature variations in the Toplica valley using data obtained by the meteorological stations located in the towns of Prokuplje, Blace and Kuršumlija. The data cover the period from 1925-2002. As seen in the Table 1, July is the hottest month in Prokuplje, while January is the coldest. Data for the Vidojevica Mountain are similar to these ones, but with the less difference between temperature extremes.

Table 1. The average monthly temperature of the air in the Toplica valley

Station	1	2	3	4	5	6	7	8	9	10	11	12	Average temperature	Amplitude
Prokuplje	-0,4	-0,6	5,7	11,6	16,1	19,8	22,6	21,7	18,2	12,3	7,4	1,3	11,4	23,0
Vidojevica	-0,8	-0,7	5,6	10,4	15,0	17,8	20,3	19,8	15,7	10,8	5,3	2,7	10,3	21,1

Table 2. The average temperature of the air per period of the year.
(Data are obtained from the meteorological station of the town of Niš, 1900-2004.)

Place	Winter	Spring	Summer	Autumn
Prokuplje	0,5	11,1	21,4	12,6
Vidojevica	- 2.5	8,3	17,5	11,7

The Table 2 shows that winter is coldest at both places, at the Vidojevica Mountain and in the town of Prokuplje. The temperature changes, as expected, with the change of the altitude; the temperature falls 6.5°C in average with every thousand meters. Further, other atmospheric phenomena occur at the mountain. For example, the temperature inversion sometimes becomes visible at the Vidojevica Mountain. Instead of temperature falling with the height, it sometimes rises up to 10°C comparing to the bottom of the mountain. The difference between seasonal extreme temperatures is high. In the summer months, the extreme temperature goes high as 40°C, while in the winter months it could be as low as -20°C.

2. RELATIVE HUMIDITY

The flow of the temperature humidity changes proportionally to the change of the temperature. It is larger in the night than in the day-time and it is smaller in summer then in winter.

Table 3. Relative humidity per months, 1950-2004.

Stations	1	2	3	4	5	6	7	8	9	10	11	12
Prokuplje	84%	81%	76%	72%	71%	68%	63%	65%	69%	75%	82%	85%
Kuršumlija	85%	83%	79%	76%	76%	70%	65%	66%	71%	79%	83%	87%
Blace	86%	84%	78%	75%	75%	71%	64%	67%	70%	80%	85%	88%
Vidojevica	82%	81%	73%	72%	71%	64%	60%	65%	69%	75%	82%	84%

According to these data, the relative humidity in Jun, July, August and September in the Toplica region does not exceed 70% and at Vidojevica it is several percents lower.

3. CLOUDINESS AND INSOLATION

The cloudiness is an important element of the climate on which the insolation obviously depends. According to the data in the Table 4, the cloudy weather in the Toplica region is mostly presented in January and December, and it is the least presented in August, July and September. For example, the cloudy weather on Vidojevica is 70% of days in January, while in August it takes only 29%.

Table 4. The average of the cloudiness per months, 1950-2004.
(The table depicts the percentage of cloudy days in one month)

Stations	1	2	3	4	5	6	7	8	9	10	11	12	yearly
Prokuplje	73	62	54	52	51	40	29	28	30	48	60	71	49
Kuršumlija	74	64	56	53	54	43	30	29	33	50	64	74	52
Blace	75	63	55	54	56	44	31	30	32	49	62	75	52
Vidojevica	70	60	49	50	50	31	28	29	31	44	59	58	49

By observing the seasonal cloudiness at the Vidojevica Mountain, we note that the cloudy weather decreases from winter to summer as it is rather low in June, July, and August, then it rises again when the winter months approach. Note that the cloudy weather is in the opposite proportion to the annual temperature flow. The total amount of insolation during the year in average is 1841 h. It is smallest in December, 45.5 h, and in January it is 75.3 h with the average daily insolation of 1.4 and 2.4. It is highest in August, 258 h, and 249 in July, with the average daily amount of 8.3 h and 8.2 h respectively. The observed data agree with the Hendman's conclusion that the insolation is directly proportional to the temperature and the humidity.

Table 5. Average insolation per months and per year in hours.

Station	1	2	3	4	5	6	7	8	9	10	11	12	Insol.
Prokuplje	75,3	77,2	107,4	155,2	203,9	210,8	248,7	258,1	229,6	152,2	76,1	45,5	1841

4. PRECIPITATION

According to data in the Table 6, the Toplica region is one of the driest in Serbia, as it gets only 580 mm of the accretion measured on the annual basis. And Vidojevica Mountain even belongs to semi-arid rain with the total precipitation amount of 533 mm on annual basis.

Table 6. The average of precipitation per month in mm, 1950-2004.

Station	1	2	3	4	5	6	7	8	9	10	11	12	year
Prokuplje	41	31	36	40	60	45	33	37	26	64	49	54	516
Kuršumlija	44	40	37	48	71	68	39	41	30	68	51	60	597
Blace	44	40	40	55	74	72	55	42	31	64	60	51	628
Vidojevica	39	33	34	41	58	59	40	39	27	63	49	51	533
Niš	49	42	39	50	70	69	42	43	33	70	54	62	624
Leskovac	47	41	40	51	71	72	43	44	32	68	55	58	622

Precipitation is comparatively unbalanced in respect to the months and the type of precipitation. Rain and snow dominate. As seen from the displayed data, the greatest amount of rain is in May and in October, whiles the least is in September and in February. Such precipitation distribution belongs to the continental evaporation regime, in this case the Danube evaporation regime. However there is a small variation in the monthly distribution of precipitation; the greatest amount of precipitation is in September, not in February. We also see that the summer months July and August especially lack precipitation (especially in the Prokuplje area). Particularly, rains often come as short downpours.

5. MICROCLIMATIC CHARACTERISTICS OF THE VIDOJEVICA MOUNTAIN

Vidojevica is deeply extended into the continental climate; therefore there is the influence of this type of climate into this region. During the winter Vidojevica is under deep snow one to three months, in average 2.5 months. As the auxiliary road which connects ASV and the main road is 2.5 km long, this may affect in the wintertime the communication between ASV and the main road.

In accordance to the temperature gradient principle the average temperature on the peak of Vidojevica is lower about 5^0 C in comparisons to the average temperature in the Toplica valley. However, the temperature inversions often appear, particularly in summer and in winter. An explanation would be that the river Toplica crosses the Toplica valley just next to the foot of Vidojevica. It is the coldest hydrological object in the summertime and it could be colder up to 7^0 C than the objects on the land. Thus, the colder air along the river keeps under the warm air along the mountain, while on the top of Vidojevica appears the stream of the colder air. This streaming can be so strong that can be sensed on the human body since the temperature difference can amount up to 10^0 C. The other explanation would be as follows. The colder air from the mountain mixes up with the air currents rising from the valley, comes near the river valley suppressing the warmer air which goes up and so becomes less dense. Thus, noticeable currents are created in the valley; the current speed is 2-3 m/s up hill. When the air current moves southwards, it reaches the first obstacle when it comes to the North West side of Vidojevica. Then the air current flows near the very top of the mountain splitting into two directions. One direction again follows the Toplica valley, while the other direction goes from the top of the mountain towards the higher sea level heights and further into the troposphere. The other current that goes back changes its direction of moving in the midday.

On the other side, during the winter time there are more sunny days on the top of the mountain than in the Toplica valley. So the temperature is higher at the top of Vidojevica than in the valley and again appears the temperature inversion. For better understanding of just described microclimatic phenomena at Vidojevica Mountain, we give the Table 7 with meteorological parameters recently measured.

Table 7. Climatologic elements at the Vidojevica Mountain

Date	Peak Bandera	Height of snow	Precipitations	Maximum °C Minimum °C	Air pressure	Relative Humidity
15.01.2004	/	14,5 cm	Jan. 39 ml	-9,0 -1,0	1016 mb	79%
02.04.2004	/		Apr. 29 ml	- 4,1 14,0	1010 mb	70%
09.06.2004	/		June 21 ml	+8,0 +31,0	1009 mb	64%
10.10.2004	/		Oct. 47 ml	+6,0 +13,0	1013 mb	73%

6. CONCLUSION

Weather prediction has always been considered an important tool for the routine operation of an observing site. This paper approaches the topic with particular reference to the new astronomical station ASV at the Vidojevica Mountain. This research is based on meteorological data for the Toplica region which are collected in last 50 years; some of them are more than 100 years old. There are often noticeable air currents at the top of the Vidojevica Mountain where the station is situated. But they appear on the North direction from the station so they should not be the significant obstacle for the future observations since most of them will be dedicated to the observations of the southern celestial objects. Long term observations of other meteorological parameters, in particular the precipitation, shows that the climate at Vidojevica becomes drier turning into almost semi arid climate. This shows that the choice of the Vidojevica Mountain as a place for building of an astronomical station was good.

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38 A better reference frame by using improved proper motions of single and double stars

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The authors used the satellite Hipparcos observations and the ground-based observations (made during the XX century of latitude variations), as their input data. The ground-based data were obtained with 26 instruments of observatories located all over the world, and the observed stars were included in the programs used for the Earth rotation study. Now, with these ground-based observations the Hipparcos proper motions can be improved. The goal of the presented research was to improve the proper motions in declination of stars and thus to improve the reference frame, because the Hipparcos Catalogue is the optical counterpart of the International Celestial Reference Frame (ICRF). The basic authors method (the linear fit for single stars) is adapted (the sinusoidal fit for double and multiple stars) and some results are presented here.

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A BETTER REFERENCE FRAME BY USING IMPROVED PROPER MOTIONS OF SINGLE AND DOUBLE STARS

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Abstract. We used the satellite Hipparcos observations and the ground-based observations (made during the last century of latitude variations), as our input data. The ground-based data were obtained with 26 instruments of observatories located all over the world, and the observed stars were included in the programs used for the Earth's rotation study. Now, with these ground-based observations we can improve the Hipparcos proper motions. So, the ground-based data of mentioned stars were used here, if these stars are contained in the Hipparcos Catalogue. The goal of our investigations was to improve the proper motions in declination of stars (which were observed ground-based and from the Hipparcos Catalogue), and thus to improve the reference frame, because the Hipparcos Catalogue is the optical counterpart of the International Celestial Reference Frame (ICRF). There is a large number of ground-based observations per year of each of the mentioned stars. Also, the observed time intervals of these stars are usually much longer than the Hipparcos ones (the satellite mission lasted less than four years). So, it is possible to correct the Hipparcos proper motions. Our basic method (the linear fit for single stars) is adapted (the sinusoidal fit for double and multiple stars) and some results are presented here.

1. INTRODUCTION

The products of the Hipparcos satellite mission (ESA 1997) were the Hipparcos (HIgh Precision PARallax COllecting Satellite) and Tycho catalogues. They were linked to the ICRF and appeared in 1997 (after the XXIII GA of IAU in Kyoto), and since 1997 the Hipparcos Catalogue is considered the primary optical counterpart of ICRF. Also, one of decisions of the IAU GA in Kyoto was to adopt the ICRF as the realization of the ICRS (the International Celestial Reference System) from the beginning of 1998. After that, it was actual to check and correct the positions and proper motions of Hipparcos Catalogue stars using ground-based observations of these stars. On the other hand, in 1995 Commission 19 (Earth Rotation) of IAU established the Working Group on Earth Rotation in the Hipparcos Reference Frame (WG ERHRF), with Dr. Vondrák as the Head, to collect the optical observations of latitude and universal time variations. These observations were made from 1899.7 to 1992.0; they were in line with the Earth orientation programmes and were used to derive the Earth Orientation

Parameters (EOP). During a few years, the WG ERHRF collected about 4.4 million observations. The data were from 33 observatories all over the world; there were about 50 instruments (Vondrák et al. 1998).

The Hipparcos Catalogue contains 118218 stars brighter than magnitude 12, and the epoch of the catalogue is 1991.25; the positions are with an accuracy of the order of 1 marcsec at 1991.25 and the proper motions in $\mu_\alpha \cos\delta$ and μ_δ are with a standard error of about 1 marcsec/yr. Some star catalogues appeared after the Hipparcos one: the ARIHIP one, the Earth Orientation Catalogue (EOC-2), etc. The EOC-2 is with more accurate star positions and proper motions than the Hipparcos ones. Also, we did our investigations to improve the Hipparcos proper motions in declination. It is $\mu_\delta = \frac{\delta_1 - \delta_2}{t_1 - t_2}$ (Eichhorn 1974), where δ_1 and δ_2 are two positions (in the same system) of the same star for the epochs t_1 and t_2, respectively. The error of μ_δ is $\epsilon_{\mu_\delta} = \frac{\sqrt{\epsilon_1^2 + \epsilon_2^2}}{t_2 - t_1}$, where ϵ_1 and ϵ_2 are standard errors of δ_1 and δ_2, respectively. So, ϵ_{μ_δ} is proportional to $\frac{1}{t}$, and with a sufficiently long interval of observations we can get a very good accuracy ϵ_{μ_δ} (better than the Hipparcos one).

2. DATA

Shortly after appearing of the Hipparcos Catalogue, it was evident that there were some problems concerning the Hipparcos data. The accuracy of the data concerning single stars is better than that for double/multiple ones. There is a different accuracy of the data on different parts of the celestial sphere. The error of apparent position has attained about 19 mas until now; the influence of proper motions is linear in time from the moment 1991.25, and the value 19 mas is one order of magnitude larger than the average position error in the Hipparcos Catalogue. These reasons are enough to start with improving the Hipparcos proper motions. So, it is of importance to combine the satellite data (as the Hipparcos ones) with the ground-based ones.

The polar motion coordinates (x_i, y_i) for our calculations are from Vondrák's file EOPOA00.dat; the $MJD = JD - 2400000.5$ is Modified Julian Date. We used the latitude variations φ_i (Vondrák et al. 1998) which Dr. Vondrák sent us via private communication. The φ_i were obtained during the last century (1899.7 – 1992.0) and primarily used to derive the EOP. The several types of classical astrometric instruments were operating. They are the visual and floating zenith-telescopes (ZT and FZT), the visual zenith tube (VZT) and the photographic zenith tube (PZT). Also, we used the Hipparcos data. There are 2347 common ground-based/Hipparcos stars. The goal was to improve the Hipparcos proper motions in declination μ_δ of stars observed with these instruments. We used the data of 26 classical instruments (14 ZT, 1 FZT, 1 VZT and 10 PZT ones) located at 19 observatories. They are 6 PZT stations: Mizusawa-MZP/MZQ, Mount Stromlo-MS, Ondřejov-OJP, Punta Indio-PIP, Richmond-RCP/RCQ, Washington-WA/W/WGQ. Than 7 ZT stations of the International Latitude Service (ILS): Carloforte-CA, Cincinnati-CI, Gaithersburg-GT, Kitab-KZ, Mizusawa-MZZ, Tschardjui-TS, Ukiah-UK. The independent stations 1 FZT (Mizusawa-MZL), 1 VZT (Tuorla-Turku-TT) and 6 ZT: Belgrade-BLZ (Damljanović 1997), Blagoveschtschensk-BK, Irkutsk-IRZ, Poltava-POL, Pulkovo-PU/PUZ, Warsaw-VJZ. More about it are in (Vondrák et al. 1998) and in (Damljanović 2007).

3. METHOD AND CONCLUSIONS

In the beginning, the values of φ_i were used to study the terrestrial rotation (Vondrák et al. 1998, Ron and Vondrák 2001, Vondrák and Ron 2003). These ground-based observations (tens of years) lasted much longer than the Hipparcos ones (shorter than 4 years). The annual number of ground-based observations for every star is on the average several hundred. So, our value of ϵ_{μ_δ} can be less than the Hipparcos one though the Hipparcos accuracy of the star coordinates is by two orders of magnitude better than the one of the star coordinates following from the φ_i observations.

First, we removed the polar motion components $\Delta\varphi_i$, the systematic (local, instrumental) changes se_i with time, and get the residuals r'_i (Damljanović 2005, Damljanović and Pejović 2005). The $\Delta\varphi_i$ were calculated by using the Kostinski formula (Kulikov 1962) $\Delta\varphi_i = x_i \cos\lambda_W + y_i \sin\lambda_W$, where λ_W is the longitude (the west direction) of the observatory. The r'_i are with systematic catalogue effects which are of our interest here. For each star, the r''_i are the averaged values of r'_i over subperiods of about 1 year (Damljanović et al. 2006, Damljanović and Pejović 2006). Then, we can use the LSM with the linear fit $r''_i = a + b(t_i - 1991.25)$, and get the corrections b to the proper motions in declination for treated Hipparcos stars. The values t_i are the times (in years) corresponding to r''_i. The value a is the correction $\Delta\delta$ and b is the correction $\Delta\mu_\delta$; both are referred to the epoch 1991.25. The points r''_i and the Hipparcos one (1991.25, 0.″0) are with the suitable weights (Damljanović et al. 2006). This procedure is valid for PZT (Damljanović and Pejović 2008, McCarthy 1970) and VZT observations, but for ZT and FZT ones there are some additional steps in order to obtain the values of a and b by using the observations of star pairs (Damljanović and Pejović 2006, Damljanović 2007, Damljanović 2008). For the case of a single star, we process star by star by using the LSM, and the input values r''_i and (1991.25, 0.″0). We add our corrections b to the corresponding Hipparcos proper motions in declination. So, we obtain the values μ_δ and their standard errors; the results (as the catalogue of 2347 stars) are contained in PhD thesis (Damljanović 2007) and few published papers (Damljanović and Pejović 2006, Damljanović and Pejović 2008).

In the case of double or multiple stars, we get the model (for star by star procedure), as the second step of our calculations,

$$r'''_i = b't'_i + A\sin(2\pi t'_i/P) + B\cos(2\pi t'_i/P), \tag{1}$$

where: the residuals $r'''_i = r''_i - a$, the time $t'_i = t_i - 1991.25$, b' is the unknown correction to the Hipparcos proper motions in declination (close to the suitable values b), the amplitude $am = (A^2 + B^2)^{1/2}$ of periodic part. The periodic part is $am\cos(2\pi t'_i/P - F)$ with the period P and phase F (from the moment 1991.25). We have got three unknowns to calculate: b', A and B. The value P (in years) is from the interval $(2; 4m')$ with step $k = 1$ year, where m' is the number of the observed years. For each star we have got $(4m' - 3)$ different solutions of the unknowns b', A and B. We use the solution when the calculated curve is the best fit for the set of the input points (r'''_i and (1991.25, 0.″0)) and value P. That fit is for the case $\sigma_0 = min.$, where σ_0 is the root-mean-square error of differences between the calculated points and suitable input ones. Like that, we can determine the values P and F.

For the single stars with a sufficient number of observations, the newly obtained values of proper motions in declination (Damljanović 2007) are substantially more

precise than those of the Hipparcos ones. Our results were compared with ARIHIP and EOC-2 data, and we found a good agreement (Damljanović et al. 2006, Damljanović 2007). For example, the value $b \pm \varepsilon_b$ for star H15334 is -0.44 ± 0.15 mas/yr, and it is $\mu_\delta = -17.13 \pm 0.15$ mas/yr. From EOC-2, it is $\mu_\delta = -17.29 \pm 0.12$ mas/yr, and from Hipparcos $\mu_\delta = -16.69 \pm 0.69$ mas/yr.

Conclusions: after removing all known effects (as the polar motion one and some local and instrumental errors) from the instantaneous latitude, the residuals are then used for our first step to calculate the corrections of the Hipparcos proper motions in declination by using the LSM with the linear fit. This step was good enough for the single stars results. The newly obtained values of proper motions in declination are substantially more precise than those of the Hipparcos Catalogue. It is because the time interval covered by the latitude observations (tens of years) is much longer than the Hipparcos one (less than four years), and because of the great number of ground-based observations made during this interval. The calculated results were compared with the ARIHIP and EOC-2 ones (Vondrák 2004) and we found a good agreement. For double and multiple stars, we developed the second step of our calculations using LSM with the sinusoidal curve. So, the Hipparcos proper motions can be improved by using together the Hipparcos data and the ground-based ones.

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39 Error analysis of proper motions in declination obtained for 807 Hipparcos stars from PZT observations over many decades

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This paper combines three related areas: astrometry, astronomical catalogues and reference systems used in astronomy. After publication of the Hipparcos catalogue (in 1997), a few new astrometric catalogues have appeared (TYCHO-2, ARIHIP, etc.), as a good combination of the Hipparcos satellite and ground-based data, to get more accurate coordinates and proper motions of stars than the Hipparcos catalogue ones. There are about 4.4 million optical observations made worldwide at 33 observatories and with 47 instruments during 1899.7. - 1992.0; our Belgrade visual zenith telescope data (for the period 1949.0-1986.0) were included. These data were used to determine the Earth Orientation Parameters. PZT data (more than 0.9 million observations made at six observatories during the time interval 1915.8.1992.0), and combine them with the Hipparcos catalogue ones, with suitable weights, in order to check the proper motions in declination for 807 common PZT/Hipparcos stars (and to construct the PZT catalogue of μ_δ for 807 stars). The authors standard standard errors in proper motions in declination of these stars are less than or equal to the Hipparcos ones for 423 stars. The main steps of the pro-

posed method and the investigations of systematic errors in determined proper motions are presented here. A comparison of the four catalogues by pairs shows that there is no significant relationship between the differences of their μ_δ values and magnitudes and color indices of the common 807 stars. All catalogues have relatively small random and systematic errors which are close to each other. However, the comparison shows that the authors formal errors are small, too.

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Error analysis of proper motions in declination obtained for 807 Hipparcos stars from PZT observations over many decades

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After publication of the Hipparcos catalogue (in 1997), a few new astrometric catalogues have appeared (TYCHO-2, ARIHIP, etc.), as a good combination of the Hipparcos satellite and ground-based data, to get more accurate coordinates and proper motions of stars than the Hipparcos catalogue ones. There are also investigations on improving the Hipparcos coordinates and proper motions by using the astrometric observations of latitude and universal time variations (via observed stars referred to Hipparcos catalogue), together with Hipparcos data, carried out during the last few years. These kind of ground-based data were collected at the end of the last century by J. Vondrák. There are about 4.4 million optical observations made worldwide at 33 observatories and with 47 instruments during 1899.7–1992.0; our Belgrade visual zenith telescope data (for the period 1949.0-1986.0) were included. First of all, these data were used to determine the Earth Orientation Parameters – EOP, but they are also useful for the opposite task – to check the accuracy of coordinates and proper motions of Hipparcos stars which were observed from the ground over many decades. Here, we use the latitude part of ten Photographic Zenith Tubes – PZT data (more than 0.9 million observations made at 6 observatories during the time interval 1915.8–1992.0), and combine them with the Hipparcos catalogue ones, with suitable weights, in order to check the proper motions in declination for 807 common PZT/Hipparcos stars (and to construct the PZT catalogue of μ_δ for 807 stars). Our standard errors in proper motions in declination of these stars are less than or equal to the Hipparcos ones for 423 stars. The mean value of standard errors of 313 stars observed over more than 20 years by PZT is 0.40 mas/yr. This is 53% of 0.75 mas/yr (the suitable value from the Hipparcos catalogue). We used the Least Squares Method – LSM with the linear model. Our results are in good agreement with the Earth Orientation Catalogue – EOC-2 and the new Hipparcos ones. The main steps of the method and the investigations of systematic errors in determined proper motions (the proper motion differences with respect to the Hipparcos values, the EOC-2 ones and the new Hipparcos ones, as a function of α, δ, and magnitude) are presented here. A comparison of the four catalogues by pairs shows that there is no significant relationship between the differences of their μ_δ values and magnitudes and color indices of the common 807 stars. All catalogues have relatively small random and systematic errors which are close to each other. However, the comparison shows that our formal errors are too small. They are underestimated by a factor of nearly 1.7 (for EOC-2, it is 2.0) if we take the new Hipparcos (or Hipparcos) data as reference.

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1 Introduction

A few of problems concerning the Hipparcos (ESA 1997) data have become clear, as some parts of the catalogue (mostly double or multiple stars) show larger errors of proper motions than others (Vondrák 2004). Also, there are about 19 years after 1991.25 (the epoch of the Hipparcos catalogue), and therefore the error in the calculated apparent positions of Hipparcos stars is more than one order bigger than the errors of their catalogue positions. About 20% of the positions and proper motions of Hipparcos stars had to be corrected (Vondrák 2004), and these corrections mostly reflected the problems in line with double/multiple stars because the interval of Hipparcos satellite observations was less than 4 years (not enough for a good proper motions accuracy of these stars), etc. All mentioned remarks were a good reason that new star catalogues appeared after the Hipparcos one; their accuracy of star positions and proper motions is close to or better than the Hipparcos one. Also, the re-reduced Hipparcos data appeared (van Leeuwen 2007). We used tens of years of PZT observations of 807 Hipparcos stars to solve some of the mentioned problems; it is useful that there are several hundreds of PZT observations (on the average) per year of each observed PZT star.

The general idea to construct the new Earth Orientation Catalogue (EOC) which is based on ARIHIP (Wielen et al. 2001), TYCHO-2 (Høg et al. 2000) and the optical data of latitude, universal time, and altitude variations (covering the interval 1899.7–1992.0) was outlined by Vondrák & Ron (2003). At Belgrade Astronomical Observatory, we used the latitude variations (referred to the Hipparcos catalogue) observed with 26 ground-based instruments to check the accuracy of proper motions in declination of Hipparcos stars. Some of our results have already been published (Da-

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mljanović & Pejović 2006; Damljanović et al. 2006; Damljanović & Pejović 2008).

Our method of calculations was developed at Belgrade (Damljanović 2005; Damljanović & Pejović 2005; Damljanović et al. 2006). We got the data from Dr. Vondrák via private communication. The Hipparcos point (1991.25, 0.″0) was also taken for our calculations, and all input points (for LSM method and the linear model) were included into our procedure with suitable weights. Here we present some of our investigations to get the proper motions in declination with the accuracy which is close to or better than the Hipparcos ones for 807 stars observed by ten PZT instruments. We show the proper motion differences with respect to other data (the comparison with other catalogues as the Hipparcos, new Hipparcos and EOC-2), these differences as a function of coordinates and magnitude, etc. Our calculated results are in good agreement with the re-reduced Hipparcos ones (van Leeuwen 2007) and EOC-2 (Vondrák 2004) ones.

2 Data

Over longer time intervals, the coordinates of nearby stars are a linear function of time ($\alpha(t)$, $\delta(t)$) because of their space motion. Therefore, we use the linear fit in our method. The unknowns are determined by using the LSM method. The proper motion of a star μ has got two parts: along the α coordinate (μ_α) and along the δ coordinate (μ_δ). It is $\mu = \sqrt{\mu_\alpha^2 \cos^2\delta + \mu_\delta^2}$. Together, μ, the distance to the star, and its radial velocity determine the space velocity of a star.

We are dealing with $\mu_\delta = \frac{\delta_1 - \delta_2}{t_1 - t_2}$, and to calculate the value μ_δ it is necessary to have at least two positions of a star (δ_1, δ_2) in the same system (Eichhorn 1974) and their epochs (t_1, t_2), respectively. The standard errors of positions are ε_1 and ε_2, respectively; the error of μ_δ is $\varepsilon_{\mu_\delta} = (\sqrt{\varepsilon_1^2 + \varepsilon_2^2})/(t_2 - t_1)$ (Eichhorn 1974). The value of ε_{μ_δ} is inversely proportional to the observed interval $t = t_2 - t_1$ of the PZT star. If t is long enough, it is possible to get a similar or better accuracy of μ_δ than the Hipparcos one.

There are more than 0.9 million observations made at 6 observatories during the interval 1915.8–1992.0, and several hundreds of observations per year (on the average) for each PZT star. The standard error of a single PZT observation ranges from 100 mas to 200 mas, and of the Hipparcos star position is near 1 mas. Approximately, if the ratio between the mentioned st. errors of single observations is at least 100, then the ratio between the error of the mean PZT position and that of the Hipparcos position is going to be 10 for one hundred PZT observations per year. The Hipparcos position accuracy is still ten times better than the PZT one. Therefore, we need about 10 years of PZT latitude observations of some star to get an accuracy (of its proper motion in declination) near the Hipparcos one. If we use the Hipparcos point (1991.25, 0.″0), as input together with the PZT ones, the necessary time interval of PZT observations could be less than 10 years. With the same consideration, but for the maximum value of the mentioned ratio (which is near 200), we need a longer period of PZT observations (up to 20 years). Anyway, with more than 20 years of PZT observations we could get a value of ϵ_{μ_δ} which is close to or less than the Hipparcos one. Because of that, we investigate three subgroups: A (313 stars with $\geq$20 years of PZT observations), B (685 stars with $\geq$10 years), C (all 807 stars). Some of N stars (about 15%) of the PZT programmes mentioned in Table are not among the 807 (Damljanović & Pejović 2008) because their data were already corrected (Vondrák et al. 1998), and some stars are presented in more than one programme.

Table 1 Ten PZT instruments.

Observatory Instr., N	Period in yr (+1900) and in MJD	$\lambda(^\circ)$, $\varphi(^\circ)$, $\lambda_W(^\circ)$
Mizusawa MZP, MZQ 137	02 I '59–21 XII '91 36570.6–48611.5 (MZP to '75.3) (MZQ from '74.2)	141.1, 39.1, 218.9
Mount Stromlo MS, 184	31 X '57–27 VIII '85 36142.5–46304.7	149.0, –35.3, 211.0
Ondřejov OJP, 285	05 II '73–14 XII '91 41718.9–48604.7	14.8, 49.9, 345.2
Punta Indio PIP, 165	03 VIII '71–29 VI '84 41166.9–45881.1	–57.3, –35.3, 57.3
Richmond RCP, RCQ 202	05 XI '49–12 V '89 33225.2-47658.1 (RCP to '87.5) (RCQ from '81.9)	–80.4, 25.6, 80.4
Washington WA, W, WGQ 188	28 X '15–26 XII '91 20798.2–48616.0 (WA to '55.3) (W '54.3–'84.8) (WGQ from '81.7)	–77.1, 38.9, 77.1

In Fig. 1, the distribution of 807 stars on the celestial sphere is presented. Only a few declination zones are covered because each PZT instrument was operational only in its zenith zone. Therefore, the proper motion differences as a function of δ (in Fig. 6) include only four points. In Fig. 2, the distribution of magnitudes of 807 stars is shown. Our PZT stars are mostly between $Hp = 6$ mag and $Hp = 10$ mag. Note that the new Hipparcos catalogue provides astrometric data with better accuracies (by a factor 2.2 in the total weight and by up to a factor 4 for nearly all stars brighter than $Hp = 8$ mag) than in the original catalogue (van Leeuwen 2007).

The procedure of reduction of latitude variations φ_i around the suitable mean latitude were described in Vondrák et al. (1998). During the twentieth century, the values of φ_i were used to monitor the polar motion of the Earth, but they are also good enough for modern astrometry tasks. Here, we are using the values of φ_i to calculate the correction to $\mu_{\delta_{\rm HIP}}$ and to get the value of μ_δ. We expect that our accuracy is close to or better than the Hipparcos one for any PZT star

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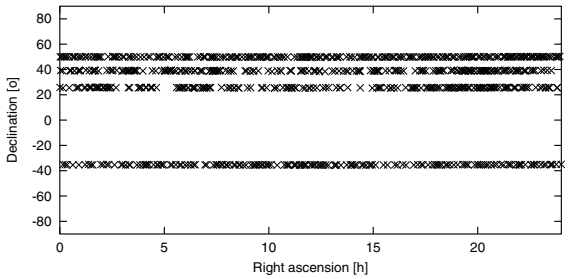

Fig. 1 Distribution of 807 stars on the celestial sphere.

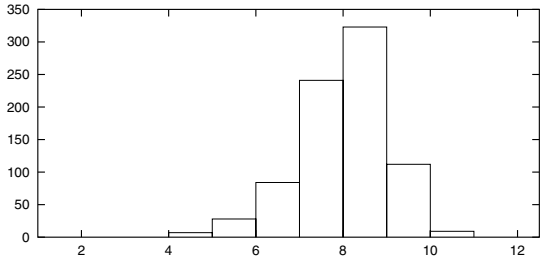

Fig. 2 Distribution of magnitudes of 807 stars.

with a sufficient number of observations (Damljanović & Pejović 2008).

In Table 1 there are some basic informations about the PZT instruments and our input data.The columns of Table 1 are: observatory – the name of the observatory, instrument – the codes of the instruments, N – the number of observed stars, period – the dates (the beginning and the end of observations) in years and MJD (Modified Julian Date, $\mathrm{MJD} = \mathrm{JD} - 2\,400\,000.5$), λ and φ are the geographic coordinates (latitude and longitude) of the observatory, (λ_{W} is also the longitude but west of the zero meridian).

The observatories Punta Indio (Argentina) and Mount Stromlo (Australia) are with nearly the same value of latitude ($\varphi = -35\overset{\circ}{.}3$), but different longitudes ($\lambda = -57\overset{\circ}{.}3$ for PIP, $\lambda = 149\overset{\circ}{.}0$ for MS), and there are 157 common stars in their observational programmes. Their intervals overlap by about 13 years (the PIP's interval is 1971.6–1984.5 and the MS's one is 1957.8–1985.7). We checked the method via the internal errors by using the calculated values of μ_δ from PIP data and MS ones separately, and also compared the results; some details and examples about it are presented in a few papers (Damljanović 2005; Damljanović & Pejović 2005; Damljanović & Pejović 2008; Damljanović 2008). The similar situation is for the observatories Mizusawa ($\varphi = 39\overset{\circ}{.}1$, $\lambda = 141\overset{\circ}{.}1$, 1959.0–1992.0, Japan) and Washington ($\varphi = 38\overset{\circ}{.}9$, $\lambda = -77\overset{\circ}{.}1$, 1915.8–1992.0, USA). Their intervals overlap by 33 years, and there are 71 common stars. Our raw sets of data are with polar motion part and different systematic variations (local, instrumental, etc.) vs. time. The first step of our method is to determine and remove both effects. So, we aim at getting the residuals (which are with the Hipparcos catalogue systematic errors of proper motions), and to calculate the corrections of star proper motions in δ.

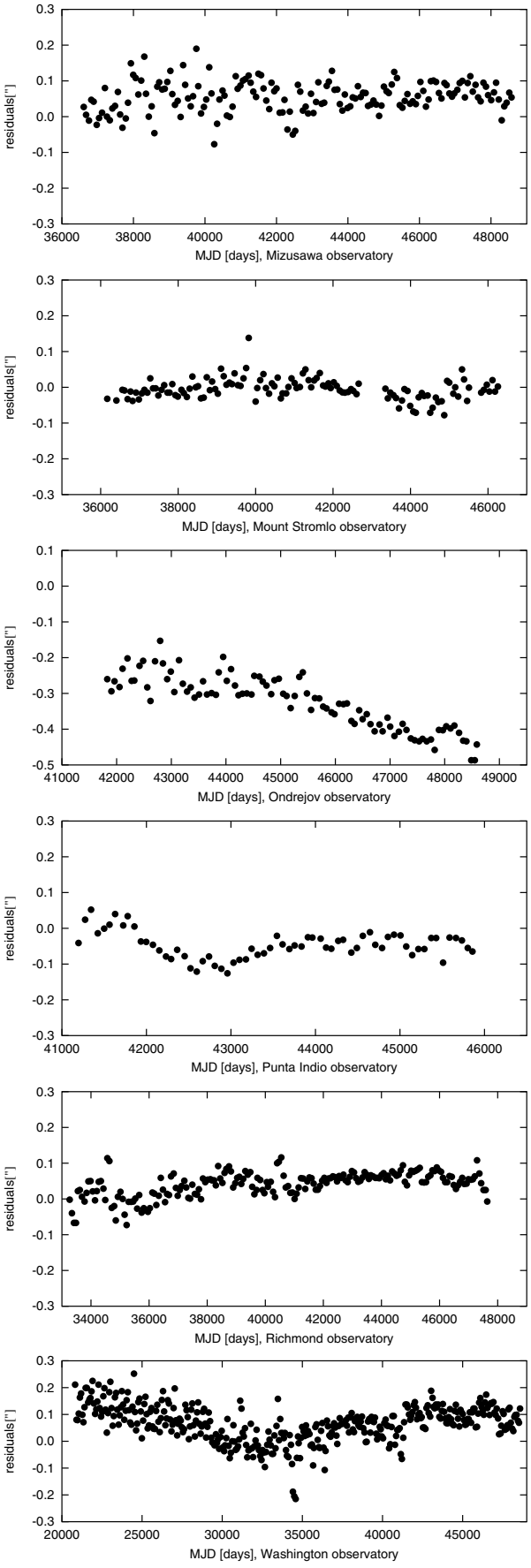

Fig. 3 Mean residuals r_i in bins of 0.2 years vs. time (MJD).

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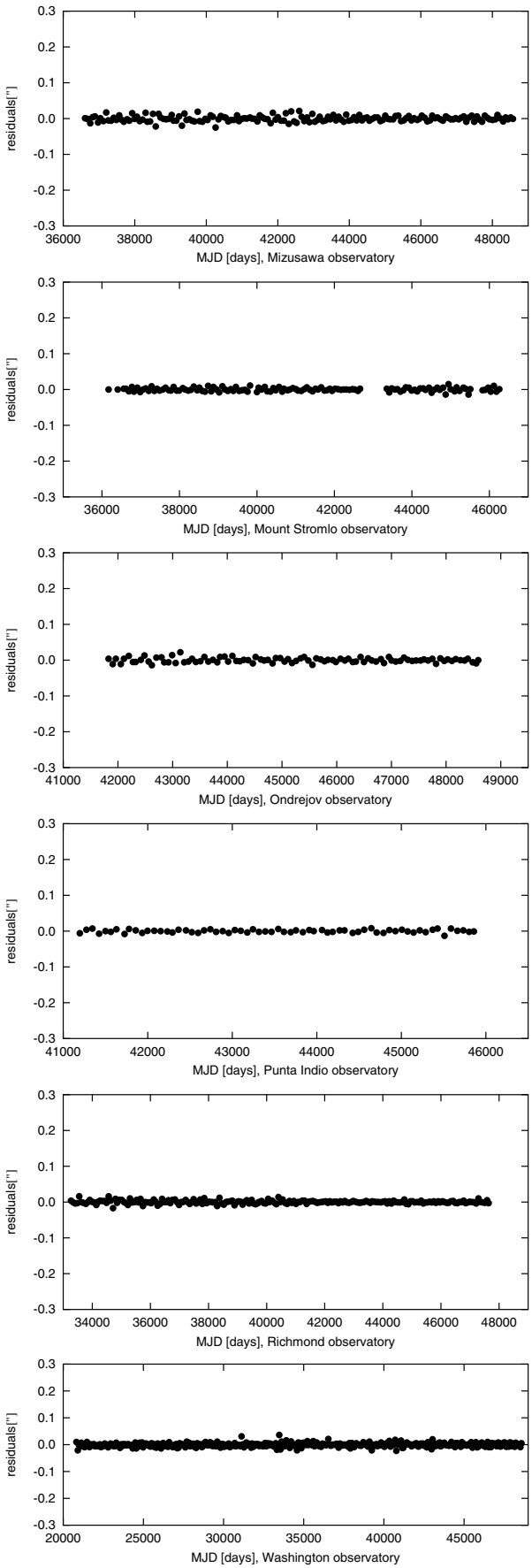

Fig. 4 Mean residuals r'_i in bins of 0.2 years vs. time (MJD).

3 Calculations and systematic errors of the results

We removed the polar motion effect $\Delta\varphi_i$ (calculated below) from received data of latitude variations φ_i to get the residuals r_i, and the systematic ones (local, instrumental, etc.) se_i (some steps of our procedure are explained below) from r_i values to calculate the residuals r'_i (Damljanović & Pejović 2008),

$$r_i = -(\varphi_i - \Delta\varphi_i), \tag{1}$$

$$r'_i = r_i - \mathrm{se}_i, \tag{2}$$

for every PZT star (Damljanović 2005; Damljanović & Pejović 2005; Damljanović et al. 2006).

The values of $\Delta\varphi_i$ were calculated by using Kostinski's formula $\Delta\varphi_i = x_i \cos\lambda_\mathrm{W} + y_i \sin\lambda_\mathrm{W}$ (Kulikov 1962). The polar motion coordinates (x_i, y_i) are from Vondrak's file EOPOA00.dat. The se_i (Eq. 2) were calculated for each instrument by using r_i (Eq. 1) of all stars of the PZT programme (Damljanović 2005; Damljanović et al. 2006).

The latitude φ_s of any observed PZT star was calculated by using McCarthy's (1970), or another similar formula,

$$\varphi_\mathrm{s} = \delta_\mathrm{app} + F, \tag{3}$$

where the value φ_s is proportional to δ_app (apparent declination of observed star referred to the Hipparcos catalogue). The difference between φ_i and φ_s is the mean latitude of PZT instrument (Vondrák et al. 1998). The values of F were taken from other observations (measurement of stellar images on photographic plates and other measurements). For a PZT star, the set of values r'_i (with the constant and linear part) is close to $\Delta\varphi_\mathrm{s} + (\mathrm{d}\varphi_\mathrm{s}/\mathrm{d}t)t$, and according to Vondrák et al. (1998),

$$\Delta\varphi_\mathrm{s} + (\mathrm{d}\varphi_\mathrm{s}/\mathrm{d}t)t \approx \Delta\delta + t\Delta\mu_\delta. \tag{4}$$

So, if we use the linear fit

$$r''_i = a + b(t_i - 1991.25), \tag{5}$$

our correction to $\mu_{\delta_{HIP}}$ is the value of b (calculated together with the value of a by using the LSM). Both, a and b are in accordance with the epoch 1991.25. The values of r''_i are the averaged values of r'_i over subperiods of about 1 year for each PZT star. The values of t_i (in years) are the times corresponding to r''_i.

The residuals r_i are without the polar motion effect, but still with the systematic ones (local, instrumental, etc.). For each observatory, the averaged values of r_i (over subperiods of 0.2 yr) of all programme stars are presented in Fig. 3. The chosen subperiods of 0.2 years were the optimal ones (Damljanović 2005). So, the mean residuals r_i in bins of 0.2 years present the systematic errors (local, instrumental, etc.) for each observatory. We calculated the values se_i of some observatory by using the mean residuals r_i (in bins of 0.2 yr) of all stars observed at that observatory, points presented in Fig. 3, and by using the linear interpolation for the moments t_i. Removing the values se_i (suitable for each observatory)

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from the r_i ones, we get the values r'_i (suitable for each observed star and its error of coordinates and proper motions). The r'_i are without both effects (the polar motion one and the mentioned systematic ones which are common for all stars of some observatory) because, in line with the statistical rules, we used the data of all observed stars (at that observatory) to get the values r'_i, but the astrometric errors of each separate star are still present in its set of values r'_i. It is what we wanted to do, to remove all effects, but not the astrometric errors of stars of the Hipparcos catalogue, in particular not the errors of star proper motions. To check the explained procedure and present mentioned systematic errors in r'_i, we repeated the calculation of the systematic effects, but by using the r'_i values; in Fig. 4, it is evident that the systematic changes (local, instrumental, etc.) and polar motion one were removed well from r'_i. Then, the values of r'_i are with astrometric errors of stars of the Hipparcos catalogue, but we continue with r''_i ones (averaged values of r'_i ones over subperiods of near 1 year). It is easy to follow the changes of μ_δ (the correction to $\mu_{\delta_{\rm HIP}}$) from year to year. By using LSM and the input points r''_i with the Hipparcos one, we calculate the value of b for each PZT star (all points are with suitable weights). The weights of the input points are inversely proportional to their formal errors.

For example, the values of $b \pm \varepsilon_b$ for the star H15334 ($Hp = 5.97$ mag) are: -0.67 ± 0.53 mas/yr (from Mizusawa data, 33 points of r''_i), and -0.41 ± 0.15 mas/yr (from Washington ones, 40 points). These results are close to each other and the internal comparison is good. From both sets of data (73 points), we get -0.44 ± 0.15 mas/yr, and the value $\mu_\delta \pm \varepsilon_{\mu_\delta}$ is -17.13 ± 0.15 mas/yr. From EOC-2, it is -17.29 ± 0.12 mas/yr, from Hipparcos -16.69 ± 0.62 mas/yr, and from re-reduced Hipparcos -17.74 ± 0.27 mas/yr. So, the external comparison of our result with the EOC-2 one and re-reduced Hipparcos one is good. Also, the formal errors are close to each other, but a few times smaller than the Hipparcos one. A similar quality of the results is achieved for the other PZT stars if these stars have sufficiently long periods of observations (see Figs. 8–10). If we compare the formal errors (of μ_δ and $\mu_{\delta_{\rm HIP}}$), we see that it is necessary to have $\geq$10 years of PZT observations per star to get the value of ε_b less than the Hipparcos error $\varepsilon_{\mu_{\delta_{\rm HIP}}}$, and if we use the F-test, we need $\geq$20 years (Damljanović & Pejović 2008). In any case, our results have formally smaller errors than the Hipparcos ones with more than 20 years of PZT observations. This is in accordance with our conclusion in the previous section. The standard errors in μ_δ of 423 PZT stars are less than or equal to the Hipparcos ones.

The values of ε_{μ_δ} are different for different observatories and PZT instruments, and the best results are the Richmond ones (Damljanović & Pejović2008). Slightly higher values of ε_{μ_δ} are obtained for Mizusawa (because of a very seismic region) and Ondřejov (this observing programme contains a lot of double and multiple stars). Also, a big shift and linear trend of OJP systematic variations is visible in Fig. 3 (Damljanović & Pejović 2008). For all PZT instruments, we removed well the mentioned systematic variations (see Fig. 4). Some results for the overlapping stars between different observatories were presented for ILS data (Damljanović & Pejović 2006; Damljanović 2008) and PZT ones (Damljanović 2005; Damljanović & Pejović 2008). For similar cases, we used all sets of data to calculate the final value μ_δ.

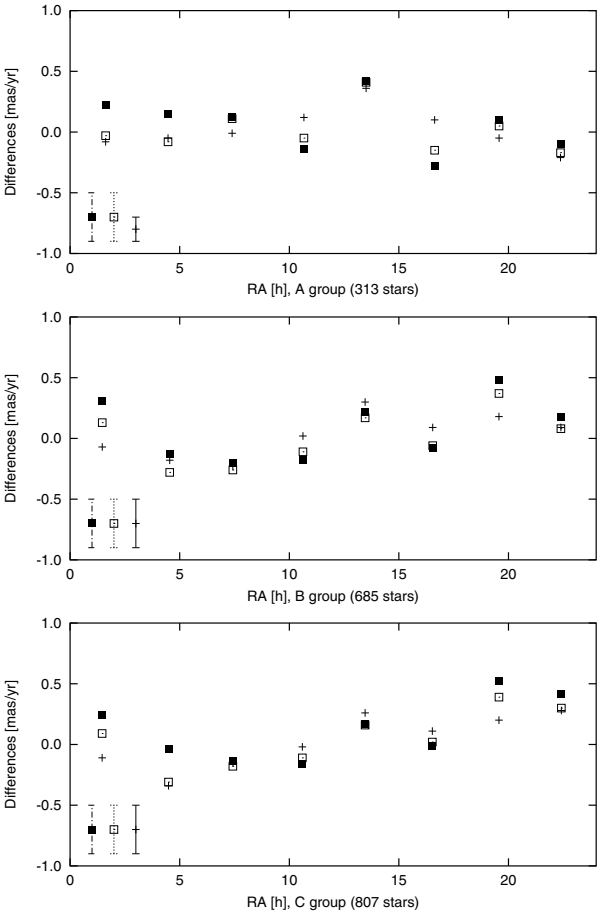

Fig. 5 Mean proper motion differences (with a typical mean error bars), as a function of α, between our results and the re-reduced Hipparcos (solid rectangle), Hipparcos (open rectangle) and EOC-2 ones (+).

From Figs. 5–7 (mean differences between our results and EOC-2, Hipparcos and re-reduced Hipparcos ones, as the function of α, δ and magnitude, respectively) is evident that our results are on the average very close to those of the three other catalogues. It is well-seen in the case of the group A, in Fig. 5, there is some bigger difference in the central part (where the α is about 12 h) than in the other parts of the picture. In Fig. 5 and in the cases of the groups B and C, there is a small sinusoidal curve which is in line with the ground-based observations (a seasonal effect which was expected); the differences are in the small domain of about $\pm$0.4 mas/yr. The seasonal effect is not present in data of the group A, because there are many years of observations (more than 20 years) for this group of stars. In Fig. 6, there are just few points, but they are consistent between

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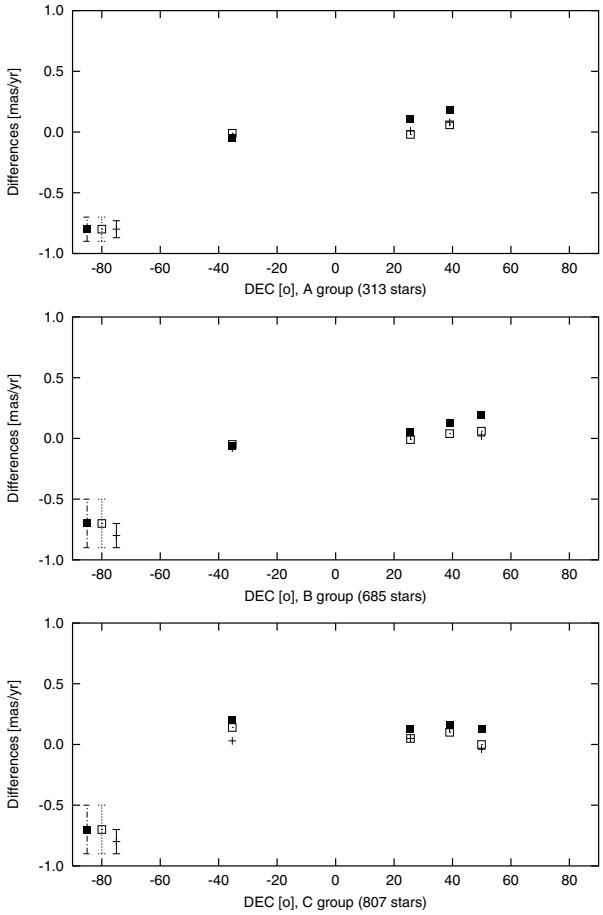

Fig. 6 Mean proper motion differences (with a typical mean error bars), as a function of δ, between our results and: the re-reduced Hipparcos (solid rectangle), Hipparcos (open rectangle) and EOC-2 ones (+).

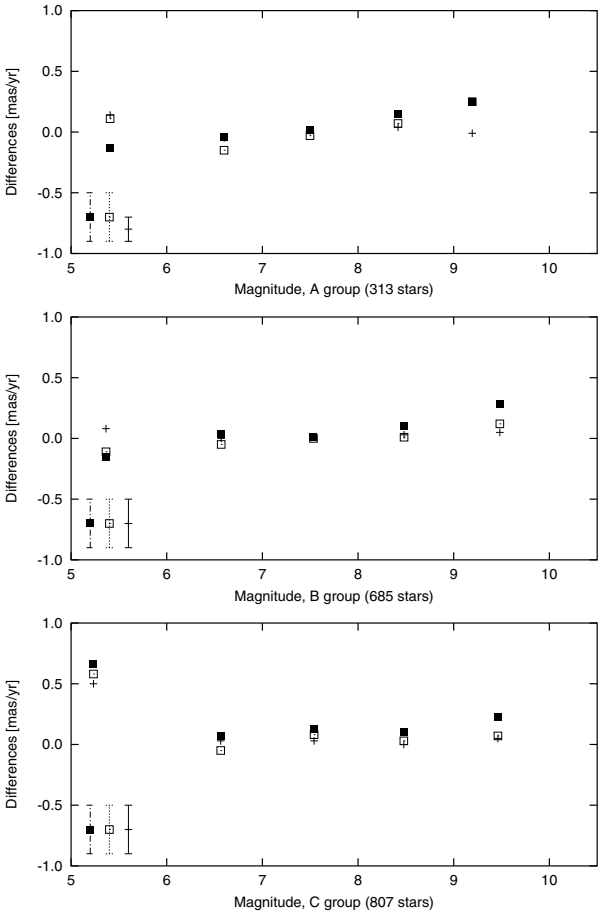

Fig. 7 Mean proper motion differences (with a typical mean error bars), as a function of magnitude, between our results and: the re-reduced Hipparcos (solid rectangle), Hipparcos (open rectangle) and EOC-2 ones (+).

each other and close to zero (for all groups of PZT stars). In Fig. 7, only in case of group C and for bright stars (with less than 6 mag) there is some bigger difference than in the other parts of the diagram. Anyway, in Figs. 5–7 which provide an external check for systematic errors in our data, our mean values of μ_δ are in good agreement with other results. In Figs. 5–7, a typical mean error bars (of presented mean values), but not standard deviations in the bins, are shown. These bars correspond to the standard deviations divided by the square root of the number of stars in each bin. These standard deviations are close to the values s_0 (in Table 2), respectively. The plotted bars are about 0.1 to 0.2 mas/yr (about ten times smaller than the s_0 values). More explanation about s_0 is given below. The standard deviations (of corresponding bars) represent the level of the combined random errors of both catalogues; in our cases: PZT-EOC2, PZT-HIP and PZT-NHIP. The mean values (in the bins in Figs. 5–7) represent the systematic errors which exist between the catalogues.

If we compare the values of standard errors presented in Figs. 8–10 we see that our results are: good in the cases of groups A (the average of ε_{μ_δ} is 0.40 mas/yr) and B (0.83 mas/yr), whereas in case of group C some PZT stars have larger values of ε_{μ_δ} (the average is 0.98 mas/yr) than in the other catalogues. For group A, the average of ε_{μ_δ} is 0.75 mas/yr from Hipparcos, 0.58 mas/yr from re-reduced Hipparcos, and 0.26 mas/yr from EOC-2 data, respectively. For group B, it is 0.80 mas/yr from Hipparcos, 0.64 mas/yr from re-reduced Hipparcos, and 0.39 mas/yr from EOC-2. And for group C, it is 0.80 mas/yr from Hipparcos, 0.63 mas/yr from re-reduced Hipparcos, and 0.42 mas/yr from EOC-2. Generally, our results are not good enough for PZT stars which were observed over less than 10 years. That limit of about 10 years of PZT data is in agreement with some information given in the previous section. In the case of group A, the mean value of our ε_{μ_δ} is 53% of the corresponding Hipparcos one, 69% of re-reduced Hipparcos one, and bigger than the EOC-2 one; so it is very good. In the case of group B, the mean value of our ε_{μ_δ} is close to the Hipparcos one, but bigger than the re-reduced Hipparcos one and the EOC-2 one. Our values of ε_{μ_δ} are less than or equal to the suitable Hipparcos ones for 423 stars. However, from the results of Table 2, we see that our formal error (for all sub-

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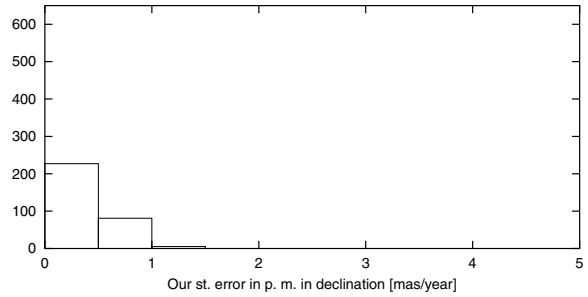
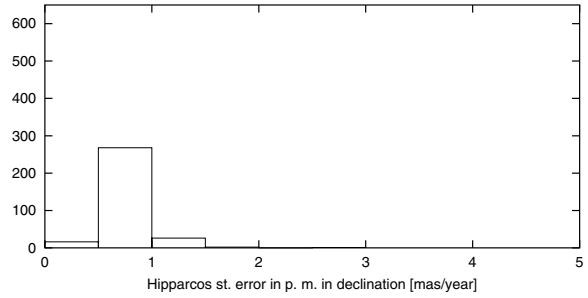
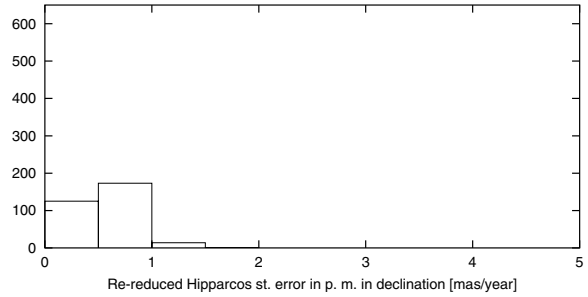






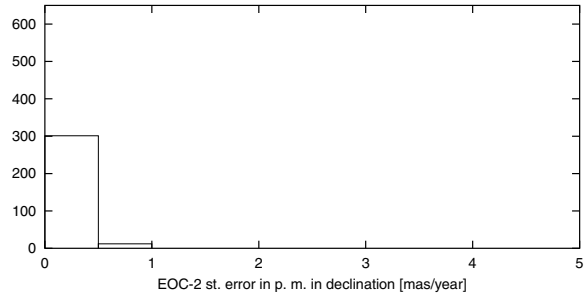
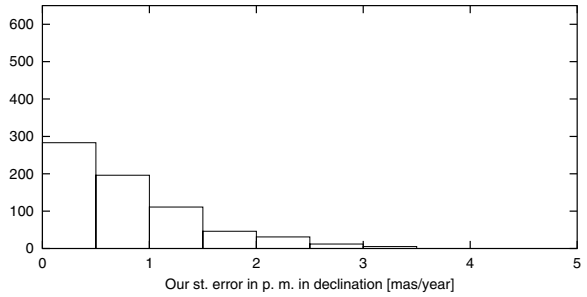
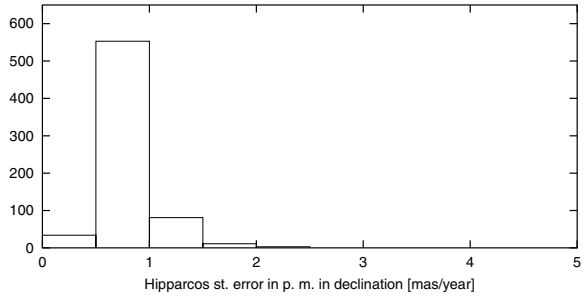
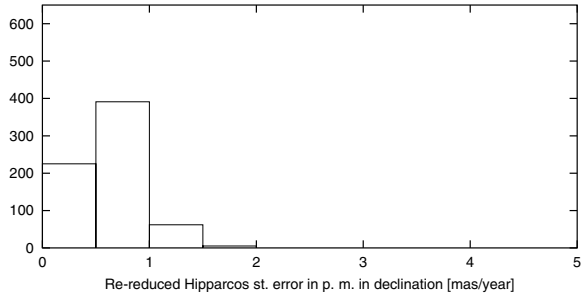


Fig. 8 Distribution of standard errors in proper motions in declination for group A.







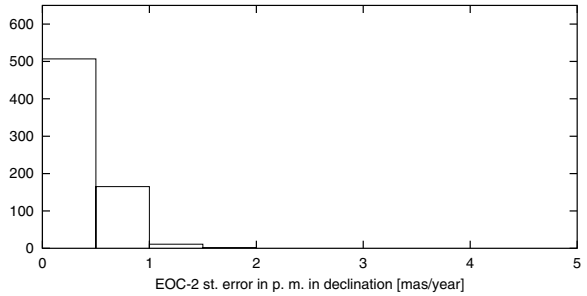


Fig. 9 Distribution of standard errors in proper motions in declination for group B.

groups A, B and C) is underestimated by factor near 1.7 (for EOC-2, it is 2.0).

Pairwise differences of μ_δ of stars from the catalogues Hipparcos, new Hipparcos, EOC-2 and PZT (our results) were used to determine their random and systematic errors. We did it by using the formula according to Ivanov & Yatsenko (2003) and LSM,

$$k_1 + k_2(m - m_0) + k_3(B - V) = \Delta, \qquad (6)$$

where: k_1, k_2 and k_3 are the unknowns (which describe the systematic part of differences Δ), m are the magnitudes of stars (m_0 is their mean value), and $B-V$ their color indices. The values of the unknowns and s_0 are presented in Table 2 for each combination of two catalogues and subgroups (A, B, C). s_0 is the error of unit weight of the solution of the system given by Eq. (6), and presents the random part of Δ; s_0 is the sum of random errors of both of the two treated catalogues.

The values of Table 2 are significantly smaller than the corresponding ones in Ivanov & Yatsenko (2003), where the random and systematic errors of positions and proper motions of stars from the catalogues Hipparcos, ACT, CMC11, FONAC, NPM1 and PPM are given. The errors of k_1, k_2 and k_3 are close to or higher than their corresponding values (see Table 2). This means that the random and systematic errors of values Δ (of catalogues presented in Table 2) are very small, less than those given in the paper by Ivanov & Yatsenko (2003), and that they are close to each other. There is no significant relationship between Δ and $m - m_0$

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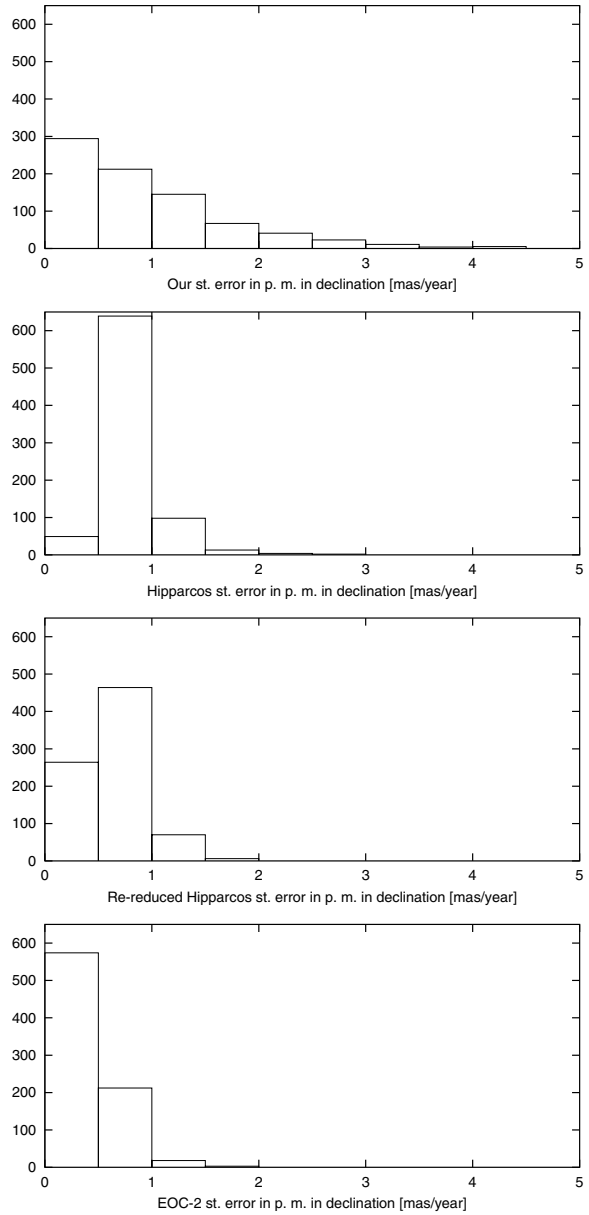

Fig. 10 Distribution of standard errors in proper motions in declination for group C.

(or $B-V$) for any pair of catalogues in this paper; our values of μ_δ have got a high accuracy (as in the other three catalogues). Also, as shown in Table 2, the level of random errors of our μ_δ is underestimated by a factor near 1.7 (it is near 2.0 for the EOC-2 case); 0.5 mas/yr $\leq s_0 \leq$ 1.7 mas/yr for PZT-EOC2 (1.1 mas/yr $\leq s_0 \leq$ 2.1 mas/yr for PZT-HIP, 1.2 mas/yr $\leq s_0 \leq$ 2.2 mas/yr for PZT-NHIP), and between EOC2, HIP and NHIP the value s_0 is near 1 mas/yr.

The $s_{0\min}$ is $\pm$ 0.49 mas/yr (PZT-EOC2, subgroup A) and after that $\pm$ 0.76 mas/yr (NHIP-HIP, subgroup B).

Table 2 Comparison of μ_δ (of subgroups A, B, C), from our results (PZT), EOC-2, Hipparcos (HIP) and new Hipparcos (NHIP), to get formal and systematic errors (in $0''.00001/\mathrm{yr}$).

Catalogue	s_0	k_1	k_2	k_3
PZT-EOC2, A	49	-2 ± 4	0 ± 3	6 ± 6
PZT-EOC2, B	131	6 ± 8	-3 ± 5	-5 ± 11
PZT-EOC2, C	168	1 ± 9	-7 ± 6	6 ± 13
PZT-HIP, A	111	-2 ± 10	7 ± 7	6 ± 14
PZT-HIP, B	180	6 ± 11	2 ± 7	-7 ± 15
PZT-HIP, C	209	5 ± 12	-5 ± 7	3 ± 16
PZT-NHIP, A	122	6 ± 11	10 ± 7	2 ± 15
PZT-NHIP, B	189	8 ± 12	6 ± 7	0 ± 16
PZT-NHIP, C	215	11 ± 12	-3 ± 7	7 ± 17
EOC2-HIP, A	88	0 ± 8	6 ± 5	0 ± 11
EOC2-HIP, B	107	0 ± 7	5 ± 4	-2 ± 9
EOC2-HIP, C	110	4 ± 6	1 ± 4	-3 ± 8
EOC2-NHIP, A	102	7 ± 9	9 ± 6	-4 ± 13
EOC2-NHIP, B	119	3 ± 7	10 ± 4	5 ± 10
EOC2-NHIP, C	132	10 ± 7	4 ± 4	1 ± 10
NHIP-HIP, A	78	-8 ± 7	-3 ± 5	4 ± 10
NHIP-HIP, B	76	-3 ± 5	-5 ± 3	-8 ± 6
NHIP-HIP, C	81	-6 ± 5	-2 ± 3	-4 ± 6

4 Conclusions

As demonstrated for the case of ILS (International Latitude Service) data by Damljanović (2008), it is still possible to satisfy the requirements of the modern astrometry by using ground-based PZT observations, and to check the present reference frame materialized via the Hipparcos catalogue data. In particular, it is possible to check the proper motions in declination of the observed PZT/Hipparcos stars. During the last century, PZT data were used to monitor the Earth orientation, but now they can also be used to check the Hipparcos satellite data (Ron & Vondrák 2001).

Here, we presented the main steps of our method (adapted for the case of PZT data) to construct the PZT catalogue of μ_δ for 807 stars: calculating and removing the polar motion and some systematic (local, instrumental, etc.) effects from the observed latitudes, using the linear fit to calculate the corrections of the Hipparcos proper motions in declination, the LSM to calculate the unknowns, including the Hipparcos point (1991.25, $0''.0$) and assigning weights to all input points in the calculations, etc.

The values of μ_δ from the PZT catalogue were compared with the EOC-2, Hipparcos and re-reduced Hipparcos data, and the consistency is good. In any case, with about 20 years of PZT observations, the proper motions in declination obtained by our method are formally on the level of the Hipparcos ones. Mostly, this is because of the big number of PZT observations (several hundreds per year) of the observed stars and the long time interval of PZT observations (much longer than with Hipparcos).

Following the procedure of determining random and systematic errors of four catalogues by pairs (Hipparcos, new Hipparcos, EOC-2 and PZT), according to Ivanov & Yat-

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senko (2003), we did not find a significant relationship between the differences of μ_δ and magnitudes and color indices of the common 807 stars. We can conclude that all catalogues have high accuracy of μ_δ, since after using our sample of 807 common stars, their random and systematic errors of μ_δ are small and close to each other. Our formal error (for all subgroups A, B and C) is too small, i.e. underestimated by a factor near 1.7 (for EOC-2, it is 2.0) if the new Hipparcos (or Hipparcos) is the first catalogue. Our results provide an independent checking of μ_δ from the catalogues Hipparcos, new Hipparcos and EOC-2.

We are planning to submit the PZT catalogue to the CDS. At present the PZT catalogue data are available at the URL: http://saj.matf.bg.ac.rs/177/pdf/Table2.dat. Also, they will be provided on request to those who are interested.

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40 Regularly varying solutions of Friedman equation

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The authors discuss solutions of the acceleration equation, the equation associated to the Friedman equation, in the light of the theory of regularly varying functions, also known as Karamata functions. As a result they obtained that the solutions of the acceleration equation might have a multiplicative term which is a slowly varying function. Under usual assumptions for the scale factor $a(t)$, such as $a(t) = t^{\alpha}$, it appears that this slowly varying term exists. Slowly varying term may explain some phenomena in the standard models of the evolution of the Universe. This paper is an announcement of the more detailed research in this area.

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REGULARLY VARYING SOLUTIONS OF FRIEDMAN EQUATION

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Abstract. We discuss solutions of the acceleration equation, the equation associated to the Friedman equation, in the light of the theory of regularly varying functions, also known as Karamata functions. As a result we obtain that the solutions of the acceleration equation might have a multiplicative term which is a slowly varying function. Under usual assumptions for the scale factor $a(t)$, such as $a(t) = t^{\alpha}$, it appears that this slowly varying term exists. Slowly varying term may explain some phenomena in the standard models of the evolution of the Universe. This paper is an announcement of the more detailed research in this area.

1. INTRODUCTION

The Friedman acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) \tag{1}$$

together with the fluid equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0 \tag{2}$$

and the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \tag{3}$$

determines the expansion scale factor $a(t)$ of the Universe. Here ρ is the mass density while p is the pressure of the material in the Universe. The nature of the solution of (1) strongly depends on the energy density term

$$\mathbf{E} = \rho + 3p/c^2, \tag{4}$$

particularly of the sign of **E**. In order to explain the acceleration of the expansion of the Universe the cosmological constant Λ is added in (1):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}. \tag{5}$$

The modified energy density term for $\Lambda \neq 0$

$$\mathbf{E}_\Lambda = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}. \tag{6}$$

admits negative values, giving more possibilities in modelling of possible scenarios of the past and the evolution of the Universe (e.g. Coles, et al., 2002; Hogan 2007, Lidle, et al., 2000; Narlikar, 2002; Peacock, 1999; Vikman 2005). We note the following two remarks in regards to Λ. First, under the transformations

$$\rho' = \rho + \Lambda/(8\pi G), p' = p - \Lambda/(8\pi G) \tag{7}$$

the equation (5) transforms into (1), but now referring it to the terms ρ' and p'. Therefore, our discussion will be concentrated further on the mathematical solutions of the equation (1). However, we shall abandon in certain situations the strong energy condition $\mathbf{E} > 0$. Secondly, it is well known that there are significant discrepancies in the predictions of what order should be the value of Λ. The reason may lay in the course tuned asymptotic description of the scale factor $a(t)$. In order to avoid this situation a better asymptotic analysis is needed. The theory of regularly varying function in the Karamata sense provides the means for such an analysis, particularly for solutions of the second order differential equations as it is (1). This theory is quite well developed (Bingham at al., 1987; Hille, 1948; Howard, et al., 1990; Marić, 2000; Marić et al, 1990; Omey, 1981; Seneta, 1976; Swanson, 1968), but it seems it has not been much applied in cosmology and in astrophysics in general. Yet, there are few to mention (Mijajlović et al., 2007; Molchanov, et al., 1997; Stern, 1997).

2. REGULAR VARIATION

In this section we shall briefly review the basic notions related to the regularly varying functions. We shall discuss also some properties of solutions of the second order differential equation of the form

$$y'' + f(x)y = 0 \tag{8}$$

related to the regularly varying functions. Observe that the acceleration equation has the form (8).

2.1. Regularly varying functions

A regular variation is related to the power-law distributions, a kind of polynomial relationship between two quantities. It exhibits the property of scale invariance represented by

$$f(x) = ax^k + o(x^k) \tag{9}$$

where a and k are constants, and $o(x^k)$ is an asymptotically small function of x^k. Here, k is the scaling exponent, i.e. a power-law function satisfies $f(\lambda x) \propto f(x)$ where λ is a constant. So, a rescaling of the function's argument changes the constant of proportionality but preserves the shape of the function itself. Power-law relations characterize a large number of natural phenomena, particularly in physics and astronomy. Examples are the Stefan-Boltzman law, gravitational potential and the scale factor $a(t)$ in various cosmological models. A particular interest in a power law can be found in the study of probability distribution and the large fluctuations that occur in the tail of the distribution – the part of the distribution representing large but rare events.

The most general form of a power law is given by

$$f(x) = L(x)x^{\alpha}, \quad \text{or} \quad f(x) \propto x^{\alpha} \tag{10}$$

where $L(x)$ is a slowly varying function i.e. $L(x)$ is positive continuous function (more generally measurable function, but in this article we are dealing anyway only with continuous functions) defined on some neighborhood $[a, \infty]$ of the infinity which satisfies

$$\lim_{x\to\infty} \frac{L(\lambda x)}{L(x)} = 1, \quad \text{for each } \lambda > 0 \tag{11}$$

The real number α is called the index of regular variation.

A positive continuous function R defined on some neighborhood $[a, \infty]$ is the regularly varying of index α if and only if it satisfies

$$\lim_{x\to\infty} \frac{R(\lambda x)}{R(x)} = \lambda^{\alpha}, \quad \text{for each } \lambda > 0 \tag{12}$$

It immediately follows that the regularly varying function $R(x)$ has the form

$$R(x) = L(x)x^{\alpha} \tag{13}$$

where $L(x)$ is slowly regular.

Jovan Karamata (Karamata, 1930) introduced the conceptions of slowly varying function and regularly varying functions. He also proved the following two fundamental theorems.

Examples of slowly varying functions includes iterated logarithms, but there are more complicated examples e.g.

$$L_1(x) = \frac{1}{x}\int_a^x \frac{dt}{\ln(t)}, \tag{14}$$

$$L_2(x) = \exp\left((\ln(x)^{\frac{1}{3}}\cos(\ln(x))^{\frac{1}{3}}\right) \tag{15}$$

The second example is interesting since $L_2(X)$ oscillates infinitely many times between 0 and infinity.

3. REGULARLY VARIATION AND ACCELERATION EQUATION

We shall represent the acceleration equation (5) in the form

$$\ddot{a} + \frac{\mu(t)}{t^2} a = 0. \tag{16}$$

The Hubble parameter $H(t)$ and the deceleration parameter $q(t)$ are defined by

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \quad q(t) = -\frac{\ddot{a}(t)}{a(t)} \frac{1}{H(t)^2} \tag{17}$$

where $a(t)$ is the expansion scale factor of the Universe. Therefore, the equation (16) can be written as

$$\ddot{a} + \frac{q(t)(H(t)t)^2}{t^2}\, a = 0, \tag{18}$$

hence

$$\mu(t) = q(t)(H(t)t)^2. \tag{19}$$

Observe that $\mu(t)$ is a dimensionless parameter. Let us remind that the density parameter $\Omega(t)$ and the density parameter for the cosmological constant Λ are defined by

$$\Omega = \frac{\rho}{\rho_c}, \quad \Omega_\Lambda = \frac{\Lambda}{3H^2}$$

where ρ_c is the critical density.

The Friedman acceleration equation (1) has two different fundamental solutions that satisfy a power law if and only if the limit

$$\gamma = \lim_{x\to\infty} x \int_x^\infty \frac{\mu(t)}{t^2} dt \tag{20}$$

exists and $\gamma < \frac{1}{4}$. According to the theory of of regularly varying functions and differential equations, if $-\infty < \gamma < 1/4$ and $\alpha_1 < \alpha_2$ are two roots of the equation

$$\alpha^2 - \alpha + \gamma = 0. \tag{21}$$

then there exist two linearly independent regularly varying solutions of the equation $y'' + f(x)y = 0$ of the form

$$y_i(x) = x^{\alpha_i} L_i(x), i = 1, 2, \tag{22}$$

if and only if $\lim\limits_{x\to\infty} x \int_x^\infty f(t)dt = \gamma$. Here, L_i, i=1,2 denote two normalized slowly varying functions. We see that this observation directly applies to the Friedman acceleration equation (16). Thus, the expansion scale factor $a(t)$ satisfies the power law if and only if $\gamma < \frac{1}{4}$.

We discuss only the existence and the possible values of the limit $\lim\limits_{t\to\infty} \mu(t)$. The existence of this limit is the sufficient condition for the existence of (20) and in this case

$\gamma = \lim_{t\to\infty} \mu(t)$. It appears that the values of the constant γ determine the asymptotical behavior at the infinity of the solutions of the acceleration equation, i.e. of the expansion scale factor $a(t)$ of the Universe. If the matter-dominated evolution of the Universe is assumed, that is, dominated by some form of pressureless material since the certain time moment t_0 then the expression $H(t)t$ depends solely on the parameter Ω. In this case we will be able to estimate the possible values of γ. We discuss also the status of the constant γ and the related asymptotic behavior of $a(t)$ for the flat Universe including the cosmological constant Λ and the open Universe with $\Lambda = 0$. Detailed proofs of will be published somewhere else.

4. CONCLUSION

A new constant γ is introduced by (20) related to the Friedman acceleration equation (1). The values of the constant γ determine the asymptotical behavior at the infinity of the solutions of the acceleration equation, i.e. of the expansion scale factor $a(t)$ of the Universe. The instance $\gamma < \frac{1}{4}$ is appropriate for the both cases, the flat and open Universe, and gives the sufficient and necessary condition that the solutions of the acceleration equation are in the Karamata class of functions; more specifically that they satisfy a power law. This property of the acceleration equation is formulated as the *power law conclusion* in the subsection 3.3. As power law functions are the most frequently occurring type of the solutions of the Friedman equation, the study of the constant γ and the related function $\mu(t)$ defined by (19) might be of a particular interest.

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41 On asymptotic solutions of Friedman equation

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The aim of the paper is to apply the theory of regularly varying functions to the asymptotical analysis at infinity of solutions of Friedmann cosmological equations. A new constant Γ is introduced related to the Friedmann cosmological equations. Determining the values of Γ we obtain the asymptotical behavior of the solutions, i.e. of the expansion scale factor $a(t)$ of a universe. The instance $\Gamma < 1/4$ is appropriate for both cases, the spatially flat and open universe, and gives a sufficient and necessary condition for the solutions to be regularly varying. This property of Friedmann equations is formulated as the generalized power law principle. From the theory of regular variation it follows that the solutions under usual assumptions include a multiplicative term which is a slowly varying function.

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On asymptotic solutions of Friedmann equations

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ABSTRACT

Our main aim is to apply the theory of regularly varying functions to the asymptotical analysis at infinity of solutions of Friedmann cosmological equations. A new constant Γ is introduced related to the Friedmann cosmological equations. Determining the values of Γ we obtain the asymptotical behavior of the solutions, i.e. of the expansion scale factor $a(t)$ of a universe. The instance $\Gamma < \frac{1}{4}$ is appropriate for both cases, the spatially flat and open universe, and gives a sufficient and necessary condition for the solutions to be regularly varying. This property of Friedmann equations is formulated as the *generalized power law principle*. From the theory of regular variation it follows that the solutions under usual assumptions include a multiplicative term which is a slowly varying function.

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1. Introduction

In this paper[1] we describe conditions under which the Friedmann equations [5] have regularly varying solutions. Strictly speaking, we found a necessary and sufficient condition for Friedmann equations, expressed by the values of a constant Γ, to have regularly varying solutions. We formulate this description as the *generalized power law principle for Friedmanns equation*. The physical formulation of this condition is that a certain form of the equation of state $p \sim w\rho c^2$ must hold. Hence, our discussion is mainly about a universe filled with the perfect fluid with constant barotropic equation of state $p = w\rho c^2$. The sufficiency of this condition is well known, e.g. Liddle and Lyth [9], Coles and Lucchin [4], Narlikar [16] and Islam [8]. However, we have not found in the literature the necessity part of the power law principle.

It appears that the mentioned constant Γ related to the Friedmann acceleration equation plays the crucial role in this analysis. Its values determine the asymptotical behavior of the solutions of the Friedmann equations, i.e. of the scale factor $a(t)$ as time t tends to ∞. Our solution is also valid for non-zero cosmological constant Λ if the pressureless spatially flat universe is assumed. This was possible due to a formula of Carroll et al. [3] for the predicted age of the universe. In the course of this analysis, mathematical singularities appearing in the solutions are classified and are clearly distinguished from those arising from the physical constrains. All solutions we found are in agreement with the results widely found in the literature on standard cosmological model.

The background for our analysis is the theory of regularly varying functions which could be considered as the mathematical counterpart of the general form of the power law, the term often used in physics. A good presentation of this subject can be found in Bingham et al. [2] and Seneta [17]. Another tool we used is the theory of regularly varying solutions of differential equations. A good source for this theory is Marić [13]. The theory of regular variation provides additional means in the asymptotical analysis of the solutions of the second order linear differential equations as (3), but it seems it has not been

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much applied in cosmology and in astrophysics. There are few such applications, e.g. Molchanov et al. [15], Stern [18] and Mijajlović et al. [14].

By $\mathbb{R}$ we denote the set of real numbers. As usually, for two real functions f and g, $f(x) \sim g(x)$ (or $f \sim g$) means that $\lim_{x\to\infty} f(x)/g(x) = 1$.

The paper is organized as follows. In the first section the history of the problem is explained and physical (Friedmann equations) and mathematical (regular variation) background is given. The main results of the paper are presented in Sections 2 and 3.

1.1. Friedmann equations

The scale factor $a(t)$ is defined by Friedmann–Lemaître–Robertson–Walker (FLRW) metric. The FLRW 4-dimensional line element in spherical comoving coordinates is given by

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]. \tag{1}$$

This metric is an exact solution of Einstein's field equations of general relativity and it describes a homogeneous, isotropic expanding or contracting universe. In this paper we shall discuss only the expanding universe. The scale factor $a(t)$ is a solution of the Friedmann equations. These equations are derived from the Einstein field equations; they are the following three differential equations. The term Friedmann equation is usually reserved for the first one.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}. \tag{2}$$

The Friedmann acceleration equation is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) \tag{3}$$

while the fluid equation is

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0. \tag{4}$$

The solutions of these equations are three fundamental parameters, the scale factor $a = a(t)$, the energy density $\rho = \rho(t)$ and $p = p(t)$, the pressure of the material in the universe. Here k is the curvature index with possible values -1 (elliptic geometry), 0 (spatially flat geometry) and 1 (hyperbolic geometry). The symbol G denotes the gravitational constant and c is the speed of light. Eqs. (2)–(4) are not independent. Eq. (3) follows from (2) and (4), while Eqs. (2) and (3) yield (4).

We shall use Karamata theory of regularly varying functions, as applied to differential equations in Maric and Tomic [12] and Marić [13]. This theory generalizes the power law in physics and we shall use it to obtain the asymptotic analysis of solutions of Friedmann equations.

In our study of the asymptotical solutions of Friedmann equations, the acceleration equation will have the central point for several reasons. First, it does not contain explicitly the curvature index k. Secondly, the theory of regularly varying solutions of such type of equations can be applied successfully, regardless if the cosmological constant Λ is added in (2) and (3):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}. \tag{5}$$

Namely, under the transformations $\rho' = \rho + \Lambda/(8\pi G)$, $p' = p - \Lambda/(8\pi G)$ Eqs. (5) yield (2) and (3), but now with respect to the parameters ρ' and p'. The fluid equation is not affected by the parameter Λ. Therefore, our discussion will be concentrated further on the solutions of the Friedmann equations in their basic form (2)–(4) if it is not otherwise stated.

From now on, we shall assume that the functions $a(t)$, $p(t)$ and $\rho(t)$ satisfy all three Friedmann equations. We shall also assume that all appearing functions are continuous in their domains and have the sufficient number of derivatives, at least that they have the continuous second derivative.

1.2. Regular variation

In this section we shall review the basic notions related to the regular variation necessary for our analysis. In particular we shall need properties of regularly varying solutions of the second order differential equation

$$\ddot{y} + f(t)y = 0, \quad f(t) \text{ is continuous on } [\alpha, \infty). \tag{6}$$

Observe that the acceleration equation (3) has the form (6). In short, the notion of a regular variation is related to the power law distributions, described by the following relationship between quantities F and t:

$$F(t) = t^r(\alpha + o(1)), \quad \alpha, r \in \mathbb{R}. \tag{7}$$

It is said that two quantities y and t^r satisfy the power law if they are related by a proportion,[2] i.e. there is a constant α so that $y = \alpha t^r$. This definition of power law can be naturally extended by use of the notion of slowly varying function.

A real positive continuous function[3] $L(t)$ defined for $x > x_0$ which satisfies

$$\frac{L(\lambda t)}{L(t)} \to 1 \quad \text{as} \quad t \to \infty, \quad \text{for each real} \quad \lambda > 0 \tag{8}$$

is called a slowly varying function.

Definition. A physical quantity $F(t)$ is said to satisfy the generalized power law if

$$F(t) = t^r L(t), \tag{9}$$

where $L(t)$ is a slowly varying function and r is a real constant.

Examples of slowly varying functions are $\ln(x)$ and iterated logarithmic functions $\ln(\ldots \ln(x) \ldots)$. More complicated examples (cf. Marić [13]) are provided by:

$$L_1(x) = \frac{1}{x}\int_a^x \frac{dt}{\ln t}, \quad L_2(x) = \exp((\ln x)^{1/3}\cos(\ln x)^{1/3}). \tag{10}$$

We note that $L_2(x)$ varies infinitely between 0 and ∞.

A positive continuous function F defined for $t > t_0$, is the regularly varying function of the index r, if and only if it satisfies

$$\frac{F(\lambda t)}{F(t)} \to \lambda^r \quad \text{as} \quad t \to \infty, \quad \text{for each} \quad \lambda > 0. \tag{11}$$

It immediately follows that a regularly varying function $F(t)$ has the form (9). So to say that $F(t)$ is regularly varying is the same as $F(t)$ to satisfy the generalized power law. By Proposition 7 in [13], if a function $F(x)$ is asymptotically equivalent to a regularly varying function, it is a regularly varying function. Hence, we may define the generalized power law also by

$$F(x) \sim t^\alpha L(t), \quad \text{as } t \to \infty. \tag{12}$$

The class of regularly varying functions of index α we shall denote by $\mathcal{R}_\alpha$. Hence $\mathcal{R}_0$ is the class of all slowly varying functions. By $\mathcal{Z}_0$ we shall denote the class of zero functions at ∞, i.e. $\varepsilon \in \mathcal{Z}_0$ if and only if $\lim_{t\to+\infty}\varepsilon(t) = 0$.

Karamata introduced in [10] the concept of regularly varying functions continuing the works of G.H. Hardy, J.L. Littlewood and E. Landau in the asymptotic analysis of real functions. The following two theorems describe fundamental properties of this class of functions.

Theorem 1.1 ([10] Representation theorem). *$L \in \mathcal{R}_0$ if and only if there are measurable functions $h(x)$ and $\varepsilon \in \mathcal{Z}_0$ and $b \in \mathbb{R}$ so that*

$$L(x) = h(x)e^{\int_b^x \frac{\varepsilon(t)}{t}dt}, \quad x \geqslant b, \tag{13}$$

and $h(x) \to h_0$ as $x \to \infty$, h_0 is a positive constant.

The function $\varepsilon(t)$ in the above theorem is not uniquely determined. If $h(x)$ is a constant function, then $L(x)$ is called normalized. We denote by $\mathcal{N}$ the class of normalized slowly varying functions. We note the following important fact for $\mathcal{N}$-functions. If $L \in \mathcal{N}$ and there is $\dot{L}$, then ε in (13) has the first order derivative $\dot{\varepsilon}$. This follows from the identity $\varepsilon(t) = t\dot{L}(t)/L(t)$.

There is also an appropriate definition of regular variation at 0 and ∞ and various generalizations such as the rapidly varying functions. Even if such solutions of Friedmann equation are possible, we will not discuss these types of solutions in this article, so we omit these definitions.

For our study of Friedmann equations we need several results on solutions of (6). There are various conditions for $f(t)$ that ensure that regularly varying solutions of $\ddot{y} + f(t)y = 0$ exist. We shall particularly use the following result, see Howard and Maric [7] and Marić [13] the Theorems 1.10 and 1.11:

Theorem 1.2. *Let $-\infty < \Gamma < 1/4$, and let $\alpha_1 < \alpha_2$ be two roots of the equation*

$$x^2 - x + \Gamma = 0. \tag{14}$$

Further let L_i, $i = 1,2$ denote two normalized slowly varying functions. Then there are two linearly independent regularly varying solutions of $\ddot{y} + f(t)y = 0$ of the form

$$y_i(t) = t^{\alpha_i}L_i(t), \quad i = 1,2, \tag{15}$$

if and only if $\lim_{x\to\infty} x\int_x^\infty f(t)dt = \Gamma$. Moreover, $L_2(t) \sim \frac{1}{(1-2\alpha_1)L_1(t)}$. □

[2] This relation is usually denoted by $y \propto t^r$.

[3] More generally it may be assumed that $L(t)$ is a measurable function, but in this article we are dealing only with continuous functions anyway.

The limit of the integral in the theorem is not always easy to compute. As $\lim_{t\to\infty} t^2 f(t) = \Gamma$ implies $\lim_{x\to\infty} x\int_x^\infty f(t)dt = \Gamma$, we see that

$$\lim_{t\to\infty} t^2 f(t) = \Gamma \tag{16}$$

gives a useful sufficient condition for the existence of regular solutions of the equation $\ddot{y} + f(t)y = 0$ as described in the previous theorem.

2. Regularly varying solutions of acceleration equations

As noted, the acceleration equation obviously has the form (6) so under appropriate assumptions, i.e. that the functions we encounter are continuously differentiable as many times as necessary, the analysis of the previous section, in particular Theorem 1.2, can be applied to it. For this reason, we shall write from now on the acceleration equation (5) in the form

$$\ddot{a} + \frac{\mu(t)}{t^2} a = 0, \tag{17}$$

where

$$\mu(t) = \frac{4\pi G}{3} t^2 \left(\rho + \frac{3p}{c^2}\right). \tag{18}$$

Our approach in the next analysis is as follows. Obviously μ is a function of ρ and p. We assumed that ρ and p are solutions of Friedmann equations, hence $\mu(t)$ is a well-defined function. Under this assumption, the theory of regular variation applied to Eq. (17) yields the asymptotic expansions of $\mu(t)$ and $a(t)$ and exact conditions on $\mu(t)$ under which these expansions exist. Using the identity (18) we will be able then to find the asymptotical expansions for $\rho,\ p$ and other cosmological parameters.

In the next the crucial role will play the following integral limit:

$$\lim_{x\to\infty} x\int_x^\infty \frac{\mu(t)}{t^2} dt = \Gamma. \tag{19}$$

Let us denote by $\mathcal{M}_\Gamma$ the class of functions μ that satisfy the integral condition (19). Further, let

$$\mathcal{M} = \bigcup_{r\in\mathbb{R}} \mathcal{M}_r.$$

Marić introduced the integral condition (19) (cf. [13]), accordingly we shall call the class $\mathcal{M}$ also as Marić class of functions. Obviously, $\mathcal{M}$ is a vector space over $\mathbb{R}$ and the map $\mathbf{M} \to \mathbb{R}$ defined by

$$\mathbf{M}(u) = \lim_{x\to\infty} x\int_x^\infty \frac{u(t)}{t^2} dt$$

is a linear functional, i.e. $\mathbf{M}(\alpha u + \beta v) = \alpha\mathbf{M}(u) + \beta\mathbf{M}(v),\ \alpha,\beta\in\mathbb{R},\ u,v\in\mathcal{M}$. It is easy to see that $\mathbf{M}(\varepsilon) = 0$ for $\varepsilon \in \mathcal{Z}_0$. By the note regarding (16), we immediately have

Proposition 2.1. *If* $\lim_{t\to\infty} u(t) = r$ *then* $\mathbf{M}(u) = r$.

Now we prove a useful representation theorem for Marić class of functions.

Theorem 2.2 (*Representation theorem for $\mathcal{M}$-functions*). *$u \in \mathcal{M}_r$ if and only if there are $\varepsilon, \eta \in \mathcal{Z}_0$ such that $u(t) = r - t\dot{\varepsilon}(t) + \eta(t)$. If $r < 1/4$ then ε is that one appearing in the representation (13) of $a(t)$, with $h(t)$ constant.*

Proof. ($\Rightarrow$) Suppose $u \in \mathcal{M}_r$, and $r < \frac{1}{4}$. By Theorem 1.2 the equation $\ddot{y} + \frac{u(t)}{t^2} y = 0$ has a solution $a(t) = t^\alpha L(t)$ where α is a root of the equation $x^2 - x + r = 0$ and $L \in \mathcal{N}$. By Theorem 1.1 there are $a_0, b \in \mathbb{R}$ and $\varepsilon \in \mathcal{Z}_0$ so that $a(t) = a_0 t^\alpha e^{\int_b^x \frac{\varepsilon(t)}{t} dt}$. As $\dot{L}(t) = \frac{\varepsilon(t)}{t} L(t)$ and $r = -\alpha(\alpha - 1)$, we have

$$\ddot{a}(t) = (-r + t\dot{\varepsilon} - (1 - 2\alpha)\varepsilon + \varepsilon^2) L(t) t^{\alpha-2}.$$

Since $-\frac{u(t)}{t^2} = \frac{\ddot{a}(t)}{a(t)}$ it follows $u(t) = r - t\dot{\varepsilon}(t) + \eta(t)$ where $\eta = \varepsilon^2 - (1 - 2\alpha)\varepsilon$.

Suppose $r \geqslant \frac{1}{4}$. As $\mathbf{M}(\frac{1}{8r}u) = \frac{1}{8}$, taking $\frac{1}{8r}u$ instead of u in the previous proof, we have $\frac{1}{8r}u(t) = \frac{1}{8} - t\dot{\varepsilon}(t) + \eta(t)$ for some $\varepsilon, \eta \in \mathcal{Z}_0$, hence $u(t) = r - t\dot{\varepsilon}_1(t) + \eta_1(t)$ where $\varepsilon_1 = 8r\varepsilon$ and $\eta_1 = 8r\eta$.

($\Leftarrow$) Suppose $u(t) = r - t\dot{\varepsilon}(t) + \eta(t)$ where $\varepsilon, \eta \in \mathcal{Z}_0$. Then

$$\mathbf{M}(u) = \mathbf{M}(r) - \mathbf{M}(t\dot{\varepsilon}) + \mathbf{M}(\eta) = r - \mathbf{M}(t\dot{\varepsilon}).$$

Further, taking $v(t) = t\dot{\varepsilon}$,

$$\int_x^\infty \frac{v(t)}{t^2}dt = \int_x^\infty \frac{d\varepsilon}{t} = -\frac{\varepsilon}{x} + \int_x^\infty \frac{\varepsilon}{t^2}dt = -\frac{\varepsilon}{x} + o\left(\frac{1}{x}\right).$$

Hence, $\mathbf{M}(t\dot{\varepsilon}) = \lim_{x\to\infty} x\int_x^\infty \frac{v(t)}{t^2}dt = \lim_{x\to\infty}(-\varepsilon + o(1)) = 0$, so $\mathbf{M}(u) = r$. □

Corollary 2.3. *Assume $u \in \mathcal{M}_r$. Then $\lim_{t\to\infty} u(t) = r$ if and only if $\lim_{t\to\infty} t\dot{\varepsilon}(t) = 0$ in above representation of u.*

Example. Let $\varepsilon(t) = \frac{\sin(t^3)}{t}$. Then for

$$\mu(t) = \frac{1}{8} - t\dot{\varepsilon}(t) - \varepsilon(t) = \frac{1}{8} - 3t^2\cos(t^3).$$

$\mathbf{M}(\mu) = \frac{1}{8}$ and all ultimately positive solutions of (17) are regularly varying, but $\lim_{t\to\infty}\mu(t)$ does not exist. Note that it follows $\lim_{x\to\infty} x\int_x^\infty \cos(t^3)dt = 0$. □

The next proposition will be useful in our further analysis. It also gives the ε-representation of the logarithmic derivative $H(t) \equiv \dot{a}(t)/a(t)$ of $a(t)$.

Proposition 2.4. *Suppose $\mu \in \mathcal{M}$ and $\Gamma \equiv \mathbf{M}(\mu) < \frac{1}{4}$ holds for Eq. (17). Then any ultimately positive solution $a(t)$ of (17) is a normalized regularly varying function, i.e. there are $L \in \mathcal{N}$ and $\alpha \in \mathbb{R}$, so that $a(t) = t^\alpha L(t)$.*

If $L(t)$ has the ε-representation as in Theorem 1.1, where $h(t)$ is a positive constant, then $H(t) = \alpha/t + \varepsilon/t$.

Proof. Suppose $a(t)$ is positive at ∞. By Theorem 1.2 there are $L_1, L_2 \in \mathcal{N}$ and $\alpha_1, \alpha_2 \in \mathbb{R}$ so that

$$a(t) = c_1L_1t^{\alpha_1} + c_2L_2t^{\alpha_2}, \quad c_1, c_2 \in \mathbb{R}, \tag{20}$$

where $\alpha_1,\ \alpha_2$ are the roots of Eq. (14). Since $\Gamma < 1/4$ we have $\alpha_1 \neq \alpha_2$, so we may assume $\alpha_1 > \alpha_2$. Suppose $c_1 \neq 0$. Hence, the term $c_1L_1t^{\alpha_1}$ dominates $c_2L_2t^{\alpha_2}$, so there is $t_0 > 0$ so that $a(t) > 0$ for $t > t_0$. Let $\delta = \alpha_2 - \alpha_1,\ c_0 = c_1/c_2$ and $L_0 = L_1/L_2$. Note that $L_0 \in \mathcal{R}_0$. By Representation Theorem 1.1 and as L_1, L_2 are normalized, there are constants $h_1, h_2, b \in \mathbb{R}$ and $\varepsilon_1, \varepsilon_2 \in \mathcal{Z}_0$ so that

$$L_i(x) = h_i e^{\int_b^x \frac{\varepsilon_i(t)}{t}dt}, \quad x \geqslant b, \quad i = 1, 2.$$

As $\dot{L}_i(t) = \frac{\varepsilon_i(t)}{t}L_i(t)$, taking the logarithmic derivative $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$ of $a(t)$ and $\alpha \equiv \alpha_1$ we obtain

$$H(t) = \frac{\alpha}{t}(1 + \varepsilon_1/\alpha)\frac{1 + c_0\frac{\alpha_2+\varepsilon_2}{\alpha_1+\varepsilon_1}L_0(t)t^\delta}{1 + c_0L_0(t)t^\delta}.$$

Since L_0 is slowly varying and $\delta < 0$, it follows $L_0t^\delta \to 0$ as $t \to 0$. Hence, there is $\varepsilon \in \mathcal{Z}_0$ so that

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{\alpha}{t} + \frac{\varepsilon(t)}{t}. \tag{21}$$

By integration of this relation we have immediately

$$a(x) = a_0 e^{\int_b^x \frac{\varepsilon(t)}{t}dt}, \quad x \geqslant b, \quad \text{where } b = t_0,\ a_0 = a(t_0).$$

Hence, by Theorem 1.1, $a(t)$ is a normalized slowly varying function. □

3. Asymptotic solutions of Friedmann equations

We proceed to the analysis of solutions of Friedman equations taking into account the physical constraints. We remind that besides the acceleration equation (17) with $\mu(t)$ defined by (18), the scale factor $a(t)$ also satisfies the other two Friedmann Eqs. (2) and (4). Note that $a(t)$ is a function of time which represents the relative expansion of the universe. This function relates the proper distance between a pair of objects, e.g. two galaxies, moving with the Hubble flow in a FLWR universe at any arbitrary time t to their distance at some reference time t_0. Thus, $d(t) = a(t)d(t_0)$ where $d(t)$ is the proper distance at epoch t. Hence $a(t) > 0$. Therefore, we shall consider only positive solutions $a(t)$ of Friedmann equations.

3.1. Cosmological parameters

The Hubble parameter $H(t)$ and the deceleration parameter $q(t)$ are defined in Cosmology by

$$H(t)=\frac{\dot{a}(t)}{a(t)},\quad q(t)=-\frac{\ddot{a}(t)}{a(t)}\cdot\frac{1}{H(t)^2} \tag{22}$$

where $a(t)$ is the scale factor. Then obviously we have the following identity.

$$\mu(t)=q(t)(H(t)t)^2. \tag{23}$$

Observe that $\mu(t)$ is a dimensionless parameter. We shall assume that $\mu(t)$ is continuous. From the physical point of view, it means that scenarios such as Big Crunch, or Big Rip are not included in our analysis. That is, in finite time t, $a(t)\neq 0$, neither $a(t)$ becomes infinite. If $\mu(t)$ is an $\mathcal{M}$-function then the real constant Γ is defined by $\Gamma=\mathbf{M}(\mu)$.

Let us remind that the density parameter $\Omega(t)$ and the density parameter for the cosmological constant Λ are defined by

$$\Omega=\Omega(t)=\frac{\rho(t)}{\rho_c},\quad \Omega_\Lambda=\Omega_\Lambda(t)=\frac{\Lambda}{3H(t)^2},$$

where ρ_c is the critical density.

Proposition 3.1. *If the limit $H_\infty=\lim_{t\to\infty}H(t)$ exists, then*

$$\Gamma=\limsup_{t\to\infty}\left((H(t)-H_\infty)-\int_t^\infty H(t)^2dt\right).$$

Proof. As $\mu(t)=-\frac{\ddot{a}}{a}t^2$ by use of partial integration we have:

$$\int\frac{\mu(t)}{t^2}dt=-\frac{\dot{a}}{a}-\int\frac{\dot{a}^2}{a^2}dt=-H(t)-\int H(t)^2dt$$

and the statement follows as $\Gamma=\lim_{x\to\infty}x\int_x^\infty\frac{\mu(t)}{t^2}dt$. □

Therefore, if the limit (19) exists then Γ depends solely on the behavior of the Hubble parameter $H(t)$ at ∞.

The next theorem describes the main property of the scale factor $a(t)$ for the non-oscillatory universe. Namely, it gives the necessary and sufficient condition for $a(t)$ to satisfy the generalized power law.

Theorem 3.2 (*Generalized power law for the scale factor $a(t)$*). *Let $a(t)$ be the scale factor, a solution of Friedmann equations, and $\alpha\in\mathbb{R}$. Then*

1. *If $\mu\in\mathcal{M}_\Gamma$ and $\Gamma<1/4$ then there is $L\in\mathcal{N}$ so that $a(t)=t^\alpha L(t)$, where α is a root of the polynomial $x^2-x+\Gamma$.*
2. *If there is $L\in\mathcal{N}$ so that $a(t)=t^\alpha L(t)$ then $\mu\in\mathcal{M}_\Gamma$, $\alpha^2-\alpha+\Gamma=0$ and $\Gamma\leqslant 1/4$.*

Proof

1. This assertion follows immediately from Proposition 2.4.
2. The next proof follows the ideas presented in [13, Section 1.4]. So, suppose $a(t)=t^\alpha L(t), L\in\mathcal{N}$. By Representation theorem 1.1 there is $\varepsilon\in\mathcal{Z}_0$ so that $\dot{L}=\frac{\varepsilon}{t}L$, hence

$$t\frac{\dot{a}(t)}{a(t)}=\varepsilon(t)+\alpha,\qquad\left(t\frac{\dot{a}(t)}{a(t)}\right)^2=\eta(t)+\alpha^2,\quad\eta\in\mathcal{Z}_0. \tag{24}$$

Using $\frac{\ddot{a}}{a}=-\frac{\mu}{t^2}$ and by integration of the identity $\frac{\ddot{a}}{a}=\left(\frac{\dot{a}}{a}\right)'+\left(\frac{\dot{a}}{a}\right)^2$ we obtain after multiplying by x

$$-x\frac{\dot{a}(x)}{a(x)}+x\int_x^\infty\left(t\frac{\dot{a}(t)}{a(t)}\right)^2t^{-2}dt+x\int_x^\infty\frac{\mu(t)}{t^2}dt=0.$$

By (24), the last identity and applying $x\to\infty$, we infer $\alpha^2-\alpha+\Gamma=0$. Since α is a real number, for the discriminant $\Delta=1-4\Gamma$ of the polynomial $x^2-x+\Gamma$ must be $\Delta\geqslant 0$, i.e. $\Gamma\leqslant 1/4$. □

Remark. Under certain conditions Theorem 3.2.1 also holds for $\Gamma=1/4$, i.e. $\alpha=1/2$. This case will be discussed in Section 3.4.

Theorem 3.3. *Assume $\mu\in\mathcal{M}_\Gamma$ where $\Gamma<1/4$. Let $a(t)=t^\alpha L(t)$ be the corresponding scale factor, where $\alpha\neq 0$ and $L\in\mathcal{N}$, with L having the ε-representation as in Theorem 1.1, where $h(t)$ is a positive constant. Then*

1. *The possible values of the curvature index k are 0 and −1, i.e. the Friedmann model of the universe is non-oscillatory.*
2. *The Hubble parameter $H(t)$ has the following representation*

$$H(t) = \frac{\alpha}{t} + \frac{\varepsilon}{t}. \tag{25}$$

3. *The deceleration parameter $q(t)$ has the following representations*

$$q(t) = \frac{\mu(t)}{\alpha^2}(1+\eta), \tag{26}$$

$$q(t) = \frac{1-\alpha}{\alpha} - \frac{t\dot{\varepsilon}}{\alpha^2}(1+\eta) + \tau, \quad \eta, \tau \in \mathcal{Z}_0. \tag{27}$$

$$q(t) = \frac{1-\alpha}{\alpha} - t\dot{\xi} + \zeta, \quad \xi, \zeta \in \mathcal{Z}_0 \tag{28}$$

Proof

1. $\mathcal{N}$-functions belong to so called Zygmund class (Bojanić and Karamata, see [2]), hence, since $\alpha \neq 0$, the scale factor $a(t)$ is ultimately monotonous function. Thus, the universe is non-oscilatory, hence $k = 0$ or $k = -1$.
2. The representation (25) follows from Proposition 2.4
3. $q(t) = -\frac{\ddot{a}}{a}\cdot\frac{1}{H^2} = \frac{\mu}{t^2}\cdot\frac{1}{(\alpha/t+\varepsilon/t)^2} = \frac{\mu}{\alpha^2}(1+\varepsilon/\alpha)^{-2} = \frac{\mu(t)}{\alpha^2}(1+\eta(t))$ for some $\eta \in \mathcal{Z}_0$, i.e. (26) holds.Further, by ε-representation for $\mu(t)$, Theorem 2.2, there is $\delta \in \mathcal{Z}_0$ so that $q(t) = (\Gamma - t\dot{\varepsilon} + \delta)(1+\eta)/\alpha^2$. As $\Gamma = \alpha(1-\alpha)$ we obtain (27) taking $\tau = \Gamma\eta/\alpha^2 + \delta(1+\eta)/\alpha^2$.

   Finally we show that $q \in \mathcal{M}_{(1-\alpha)/\alpha}$. According to Theorem 2.2 this will prove the representation (28). So, we have

$$q(t) = \frac{\mu}{\alpha^2}(1+\varepsilon/\alpha)^{-2} = \frac{\Gamma - t\dot{\varepsilon} + \delta}{\alpha^2}(1+\varepsilon/\alpha)^{-2},$$

hence for $v(t) = (1+\varepsilon(t)/\alpha)^2$ we have

$$\mathbf{M}(q) = \frac{\Gamma}{\alpha^2}\mathbf{M}(1/v) - \frac{1}{\alpha^2}\mathbf{M}(t\dot{\varepsilon}/v) + \frac{1}{\alpha^2}\mathbf{M}(\delta/v).$$

Further, $\mathbf{M}(1/v) = 1$ since $x\int_x^\infty \frac{1}{(1+\varepsilon/\alpha)^2}\cdot\frac{dt}{t^2} \to 1 \quad \text{as } x \to \infty$.

$\mathbf{M}(t\dot{\varepsilon}/v) = 0$ since $x\int_x^\infty \frac{t\dot{\varepsilon}}{v}\cdot\frac{dt}{t^2} = -x\alpha\int_x^\infty \frac{1}{t}\,d\frac{1}{1+\varepsilon/\alpha} = \frac{\alpha}{1+\varepsilon(x)/\alpha} - x\alpha\int_x^\infty \frac{1}{1+\varepsilon/\alpha}\cdot\frac{1}{t^2}dt = \frac{\alpha}{1+\varepsilon(x)/\alpha} - \alpha + o(1) \to 0$ as $x \to \infty$.

$\mathbf{M}(\delta/v) = 0$ since $\delta(t)(1+\varepsilon(t)/\alpha)^{-2}$ is a $\mathcal{Z}_0$-function.

Therefore $\mathbf{M}(q) = \Gamma/\alpha^2 = (1-\alpha)/\alpha$. □

Now we introduce a new constant w related to the scale factor $a(t)$ which satisfy the generalized power law. It will appear that w is in fact the equation of state parameter. So assume $a(t) = t^\alpha L(t),\ L \in \mathcal{N}$ and $\alpha \neq 0$. We define w by

$$w \equiv w_\alpha = \frac{2}{3\alpha} - 1 \quad \text{(equation of state parameter)}. \tag{29}$$

Note that $w \neq -1$. As $\Gamma = \alpha(1-\alpha)$, we have the following statement:

Proposition 3.4

1. $\Gamma = \frac{2}{9}\cdot\frac{1+3w}{(1+w)^2}$.
2. $w = \frac{1-3\Gamma+\sigma_\alpha\sqrt{1-4\Gamma}}{3\Gamma}$, *where* $\sigma_\alpha \in \{1,-1\}$.

The sign σ_α is determined as follows. Suppose $\Gamma \neq 1/4$. Then the polynomial $x^2 - x + \Gamma$ has two different roots α, β. As $\alpha + \beta = 1$, we see that $\alpha > \beta$ if and only if $\alpha > 1/2$. Since w in decreasing in α we have:

Case $\alpha > 1/2$. Then: if $1/4 > \Gamma > 0$ then $\sigma_\alpha = -1$; if $\Gamma < 0$ then $\sigma_\alpha = +1$.
Case $\alpha < 1/2$. Then: if $1/4 > \Gamma > 0$ then $\sigma_\alpha = +1$; if $\Gamma < 0$ then $\sigma_\alpha = -1$.

According to Theorem 3.3 we have also the following statement.

Theorem 3.5. *Under the assumptions of Theorem 3.3 there are the following relations*

$$\alpha = \frac{2}{3(1+w)}, \quad a(t) = a_0 L(t) t^{\frac{2}{3(1+w)}}, \quad H(t) \sim \frac{2}{3(1+w)t}, \quad \mathbf{M}(q) = \frac{1+3w}{2}. \tag{30}$$

For determination of energy density $\rho(t)$ and pressure $p(t)$ more information on the geometry of the universe are needed. We proceed to study cosmological parameters of the universe with the specific curvature index k.

3.2. Asymptotic solution for universe with curvature index $k = 0$

In this subsection we shall discuss cosmological parameters for spatially flat universe. Hence $k = 0$ where k is the curvature index. We also assume that the scale factor $a(t)$ satisfies the generalized power law. This allows us to estimate at ∞ parameters $\rho = \rho(t)$ and $p = p(t)$. The symbol w denotes the equation of state parameter as defined in the previous subsection.

Theorem 3.6. *Assume $\mu \in \mathcal{M}_\Gamma$ where $\Gamma < 1/4$. Let $a(t) = t^\alpha L(t)$ be the corresponding scale factor, where $\alpha \neq 0$ and $L \in \mathcal{N}$, with L having the ε-representation as in Theorem 1.1. Then*

1. $\rho = \frac{1}{6\pi G(1+w)^2 t^2} + \frac{\eta}{t^2}, \quad \eta \in \mathcal{Z}_0$. 2. $\mathbf{M}\left(\frac{p}{\rho c^2}\right) = w$.

Proof

1. As $k = 0$, the Friedmann equation (2) becomes $H^2 = 8\pi G\rho/3$. As $H(t) = \alpha/t + \varepsilon/t$ and $w = \frac{2}{3\alpha} - 1$, the statement follows if $\eta = \frac{3(2\varepsilon+\varepsilon^2)}{8\pi G}$.
2. By (18), Theorem 2.2 and the above representation of ρ, we have

$$\frac{p}{\rho c^2} = \frac{2\mu}{3\alpha^2(1+\varepsilon/\alpha)^2} - \frac{1}{3}, \quad \mu = \Gamma - t\dot{\varepsilon} + \eta, \ \eta \in \mathcal{Z}_0.$$

Let us take $v(t) = (1+\varepsilon(t)/\alpha)^2$. Then

$$\mathbf{M}(p/\rho c^2) = \frac{2\Gamma}{3\alpha^2}\mathbf{M}(1/v) - \frac{2}{3\alpha^2}\mathbf{M}(t\dot{\varepsilon}/v) + \mathbf{M}(\eta/v) - \frac{1}{3}.$$

As in the proof of Theorem 3.3.3, we have $\mathbf{M}(1/v) = 1$, $\mathbf{M}(t\dot{\varepsilon}/v) = 0$ and $\mathbf{M}(\eta/v) = 0$. Hence, $\mathbf{M}(p/\rho c^2) = 2\Gamma/3\alpha^2 - 1/3 = 2/3\alpha - 1/3 = w$. □

Corollary 3.7. *Under assumptions of Theorem 3.6 there are $\xi, \zeta \in \mathcal{Z}_0$ so that $p = \hat{w}\rho c^2$, where $\hat{w}(t) = w - t\dot{\xi} + \zeta$.*

Hence, the assumption that the scale factor $a(t)$ satisfies the generalized power law implies a certain form of equation of state, $p = \hat{w}\rho c^2$. If $\Gamma = \lim_{t\to\infty}\mu(t)$ exists, then $\mathbf{M}(\mu) = \Gamma$ and by the proof of Theorem 3.6 it follows $\hat{w} = w$, i.e. the classical form of the equation of state is valid. In the next subsection we shall see that the assumption of the existence of $\Gamma = \lim_{t\to\infty}\mu(t)$ leads to the classical formulas for cosmological parameters.

3.3. Solution for $\mu(t)$ constant at ∞

In this section we shall discuss conditions under which the parameter $\mu(t)$ introduced by (23) is constant at ∞ and how this relates to the solutions of the Friedmann equations. Therefore we assume $\lim_{t\to\infty}\mu(t) = \Gamma$. Hence, by Proposition 2.1, $\mathbf{M}(\mu) = \Gamma$. So, all up to now derived properties of cosmological parameters related to the scale factor $a(t)$ which satisfies the generalized power law are valid. By Theorem 3.2 this will be the case if $\Gamma < 1/4$ and under under additional assumptions if $\Gamma = 1/4$. In this subsection we shall assume $\Gamma < 1/4$.

3.3.1. Case $k = 0$, spatially flat universe

By Corollary 2.3 $\lim_{t\to\infty}\mu(t) = \Gamma$ if and only if $\lim_{t\to\infty} t\dot{\varepsilon} = 0$ in ε-representation of $a(t)$ described by Theorem 1.1. Hence, according to Theorems 3.3, 3.5, and 3.6 we immediately obtain:

$$\alpha = \frac{2}{3(1+w)}, \quad a(t) = a_0 L_\alpha(t) t^{\frac{2}{3(1+w)}}, \quad \rho(t) \sim \frac{1}{6\pi G(1+w)^2 t^2}, \quad p(t) \sim wc^2\rho, \quad H(t) \sim \frac{2}{3(1+w)t}, \quad q(t) \sim \frac{1+3w}{2} \tag{31}$$

and Eqs. (3), (4) and (2) are satisfied.

First we suppose that α is greater of the roots of the polynomial $x^2 - x + \Gamma$, hence $\alpha > 1/2$. Then by (31) immediately follows $\frac{1}{3} > w > -1$, hence the set of admissible values of w is the interval

$$I_\alpha = (-1, 1/3). \tag{32}$$

The value $w = -1$ yields singularity; for such w there is no corresponding α neither Γ. If $p = -\rho c^2$ is anyway assumed, then by fluid equation we have $\dot{\rho} = 0$, i.e ρ is constant. This case corresponds to the cosmological constant, so $\rho = \rho_\Lambda = \frac{\Lambda}{8\pi G}$. The constant Λ has a negative effective pressure, and as the universe expands, work is done on the cosmological constant fluid. Hence energy density remains constant in spite of the fact that universe expands.

If $w = 1/3$ then $\alpha = \beta = 1/2,\ \Gamma = 1/4$ and in this case (20) is not the general solution for Friedmann equations. This case will be discussed later.

If $w = -1/3$, then $\alpha = 1,\ \Gamma = 0$ and the acceleration equation reduces to $\ddot{a} \sim 0$. If ρ is computed using the acceleration equation, assuming the asymptotic value for $a(t)$ in (31), then the following asymptotic formula for ρ holds, except for $w = -1/3$,

$$\rho \sim \frac{3\Gamma}{4\pi G(1+3w)} \cdot \frac{1}{t^2}.$$

Hence, $w = -1/3$ is a kind of singularity, but not the proper one, as it can be replaced by the second formula for ρ in (31).

Let us take into account the physical constraints on the parameters occurring in our calculations. For example, the universe is decelerating if and only if $q > 0$ hence, by (27), this is equivalent to $\alpha < 1$. On the other hand, $\alpha < 1$ is equivalent to $w > -\frac{1}{3}$ by definition (29) of w. Therefore,

$$\frac{1}{2} < \alpha < 1 \quad \text{if and only if} \quad -\frac{1}{3} < w < \frac{1}{3} \tag{33}$$

and the universe decelerates in all cases. From $p \sim wc^2\rho$ we see that the pressure $p > 0$ if and only if $w > 0$. Hence, using (29), we see that $p > 0$ if and only if $\alpha < \frac{2}{3}$ and the interval for α in (33) reduces to $\frac{1}{2} < \alpha < \frac{2}{3}$.

Let us consider the second fundamental solution in (20) of the acceleration equation with the index $\beta \equiv \alpha_2 < 1/2$. First we introduce the constant w_β by

$$w_\beta = \frac{2 - 3\beta}{3\beta}. \tag{34}$$

Then as $\alpha + \beta = 1$ and the following symmetric identity holds:

$$w_\alpha + w_\beta + 3w_\alpha w_\beta = 1. \tag{35}$$

Let (31_β) be the set of parameters obtained from (31) by replacing $w(= w_\alpha)$ by w_β. Using (35) one can show that $b(t)$ satisfies all three Eqs. (3), (4) and (2). Now having $\beta < \frac{1}{2}$ we can extend the interval for w in (33). As $\beta < \frac{1}{2}$ we find from (34) that $w \equiv w_\beta > \frac{1}{3}$ or $w \equiv w_\beta < -1$. Therefore, by (31_β) the set of admissible values for $w = w_\beta$ is the set

$$I_\beta = (-\infty, -1) \cup \left(\frac{1}{3}, +\infty\right). \tag{36}$$

Putting together (32) and (36) we see that the set of all admissible values for w corresponding to all possible solutions of Eqs. (3), (4) and (2) is the set

$$I = \mathbb{R} \setminus \left\{-1, \frac{1}{3}\right\}, \quad \mathbb{R} \text{ is these to f real numbers.} \tag{37}$$

The physical constraints on physical parameters for $\beta < \frac{1}{2}$ give the narrower bound for w_β. By (31_β) the adiabatic sound speed $v_s = \left(\frac{\partial p}{\partial \rho}\right)^{1/2} = w_\beta c^2$, hence $w_\beta < 1$ and so $\beta > \frac{1}{3}$. Therefore, including also our previous discussion on physical constraints for $\alpha > \frac{1}{2}$, we have the following bounds for α and w:

$$\frac{1}{3} < \alpha < \frac{2}{3} \quad \text{and} \quad 0 < w < 1 \quad (\text{Zel}'\text{dovich interval}). \tag{38}$$

3.3.2. Case $k = -1$, spatially open universe

We shall only briefly discuss this case. We remind that if we assume $\Gamma = \lim_{t\to\infty}\mu(t)$ exists and $\Gamma < 1/4$, then $t\dot{\varepsilon}(t) \to 0$ as $t \to \infty$.

Lemma 3.8. *Let $a(t) = t^\alpha L(t)$ be a solution of Friedmann equations, $L \in \mathcal{N}$, with ε-representation (13) and $k = -1$ the spatial index. Then*

$$\frac{p}{\rho c^2} = \frac{2}{3} \cdot \frac{\alpha + \varepsilon + kc^2 a^{-2} t^2 - t\dot{\varepsilon}}{(\alpha+\varepsilon)^2 + kc^2 a^{-2} t^2} - 1. \tag{39}$$

Proof. Taking the logarithmic derivative of ρ using (2) we obtain

$$\frac{\dot{\rho}}{\rho} = 2 \cdot \frac{H\dot{H} - kc^2 \dot{a} a^{-3}}{H^2 + kc^2 a^{-2}}.$$

Using $\dot{a} = Ha$ and by (25), $\dot{H} = \dot{\varepsilon}/t - (\alpha+\varepsilon)/t^2$, we get

$$\frac{\dot{\rho}}{\rho} = -2H \cdot \frac{\alpha + \varepsilon + kc^2 a^{-2} t^2 - t\dot{\varepsilon}}{(\alpha+\varepsilon)^2 + kc^2 a^{-2} t^2}. \tag{40}$$

By fluid equation (4) we have $p/\rho c^2 = -\frac{1}{3H}\cdot\frac{\dot{\rho}}{\rho} - 1$, hence (39) follows. □

Corollary 3.9. *Suppose $k = -1$ and $\alpha < 1$. Then $p/\rho c^2 \to -1/3$ as $t \to \infty$.*

Proof. As $\alpha < 1$ and L is slowly varying, we have $a^{-2}t^2 = L(t)t^{2(1-\alpha)} \to \infty$ as $t \to \infty$. Since also $\varepsilon, t\dot{\varepsilon} \to 0$ as $t \to \infty$, by (39) the assertion follows. □

Theorem 3.10. *Suppose $\Gamma = \lim_{t\to\infty}\mu(t)$ exists, $\Gamma < 1/4$, and $a(t) = t^\alpha L(t),\ \alpha \neq 0$, is a corresponding scale factor for an open universe* (i.e. $k = -1$). *Then $\alpha = 1$ and $w = -1/3$.*

Proof. Suppose $\alpha = 1$. Then it easy to see that in this case, by (39),

$$\frac{p}{\rho c^2} = \frac{2}{3}\cdot\frac{1+\varepsilon+kc^2L^{-2} - t\dot{\varepsilon}}{(1+\varepsilon)^2 + kc^2L^{-2}} - 1 \to -\frac{1}{3}, \quad \text{as} \quad t \to \infty.$$

Assume $\alpha < 1$. Then, by Corollary 3.9, we have $p/\rho c^2 \to -1/3$, as $t \to \infty$. Further, as $a^{-2}t^2 \to \infty$ for $t \to \infty$, by (40) it follows $\dot{\rho}/\rho \to -2H$, as $t \to \infty$. Then by (25) it follows $\rho(t) = L_1 t^{-2\alpha}$ for some $L_1 \in \mathcal{N}$. By (25), Friedman equation (2), some constants c_1, c_2 and $L_2 \in \mathcal{N}, L_2 = L^{-2}$, it follows

$$(\alpha+\varepsilon)^2 = (c_1L_1 + c_2L_2)t^{2(1-\alpha)}. \tag{41}$$

Since $2(1-\alpha) > 0$ and L_1, L_2 are slowly varying, it follows

$$L_1 t^{2(1-\alpha)}, L_2 t^{2(1-\alpha)} \to \infty \quad \text{as} \quad t \to \infty,$$

contradicting the identity (41), as $(\alpha+\varepsilon)^2 \to \alpha^2$ for $t \to \infty$. Thus, $\alpha \geqslant 1$.

Suppose $\alpha > 1$ and $t \to \infty$. Then for the second fundamental solution u_β we would have $\beta < 1$ as $\alpha + \beta = 1$. But this solution is impossible as it was just proved in the previous case $\alpha < 1$.

Therefore $\alpha = 1$. □

Hence, if the power law is assumed for the scale factor $a(t)$ for an open universe, then $a(t) \sim a_0 t$ as $t \to \infty$. Also, $\Gamma = 0$ since $\Gamma = \alpha(1-\alpha)$. Our analysis in this subsection leads to the following conclusions.

1° **The values of the equation of state parameter** w. Let us discuss the values of the parameter w excluded by (37). In the following we shall use the relation (35). We see that $w = -1$ leads to the singularities in (31). Also, $w_\alpha = -1$ if and only if $w_\beta = -1$ and in this case there is no corresponding Γ. This case corresponds to cosmic inflation. The value $w = -\frac{1}{3}$ yields a kind of singularity in (31), while the relation (35) is inconsistent. In this case w_β does not exists and the symmetry between w_α and w_β is broken. Also $\Gamma = 0$ and the corresponding $w = -\frac{1}{3}$ appears in the solution for the open universe. If $w = \frac{1}{3}$, then $\Gamma = \frac{1}{4},\ w_\alpha = w_\beta$ and $\alpha = \beta$. This case will be analyzed in the next subsection. Finally, let us consider the cases $w = 0, 1$, the values that appear as limits in (38). If $w = 0$, then $\alpha = \frac{2}{3}$ and from the definition of w we see that $p = 0$ (the matter dominated universe). If $w = 1$, then $\alpha = \frac{1}{3}$ and this value of w corresponds to the universe with the mixture of dust and radiation. This is possible only if the integration constant c_2 in (20) of the dominant fundamental solution is equal to 0.

2° By discussion in this subsection and the remarks in **1°** we arrive to the following conclusion: *For the spatially flat universe the assumption that $\mu(t)$ is constant at ∞ leads to the classic solution of the Friedmann equation.*

3° For the sake of completeness we give a rather short derivation of solution for the spatially flat universe assuming the equation of state $p = wc^2\rho,\ w$ is nonsingular. Under this assumption the acceleration equation is reduced to $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1+3w)\rho$, while the Friedmann equation becomes $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$. Dividing the acceleration equation by the Friedmann equation, we obtain $\frac{\ddot{a}}{a} = \lambda\frac{\dot{a}^2}{a^2}$ where $\lambda = -\frac{1+3w}{2}$. Hence $\frac{d\dot{a}}{\dot{a}} = \lambda\frac{da}{a}$, i.e. $\log\dot{a} = \log(c_0a^\lambda)$ and so $\frac{a^{1-\lambda}}{1-\lambda} = c_0t + c_1$, where $c_0,\ c_1$ are the integration constants. Taking $a(0) = 0$, we find $a(t) = a_0t^{\frac{1}{1-\lambda}} = a_0t^{\frac{2}{3(1+w)}}$ for some constant a_0.

4° **Generalized power law principle**. Putting together all results presented up to now, we see that the following are equivalent:

a. The integral limit Γ in (19) exists and $\Gamma < \frac{1}{4}$.

b. The solutions $a(t)$ of the Friedmann equation satisfy the generalized power law with index $\alpha \neq 1/2$.

c. The equation of state holds at ∞ as described by Corollary 3.7, $w \neq -1, \frac{1}{3}$. If $\mu(t)$ is constant at ∞ then $p \sim wc^2\rho$, as $t \to \infty$.

5° Power law principle and cosmological constant Λ. If $\Lambda \neq 0$ is assumed, all the asymptotic formulas (31) for the cosmological parameters are valid, except for $w = -1$. This follows from the fact that by the appropriate substitutions the Friedmann equations with the parameter Λ transform to their basic form (2)–(4).

3.4. Case $\Gamma = \frac{1}{4}$, adjacent case

In this subsection we shall assume $\Gamma = \frac{1}{4}$ in the limit (19). Then the polynomial $x^2 - x - \frac{1}{4}$ has the double root $\alpha = \frac{1}{2}$. In discussion of the acceleration equation (17) for this case we shall use the following criterion, see Marić [13, p. 37] and Kusano and Marić [11, Theorem 2.2]:

Theorem 3.11. *Let* $\phi(x) = x\int_x^\infty \frac{\mu(t)}{t^2}dt - \frac{1}{4}$, *let the integral*

$$\psi(x) = \int_x^\infty \frac{|\phi(t)|}{t}dt, \quad x > x_0 > 0, \ converge \tag{42}$$

and assume

$$\int_x^\infty \frac{\psi(t)}{t}dt < \infty, \quad x > x_0. \tag{43}$$

Further, let L_1, L_2 *denote two normalized slowly varying functions. Then there exist two fundamental solutions of the acceleration equation* (17)*:*

$$u(t) = t^{\frac{1}{2}}L_1(t) \quad v(t) = t^{\frac{1}{2}}\log(t)L_2(t) \tag{44}$$

if and only if the condition (19) *holds (for* $\Gamma = \frac{1}{4}$*). Also* L_1, L_2 *tend to a constant as* $t \to \infty$ *and* $L_2(t) \sim 1/L_1(t)$. □

As $\log(t)L_2(t)$ is also a slowly varying, we see that both fundamental solutions $u(t)$ and $v(t)$ satisfy the general form of power law. Hence, each solution $a(t) = c_1u(t) + c_2v(t)$ of the acceleration equation is regularly varying of index $\frac{1}{2}$. By the results in the previous subsection when $\Gamma < \frac{1}{4}$ was assumed, we see, if the conditions (42) and (43) are satisfied, that $a(t)$ is regularly varying of index $\frac{1}{2}$ if and only if $w \sim \frac{1}{3}$ as $t \to \infty$, i.e. $p \sim \frac{1}{3}c^2\rho$ holds asymptotically. This is the second classic cosmological solution.

3.5. Asymptotic solution for spatially flat universe with matter-dominated evolution

We have seen that the constant $\Gamma = \mathbf{M}(\mu)$ determine the asymptotical behavior at the ∞ of the scale factor $a(t)$. If the matter-dominated evolution of the universe is assumed, i.e. dominated by some form of pressureless material after the certain time moment t_0 then it appears that the expression $H(t)t$ depends solely on the parameter Ω. In this case we are able to estimate possible values of Γ. We shall discuss also the status of the constant Γ and the related asymptotic behavior of $a(t)$ for the spatially flat universe including the cosmological constant Λ. Therefore, in this section we discuss asymptotic solutions and Friedmann equations, the related parameter $\mu(t)$, and the constant Γ assuming the pressureless spatially flat universe with the cosmological constant Λ.

Using the formula for the age of the spatially flat universe with the cosmological constant Λ Carroll et al. [3], see also Liddle and Lyth [9] and Narlikar [16], the expression $H(t)t$ in this case is given by

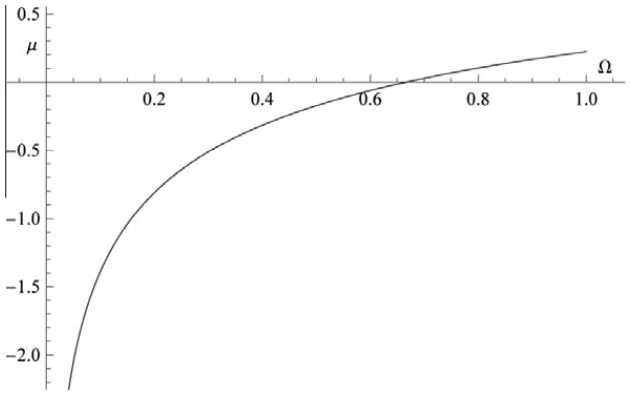

Fig. 1. Graph of $\bar{\mu}(\Omega)$.

$$H(t)t = \frac{2}{3}\cdot\frac{1}{\sqrt{1-\Omega}}\ln\left(\frac{1+\sqrt{1-\Omega}}{\sqrt{\Omega}}\right), \tag{45}$$

while the deceleration parameter $q(t)$ is given by

$$q(t) = \frac{\Omega}{2} - \Omega_\Lambda. \tag{46}$$

In the model of the spatially flat universe we have $\Omega + \Omega_\Lambda = 1$ hence, $q(t) = \frac{3\Omega}{2} - 1$. Therefore, by (23) and (45) it follows

$$\mu(t) = \frac{2}{9}\cdot\frac{3\Omega-2}{1-\Omega}\left(\ln\left(\frac{1+\sqrt{1-\Omega}}{\sqrt{\Omega}}\right)\right)^2, \tag{47}$$

where $\Omega = \Omega(t)$. We see that the parameter $\mu(t)$ in the model for the pressureless spatially flat universe depends solely on Ω. The graph of the parameter $\mu(t)$ is presented in Fig. 1, as a function of Ω.

The limit value

$$\Omega_\infty = \lim_{t\to\infty}\Omega(t) \tag{48}$$

can be in principle any value in the interval $[0,1]$. Let us introduce the parameter $\bar{\mu}(\Omega)$ by the expression on the right hand side of (47). Hence $\mu(t) = \bar{\mu}(\Omega(t))$ and

$$\Gamma = \lim_{t\to\infty}\mu(t) = \lim_{\Omega\to\Omega_\infty}\bar{\mu}(\Omega). \tag{49}$$

We see that $\bar{\mu}(\Omega)$ is an increasing function in Ω and that its values lay in the interval $[-\infty, \frac{2}{9}]$, as $\lim_{\Omega\to 1-0}\bar{\mu}(\Omega) = \frac{2}{9}$. Hence $\mu(t) < \frac{2}{9}$. Suppose the limit $\Gamma = \lim_{t\to\infty}\mu(t)$ exists. Then $\Gamma \leqslant \frac{2}{9} < \frac{1}{4}$. Thus, assuming the pressureless spatially flat universe with the cosmological constant, all possible values of Γ are less than $2/9$, hence Theorem 3.2 can be applied. This analysis leads to the following conclusions for the solutions of Friedmann equations for the pressureless spatially flat universe with the cosmological constant Λ.

1° The scale factor $a(t)$ satisfies the generalized power law. More specifically, $a(t) = t^\alpha L(t)$ where $L \in \mathcal{N}$ and α is a root of the polynomial $x^2 - x + \Gamma$.

2° Suppose $\Omega_\infty = 1$. By the identity $\Omega + \Omega_\Lambda = 1$ it follows $\Omega_\Lambda \sim 0$ as $t \to \infty$ and $\Gamma = \frac{2}{9}$. Then Eq. (14) becomes $x^2 - x + \frac{2}{9} = 0$ and it has the solutions $\alpha_1 = \frac{1}{3},\ \alpha_2 = \frac{2}{3}$. According to the conclusion **1**°, $a(t)$ regularly varying of index $\frac{2}{3}$ and by (45) and (46), $H(t)t \sim \frac{2}{3}$ and $q \sim \frac{1}{2}$ as $t \to \infty$. This result corresponds to the classic solution of the Friedmann equation for the pressureless spatially flat universe with the cosmological constant $\Lambda = 0$.

3° The formula (47) for $\bar{\mu}$ shows that the evolution of the expansion scale factor $a(t)$ depends only on the evolution of the density parameter Ω. The nature of this evolution is determined by the constant Γ but in all instances it satisfies the power law represented by some regularly varying function. The introduction of the cosmological constant only changes the index of the regular variation with respect to the model with $\Lambda = 0$.

4° Let us consider the possible values of Γ. The value $\Omega_0 = 0.3$ (e.g. [9]) for the present epoch is close to the value preferred by the observation. If we assume that the energy density ρ becomes lower as the age of the universe becomes

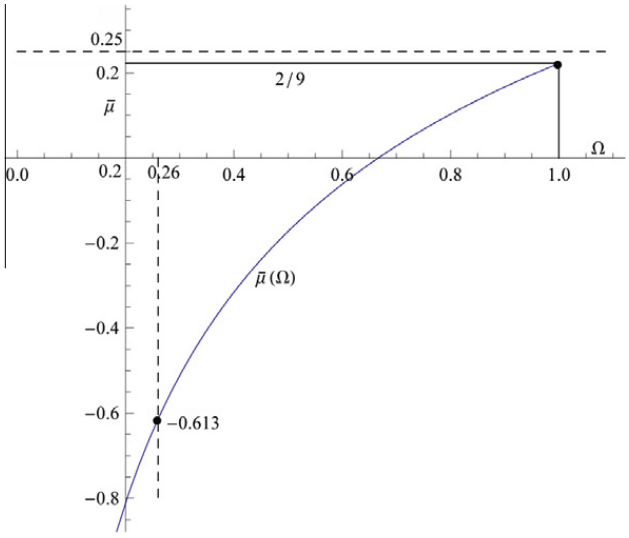

Fig. 2. Graph of $\bar{\mu}(\Omega)$ in the preferred interval.

older, we may suppose that the possible range for the constant Ω_∞ is the interval $[0.3, 1]$, i.e. $0.3 \leqslant \Omega_\infty \leqslant 1$. The graph of $\bar{\mu}$ for this interval is presented in Fig. 2. We see that $\Gamma \leqslant 2/9$.

5° The solution $a(t)$ is regularly varying of some index α, i.e. $a(xt)/a(t) \to x^\alpha$ as $t \to \infty,\ x > 0$. So for relatively large[4] t_0 with respect to $x > 0$, we have $a(xt_0) \sim a(t_0)x^\alpha$ and we may take that the time instances t_0 and xt_0 belong to the same epoch in the evolution of the universe. So, taking $t = xt_0$ and eliminating x from the last asymptotic relation, we find the asymptotic estimation for $a(t)$ for an epoch with respect to the initial value $a(t_0)$:

$$a(t) \sim a(t_0)\left(\frac{t}{t_0}\right)^\alpha. \tag{50}$$

Also, $a(t) = t^\alpha L(t)$ where $L(t)$ is slowly varying. Hence $L(t) \sim L(t_0)$ for an epoch, so it is hard to measure $L(t_0)$ and the influence of $L(t)$ on $a(t)$. However, the influence of $L(t)$ on the large scale might be substantial, particularly if $\alpha \approx 0$, as the example (10) shows.

3.6. Case $\Gamma > \frac{1}{4}$

Assume $\Gamma > \frac{1}{4}$ in the limit (19). Then the solution $a(t)$ of the acceleration equation is oscillatory. This immediately follows from Hille's classical theorem (see Hille [6] and Marić [13, Theorem 1.8]). Therefore, for these values of Γ the expansion scale factor $a(t)$ of the universe does not satisfy the power law and the approach presented in the paper is not appropriate for this case. Since $a(t)$ has in this case (infinitely many) zeros, then there is t_0 such that $\dot{a}(t_0) = 0$. So, from the Friedmann equation (2) it follows that $k > 0$, i.e. the universe must be closed. This is obviously true even if the Friedmann equation is modified by adding the cosmological constant $\Lambda > 0$.

We see that the constant 1/4 plays an important role as a possible value of Γ in the limit (19). This constant provides a sharp "threshold," or "cut-off point," at which the oscillation of $a(t)$ takes place.

4. Conclusion

It has been shown that a dimensionless constant Γ related to the Friedmann acceleration equation and the theory of regularly varying functions play the key role in the formulation of the power law principle for solutions $a(t)$ of the Friedmann equations. The constant Γ is defined by

$$\Gamma = \limsup_{x\to\infty} \int_x^\infty \frac{\mu(t)}{t^2}\,dt, \tag{51}$$

where $\mu(t) = q(t)(H(t)t)^2$ as $t \to \infty$, $q(t)$ is the deceleration parameter and $H(t)$ is the Hubble parameter. We have shown that the generalized power law principle for the scale factor $a(t)$ holds if and only if the integral limit (51) exists and $\Gamma < \frac{1}{4}$. The cosmological constants were also discussed under relaxed condition $\lim_{t\to\infty}\mu(t) = \Gamma$ which implies (51), Under this condition we have shown that the power law principle is equivalent to the equation of state $p \sim wc^2p,\ w \neq -1, 1/3$. The values of Γ determine the asymptotical behavior of the scale factor $a(t)$ as time t tends to ∞. The constant Γ also uniquely determines other cosmological parameters such as the Hubble parameter and the equation of state parameter w. Particularly is discussed the pressureless spatially flat universe with non-zero cosmological constant. Further, the value of Γ determines the type of the universe; for $\Gamma < \frac{1}{4}$ the universe is spatially flat or open, while for $\Gamma > \frac{1}{4}$ the universe is oscillatory. The boundary case $\Gamma = \frac{1}{4}$ is also analyzed. All solutions we found are in agreement with the results found widely in the literature on standard cosmological model. As power law functions are the most frequently occurring type of the solutions of the Friedmann equation, the study of the constant Γ and the related function $\mu(t)$ might be of a particular interest.

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42 Astrometric positions of ICRF2 radio sources with different reference catalogues

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M. Martynov and N. Pejović,
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Results of an investigation of astrometric positions of a few extragalactic radio sources (ERS) from the ICRF2 list are presented. The celestial system is based on IAU (International Astronomical Union) Resolution A4 (1991). The fundamental celestial reference frame (International Celestial Reference Frame - ICRF) was adopted by the IAU (1997), with its original list of radio objects and two extensions (ICRF-ext1 and ICRF-ext2); referred to as ICRF1. At the IAU XXVII GA (2009), the second realization of the ICRF (ICRF2) was adopted with the list of precise positions for 3414 compact radio astronomical sources. It is more then five times the number as in the ICRF1. The ICRF2 has a noise threshold of about 0.04 mas (nearly 6 times better than ICRF1) and an axis stability of about 0.01 mas (nearly twice as stable as ICRF1). Comparison is done using the authors CCD observations of a few ERS made with the 2 m RCC telescope1 (with the focal length of 16 m) of Rozhen National Astronomical Observatory. About 30 fields around ERS were observed with CCD camera VersArray 1300B (1340x1300 pixels, the pixel size is 20x20 micrometers, one pixel is 0.258 arcsec) in the end of March 2011. The preliminary results are presented.

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ASTROMETRIC POSITIONS OF ICRF2 RADIO SOURCES WITH DIFFERENT REFERENCE CATALOGUES

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Abstract. We present the results of an investigation of astrometric positions of a few extragalactic radio sources (ERS) from the ICRF2 list. The reference systems are based on the resolutions of the international scientific unions. The celestial system is based on IAU (International Astronomical Union) Resolution A4 (1991). It was officially initiated and named International Celestial Reference System (ICRS) by IAU Resolution B2 (1997). Its definition was further refined by IAU Resolution B1 (2000) and Resolution B3 (2009). The fundamental celestial reference frame (International Celestial Reference Frame – ICRF) was adopted by the IAU (1997), with its original list of radio objects and two extensions (ICRF-ext1 and ICRF-ext2); hereafter referred to as ICRF1. Alltogether, there were 717 sources: 212 defining ones, 109 new ones, 294 candidate ones, and 102 additional sources. At the IAU XXVII GA (2009), the second realization of the ICRF (the ICRF2) was adopted with the list of precise positions for 3414 compact radio astronomical sources. It is more then five times the number as in the ICRF1. At that moment there were nearly 30 years of VLBI (Very Long Baseline Interferometry) observations of some radio sources. The ICRF2 has a noise threshold of about 0.04 mas (nearly 6 times better than ICRF1) and an axis stability of about 0.01 mas (nearly twice as stable as ICRF1). Also, a search for a relation between optical and radio reference frames is important. To do that, it is necessary to make the observations of some ICRF2 ERS which are visible in the optical domain, and to compare their optical and radio positions (VLBI ones). The optical positions (α and δ) could be calculated using reference stars from some of nowadays big star catalogues. The XPM, 2MASS (with XC1) and DR7 SDSS ones were used here, and the relative method was applied. We started to do that comparison using our CCD observations of a few ERS made with the 2 m RCC telescope[1] (with the focal length of 16 m) of Rozhen National Astronomical Observatory (Bulgarian Academy of Sciences). About 30 fields around ERS were observed with CCD camera VersArray 1300B (1340x1300 pixels, the pixel size is 20x20 micrometers, one pixel is 0.258 arcsec) in the end of March 2011. The main steps of our reduction and preliminary results are presented here.

[1]Based on observations with the 2 m RCC telescope of the Rozhen National Astronomical Observatory operated by the Institute of Astronomy, Bulgarian Academy of Sciences.

1. INTRODUCTION

The fundamental celestial reference frame, ICRF1 (Ma et al. 1998), was adopted by the IAU in 1997 and its extensions ICRF-ext1 (IERS 1999) and ICRF-ext2 (Fey et al. 2004) were later issued. The revised coordinates of candidate and other sources were given by Fey et al. (2004). The ICRF2 (IERS 2009) was presented at the GA of IAU (2009). The HCRF, Hipparcos Celestial Reference Frame, is the optical one. It was linked to the ICRF1 (radio one) with an accuracy of ± 0.6 mas in position (for the epoch 1991.25) and ± 0.25 mas per year in rotation (Kovalevsky et al. 1997), but that accuracy decreases over time. The reason is the error in the proper motions of stars. Because of it, it is necessary to verify and refine the relation between the HCRF and ICRF2 by using different telescopes and methods.

The ICRF1 consists of the precise coordinates of compact extragalactic objects, mostly quasars (QSO), BL Lacertae (BL Lac) sources and a few active galactic nuclei (AGNs). These sources are far away and their proper motions should be negligibly small. The current positions are known to be better than 1 mas. Also, there is the structure instability of the sources in radio wavelengths and because of it the ultimate accuracy is limited. The origin of ICRF1 is at the barycenter of the solar system. The directions of the axes should be fixed with respect to the quasars. Its principal plane and the origin of this plane are as close as possible to the mean equator at J2000.0 and to the dynamical equinox of J2000.0, respectively. The ICRF1 replaced the optical FK5 reference frame on January 1, 1998. The Hipparcos reference frame was realizing the ICRS in optical wavelengths; the accuracy is significantly better than that of FK5.

The reference directions of ICRS should be fixed and there is the continuing improvement in the ERS coordinates via VLBI observations; a regular maintenance of the system and improvement of the frame are necessary. Because of it, there are many observations which define ERS over a long data span, and they are useful to maintain the axes of the ICRS via good position stability. The frame stability is based upon the assumption that the proper motions of ERS are negligible. The ICRF1 is based on positions of 212 'defining' ERS, but to densify the frame, we added 294 less observed 'candidate' and 102 less suitable 'other' sources. The total number of objects is 667 in ICRF-Ext.1, 717 in ICRF-Ext.2 and 3414 in ICRF2 (295 'defining' and 3119 additional ones). The alignment of ICRF2 with the ICRS was made by using common ICRF2/ICRF-Ext.2 138 stable ERS. The two largest weaknesses of ICRF1 were eliminated: more uniform sky distribution of ERS and the position stability of the 295 ICRF2 defining sources. From 1 January 2010, the realization of the ICRS is the ICRF2.

The Hipparcos Catalogue provides the equatorial coordinates for 117955 star on the ICRS for epoch 1991.25; also, their proper motions, parallaxes and magnitudes. The stars are brighter than $V = 12$, mostly from $V = 7$ to $V = 9$. The median uncertainties are (for bright stars with magnitude $V < 9$): 0.8 mas in α, 0.6 mas in δ, 0.9 mas/yr in μ_α and 0.7 mas/yr in μ_δ. The proper motions of many double or multiple stars are unreliable due to the short epoch span of Hipparcos observations (less than 4 years). Van Leeuwen (2007) did a new reduction of the Hipparcos data. It resulted in significant improvements (mainly for the parallaxes) which had effect on the ICRS. The densification catalogues were derived from the HCRF because the

HCRF was primary realization of the ICRS at optical wavelengths. Some of these catalogues are: Tycho-2 (as the first step of the densification, the AC was included in Tycho-2 proper motions data), UCAC3, 2MASS (near-IR), PPMXL, XPM, etc.

It is possible to investigate the relation between optical and radio frames using the differences between optical and radio (VLBI) coordinates $((O-R)_\alpha, (O-R)_\delta)$ of ERS. It means, we can use the secondary reference frame to derive the optical positions of ERS. Partly, the optical/radio frames relation is dependent of star catalogues. Here, the positions (α, δ) of the optical counterparts to ERS were determined, and the relative method was applied with respect to the XPM (Fedorov et al. 2010), 2MASS (Cutri at al. 2003) and DR7 SDSS (Abazajian et al. 2009) catalogues. We used the 2MASS positions with XC1 proper motions of stars, and DR7 SDSS (The Seventh Data Release of the Sloan Digital Sky Survey) positions without proper motions (because the epoch difference between DR7 SDSS observations and our ones is small). The HCRF was used to calculate astrometric positions of ERS via small fields of view of our CCD observations. We assume that the centers of emission (of radio/optical sources) coincide between each other in line with the accuracy level of the optical observations.

2. DATA AND CALCULATION

To align radio frame (ICRF2) and optical one (HCRF) with high accuracy, we need to observe the common objects and to calculate their accurate optical positions. Because of it, the ground based astrometric observations of ERS are very important. In the end of March 2011, we spent a few nights at the Rozhen National Astronomical Observatory (Bulgarian Academy of Sciences). After the summer of 2010, the Rozhen telescope ($D/F = 2m/16m$) was fully automated. Its geographic coordinates are: $\lambda = 1^h38^m58^s(24^o44'38'')$, $\varphi = 41^o41'35''$. The altitude is 1759 m. Our observations were carried out with that RCC telescope equipped with CCD camera VersArray 1300B. The main characteristics of CCD camera are: the size of CCD chip is 1340x1300 pixels, the pixel size is 20x20 mkm, the scale is 0.258 arcsec/pix, and the field of view (FOV) is about 5.5x5.5 arcmin.

A total of 5 optical counterparts of the ICRF2 radio sources were observed (see Table 1):Q 1252+119(ICRF J125438.2+114105), L 1215+303 (ICRF J121752.0+300700), Q 1240+381 (ICRF J124251.3+375100), Q 1219+044 (ICRF J122222.5+041315) and L1221+809 (ICRF J122340.4+804004). We made 6 frames per source (3 at R filter and 3 at V one). These ERS are visible in optical part of wavelengths and the magnitudes ranged from 15.7 to 19.0 (ICRF, V domain) or from 15.2 to 18.8 (GSC, R domain). All exposures were guided, and the exposure time ranged from 5^s to 90^s (see Table 1). The contents of some columns in Table 1 is self-evident; for this reason each of them is not mentioned. The source designation and ERS type are in the second column, Q means quasar, L BL Lac, the next two columns give α and δ (ICRF2 coordinates) of ERS, the fourth one the magnitude for V and R domains, whereas the last two columns give our exposure time also for V and R domains.

The bias and flat-field frames were not applied to our raw frames. The dark correction is not significant because the CCD chip was cooled to -110° C (good for ERS fainter objects). The APEXII image processing package (Kouprianov 2008) was applied for CCD observations, and the DR7 SDSS and 2MASS (+XC1) catalogues

Table 1: Observed ERS.

No	Type of ERS, source name	RA(ICRF2) (h,m,s)	DEC(ICRF2) (°,′,″)	Mag V, R	Exp(s) for V	Exp(s) for R
1	Q 1252+119	12 54 38.256	11 41 05.90	16.2 15.2	3,5	5
2	L 1215+303	12 17 52.082	30 07 00.64	15.7 15.2	5	5
3	Q 1240+381	12 42 51.369	37 51 00.03	19.0 18.3	30,60,90	10,15
4	Q 1219+044	12 22 22.550	04 13 15.78	18.0 16.8	20	20
5	L 1221+809	12 23 40.494	80 40 04.34	18.0 18.8	20	20

were used for reduction of data. Also, the software AIP4WIN (Berry and Burnell 2002) was used together with XPM catalogue. All frames were reduced individually.

The main steps for processing the CCD images are: the identification of star-like objects (ERS) and reference stars, measuring the positions (x, y) of their centers, and the reduction (to get tangential and equatorial coordinates). We used the linear model, a standard astrometric "plate" reduction with the available reference stars,

$$\xi = ax + by + c$$
$$\eta = dx + ey + f$$

to transform the measured CCD coordinates (x, y) into tangential ones (ξ, η). The Least-Squares Method (LSM), the unweighted one, was applied to calculate the unknown values of the parameters a, b and c (to determine α), and d, e, and f (to determine δ). The FOV of CCD frames was small (5.5x5.5 arcmin) and we did not apply the corrections for apparent displacements, such as differential refraction (Aslan at al. 2010, Kiselev 1989). Because of small FOV, the big problem during astrometric reduction is that there are not enough reference stars (at least 3) with accurate coordinates and proper motions to calculate the values of α and δ of ERS; it was the case for the ERS L 1221+809. Also, if we have the low accuracy of astrometric data for reference stars from a catalogue, we cannot determine a good link between the radio and optical frames.

We used the positions from 2MASS together with XC1 proper motions, because the 2MASS catalogue has no proper motions. The Two Micron All Sky Survey (2MASS) covers all sky and contains positions and photometry (J, H, K_s) for about 470 million point sources; the observations were made very close to 2000. Another possibility was the DR7 SDSS positions without proper motions (because of small epoch difference between our observations and catalogue ones). Also, we used the XPM catalogue which contains the positions and proper motions for 314 million stars distributed all over the sky for the epoch 2000.0. The 2MASS and XPM are in ICRS. The magnitudes of our reference stars are from 13.5 to 19.5.

In the Figure 1, as an example, one CCD frame (at R filter) of ERS Q 1252+119 is presented. The ERS is near the central part of the image. It is marked with direction arrow and circle. The reference star is marked with a circle; there are six ones.

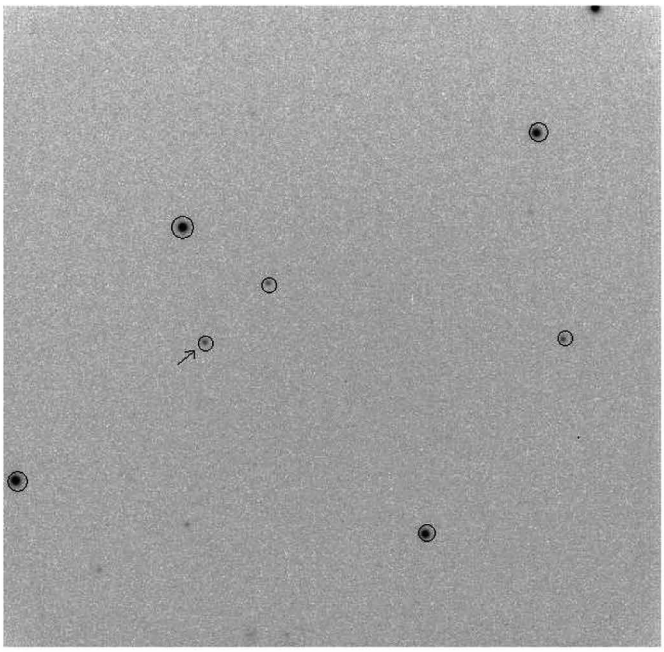

Figure 1: The observation of ERS Q 1252+119 (ICRF J125438.2+114105) in R filter.

3. RESULTS AND CONCLUSIONS

The positions of ERS were calibrated with respect to the XPM, 2MASS+XC1, and DR7 SDSS catalogues by using CCD observations. To calculate the stellar apparent positions we used some programmes from SOFA package. The ERS coordinates are from ICRF2 list. Some of XPM stars (or 2MASS, DR7 SDSS ones) were not taken into account because of their very low signal-to-noise ratio. By using 2MASS (or DR7 SDSS), the reduction was made with different reference stars (than in the case of XPM), and a different software (APEXII) was applied to get x and y coordinates (AIP4WIN in the case of XPM). After calculation of the optical positions of ERS we compared their optical positions and radio ones (to determine the values $(O-R)_\alpha$ and $(O-R)_\delta$); see Tables 2 and 3. From the results of Tables 2 and 3 we get the unweighted mean offsets of $(O-R)_\alpha$ and $(O-R)_\delta$ values. Relative to the XPM catalogue they are $-0''.06$ in α and $-0''.05$ in δ. In the case of 2MASS+XC1 it is $0''.15$ and $0''.13$, and of DR7 SDSS it is $-0''.01$ and $0''.05$, respectively. These offsets in α are close one to another; also, in δ.

We can conclude that the optical observations of ERS are possible by using 2m Rozhen telescope and a good CCD camera. The offsets (in α and in δ) are small and very similar for catalogues (XPM, 2MASS and DR7 SDSS). So, it is possible to use XPM, 2MASS and DR7 SDSS as the reference catalogues for astrometric reduction in small FOV of CCD observations. The XPM, 2MASS and DR7 SDSS are with high star density and a good densification of HCRF, but a higher accuracy catalogue of higher star density is required. The link between HCRF and ICRF2 is good enough at the mean epoch of our observations; XPM, 2MASS and DR7 SDSS are similar frames. Also, some problems during calculation of ERS optical positions can be caused by: faintness of the optical counterparts to ERS, atmospheric influences and technical problems.

Table 2: The values $(O-R)_\alpha$ and $(O-R)_\delta$ from 2MASS+XC1 and DR7 SDSS.

Source name	2MASS+XC1 $(O-R)_\alpha[\prime\prime]$	2MASS+XC1 $(O-R)_\delta[\prime\prime]$	DR7 SDSS $(O-R)_\alpha[\prime\prime]$	DR7 SDSS $(O-R)_\delta[\prime\prime]$
1252+119	0.441±0.027	0.051±0.020	0.096±0.040	0.007±0.038
1215+303	-0.025±0.008	0.088±0.027	-0.114±0.015	0.010±0.036
1240+381	0.056±0.038	0.132±0.052	-0.036±0.021	0.084±0.018
1219+044	0.117±0.014	0.231±0.020	0.031±0.051	0.095±0.036

Table 3: The values $(O-R)_\alpha$ and $(O-R)_\delta$ from XPM.

Source name	XPM $(O-R)_\alpha[\prime\prime]$	XPM $(O-R)_\delta[\prime\prime]$
1252+119	0.181±0.047	-0.014±0.014
1215+303	-0.479±0.394	-0.223±0.581
1240+381	0.041±0.011	-0.082±0.011
1219+044	0.015±0.009	0.137±0.003

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43 On the ε cosmological parameter

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In the previous paper *On asymptotic solutions of Friedmann equations* (Mijajlović et al. 2012), the theory of regularly varying functions in the sense of Karamata is applied in an asymptotic analysis of solutions of Friedmann equations. As is well known, solutions of these equations are used to represent cosmological parameters. Therefore, according to the theory of regularly varying functions all cosmological parameters depend on a function $\varepsilon(t)$ such that $\lim_{t\to\infty}\varepsilon(t) = 0$ and which appears in their integral representation. In this paper we derive a differential equation for the parameter $\varepsilon(t)$, discuss its solutions and give some physical interpretations.

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ON THE ε COSMOLOGICAL PARAMETER

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SUMMARY: In the paper *On asymptotic solutions of Friedmann equations* **(Mijajlović et al. 2012), the theory of regularly varying functions in the sense of Karamata is applied in an asymptotic analysis of solutions of Friedmann equations. As is well known, solutions of these equations are used to represent cosmological parameters. Therefore, according to the theory of regularly varying functions all cosmological parameters depend on a function $\varepsilon(t)$ such that $\lim_{t\to\infty}\varepsilon(t)=0$ and which appears in their integral representation. In this paper we derive a differential equation for the parameter $\varepsilon(t)$, discuss its solutions and give some physical interpretations.**

Key words. cosmological parameters, large-scale structure of Universe

1. INTRODUCTION

By cosmological parameters are usually meant some global physical quantities linked to the Universe. A good review of such approach for the ΛCDM model can be found, for example, in Lahav and Liddle (2014). Here we shall adopt a somewhat formalistic definition of cosmological parameters. By them we shall mean first of all solutions of Fiedmann equations, then any functions derived from these solutions and parameters by which the basic cosmological parameters can be expressed. While examples of the first two types of cosmological parameters are widely known, this is not the case for the third type. In our paper (Mijajlović et al. 2012) we exhibited such a parameter for the expanding universe with the cosmological constant Λ. This parameter is a continuous function $\varepsilon(t)$ having the limit 0 at infinity[1] and which appears in the Karamata representation of regularly varying functions (see Karamata 1930).

[1] Greek letters ε, ξ ζ, η and τ in this paper will be exclusively reserved for the names of continuous functions which have the limit 0 at infinity, e.g. $\lim_{t\to\infty}\varepsilon(t)=0$. Therefore, any mention or use of a function with this name assumes the mentioned property. We shall often write shortly ε, η and so on instead of $\varepsilon(t)$, $\eta(t)$, etc.

The scale factor $a(t)$, the energy density $\rho(t)$, and material pressure in the universe $p(t)$ are usually taken as fundamental or basic cosmological parameters. These parameters are solutions of the Friedman equations (see Friedman 1924). We remind the reader that the Friedman equations are derived from the Einstein field equations. The Friedman equations are the following three ordinary differential equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}, \qquad \text{Friedman equation,}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right), \qquad \text{Acceleration equation,}$$
$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0, \qquad \text{Fluid equation.}$$

Let us mention that these three equations are not independent. For example, the fluid equation can be inferred from the other two equations. Therefore, for solving this system of essentially of two equations and three unknowns some additional condition is needed. Usually the equation of state $p = w\rho c^2$ is assumed. Here we shall discuss also some other conditions that are set to the unknowns $a(t)$, $p(t)$ and $\rho(t)$.

In our paper (Mijajlović et al. 2012), we applied the theory of regularly varying functions in asymptotic analysis of solutions of Friedman equations for cosmological parameters of the expanding universe. We proved that under certain but broad condition relating the acceleration equation, some of these parameters, including the acceleration parameter $a(t)$ and the Hubble parameter $H(t)$, are regularly varying functions. This is an integral limit condition which is a part of the Howard-Marić theorem, stated at the end of this section. One consequence is that, according to the representation theory for regularly varying functions, all these parameters depend on a function $\varepsilon(t)$ which is "hidden" in the integral representation of regularly varying functions. We shall see that the parameters $a(t)$, $\rho(t)$ and $H(t)$ uniformly depend only on $\varepsilon(t)$, while the parameters $p(t)$ and $q(t)$, besides $\varepsilon(t)$, depend explicitly on terms $\dot{\varepsilon}(t)$ and $\dot{\varepsilon}(t)t$ as well. While $\varepsilon(t) \to 0$ as $t \to \infty$, this is not necessary so for $\dot{\varepsilon}(t)$, or $\dot{\varepsilon}(t)t$ which may lead to various evolutions of cosmological parameters. Here we shall derive a differential equation for $\varepsilon(t)$ and discuss its possible solutions. This differential equation was announced in Mijajlović and Pejović (2015).

For better understanding we shall review briefly some basic concepts related to regular variation. Even though the modern theory of regular variations deals mainly with measurable functions, we shall assume here that all appearing functions are continuous in their domains and have a sufficient number of derivatives, at least a continuous second derivative. This assumption is clearly pursuant with the physical meaning of cosmological parameters. From the physical point of view, it means that events such as Big Crunch, or Big Rip are not included in our analysis. That is, in finite time t, $a(t) \neq 0$ and $a(t)$ does not become infinite. We are particularly concerned with the properties of regularly varying solutions of the second order differential equation

$$\ddot{y} + f(t)y = 0, \tag{1}$$

assuming that $f(t)$ is continuous on some time interval $[\alpha, \infty]$. Note that the acceleration equation has the form Eq. (1) if we take $y(t)$ for $a(t)$ and:

$$f(t) = \frac{4\pi G}{3}\left(\rho(t) + \frac{3p(t)}{c^2}\right). \tag{2}$$

This fact makes the acceleration equation central in our consideration. The reason is that the asymptotic analysis of solutions of Friedman equations reduces mainly to an analysis of solutions of the acceleration equation.

The notion of a regular variation is a form of a power law distribution. A weaker form of regular variation is described by the following power law relationship between quantities[2] F and t:

$$F(t) = t^r(\alpha + o(1)), \quad \alpha, r \in R. \tag{3}$$

Therefore, the simplest form of the power law is given by the equation $y = \alpha t^k$. Definition Eq. (3) of the power law can naturally be extended by use of the notion of a slowly varying function introduced by J. Karamata.

A real positive continuous function $L(t)$ defined for $x > x_0$ which satisfies:

$$\frac{L(\lambda t)}{L(t)} \to 1 \quad as \quad t \to \infty, \quad \lambda > 0, \tag{4}$$

is called a slowly varying function. A quantity F(t) is said to satisfy the generalized power law if:

$$F(t) = t^r L(t)\,, \tag{5}$$

where $L(t)$ is a slowly varying function and r is a real constant, so called the index of function $F(t)$. Hence, $F(t)$ is a regularly varying function if and only if $F(t)$ satisfies the generalized power law. Examples of slowly varying functions include $\ln(x)$ and iterated logarithmic functions $\ln(\ldots\ln(x)\ldots)$. In the rest of the paper the regularly varying functions will be abbreviated by RV functions, while the term "slowly varying" will be denoted by SV.

The following representation, according to Jovan Karamata, of RV and SV functions is of the great importance. It says that a function L is SV if and only if there are measurable[3] functions $h(x)$, a function $\varepsilon(t)$ and $b \in R$ so that:

$$L(x) = h(x)e^{\int_b^x \frac{\varepsilon(t)}{t}dt}, \quad x \geq b, \tag{6}$$

[2]In the following, R will denote the set of real numbers.

[3]For our purpose it is safe to assume $h(x)$ be continuous.

and $h(x) \to h_0$ as $x \to \infty$, h_0 is a positive constant. For further properties of RV functions, one may see Bigham et al. (1987).

Solutions L of differential equations that we are working with represent mechanical phenomena. Hence, as already mentioned, we may assume that L is a twice differentiable function. The function $\varepsilon(t)$ is not uniquely determined and due to the representation Eq. (6) where it is "covered" by the integral sign, we shall often call it a hidden parameter. Further, it is assumed that $L(t)$ is normalized, i.e. that $h(x)$ is a constant function. The class of normalized SV functions is denoted by $\mathcal{N}$. In Mijajlović et al. (2012) it was proved in a sequence of representation theorems that the fundamental cosmological parameters for the expanding universe depend essentially on the hidden parameter $\varepsilon(t)$. As the Friedman equations are invariant under translation transformation, this is also true for the expanding universe with the cosmological parameter Λ.

In our study of Friedman equations we used several results on RV solutions of the Eq. (1). There are various conditions for $f(t)$ that ensure that RV solutions of $\ddot{y} + f(t)y = 0$ to exist. We particularly used the following result, according to Howard and Marić (see Marić 2000) and (Kusano and Marić 2010):

Theorem *Let $-\infty < \Gamma < 1/4$, and let $\alpha_1 < \alpha_2$ be two roots of the equation:*

$$x^2 - x + \Gamma = 0. \tag{7}$$

Further, let L_i, i=1,2 denote two normalized SV functions. Then there are two linearly independent RV solutions of $\ddot{y} + f(t)y = 0$ of the form:

$$y_i(t) = t^{\alpha_i} L_i(t), \quad i = 1, 2, \tag{8}$$

if and only if $\lim\limits_{x\to\infty} x \int_x^\infty f(t)dt = \Gamma$. Moreover:

$$L_2(t) \sim \frac{1}{(1-2\alpha_1)L_1(t)}.$$

The limit of the integral in the above theorem is central in our analysis and, in general, it is not easy to compute. However, it is easy to see that $\lim\limits_{t\to\infty} t^2 f(t) = \Gamma$ implies $\lim\limits_{x\to\infty} x \int_x^\infty f(t)dt = \Gamma$. Hence:

$$\lim_{t\to\infty} t^2 f(t) = \Gamma \tag{9}$$

gives a useful sufficient condition for the existence of RV solutions of the equation $\ddot{y} + f(t)y = 0$ as described in the previous theorem.

2. RV COSMOLOGICAL PARAMETERS

We already noted that the acceleration equation obviously has the form Eq. (1). Hence, under appropriate assumptions, we can apply the analysis of the previous section, in particular the Howard-Marić theorem. For this reason, we shall write from now on the acceleration equation in the form:

$$\ddot{a} + \frac{\mu(t)}{t^2} a = 0, \tag{10}$$

where:

$$\mu(t) = \frac{4\pi G}{3} t^2 \left(\rho(t) + \frac{3p(t)}{c^2} \right). \tag{11}$$

Then, the integral limit in the Howard-Marić theorem for the Eq. (10) looks like:

$$M(\mu) = \lim_{x\to\infty} x \int_x^\infty \frac{\mu(t)}{t^2} dt. \tag{12}$$

Functions for which this integral limit converges define the so called Marić class of functions $\mathcal{M}$. It is easy to see that M is a functional defined on $\mathcal{M}$. Also, according to the discussion following this theorem, we have:

$$\text{if} \quad \lim_{t\to\infty} \mu(t) = \Gamma \quad \text{then} \quad M(\mu) = \Gamma. \tag{13}$$

We note that the opposite of Eq. (13) does not hold. It is easy to find a function $\mu(t)$ such that $M(\mu)$ exists and is finite, but $\lim_{t\to\infty} \mu(t)$ does not exist[4].

In Mijajlović et al. (2012) Theorems 3.2 and 3.3, RV solutions of Friedman equations are found and in accordance with that, the cosmological parameters for Friedman non-oscillatory universe are determined. In fact, assuming that the integral limit $M(\mu)$ is convergent, say $M(\mu) = \Gamma$, the following is proven:

- if $\Gamma < 1/4$ then the universe is non-oscillatory.
- The converse is almost true, namely, if the universe is non-oscillatory then $\Gamma \leq 1/4$.
- If $\Gamma < 1/4$ and in some special cases for $\Gamma = 1/4$, the acceleration parameter $a(t)$, a solution of Friedman equations, is an RV function.

Assume that α is a root of the polynomial $x^2 - x + \Gamma$. Therefore:

$$\Gamma = \alpha(1 - \alpha). \tag{14}$$

Then the cosmological parameters are represented as follows:

scale factor $a(t)$: $a(t) = t^\alpha L(t)$, $\alpha \neq 0$ and L is an SV function. In other words, $a(t)$ is a regularly varying function of index α.

[4]Example from Mijajlović et al. (2012): $\mu(t) = \frac{1}{8} - 3t^2 \cos(t^3)$.

Hubble parameter $H(t)$:

$$H(t) = \frac{\alpha}{t} + \frac{\varepsilon}{t}. \tag{15}$$

Deceleration parameter $q(t)$:

$$q(t) = \frac{\mu(t)}{\alpha^2}(1+\eta) = \frac{1-\alpha}{\alpha} - \frac{t\dot{\varepsilon}}{\alpha^2}(1+\eta) + \tau. \tag{16}$$

Assuming the scale factor $a(t)$ satisfies the generalized power law we can introduce a new and useful constant w. It will appear that w is in fact the equation of state parameter. So, assume $a(t) = t^{\alpha}L(t)$, $L \in \mathcal{N}$ and $\alpha \neq 0$. We define w by:

$$w \equiv w_{\alpha} = \frac{2}{3\alpha} - 1. \tag{17}$$

Then the cosmological parameters can be put in a more standard form:

$$\begin{aligned} \alpha &= \frac{2}{3(1+w)}, & a(t) &= a_0 L(t) t^{\frac{2}{3(1+w)}} \\ H(t) &\sim \frac{2}{3(1+w)t}, & M(q) &= \frac{1+3w}{2} \end{aligned} \quad . \tag{18}$$

Formulas for the exponent α and the Hubble parameter $H(t)$ are widely found in the literature. Formulas for $a(t)$ and $q(t)$ are also reduced to the standard formulas found in the literature if $L(t)$ and $q(t)$ are constant at infinity, or if the equation of state $p = w\rho c^2$ is assumed, or $\lim_{t\to\infty} t\dot{\varepsilon}(t) = 0$. We note that we did not assume any of these assumptions in derivation of Eq. (18). In fact, we found asymptotics for solutions of Friedman equations only by assuming $M(\mu) = \Gamma < 1/4$, and in certain cases for $\Gamma = 1/4$. As far as we know, it is widely assumed (implicitly) that the limit $\lim_{t\to\infty}\mu(t)$ exists and is finite, what is a much stronger assumption than that the integral limit $M(\mu)$ is convergent.

For the universe having flat curvature, one can infer the following weak form of the equation of state which gives a relation between pressure and density parameters.

There are functions ξ *and* ζ *such that* $p = \hat{w}\rho c^2$, *where* $\hat{w}(t) = w - t\dot{\xi} + \zeta$.

Therefore, if $t\dot{\xi} \to 0$ as $t \to \infty$, then $\hat{w}(t) \approx w$, which leads to $p = w\rho c^2$ the standard equation of state and classical asymptotics for cosmological parameters. In Mijajlović et al. (2012), Proposition 3.4, we found:

$$M(\mu) = \Gamma = \frac{2}{9}\cdot\frac{1+3w}{(1+w)^2}. \tag{19}$$

This allows us to give short derivations of asymptotics for cosmological parameters if the equation of state is assumed. We illustrate the method for the Hubble parameter $H(t)$. The acceleration equation is reduced to:

$$\ddot{a} + \frac{\Gamma}{t^2}a = 0, \tag{20}$$

and also we have the well known equation for the Hubble parameter:

$$\dot{H} + H^2 + \frac{\mu}{t^2} = 0. \tag{21}$$

By substitution $y(t) = H(-t)$ this equation is reduced to the Riccati equation:

$$\dot{y} = y^2 + \frac{\Gamma}{t^2}. \tag{22}$$

This equation has the general solution (see Polyanin and Zaitsev 2003 case 1.2.2-13)

$$y = \frac{\lambda}{t} - t^{2\lambda}\left(\frac{t^{2\lambda+1}}{2\lambda+1} + C\right)^{-1} \sim -\frac{\lambda+1}{t} \tag{23}$$

where λ is a root of $x^2 + x + \Gamma = 0$ and $t \to \infty$. Therefore:

$$H(t) = y(-t) \sim \frac{\lambda+1}{t} = \frac{\alpha}{t}, \tag{24}$$

where α is a root of $x^2 - x + \Gamma = 0$. Solving this quadratic equation in α for Γ given by Eq. (19) (or by Eqs. (17) and (19)) we immediately infer the formula for $H(t)$ in Eq. (18). In a similar manner, we may infer the asymptotics for other cosmological parameters. However, if the equation of state is not assumed, then for the solving of the Eq. (10) in general case for the non-oscillatory universe, i.e. with the condition $M(\mu)$ be convergent (and $M(\mu) < 1/4$), we need a much more subtle technique such as the theory of regular variations and Tauberian theory (see Korevar 2004), as shown in Mijajlović et al. (2012).

At this point we make a small historical digression. Professor John Barrow in a letter from October 2013 addressed our attention to his paper (Barrow 1998) where he established possible asymptotes for polynomial pressure and density behaviors. To find these asymptotics, Barrow applied some general theorems of Hardy and Fowler, which give the asymptotic behavior of all ultimately monotonic solutions of the first- and second-order polynomial differential equations, to the Einstein equations describing expanding universes. This theorem says that any rational function $H(x, y, y')$ is ultimately monotonic along a solution $y(x)$ of an algebraic differential equation of the form $f(x, y, y') \equiv \sum Ax^m y^n y'^p = 0$. It is interesting that the asymptotics described by the Hardy theorem was also used by Chandrasekhar in the study of gravitational equilibrium of a gaseous configuration in stars in which the pressure and the density are related by a form of the equation of state (see Chandrasekhar 1938). The Hardy theorem comes from his theory *Orders of infinity* which preceded and influenced the Karamata theory of regular variation.

We see that the deceleration parameter $q(t)$, equation of state, and pressure $p(t)$ contain not only the "hidden" parameter $\varepsilon(t)$, but $\dot{\varepsilon}(t)$ and $t\dot{\varepsilon}(t)$ as well. While $\varepsilon(t) \to 0$ as $t \to \infty$, functions $\dot{\varepsilon}(t)$ and

$t\dot{\varepsilon}(t)$ may not have this property. In fact, they can be unbounded and oscillatory as well. This means that $q(t)$ and $p(t)$ can be unbounded and oscillatory as well. It seems that this fact is overlooked in classical cosmology, mainly due to the absence of "microscopic" analysis which gives us the theory of regularly varying functions. Therefore, it is of an interest to describe the hidden parameter ε in more details.

3. DIFFERENTIAL EQUATION FOR ε

Let $a(t)$ be a solution of the acceleration Eq. (10). Suppose that the integral limit $M(\mu)$ is convergent and that $M(\mu) = \Gamma < 1/4$. According to the previous section, these assumptions ensure that the acceleration parameter $a(t)$ is a normalized RV function. Finally, assume that ε is a "hidden" parameter in the representation Eq. (6) of $a(t)$. Under these conditions we shall prove that the function $\varepsilon(t)$ is a solution of the following general Riccati differential equation:

$$t\dot{\varepsilon} = (1-2\alpha)\varepsilon - \varepsilon^2 + \alpha(1-\alpha) - \mu(t), \qquad (25)$$

where α is a root of the polynomial $x^2 - x + \Gamma$. To prove this, first note that $a(t) = t^\alpha L(t)$ and

$$L(x) = c_0 e^{\int_b^x \frac{\varepsilon(t)}{t} dt}, \quad x \geq b,$$

for some constants c_0 and $b > 0$. Then we have $\dot{L} = \frac{\varepsilon}{t} L$, where from we find:

$$\ddot{L} = (\varepsilon^2 + \dot{\varepsilon}t - \varepsilon) L t^{-2}. \qquad (26)$$

After a short derivation directly from $a(t) = t^\alpha L(t)$ we find:

$$\ddot{a} = \ddot{L} t^\alpha + 2\alpha \dot{L} t^{\alpha-1} + \alpha(\alpha-1) L t^{\alpha-2}. \qquad (27)$$

Substituting $\dot{L}$ and $\ddot{L}$ in this identity, we find

$$\ddot{a} = (\varepsilon^2 + \dot{\varepsilon}t - \varepsilon + 2\alpha\varepsilon + \alpha(\alpha-1)) L t^{\alpha-2}. \qquad (28)$$

Substituting so obtained $\ddot{a}$ and a in the acceleration Eq. (10) and canceling appropriate terms we obtain the differential equation Eq. (25).

We note that by Eq. (14) there is another form of the Eq. (25):

$$t\dot{\varepsilon} = (1-2\alpha)\varepsilon - \varepsilon^2 + \Gamma - \mu(t). \qquad (29)$$

We also remark the striking similarity of the Eq. (25) to Emden's differential equation, well known to play a fundamental role in the study of the internal structure of the stars, see for example Hopf (1931).

We continue now to discuss properties of solutions of the Eq. (25). Functions having the limit 0 at infinity are mapped by the functional M into 0. Hence $M(\varepsilon) = 0$ and $M(\varepsilon^2) = 0$. Also $M(\mu) = \Gamma$ and $M(\Gamma) = \Gamma$. We have now to distinguish between two cases:

a. There exists $\lim_{t\to\infty} \mu(t)$.
b. $\lim_{t\to\infty} \mu(t)$ does not exist but, of course, the integral limit $M(\mu)$ does exist.

In the former case, $t\dot{\varepsilon}(t)$ tends to 0 as $t \to \infty$. Hence, applying functional M on the Eq. (29), we obtain:

$$M(t\dot{\varepsilon}) = 0. \qquad (30)$$

Actually, this formula is valid for any continuously differentiable function which tends to zero as $t \to \infty$ and this can be proved by partial integration.

By remark Eq. (13), which in fact holds for arbitrary continuous functions, we infer the following result:

$$\textit{if } \lim_{t\to\infty} t\dot{\varepsilon}(t) \textit{ exists, then } \lim_{t\to\infty} t\dot{\varepsilon}(t) = 0. \qquad (31)$$

In particular, if $\lim_{t\to\infty} \mu(t) = \Gamma = \alpha(1-\alpha)$, then $t\dot{\varepsilon}$ has the limit 0 at infinity, hence $\dot{\varepsilon}$ and $t\dot{\varepsilon}$ can be neglected in representation of cosmological parameters. In this case, let us call it the tame case, $q(t)$ and the state equation reduce to their standard form in classical cosmology:

$$q(t) = \frac{1+3w}{2}, \quad p(t) = w\rho(t)c^2.$$

The later, non-tame and particularly interesting case is when $\lim_{t\to\infty} \mu(t)$ does not exist but anyhow $M(\mu) = \Gamma = \alpha(1-\alpha)$. Then the "hidden" parameter ε, in fact its derivative $\dot{\varepsilon}$, might have a strong influence in asymptotical behavior of parameters $q(t)$, $p(t)$ and the equation of state. In this case one can show, for example, that the function $\xi(t) = t\dot{\varepsilon}(t)$ oscillates infinitely many times, i.e. it intersects the time-axes infinitely many times. This property of ξ would induce variations of deceleration parameter $q(t)$ around the value:

$$\frac{1-\alpha}{\alpha} = \frac{1+3w}{2} \qquad (32)$$

see Eqs. (16) and (18), and also of the energy pressure $p(t)$, see the weak form of the equation of state.

If $\zeta = (1-2\alpha)\varepsilon - \varepsilon^2$, then Eq. (29) may be written as:

$$t\dot{\varepsilon} = \zeta + \Gamma - \mu(t)\,. \qquad (33)$$

The function $\zeta(t)$ has the limit 0 at infinity, wherefrom we have the following relation:

$$t\dot{\varepsilon} \sim \Gamma - \mu(t)\,, \qquad (34)$$

which is particularly interesting when $\lim_{t\to\infty} t\dot{\varepsilon}(t)$ does not exist. From the relation Eq. (34) we see once again that if we assume $t \to \infty$ then $t\dot{\varepsilon}(t) \to 0$ if and only if $\mu(t) \to \Gamma$.

We also observe that under the substitution $\eta = (\varepsilon + \alpha)/t$ the Eq. (25) is reduced to:

$$\dot{\eta} + \eta^2 + \frac{\mu}{t^2} = 0. \qquad (35)$$

Note that this the same differential equation Eq. (21) whose solution is the Hubble parameter $H(t)$. Therefore, $H(t) \sim \eta(t)$, pursuant to the asymptotics of $H(t)$ we found in Eq. (24).

4. DISCUSSION

We shall discuss briefly some possible physical explanations or models assuming the non-tame case. The explanation is proposed taking into account properties of the "hidden" parameter $\varepsilon(t)$.

1. Dark matter and dark energy are in the equilibrium but fluctuations of this state produce variation of $\hat{w}(t)$, $q(t)$ and $p(t)$.

2. Variations of $q(t)$ and $p(t)$ are consequences of the extremely rapid expansion of the Universe which appeared in the inflationary epoch (about 10^{-36} seconds after the Big Bang). We may think of these variations as an echo effect due to thermalization which appeared when the inflation epoch ended (about 10^{-32} seconds after the Big Bang).

3. Variations of $q(t)$ and $p(t)$ are consequences of the existence and influence of the dual universe. Even though it may sound as a mathematical fiction, we can easily and explicitly find the dual set of "cosmological parameters" starting from the second fundamental solution $L_2(t)$ in the Howard - Marić theorem applied to the acceleration equation. To find the dual set of cosmological parameters we take the second root $\beta = 1 - \alpha$ of the quadratic equation $x^2 - x + \Gamma = 0$ appearing in this theorem. Now we use β instead of α for the index of the RV solution $a(t)$ - deceleration parameter and for determination of other constants and cosmological parameters. For example, we derived in [Mijajlović at al., 2012] the following symmetric identity for equation of state parameters:

$$w_\alpha + w_\beta + 3w_\alpha w_\beta = 1 \tag{36}$$

For our universe we have $w = w_\alpha$, while for the dual universe the corresponding equation of state parameter is w_β. In a similar manner one can deduce formulas for other parameters. If one wants to give any physical meaning to the so obtained dual set of functions, it is rather natural to interpret them as cosmological parameters of the dual universe.

We note that any of the proposed explanations does not exclude the validness of the other two. If these models, or some of them are valid, then the variations of cosmological parameters in the non-tame case may be seen also as the resultant of their mutual interference.

5. CONCLUSION

This paper is a continuation of the asymptotical analysis of solutions of Friedman equations that we started in our paper (Mijajlović et al. 2012), using the theory of regularly varying functions. Here we discussed the possible importance of properties of the ε parameter which appears in the representation of regularly varying functions. Clearly two possibilities are distinguished: the tame case described by $\lim_{t\to\infty} t\dot{\varepsilon}(t) = 0$, and the opposite, when this limit does not exist. We also found the differential equation that $\varepsilon(t)$ must satisfy and we discussed properties of solutions to this equation.

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О КОСМОЛОШКОМ ПАРАМЕТРУ ε

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Оригинални научни рад

У раду *On asymptotic solutions of Friedmann equations* (Mijajlović et al. 2012), применили смо теорију регуларно променљивих функција у асимптотској анализи решења Фридманових једначина. Као што је добро познато, решења ових једначина репрезентују космолошке параметре. Дакле, према теорији регуларно променљивих функција, сви космолошки параметри зависе од функције $\varepsilon(t)$ која се појављује у њиховој интегралној репрезентацији и за коју важи $\lim_{t\to\infty}\varepsilon(t)=0$. У овом раду извели смо диференцијалну једначину за $\varepsilon(t)$, дискутовали решења те једначине и дали неке физичке интерпретације.

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Authors of this paper recently applied the theory of regularly varying functions in asymptotic analysis of cosmological parameters for the expanding universe. According to this theory, all cosmological parameters depend on a 0-function $\varepsilon(t)$ which is "hidden" in the integral representation of regularly varying functions. We derived a differential equation for $\varepsilon(t)$ and discussed possible solutions.

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ON A HIDDEN PARAMETER IN FRIEDMAN EQUATIONS

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Abstract. Authors of this paper recently applied the theory of regularly varying functions in asymptotic analysis of cosmological parameters for the expanding universe. According to this theory, all cosmological parameters depend on a a 0-function $\varepsilon(t)$ which is "hidden" in the integral representation of regularly varying functions. We derived a differential equation for $\varepsilon(t)$ and discussed possible solutions.

1. INTRODUCTION

In our paper [Mijajlović et al., 2012], we applied the theory of regularly varying functions in asymptotic analysis of cosmological parameters for the expanding universe. For this analysis we used Friedman equations [see Friedman, 1924], which are derived from the Einstein field equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}, \qquad \text{Friedman equation,}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right), \qquad \text{Acceleration equation,}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0, \qquad \text{Fluid equation.}$$

Cosmological parameters appearing in these equations are:

$a = a(t)$, the scale factor,
$\rho = \rho(t)$, the energy density, and
$p = p(t)$, the pressure of the material in the universe.

It appears that these parameters, including Hubble parameter, $H(t)$, and deceleration parameter, $q(t)$, are regularly varying functions. According to the representation theory for regularly varying functions, all these parameters depend on a 0-function $\varepsilon(t)$ which is "hidden" in the integral representation of regularly varying functions. Parameters $a(t)$, $\rho(t)$ and $H(t)$ uniformly depend only on $\varepsilon(t)$, but the parameters $p(t)$ and $q(t)$ depend on $\dot{\varepsilon}$ as well. While $\varepsilon(t) \to 0$ as $t \to \infty$, this is not necessary for

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$\dot{\varepsilon}$, what may lead to various evolutions of these parameters. We derive a differential equation for $\varepsilon(t)$ and discuss possible solutions.

2. REGULAR VARIATION

We shall review briefly basic notions related to regular variation. We shall need particularly properties of regularly varying solutions of the following second order differential equation

$$\ddot{y} + f(t)y = 0, \quad f(t) \text{ is continuous on } [\alpha, \infty]. \tag{1}$$

Observe that the acceleration equation has the form (1). In short, the notion of a regular variation is a form of the power law distribution, described by the following relationship between quantities F and t:

$$F(t) = t^r(\alpha + o(1)), \quad \alpha, r \in \mathbb{R}. \tag{2}$$

Obviously, the most simple form of the power law is given by the equation $y = t^k$. It is said that two quantities y and t^r satisfy the power law if they are related by a proportion, i.e. there is a constant α so that $y = \alpha t^r$. This definition of power law can be naturally extended by use of the notion of slowly varying function introduced by J. Karamata, (1930).

A real positive continuous function $L(t)$ defined for $x > x_0$ which satisfies

$$\frac{L(\lambda t)}{L(t)} \to 1 \quad \text{as} \quad t \to \infty, \quad \text{for each real } \lambda > 0. \tag{3}$$

is called a slowly varying function. A physical quantity F(t) is said to satisfy the generalized power law if

$$F(t) = t^r L(t) \tag{4}$$

where $L(t)$ is a slowly varying function and r is a real constant. So to say that $F(t)$ is regularly varying is the same as $F(t)$ to satisfy the generalized power law. The simplest example of slowly varying functions are $\ln(x)$ and iterated logarithmic functions $\ln(\ldots \ln(x) \ldots)$.

Regularly varying function have the following representation. Namely, a function L is slowly varying if and only if there are measurable functions $h(x)$ and a zero function ε and $b \in \mathbb{R}$ so that

$$L(x) = h(x)e^{\int_b^x \frac{\varepsilon(t)}{t} dt}, \quad x \geq b, \tag{5}$$

and $h(x) \to h_0$ as $x \to \infty$, h_0 is a positive constant. For further properties of regularly varying functions, one may consult Bingham et al., 1987.

As we are dealing with solutions L of differential equations which model mechanical phenomena, it is quite safe to assume that L is a twice differentiable function. The function $\varepsilon(t)$ is not uniquely determined and in our context we shall call it a hidden parameter. We shall also assume that $L(t)$ is normalized, i.e. that $h(x)$ is a constant function. The class of normalized slowly varying functions will be denoted by $\mathcal{N}$. We shall see that the fundamental cosmological parameters depend essentially on the hidden parameter $\varepsilon(t)$.

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For our study of Friedman equations we need several results on solutions of equation (1). There are various conditions for $f(t)$ that ensure that regularly varying solutions of $\ddot{y} + f(t)y = 0$ exist. We shall particularly use the following result, due to Howard and Marić, see Marić, 2000 and Kusano-Marić, 2010:

Theorem *Let* $-\infty < \Gamma < 1/4$, *and let* $\alpha_1 < \alpha_2$ *be two roots of the equation*

$$x^2 - x + \Gamma = 0. \tag{6}$$

Further let L_i, *i=1,2 denote two normalized slowly varying functions. Then there are two linearly independent regularly varying solutions of* $\ddot{y} + f(t)y = 0$ *of the form*

$$y_i(t) = t^{\alpha_i} L_i(t), \quad i = 1, 2, \tag{7}$$

if and only if $\mathbf{M}(f) = \lim_{x\to\infty} x \int_x^\infty f(t)dt = \Gamma$. *Moreover,* $L_2(t) \sim \dfrac{1}{(1-2\alpha_1)L_1(t)}$.

The limit of the integral in the theorem is crucial in our analysis and it is not always easy to compute.

As $\lim_{t\to\infty} t^2 f(t) = \Gamma$ implies $\lim_{x\to\infty} x \int_x^\infty f(t)dt = \Gamma$, we see that

$$\lim_{t\to\infty} t^2 f(t) = \Gamma \tag{8}$$

gives a useful sufficient condition for the existence of regularly varying solutions of the equation $\ddot{y} + f(t)y = 0$ as described in the previous theorem. By r.v. we shall denote the term "regularly varying".

3. R. V. SOLUTIONS OF FRIEDMAN EQUATIONS

As noted, the acceleration equation obviously has the form (1) so under appropriate assumptions, i.e. that the functions we encounter are continuously differentiable as many times as necessary, the analysis of the previous section, in particular the Howard-Marić theorem, can be applied to it. For this reason, we shall write from now on the acceleration equation in the form

$$\ddot{a} + \frac{\mu(t)}{t^2} a = 0, \tag{9}$$

where

$$\mu(t) = \frac{4\pi G}{3} t^2 \left(\rho + \frac{3p}{c^2}\right). \tag{10}$$

We found in [Mijajlovic et al. 2012] r.v. solutions of Friedman equations and determined cosmological parameters:

Scale factor $a(t)$: $a(t) = t^\alpha L(t)$, where $\alpha \neq 0$ with L having the ε-representation as in Karamata Representation theorem.

Hubble parameter $H(t)$:

$$H(t) = \frac{\alpha}{t} + \frac{\varepsilon}{t}. \tag{11}$$

Deceleration parameter $q(t)$:

$$q(t) = \frac{\mu(t)}{\alpha^2}(1+\eta) = \frac{1-\alpha}{\alpha} - \frac{t\dot{\varepsilon}}{\alpha^2}(1+\eta) + \tau, \tag{12}$$

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where η and τ are zero functions.

Now we introduce a new constant w related to the scale factor $a(t)$ which satisfy the generalized power law. It appears that w is in fact the equation of state parameter. So assume $a(t) = t^{\alpha}L(t)$, $L \in \mathcal{N}$ and $\alpha \neq 0$. We define w by

$$w \equiv w_{\alpha} = \frac{2}{3\alpha} - 1 \quad \text{(equation of state parameter)}. \tag{13}$$

Then cosmological parameters can be put in a more standard form, widely found in the cosmology literature.

$$\begin{aligned} \alpha &= \frac{2}{3(1+w)}, & a(t) &= a_0 L(t) t^{\frac{2}{3(1+w)}} \\ H(t) &\sim \frac{2}{3(1+w)t}, & \mathbf{M}(q) &= \frac{1+3w}{2} \end{aligned} \tag{14}$$

By the representation theorem for regularly varying functions we see that the scale factor $a(t)$ and the deceleration parameter $q(t)$ depend on a hidden parameter ε and its derivative $\dot{\varepsilon}$. For the universe having the flat curvature one can infer the following relation between pressure and density parameters:

There are 0*-functions* ξ, ζ *such that* $p = \hat{w}\rho c^2$*, where* $\hat{w}(t) = w - t\dot{\xi} + \zeta$.

We see that the deceleration parameter $q(t)$, the equation of state and the pressure $p(t)$ contain not only the "hidden" parameter $\varepsilon(t)$, but $\dot{\varepsilon}(t)$ and $t\dot{\varepsilon}(t)$ as well.

While $\varepsilon(t)$ is a 0-function, $\dot{\varepsilon}(t)$ and $t\dot{\varepsilon}(t)$ need not to be. In fact they can be unbounded and oscillatory as well. Examples of this kind are:

$$\varepsilon(t) = \cos(t^2)/t, \quad \varepsilon(t) = \sin(t^3)/t.$$

It means that $q(t)$ and $p(t)$ can be unbounded and oscillatory as well. It seems that this fact is overlooked in the classical cosmology, mainly due to the absence of "microscopic" analysis which give us the theory of regularly varying functions.

Therefore, it is of interest to describe in more detail the hidden parameter ε. We found that ε is a solution of the Riccati differential equation:

$$t\dot{\varepsilon}(t) = (1-2\alpha)\varepsilon - \varepsilon^2 + \alpha(1-\alpha) - \mu(t). \tag{15}$$

where $\mathbf{M}(\mu) = \alpha(1-\alpha)$ and ε is a 0-function. If $\lim_{t\to\infty}\mu(t) = \alpha(1-\alpha)$, then $t\dot{\varepsilon}$ is a 0-function, hence $\dot{\varepsilon}$ and $t\dot{\varepsilon}$ can be neglected in the representation of cosmological parameters. In this case $q(t)$ and equation of state reduce to their standard form in classical cosmology:

$$q(t) = \frac{1+3w}{2}, \quad p(t) = wc^2\rho(t)$$

where w is a constant (c is the the speed of light). Hence, the only interesting case would be when $\mathbf{M}(q) = \alpha(1-\alpha)$ and $\lim_{t\to\infty}\mu(t)$ does not exist. Then the "hidden" parameter ε, in fact its derivative $\dot{\varepsilon}$, might have the strong influence in the asymptotical behavior of the parameters $q(t)$ and $p(t)$.

We note that J. Barrow (see, for example, J. Barrow, D. Shaw, 2008) also discussed the asymptotic behavior of cosmological parameters based on the theory of Hardy fields. We note that this theory precedes Karamata's theory of regular variation.

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Several physical models can be proposed taking into account the "hidden" parameter $\varepsilon(t)$:

1. Dark matter and dark energy are in the equilibrium but small fluctuation in this state produce variation of $q(t)$ and $p(t)$.

2. Variations of $q(t)$ and $p(t)$ are the consequences of the echo effect which appeared in the inflationary epoch which lasted from 10^{-36} seconds after the Big Bang until 10^{-32} seconds (Alan Guth and Andrei Linde, 1981).

3. Variations of $q(t)$ and $p(t)$ are the consequences of the existence and the influence of the dual universe.

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45 Asymptotic solution for expanding universe with matter-dominated evolution

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The authors applied the theory of regularly varying (RV) functions to the analysis of the cosmological parameters for the ΛCDM model with the matter-dominated evolution. Carroll et al. proved in 1992 that for this type of universe with the curvature $k = 0, -1$, the expression $H(t)t$ (H(t) is the Hubble parameter) depends solely on the density parameter $\Omega(t)$. Using this result, the authors infer for such universe the complete asymptotics of all main cosmological parameters.

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Asymptotic solution for expanding universe with matter-dominated evolution

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We applied the theory of regularly varying (RV) functions to the analysis of the cosmological parameters for the ΛCDM model with the matter-dominated evolution. Carroll *et al.* proved in 1992 that for this type of universe with the curvature $k = 0, -1$, the expression $H(t)t$ ($H(t)$ is the Hubble parameter) depends solely on the density parameter $\Omega(t)$. Using this result, we infer for such universe the complete asymptotics of all main cosmological parameters. More specifically, the following is derived. If the limit $\omega = \lim_{t\to\infty} \Omega(t)$ does exist and $\omega \neq 0$, then the cosmological constant Λ is equal to 0. If $\omega = 0$, then for the expansion scale factor $a(t)$ of a flat universe, we have $a(t) \sim e^{\sqrt{\Lambda/3}t}$. On the other hand, if the limit $\lim_{t\to\infty} \Omega(t)$ does not exist, then $a(t)$ bounces between two power functions and therefore has infinitely many flexion points. Hence, the deceleration parameter in this case changes the sign infinitely many times.

Keywords: Cosmological parameters; ΛCDM.

Mathematics Subject Classification 2010: 83F05, 83C99, 26B99

1. Introduction

In our papers [1, 2], we applied the theory of regularly varying (RV) functions in the asymptotic analysis of cosmological parameters of the expanding universe. The main aim of the present paper is to apply this technique in the study of the ΛCDM model with a matter-dominated evolution. As a result, we obtained the complete asymptotics of solutions of Friedmann equations and the related cosmological parameters. Our secondary goal is to present further techniques of the theory of regular variation as a natural method in the theoretical studies in cosmology.

The paper is organized as follows. In the introduction, we present some basic facts on Friedmann equations, short history and elements of regular variation needed in the rest of the paper. Due to the variety of appearance in the literature,

in Sec. 2, notation and the meaning of cosmological parameters are fixed. Also, properties of the constant Γ introduced in [1] are reviewed. This constant and the related operator $\mathbf{M}$ will have to play crucial roles in our analysis of asymptotics of cosmological parameters. Section 3 is central in this paper. There we inferred complete asymptotics of cosmological parameters for a matter-dominated ΛCDM model. By complete inference, we mean that we resolved the asymptotics when the limit $\lim_{t\to\infty}\Omega(t)$ exists, but in the opposite case, too. This analysis is based on Carroll, Press and Turner formulas for $\Omega(t)$ [3] for this type of universe. In the last section, we demonstrated a use of the theory of regular variation in possible descriptions of cosmological parameters of the dual universe.

We remind that the cosmological parameters are solutions of Friedmann equations:

$$\begin{aligned}
&\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}, && \text{Friedmann equation,}\\
&\frac{\ddot a}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right), && \text{Acceleration equation,}\\
&\dot\rho + 3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right) = 0, && \text{Fluid equation.}
\end{aligned} \tag{1}$$

Functions appearing in these equations are the expansion scale factor $a = a(t)$, the energy density $\rho = \rho(t)$ and the pressure of the material in the universe $p = p(t)$. We shall show that all these parameters, including the Hubble parameter $H = H(t) = \dot a/a$ and deceleration parameter $q = q(t) = -a\ddot a/\dot a^2$, are RV functions under standard assumptions. We shall assume in our derivations that the scale factor a is normalized in respect to the present epoch. In other words, we take $a(t_0) = 1$ where t_0 is set to the present age of the universe $13.799 \pm 0.021\,\text{Gyr}$ (see [4]).

The system (1) involves three unknown functions, but only two of them are independent, for example the first and the third equation. However the acceleration equation is central in our study of asymptotics of solutions of Friedmann equations for several reasons. For instance it does not depend on the curvature index k. Further, the theory of RV solutions of such type of equations can be applied successfully in the analysis of Friedmann equations even if the cosmological constant Λ is included:

$$\begin{aligned}
\left(\frac{\dot a}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{kc^2}{R_0^2 a^2} + \frac{\Lambda}{3},\\
\frac{\ddot a}{a} &= -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}.
\end{aligned} \tag{2}$$

Namely, under the transformations

$$\rho' = \rho + \Lambda/(8\pi G), \quad p' = p - \Lambda/(8\pi G) \tag{3}$$

the Friedmann equations (1) are invariant, while the fluid equation is not affected by the parameter Λ.

Theory of RV functions is introduced mainly for studying behavior of real functions at infinity and also behavior of the functions satisfying the power law. This theory was started in the thirties of the last century by Karamata in [5]. Many other authors continued to develop it [6, 7]. At the present time, this theory is used in many areas of mathematics particularly in the asymptotic analysis of functions and probability theory. There were also some uses of this theory in cosmology, particularly in the study of asymptotic behavior of cosmological parameters, e.g. [1, 2], but also by [8, 9]. Papers [10, 11] had a similar approach in studies of asymptotic behavior of solutions to the Einstein equations describing expanding universes. They used there a theory of Hardy and Fowler which preceded the theory of regular variation. We give a short review of main notions related to the regular variation and some extensions of this theory that we shall need in the rest of the paper.

1.1. *Regularly varying functions*

A real positive continuous function $L(t)$ defined for $x > x_0$ and which satisfies for each $\lambda > 0$

$$\frac{L(\lambda t)}{L(t)} \to 1 \quad \text{as } t \to \infty, \tag{4}$$

is called a slowly varying (SV) function. Continuing works of Hardy *et al.* [5] originally defined and studied this notion for continuous functions. Later this theory was extended to measurable functions. Due to physical constraints, we assume here that all functions are continuously differentiable.

Definition 1. A function $F(t)$ is said to satisfy the generalized power law if

$$F(t) = t^r L(t), \tag{5}$$

where $L(t)$ is a SV function and r is a real constant.

Logarithmic function $\ln(x)$ and iterated logarithmic functions $\ln(\ldots \ln(x) \ldots)$ are examples of SV functions. More complicated examples are provided in [6, 7, 12].

A positive continuous function F defined for $t > t_0$, is a RV function of the index r, if and only if it satisfies

$$\frac{F(\lambda t)}{F(t)} \to \lambda^r \quad \text{as } t \to \infty, \quad \text{for each } \lambda > 0. \tag{6}$$

It immediately follows that a RV function $F(t)$ has the form of (5). Therefore $F(t)$ is RV if and only if it satisfies the generalized power law. By $\mathcal{R}_\alpha$ we denote the class of RV functions of index α. Hence $\mathcal{R}_0$ is the class of all SV functions. By $\mathcal{Z}_0$ we shall denote the class of zero functions at ∞, i.e. $\varepsilon \in \mathcal{Z}_0$ if and only if $\lim_{t\to+\infty} \varepsilon(t) = 0$. The following representation theorem [5] describes the fundamental property of these functions.

Theorem 2 (Representation theorem). *$L \in \mathcal{R}_0$ is SV function if and only if there are measurable functions $h(x)$, $\varepsilon \in \mathcal{Z}_0$ and $b \in \mathbb{R}$ so that*

$$L(x) = h(x)e^{\int_b^x \frac{\varepsilon(t)}{t}dt}, \quad x \geq b, \tag{7}$$

and $h(x) \to h_0$ as $x \to \infty$, h_0 is a positive constant.

If $h(x)$, defined in representation theorem is a constant function, then $L(x)$ is called normalized. A RV function having normalized SV part will be also called normalized. Let $\mathcal{N}$ denote the class of normalized SV functions. The next fact on $\mathcal{N}$-functions will be useful for our later discussion. If $L \in \mathcal{N}$ and there is $\ddot{L}$, then ε in (7) has the first-order derivative $\dot{\varepsilon}$. This follows from the identity $\varepsilon(t) = t\dot{L}(t)/L(t)$.

There are various classes of positive measurable functions with similar asymptotic behavior to RV function such as the class ER — extended RV functions (or Matuszewska class of functions), OR — O-RV functions, or recently introduced interesting class $\mathcal{M}$ [13]. Here, we shall particularly use the classes ER and OR. Extensive literature on these functions is available, e.g. [14, 6, 15, 7]. We review here, following [6], their definitions and very basic notions and properties related to these functions. As in the case of regular variation, we assume that all mentioned functions are continuous.

The limit in (4) does not exist always for an arbitrary function, but the limit superior and the limit inferior do exist. So let

$$f^*(\lambda) = \limsup_{x\to\infty} \frac{f(\lambda x)}{f(x)}, \quad f_*(\lambda) = \liminf_{x\to\infty} \frac{f(\lambda x)}{f(x)}. \tag{8}$$

Note that the difference $f^*(\lambda) - f_*(\lambda)$ measures the oscillation of $f(\lambda x)/f(x)$ at infinity. It is said that a real positive measurable function f belongs to the ER class if and only if there are constants d and c such that

$$\lambda^d \leq f_*(\lambda) \leq f^*(\lambda) \leq \lambda^c, \quad \lambda \geq 1. \tag{9}$$

The class OR functions is the set of positive measurable f such that

$$0 < f_*(\lambda) \leq f^*(\lambda) < \infty, \quad \lambda \geq 1. \tag{10}$$

We see at once that RV $\subseteq$ ER $\subseteq$ OR. There are examples that show that both extensions are proper. Note that f is RV if and only if $f_*(\lambda) = f^*(\lambda)$, $\lambda \geq 1$. Therefore, RV functions are exactly OR (or ER) functions which do not oscillate at infinity.

Proposition 3. *The following statements are equivalent for a real function f:*

(1) *f belongs to the class ER;*
(2) *f has the representation: $f(x) = \exp(C + \eta(x) + \int_1^x \xi(t)dt/t)$, $x \geq 1$, with C constant, $\eta(x) \to 0$ as $x \to \infty$, ξ bounded and ξ, η both measurable.*

Similar representation theorem holds for OR functions: $f \in \mathrm{OR}$ if and only if

$$f(x) = \exp\left(\eta(x) + \int_1^x \xi(t)dt/t\right), \quad x \geq 1, \tag{11}$$

where $\eta(x)$ and $\xi(x)$ are measurable and bounded.

2. Cosmological Parameters and Cosmological Constants

In this section, we review some basic cosmological parameters and constants and their mostly well-known properties. Our only aim here is to establish their exact notation and meaning used in the rest of the paper.

A cosmological parameter basically is a real function $P = P(t)$ of the time variable t and it usually represents an essential physical value related to the standard cosmological model. In the literature there are variations of their notation and even definitions that may lead to ambiguities. Therefore, we fix here their notation, at least for some of the parameters, but we assume their standard meaning. Also, for the simplicity of the notation from now on for the speed of light we shall take $c = 1$.

$$\begin{aligned}
&\rho_c = \rho_c(t) = \frac{3H^2}{8\pi G} && \text{critical density,}\\
&\Omega = \Omega(t) = \rho/\rho_c && \text{density parameter,}\\
&\Omega_\Lambda = \Omega_\Lambda(t) = \frac{\Lambda}{3H^2} && \Lambda \text{ density parameter,}\\
&\Omega_k = \Omega_k(t) = -\frac{k}{a^2H^2} && \text{curvature density parameter,}\\
&q = q(t) = -\frac{\ddot{a}}{aH^2} && \text{deceleration parameter.}
\end{aligned} \tag{12}$$

Derived constants are obtained from cosmological parameters fixing their values at $t = t_0$. The time value t_0 usually stands for a certain moment in the present epoch. If P is a cosmological parameter, then we often write shortly P_0 instead of $P(t_0)$. Derived constants inherit their names from the parameters from which they were obtained, e.g. Hubble constant H_0. Here are some examples of derived constants.

$$\Omega_0 = \frac{8\pi G}{3H_0^2}\rho_0, \quad \Omega_{\Lambda 0} = \frac{\Lambda}{3H_0^2}, \quad \Omega_{k0} = -\frac{k}{a_0^2H_0^2}. \tag{13}$$

The following form of Friedmann equation is useful in determining relations linking basic cosmological parameters for pressureless universe [3]. Its usefulness follows from the explicit appearance of a term which relates the scale factor and the density for this type of universe. This theorem and its proof also illustrate the use of derived cosmological constants as the derived equation is in the parametrized form. For the simplicity of the computation we took the speed of light for unit, hence $c = 1$.

Theorem 4. *The first Friedmann equation with nonzero cosmological constant* Λ *is equivalent to the following equation*:

$$\dot{a}^2 + \frac{8\pi G}{3}(\rho_0/a^3 - \rho)a^2 = H_0^2(1 + \Omega_0(1/a - 1) + \Omega_{\Lambda 0}(a^2 - 1)). \tag{14}$$

Proof. If $L = \dot{a}^2 + \frac{8\pi G}{3}(\rho_0/a^3 - \rho)a^2$ then using (2) and definitions (13) of Ω_0 and $\Omega_{\Lambda 0}$, we have

$$\begin{aligned}
L &= a^2\left(\frac{8\pi G}{3}\cdot\frac{\rho_0}{a^3} + \frac{\dot{a}^2}{a^2} - \frac{8\pi G}{3}\rho\right)\\
&= a^2\left(\frac{8\pi G}{3}\cdot\frac{\rho_0}{a^3} - \frac{k}{R_0^2a^2} + \frac{\Lambda}{3}\right)\\
&= \frac{H_0^2}{a} + \frac{1}{a}\left(\frac{8\pi G}{3}\rho_0 - H_0^2\right) - \frac{k}{R_0^2} + \frac{\Lambda}{3}a^2\\
&= \frac{H_0^2}{a} + \frac{1}{a}\left(\frac{k}{R_0^2} - \frac{\Lambda}{3}\right) - \frac{k}{R_0^2} + \frac{\Lambda}{3}a^2\\
&= H_0^2 + \left(H_0^2 + \frac{k}{R_0^2} - \frac{\Lambda}{3}\right)\left(\frac{1}{a} - 1\right) + \frac{\Lambda}{3}a^2 - \frac{\Lambda}{3}\\
&= H_0^2 + \frac{8\pi G}{3}\rho_0(1/a - 1) + \frac{\Lambda}{3}(a^2 - 1)\\
&= H_0^2\left(1 + \frac{8\pi G}{3H_0^2}\rho_0(1/a - 1) + \frac{\Lambda}{3H_0^2}(a^2 - 1)\right)\\
&= H_0^2(1 + \Omega_0(1/a - 1) + \Omega_{\Lambda 0}(a^2 - 1)).
\end{aligned}$$

□

It is well known that a universe is pressureless (matter-dominated) if and only if $\rho = \rho_0/a^3$. Therefore, in this case the second term in L vanishes and we have the following statement.

Corollary (see [3]). *If matter-dominated universe is assumed, then the first Friedmann equation reduces to the following equation with the constant coefficients*:

$$\frac{\dot{a}^2}{H_0^2} = 1 + \Omega_0(1/a - 1) + \Omega_{\Lambda 0}(a^2 - 1). \tag{15}$$

If a flat universe is assumed, i.e. $\Omega_k = 0$, due to the formula $\Omega + \Omega_\Lambda + \Omega_k = 1$ the relation (15) obviously obtains more simple form:

$$\frac{\dot{a}^2}{H_0^2} = \Omega_0 a^{-1} + (1 - \Omega_0)a^2. \tag{16}$$

Note. By careful examination of the proof of Theorem 4, we see that no infinitesimal transformation is applied to Λ, i.e. only algebraic transformations were used in this proof. We have the same conclusion for formulas (15) and (16).

It is interesting that from (15) a rough estimation of $a(t)$ can be obtained in regards to the various epochs in the development of the universe. If the gravitation domination is assumed, i.e. the rate of the universe expansion is small, then the scale factor a is small. Therefore the term $1/a$ dominates on the right-hand side of Eq. (15) and so $\dot{a}^2 \propto a^{-1}$. From this relation one obtains at once $a(t) \propto t^{\frac{2}{3}}$, a well-known asymptotics for a matter-dominated universe. On the other side, if the dark energy prevails, then the rate of the universe expansion is large, i.e. now the term a^2 dominates the right-hand side of Eq. (15). Hence $\dot{a}^2/H_0^2 \sim \Omega_{\Lambda 0} a^2$, i.e. $(\dot{a}/a)^2 \sim \Lambda/3$, wherefrom we obtain $a(t) \sim \exp(\sqrt{\frac{\Lambda}{3}}t)$, a well known result for this epoch of the universe evolution.

2.1. *The constant* Γ

The constant Γ is an additional cosmological constant. We introduced it in [1] and it has an important role in our analysis in this paper. Namely, assuming the theory of RV solutions of linear second-order differential equations, Γ appears in the description of cosmological parameters, see [1, 2, 12]. Moreover, the asymptotical behavior of parameters $a(t)$, $\rho(t)$ and $p(t)$ strongly depends on the value of Γ. On the other hand, Γ is dimensionless, but physically speaking, it is a constant vacuum energy term. In this subsection, we briefly describe main properties of Γ.

The constant Γ is naturally related to the Friedmann equations, particularly to the acceleration equation which we consider as the second-order linear differential equation

$$\ddot{a} + \frac{\mu(t)}{t^2} a = 0. \tag{17}$$

Here,

$$\mu(t) = \frac{4\pi G}{3} t^2 \left(\bar{\rho}(t) + \frac{3\bar{p}(t)}{c^2} \right), \tag{18}$$

where $\bar{\rho}(t)$ and $\bar{p}(t)$ are some particular solutions of Friedmann equations. Then Γ is defined as the integral limit

$$\Gamma = \Gamma_\mu = \lim_{x \to \infty} x \int_x^\infty \frac{\mu(t)}{t^2} dt \equiv \mathbf{M}(\mu) \tag{19}$$

assuming the existence of the limit integral on the right-hand side of (19). The function $\mathbf{M}(\mu)$ is a linear functional $\mathbf{M} : \mathcal{M} \to R$, where $\mathcal{M}$ is the space of all real functions μ for which the limit integral in (19) exists. $\mathcal{M}$ is called a Marić class of functions (see [1]). If this integral limit does not exist, we say that Γ does not exist.

The limit integral in (19) in general is not easy to compute. However, as

$$\lim_{t \to \infty} \mu(t) = \Gamma \tag{20}$$

implies (19), we see that (20) gives a useful sufficient condition for the existence of regular solutions of Eq. (17).

The significance of the integral limit (19) is seen in the following facts. We have shown in [1] that for any function $\mu \in \mathcal{M}$ such that $\Gamma_\mu < 1/4$, the Friedmann equations has a normalized RV solutions and the universe modeled by these solutions must be spatially flat or open. On the other hand, if $\Gamma_\mu > 1/4$ then the universe is oscillatory. Therefore, Γ provides a kind of sharp threshold, or cut-off point, from which the oscillation of $a(t)$ takes place [16]. It was also proved that $\Gamma < 1/4$ implies a weak form of the equation of state. The converse also holds. Namely, we show the following proposition.

Proposition 5. *For any normalized RV solution of Friedman equations the integral limit* (19) *must exist and* $\Gamma \leq 1/4$.

Proof. So suppose $a(t) = t^\alpha L$ is a solution of Friedmann equations where L is a normalized slowly regular function and let $\beta = 1 - \alpha$. In accordance with the representation (7), Theorem 1, of SV functions and as L is normalized, we have $\dot{L} = \varepsilon L/t$, $\varepsilon \in \mathcal{Z}_0$. From there immediately follows the next derivation:

$$\dot{a} = \alpha t^{\alpha-1} L + t^\alpha \dot{L} = (\alpha + \varepsilon) t^{\alpha-1} L,$$

$$\ddot{a} = \dot{\varepsilon} t^{\alpha-1} L - (\alpha + \varepsilon)(\beta - \varepsilon) t^{\alpha-2} L,$$

wherefrom we obtain

$$\frac{\ddot{a}}{a} = (\dot{\varepsilon} t - (\alpha + \varepsilon)(\beta - \varepsilon))/t^2.$$

As $a(t)$ is a solution of Friedmann equations, in particular of Eq. (17), it must be $\mu = -\dot{\varepsilon} t + (\alpha + \varepsilon)(\beta - \varepsilon)$ and also

$$\begin{aligned} \mathbf{M}(\mu) &= \mathbf{M}(-\dot{\varepsilon} t + (\alpha + \varepsilon)(\beta - \varepsilon)) \\ &= \mathbf{M}(-\dot{\varepsilon} t) + \mathbf{M}((\alpha + \varepsilon)(\beta - \varepsilon)). \end{aligned}$$

Further, $\mathbf{M}(-\dot{\varepsilon} t) = 0$ (see [1, Proof of Theorem 2.2]). Since $(\alpha + \varepsilon)(\beta - \varepsilon) \to \alpha\beta$ as $t \to \infty$, it follows $\mathbf{M}((\alpha + \varepsilon)(\beta - \varepsilon)) = \alpha\beta$. Hence $\mathbf{M}(\mu) = \alpha\beta$ and so $\Gamma = \alpha\beta$, which proves the existence of Γ. Further, if α, $\beta \geq 0$ then $\sqrt{\alpha\beta} \leq (\alpha + \beta)/2 = 1/2$, hence $\Gamma \leq 1/4$. If $\alpha < 0$ then $\beta > 0$, hence $\Gamma < 0 \leq 1/4$. □

Observe that α and β are the roots of the equation $x^2 - x + \Gamma = 0$ which defines the fundamental solutions of the acceleration equation, see [12, 1].

3. Density Parameter $\Omega(t)$ and Regular Variation

As already noted, the two Friedmann equations are not enough to fully solve for the energy density, the pressure and the scale factor. Hence, we need an additional relation which connects these parameters. In the literature almost without exceptions the equation of state $p = f(\rho)$ is supposed, usually in the form $p = w\rho$, or $p_i = w_i \rho_i$, if the universe composed of several components i is assumed (see

p. 6 in "The accelerating universe"[a]). However, if the pressureless ΛCDM model is assumed, the regular variation property immanent to the solutions of the system (1) and to other cosmological parameters as well, enables us to use another approach. Namely, we can give without additional assumptions a complete asymptotics for this type of universe using the theory of RV functions. This section, including Secs. 3.1 and 3.2 are devoted to this analysis. Hence, from now on we shall assume a pressureless universe with nonzero cosmological constant Λ. For this type of universe Carroll *et al.* developed in [3] formulas of the form

$$H(t)t = F(\Omega), \quad \Omega = \Omega(t) \text{ is a density parameter.} \tag{21}$$

Using Carroll, Press and Turner formulas, we show that the main cosmological parameters $a(t)$ and $\mu(t)$ for a pressureless universe with $\Lambda \neq 0$ depend solely on the density parameter $\Omega(t)$, see (28) and (29). This enables us to consider asymptotics for cosmological parameters in respect to Ω instead of time t and to infer asymptotics regardless of the convergence of the limit integral (19). Section 3.1 is devoted to finding dependencies between $\mu(t)$, $a(t)$ and $\Omega(t)$.

3.1. *Dependencies between $\mu(t)$, $a(t)$ and $\Omega(t)$*

The functions $F(\Omega)$ introduced in [3] are defined as follows:

$F(\Omega)$ for flat universe ($k = 0$):

$$F(\Omega) = \frac{2}{3}(1-\Omega)^{-\frac{1}{2}} \ln\left(\frac{1+\sqrt{1-\Omega}}{\sqrt{\Omega}}\right), \tag{22}$$

$F(\Omega)$ for open universe ($k = -1$):

$$F(\Omega) = \frac{1}{1-\Omega} - \frac{\Omega}{2(1-\Omega)^{\frac{3}{2}}} \cosh^{-1}\left(\frac{2-\Omega}{\Omega}\right). \tag{23}$$

Observe that $F(\Omega)$ is a dimensionless physical quantity.

We note that the functions $F(\Omega)$ are easily inferred from (15). The acceleration equation with Λ is written as follows:

$$\frac{\ddot{a}}{a} = -\frac{\mu}{t^2} + \frac{\Lambda}{3}. \tag{24}$$

Applying substitutions (3) in identity (18), the acceleration equation is reduced to the standard form (1) where μ is replaced by

$$\mu_\Lambda = (3\mu - \Lambda t^2)/3 = 4\pi G t^2(\rho^* + 3p^*)/3. \tag{25}$$

In other words

$$\frac{\ddot{a}}{a} = -\frac{\mu_\Lambda}{t^2} \quad \text{and} \quad q = \frac{\mu_\Lambda}{F(\Omega)^2}. \tag{26}$$

It is well-known (see [17] and link https://ned.ipac.caltech.edu/level5/Carroll2) that in the matter-dominated universe with the cosmological constant the following

[a]The Accelerating Universe, "The Nobel Prize in Physics 2011 — Advanced Information", https://www.nobelprize.org/nobel_prizes/physics/laureates/2011/advanced-physicsprize2011.pdf.

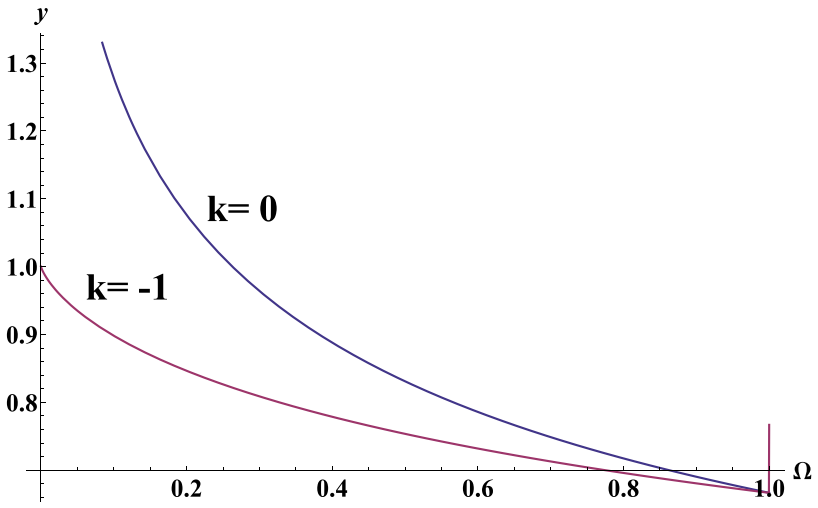

Fig. 1. (Color online) Graphs of $y = F(\Omega)$ introduced in [3]. Blue graph shows $F(\Omega)$ for the flat universe ($k = 0$), while the purple one shows $F(\Omega)$ for the open universe ($k = -1$).

holds:

$$q = \Omega/2 - \Omega_\Lambda. \tag{27}$$

Thus, for such universe, by definition of $q(t)$, (24) and (27), we have

$$\Omega/2 - \Omega_\Lambda = q = \frac{\mu}{(tH)^2} - \frac{\Lambda}{3H^2} = \frac{\mu}{F(\Omega)^2} - \Omega_\Lambda.$$

Hence (see Fig. 2)

$$\mu = \frac{\Omega}{2} F(\Omega)^2. \tag{28}$$

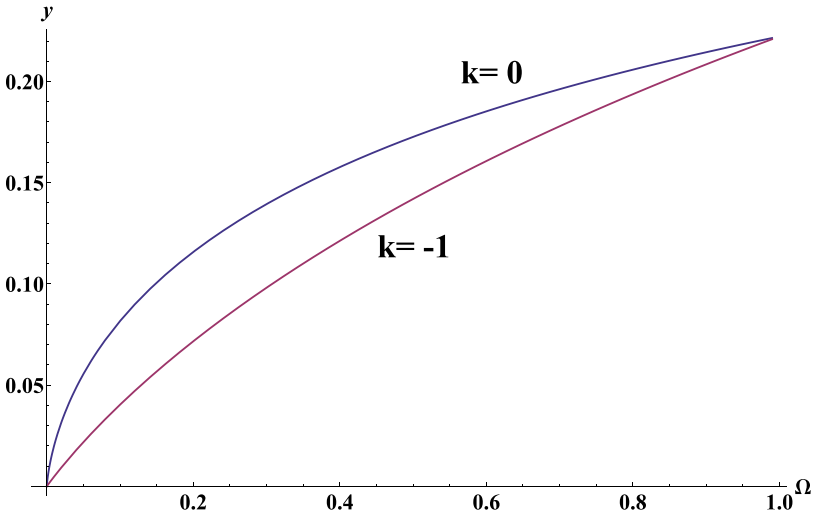

Fig. 2. (Color online) Graphs of $\mu(\Omega) = \frac{\Omega}{2} F(\Omega)^2$ for the flat universe (blue graph), while the purple one shows function $\mu(\Omega)$ for the open universe ($k = -1$).

We show how to infer asymptotic formulas for the scale factor $a(t)$ using $F(\Omega)$. For this, we note that the Hubble parameter is the logarithmic derivative of $a(t)$:

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{d \ln a(t)}{dt}.$$

Hence,

$$a(t) = \exp\left(\int_{t_0}^{t} H(t)dt\right) = \exp\left(\int_{t_0}^{t} \frac{H(t)t}{t}dt\right) = \exp\left(\int_{t_0}^{t} \frac{F(\Omega)}{t}dt\right). \quad (29)$$

We shall prove the following crucial characteristic of the scale factor $a(t)$.

Proposition 6.

The function $a(t)$ belongs to the class ER. (30)

Proof. This property will follow from (29) if we show that $F(\Omega(t))$ is bounded on an interval $[t_0, \infty]$ for some $t_0 > 0$. So first suppose that the universe is open. Then the function $F(\Omega(t))$ is obviously bounded, see Fig. 1. If the universe is flat, then $\Omega_\infty = 1$ and so $F(\Omega(t))$ is bounded at the infinity, too. Further, the value $\Omega_0 = 0.3$ is widely taken for the present epoch (see e.g. [17]) and is close to the value preferred by the observation. If we assume that the energy density ρ becomes lower as the age of the universe becomes older, we may suppose that the possible range for the constant Ω_∞ is the interval $[0.3, 1]$ and so $F(\Omega(t))$ is bounded, too. Therefore in all cases, we may take that $F(\Omega(t))$ is bounded on the every interval $[t_0, \infty]$ for every $t_0 > 0$. Hence, due to (29), $a(t)$ is an extended RV function. □

In deriving asymptotic formula for $a(t)$ and other cosmological parameters we shall distinguish the following two opposite cases. The first one is when there is $\lim_{t\to\infty} \Omega(t)$. The second one is when this limit does not exist. Obviously, there is no third possibility and so, assuming Einstein field equations ant therefore EFT, these lead us to two cosmological models. The first assumption forces $\Omega(t) \to 0$, as $t \to 0$, and this leads to the more or less classical solution for ΛCDM model. However, we got it not assuming equation of state, in fact a form of it is obtained. We must say that this solution (model) is preferable with the majority of cosmological specialists. We also considered in depth the second alternative, when there is no limit $\lim_{t\to\infty} \Omega(t)$. As $a(t)$ belongs to the ER class (Theorem 6), we got a somewhat unusual result that the acceleration parameter in this case varies between two power functions and therefore the deceleration parameter $q(t)$ changes the sign infinitely many times.

3.2. *There is a limit* $\lim_{t\to\infty} \Omega(t)$

In this section, we discuss the cosmological parameters for a pressureless universe with the cosmological constant Λ, assuming that $\Omega(t)$ converges as $t \to \infty$. Under

this assumption asymptotics of main cosmological constants for flat and open universe are inferred. We shall briefly discuss also asymptotics for an arbitrary universe ($k = 0, -1$), not assuming the equation of state.

Suppose a pressureless universe and let there be a limit $\lim_{t\to\infty} \Omega(t) = \omega$. From (22), we see that it must be $0 \leq \omega \leq 1$. We shall distinguish two cases, $0 < \omega \leq 1$ and $\omega = 0$. Note that the functions $F(\Omega)$ are defined and continuous in the interval $0 < \Omega < 1$. Suppose first $0 < \omega < 1$. By the continuity of $F(\Omega)$, there exists α such that

$$\alpha = \lim_{t\to\infty} F(\Omega(t)) = \lim_{\Omega\to\omega} F(\Omega) = F(\omega) \tag{31}$$

and therefore by (21)

$$H(t) = \frac{\alpha}{t} + \frac{\varepsilon}{t}, \quad \varepsilon \in \mathcal{Z}_0. \tag{32}$$

Further, using (29) it follows:

$$\frac{a(t)}{a(t_0)} = \exp\left(\int_{t_0}^{t} \frac{\alpha}{t} dt + \int_{t_0}^{t} \frac{\varepsilon}{t} dt\right) = \left(\frac{t}{t_0}\right)^{\alpha} \exp\left(\int_{t_0}^{t} \frac{\varepsilon}{t} dt\right). \tag{33}$$

Hence,

$$a(t) = a(t_0) \left(\frac{t}{t_0}\right)^{\alpha} L_0(t), \quad t > t_0 > 0, \tag{34}$$

where $L_0(t) = \exp(\int_{t_0}^{t} \frac{\varepsilon(t)}{t} dt)$, $\varepsilon(t) \to 0$ if $t \to \infty$. Hence $L_0(t)$ is slowly regular, and so $a(t)$ is a normalized RV function.

In conclusion, if $\omega = \lim_{t\to\infty} \Omega(t)$ exists where $0 < \omega < 1$ and $\lim_{t\to\infty} F(\Omega(t)) = F(\omega) = \alpha$, then the scale factor $a(t)$ satisfies generalized power law:

$$a(t) = t^{\alpha} L_0(t), \quad L_0(t) \text{ is a SV function.}$$

According to [1] then all cosmological parameters are uniquely determined and a form of equation of state holds. Also, by Proposition 5, the integral limit Γ defined by (19) must exist and $\Gamma \leq 1/4$.

For the border case $\Omega \sim 1$, we have

$$F(\Omega) = \frac{2}{3}\left(1 + \frac{\sqrt{1-\Omega}}{1+\sqrt{\Omega}}\right) + o(1-\Omega), \quad (k = 0),$$

$$F(\Omega) = \frac{2}{\Omega} - \frac{4}{3\Omega^2} + \frac{2\sqrt{1-\Omega}}{\Omega} + o(1-\Omega), \quad (k = -1).$$

Hence, in both cases $F(\Omega) \to 2/3$ as $\Omega \to 1^-$. Further, by (21) we obtain $H(t) \sim \frac{2/3}{t}$ and so $a(t) \sim (t/t_0)^{\frac{2}{3}}$ as $t \to +\infty$. This is expected and a well-known result for a matter-dominated universe but with $\Lambda = 0$, even if we did not assumed so. We shall see that in all, except one case, indeed $\Lambda = 0$.

At this point, we should mention that this result agrees with [19, 18] where the authors showed that it is possible to discard the existence of the cosmological constant Λ by using ordinary thermodynamics.

Before we proceed further, we note

$$\text{If } \ 0 < \omega \leq 1 \ \text{ then } \alpha \neq 0. \tag{35}$$

This follows from the fact that each function $F(\Omega)$ is decreasing and the right endpoint of its domain is 1 (see Fig. 1). As we have seen, the left limit of $F(\Omega)$ at this endpoint is $2/3$, so $\alpha \geq 2/3$ and thus $\alpha \neq 0$.

For the second case, $\omega = 0$, it is easy to see that

$$\begin{aligned} &F(\Omega) \to +\infty \quad \text{as } \Omega \to 0^+ \quad (k = 0), \\ &F(\Omega) \to 1 \qquad \text{as } \Omega \to 0^+ \quad (k = -1). \end{aligned} \tag{36}$$

The following statement is one of the main results in this paper. It says that the convergence of $\Omega(t)$ to $\omega \neq 0$ at infinity is a strong assumption.

Proposition 7. *Assume a pressureless universe, open or flat, with the Cosmological constant* Λ *and suppose the limit* $\lim_{t\to\infty} \Omega(t) = \omega$ *exists. Then,* $\omega > 0$ *implies* $\Lambda = 0$.

To prove this proposition, we shall consider separately two cases: $k = 0$ (flat universe) and $k = -1$ (open universe). We prove: if $\omega \neq 0$ then $\Lambda = 0$. So suppose $\omega \neq 0$. Then the admissible values of ω are $0 < \omega \leq 1$, and so $F(\omega) = \alpha \neq 0$, see (35) and Fig. 1.

Case $k = 0$.

From the identity $\Omega + \Omega_\Lambda = 1$ it follows that there exists $\omega_\Lambda = \lim_{t\to\infty} \Omega_\Lambda$. Hence,

$$\omega + \omega_\Lambda = 1. \tag{37}$$

As $\Lambda = 3\Omega_\Lambda H^2$, by (32) there is a zero function $\xi = \xi(t)$ such that

$$\Lambda = \frac{3(\omega_\lambda \alpha^2 + \xi)}{t^2}. \tag{38}$$

The constant Λ is time independent and the right-hand side of (38) converges to 0, so we have $\Lambda = 0$, $\omega_\Lambda = 0$ and $\omega = 1$. In fact, we have $\Omega = 1$.

Case $k = -1$.

Derivation in this case is more complicated, but follows the idea presented in the case $k = 0$ — approximation of cosmological parameters with simpler terms for which asymptotics can be found. Friedmann equation also has the following well-known form

$$\Omega + \Omega_\Lambda + \Omega_k = 1. \tag{39}$$

There is a zero function ε_1 so that $\Omega = \omega - \varepsilon_1$. Hence by (32), (34), some SV function L_0, some zero function ε_2, (39) and definitions of Ω_Λ and Ω_k we get identities

$$H = \frac{\alpha}{t} + \frac{\varepsilon_2}{t}, \quad a(t) = t^\alpha L_0(t), \quad \frac{\Lambda}{3H^2} + \frac{1}{a^2H^2} = 1 - \omega + \varepsilon_1. \tag{40}$$

By the first identity in (40) there are zero functions η_1 and η_2 so that

$$H^2 = (\alpha^2 + \eta_1)t^{-2}, \quad H^{-2} = (\alpha^{-2} + \eta_2)t^2. \tag{41}$$

If $L(t)$ is a SV function, it is well known that $L(t)^2$ and $L(t)^{-1}$ are also SV. Hence, there is a SV function $L_1(t)$ such that

$$\frac{1}{a^2H^2} = (\alpha^{-2} + \eta_2)t^{2-2\alpha}L_1(t). \tag{42}$$

We shall use the following important properties of a SV function $L(t)$:

$$\text{For any } \beta > 0, \quad \lim_{t\to\infty} t^{-\beta}L(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} t^{\beta}L(t) = \infty. \tag{43}$$

Using so far obtained relations, from the third identity in (40), we obtain

$$\frac{\Lambda}{3} = \frac{(\alpha^2 + \eta)(1 - \omega - (\alpha^{-2} + \eta_2)t^{2-2\alpha}L_1(t) + \varepsilon_1)}{t^2}. \tag{44}$$

As $\alpha > 0$, for some zero function ξ this equation reduces to

$$\frac{\Lambda}{3} = -t^{-2\alpha}L_1(t) + \xi. \tag{45}$$

By (43) the right-hand side of this equation converges to 0 as $t \to \infty$ and as Λ is constant, it follows $\Lambda = 0$.

It is easy to see that $\Lambda = 0$ for open universe even for $\omega = 0$. Namely, assuming $\omega = 0$ we have $\alpha = F(\omega) = F(0) = 1$ (see Fig. 1), and the same argument applied to (45) yields $\Lambda = 0$, too. In other words $\Lambda \neq 0$ is incompatible with the pressureless, open universe with convergent Ω. Therefore, it remains to see what happens with pressureless flat universe assuming $\omega = 0$.

Assume pressureless flat universe and $\lim_{t\to\infty} \Omega(t) = 0$. Then, using (22), we find $F(\Omega) \sim \frac{2}{3}(\ln(2) - \frac{1}{2}\ln(\Omega))$ as $\Omega \to +0$, or

$$F(\Omega) \sim -\frac{1}{3}\ln(\Omega) \quad \text{as } \Omega \to +0. \tag{46}$$

Hence, by (21) we have immediately

$$H(t) \sim -\frac{1}{3t}\ln(\Omega) \quad \text{as } t \to \infty. \tag{47}$$

As $H = \dot{a}/a$ we also find for large t

$$a(t) = a(t_0)e^{-\int_{t_0}^{t} \frac{\ln(\Omega)}{3t} dt}. \tag{48}$$

We can find a more exact formula for the scale factor $a(t)$. By formulas (28) and (46), we see that $\mu \sim \Omega \ln(\Omega)^2$ as $\Omega \to +0$. Therefore (see also Fig. 2)

$$\lim_{\Omega\to+0} \mu = 0. \tag{49}$$

Hence, the term μ/t^2 may be neglected in Eq. (24). Assuming $\Lambda > 0$, the fundamental solutions of so simplified equation $\ddot{a}/a = \Lambda/3$ are $\exp(\sqrt{\Lambda/3}t)$ and $\exp(-\sqrt{\Lambda/3}t)$. As the first fundamental solution dominates at infinity the second one, we find

$$a(t) \sim \exp(\sqrt{\Lambda/3}t) \quad \text{as } t \to +\infty. \tag{50}$$

Interestingly, we obtained the same asymptotics for the scale factor $a(t)$ as in the case of cosmic inflation, perfectly consistent with the ultimate fate of the universe such as Big Rip.

Now, we shortly discuss asymptotics for $\Lambda = 0$ of an arbitrary flat or open universe i.e. without assumptions of the matter dominance. In this derivation, we do not assume the equation of state. In fact, we obtained a weak form of its. First, we note that the threshold constant Γ (see Sec. 1.2) is related to the acceleration equation and it is defined by (19). In [1] it is shown that the exponent α of the expansion scale factor (34) is a solution of $x^2 - x + \Gamma = 0$, hence

$$\Gamma = \alpha(1 - \alpha). \tag{51}$$

For the curvature index $k = 0, -1$ it must be $\Gamma \leq 1/4$, see Proposition 5 and comments that precedes it. In [1] exhaustively are discussed admissible values for α and related cosmological models. For example, the Zel'dovich interval $0 < w < 1$ of the equation of state parameter w corresponds to the α — interval $1/3 < \alpha < 2/3$. Following ideas in [1], we formally define the equation of state constant w by

$$w = \frac{2}{3\alpha} - 1. \tag{52}$$

Then the formulas for other cosmological parameters can be obtained and they are written as follows (see [1, Theorem 3.5]):

$$\begin{aligned} &\alpha = \frac{2}{3(1+w)}, \qquad a(t) = a_0 t^{\frac{2}{3(1+w)}} L_0(t), \\ &H(t) \sim \frac{2}{3(1+w)t}, \quad \mathbf{M}(q) = \frac{1+3w}{2}. \end{aligned} \tag{53}$$

We see that these formulas give the standard solutions of Friedman equations for the case $k = 0$. Also, there are functions $\hat{w}(t)$, $\xi(t)$ and $\zeta(t)$ such that

$$p = \hat{w}\rho c^2, \quad \text{(equation of state)}, \tag{54}$$

where $\hat{w}(t) = w - t\dot{\xi} + \zeta$, $\xi, \zeta \in \mathcal{Z}_0$. Hence a weak form of the equation of state holds. If $t\dot{\xi} \to 0$ as $t \to \infty$, then $\hat{w}(t) \approx w$, which leads to $p = w\rho c^2$, the standard equation of state and classical asymptotics for cosmological parameters. In [1] it is also found

$$\mathbf{M}(\mu) = \Gamma = \frac{2}{9} \cdot \frac{1+3w}{(1+w)^2}. \tag{55}$$

For more details of this approach the reader is directed to [1].

However, there are several evidences measured in the last two decades that are against the $\Lambda = 0$. The first one is the estimated large value $\Omega_\Lambda = 0.6911 \pm 0.0062$, according to results published by the Planck Collaboration in 2016 [4], and the second one is the accelerated universe [20–22]. If pressureless universe with $\Lambda \neq 0$ is supposed, by discussion in this section, such a model is possible only if $\omega = 0$ or divergent Ω is assumed.

Now, we turn to the next section where we consider the non-convergent $\Omega(t)$ at infinity.

3.3. *There is no limit* $\lim_{t\to\infty} \Omega(t)$

In this section, we shall suppose that the limit $\lim_{t\to\infty} \Omega(t)$ does not exist. By (6) $a(t)$ belongs to the class ER, so we shall apply the theory of these functions in study of the asymptotics of $a(t)$. We remind that we assumed expanding universe, i.e. that $a(t)$ is unbounded and strictly increasing and so $\dot{a}(t) > 0$.

The following simple statement will be useful in the following consideration.

Proposition 8. *Let $g(t)$, $f(t)$, $h(t)$ be real functions defined on a real interval $[t_0, \infty]$ and suppose they satisfy*:

$$g(t) \leq f(t) \leq h(t), \quad t \in [t_0, \infty].$$

Then there is a real function $u(t)$ such that

$$f(t) = g(t)\cos(u(t))^2 + h(t)\sin(u(t))^2, \quad t \in [t_0, \infty]. \tag{56}$$

If f, g and h are continuously differentiable functions such that $g(t) < h(t)$, $t \in [t_0, \infty]$, then $u(t)$ is also continuously differentiable on $[t_0, \infty]$.

Proof. For any triple $a \leq x \leq b$ of real numbers there are $\alpha, \beta \geq 0$ such that $\alpha + \beta = 1$ and $x = \alpha a + \beta b$. Then there is u such that $\alpha = \cos(u)^2$ and $\beta = \sin(u)^2$. Taking for each t, $a = g(t)$ and $b = h(t)$, we obtain (56). Further, it is easy to see that from (56) it follows:

$$f = \frac{1}{2}(g + h) + \frac{1}{2}(g - h)\cos(2u), \tag{57}$$

wherefrom

$$u = \frac{1}{2}\arccos\left(\frac{h + g - 2f}{h - g}\right), \tag{58}$$

hence $u(t)$ is a continuously differentiable function. □

The function $u(t)$ is not uniquely determined. Functions $\cos(u)^2$ and $\sin(u)^2$ have a period π, so we can choose a particular $u_0(t)$ such that $0 \leq u_0(t) \leq \pi$. We are interested in functions $f(t)$ which infinitely many times touches or approaches arbitrary close $g(t)$ and $h(t)$. Then $f(t)$ consists of infinitely many bounces between $g(t)$ and $h(t)$, each bounce b, see Fig. 3, divided into two flows, from $g(t)$ to $h(t)$

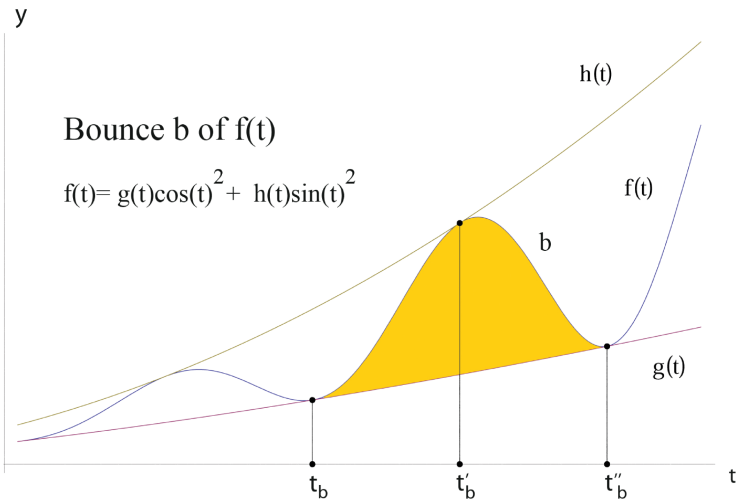

Fig. 3. Bounce of the function $f(t) = g(t)\cos(t)^2 + h(t)\sin(t)^2$.

with $0 \leq u_0(t) \leq \pi/2$, $u_0(t_b) = 0$ and $u_0(t'_b) = \pi/2$ for some instances of time t_b, t'_b and from $h(t)$ to $g(t)$ with $\pi/2 \leq u_0(t) \leq \pi$ and $u_0(t''_b) = \pi$ for some t''_b. For the chosen t_b, t'_b, t''_b, we obviously have $f(t_b) = g(t_b)$, $f(t'_b) = h(t'_b)$ and $f(t''_b) = g(t''_b)$. If bounces are enumerated with non-negative integers k, then we can take such a function $u(t)$ so that $u(t) = k\pi + u_0(t)$ for the domain interval $t_{b_k} \leq t \leq t_{b''_k}$ of the bounce b_k. Obviously, $u(t) = u_0(t) \pmod{k\pi}$. Such functions $u(t)$ we shall call time-like and in the rest of this section, we shall refer to the time-like functions $u(t)$.

We have shown that $a(t)$ belongs to the class ER and that $F(\Omega)$ is a bounded function for $t \geq t_0$ for $t_0 > 0$. Hence there are real numbers ω and ω' such that $\omega \leq \Omega(t) \leq \omega'$. By assumption $\Omega(t)$ is divergent so we can take

$$\omega = \liminf_{t\to\infty} \Omega(t) \quad \text{and} \quad \omega' = \limsup_{t\to\infty} \Omega(t). \tag{59}$$

Further, let $\alpha = F(\omega')$ and $\beta = F(\omega)$. The functions $F(\Omega)$ are monotonously decreasing, see Fig. 1, hence $\alpha < \beta$. Such a choice of α and β assures that $F(\Omega)$ infinitely many times takes values α and β or approaches them arbitrarily close. This guarantees that $u(t)$ is not a constant function, moreover that $u(t)$ takes all values in the open interval $(0, \pi)$, and makes the next discussion sound. Alternatively, to achieve this goal, we can use the more visualizing property (70) inferred at the end of this subsection and which is a characteristic of ER functions and so of the expansion scale factor $a(t)$, too.

By (6) and representations (21) and (29), we immediately infer

$$(t/t_0)^\alpha \leq a(t) \leq (t/t_0)^\beta, \quad t \geq t_0. \tag{60}$$

Then by the previous proposition there is a continually differential function $u(t)$ such that

$$a(t) = \frac{1}{2}\left(\left(\frac{t}{t_0}\right)^\alpha + \left(\frac{t}{t_0}\right)^\beta\right) + \frac{1}{2}\left(\left(\frac{t}{t_0}\right)^\alpha - \left(\frac{t}{t_0}\right)^\beta\right)\cos(2u(t)) \tag{61}$$

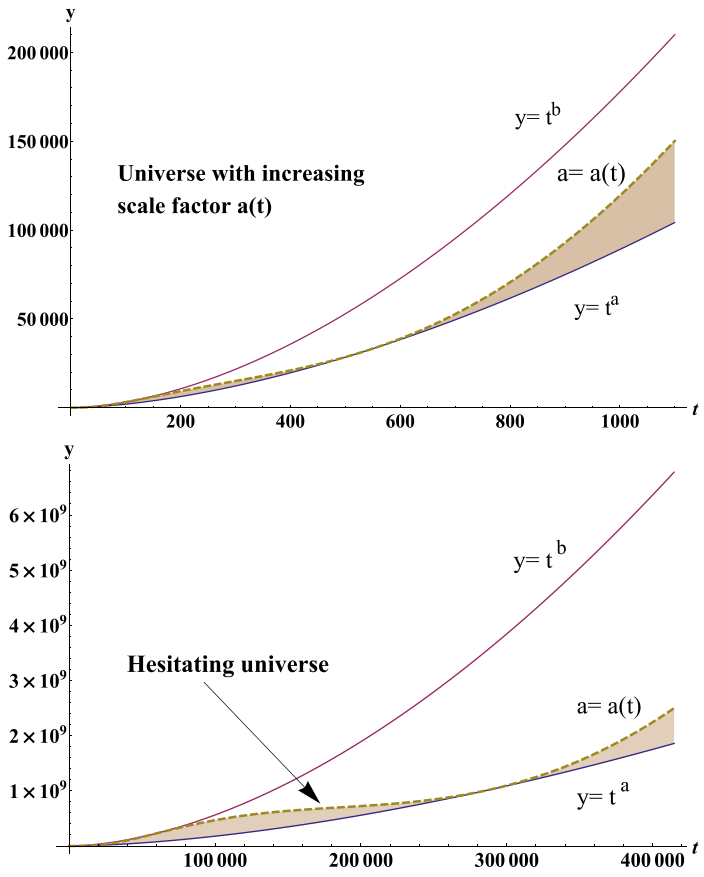

Fig. 4. Depending on the nature of the function $u(t)$ there are several possible cases in the evolution of the scale factor (see Eq. (62)) for the case when there is no limit $\lim_{t\to\infty}\Omega(t)$. Top image shows the evolution of the universe with increasing $a(t)$, and the bottom shows hesitating scale factor.

or by normalizing, i.e. putting $t_0 = 1$, we obtain a somewhat simpler form

$$a(t) = \frac{1}{2}(t^\alpha + t^\beta) + \frac{1}{2}(t^\alpha - t^\beta)\cos(2u) \equiv t^\alpha \cos(u)^2 + t^\beta \sin(u)^2. \tag{62}$$

Depending on the nature of the function $u(t)$, there are several possible cases in the evolution of the scale factor $a(t)$. These cases are depicted in Figs. 4 and 5. Note that $a(t)$ approaches t^α, respectively, t^β infinitely many times arbitrary close, while by (62) $u(t)$ takes both values $k\pi$ and $k'\pi+\pi/2$, or approaches them arbitrary close infinitely many times, where k and k' are positive integers.

We have seen that $F(\Omega) \geq 2/3$, and so, by the choice of α and β, we have $2/3 \leq \alpha < \beta$. Further,

$$\dot{a} = \alpha t^{\alpha-1}\cos(u)^2 + \beta t^{\beta-1}\sin(u)^2 + (t^\beta - t^\alpha)\dot{u}\sin(2u), \tag{63}$$

$$\ddot{a} = \frac{\alpha(\alpha-1)}{2}t^{\alpha-2}\cos(u)^2 + \frac{\beta(\beta-1)}{2}t^{\beta-2}\sin(u)^2 + 2(\beta t^{\beta-1} - \alpha t^{\alpha-1})\dot{u}\sin(2u) + (t^\beta - t^\alpha)(2\dot{u}\cos(2u) + \ddot{u}\sin(2u)). \tag{64}$$

Let us consider properties of $a(t)$ and it's bounce b_k over the interval $(k\pi, k\pi+\pi)$ in more details. The power term t^β dominates at infinity in (63), hence $\dot{a} \sim t^\beta \dot{u}\sin(2u)$

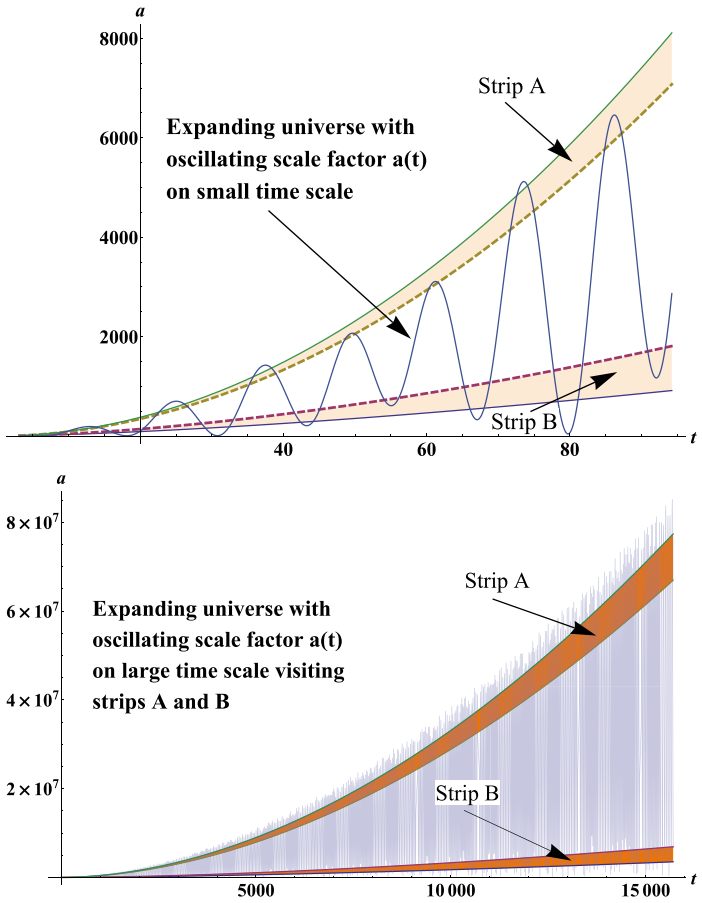

Fig. 5. Evolution of the oscillating scale factor $a(t)$ on small (top) and large (bottom) timescale for the case when there is no limit $\lim_{t\to\infty}\Omega(t)$.

for $\dot{u} \neq 0$ and $u \neq k\pi$. As $\dot{a} > 0$, we see that $\dot{u} > 0$ for $k\pi < u < k\pi + \pi/2$. Hence, using (64), we infer the following properties for large t:

- If $u(t) \sim k\pi^+$, i.e. $u(t)$ is close to $k\pi$ from the right, then $\ddot{a} \sim 2t^\beta \dot{u}\cos(2u) > 0$, so $\ddot{a} > 0$.
- If $u(t) \sim k\pi + \pi/2^-$, i.e. $u(t)$ is close to $k\pi + \pi/2$ from the left, then $\ddot{a} \sim 2t^\beta \dot{u}\cos(2u) < 0$, so $\ddot{a} < 0$.

We conclude that $\ddot{a}$ changes the sign in the interval $(k\pi, k\pi + \pi/2)$ and so $\ddot{a}(t) = 0$ for some t in this interval and the scale factor a has an inflection point, too.

Summarizing and using (62)–(64), we can reckon for large t.

Proposition 9. (1) *The scale factor $a(t)$ bounces between bounding functions t^α and t^β.*
(2) *If $\sin(u) \approx 0$, i.e. $u \approx k\pi$ for some integer k, then $a(t) \approx t^\alpha$.*
(3) *If $\cos(u) \approx 0$, i.e. $u \approx k\pi + \pi/2$ for some integer k, then $a(t) \approx t^\beta$.*
(4) *If $\sin(u) \not\approx 0$, then $a(t) \approx t^\beta \sin(u)^2$.*
(5) *Every jump of $a(t)$ between t^α and t^β contains an inflection point, i.e. $a(t)$ change from concave to convex, or vice versa. Therefore, $\ddot{a}$ changes the sign in*

the future infinitely many times and so does the deceleration parameter $q(t)$. Hence the universe's expansion alternatively decelerates and accelerates.

Further, we discuss the monotonicity of $a(t)$ at infinity (i.e. for large t). If in some point t_1, $\dot{u}(t_1) = 0$, or $\sin(2u(t_1)) = 0$, then by (63), we have $a(t_1) = \alpha t_1^{\alpha-1}\cos(u(t_1))^2 + \beta t_1^{\beta-1}\sin(u(t_1))^2$, so

$$\alpha t_1^{\alpha-1} \leq \dot{a}(t_1) \leq \beta t_1^{\beta-1}.$$

Therefore $\dot{a}(t_1) > 0$ and $a(t)$ is increasing in a neighborhood of t_1.

If $\dot{u}(t)\sin(2u(t)) \not\approx 0$, then

$$\dot{a}(t) \approx t^{\beta}\dot{u}(t)\sin(2u(t)).$$

In the next paragraph, we omit the assumption that $a(t)$ is monotonous. Suppose $u(t)$ is a time-like function and monotonously increasing. Then $\dot{u}(t) > 0$ and so the sign of $\dot{a}(t)$ depends solely on the term $\sin(2u(t))$. As $u(t)$ is unbounded, $\sin(2u(t))$ certainly change the sign and so does $\dot{a}(t)$. Hence $a(t)$ oscillates between t^{α} and t^{β} and the universe alternatively slows down and increases the expansion. But if $\dot{u}(t)$ and $\sin(2u(t))$ are functions of the same sign then due to (63) for the sufficiently large t we have $\dot{a}(t) > 0$, i.e. $a(t)$ is increasing.

According to the previous discussion, all of the following cases are possible for the scale factor $a(t)$:

(1) For large t, $a(t)$ is increasing.
(2) In some intervals $\dot{a}(t) \approx 0$, i.e. $a(t)$ is an almost constant function on these intervals. This case corresponds to the so-called Lemaitre hesitating universe.
(3) The scale factor $a(t)$ oscillatory varies between t^{α} and t^{β}.

All these situations are depicted in the diagrams of Figs. 3 and 4. We also note that if $2\dot{u}\cos(2u) + \ddot{u}\sin(2u) \not\approx 0$, then for large t

$$\ddot{a} \approx t^{\beta}(2\dot{u}\cos(2u) + \ddot{u}\sin(2u)). \tag{65}$$

Equation (21) tell us that $F(\Omega) = H(t)t$. Since $H = \dot{a}/a$, by (62) and (63) taking $\gamma = \alpha - \beta$, we have

$$F(\Omega) = \frac{\alpha t^{\gamma}\cos(u)^2 + \beta\sin(u)^2 + t(1-t^{\gamma})\dot{u}\sin(2u)}{t^{\gamma}\cos(u)^2 + \sin(u)^2}. \tag{66}$$

For large t we have $t^{\gamma} \approx 0$ as $\gamma < 0$. Therefore, if $u \not\approx k\pi$, k is an integer, then we have

$$F(\Omega) = \beta + 2t\dot{u}\cot(u). \tag{67}$$

On the other hand, if $u \approx k\pi$, then we immediately find $F(\Omega) \approx \alpha$. Further, if we integrate Eq. (67), we find

$$\ln|\sin(u)| = \frac{1}{2}\int_{t_0}^{t}\frac{F(\Omega)}{t}dt - \frac{1}{2}\beta\ln(t), \tag{68}$$

wherefrom and (29), we obtain $a(t) \approx t^{\beta}\sin(u)^2$ for $u \not\approx k\pi$, an already noted result in Proposition 9.3.

Finally let us mention the following useful result on ER functions. According to the theory of RV functions [6] for an extended RV function f, there are numbers $c'(f)$ and $d'(f)$, so-called Karamata indices and $\alpha'(f)$ and $\beta'(f)$, so-called Matuszewska indices so that (see [6, Theorem 2.1.8])

$$\lambda^{d'(f)} \leq f_*(\lambda) \leq \lambda^{\beta'(f)} \leq \lambda^{\alpha'(f)} \leq f^*(\lambda) \leq \lambda^{c'(f)}, \quad \lambda \geq 1. \tag{69}$$

Using this property and definition (8) of functions f_* and f^* it is easy to show that there are numbers α, β, a and b which naturally define strips A and B in the euclidean plane:

$$A = \{(t, y) \in R^2 : t \geq 0, t^b \leq y \leq t^\beta\},$$

$$B = \{(t, y) \in R^2 : t \geq 0, t^a \leq y \leq t^\alpha\}$$

so that

$$f(t) \text{ visits (intersects) strips } A \text{ and } B \text{ infinitely many times.} \tag{70}$$

As the scale factor $a(t)$ belongs to the class ER, we see that $a(t)$ has property (70). This characteristic of $a(t)$ is depicted for a short time scale and a large time scale on graphs, the Fig. 4.

Even if the auxiliary function $u(t)$ in the previous analysis is unknown, it has useful properties forced by the ER property of $a(t)$ and the bounding functions t^a, t^α, t^b and t^β. These properties were enough to deduce for divergent case of $\Omega(t)$, that $a(t)$ is oscillating between the two indicated strips, has infinitely many flection points and that the deceleration parameter $q(t)$ changes the sign infinitely many times. These types of variation of the cosmological parameters is difficult to explain by some inner evolution processes of the universe or the relicts from the early universe as it was indicated in the convergent case. A possible explanation is given in the following section.

4. Discussion

Our goal in this paper was to state and resolve the dichotomy between two possible alternatives in the evolution of the pressureless universe with cosmological constant Λ. We have shown that there are only two possibilities in the asymptotic of such a universe. $\lim_{t\to\infty} \Omega(t) = 0$ and the second one is when the limit $\lim_{t\to\infty} \Omega(t)$ does not exist (see Sec. 3). Asymptotics of the main cosmological parameters are found in both cases and the results agree with predictions found in the literature. A major novelty not found in the literature is the second non-convergent case (see Sec. 3.2). Namely, as an outcome we have that the universe mast pass through the infinite number of flection points. It means that the universe will alternate infinitely many times between acceleration and deceleration phases. In the later parts of this section, we discussed possible causes of this behavior. We have to mention that our analysis is based on the theory of regular variation, a mathematical method appropriate for studies in general asymptotics at infinity. The usage of this,

purely mathematical theory permitted us to find a different solutions regardless of a chosen cosmological model. Therefore, theory of a RV functions could be applied to the analysis which involves only the cosmological principle, general relativity and Friedmann equations. Other possible approaches that are model independent can be found in the literature, like cosmography [23]. Cosmography or cosmo-kinetics can be used in order to test possible models without previous model assumptions. In [24], it is shown that observational data are consistent with the ΛCDM model, but also with alternative fluid models.

Finally but certainly most important, our motivation also lies in recent evidence for the existence of dark matter and dark energy, dominant entities of our universe which governs its evolution. The observational data suggest that an important part of the universe is smoothly distributed dark energy component [25]. A possible candidate for this dark energy (or cosmological constant Λ) is a vacuum energy with negative pressure $p_\Lambda = -\rho_\Lambda$. Some authors suggest that cosmological constant should be discarded, as there are other explanations for a negative pressure, like vanishing speed of a sound [19]. We shall discuss in more details the current status of dark energy in cosmology and how it is related to results presented in this paper.

Discovery of the accelerated expansion of the universe through observing type Ia supernovae (SNe Ia) was the first evidence for existence of the dark energy [20–22]. As SNe Ia have a very regular absolute luminosity, they can be used as "standard candles" in order to measure distances throughout the universe. Papers [20, 22] found that supernovae at the same redshift were less luminous then it had been predicted by a model of the universe with only matter. Difference in the observed and predicted luminosity was explained with a presence of dominant, unknown dark energy. It should be noted that some authors found that at low redshift ($z < 0.05$) SNe Ia data are inconsistent with ΛCDM model [26]. Except the SNa Ia, baryonic acoustic oscillations (BAO) can be used to measure distances. BAO was detected at different redshifts with BOSS data and it supported existence of dark energy [27]. Results published by the Planck Collaboration in 2016 [4] ($\Omega_\Lambda = 0.6911 \pm 0.0062$) are also consistent with ΛCDM model.

In order to obtain more information about dark energy we will look at its equation of state. The equation of state of the cosmological constant is $w = -1$ which is consistent with the obtained observational data $w = -1.0006^{+0.067}_{-0.068}$ [28].

The equation of state parameter w is directly connected to the accelerated expansion of the universe (see accelerated equation (1.1)):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + \rho_\Lambda + 3p_\Lambda), \tag{71}$$

where ρ_Λ and p_Λ are density and pressure of dark energy, respectively, and ρ_m is a density of the matter in the universe. Accelerated expansion of the universe can happen only if pressure component is strongly negative or $w < -\frac{1}{3}(1 + \rho_m)$. Hence, acceleration will not occur if $w > \frac{1}{3}$ (see for example [29]). In the case when

$-1 \leq w < -1/3$ (accelerated universe) the future of the universe is accelerated expansion but structures as galaxies, star systems, etc. remain unaffected. But, what about $w < -1$? In this case, the universe will become increasingly dark energy dominated which will lead to the Big Rip scenario [30]. Current value of w is near the border, therefore it is not possible to precisely determine its fate [31]. Our approach gives interpretation for both cases as it is shown in Sec. 3.

Also, it is possible that w is not a constant but a time-dependent function. Using generalized, time-dependent, form of the equation of the state a cosmology which describes the transition from decelerating universe to accelerating one or vice versa can be constructed [32]. This could lead to some interesting models such as with weak form of equation of state (54) which is also the result of our analyzes using RV functions.

Now, we shall discuss certain physical hypotheses for the universe models that might explain variation of cosmological parameters we described in previous sections. We briefly indicated some of these possibilities in [2].

4.1. *Echoing*

One interesting possibility for the variations of $q(t)$ and $p(t)$ for the convergent $\Omega(t)$ could be a kind of reverberation due to the extremely rapid expansion of the universe which appeared in the inflationary epoch, about 10^{-36} s after the Big Bang. One can speculate that these variations are consequences of an echo effect due to thermalization which appeared when the inflation epoch ended. However a digressive variation for divergent $\Omega(t)$ of $a(t)$ and $q(t)$ on large time scale is difficult to explain only with inner processes in the course of the universe evolution. We prefer the idea that this variation comes from an external cause. Namely, that it is an effect of the existence of a dual universe and that it is a resultant of their mutual interference. We discuss this possibility in more details.

4.2. *Dual universe*

One of the concepts of modern string theory and hence M-theory is that the big bang was a collision between two membranes [33–35]. The outcome was the creation of two universes, one in the surface of each membrane. Using the Large Hadron Collider (LHC) located in CERN, some data are collected that this might lead to the conclusion that the parallel universe exist. Specifically, if the LHC detects the presence of miniature black holes at certain energy levels, then it is believed [36] that these would be the fingerprints of multiple universes. Collected data are still analyzed, but there are also other research in this direction, e.g. [37].

We will not enter here into the full discussion on the existence of the multiverse. But if it is assumed that a parallel universe exists, we can explicitly find, at least for RV cosmological parameters, a set of formulas that might represent cosmological parameters of a dual universe. Our argument for supporting this approach we

found in the following theorem and the symmetry which exists between formulas expressing cosmological parameters for this pair of universes. The symmetry is represented by the Galois group of the associated algebraic Eq. (72). This group and the translation of the set of formulas of the primary universe to its dual is also described in this section. We obtain them using the second fundamental solution $L_2(t)$ in the Howard — Marić theorem see [12, 1], applied to the acceleration equation.

Theorem 10 (Howard-Marić). *Let $-\infty < \Gamma < 1/4$, and let $\alpha_1 < \alpha_2$ be two roots of the equation*

$$x^2 - x + \Gamma = 0. \tag{72}$$

Further, let L_i, $i = 1, 2$ denote two normalized SV functions. Then there are two linearly independent RV solutions of $\ddot{y} + f(t)y = 0$ of the form

$$y_i(t) = t^{\alpha_i} L_i(t), \quad i = 1, 2, \tag{73}$$

if and only if $\lim_{x\to\infty} x \int_x^\infty f(t)dt = \Gamma$. Moreover, $L_2(t) \sim \frac{1}{(1-2\alpha_1)L_1(t)}$.

The second fundamental solution and so the dual set of these formulas is determined by the second root $\beta = 1 - \alpha$ of the quadratic equation $x^2 - x + \Gamma = 0$ appearing in this theorem. To avoid singularities, we assume $\alpha, \beta \neq 0$. Now, we use β instead of α for the index of RV solution $a(t)$ — deceleration parameter and for determination of other constants and cosmological parameters. As in (52), we introduce $w_\beta = \frac{2}{3\beta} - 1$. Then, we have the following symmetric identity for the equation of state parameters:

$$w_\alpha + w_\beta + 3w_\alpha w_\beta = 1. \tag{74}$$

For our universe we have $w = w_\alpha$, while for the dual universe the corresponding equation of state parameter is w_β. Then the dual formulas are obtained by replacing α with β and w_α with w_β in (32), (54) and (53). If one wants to give any physical meaning to the so obtained dual set of functions, it is rather natural to interpret them as the cosmological parameters of the dual universe.

As we shall see, these two universes are isomorphic in the sense that there is an isomorphism which maps cosmological parameters into their dual forms. Hence, this isomorphism introduces a symmetry between cosmological parameters and their dual forms. It would mean that both universes have the same or similar physics as anticipated in [35]. In this derivation, we use some elements of the Galois theory. For the basics of this theory the reader may consult for example [38].

Our assumption $\Gamma < \frac{1}{4}$ and that the solutions α and β of Eq. (72) differ, say $\alpha < \beta$, introduces the following kind of symmetry. Let $F = \mathbf{R}(t, \Gamma)$ be the extension algebraic field where $\mathbf{R}$ is the field of real numbers and t and Γ are letters (variables). It is easy to see that for such Γ the polynomial $x^2 - x + \Gamma$ is irreducible over the field F. Hence, the Galois group $\mathbf{G}$ of Eq. (72) is of the order 2 and has a nontrivial automorphism σ. Let α and β be the roots of the polynomial $x^2 - x + \Gamma$. Then

$\sigma(\alpha) = \beta$ and $\sigma(\beta) = \alpha$. Further, let $\Gamma = \frac{2}{9} \cdot \frac{1+3w}{(1+w)^2}$ where w is a parameter. Then, we can take $\alpha = \frac{2}{3(1+w)}$ and $\beta = \frac{1+3w}{3(1+w)}$. Let $w_\alpha \equiv w$ and $w_\beta \equiv \frac{1-w}{1+3w}$. Then $\sigma(w_\alpha) = w_\beta$ since w_α and w_β are rational expressions, respectively, in α and β. Further, the time t and the constant Γ are invariant under σ i.e. $\sigma(t) = t$ and $\sigma(\Gamma) = \Gamma$ since t and Γ are the elements of the ground field F. The cosmological parameters (32) and (53) are rational expressions of w so if P_α is the corresponding parameter to the solution α, then $\sigma(P_\alpha) = P_\beta$. For example, for the Hubble parameters we have $\sigma(H_\alpha) = H_\beta$. Hence, not only solutions (isomorphic via σ) come into the pairs but the sets of all cosmological parameters come as well. At this point one may speculate about two dual mutually interacting universes having the same time t and the constant Γ and the conjugated parameters w_α and w_β connected by relation (74).

Of course, there is a question what are the values of the constants appearing in cosmological parameters, for example of $w = w_\alpha$. During the last two decades [39] there was a great progress in fundamental measurements of cosmic data and estimation of cosmological constants. In particular collected precision data in WMAP[b] (Wilkinson Microwave Anisotropy Probe) mission. enabled accurate testing of cosmological models. It was found that in the emerging standard model of cosmology, a flat Λ-dominated universe seeded by a nearly scale-invariant adiabatic Gaussian fluctuations fits the WMAP data [40]. According to these observations, the value of w, also called equation of state of cosmological constant (or the dark energy equation of state), is near -1. More precisely, the collected data are consistent with the density being time-independent as for a simple cosmological constant ($w = -1$), with uncertainties in w at the 20% level [41]. Other results from experimental cosmology, such as the Baryon Oscillation Spectroscopic Survey (BOSS) of Luminous Red Galaxies (LRGs) in the Sloan Digital Sky Survey (SDSS) are also in favor of $w = -1$, [27], Particle Data Group: http://pdg.lbl.gov. However, the value $w = -1$ yields singularity in (53). For such w there is no corresponding α neither Γ. Equation of state is $p = -\rho c^2$ and then by fluid equation we have $\dot{\rho} = 0$, i.e. ρ is constant. This case corresponds to the cosmological constant, so $\rho = \rho_\Lambda = \frac{\Lambda}{8\pi G}$. In the absences of α and β for dual w_β of $w = w_\alpha$ we may take (74) for defining relation. Putting $w_\alpha = -1$ in this identity we obtain $w_\beta = -1$. Hence, if the ΛCDM model is assumed, dual universe is also equipped with cosmological constant and its expansion is also governed by dark energy.

The other values of w are also considered. For example if $w = 1/3$ then $\alpha = \beta = 1/2$, $\Gamma = 1/4$ and in this case the Howard-Marić theorem cannot be applied since functions $L_1(t)$ and $L_2(t)$ from this theorem are not fundamental solutions. But there is a variant of this theorem appropriate for this case [12], and applying it one can show that $a(t)$ is RV of index $\frac{1}{2}$ if and only if $w \sim \frac{1}{3}$ as $t \to \infty$, i.e. $p \sim \frac{1}{3}c^2\rho$ holds asymptotically. This is the second classic cosmological solution. For more details one can consult [1].

[b]https://lambda.gsfc.nasa.gov/product/map/dr5/.

5. Conclusion

We analyzed Friedmann equations and cosmological parameters for flat or open universe from the point of view of regular variation. Our discussion mostly was related to a pressureless universe with nonzero cosmological constant Λ. For this type of universe Carroll *et al.* introduced in [3] a function $F(\Omega)$ which depends only on Ω and by which other cosmological parameters can be expressed. This allowed us, using the theory of regular variation, to describe complete asymptotics for cosmological parameters when $\omega = \lim_{t\to\infty} \Omega$ does exist, but also in the case when this limit does not exist. As a conclusion, we obtained (Sec. 3.2) for the convergent $\Omega(t)$ at infinity and $\omega \neq 0$ that $\Lambda = 0$. We also obtained asymptotics for flat universe of all main cosmological parameters if $\omega = 0$. The other result says that $\Lambda \neq 0$ is incompatible with the (pressureless) open universe with convergent Ω. On the other hand, (Sec. 3.3), it appears that for divergent Ω the expansion scale factor $a(t)$ varies between two strips bounded by power functions and that $a(t)$ changes its convexity and concavity infinitely many times. Hence, in this case the deceleration parameter changes its sign infinitely many times and $a(t)$ must accelerate and decelerate infinitely many times, too. This property can be attributed to the existence and effect of dark energy. There are strong evidences in favor of the existence of dark energy which makes this model with non-convergent Ω interesting for further studies. For example [41], list three such observational proofs: supernovas of type Ia, power spectrum analysis as done in [41] and the late ISW effect (Integrated Sachs–Wolfe effect). Therefore if the ΛCDM model is assumed, for flat universe the case $\omega \neq 0$ is excluded, while for open universe Ω must be divergent. We also presented asymptotic formulas that might represent the cosmological parameters of the interacting dual universe.

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PART III

History of Science and Digitization

Overview

This book is the second part of collected works of Nadežda Pejović, a retired professor of astronomy at the Faculty of mathematics, University of Belgrade. Papers in this collection are related to the history of science and digitization of scientific works. Professor Pejović was since 2005 one of the leading researchers on a comprehensive and interdisciplinary project related to digitization of scientific works of Serbian scientists from the past who were working in the field of mathematics and the related sciences. The Virtual Library of the Faculty of Mathematics, University of Belgrade, was built during the project. The library itself is an electronic repository of digitized books in these sciences containing today more than 5000 items and many important collections. Professor Pejović has the great merit for including in the library most of the old astronomical books. Papers in this collection are mostly reviews of some of these books that also include biographical notes on their authors. The work on the project was of a collective character, so understandably a large number of the works presented here were co-authored.

46 Digitalizacija gimnazijskih udžbenika Riste Karljikovića

N. Pejović
NCD Review, vol. 11, pp. 48-56, 2007.

Rista Karljiković was a prominent gymnasium professor between two World wars. He wrote several textbooks on mathematics that were used in Serbian secondary schools at that time, (1922 - 1941). Four of his books: Trigonometry, Algebra, Analytical geometry and Geometry are digitized and they are deposited in the Virtual Library. Karljiković's books satisfied very high scholar standards and they represent very well classroom mathematics taught in the most developed European countries in this period. Contents of the books are shortly presented.

Преглед НЦД 11 (2007), 48–56

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(Математички факултет, Београд)

ДИГИТАЛИЗАЦИЈА ГИМНАЗИЈСКИХ УЏБЕНИКА РИСТЕ КАРЉИКОВИЋА

Сажетак. Риста Карљиковић, гимназијски професор између два светска рата, написао је неколико уџбеника из математике који су се користили у српским гимназијама у том периоду (1922–1941). Четири његова уџбеника Тригонометрија, Алгебра, Аналитичка геометрија и Геометрија су дигитализоване и налазе се у електронском облику у Виртуелној библиотеци Националног центра за дигитализацију (Virtual library, http://elib.matf.bg.ac.yu:8080/virlib/). Дигитализација ових књига део је пројекта електронског архивирања српских књига са математичким садржајем штампаних у прошлости. У овом чланку пишемо зашто су изабране Карљиковићеве књиге за укључење у Виртуелну библиотеку. Такође су укратко представљени садржаји ових књига.

Виртуелна библиотека Националног центра за дигитализацију основана је са циљем да се произведе што комплетнија и репрезентативнија колекција ретродигитализованих књига и других докумената, превасходно српских аутора. Мада се у овој библиотеци налазе књиге из различитих области, ипак основни циљ овог пројекта је да се сачувају и представе општој јавности пре свега дела из математичких наука: математике, астрономије, механике, теоријске физике, математичке географије и других наука у којима математички апарат има доминирајућу улогу. Сматрамо да у тој литератури важно место имају образовне књиге, посебно уџбеници који су се користили у настави у средњим школама и на Београдском универзитету. Има више разлога за избор ове врсте књига за дигитализацију и рачунарско архивирање. Наравно, свака писана или штампана реч у прошлости, нарочито у даљој, део је културног наслеђа и већ тиме се ова дела квалификују за овакву врсту њиховог чувања. Наведимо и неке више прагматичне разлоге.

Најпре, лак и једноставан доступ овим књигама путем Интернета, омогућава се историчарима образовања, али и едукаторима да изучавају образовање у прошлости у српским школама.. Читалац се већ простим прегледањем ових књига може одмах уверити да у том погледу Србија није била изолована, напротив, школски систем и образовање у Србији, бар кад је реч о математичким наукама, били су сагласни оним најбољим у Европи. Ралог за то можемо потражити у чињеници да су најученији Срби из 18. и 19. века углавном живели и школовали се у дијаспори, пре свега у Аустроугарској. Исто тако увериће се да није исправно често присутно мишљење, да су савремени уџбеници бољи од оних у прошлости. Напротив, садржаји у већини старих уџбеника представљани су на занимљив начин, методички исправно, уз пуно уредних графичких прилога и лепих илустрација. Примери и задаци нису били уско везани за изложену тему, већ су често илустровали примене у другим наукама, на пример у геодезији и астроно-

мији, ако је тема била из тригонометрије. Наводили су се извори задатака, затим имена математичара - аутора важнијих теорема, а често можемо наћи и доста детаљне историјске напомене. У овим књигама обрађиване су важне теме које се данас изостављају из наставе математике у средњим школама, а неких од њих нема нити у настави на факултетима. Споменимо само два таква примера, решавање кубне једначине и сферну тригонометрију. И сви ови садржаји били су написани у књигама невеликог обима, обично око стотинак страница, ретко кад коју десетину страница преко тога.

Ове књиге занимљиве су, нарочито оне из старијих времена, јер у њима можемо пратити развој српског језика, бар када је реч о оном делу који се односио на математичке науке. Оне најстарије, јер су писане на старосрпском језику и предвуковским писмом. Споменимо овде Аритметику Василија Дамјановића, прву књигу из математике штампану на српском (Венеција, 1767) и Численицу Јована Дошеновића (Будимпешта, 1809), уџбеник који се користио на Великој школи у ослобођеном Београду неколико година после Првог српског сутанка. У оним нешто новијим, уверавамо се да се терминологија мењала, и није увек једноставно препознати на шта се одређен термин односио. То је свакако предмет посебног изучавања, али споменимо, на пример, да се реч казаљка користила све до Првог светског рата за данас савремени термин индекс, слично речи склоп за скуп, прецрт за пресликавање, стваран број за реалан број, космографија често за астрономију. Најзад споменимо ћириличне фонтове којим су писане ове књиге. У основном тексту то су лепа слова Ћирилице, укључујући све врсте фонтова, курзив и masna, и величину, од оних великих која су се користила за наслове до текстова у петиту. Латинично писмо у којем је записиван математички текст, било је сагласно по облику и величини основном тексту. Све то доприносило је естетској вредности књига у графичком погледу и лакоћи читања. Споменимо да су оловне плоче – носачи слова, које су се користиле у штампи пре Другог светског рата претопљене, тако да је једини извор ових слова штампани материјал. Дигаталне копије ових текстова представљају добар извор за реконструкцију старих фонтова и обогаћивање презентације Ћириличног писма.

У овој прилици одлучили смо се да представимо четири уџбеника из математике Ристе Карљиковића, који су се користили у средњим школама између два светска рата, не само у Србији, већ и у другим деловима некадашње Краљевине Југославије. Напреднија поглавља у овим књигама користила су се у гимназијама реалкама, које одговарају данашњим гимназијама природно-математичког смера. У његовим уџбеницима види се заиста висок ниво наставе математике. По нашем мишљењу, ове књиге најбоље представљају уџбеничку литературу из математике у Србији у овом периоду. Запарво, његове књиге приказују како је изгледала математика која се предавала у Европи у двадесетим и тридесетим годинама двадесетог века. Прегледањем ових књига можемо се уверити да су то квалитетни уџбеници и да неке од њих, нарочито његова Тригонометрија, може бити занимљива за читање и коришћење у настави и данас.

О Ристи Карљиковићу не знамо много. Знамо толико да је био гимназијски професор математике и директор II Женске гимназије у Београду између два светска рата. Пронашли смо да је написао четири гимназијска уџбеника Тригонометрију, Алгебру, Аналитичку геометрију и Геометрију.


РИСТА КАРЉИКОВИЋ
ДИРЕКТОР II ЖЕН. ГИМН. У БЕОГРАДУ

ГЕОМЕТРИЈА

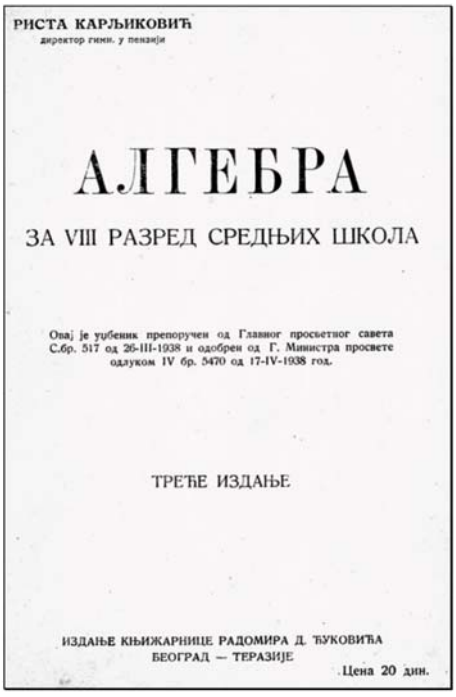
ЗА ВИШЕ РАЗРЕДЕ СРЕДЊИХ ШКОЛА

ТРЕЋИ ДЕО
ТРИГОНОМЕТРИЈА

ДРУГО ПОПРАВЉЕНО И ДОПУЊЕНО ИЗДАЊЕ

Овај је уџбеник препоручен од Главног просветног савета и одобрен од Г. Министра просвете одлуком С.н.бр. 23235 од 6 августа 1931 год.

ИЗДАЊЕ КЊИЖАРНИЦЕ РАЈКОВИЋА И ЂУКОВИЋА
БЕОГРАД — ТЕРАЗИЈЕ


РИСТА КАРЉИКОВИЋ
директор гимн. у пензији

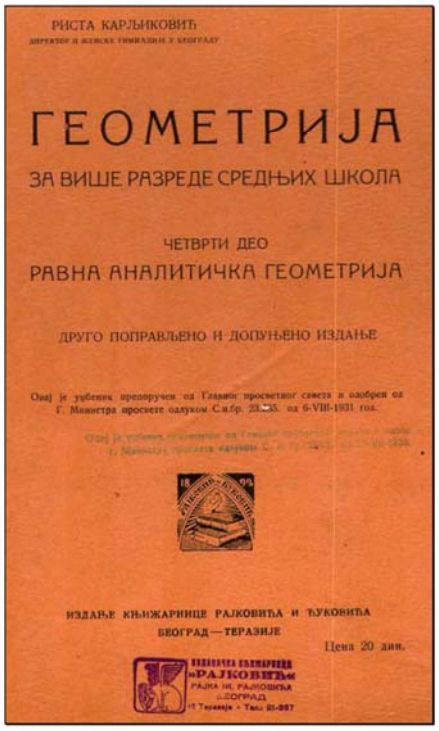
АЛГЕБРА

ЗА VIII РАЗРЕД СРЕДЊИХ ШКОЛА

Овај је уџбеник препоручен од Главног просветног савета С.бр. 517 од 26-III-1938 и одобрен од Г. Министра просвете одлуком IV бр. 5470 од 17-IV-1938 год.

ТРЕЋЕ ИЗДАЊЕ

ИЗДАЊЕ КЊИЖАРНИЦЕ РАДОМИРА Д. ЂУКОВИЋА
БЕОГРАД — ТЕРАЗИЈЕ
Цена 20 дин.


РИСТА КАРЉИКОВИЋ

ГЕОМЕТРИЈА

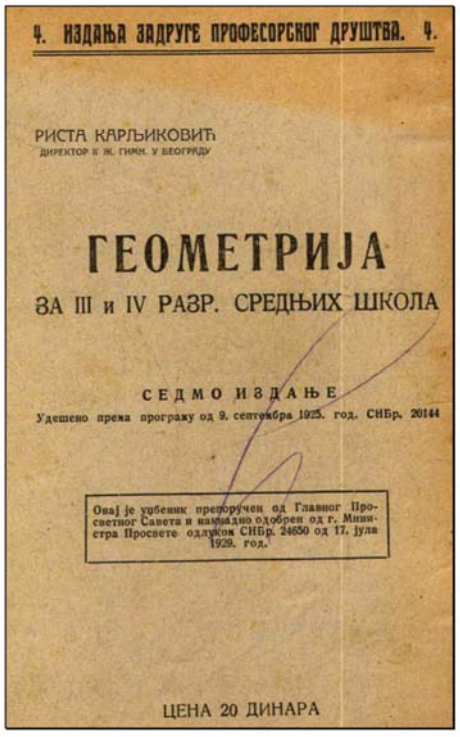
ЗА ВИШЕ РАЗРЕДЕ СРЕДЊИХ ШКОЛА

ЧЕТВРТИ ДЕО
РАВНА АНАЛИТИЧКА ГЕОМЕТРИЈА

ДРУГО ПОПРАВЉЕНО И ДОПУЊЕНО ИЗДАЊЕ

Овај је уџбеник препоручен од Главног просветног савета и одобрен од Г. Министра просвете одлуком С.н.бр. 23235 од 6-VIII-1931 год.

ИЗДАЊЕ КЊИЖАРНИЦЕ РАЈКОВИЋА И ЂУКОВИЋА
БЕОГРАД—ТЕРАЗИЈЕ
Цена 20 дин.


4. ИЗДАЊА ЗАДРУГЕ ПРОФЕСОРСКОГ ДРУШТВА. 4.

РИСТА КАРЉИКОВИЋ
ДИРЕКТОР X. Ж. ГИМН. У БЕОГРАДУ

ГЕОМЕТРИЈА

ЗА III и IV РАЗР. СРЕДЊИХ ШКОЛА

СЕДМО ИЗДАЊЕ
Удешено према програму од 9. септембра 1925. год. СИБр. 20144

Овај је уџбеник препоручен од Главног Просветног Савета и накнадно одобрен од г. Министра Просвете одлуком СИБр. 24650 од 17. јула 1929. год.

ЦЕНА 20 ДИНАРА

Тригонометрија Ристе Карљиковића користила се као уџбеник за више разреде средњих школа. У оквиру предмета Геометрија представљала је њен трећи део. Књигу је препоручио као уџбеник Главни просветни савет и одобрио тадашњи Министар просвете одлуком С.н. бр. 23235 од 6 августа 1931 године. Књигу је издала Књижарница Рајковића и Ћурковића у Београду. Такође нам је

познато да је књига доживела више издања и да се користила до почетка Другог светског рата. Одлучила сам се да представим ову књигу имајући у виду њен, делимично необичан садржај. Док се у првим деловима ове књиге излажу стандардни садржаји тригонометрије последњи део посвећен је Сферној тригонометрији. Књига се састоји из следећих поглавља: Увод, Гониометрија, Равна тригонометрија и Сферна тригонометрија.

13) *Наћи отстојање једног небеског тела (месеца, сунца) до земљиног средишта.*

Нека тачка O (сл. 57) претставља центар Земље, EK пречник екватора, а кружна периферија меридијан места B и C на Земљиној површини одакле се посматрање врши. Ако су углови $BOK = \varphi$ и $COK = \varphi'$, географске ширине места B и C, а углови Z и Z' зенитне раздаљине небеског тела, који се углови одређују у истом тренутку од посматрача када се небеско тело налази над меридијаном места B и C, R полупречник Земље, онда се раздаљина $OS = d$ одређује из четвороугла $SBOC$ помоћу количина: R, φ, φ', Z и Z'. Ако означимо углове BSO и CSO са x и y, онда најпре одређујемо те углове помоћу њиховог збира и њихове разлике.

Њихов збир из четвороугла $SBOC$ јесте:

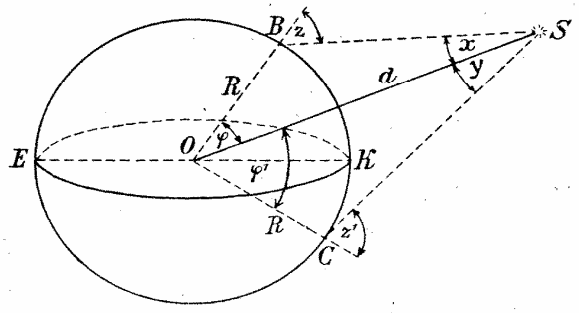
$$x+y=360^0-(180^0-z)-(180^0-z')-(\varphi+\varphi')=(z+z')-(\varphi+\varphi')\ldots(1)$$

Њихову разлику одређујемо на следећи начин: Из $\triangle SOB$ имамо $R : d = sinx : sin\ (180^0 - z)$, а из $\triangle SOC$ имамо

$$R : d = siny : sin\ (180^0 - z').$$

Упоређивањем ових двеју пропорција налазимо да је

$$sinx : sinz = siny : sinz', \text{ или } sinx : siny = sinz : sinz'.$$



Сл. 57

Применом изведених пропорција добијамо:

$$(sinx + siny) : (sinx - siny) = (sinz + sinz') : (sinz - sinz'), \text{ или}$$

$$2\sin\frac{x+y}{2}\cos\frac{x-y}{2} : 2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = 2\sin\frac{z+z'}{2}\cos\frac{z-z'}{2} :$$

$$: 2\cos\frac{z+z'}{2}\sin\frac{z-z'}{2}, \text{ или } tg\frac{x+y}{2} : tg\frac{x-y}{2} = tg\frac{z+z'}{2} : tg\frac{z-z'}{2}.$$

Одавде је

$$tg\frac{x-y}{2} = tg\frac{\frac{x+y}{2}\cdot tg\frac{z-z'}{2}}{tg\frac{z+z'}{2}} = \frac{tg\frac{z+z'-\varphi-\varphi'}{2}\,tg\frac{z+z'}{2}}{tg\frac{z+z'}{2}}.$$

Из ове једначине, употребом логаритама, налазимо угао $(x - y)$. За $x - y = \omega$ и $x + y = (z + z') - (\varphi + \varphi') = \varepsilon$, налазимо да је $x = \frac{\omega+\varepsilon}{2}$ и $y = \frac{\varepsilon-\omega}{2}$.

Најзад из троугла SBO и SCO налазимо да је:

$$d = \frac{R\sin z}{sinx}, \text{ или } d = \frac{R\sin z'}{siny}.$$

Бројни пример. Зенитни угао Месеца измерен у Берлину (северна географска ширина $\varphi = 52^0\ 31'\ 33''$) је $z = 32^0\ 3'\ 51''$, а зенитни угао месеца, измерен у истом тренутку на гребену Добре Наде (јужна географска ширина $\varphi' = 33^0\ 56'\ 3''$), је $z' = 55^0\ 42'\ 48''$; наћи одстојање месеца до центра наше Земље

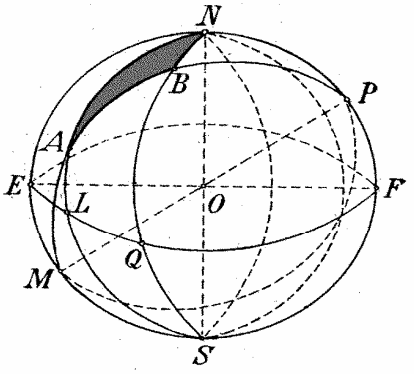
Занимљиво је да је у поглављу Равна тригонометрија дата примена тригонометрије на решавање задатака из стереометрије као и примена тригонометрије на решавање задатака из практичне геометрије и астрономије. То се нарочито може видети у задацима под бројевима 7, 8, 9, 10, 11, 12, 13, 14 и 15. Издвојамо задатак 13, стр. 110, у којем је наведен поступак за одређивање даљине Месеца од Земље (види горе).

Напомена. Аристарх је већ у III веку пре нове ере нашао геометријску методу да одреди Месечеву даљину, али вредност коју је тако добио била је превише груба. Прво тачно одређивање даљине једног небеског тела на примеру Месеца урадили су француски астрономи Лаланд и Лакај у XVIII веку, управо мерећи Месечеве координате с крајева основице која се протезала од Рта Добре Наде до Берлина. Видимо да је Карљиковић узео управо овај бројни пример.

Истичемо поглавље Сферна тригонометрија, у којем су описани сферни троуглови, њихове врсте и особине. Затим је дато решавање елемената правоуглог сферног троугла, решавање елемената косоуглог сферног троугла, као и примена сферне тригонометрије у астрономији и геодезији. Тако, на страни 140 Тригонометрије, имамо пример одређивања сферне раздаљине између два места на Земљи .

III. Сферна раздаљина између два места на земљи

Да бисмо израчунали сферну раздаљину места A и B на земљи, треба да знамо географске ширине и географске дужине тих места. Ако су φ_1 и λ_1 географска ширина и дужина места A, φ_2 и λ_2 географска ширина и дужина места B, N и S полови земље, круг $NESF$ главни меридијан, а круг $ELQF$ екватор, кругови $NALS$ и $NBQS$ меридијани места A и B, онда је: $\varphi_1 = AL$, $\lambda_1 = EL$, $\varphi_2 = BQ$ и $\lambda_2 = EQ$, тада је тражена сферна раздаљина AB страна сферног троугла ABN у коме знамо две стране NA и NB ($NA = 90^0 - \varphi_1$ и $NB = 90^0 - \varphi_2$) и захваћени угао $N = \lambda_2 - \lambda_1$. Задатак се, дакле, своди на пети случај решавања сферног троугла из претходног параграфа.



Сл. 70

Напомена. Ако желимо да израчунамо сферну раздаљину двају места на земљи не у степенима већ у дужинској јединици (у метрима, километрима, миљама), онда примењујемо пропорцију:

$$40000000 : x = 360^0 : \sphericalangle AB, \quad \text{одакле је:}$$

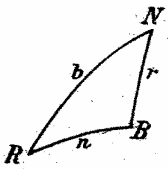
$$x = \frac{40000000 \cdot \sphericalangle AB}{360^0}\, m = \frac{40000 \cdot \sphericalangle AB}{360^0}\, km,$$

где је 40000000 обим великог круга у метрима (приближно), x растојање места A и B у метрима, а $\sphericalangle AB$ растојање места (страна сферног троугла NAB у степенима).

Поред општег решења, Карљиковић наводи и рачунски пример решавања сферног троугла. Пример се односи на одређивање раздаљине између два места на

Земљи. Карљиковић овде узима да су географске дужине Рима и Беча позитивне. Дакле, већ 1930, много раније него што је Међународна унија усвојила ту конвенцију 1977. године.

Пример. Наћи сферну раздаљину између Рима и Беча, ако је геогр. ширина Рима $\varphi_1 = 41^0\,53'\,54''$ *а Беча* $\varphi_2 = 48^0\,12'\,35''$, *геогр. дужина Рима* $\lambda_1 = 12^0\,28'\,48''$, *а Беча* $\lambda_2 = 16^0\,22'\,42''$, *рачунајући од гриничког меридијана.*



Сл. 71

Ако сферни пол N, Рим (R) и Беч (B) дају сферни троугао NRB, онда је страна $NR = 90^0 - \varphi_1 = 48^0\,6'\,16''$, страна $NB = 90^0 - \varphi_2 = 41^0\,47'\,25''$, а угао код $N = \lambda_2 - \lambda_1 = 3^0\,53'\,54''$.

Тада је :

$$tg\frac{B+R}{2} = \frac{\cos\frac{b-r}{2}}{\cos\frac{b+r}{2}}\operatorname{cotg}\frac{N}{2} \text{ и } tg\frac{B-R}{2} = \frac{\sin\frac{b-r}{2}}{\sin\frac{b+r}{2}}\operatorname{cotg}\frac{N}{2} \text{ или}$$

$$tg\frac{B+R}{2} = \frac{\cos 3^0\,9'\,25{,}5''\cdot \operatorname{cotg} 1^0\,56'\,57''}{\sin 44^0\,56'\,50{,}5''} \text{ и}$$

$$tg\frac{B-R}{2} = \frac{\sin 3^0\,9'\,25{,}5''\cdot \operatorname{cotg} 1^0\,56'\,57''}{\sin 45^0\,56'\,50{,}5''};$$

$$\log tg\frac{B+R}{2} = 1{,}48340 \text{ и } \log tg\frac{B-R}{2} = 0{,}79754;$$

$$\frac{B+R}{2} = 88^0\,7'\,6''; \frac{B-R}{2} = 80^0\,56'\,38''; \quad B+R = 176^0\,14'\,12'',$$

$$B-R = 161^0\,53'\,16''; \; B = 169^0\,3'\,44'', \; R = 7^0\,10'\,28''.$$

$$tg\frac{n}{2} = \frac{\sin\frac{B+R}{2}}{\sin\frac{B-R}{2}}tg\frac{b-r}{2} = \frac{\sin 88^0\,7'\,6''\cdot tg\,3^0\,9'\,25{,}5''}{\sin 80^0\,56'\,38''}; \; \log tg\frac{n}{2} =$$

$= 2{,}74682; \frac{n}{2} = 3^0\,11'\,42''; n = 6^0\,23'\,24''$. Дакле, сферна раздаљина Беч—Рим износи $6^0\,23'\,24''$, или у дужинској јединици:

$$RB = \frac{40000000\cdot 6^0\,23'\,34''}{360^0} = \frac{40000000\cdot 23004}{1\,296\,000} =$$

$= 710\,000\,m = 710\,km.$

Напомена. На сличан начин се одређује и сферна раздаљина између било која два небеска тела на небеској сфери..

Карљиковић у књизи Тригонометрија поред занимљивих примера даје велики број задатака. За неке задатке, нарочито теже, наводи њихово порекло и годину. Углавном су то задаци који су се давали на матури у предратној Југославији (Београд, Ужице, Карловац, Приштина, Шибеник, итд) и Француској (Sorbonne, Caen, Marseille, итд). Неке познате теореме наводи по имену аутора: Косинусна теорема – Карнотова теорема, Гасуове обрасце – Молвајдове једначине. Занимљиво је да Херонов образац за површину троугла назива и Брамагуптов образац. У књизи се појављују речи које су данас готово изгубљене, на пример речи космографија и гониометрија.

Сферна тригонометрија која је представљена у Карљиковићевој књизи предавала се само у гимназијама реалкама, које би сада одговарале гимназијама природноматематичког смера. Данас се ови садржаји не предају у средњим школама.

Алгебра Ристе Карљиковића користила се као уџбеник у завршном разреду средњих школа пре Другог светског рата. Књигу је прихватио Главни просветни

савет и одобрио тадашњи Министар просвете одлуком IV бр. 5470 од 17. априла 1938 године. Уџбеник је издала Књижарница Ђурковића у Београду. Књига је доживела више издања а дигитализовано је треће издање. Карљиковићева Алгебра садржи следећа поглавља: Терија извода и појам максимума и минимума, Основи интегралног рачуна, Комплексни бројеви, Моавров образац, једначине трећег степена и Додатак. Најзанимљивије поглавље у овој књизи је Комплексни бројеви, Моавров образац, једначине трећег степена које се предавало само у гимназијама реалкама. Овде се излаже тригонометријски облик комплексног броја као и операције са комплексним бројевима. За степеновање комплексног броја користио је Моавров образац и дата је његова примена за израчунавање функција n-тостроког угла помоћу синуса и косинуса тог угла.

Из свега овога види се да једначина $x^3+px+q=0$ има један корен стваран а два имагинарна, ако је испуњен услов $27q^2+4p^3>0$. Образац:

$$x=\sqrt[3]{\frac{-27q+\sqrt{729q^2+108p^3}}{54}}+\sqrt[3]{\frac{-27q-\sqrt{729q^2+108p^3}}{54}},\ \text{или}$$

$$x=\sqrt[3]{\frac{-9q+\sqrt{81q^2+12p^3}}{18}}+\sqrt[3]{\frac{-9q-\sqrt{81q^2+12p^3}}{18}},\ \text{или}$$

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}\quad(11)$$

зове се ***Кардан-ов.*** Овај образац употребљава се при добивању корена оне једначине трећег степена облика $x^3+px+q=0$, која има имагинарне корене, за шта је потребан услов $\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3>0$ или $27q^2+4p^3>0$. Овај се образац не примењује при решавању једначине $x^3+px+q=0$, који има само

У књизи Карљиковић излаже решавање кубне једначине, садржаја који се данас не учи у средњим сколама. Наводи разне облике једначина трећег степена и њихове особине, врсте корена (сва три реална или један реалан и два комплексна конјугована броја) и веза између корена и коефицијената. Изложен је поступак свођења опште једначине трећег степена на простији облик. Изведен је Карданов образац који је користио за добијање сва три корена једначине трећег степена.

Напомена. Карљиковић у поглављу Додатак наводи да Карданов образац неправедно носи назив према Кардану и да је стварни творац овог обрасца Тартаљ.

У истом поглављу Карљиковић даје приказ историјског развитка аритмеике и алгебре, почев од Феничана, Египћана, Индијаца, Вавилонаца и Халдејаца, што је неуобичајено у новијим средњошколским уџбеницима. Тако сазнајемо да су Феничани творци аритметике, Египћани геометрије, Халдејци и Вавилонци астрономије. Даље, Карљиковић пише да најстарија пронађени аритметички списи воде порекло из Египта и да су они писани хијероглифским цифрама на папирусу између 20. и 17. века пре нове ере. Пише да се први елементи алгебре налазе код Еуклида у његовом делу Елементи, и да је прво дело из алгебре објавио у 4. веку Диофант Александријски (325–409). Карљиковић потом излаже развој алгебре у средњем веку и наводи да је Италијан Леонардо из

Пизе пренео у 12. веку арапску и индијску аритметику и алгебру у Западну Европу, док је творац модерне алгебре Франсоа Вијет (1540–1603), творца алгебарског симболизма, што је знатно упростило математичке операције и подигло ниво алгебре. Такође излаже доприносе других математичара тог времена, да је Декарт (1596–1650) творац аналитичке геометрије и да је усавршио примену алгебре у геометрији. Описује доприносе и математичара из каснијих вемена, да су Лајбниц и Њутн у 17. веку створили теорију инфинитезималног рачуна и да у 18. и 19. веку постоји читава плејада славних математичара: Ојлер, Даламбер, Безут, Лаплас, Лагранж, Монж, Карно, Аргон, Муреј, Гаус, Коши и Јакоби.

Карљиковић на крају књиге даје и кратак преглед увођења математичких симбола. Тако сазнајемо да су данашње цифре индијског порекла и да су у Европу стигле преко Арабљана, па стога и носе назив арапске цифре. Затим да је знакове сабирања и одузимања увео Леонардо из Пизе, а да се први пут се у штампаном облику појављују у Немачкој 1489, да је знак једнакости увео Рекорд 1552, а знаке неједнакости веће и мање (>, <) увео Хариот 1600 и знак различитости (≠) Христофен.

Аналитичка геометрија Ристе Карљиковића користила се као уџбеник за више разреде средњих школа. Књигу је препоручио као уџбеник Главни просветни савет а одобрио тадашњи Министар просвете одлуком С.н. бр. 23235 од 6. августа 1931. године. Књигу је издала Књижарница Рајковића и Ђуковића у Београду. Књига је доживела више издања, дигитализовано је друго поправљено и допуњено издање.

Књига се састоји од следећих поглавља: Увод, Тачка, Права линија, Круг, Елипса, Хипербола, Парабола и Тангенте и нормале кривих линија.

Неке параграфе у књизи обележава звездицом, јер су били предвиђени само за ученике Реалке, као на пример: Поларна једначина елипсе, Поларна једначина хиперболе, Поларна једначина параболе, Полови и поларе кривих линија, општи облик једначина кривих другог реда.

Геометрија Карљиковића користила се као уџбеник у нижим разредима средњих школа до Другог светског рата. Књигу је као уџбеник препоручио Главни просветни савет а одобрио тадашњи Министар просвете одлуком С.н. бр. 24650 од 17. јула 1929. године. Књигу је издала штампарија Јовановић у Београду. Уџбеник Геометрија је доживео велики број издања, дигитализовано је седмо издање. У књизи су покривени стандардни садржаји из геометрије за ђачки узраст.

Закључак

Карљиковићеве књиге су методички лепо написане и градиво се у њима постепено уводи. Карљиковић даје много урађених примера тако да пратећа збирка није ни поребна. Посебно бих истакла једноставност извођења основних теорема Сферне тригонометрије у његовој Тригонометрији, као и налажење решења кубне једначине у његовој Алгебри. Тај део математике био је приступачан у сваком погледу ђацима гимназија реалке у оно време. Штампарских грешака у његовим књигама готово да нема.

Имајући у виду методички начин писања књига, њихов сдржај, примере и задатке, препоручила бих ове књиге и данашњим ђацима, средњошколким

професорима па и студентима. Књиге су дигитализоване и налазе се на интернет страници Математичког факултета у оквиру виртуелне библиотеке на адреси http://elib.matf.bg.ac.yu:8080/virlib/.

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Nadežda Pejović

DIGITIZATION OF HIGHSCHOOL TEXTBOOKS OF RISTA KARLJIKOVIĆ

Abstract. Rista Karljiković was a gymnasium professor between two World wars. He wrote several textbooks on mathematics that were used in Serbian secondary schools at that time, (1922–1941). Four of his books: Trigonometry, Algebra, Analytical geometry and Geometry are digitized and they can be found in the electronic form in the Virtual Library, http://elib.matf.bg.ac.yu:8080/virlib/, of the National Center for Digitization. Retro-digitization of these books is a part of the project on archiving of Serbian mathematical books printed in the past. We explain in this article why Karljiković's books were selected for digitization and inclusion in the Virtual Library. It appears that Karljiković's books satisfied very high scholar standards and that they represent very well classroom mathematics taught in the most developed European countries in this period. Also, contents of the books are shortly presented.

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47 Sferna trigonometrija Riste Karljikovića

N. Pejović,
Publ. Astron. Druš. Rudjer Bošković,
Beograd, vol. 7, pp. 189 - 198, 2007.
Zb. konf. *Razvoj astronomije kod Srba* IV,
Beograd, 22-26 april 2006, (M. Dimitrijević, ed).

The book *Trigonometry* of Rista Karljiković was used as a textbook in Serbian gymnasiums between two World wars (1924-1941). Interestingly, the significant part of the book is devoted to spherical trigonometry and it's usage in astronomy and geodesy. This part of mathematics does not exist in contemporary teaching of mathematics, neither in secondary schools, nor in university lessons.

Зборник радова конференције "Развој астрономије код Срба IV"
Београд 22-26. април 2006,
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 7, 2007, 189-198

СФЕРНА ТРИГОНОМЕТРИЈА РИСТЕ КАРЉИКОВИЋА

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Резиме. Књига *Тригонометрија* Ристе Карљиковића користила се као уџбеник за гимназије реалке између два светска рата (1924-1941). Занимљива је чињеница да је значајан њен део посвећен Сферној тригонометрији и њеним применама у астрономији и геодезији. Ови садржаји данас не постоје у настави математике не само у средњим школама, већ не постоје нити на факултетима.

Тригонометрија Ристе Карљиковића користила се као уџбеник за више разреде средњих школа између два светска рата. У оквиру предмета Геометрија представљала је њен трећи део. О аутору знамо толико да је био директор II Женске гимназије у Београду. Књигу је препоручио као уџбеник Главни просветни савет а на другом поправљеном и допуњеном издању, које сам користила, пише да га је одобрио тадашњи Министар просвете одлуком С.н. бр. 23235 од 6. августа 1931. године. Књигу је издала Књижарница Рајковића и Ћурковића у Београду. Такође нам је познато да је Књига доживела више издања и да се користила до почетка Другог светског рата.

Одлучила сам се да представим ову књигу имајући у виду њен, делимично необичан садржај. Док се у првим деловима ове књиге излажу стандардни садржаји тригонометрије последњи део посвећен је Сферној тригонометрији.

Књига се састоји из следећих поглавља: *Увод, Гониометрија, Равна тригонометрија и Сферна тригонометрија.*

У *Уводу* аутор говори о функцијама уопште, а посебно о гониометријским или тригонометријским функцијама. Затим о израчунавању функција углова од тридесет, шесдесет и четрдесет пет степени, као и о задатку тригонометрије и њеној подели. Запазила сам да аутор углавном користи назив гониометријске функције који се до данас готово изгубио и уместо тога се користи назив тригонометријске функције.

У првом поглављу, *Гониометрија,* аутор даје графичко представ-љање тригонометријских функција код круга полупречника $r=1$ (тригоно-

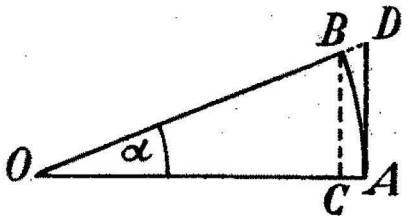
метријски круг), међусобни однос тригонометријских функција истог угла, као раст и опадање тригонометријских функција кад угао расте, затим формуле за претварање тригонометријских функција неоштрих углова у тригонометријске функције оштрих и негативних углова. Дате су адиционе теореме два угла као и идентитети за претварање збирова и разлика тригонометријских функција у производ ради њиховог логаритмовања. Доказује разне друге тригонометријске идентитете, даје поступак за израчунавање вредности тригонометријских функција, као и израчунавање помоћу њихових логаритама и наводи примере решавања тригонометријских једначина.

Посебно бих нагласила са којом тачношћу Карљиковић врши израчунавање вредности тригонометријских функција. Занимљиво је да су ђаци учили израчунавање ових вредности коришћењем њихових аналитичких особина. На пример, на страни 42 налазимо коришћење неједнакости за угао $0 < \alpha < 90^0$

$$\textbf{arc } \alpha - \textbf{sin } \alpha < \textbf{arc } \alpha \textbf{ sin}^2(\alpha/2) \qquad (1)$$

Наиме, аутор показује да је израчунавање тригонометријских функција елементарном методом засновано на принципу: *да је код круга полупречника r= 1, разлика између tangens-a, sinus-a и arcus-a, када је средишњи угао сасвим мали, незнатна.*

На слици 25 је дато AD=tg α, и BC= sin α и AB= arc α. Са слике се види да је разлика између тангенса AD, аркуса AB и синуса BC утолико мања у колико је средишњи угао α мањи. За α= 1′ разлика између вредности tg 1′, sin 1′ и arc 1′ је готово једнака нули. Аутор доказује да је *разлика између аркуса и синуса једног оштрог угла мања од четвртине трећег степена аркуса тога угла.*



Сл. 25

Да би ово показао аутор полази од чињенице да је лук оштрог угла мањи од његовог тангенса, тј. да је $\text{arc}(\alpha/2) < \text{tg}(\alpha/2)$. Ако обе стране ове неједнакости помножи са $2\cos^2(\alpha/2)$ добија

$$\textbf{2arc}(\alpha/2)\textbf{cos}^2(\alpha/2) < \textbf{2tg}(\alpha/2)\textbf{cos}^2(\alpha/2), \text{ или } \textbf{arc } \alpha - \textbf{sin } \alpha < \textbf{arc}(\alpha)\textbf{sin}^2(\alpha/2).$$

неједнакост која је наведена под бројем (1). Ако се овде замени $\sin(\alpha/2)$ са $\text{arc}(\alpha/2)$ онда она постаје већа (видети сл. 25). Тада је тим пре њена лева страна мања од $\text{arc}(\alpha)(\text{arc}(\alpha)/2)^2$ када је била мања од $\text{arc}(\alpha)\sin^2(\alpha/2)$. Даље Карљиковић пише:

Стога је заиста:

$$arc\ \alpha - sin\ \alpha < \frac{(arc\ \alpha)^3}{4} \ldots . \quad (2)$$

Узимајући да је $\alpha = 1'$ имамо $arc\ 1' = \frac{r\pi\alpha}{180^0} = \frac{1 \cdot 3{,}14 \cdot 1'}{10800'} =$ $= 0{,}0002908882$. Тада према неједначини (2) имамо:

$$arc\ 1' - sin\ 1' < \frac{0{,}0002908882^3}{4} \ldots . . \quad (3)$$

Па како је $0{,}0002908882^3 < 0{,}0000000001$, то је тим пре:

$$arc\ 1' - sin\ 1' < 0{,}000000000025 \ldots . \quad (4)$$

Ова нам неједначина показује да је разлика између $arc\ 1'$ и $sin\ 1'$ тако мала да је без велике грешке сматрамо равном нули. Стога је $arc\ 1' = sin\ 1'$. А како смо нашли да је $arc\ 1' = 0{,}0002908882$, то је и

$$\boldsymbol{sin\ 1' = 0{,}0002908882.}$$

Помоћу основних образаца налазимо из једначине:

$sin\ 1' = 0{,}0002908882$ и $cos\ 1'$, $tg\ 1'$ и $cotg\ 1'$ а затим, употребом образаца за функције збира двају углова и образаца за функције удвојених углова, налазимо: $sin\ 2'$, $sin\ 3'$, $sin\ 4'$, ... ; $cos\ 2'$, $cos\ 3'$, $cos\ 4'$, ... ; $tg\ 2'$, $tg\ 3'$, $tg\ 4'$, ... ; итд. док не добијемо вредности функција свију углова закључно до 45^0.

Да бисмо одредили вредности функција углова који имају само секунде, можемо утолико пре да заменимо *arcus* угла са *sinus*-ом тога угла. Па како је $arc\ 1'' = \frac{arc\ 1'}{60}$, то је и

$$sin\ 1'' = \frac{0{,}0002908882}{60} = 0{,}0000048481 \ldots .$$

Тада је: $sin\ 2'' = 2 \cdot sin\ 1''$, $sin\ 3'' = 3 \cdot sin\ 1''$, $sin\ 4'' = 4 \cdot sin\ 1''$ итд., а употребом основних образаца израчунавамо и остале функције углова који имају само секунде.

Како су вредности гониометриских функција понајвише ирационалнй бројеви, то су ове вредности утолико тачније израчунате уколико имају више децимала. Али су тада утолико више отежане математичке радње са њима. Да бисмо избегли ову тешкоћу, особито при множењу, дељењу, степеновању и кореновању, служимо се употребом логаритама. Из овога разлога, таблице у којима се налазе вредности гониометриских функција, имају мању примену од таблица у којима се налазе логаритми бројних вредности гониометриских функција.

Напомена. За јединично растојање у Сунчевом систему узима се растојање Земље од Сунца (велика полуоса Земљине путање) и назива се Астрономска јединица, 1AJ = 149 597 870 km. Угао у правоуглом троуглу (Сунце – Земља – звезда) под којим се ово растојање види са неке звезде, назива се паралакса звезде и паралксе су мање од 1". Парсек је растојање до објекта које би имало паралаксу од 1" и та мера се узима за јединично растојање у нашој галаксији. Отуда се у астрономији појављује једнакост 1парсек= 206265AJ. Занимљиво је да Карљиковић у својој књизи даје са високом тачношћу вредност: **sin 1" = 0.0000048481 ≈ 1/206265.**

У другом поглављу *Равна тригонометрија* аутор даје решавање елемената код правоуглог троугла, решавање елемената код једнакокраког троугла, решавање елемената код правилних многоуглова, решавање код

круга, однос између страна и функција углова једнакостраног троугла и израчунавање елемената код четвороуглова. Дата је примена тригонометрије на решавање задатака из стереометрије као и примена тригонометрије на решавање задатака из практичне геометрије.

Занимљиво је да у овом поглављу аутор на страни 107 даје задатке из космографије. То су задаци под бројевима 7, 8, 9, 10, 11, 12, 13, 14 и 15. Издвојила бих следећи задатак (задатак 13, стр. 110).

13) *Наћи отстојање једног небеског тела (месеца, сунца) до земљиног средишта.*

Нека тачка O (сл. 57) претставља центар Земље, EK пречник екватора, а кружна периферија меридијан места B и C на Земљиној површини одакле се посматрање врши. Ако су углови $BOK = \varphi$ и $COK = \varphi'$, географске ширине места B и C, а углови Z и Z' зенитне раздаљине небеског тела, који се углови одређују у истом тренутку од посматрача када се небеско тело налази над меридијаном места B и C, R полупречник Земље, онда се раздаљина $OS = d$ одређује из четвороугла $SBOC$ помоћу количина: R, φ, φ', Z и Z'. Ако означимо углове BSO и CSO са x и y, онда најпре одређујемо те углове помоћу њиховог збира и њихове разлике.

Њихов збир из четвороугла $SBOC$ јесте:

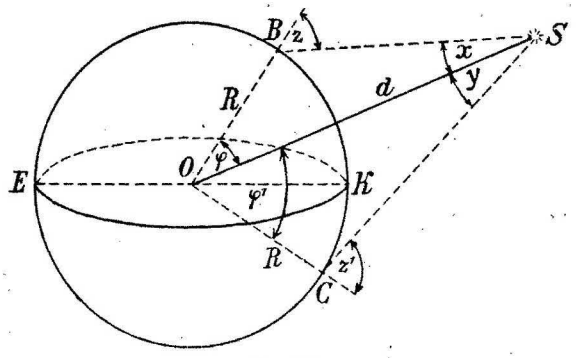
$$x+y=360^0-(180^0-z)-(180^0-z')-(\varphi+\varphi')=(z+z')-(\varphi+\varphi')\ldots(1)$$

Њихову разлику одређујемо на следећи начин: Из $\triangle SOB$ имамо $R : d = sinx : sin\ (180^0 - z)$, а из $\triangle SOC$ имамо

$$R : d = siny : sin\ (180^0 - z').$$

Упоређивањем ових двеју пропорција налазимо да је

$$sinx : sinz = siny : sinz', \text{ или } sinx : siny = sinz : sinz'.$$



Сл. 57

Применом изведених пропорција добијамо:

$$(sinx + siny) : (sinx - siny) = (sinz + sinz') : (sinz - sinz'), \text{ или}$$

$$2\sin\frac{x+y}{2}\cos\frac{x-y}{2}:2\cos\frac{x+y}{2}\sin\frac{x-y}{2}=2\sin\frac{z+z'}{2}\cos\frac{z-z'}{2}:$$

$$:2\cos\frac{z+z'}{2}\sin\frac{z-z'}{2},\ \text{или}\ tg\frac{x+y}{2}:tg\frac{x-y}{2}=tg\frac{z+z'}{2}:tg\frac{z-z'}{2}.$$

Одавде је

$$tg\frac{x-y}{2}=tg\frac{\frac{x+y}{2}\cdot tg\frac{z-z'}{2}}{tg\frac{z+z'}{2}}=\frac{tg\frac{z+z'-\varphi-\varphi'}{2}\,tg\frac{z+z'}{2}}{tg\frac{z+z'}{2}}.$$

Из ове једначине, употребом логаритама, налазимо угао $(x-y)$. За $x-y=\omega$ и $x+y=(z+z')-(\varphi+\varphi')=\varepsilon$, налазимо да је $x=\frac{\omega+\varepsilon}{2}$ и $y=\frac{\varepsilon-\omega}{2}$.

Најзад из троугла SBO и SCO налазимо да је:

$$d=\frac{R\sin z}{\sin x},\ \text{или}\ d=\frac{R\sin z'}{\sin y}.$$

Бројни пример. Зенитни угао Месеца измерен у Берлину (северна географска ширина $\varphi=52^0\,31'\,33''$) је $z=32^0\,3'\,51''$, а зенитни угао месеца, измерен у истом тренутку на гребену Добре Наде (јужна географска ширина $\varphi'=33^0\,56'\,3''$), је $z'=55^0\,42'\,48''$; наћи одстојање месеца до центра наше Земље када је њен полупречник $R=6370{,}308\ km$.

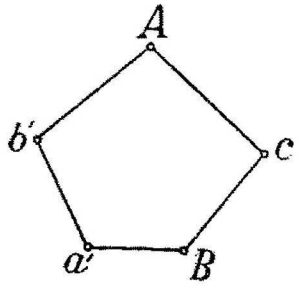
Напомена. Аристарх је већ у III веку пре нове ере нашао геометријску методу да одреди Месечеву даљину, али вредност коју је тако добио била је превише груба. Прво тачно одређивање даљине једног небеског тела на примеру Месеца урадили су француски астрономи Лаланд и Лакај у XVIII веку, управо мерећи Месечеве координате с крајева основице која се протезала од Рта Добре Наде до Берлина. Видимо да је Карљиковић узео управо овај бројни пример.

У трећем поглављу, *Сферна тригонометрија*, излажу се сферни троуглови, њихове врсте и особине. Затим је дато решавање елемената правоуглог сферног троугла, решавање елемената косоуглог сферног троугла, као и примена сферне тригонометрије у астрономији и геодезији.

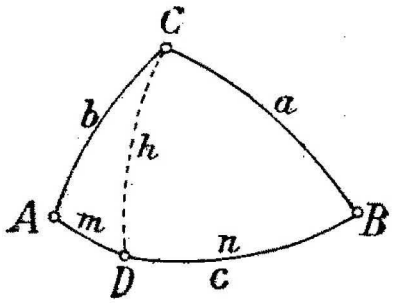
Осврнула бих се посебно на ово поглавље. Наиме, аутор до Неперовог правила долази на једноставан начин, применом равне тригонометрије. Неперово правило гласи: *Косинус ма ког елемента једнак је производу синуса супротних елемената или производу контангенса суседних елемената.* Ово правило се користи за решавање елемената правоуглог сферног троугла. Да би се оно применило, прав угао не треба сматрати елементом а уместо катета треба узети њихове комплементе a'= a - 90^0, b'= b - 90^0. Ових пет елемената представљени су на сл. 64, стр 118.

Да би решио елементе косоуглог сферног троугла Карљиковић на једноставан начин, спуштањем сферне нормале из једног темена, дели косоугли сферни троугао на два правоугла сферна троугла, (видети сл. 66, стр. 123). Применом Неперовог правила на та два правоугла сферна троугла долази до основних теорема сферне тригонометрије. То су синусна теорема и

косинусна теорема за стране и косинусна теорема углове. Исказе ових теорема наводимо онако како их је Карљиковић написао.



Сл. 64



Сл. 66

1) **Синусна теорема:** *У сваком сферном троуглу имају се синуси страна као што се имају синуси њихових супротних углова*

sin(a) : sin(b) : sin(c) = sin(A) : sin(B) : sin(C)

2) **Косинусна теорема**: *У сваком сферном троуглу је косинус једне стране једнак производу косинуса осталих двеју страна више производу синуса тих страна помножен косинусом захваћеног угла.*

cos(a) = cos(b)cos(c) + sin(b)sin(c)cos(A)
cos(b) = cos(a)cos(c) + sin(a)sin(c)cos(B)
cos(c) = cos(a)cos(b) + sin(b)sin(c)cos(C)

3) **Косинусна теорема за углове гласи:** *У сваком сферном троуглу је косинус једног угла једнак негативном производу косинуса осталих углова више производу синуса тих углова помножен косинусом стране на којој се ти углови налазе.*

cos(A) = -cos(B)cos(C) + sin(B)sin(C)cos(a)
cos(B) = -cos(aA)cos(C) + sin(A)sin(C)cos(b)
cos(C) = -cos(A)cos(B) + sin(B)sin(C)cos(c)

Напоменула бих да аутор до ових основних теорема сферне тригонометрије долази на изузетно једноставан начин. У књигама руских аутора, у књизи *Општа астрономија* од Б Шеварлића и З. Бркића као и у књизи *Spherical Astronomy* од Робина Грина до ових теорема се долази доста

компликованије. Да би се добили основни обрасци сферне тригонометрије користи се ротација правоуглог координатног система чији је почетак смештен у центар сфере на којој се налази сферни троугао. Затим се за косинусну теорему за угао уводи појам поларног сферног троугла да би се преко њега дошло до ове теореме.

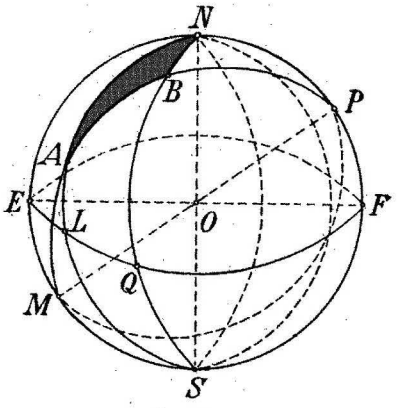
Помоћу основних теорема (1), (2) и (3) аутор долази до сложенијих израза за решавање сферног троугла као што су Гаусове једначине (стране 127 и 128) и Неперове аналогије (стране 129, 130 и 131).

На странама 138 и 139 дати су изрази за израчунавање полупречника описаног круга око сферног троугла и полупречника уписаног круга у сферни троугао.

Примена сферне тригонометрије у астрономији је велика. Аутор у књизи на страни 140 даје пример одређивања сферне раздаљине између два места на Земљи.

III. Сферна раздаљина између два места на земљи

Да бисмо израчунали сферну раздаљину места A и B на земљи, треба да знамо географске ширине и географске дужине тих места. Ако су φ_1 и λ_1 географска ширина и дужина места A, φ_2 и λ_2 географска ширина и дужина места B, N и S полови земље, круг $NESF$ главни меридијан, а круг $ELQF$ екватор, кругови $NALS$ и $NBQS$ меридијани места A и B, онда је: $\varphi_1 = AL$, $\lambda_1 = EL$, $\varphi_2 = BQ$ и $\lambda_2 = EQ$, тада је тражена



Сл. 70

сферна раздаљина AB страна сферног троугла ABN у коме знамо две стране NA и NB ($NA = 90^0 - \varphi_1$ и $NB = 90^0 - \varphi_2$) и захваћени угао $N = \lambda_2 - \lambda_1$. Задатак се, дакле, своди на пети случај решавања сферног троугла из претходног параграфа.

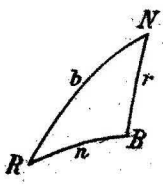
Напомена. Ако желимо да израчунамо сферну раздаљину двају места на земљи не у степенима већ у дужинској јединици (у метрима, километрима, миљама), онда примењујемо пропорцију:

$$40000000 : x = 360^0 : \sphericalangle AB, \quad \text{одакле је:}$$

$$x = \frac{40000000 \cdot \sphericalangle AB}{360^0}\, m = \frac{40000 \cdot \sphericalangle AB}{360^0}\, km,$$

где је 40000000 обим великог круга у метрима (приближно), x растојање места A и B у метрима, а $\sphericalangle AB$ растојање места (страна сферног троугла NAB у степенима).

Пример. Наћи сферну раздаљину између Рима и Беча, ако је геогр. ширина Рима $\varphi_1 = 41^0\,53'\,54''$ *а Беча* $\varphi_2 = 48^0\,12'\,35''$, *геогр. дужина Рима* $\lambda_1 = 12^0\,28'\,48''$, *а Беча* $\lambda_2 = 16^0\,22'\,42''$, *рачунајући од гриничког меридијана.*



Сл. 71

Ако сферни пол N, Рим (R) и Беч (B) дају сферни троугао NRB, онда је страна $NR = 90^0 - \varphi_1 = 48^0\,6'\,16''$, страна $NB = 90^0 - \varphi_2 = 41^0\,47'\,25''$, а угао код $N = \lambda_2 - \lambda_1 = 3^0\,53'\,54''$.

Тада је:

$$tg\frac{B+R}{2} = \frac{\cos\frac{b-r}{2}}{\cos\frac{b+r}{2}} cotg\frac{N}{2} \text{ и } tg\frac{B-R}{2} = \frac{\sin\frac{b-r}{2}}{\sin\frac{b+r}{2}} cotg\frac{N}{2} \text{ или}$$

$$tg\frac{B+R}{2} = \frac{\cos 3^0\,9'\,25{,}5'' \cdot cotg\,1^0\,56'\,57''}{\sin 44^0\,56'\,50{,}5''} \text{ и}$$

$$tg\frac{B-R}{2} = \frac{\sin 3^0\,9'\,25{,}5'' \cdot cotg\,1^0\,56'\,57''}{\sin 45^0\,56'\,50{,}5''};$$

$$\log tg\frac{B+R}{2} = 1{,}48340 \text{ и } \log tg\frac{B-R}{2} = 0{,}79754;$$

$$\frac{B+R}{2} = 88^0\,7'\,6''; \frac{B-R}{2} = 80^0\,56'\,38''; \quad B+R = 176^0\,14'\,12'',$$

$$B-R = 161^0\,53'\,16''; \; B = 169^0\,3'\,44'', \; R = 7^0\,10'\,28''.$$

$$tg\frac{n}{2} = \frac{\sin\frac{B+R}{2}}{\sin\frac{B-R}{2}} tg\frac{b-r}{2} = \frac{\sin 88^0\,7'\,6'' \cdot tg\,3^0\,9'\,25{,}5''}{\sin 80^0\,56'\,38''}; \; \log tg\frac{n}{2} =$$

$= 2{,}74682; \frac{n}{2} = 3^0\,11'\,42''; n = 6^0\,23'\,24''$. Дакле, сферна раздаљина Беч—Рим износи $6^0\,23'\,24''$, или у дужинској јединици:

$$RB = \frac{40000000 \cdot 6^0\,23'\,34''}{360^0} = \frac{40000000 \cdot 23004}{1\,296\,000} =$$

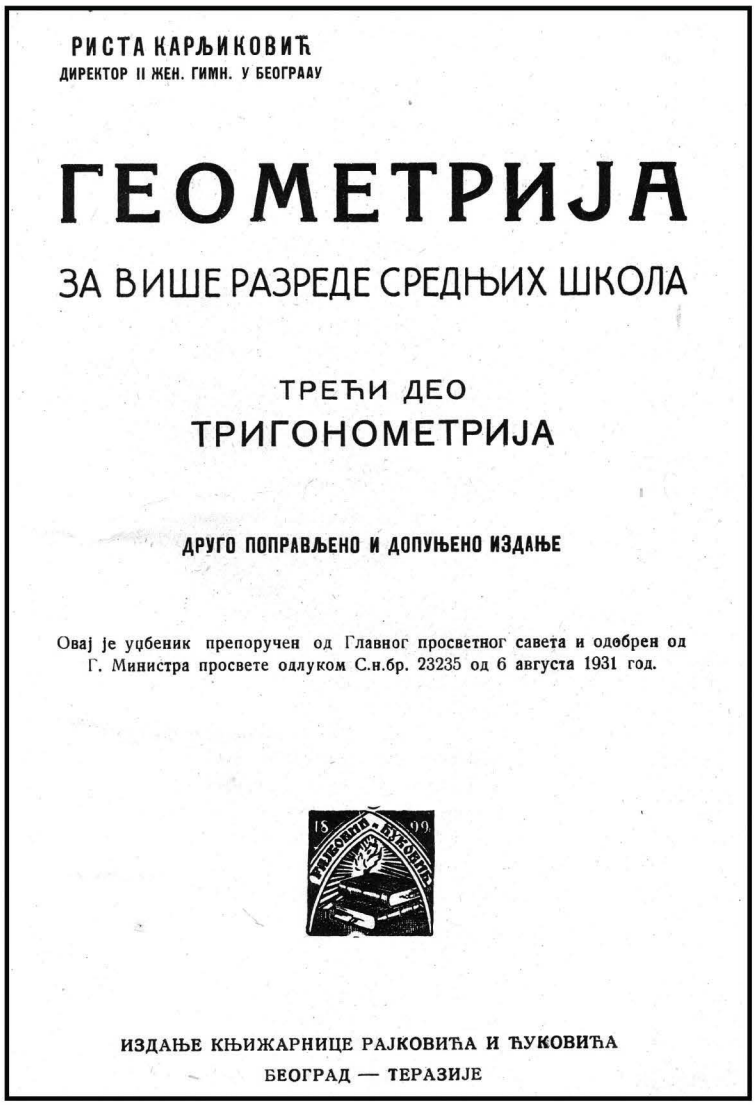
$= 710\,000\,m = 710\,km.$

Напомена. На сличан начин се одређује и сферна раздаљина између два небеска тела на небеској сфери..

Карљиковић у својој књизи поред занимљивих примера даје велики број задатака. За неке задатке, нарочито теже, наводи њихово порекло и годину. Углавном су то задаци који су се давали на матури у предратној Југославији (Београд, Ужице, Карловац, Приштина, Шибеник, итд) и Француској (Sorbonne, Caen, Marseille, итд). Неке познате теореме наводи по имену аутора: Косинусна теорема – Карнотова теорема, Гасуове обрасце – Молвајдове једначине. Занимљиво је да Херонов образац за површину троугла назива и Брамагуптов образац. У књизи се појављују речи које су данас готово изгубљене, на пример речи *космографија* и *гониометрија*.

Споменимо да се сферна тригонометрија која је представљена у Карљиковићевој књизи предавала само у гимназијама реалкама, које би сада

одговарале гимназијама природноматематичког смера. Данас се ови садржаји не предају у средњим школама.



РИСТА КАРЉИКОВИЋ
ДИРЕКТОР II ЖЕН. ГИМН. У БЕОГРААУ

ГЕОМЕТРИЈА

ЗА ВИШЕ РАЗРЕДЕ СРЕДЊИХ ШКОЛА

ТРЕЋИ ДЕО
ТРИГОНОМЕТРИЈА

ДРУГО ПОПРАВЉЕНО И ДОПУЊЕНО ИЗДАЊЕ

Овај је уџбеник препоручен од Главног просветног савета и одобрен од Г. Министра просвете одлуком С.н.бр. 23235 од 6 августа 1931 год.

ИЗДАЊЕ КЊИЖАРНИЦЕ РАЈКОВИЋА И ЋУКОВИЋА
БЕОГРАД — ТЕРАЗИЈЕ

ЗАКЉУЧАК

Књига је методички лепо написана, градиво се постепено уводи. Аутор даје много урађених примера тако да пратећа збирка није ни поребна. Посебно бих истакла једноставност извођења основних теорема Сферне тригонометрије, тако да је тај део математике заиста био приступачан ђацима гимназија реалке у оно време.

Имајући у виду методички начин писања књиге, њен сдржај, примере и задатке, препоручила бих ову књигу и данашњим ђацима, средњошколким професорима па и студентима. Књига је дигитализована и ускоро ће се наћи на интернет страници Математичког факултета у оквиру виртуелне библиотеке на адреси http://alas.matf.bg.ac.yu/biblioteka/home.jsp.

Захвалница

Користим прилику да се захвалим организатору др Милану С. Димитријевићу на позиву за учешће на конференцији *Развој астрономије код Срба IV*. Захваљујући томе, овај текст је написан.

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Шеварлић, Б и Бркић, З.: *Општа астрономија*, Савремена администрација, Београд, 1971, 1981.

SPHERICAL TRIGONOMETRY OF RISTA KARLJIKOVIĆ

The book Trigonometry by Rista Karljiković was used as a textbook in secondary schools (gymnasium) between two World wars (1924-1941). The most interesting fact is that the significant part of the book was devoted to spherical trigonometry and it's usage in astronomy and geodesy. This part of mathematics does not exist in contemporary teaching of mathematics, neither in secondary schools, nor in university lessons.

48 Review of the digitized book *Fundamentals of mathematical and physical geography* by Pavle Vujević

N. Pejović and Ž. Mijajlović,
NCD Review,no. 13, pp. 74 -90, 2008.

The book "Fundamentals of Mathematical and Physical Geography" is one of the first university textbooks on geography written in Serbian. The author is Pavle Vujević. (1881-1966), a notorious Serbian climatologist, professor at the Belgrade University and the member of the Serbian Academy of Science and Arts. For many generations of students of geography this textbook was an entering point into mathematical and physical geography and astronomy. The book, written several years after First World War, is rather comprehensive; it consists of 815 pages and four sections. The first volume is made up of two sections, Mathematical Geography and Physical Geography, while the second volume covers The atmosphere and Oceans. The first book is in fact an introduction into astronomy. In the second volume there are explanations of the chemical composition of the Earths atmosphere, climate types and their secular changes, then composition and physical characteristics of World oceans, the sea-tide phenomena and the characteristics of the seabed. This review of the book also include some interesting facts related to the book, including changes in terminology and definitions in this area since the book was published.

Преглед НЦД 13 (2008), 74–90

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University of Belgrade

REVIEW OF THE DIGITIZED BOOK *FUNDAMENTALS OF MATHEMATICAL AND PHYSICAL GEOGRAPHY* BY PAVLE VUJEVIĆ

Abstract. The book *Fundamentals of Mathematical and Physical Geography* (in Serbian *Основи математичне и физичке географије*) is one of the first university textbooks on geography written in Serbian. The author of the book is Pavle Vujević (1881-1966), a notorious Serbian climatologist, Professor at the Belgrade University and member of the Serbian Academy of Science and Arts. The book consists of two volumes and it was printed several years after the First World War. For many generations of students of geography this textbook was an entering point into mathematical and physical geography and astronomy. This book is rather comprehensive; it consists of 815 pages and four sections. The first volume is made up of two sections, *Mathematical Geography* and *Physical Geography*, while the second volume covers *The atmosphere* and *Oceans*. The first book is in fact an introduction into astronomy. It covers in details all basic notions and facts of this science known at the time when the book was written. In the second volume there are explanations of the chemical composition of the Earth's atmosphere, climate types and their secular changes, then composition and physical characteristics of World oceans, the ebb-tide phenomena and the characteristics of the seabed. The book is digitized and it is included into the NCD *Virtual library* (http://elib.matf.bg.ac.yu:8080/virlib/). We present the contents of the book, also some interesting facts related to the book, including changes in terminology and definitions in this area since the book was published.

Key words: mathematical geography, geophysics, atmosphere, oceans

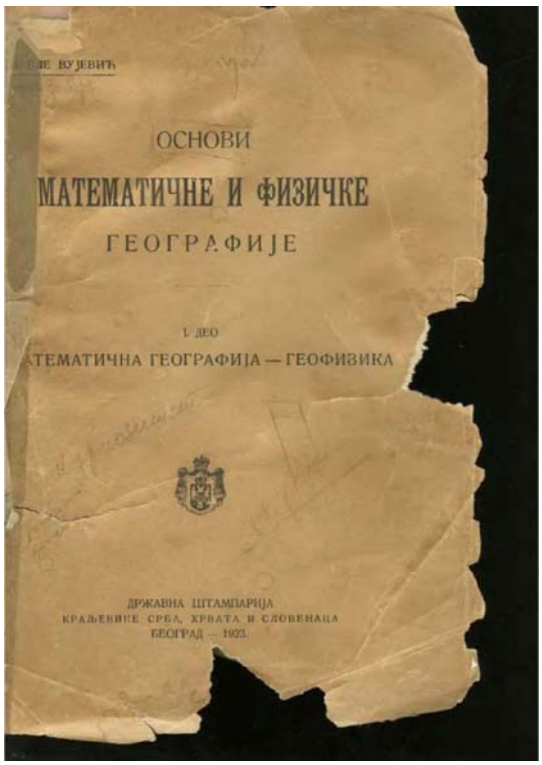
Introduction

We consider that one of the significant aims of digitization is to describe and comment digitized items. On this occasion we decided to write about an important university textbook which influenced a lot of development and a way of teaching of mathematical geography in Serbia at the university level. This is the book *Fundamentals of Mathematical and Physical Geography* (in Serbian *Основи математичне и физичке географије*), written by Professor Pavle Vujević. The book is one of the first university textbooks on geography written in Serbian and it was printed several years after the First World War in the newly built Kingdom of Serbs, Croats and Slovenienes.

In the country pauperized and destroyed by war it was difficult to find a foreign university textbook. Therefore, it was necessary to write own textbooks. Having written this textbook Pavle Vujević fulfilled this purpose. The book was printed, as its author mentions in the Preface, due to the great engagement of Jovan Cvijić who was then Rector of the Belgrade University. The textbook *Основи математичне и физичке географије* consists of two parts organized in the framework of four chapters. The first part *Математична географија* (Mathematical Geography) contains only one chapter with the same title, whereas in the second part *Физичка географија* (Physical Geography) there are three chapters: *Геофизика* (Geophysics), *Атмосфера*

(Atmosphere) and *Океани* (Oceans). The textbook was printed in two volumes with a total of 815 pages. The first volume contains the first two chapters, the second one the third and fourth chapters. The first volume *Математична географија и Геофизика* was printed in the Printing House of the Kingdom of Serbs, Croats and Slovenes in Belgrade in 1923, whereas the second volume *Атмосфера и Океани* was printed in the same printing house in 1926. Since Pavle Vujević was educated in Western Europe and knew foreign languages, in the course of writing this textbook the very contemporaneous textbook and scientific literature was used.

Contents of the Volumes



The first volume *Математична географија и Геофизика* contains 348 pages and it consists of an Introduction, first and second chapters. The first chapter *Математична географија* has 15 sections, whereas the second chapter *Геофизика* has six sections. The first chapter *Математична географија* is composed of the following sections: *Оријентација на хоризонту* (Horizon Orientation), *Оријентација на небу* (Orientation on the Sky), *Облик и величина Земље* (Shape and Size of the Earth), *Оријентација на Земљиној површини* (Orientation on the Earth Surface), *Географско одређење места на Земљи* (Geographic Determination of a Position on the Earth's Surface), *Појмови о светском систему* (Notions on the World System), *Закони планетарног кретања* (Laws governing Planetary Motions), *Карактеристика планетарних путања* (Characteristics of Planetary Orbits) , *Одређивање звезданих удаљења* (Determination of Distances to Stars), *Земљина кретања* (Motions of the Earth), *Положај Земље и Земљине осовине према еклиптици* (Positions of the Earth and its Axis with respect to Ecliptics), *Гравитациони утицај небеских тела на Земљина кретања* (Gravitational Influence of Celestial Bodies on the Motions of the Earth), *Време и одређивање времена* (Time and its Determination), *Апсолутно кретање Сунчевог система* и *непомичних звезда* (Absolute Motion of the Solar System and immovable stars) and *Положај Земље у васиони* (Earth's Position in the Universe). The second chapter *Геофизика* is composed of the following sections: *Земљина густина и маса* (The density and Mass of the Earth), *Земљина кора и језгро* (The Crust and Core of the Earth), *Подела копна и мора* (Subdivision of Land and Sea), *Земљотреси* (Earthquakes), *Земљина магнетичност* (Terrestrial Magnetism) and *Електрицитет у ваздуху и земљи* (Air and Ground Electricity).

The second volume *Атмосфера и Океани* has 467 pages numerated as continuation of the first one by numbers from 349 till 815. It contains the third and fourth chapters.

The third chapter *Атмосфера* has 11 sections; the fourth one *Океани* has seven sections. The third chapter *Атмосфера* is composed of the following sections: *Општи појмови о ваздуху* (General Notions about the Air), *Топлота* (Heat), *Температуре* (Temperatures), *Вертикална и хоризонтална подела температура* (Vertical and Horizontal Subdivisions of Temperatures), *Ваздушни притисак* (Air Pressure), *Ветрови* (Winds), *Кружење воде* (Water Circulation), *Поремећаји у ваздуху* (Perturbations in the Air), *Климатски типови* (Climate Types), *Периодске промене климата на Земљи* (Periodical Climate Changes on the Earth) and *Географска подела животиња и биљака на копну* (Geographic Subdivision of Animals and Plants on Land). The fourth chapter *Океани* (Oceans) consists of the following sections: *Топографија океана* (Topography of Oceans), *Хемијски састав и физичке особине океанске воде* (Chemical Structure and Physical Properties of Oceans Water), *Живот у океанима* (Life in Oceans), *Састав океанског дна* (Structure of Ocean Floor), *Таласи* (Waves), *Океанске струје* (Ocean Currents) и *Плима и осека* (Tide and Ebb).

Already a mere inspection of these titles indicates that Vujević's textbook is comprehensive and large. In addition to geography it also treats in details other sciences overlapping with it: astronomy, meteorology, climatology, oceanography and geophysics. For a majority of notions their historical roots are given which is rarely the case for modern textbooks. With regard to the enormous quantity of information and use of the very contemporaneous references for that time Vujević's textbook has all properties of an encyclopedia. In addition to all of this it contains many nice illustrations.

Biography of Academician Pavle Vujević



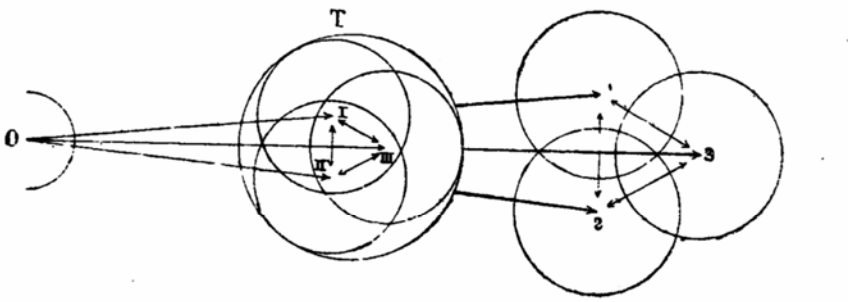
Pavle VUJEVIĆ (1881–1966)

Pavle Vujević (1881–1966) was born in Ruma. His education started in Novi Sad where he finished the Serbian Orthodox Gymnasium in 1899. He studied geography in Vienna at *Wiener Geographische Schule* (Vienna School of Geography) *famous* in that time. During his stay in Vienna he made acquaintance and became friend of Milutin Milanković who was then also a student. After finishing his studies he wrote his PhD thesis in geography. He took his PhD degree in 1904 at the age of 23. He improved his knowledge in climatology in Berlin and Potsdam. Invited by Jovan Cvijić he came to Belgrade in 1907 to become an assistant-professor on the subject of climatology and meteorology at the Faculty of Philosophy of the Belgrade University founded shortly before. After the retirement of its Director Milan Nedeljković the Astronomical and Meteorological Observatory was divided into two separate establishments. Vojislav Mišković was appointed as Director of the Astronomical Observatory, whereas Pavle Vujević became Director of the Meteorological Observatory. With his textbook *Основи математичне и физичке географије* Vujević contributed significantly to the improvement of teaching geography and meteorology at the Belgrade University. This textbook served to numerous generations of students for the purpose of learning not only geography and meteorology, but also climatology, oceanography, geophysics and geodesy. More details con-

cerning Vujević's biography can be found in [**6**]. A full member of the Serbian Academy of Sciences and Arts he became in 1958.

The First Volume: Mathematical Geography And Geophysics
I Part: *Mathematical Geography*

In Introduction**,** Vujević presents Laplace's hypothesis about formation of the Solar System and the planet Earth [Fig. 1]. (The references to the plots contained in the present article the reader can find in the digital copy of Vujević's book at the following address http://elib.matf.bg.ac.yu:8080/virlib/). He gave a short presentation of the development of natural studies, as well as their relationship with geography. In his second figure he presents a nice plot from which it is seen that *astronomy* appears as a cradle of studying nature. The Earth as a celestial body occupies the central point in this and together with many other celestial bodies appears as the subject in astronomical studies.



T = Земља
O = Астрономија = Физика = Хемија
I. Копно; Геологија;
II. Воде; Хидрологија:
III. Ваздух; Метеорологија
1. Геоморфологија (Физиогеографија)
2. Океанографија
3. Климатологија.

Скица 2.

Plot 2 Explanation T=Earth

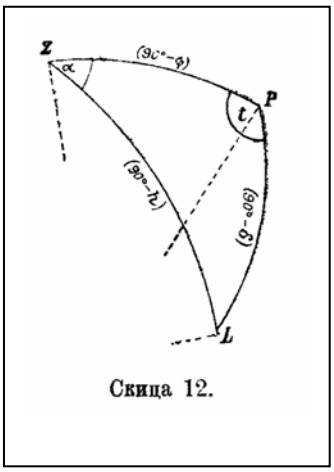
O = astronomy	I. Land Geology	1. Geomorphology (physical geography
= physics	II. Waters Hydrology	2. Oceanography
= chemistry	III. Air=Meteorology	3. Climatology

On the other hand, in geography the basic subject is the terrestrial spheres. In this science the outer terrestrial envelops, land, water and air, are presented as spheres [Fig. 2] intersecting one another, thus appear as the point of overlapping of scientific disciplines studying them. These disciplines are *geology, hydrology* and *meteorology.* Their mutual influences have contributed to developing of new scientific branches: *geomorphology, oceanography* and *climatology,* also presented by circles in Fig. 2. Just this relationship is described in Vujević's book. This textbook was an indispensable reading to many generations of students of many specialties whose interest included not only geography, but astronomy, meteorology, climatology, oceanography, geophysics and geodesy as well.

In the first chapter *Математичка географија* Vujević presents systematically the most part of what was known in astronomy then. He described the Earth, as a planet of the Solar System, in details: its formation, shape, size, motions, orientation, mass, density, day-night cycle, twilights, cycle of annual seasons, gravitation, seismity, its position among the planets, as well as the position of the Sun among stars. For this reason this chapter could be equally entitled as *Astronomy.*

In the first section of this chapter *Оријентација на хоризонту* one introduces and describes the following notions: sky line, horizon, main points of the horizon, gnomon and determination of distances and direction at the horizon.

In the second section *Оријентација на небу* one presents the astronomical spherical coordinate systems: horizon, equatorial and ecliptic ones and the relations between the horizon and equatorial coordinates and between the equatorial and ecliptic ones. In the book motions of stars, culmination and circumpolar stars are also mentioned. In addition, the apparent motion of the Sun and zodiac constellations through which the Sun passes apparently within a year following a great circle across the celestial sphere, the ecliptic, are also described. The descriptions are more frequent than the formulae. For instance, the apparent diurnal paths of stars and their upper culminations for an observer at the North Terrestrial Pole, Equator and a medium latitude are plotted in [Fig. 9].



Скица 12.

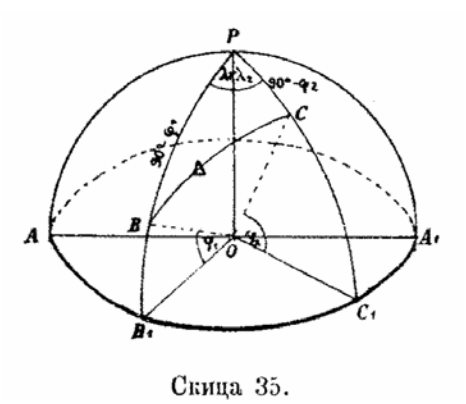
The author probably supposed that the descriptive way was more acceptable to most of the students who possessed a modest mathematical knowledge. It is curious that the spherical triangle in [Fig. 12] is referred to as *astronomical triangle,* being a better name indeed because that notion was introduced by astronomers. Vujević often gives the origin for many words, i. e. notions, such as, for instance, *tropical belt*. Namely, it is written in the book that the solar *tropics* were used for the first time by Homer in his Odyssey because he writes that the island of Syros, belonging to the Cyclades, lies north of tropical *lands,* thus north of the solar tropic. For this reason the zone between the tropics was named *tropical zone*.

In the third section *Облик и величина Земље* the Earth is modeled as a spherical body. In addition the historical development of this model was presented, including the method of Eratosthenes, Earth measuring by Arabs, the triangulation method of Snellius and Kepler's one. Further on the Earth spheroidal model is mentioned where the proofs for this approximation are given: the Huygens law of centrifugal force, Newton's proofs for the spheroidal shape of the Earth, Cassini's measurements in France, Clairaut's theorem about the Earth's flattening and geodetic measurements in the XIX century. Finally, he presents the Earth's model in the form of geoid giving the proofs for this model by presenting the procedure of measuring the perturbations in the Earth's gravity. He mentions the measurements of the Paris meridian on the basis of which the basic length unit, *meter,* was defined.

The book contains many historical matters of interest. An example is the Earth measuring by *Eratosthenes* (276–194 BC). The distances between two places were expressed then in *stadia* (1 stadion = 158 m) and determined on the basis of camel route. The distance between Alexandria and Asuan was known to Eratosthenes. He knew that in Asuan there was a deep well in which the Sun was reflected only once within a year. Very cutely Eratosthenes concluded that this was the day of summer solstice. Using this astronomical fact he found the perimeter of the Earth to be equal to 39 815 km which is very near the true value. This was a result of a genius. Only in the XVIII century the measurements yielded for the perimeter of the Paris meridian a value of 40 000 km and from it the length unit, *meter,* was derived as $1/(40 \times 10^6)$ the

length of the Paris meridian perimeter. It should be mentioned that in the book for the second pendulum one uses an expression now almost forgotten in the Serbian language. The measurements of the Earth's flattening by using the second pendulum are presented in the table (p. 46 in the book).

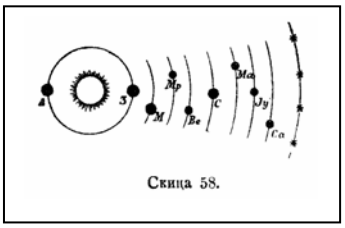
In the fourth section *Оријентација на Земљиној површини* the author presents the network of parallels and meridians, the coordinate system taken from astronomy. Vujević also writes about the distance on the surface of a sphere applied to places on the terrestrial surface, thus the elements of spherical trigonometry are also present.



Скица 35.

Of interest is the history of adopting the prime meridian for the Earth described by Vujević on page 55, as well as the origin of the names geographic longitude and latitude, also of the notions of *orthodrom* concerning direction and *loxodrom* concerning slope used in the navigation. In [Fig. 35] the determination of the shortest distance between two places on the terrestrial sphere is presented by using *astronomical triangle* introduced by astronomers for the purpose of determining the angular distances between two celestial bodies on the celestial sphere.

In the fifth section *Географско одређивање места на Земљи* the author derives the methods of determining geographic latitude such as the Horrebow–Talcott, the methods using culminations of stars, circumpolar stars, measuring shadows and measuring time. He also derives the methods of determining geographic longitude: by using light signals, telegraph, chronometer, lunar and solar eclipses, culminations of Moon and eclipses of satellites of other planets. The author presents the methods of geodetic determination of geographic coordinates such as triangulation, spherical excesses and photogrammetry. At the end he presents the procedures for determination of altitude above the sea level: by leveling and by applying trigonometric, barometric and thermometric methods. It is interesting that in this section in addition to the notion of *astronomical refraction* [Fig. 38] the author also mentions that of *terrestrial refraction* [Fig. 39] which arises on the occasion of determining the place positions on the terrestrial surface. One should remind that this phenomenon concerns the light refracted in some air layers without passing through the entire atmosphere of the Earth.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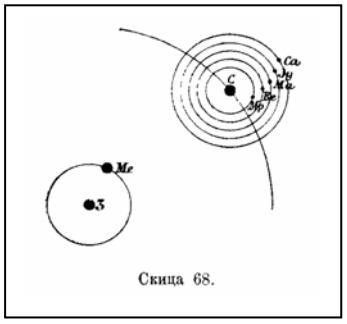
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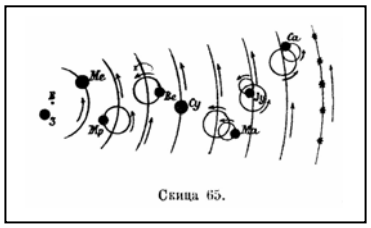

Скица 65.

The sixth section *Појмови о светском систему* concerns the historical review of comprehending the world. In the following order the systems proposed by

Pythagoras, Eudox, Hipparchus, Ptolemy, Heraclides, Copernicus, Tycho Brahe and Galileo are mentioned. The authors of the present article have selected a few interesting figures presenting the geocentric systems of Pythagoras [Fig. 58], Hipparchus-Ptolemy [Fig. 65] and Tycho Brahe [Fig. 68]. The heliocentric system of Copernicus is described in details, as well as Galileo's exact proofs which contributed significantly for this system to be generally accepted.

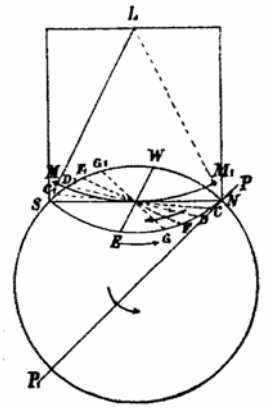
In the seventh section *Закони планетарног кретања* one presents Kepler's laws derived by Kepler empirically and after this Newton's law of general gravitation followed by a description of the way in which Newton obtained Kepler's laws theoretically starting from his gravitation law. The calculation of perturbations and the Titius–Bode progression explaining the heliocentric distances of the planets given in *astronomical units* (distance between Earth and Sun) are also mentioned. In this connection the author mentions the discoveries of minor planet Ceres (1801) and new planet Neptune (1846) where the latter one was discovered *with the point of a pen.*

We should note that this chapter devoted to astronomy was written for students of geography. For this reason the derivation of Kepler's laws from Newton's gravitation law was given descriptively and without any complicated mathematical calculations. We should remind that Newton for the purpose of this derivation introduced *the differential calculus* [Fig. 73].

The eighth section *Карактеристика планетарних путања* concerns general characteristics of planet motions, the motion of planets in the heliocentric reference frame, true motion of the Moon, its phases and the apparent motion of the Moon and finally the eclipses of the Moon and Sun.

The presentation of the true motions of planets is systematic and clear being followed by an explanation of their apparent motions. Of interest is the explanation of the apparent motion for the outer planets (those farther from the Sun than Earth) given by Copernicus and which was the strongest proof of his heliocentric theory. In the case of eclipses Vujević uses words *umbra* and *penumbra* in the Serbian language instead of the corresponding words of Slav origin.

In the ninth section *Одређење звезданих удаљења* the methods for determining the solar parallax are presented: Aristarchus' one, Hipparchus' one and that using planet transits. The trigonometric method of determining the parallax of the Moon and the ones for determining star parallaxes are also presented. To remind: to determine the parallax of a celestial body means to determine its distance. In an interesting paragraph Vujević mentions that Kepler influenced astronomers to observe the transits of Mercury and Venus over the disc of the Sun. As a response to this initiative Halley predicted that the transits of Venus would take place in 1761 and in 1769, whereas Ruđer Bošković on Halley's initiative calculated the positions of places which offered the most favorable conditions of observing this phenomenon. In the book the method, used by Lalande and Lacaille in 1751 for the purpose of determining the distance of the Moon, is also presented. In describing how distances to stars are determined, not only the method of annual parallax is presented, but also that of determining star distances by using relative parallaxes.

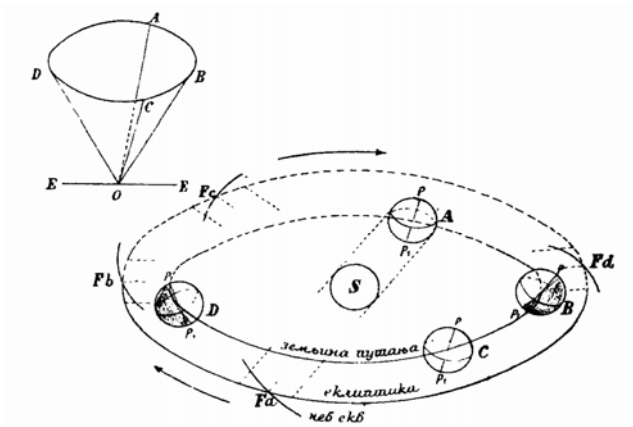


Скица 87.

In the tenth section *Земљина кретања* by using a historical approach Vujević gives proofs in favour of the diurnal motion of the Earth where he presents experiments concerning the terrestrial gravity, flattening of the Earth, free fall and deviations of

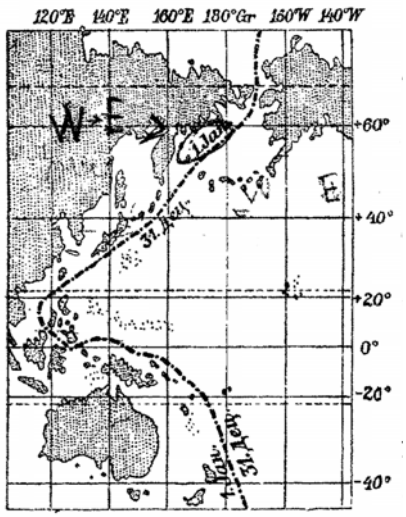
horizontal motions. He also gives proofs in favour of the annual motion of the Earth by mentioning the determination of the annual parallax of stars, light aberration, eclipses of Jupiter's satellites and "shooting stars" or meteors. The plot presenting Foucault's pendulum concerning the terrestrial rotation [Fig. 87] is interesting.

In the eleventh section *Положај Земље и Земљине осовине према еклиптици* the consequences of the diurnal and annual motions of the Earth are described : day-night cycle and cycle of annual seasons. Some terms used by Vujević seem obsolete now. They concern notions like twilights, white nights, etc. Nowadays astronomers define two kinds of twilights: dawn (in the morning) and dusk (in the evening).



Скица 113.

In the twelfth section *Гравитацијони утицај небеских тела на Земљина кретања* the author describes periodical perturbations, precession, nutation and motion of the Earth's poles, as well as secular perturbations, oscillations of ecliptic inclination, motion of line of apsides, changes of eccentricity of the Earth's orbit. In this section the precessional and nutational motions of the terrestrial rotation axis, [Fig. 113] and [Fig. 114], as well as the motion of the terrestrail poles [Fig. 117], are described.



Скица 125. — Датумска граница до 1845. године.

In the thirteenth section *Време и одређивање времена* the author presents the systems of sidereal time and of mean solar time, to describe the astronomical concept of year (tropic, sidereal, lunar, civil), notion of month (sidereal, synodic, tropic, draconistic, calendar) and the two calendars, Julian and Gregorian, afterwards. Further on the systems of local, international and zonal times, as well as the International Date Line, named also *nautical date line* by him, are described. In [Fig. 125] the International Date Line used by 1845 is presented. From 1845 the meridian opposite to the Prime Meridian has been adopted as the new International Date Line.

In the fourteenth section *Апсолутно кретање Сунчевог система и непомичних звезда* unlike the preceding ones, the topic is the true motion of the Solar System

where the coordinates of the apex of that motion are given. The true motions of stars, in astronomy known as their proper motions, are also mentioned. Herschel's very interesting method based on plots [Fig. 128] for determining the positions of the apex and antiapex for the motion of the Solar System is also present in this book.

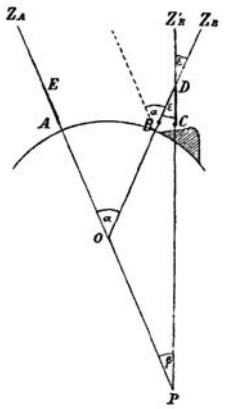
In the fifteenth section *Положај Земље у васиони* the author gives the idea of distances both within the Solar System and also of enormous ones within our Galaxy. After this there is a description of the structure of our Galaxy, of star types, star luminosities, of star clusters and nebulae.

The way in which Vujević tries to inform the readers on enormous distances in the universe is very interesting. The distances within the Solar System are described through the time needed to a human being (speed 30 km per day) and a train (speed 60 km per hour) to travel them. In this way one obtains that a walking human being needs 36 years to reach the Moon and a rapid train 9 months. In another example a person would walk to reach the Sun 13,600 years, a rapid train 285 years. The space of the Solar System is compared wth a vast desert in which a few bodies, like oases, can be found at some places. However, a real desert, according to him, could be the Milky Way only, because the distance between the Sun and the nearest star is 200,000 times that between the Sun and the Earth.

II Part: *Physical geography*

The part of the book *Физичка географија* occupies in Vujević's book a much larger space than *Математична географија.* Whereas Математична *географија* coincides with the first chapter, *Физичка географија* comprises the second (*Геофизика*), the third (*Атмосфера*) and the fourth (*Океани*) ones.

Second Chapter *Geophysics*. Geophysics is the science studying the interior of the Earth, its density, mass, the state of heat in the core and the terrestrial magnetism. The basic task of geophysics is, as said by Vujević, the determination of the Earth's density.



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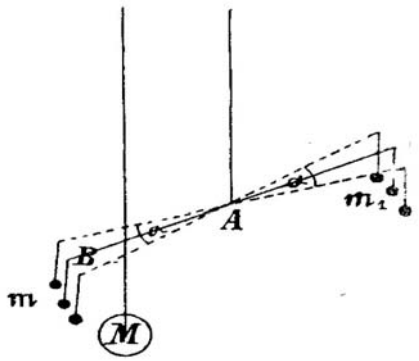
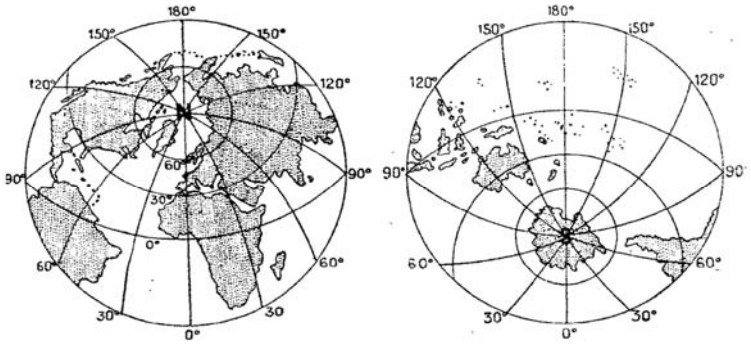


Скица 131.

The first section of this chapter *Земљина густина и маса* concerns the methods of determining the density and the mass of the Earth. In the book one finds interesting descriptions of the first methods which is practically absent in modern textbooks. An example is on p. 197 where one presents Sir Isaac Newton's (1643–1727) procedure of determining the density and mass of the Earth on the basis of the plumb deflection

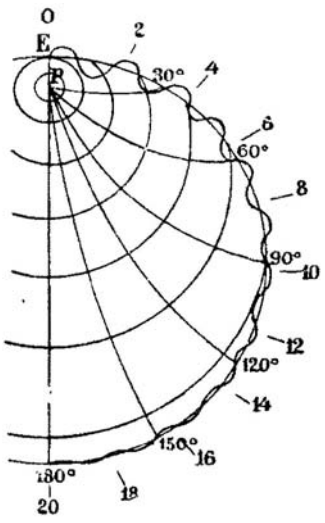
in the vicinity of a hill. On p. 198 one finds the *torsion balance method* [Fig. 131] proposed by Henry Cavendish (1731–1810) in late XVIII century.



Скица 136. — Копнена земљина полукугла Скица 137. — Водена земљина полукугла.

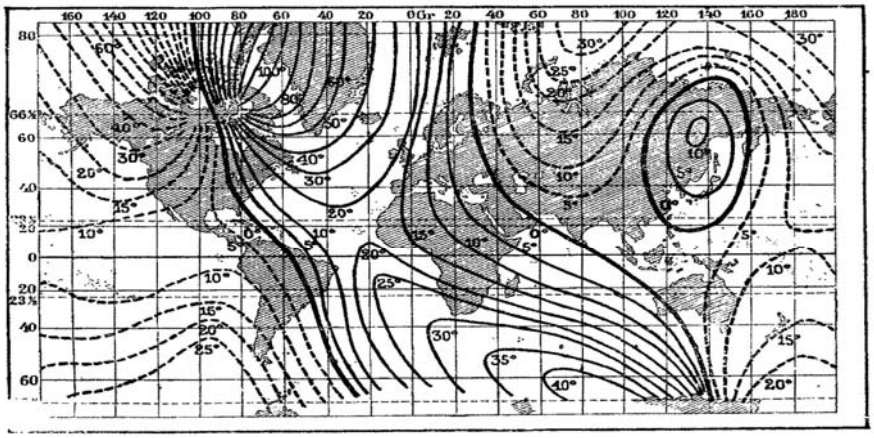
In the second section *Земљина кора и језгро* the composition of the Earth's crust and its density are described, then the chemical composition of the core and its density. Here one finds the explanations and data on the pressure and temperature in the Earth's interior, also on the elasticity of the Earth, as well as on the non-uniform distribution of gravity on the Earth and isostasy. The origin of oceans, mountains and volcanoes is described. Vujević also writes about less known phenomena. For instance, on p. 220 one finds that due to the action of the Moon and Sun in addition to the two diurnal water tidal waves (well known) there are also in the Earth's crust two ground tidal waves (less known) causing the ground beneath our feet to move alternatively up and down twice a day with a difference of 20 cm.

In the third section *Подела копна и мора* the horizontal and vertical distributions of the land and sea on the terrestrial surface are presented. Vujević's description of the land-sea distribution is interesting [Fig. 136, Fig. 137]. In the former figure it is seen that the land hemisphere of the Earth contains more water (54.5%) than land (45.5%), whereas the water hemisphere is dominated by water with 88.7%, land is 11.3% only. In the same figures one finds that the pole of the land hemisphere is situated somewhere between London and Paris, whereas the pole of the water hemisphere is east of New Zealand.



Скица 141.

In the fourth section *Земљотреси* the elements of the earthquake field are described in an illustrative way, the propagation of an earthquake and the instruments for its measuring. The phenomena concerning earthquakes are presented in details: seismic waves and earthquake phases, duration and power of an earthquake, seismic activity, the size and form of the area under earthquake and frequency of earthquakes. The ways of determining the epicentre and hypocentre are described, as well as the causes and periods of earthquakes, also seismic areas on the Earth. Here one finds that on the Earth on the average 26,000 earthquakes a year take place, i.e., 70 a day. In [Fig. 141] the way of earthquake propagation is presented, also types of internal and external waves, as well as the epicentre and hypocentre.



Скица 145. — Изогоне за почетак 1900. год. (по Neumayer-y). — Линије западних (——) и источних (– – –) деклинација.

In the fifth section *Земљина магнетичност* the elements of terrestrial magnetism and the determination of the magnetic intensity are presented. The regular and irregular terrestrial magnetic fields are described, as well as the local and regional areas of a perturbation, then variations in the terrestrial magnetism: secular, annual, diurnal followed by the theories of their origin and magnetic perturbations. The influence of the Sun on the terrestrial magnetism is explained: eleven-year period, 27-days period and the causes of magnetic perturbations. A description of the solar magnetic field is given and hypotheses on the origin of the terrestrial magnetic field.

At this place one finds interesting notes concerning the etymology of the word *magnet*. Namely, on p. 288 one finds that as early as in the Antiquity a stone capable of attracting small iron objects was known. The stone was named magnetite after the locality of Magnesia in Asia Minor where it was discovered. Further on, on p. 289 there is a note that Chinese as early as in the tenth century BC knew the properties of magnetic needle, that its point is always directed towards the north, but that it is unknown if this knowledge was used in everyday practice. One finds that the sailors from the XII century began to use the magnetic needle for the purpose of orientation. In addition, Vujević says that in the XV century Christopher Columbus navigating in the Atlantic Ocean discovered what we call today magnetic declination, the angle between a meridian on the surface of the Earth and the direction of magnetic needle. The lines along which the magnetic declination has the same value are known as *isogones* (*isogonic lines*). In Fig. [145] the complicated system of isogones is presented because the geographic and magnetic poles of the Earth do not coincide. From this figure it is seen that the north magnetic pole is in the arctic part of North America.



Скица 158. — Поларна светлост у облику драперије.

In the sixth section *Електрицитет у ваздуху и земљи* the author describes the elements of electricity, changes of tension, theory of ions, the annual and diurnal runs of electricity, then electricity in land and air, as well as the causes of positive electricity in the air and negative one in land. The part of the sixth section *Поларне светлости* (*Aurorae Borealis*) is of special interest. In it one describes the types of aurorae, their altitude, physical properties, as well as the geographic distribution. Their diurnal and annual periods, as well as the influences of the Sun

and the Moon on the frequency of their occurring, are presented. The hypotheses on the origin of this interesting and beautiful phenomenon are also included.

Second Volume: *Atmosphere and oceans*

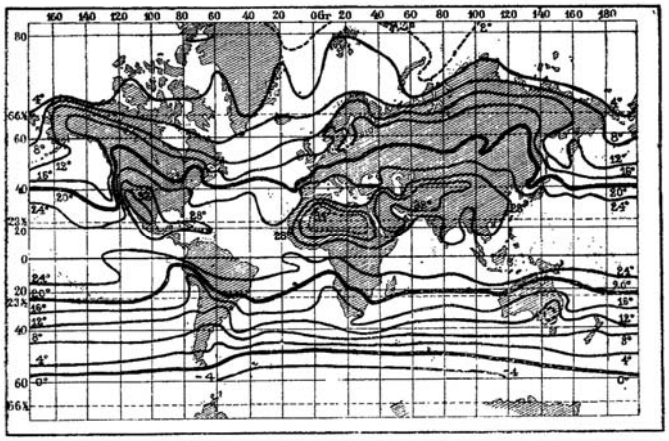
III part *Atmosphere*

In *Увод* (*Introduction*) to this chapter Vujević describes the gaseous envelope of the Earth and the scientific branches studying the atmosphere. That branch having the main atmospherical phenomena as its subject is meteorology. The branch studying the climat in different parts of the Earth is climatology. In Vujević's words climatology appears as a link between meteorology and geography.

The first section *Опити појмови о ваздуху* describes the air composition just above the surface of the Earth, as well as the air composition at higher altitudes, to continue with the air pressure and its decrease with altitude, also the thickness of the atmosphere and its mass.

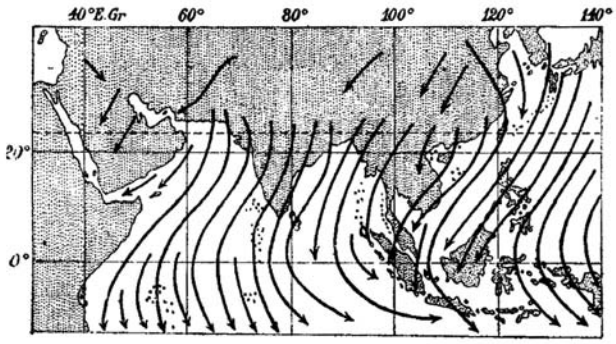
In the second section *Топлота* the heat sources on the Earth are the subject, as well as the solar constant, solar climate, the air influence on the solar climate and the factors changing the theoretical heat conditions. Here one obtains information on the heating and cooling of the land, sea and atmosphere, also on the air influence on the black-body emission from the surface of the Earth.

In the third section *Температуре* the relationship between the heat and temperature is described. In addition we have the methods of temperature measuring, the influences of land, sea and topography on the temperature of the atmosphere, the diurnal and annual run of temperature and the study of mean temperatures. The temperature types, equatorial, tropical, moderate and polar ones, as well as the perturbations in the annual temperature run, diurnal variability and the mean variability of month temperatures, are presented.



Скица 178. — Подела ваздушних температура у јулу (по *Кöppen*-у).

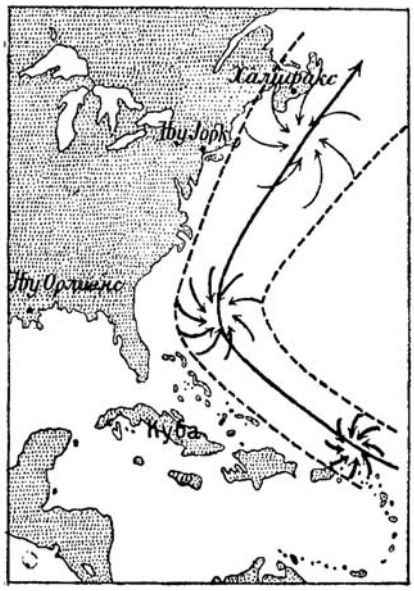
In the fourth section *Вертикална и хоризонтална подела температура* at first the temperatures within atmospheric layers are described, to be followed by a description of the heat conditions at various geographic latitudes. The other aspects concerning the air temperature are presented in details: mountain temperatures, the amount of the vertical temperature gradient, its diurnal and annual periods. At the end the subdivision of temperatures at the surface of the Earth is given, followed by the importance of maps of isotherms, subdivision of air temperatures at the sea level, mean temperatures of parallels and of the Earth as a whole, as well as temperature anomalies, continentality and oceanity for a given place. Vujević describes how *isotherms*, lines of the same temperature, are obtained from hard measurements carried out every day throughout the world. The isotherms for July are presented in [Fig. 178].



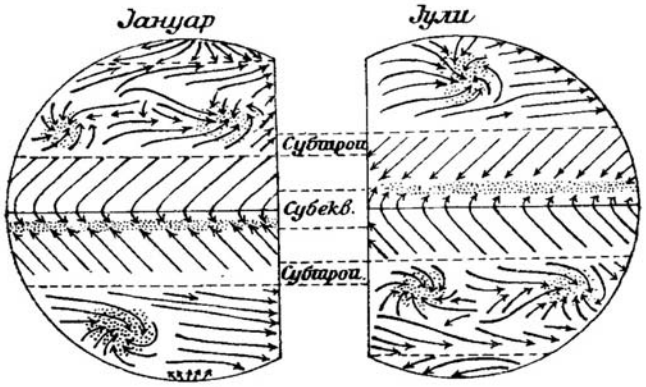
Скица 190. — Зимски монсуни у јужној Азији.

The fifth section *Ваздушни притисак* deals with the diurnal and annual oscillations of the air pressure, reduction of the barometric state to the normal values and sea level and subdivision of air pressure along the surface of the Earth. Like the preceding section the text is followed by nice plots and illustrations which give the distribution of air pressure in various regions. For instance, from [Fig. 182?] it is seen that the lowest pressure upon the Earth in January takes place within the belt of the warmest continental regions, therefore in the central parts of Brazil, South Africa and Northern Australia.

The sixth section *Ветрови* contains detailed descriptions of the wind direction and speed, frequency and velocity of winds from various directions, diurnal and annual periods of wind speed, what causes a wind, cyclones and anticyclones, the general system of winds at the surface of the Earth, equatorial belt (called *belt of doldrums* by sailors), zones of passat winds subtropic zones, zones of western winds, subpolar wind system and zone of monsoon winds. The theory of general air circulation on the Earth and the nomenclature of diurnal winds: coastal light winds, valley breeze and mountain breeze. A nice schematic presentation of theoretical systems of planetary winds in January and July is given in [Fig. 191].



Скица 211. — Средњи правац оркана при источним обалама Северне Америке.



Скица 191. — Теоријски систем планетарних ветрова у јануару и јулу (по *W. M. Davis*-у).

The seventh section *Кружење воде* is devoted to all aggregate states of water and the belts concerning water in the atmosphere: evaporation, water vapour in the air, diurnal and annual run for the absolute and relative humidity, condensation of water vapour, fogs and clouds, atmospheric precipitation (dew and hoar-frost, hard rime, ice, rain, snow, hale), measuring atmospheric precipitation, then rain distribution during a day and in months (for equatorial belt, tropical belt beyond equator, monsoon rains, passat rains, winter rains of western coasts, summer continental rains, and subtropical winter rains).

The eighth section *Поремећаји у ваздух**у*** treats cyclones and their influence on weather and tropical storms, air tubes and cascade winds as consequences of cyclones (fen and storm). Nice presentation of *hurricanes* near the eastern shores of North America is given, Fig. [211].

The ninth section *Климатски типови* gives a detailed description of all climate types: equatorial climates (oceanic and continental), tropical climates (oceanic, monsoon, continental and mountain), subtropical climates (rainy, monsoon, Mediterranean, desert and mountain), climates of the temperate zones (patagonic, western and eastern coastal regions, continental) and polar climates.

In the tenth section *Периодске промене климата на Земљи* all important periods of climate change are described – 11-years, 7-years, 35-years and the secular one.

In the eleventh section *Географска подела животиња и биљака на копну* the influence of the climate factors on the biosphere is described, on the life of plants and animals, on life zones and the causes of origin for the area of given biological species.

IV part *Oceans*

In this chapter Vujević writes about *oceanography* the science having studying of oceans, and which applies physics, chemistry and mathematics.

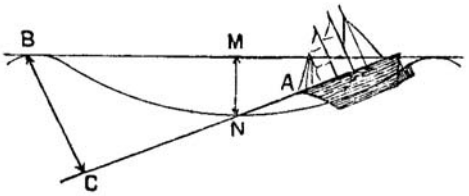
In the first section of this chapter *Топографија океана* examinations of the ocean are described, as well as the terminology of submarine forms. The reliefs of the Atlantic, Pacific and Indian Oceans, as well as that of the Mediterranean Sea and its bordering branches are presented.

In the second section *Хемијски састав и физичке особине океанске воде* a detailed description of the composition and salinity of the oceanic water is given, as well as the geographic salinity subdivision on the ocean surface and salinity variations with ocean depth. The salinity of closed and bordering seas, such as the Mediterranean Sea, Black Sea, Adriatic Sea, Persian Gulf, Red Sea and the North Sea, is described. The causes of the salinity differences upon the surfaces of oceans and seas are discussed, as well as the relationship between the salinity and density of ocean waters. The temperature distribution on the ocean surfaces and within oceans is described, as well as the causes of low temperatures of ocean water at large depths. The transparency, colour and phosphorence of the oceanic water are explained.

In the third section *Живот у океанима* Vujević mentions that oceans are craddle of life and that the life in oceans is much more diverse than on land. According to the conditions of propagation of sunlight or permanent darkness the oceans are divided into two regions: illuminated where both plants and animals live and the dark one inhabited by animals only. At this place readers can learn that within oceans there are three life zones: littoral (coasts and shallow seas), pelagic (open sea) and abyssal (dark zone deep inside).

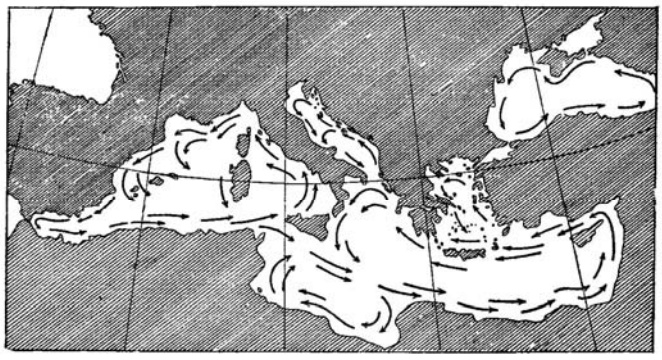
The fourth section *Састав океанског дна* describes the classification of oceanic sediments. Vujević also here has some interesting historical notes, for instance that something concerning the composition of sea bottom was known as early as in the antiquity. So, *Herodotus* writes that on the bottom of the Mediterranean Sea, at a depth of 20 m, on the day of navigating from the Nile mouth towards open sea, there is ooze. In Herodotus' opinion this sediment ioriginated from land. Only in the XVIII century the first theory concerning the composition of sea bottom was proposed. Later the patterns from the sea bottom were studied microscopically and in this way their

structure became well known. After this it was established that also land lyme rocks with fossile shellfish and snails must have been bottoms of earlier seas.



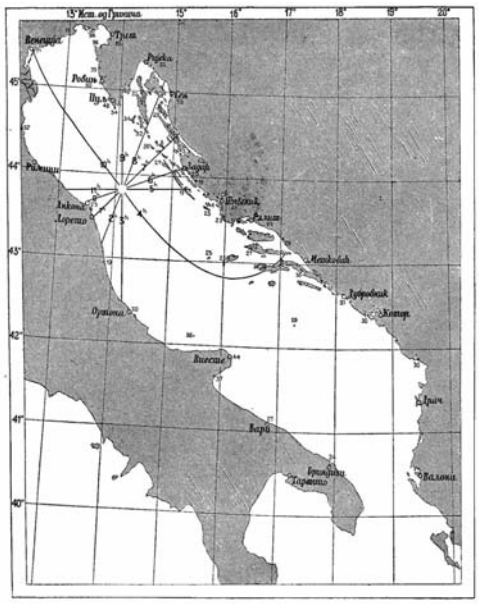
Скица 231.

In the fifth section *Таласи* the theory of formation of ocean waves, including their size and propagation speed, are presented. One describes wind waves, earthquake and explosion waves, immovable and internal oceanic waves, as well as the waves in shallow waters and hitting seashore by sea water, its power and action. In Fig. 231 we find an interesting presentation of the relative altitude determination for an ocean wave in open sea by observing. Since a ship is always somewhat inclined towards the wave trough, so observed from the deck the wave seems higher than it really is. Hitting seashore by sea water is described at the same place [Fig. 237]. At shallow shores we have breaking, foaming and splashing of waves, whereas in the case of steep shores no real hitting takes place, instead the waves move vertically and the sea level begins to oscillate. In this way during great winter storms enormous water jets can attain a height of 30 m.



Скица 245. — Струје у Средземном и Црноме Мору.

The sixth section *Океанске струје* treats observing and measuring of ocean streams, as well as the causes of their arising. The streams in the Atlantic, Pacific, Indian and Arctic Oceans, also in closed seas: Mediterranean, Black, Aegean, Adriatic and North ones, are described.



Скица 256. — Изорахије Јадранскога Мора са лучним добима појединих места (по R. v. Sterneck-y). Бројеви крај обала и острва означују амплитуду полудневних морских доба при сизигијама, у центиметрима.

In the seventh section *Плима и осека* two enormous waves moving on ocean surfaces from east to west across the entire Earth from the ocean surface to the bottom are described. Due to these waves the sea surface near shores rises and falls twice a day. These phenomena are known as *flood* and *ebb,* respectively, or *tide.* The effects of the Moon are additionally described, a relationship between the lunar culminations and high water level during flood at the same place. For instance, if there is a time lag t of the high water with respect to lunar culminations on one day, it will be also repeated on the next days with the same value. This is the effect of the Moon. After this we have a dynamical theory of tides, influence of the terrestrial rotation and interference followed by a mathematical theory of harmonic analysis. The tides taking place in the Mediterranean Sea and its bordering branches are described. In [Fig. 256] we have the Adriatic Sea with given lunar lag effect for some places.

Conclusion

In the first part Математична *географија* of university textbook *Основи математичне и физичке географије* by Pavle Vujević *astronomy* is represented very well. This chapter on astronomy is equipped with figures, tables, maps and diagrams. Vujević's simplicity and easiness of writing in presenting complicated astronomical phenomena are to be specially emphasized. Some formulae are derived, others given only. Of course, in astronomy there are very complicated formulae where the derivation requires a lot of space and mathematical knowledge. Astronomical phenomena are presented by Vujević in most cases descriptively so that students who are not mathematicians can understand the text.

In the second part *Физичка географија* we find pages containing descriptions of some geophysical, atmospherical and oceanic phenomena. These descriptions are written in a manner admitting to those who are not physicists to understand them easily. In this matter Vujević realized his intention to make a student of geography more familiar with basic knowledges of astronomy, mathematics and physics.

The book contains a large number of historical notes concerning the source and origin of many notions concerning geophysics, astronomy and other disciplines dealing with our planet. Many words used by Vujević in his book have been already forgotten or they have not been used since a long time. For this reason this book appears as a good place for studying the development and changes in our terminology concerning the given disciplines. On the other hand, as shown by these volumes, it is most likely Vujević's merit that many technical words and notions of these disciplines have been introduced in the Serbian language.

Bearing in mind the methodical way of writing for this geography textbook, its contents, clearness and easiness of expressing we recommend it also to the students and teachers of secondary schools of our times, not only in geography, but also in mathematics, astronomy, physics, geophysics and geodesy. University textbook *Основи математичне и физичке географије* by Pavle Vujević has been digitalized and it is in the Virtual Library of the National Digitalization Centre http://elib.matf.bg.ac.yu:8080/virlib/.

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ПРИКАЗ ДИГИТАЛИЗОВАНЕ КЊИГЕ *ОСНОВИ МАТЕМАТИЧНЕ И ФИЗИЧКЕ ГЕОГРАФИЈЕ* ПАВЛА ВУЈЕВИЋА

Резиме. У овом чланку представљамо део једног од првих универзитетских уџбеника из географије писаних на нашем језику. Написан је после Првог светског рата. Наиме, реч је о уџбенику *Основи математичне и физичке географије* професора Павла Вујевића који је штампан редом у две књиге 1923. и 1926. у Београду. Уџбеник се састоји од четири поглавља: *Математична геофрафија, Геофизика, Атмосфера и Океани* у обиму од 815 страница. Иако веома велики, уџбеник је методички лепо написан, попут монографије. У овом чланку је представљен посебно занимљив I део *Математична географија* који се састоји од истоименог првог поглавља посвећеног астрономији. Енциклопедијски су побројани и објашњени до тада уведени астрономски појмови са детаљним и јасним описима. II део *Физичка географија* је много обимнији, обухвата три поглавља: *Геофизика, Атмосфера* и *Океани*. У овом чланку укратко су представљени садржаји ова три поглавља као и њихове занимљивости. Уџбеник *Основи математичне и физичке географије* Павла Вијевића је дигитализован и налази се у Виртуелној библиотеци Националног центра за дигитализацију (Virtual library, http://elib.matf.bg.ac.yu:8080/virlib/). Дигитализација ових књига део је пројекта електронског архивирања српских књига са математичким и астрономским садржајем штампаних у прошлости. У чланку су поменути и други разлози зашто је овај уџбеник изабран за укључење у Виртуелну библиотеку.

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49 *Zvezdano nebo nezavisne Srbije* **Djordja Stanojevića**

Starry Sky of independent Serbia of
Djordje Stanojević
N. Pejović
Zbornik radova naučnog skupa
"Djordje Stanojević - život i delo -
povodom 150 godina od rodjenja"
Novi Sad, 10-11. april 2008, pp. 163-171
SANU, Novi Sad, 2008.

This review is on a book "Starry sky of independent Serbia" of Djordje Stanojević (1858-1921). This is one of the first Serbian book on astronomy, printed in 1882 in Belgrade. The book was intended to the general public and has nice explanations and illustrations as well of astronomical events and phenomena. Besides that, the review gives a short biography of the author, who was the astronomer, physicist, university professor, but the electrical engineer too. He is remembered as a men who built the first electrical power plants in Serbia.

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ЗВЕЗДАНО НЕБО НЕЗАВИСНЕ СРБИЈЕ ЂОРЂА СТАНОЈЕВИЋА

ON ĐORĐE STANOJEVIĆ'S BOOK "STARRY SKY OVER INDEPENDENT SERBIA"

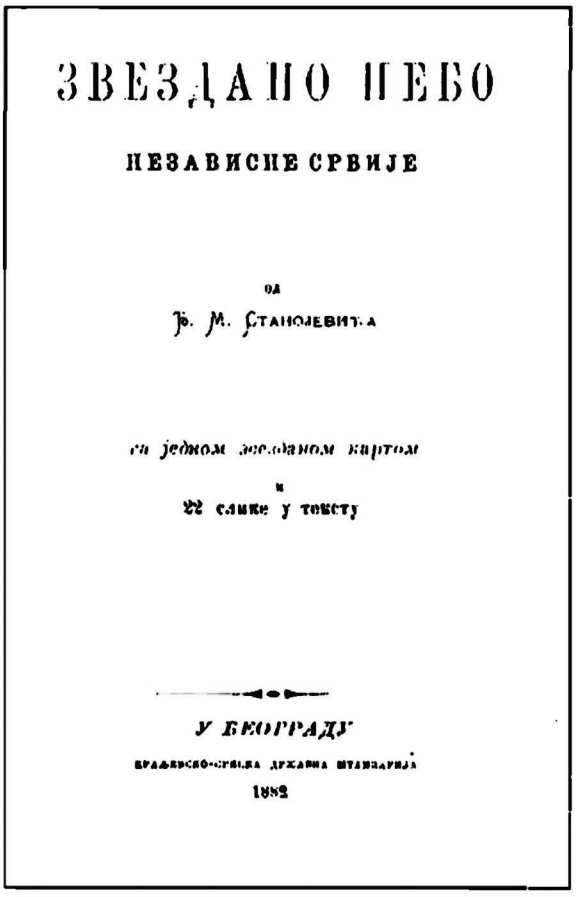
САЖЕТАК: У овом чланку представљамо једну од првих књига из астрономије писаних на српском језику. Написана је у другој половини 19. века, када стручне и научне литературе у нашој земљи готово да није ни било. Наиме, реч је о књизи професора Ђорђа Станојевића *Звездано небо независне Србије*, која је штампана 1882. године у Београду. Мада по обиму мала, књига је садржајна, методички веома јасно и прегледно написана. Астрономски појмови су систематски уведени и на једноставан и разумљив начин објашњени. У књизи нема формула нити математичких једначина него су појмови појашњени лепим графичким приказима. Књига је била намењена широј читалачкој публици оног времена. *Звездано небо независне Србије* Ђорђа Станојевића је дигитализована и налази се у Виртуелној библиотеци Националног центра за дигитализацију (Virtual library, http://elib.matf.bg.ac.yu:8080/virlib/). Дигитализација ових књига део је пројекта електронског архивирања српских књига из математичких и астрономских наука штампаних у прошлости. У чланку су наведени и други разлози зашто је ова књига изабрана за укључење у Виртуелну библиотеку.

SUMMARY: We present here the book *Starry sky over independent Serbia* written by Đorđe Stanojević. The book was printed in 1882 in Belgrade and was one of the first books in astronomy written in Serbian. It should be mentioned that at this time in Serbia there

were almost no scientific books. The book is intended to the general readers of the time. Even though the book is small in size, it contains a lot of information and is written in a clear and methodical manner. Astronomical notions are systematically introduced and explained in a simple and understandable way. The book does not contain mathematical formulas, but astronomical ideas are described with nice graphical presentations. The book is digitalized and the digital copy is deposited in the Virtual library of the National Center for Digitalization http://elib.matf.bg.ac.yu:8080/virlib/. Digitalization of the books of this kind is a part of the project of electronic archiving of the books by the Serbian authors in the area of mathematical and astronomical sciences printed in the past.

Увод

Звездано небо независне Србије професора Ђорђа Станојевића је једна од првих књига из астрономије писана на српском језику. Књигу је издала Краљевско-српска државна штампарија 1882. у Београду. Дигитализован је примерак књиге који се налази у Универзитетској библиотеци у Београду, заведен под инвентарским бр. 13006. На овом примерку Станојевић је написао посвету извесном господину Ђ. С. Симићу. Књига је по обиму мала али садржајна и представља прави бисер наше научне литературе деветнаестог века. Ђорђе Станојевић учио је школе у европским центрима и био је добар познавалац страних језика, тако да је књигу написао коришћењем тада доступне стране литературе. У предговору Станојевић као референце наводи две француске и једну немачку књигу.



ЗВЕЗДАНО НЕБО

НЕЗАВИСНЕ СРВИЈЕ

од

Ђ. М. СТАНОЈЕВИЋА

са једном звезданом картом

и

22 слике у тексту

У БЕОГРАДУ

КРАЉЕВСКО-СРПСКА ДРЖАВНА ШТАМПАРИЈА

1882

Садржај књиге

Књига ***Звездано небо независне Србије*** написана је на 70 страница и састоји се од предговора, десет поглавља и једне звездане карте. У тексту се налазе и 22 слике. Књигу чине поглавља: *Звезде уопште, Астрономска подела неба, Број,*

даљина и подела звезда по величини; звездане карте, Звездана јата и њихове звезде, Промена звезданог неба услед прецесије, нутације, аберације и рефракције, Право кретање звезда, Променљиве и нове звезде, Двогубе звезде, Звездане гомиле и маглине и *Млечни пут.*

Књига је намењена општој читалачкој публици. Отуда можемо закључити да већ из тог разлога Станојевић у књигу није укључио математичке формуле нити извођења. Ипак, мада по обиму није велика, по називима поглавља можемо закључити да је читалац оног времена могао да стекне солидан увид у астрономију. У књизи су астрономски појмови објашњени на једноставан и прегледан начин сликовитим језиком уз лепе пратеће цртеже и илустрације. Станојевић је посебну пажњу посветио звезданом небу, описима сазвежђа, оријентацији на небу помоћу сазвежђа и коришћењу звездане карте.

Биографија професора Ђорђа Станојевића



Ђорђе Станојевић[1] је рођен 1858. у Неготину, а умро је 1921. у Паризу. Завршио је Велику школу у Београду. Потом је био асистент Велике школе код свог професора Косте Алковића (1836-1909) а затим суплент у Првој београдској гимназији 1883. Као питомац војног Министарства био је 1883-1887. на студијама и раду у најпознатијим астрономским и метеоролошким опсерваторијама Европе (Потсдам, Хамбург, Медон, Гринич, Кју, Пулково). У то време, и нешто касније, објавио је неколико научних радова из астрономије у издањима Париске академије наука. То су први научни радови из астрономије код Срба. Можемо закључити да је његово занимање за астрономију било велико; на пример, своје предавање, као професор за физику и механику Војне академије, по повратку у земљу 1887, одржао је из астрономије. Касније се у науци углавном посветио физици. Био је професор Велике школе и Универзитета на том предмету. У астрономији радио је на реформи Јулијанског календара. Тај његов рад био је запажен у иностранству. Годину дана био је управник Опсерваторије Велике школе.

1 Према поглављу Астрономија Јована Симовљевића у књизи *Тридесет година Природно-математичког факултета у Београду*, ПМФ, Београд 1980.

Предговор

Станојевић на овом месту објашњава да је књигу написао са циљем да српској читалачкој публици представи науку о звездама. Можемо слободно рећи да Станојевић о звезданом небеском своду пише са извесним одушевљењем. Његов језик је леп и сликовит, стил украшен, што је одлика већине писаца о науци тог времена. На пример, када најављује о којим ће све астрономским појавама писати, на једном месту Станојевић каже:

Шта бива дању од звезда, које ноћу видимо, шта су те многе светле тачке, које се указују на сваком крају неба? Да нису то прости украси или, као што наш један песник вели, кандила, која су за небески свод утврђена, или су то тела огромне величине и велике сјајности?

Већ овде можемо закључити да је књига без сумње била значајна за образовање и популаризацију астрономије у Србији у другој половини деветнаестог века.

Прво поглавље *Звезде уопште*

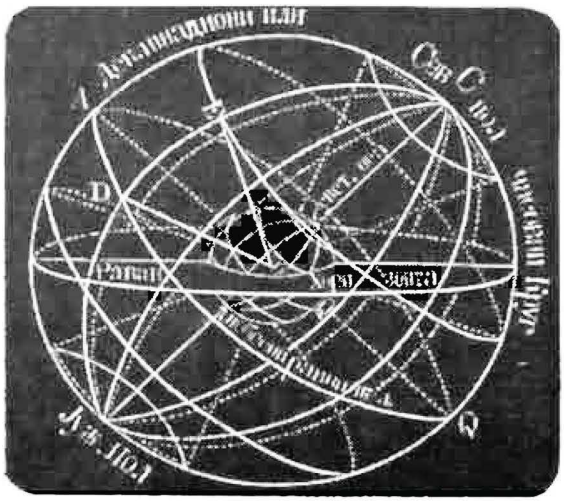
За ово поглавље истакли бисмо као посебну занимљивост поделу звезда на основу посматрања звезданог неба. Гледајући небо голим оком, већ су стари народи запазили да на небу има четири врсте *звезда*. Станојевић наводи да су то *сталне звезде*, *фиксне* или ***некретнице***, затим *покретне звезде* или ***покретнице*** у које спадају *планете* и *споредне планете* (*месеци, пратилице, трабанти*), треће су *комете* или ***луталице*** и четврте су *метеори*, ***летеће звезде*** или *ватрене кугле*. Називи ***некретнице***, ***покретнице***, ***лутајуће*** и ***летеће звезде*** су одомаћени у нашем народу и остали су непромењени до данас. Напоменимо да се за летеће звезде у народу често користи и назив ***звезде падалице***, док се реч ***трабанти*** за сателите планета готово изгубила.

Друго поглавље *Астрономска подела неба*

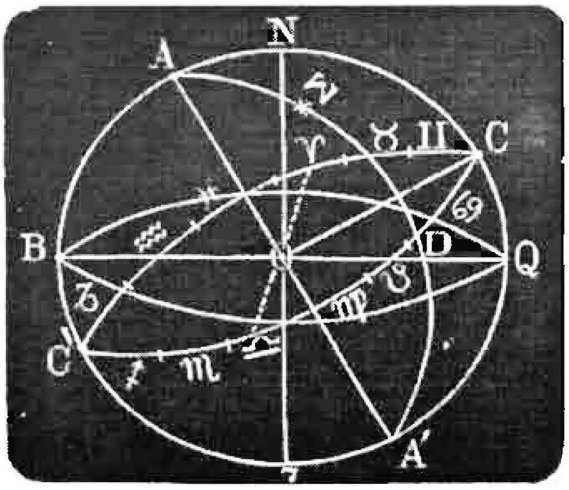
У овом поглављу Станојевић поступно дефинише и уводи основне равни, линије и тачке, односно сферне координатне системе који се користе у астрономији. То су хоризонтски и екваторски системи представљени лепом и јасном скицом на [сл. 3] и еклиптички на [сл. 5].

Слике су урађене на тамној подлози са белим линијама, ознакама и натписима како се углавном радило у оно време. Еклиптика је изванредно представљена са ознакама свих дванаест зодијачких сазвежђа. Поменимо да ***зенит*** као највишу тачку изнад главе посматрача назива и ***надтемена тачка***, док за ***екватор*** користи и називе ***полутар*** и

равнитељ. Јер, екватор полови небеску сферу на два једнака (равна) дела. Подсетимо да се у 19. веку користио Јулијански календар, који сада називамо стари, па Станојевић у књизи за привидни пролазак Сунца кроз тачку пролећне равнодневице наводи 8. марта, а за пролазак кроз јесењу тачку 10. септембра. Додавањем 13 дана, што је разлика између старог и новог (Грегоријанског) календара, добијамо датуме по новом календару, 21. марта и 23. септембра. Поменимо и сада већ заборављене називе ***равнодневични*** или ***еквинокцијални Колур*** (велики круг небеске сфере који пролази кроз небеске полове и обе равнодневичке тачке) и ***солстициони Колур*** (круг који садржи оба пола и обе тачке солстиција). У приложеној карти на крају књиге уцртана су оба ***Колура***.



Сл. 5.

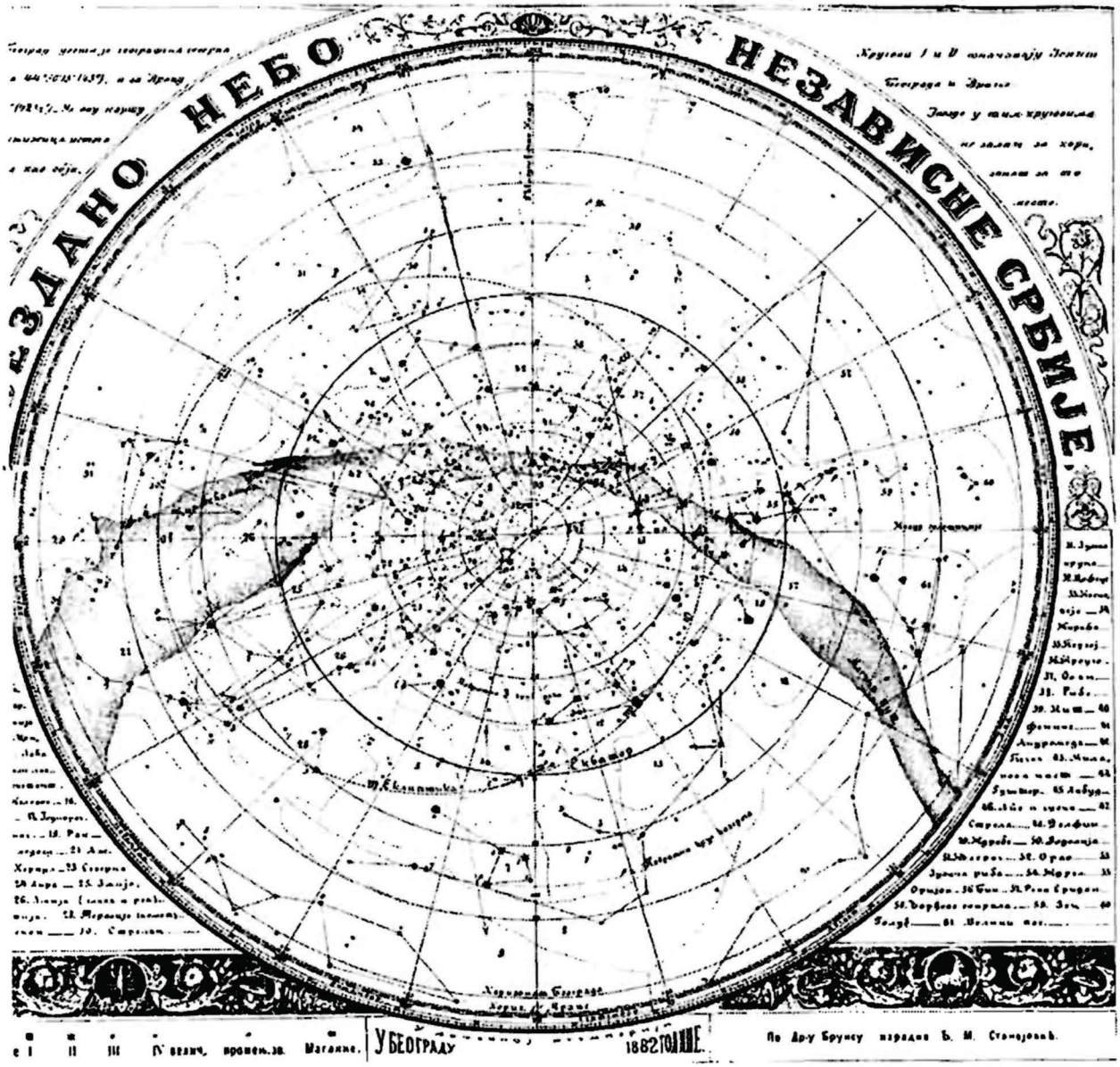


Сл. 6.

Треће поглавље *Број, даљина и поделa звезда по величини; звездане карте*

Најзанимљивији део у овом поглављу је опис звездане карте која је приложена на крају књиге. Ову карту урадио је Ђорђе Станојевић по Брунсу.

Звездана карта представља део неба који је могао да се види из ондашње Србије, из Врања као најјужније тачке и Београда као најсевернније тачке у њој. На карти су због јасноће уцртане звезде само до четврте привидне величине. Обележени су упоредници са поделом од 15 степени, екватор, повратни кругови *Рака* и *Јарца* (*Козерог*) и северни *обртни* (поларни) круг на којем је с десне стране пол еклиптике. Часовни кругови означени су за сваки сат. Уцртана је еклиптика са оба ***Колура***, равнодневичким и солстицијским. Имена и поједини дани месеца по ободу карте показују време када се Сунце привидно налази у дотичном зодијачком сазвежђу.



Четврто поглавље *Звездана јата и њихове звезде*

Ово поглавље је најобимније и односи се на сазвежђа. У 19. веку уместо речи **сазвежђе** користио се назив ***звездано јато***, па отуда и овај назив поглавља. У поглављу је наведен списак најпознатијих сазвежђа северно од зодијачког појаса (према Станојевићу *животињског круга*), сазвежђа зодијачког појаса и нека сазвежђа јужно од зодијачког појаса[2]. Станојевић спомиње 108 сазвежђа са напоменом да нека од њих многи астрономи не признају. Знамо да је сада од Међународне астрономске уније признато 88 сазвежђа. Стога се у књизи појављују сазвежђа као

2 Стране 24, 25 и 26 у књизи.

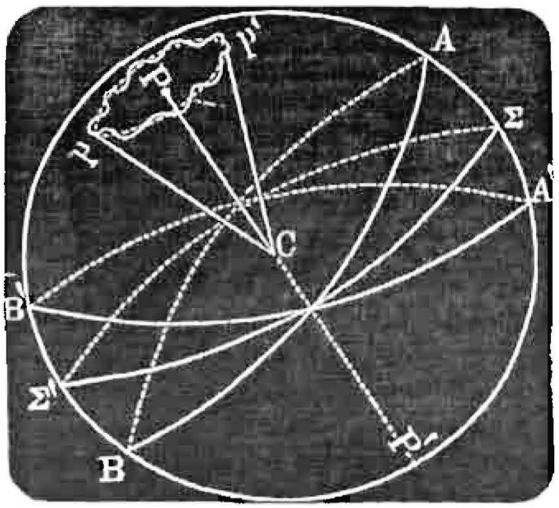


Миланова част (Honores Milani) и Ђорђева свирала (Harpha Georgii) којих данас у званичном списку сазвежђа нема. Многа сазвежђа су у овом поглављу детаљније описана, наведене су њихове најсјајније звезде и дато је објашњење како се помоћу најсјајнијих звезда и њихових астеризама сазвежђа могу препознати и пронаћи на звезданој карти и на небу. Свако сазвежђе носи број који је уписан на приложеној карти, јер би писање њихових имена карту учинило мање прегледном. Станојевић овде даје објашњење зашто се при навођењу зодијачких сазвежђа увек полази од *Овна*. Наиме, Теразије су стари симбол једнакости дана и ноћи јер је пре 2000. године у њима била тачка јесење равнодневице. Како се услед прецесије тачка јесење равнодневице померила из *Теразија* у сазвежђе *Девица*, тако се и тачка пролећне равнодневице (гама тачка) померила из *Овна* у *Рибе*. Сазвежђе *Ован* је тада било прво сазвежђе, јер је у њему била гама тачка, што је остало као наслеђе из прошлости. Тако се и данас *Ован* наводи као прво, мада је сада прво сазвежђе *Рибе*, у којем се налази гама тачка. На [сл. 15] и [сл.16][3] виде се графички прикази сазвежђа *Ован* и *Водолија*.

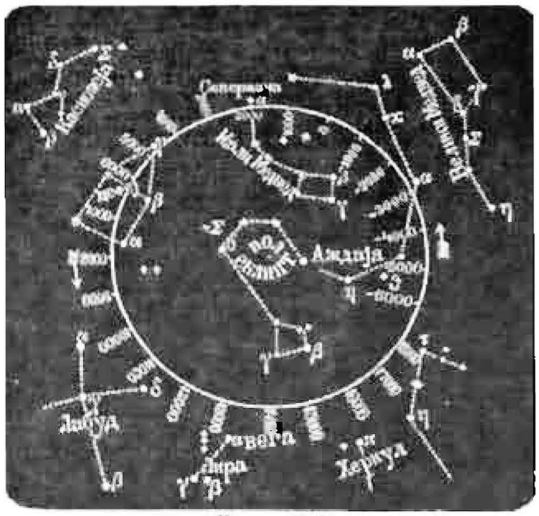
Пето поглавље *Промена звезданог неба услед прецесије, нутације, аберације и рефракције*

Овде се описује прецесионо и нутационо кретање небеског екваторског система. Све то лепо је илустровано на [сл. 20]. На [сл. 21] представљено је прецесионо кретање северног небеског пола кроз векове. На овом месту сазнајемо како је звезда *Киносура* добила ново име, *Северњача*. Северни небески пол на карти обележен је звездом Северњачом. Услед прецесије северни небески пол кроз 12000 година налазиће се у близини још сјајније звезде *Вега*. То значи да ће се звезда Вега за 12000 година звати *Северњача*. Наша звезда *Северњача*, како пише Станојевић, пре Христовог рођења звала се *Киносура*, јер северни небески пол није био близу ње.

3 Страна 35 у књизи.



Сл. 20.



Сл. 21.

Шесто, седмо и осмо поглавље *Право кретање звезда, Променљиве и нове звезде* и *Двогубе звезде*

Ова поглавља су најкраћа у књизи. Овде Станојевић пише о правим (сопственим) кретањима звезда, затим о променљивим звездама, врстама променљивих звезда, о новим звездама, као и о двојним звездама и врстама двојних звезда. Занимљиво је да Станојевић *двојне* звезде назива *двогубим* или *близним* звездама. Ни један од ових термина се није задржао у употреби, него се поред српске речи *двојна* све чешће користи туђа реч *бинарна* звезда.

Девето поглавље *Звездане гомиле и маглине*

За звездана јата у 19. веку користио се назив звездане гомиле. Станојевић наводи нека од њих: *Влашиће* и *Хијаде* у *Бику* и *Пресеће* у *Раку*. Све остале магличасте објекте Станојевић назива маглинама и дели их у осам класа; сјајне магле, слабе магле, врло слабе магле, планетске магле, врло велике магле, врло збијене звездане гомиле, прилично збијене гомиле и растурене звездане гомиле. Захваљујући наглом и великом развоју моћних астрономских инструмената, све до Хабловог телескопа крајем 20. века, ове нејасне магле блистају у свој својој лепоти као прави драгуљи дубоког неба.

Десето поглавље *Млечни пут*

У овом поглављу сликовито је описан *Млечни пут* или *Кумова слама* онако како је видимо на небу. Како Станојевић пише, у ведрим

ноћима то је светла трака неједнаке ширине која се пружа преко целокупног небеског свода (видети звездану карту). *Кумова слама* представљала је вишевековну загонетку почев од старих народа па све до 17. века. Тек је Галилео Галилеј 1609. по први пут својим дурбином у Млечном путу издвојио поједине звезде. Подсетимо се да је у част Галилеја 2009. проглашена годином астрономије.

Закључак

Књига *Звездано небо независне Србије* Ђорђа Станојевића лепо и сликовито описује звезде, сазвежђа, звездано небо као и друга тадашња знања из астрономије. Посебно истичемо Станојевићеву једноставност и лакоћу писања при излагању често сложених астрономских појава. Ове појаве, мада сложене, углавном описно представља без математичких израза уз коришћење веома лепих графичких приказа. Стога је ова књига пре свега општеобразовна и намењена широј читалачкој публици. У том погледу Станојевић је постигао свој циљ, да што већем броју читалаца приближи основна знања из астрономије. Велико ми је задовољство што сам ову драгоцену књигу извукла из таме библиотечког магацина на светлост дана Виртуелне библиотеке и учинила је лако доступном будућим читаоцима.

Имајући у виду методички начин писања астрономије у књизи *Звездано небо независне Србије*, њен садржај, јасноћу и лакоћу излагања, препоручујем ову књигу и данашњим студентима и средњошколским професорима не само астрономије него и математике, физике, географије. Књига се у електронском облику налази у Виртуелној библиотеци Националног центра за дигитализацију http://elib.matf.bg.ac.yu:8080/virlib/

50 Digitization of mathematical textbooks used in Serbia in the past

N. Pejović
NCD Review, vol. 12, pp. 55-64, 2008.

This article is a review of six mathematical books that are digitized and are available online in the Virtual Library of the Mathematical faculty. The first two books are the oldest printed mathematical Serbian textbooks, "Arithmetic" by Vasilije Damjanović. printed in 1767 and "Čislenica" (old form in Serbian of the word arithmetic) by Jovan Došenović. printed in 1809. These books are not of an interest only to the history of the Serbian people, but also for the study of the development of Serbian language, as they were written before Vuk Karadžić's reform of the language and orthography. The other four books are gymnasium textbooks written by Rista Karljiković. These excellent gymnasium textbooks were published between the two World Wars (1922-1941).

Преглед НЦД 12 (2008), 55–64

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DIGITIZATION OF MATHEMATICAL TEXTBOOKS USED IN SERBIA IN THE PAST

Abstract: We describe six mathematical books digitized and available online in the Virtual Library of the National Digitization Centre (Virtual Library, http://elib.matf.bg.ac.yu:8080/virlib/). The first two books are the oldest printed mathematical textbooks, *Аритметика* (arithmetic) by Vasilije Damjanović printed in 1767 and *Числениια* (old form in Serbian of the word arithmetic) by Jovan Došenović printed in 1809. These old books are elementary from the mathematical point of view, but, certainly, of interest to the history of the Serbian people, especially to studies of the Serbian language because they were written before Vuk Karadžić's reform of language and orthography. The other four books are gymnasium textbooks written by Rista Karljiković: *Тригонометрија, Алгебра, Аналитичка геометрија* and *Геометрија* (Trigonometry, Algebra, Analytic Geometry and Geometry). These excellent textbooks were used in Serbian gymnasiums and other secondary schools between the two World Wars (1922–1941). The digitization of these books is part of the project of forming electronic archives containing Serbian mathematical books printed in the past. In the article other reasons why these books are selected to be included in the Virtual Library are also given. The contents of these books are briefly presented together with some interesting points.

1. Introduction

The Virtual Library of the National Digitization Centre was founded with objective to form a collection as complete and representative as possible which contains retrodigitized books and other documents, preferentially of Serbian authors. Though in this library there are books from various fields, the main objective of the project is to preserve and present to the publicity, first of all, works in the field of mathematics and related sciences, such as mathematics, astronomy, mechanics, theoretical physics, mathematical geography and others in which the mathematical apparatus has a dominant role. In the present author's opinion in this literature an important place belongs to educational books, especially textbooks used in the teaching process in secondary schools and at the Belgrade University. There are many reasons to select this kind of books to be digitized and kept in computer archives. Of course, any word written or printed in the past, especially long ago, is part of cultural inheritance and this is a sufficient reason to qualify these works to be stored in such a way. Some more pragmatical reasons should be also given.

At first, the easy and simple access to these books via Internet enables historians of education and educators to study the education process in the past in Serbian schools. Any reader already by simple inspection of these books can make sure immediately that in this matter Serbia was not isolated, on the contrary the schooling system and education in Serbia, at least when mathematics and related sciences are the topic, followed the best ones in Europe. The reason may be looked for in the fact that the most educated Serbs from the XVIII and XIX centuries largely lived and were educated in the diaspora, above all in the Austrian Empire. It is also possible to make sure that the frequently present opinion that modern textbooks are better than those from the past is not correct. On the contrary, the themes in a

majority of old textbooks were presented in an interesting and methodically correct way followed by a lot graphics and nice illustrations. The examples and problems were not closely connected to the topic, but they often illustrated applications in other fields, for instance in geodesy and astronomy if the topic concerned trigonometry. The sources of the problems were cited, also the names of mathematicians, authors of important theorems and often one can find detailed historical notes. In these books important themes were treated, today omitted from the teaching process in secondary schools; some of them are absent in the teaching process even at universities. Here only two such examples are to be mentioned: solving the cube equation and spherical trigonometry. All these topics were presented within not large books, usually about hundred pages, rarely by about ten pages more.

These books are of interest, especially those written long ago, because in them one can follow the development of the Serbian language, at least in that part concerning mathematics and related sciences. The oldest ones were written in an ancient Serbian and by using a script preceding Vuk Karadžić's reform. Here *Аритметика* by Vasilije Damjanović, the first mathematical book published in Serbian (Venice, 1767), and *Численица* by Jovan Došenović (Budapest, 1809), the textbook used at theBelgrade Grand School a few years after the First Serbian National Revolution. In somewhat more recent books one makes sure that the terminology changed and it is not always easy to recognise what a given term meant. This is, certainly, a special subject of studying, but it is enough to mention here that, for instance, the Serbian word for *index* by the time of the First World War had been identical to the word for *hand* of watches (clocks). There are other examples concerning *set, mapping, real number* etc; to add that *cosmography* often meant *astronomy*.

At last to mention the Cyrillic fonts used in printing these books. In the main text we have nice Cyrillic letters including all kinds of fonts, italic and bold, the size, from big fonts used in the titles to small fonts used in the petit. The Latin letters in which the mathematics was printed were consistent in the shape and size with the main text. All of this contributed to the esthetical value of the books from the graphical point of view and to an easy reading. It should be mentioned that lead plates - letter carriers used in printing before the Second World War have been remelted so that the printed material is the only source for these fonts. The digitized copies of these texts appear as a good source for the reconstruction of the old fonts and enriching the presentation of the Cyrillic script.

On this occasion our decision is to present six books. These are the two oldest mathematical books in Serbian, *Аритметика* by Damjanović and *Численица* by Došenović and the four mathematical textbooks written by Rista Karljiković used between the two World Wars.

2. *Аритметика* by Damjanović and *Численица* by Došenović

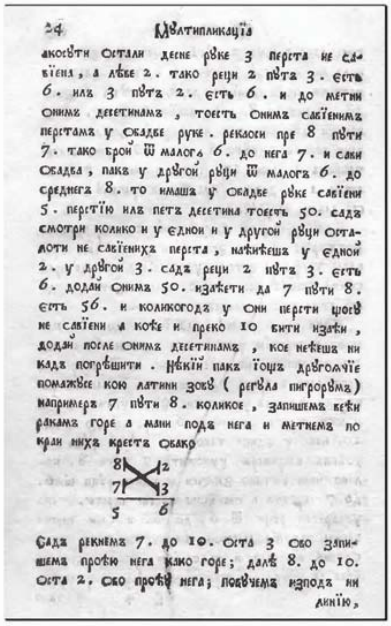
About Vasilije Damjanović (1734–1792) not much is known. It is known that he was born in Sombor, that he was a Sombor Senator, that he lived in diasphora and that he wrote *Аритметика*, the oldest Serbian book in mathematics. The book has been digitized and in this way preserved as a cultural treasure of the XVIII century.

Vasilije Damjanović's *Аритметика* is the first mathematical book printed in Serbian in Venice in 1767. As far as known to the present author, the only copy of this old book is in the Library of Matica Srpska in Novi Sad. The book has 368 pages and consists of two parts. The first part has six chapters, the second one seven chapters. In the first part the elementary arithmetical operations with natural numbers are presented- addition, subtraction, multiplication and division, whereas in the second part one has the fraction calculus, proportion calculus, interest rate calculus and conversions of units and currencies. The book

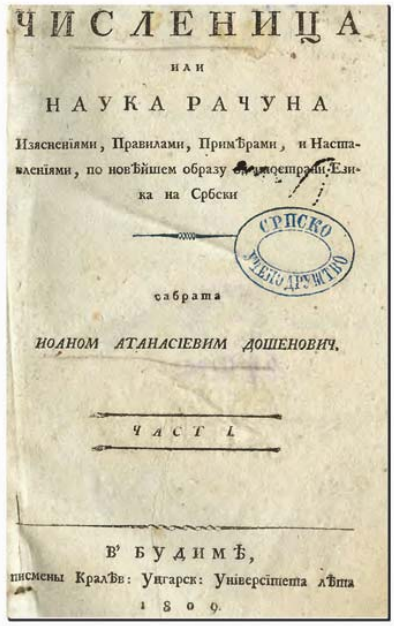
contains a large number of examples demonstrated and explained. Its purpose is practical, preferentially in trade. In the book various measuring units for goods used in the trade in that time and numerous kinds of currencies are presented.

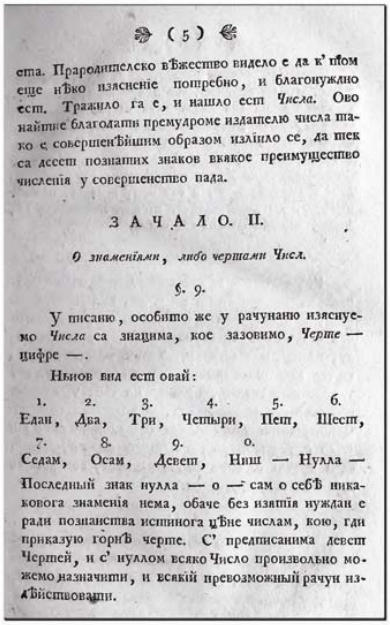
Jovan Došenović (1781–1813) was born in Počitelj in Lika. He was educated in Latin schools in Zadar and Novi Sad. In Padova he studied philosophy and took PhD in it. As a book keeper he worked in Triest and later in Budapest. His *Численица* was printed in Budapest in 1809 and was dedicated to a Budapest merchant Ignjatije Stanković. More data from Došenović's biography can be found in K. Došen's article [12].


Новам
СЕРБСКАМ
АРИѲМЕТИКА
ИЛИ
Простое наставленіе къ Хегапу
Изъ разныхъ Книгъ собрано, новыми примѣры кратчайшимъ образомъ изложено, и къ употребленію Сербскїя учащїямъ юности издано
Отъ
ВАСИЛІЯ ДАМЯНОВИЧА.
Въ Млеткахъ 1767.


24 Мултипликацїя
якостїи Остали десне рѹке 3 перста не савїенз, а лѣве 2. тако реци 2 пѹта 3. єсть 6. или 3 пѹта 2. єсть 6. и до метни Оними деветинама, тоесть Оними савїенима перстама у Обадве руке. реклоси пре 8 пѹти 7. тако брои ѿ малога 6. до нега 7. и сами Обадва, пака у дрѹгой рѹци ѿ малога 6. до среднега 8. то имаша у Обадве рѹке савїени 5. перстїю или петь десетина тоесть 50. сада смотри колико и у єднои и у другой рѹци Остаости не савїенихъ перста, наћићеша у єднои 2. у дрѹгой 3. сада реци 2 пѹта 3. єсть 6. додаи Оними 50. изаћети да 7 пѹти 8. єсть 56. и коликогода у Они персти цоуе не савїени а коће и преко 10 бити изаћи, додаи после Оними десетинама, кое неће ши кадь погрѣшити. Нѣкїи пака тоуе дрѹголчїе помажѹе кою латини зовѹ (регѹла пигрорѹма) напримеръ 7 пѹти 8. колико, запишема кеће рѣкама горе а мани подъ нега и метнема по кои нихъ крести Обако

Сада рекнема 7. до 10. Остаз 3 Обо запишема протїю нега како горе; далѣ 8. до 10. Остаз 2. Обо протїв нега; поквчема изподъ ни линїю,


ЧИСЛЕНИЦА
ИЛИ
НАУКА РАЧУНА
Изъясненіями, Правилами, Примѣрами, и Наставленіями, по новѣйшем образу [illegible] Езика на Србски
СРПСКО ДРУШТВО
сабрата
ИОАНОМ АТАНАСІЕВИМ ДОШЕНОВИЧ.
ЧАСТ I.
В' БУДИМѢ,
писмены КралѢв: Угарск: Универсітета лѣта
1809.


(5)
ета. Прародителско вѣжество видело е да к' пом еще нѣко изъясненіе потребно, и благонужно ест. Тражило га е, и нашло ест *Числа*. Ово найпре благодати премудроме издателю числа шако с совершенѣйшим образом изліло се, да шек са десет познаних знаков всякое преимущество численія у совершенство пада.

ЗАЧАЛО. II.

О знаменіями, либо чертами Числ.

§. 9.

У писаню, особито же у рачунаню изясняемо *Числа* са знацима, кое зазовимо, *Черте* — цифре —.

Њихов вид ест овай:

1. 2. 3. 4. 5. 6.
Едан, Два, Три, Четыри, Пеш, Шест,
7. 8. 9. 0.
Седам, Осам, Девеш, Ниш — Нулла —

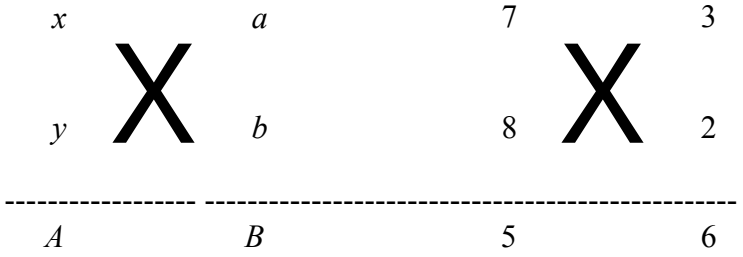
Последный знак нулла — о — сам о себѣ никаковога знаменія нема, обаче без изшпія нуждан е ради познанства испшнога цѣне числам, кою, гди приказую горнѣ черше. С' предписанима девет Чершей, и с' нуллом всяко Число произвольно можемо назначити, и всякій превозможный рачун изѣвйствовати.

Jovan Došenović's *Численица* has 327 pages and consists of nine chapters. The first chapter is about natural numbers, the second and third ones about elementary arithmetical operations – addition, subtraction, multiplication and division – the fourth about operations with mixed units, the fifth about fractions, the sixth about proportion calculus, the seventh

about conversions of units of length and weight, as well as about converting currencies, the eighth about calculus of successive proportions (верижни fractions) and the nineth about percents and interest rates. Like *Аритметика Численица* also contains a large number of examples demonstrated and explained. In it one finds a list of names and professions of its subscribers. It is seen that they were mostly merchants. It is interesting that this list contains the names of eleven pupils of the Grand School in Belgrade, just liberated from the Turks. In his *Численица* Došenović announced the second volume for educated Serbs which was never printed.

Аритметика from the XVIII century and *Численица* from the XIX one cannot be matched in their contents and level with later books which appeared in the early XX century, as is the case with, for instance, Karljiković's works. On the other hand these old books are important because in addition to the calculus of that time, one also gets insight into the development of the Serbian language and the information where mathematics was largely used then. A similar situation was met in the case of other, more developed, peoples who published such books earlier than the Serbs, some of them not much earlier than the Serbs.

A special point of interest is that these books are the proper place where one can see the origin of the symbol «times» (×) for multiplication. Namely, to avoid memorizing the complete multiplication table for one-digit numbers, such numbers exceeding five were multiplied as shown in the figure below. The multiplication procedure in a general case was like this. Let x, y be one-digit numbers exceeding 5 and a, b their complements to 10 (for instance, if $x = 7$, then $a = 3$). Furthermore, if $a \cdot b < 10$, then let it be $B = a \cdot b$. If $a \cdot b \geq 10$, let B be the last digit of this product and p the first digit (transfer). Finally, let it be $A = x - b$ ($=y - a$), i.e. $A = x - b + p$ if there is no transfer. Then it is $x \cdot y = AB$, AB is a two-digit number with digits A and B. For instance, if $x = 7$ and $y = 8$, then $a = 3$ and $b = 2$, $B = 2 \cdot 3 = 6$ and $A = x - b = 7 - 2 = y - a = 8 - 3 = 5$, thus $x \cdot y = AB = 56$.



We see that «times» is here in fact cross subtraction. It is not difficult to verify the correctness of this procedure. In Došenović's book «times» already acquires its modern shape and size and it is written in the same line as the first factor whereas the second factor was still written under the first one.

3. Rista Karljiković's Textbooks

About Rista Karljiković not much is known. It is known only that he was a gymnasium teacher of mathematics and Director of the II Female Gymnasium in Belgrade between the two World Wars. We have found that he wrote four gymnasium textbooks *Тригонометрија, Алгебра, Аналитичка геометрија* and *Геометрија*.

Karljiković's textbooks between the two World Wars were used not only in Serbia, but also in other parts of the Kingdom of Yugoslavia. The advanced chapters in these books were used in real gymnasiums which correspond to the scientific direction in gymnasiums in Serbia today. In his textbooks a really high level of mathematics teaching is seen. In the present

author's opinion these books are the best representatives of mathematical textbooks in Serbia of the given period. In fact, his books show how the mathematics taught in Europe in the twenties and thirties of XX century looked like. By inspecting these books we can make sure that they are qualitative textbooks and that some of them, especially his *Тригонометрија*, can be of interest to be read and used in the teaching process also today.


РИСТА КАРЉИКОВИЋ
ДИРЕКТОР II МЕШ. ГИМН. У БЕОГРАДУ

ГЕОМЕТРИЈА

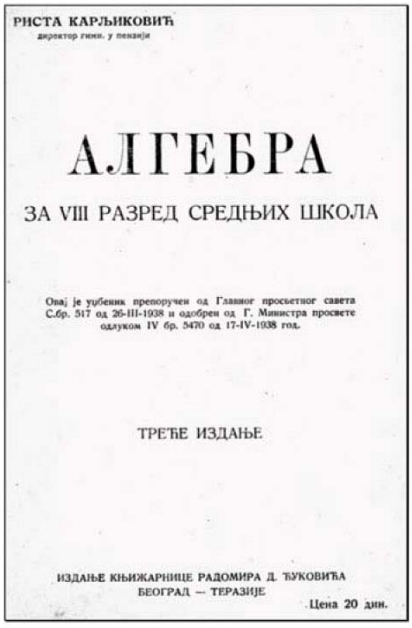
ЗА ВИШЕ РАЗРЕДЕ СРЕДЊИХ ШКОЛА

ТРЕЋИ ДЕО
ТРИГОНОМЕТРИЈА

ДРУГО ПОПРАВЉЕНО И ДОПУЊЕНО ИЗДАЊЕ

Овај је уџбеник препоручен од Главног просветног савета и одобрен од Г. Министра просвете одлуком С.и.бр. 23235 од 6 августа 1931 год.

ИЗДАЊЕ КЊИЖАРНИЦЕ РАЈКОВИЋА И ЂУКОВИЋА
БЕОГРАД — ТЕРАЗИЈЕ


РИСТА КАРЉИКОВИЋ
директор гимн. у пензији

АЛГЕБРА

ЗА VIII РАЗРЕД СРЕДЊИХ ШКОЛА

Овај је уџбеник препоручен од Главног просветног савета С.бр. 517 од 26-III-1938 и одобрен од Г. Министра просвете одлуком IV бр. 5470 од 17-IV-1938 год.

ТРЕЋЕ ИЗДАЊЕ

ИЗДАЊЕ КЊИЖАРНИЦЕ РАДОМИРА Д. ЂУКОВИЋА
БЕОГРАД — ТЕРАЗИЈЕ
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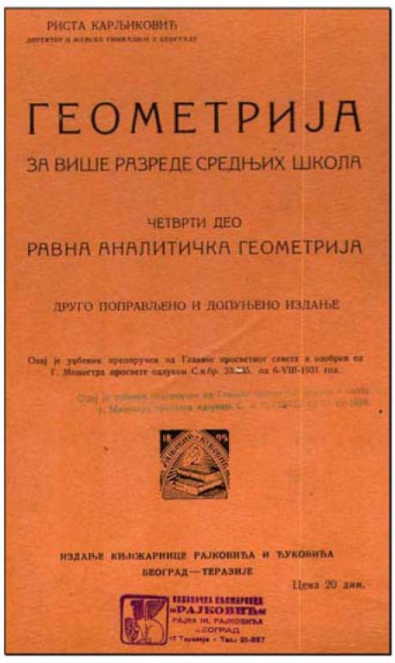

РИСТА КАРЉИКОВИЋ

ГЕОМЕТРИЈА

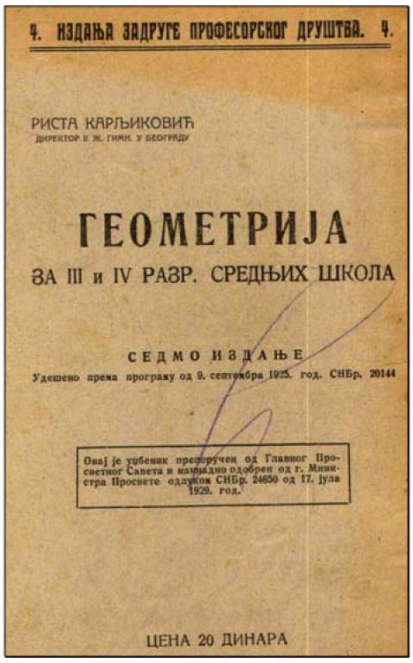
ЗА ВИШЕ РАЗРЕДЕ СРЕДЊИХ ШКОЛА

ЧЕТВРТИ ДЕО
РАВНА АНАЛИТИЧКА ГЕОМЕТРИЈА

ДРУГО ПОПРАВЉЕНО И ДОПУЊЕНО ИЗДАЊЕ

Овај је уџбеник препоручен од Главног просветног савета и одобрен од Г. Министра просвете одлуком С.и.бр. 23235. од 6-VIII-1931 год.

ИЗДАЊЕ КЊИЖАРНИЦЕ РАЈКОВИЋА И ЂУКОВИЋА
БЕОГРАД—ТЕРАЗИЈЕ
Цена 20 дин.


4. ИЗДАЊА ЗАДРУГЕ ПРОФЕСОРСКОГ ДРУШТВА. 4.

РИСТА КАРЉИКОВИЋ
ДИРЕКТОР II Ж. ГИМН. У БЕОГРАДУ

ГЕОМЕТРИЈА

ЗА III и IV РАЗР. СРЕДЊИХ ШКОЛА

СЕДМО ИЗДАЊЕ
Удешено према програму од 9. септембра 1925. год. СНБр. 20144

Овај је уџбеник препоручен од Главног Просветног Савета и најнадно одобрен од г. Министра Просвете одлуком СНБр. 24650 од 17. јула 1929. год.

ЦЕНА 20 ДИНАРА

Тригонометрија by Rista Karljiković was in use for higher forms of secondary schools. In the framework of the subject of geometry it was its third part. The book was recommended as a textbook by the Main Council of Education and approved by the Education Minister, Decision No 23235 of August 6, 1931. The book was published by the House of Rajković and Đurković in Belgrade. It is also known that the book had several editions and

was in use till the beginning of the Second World War. The present author's decision to present this book was made by bearing in mind its, partly unusual, contents. Whereas in its first parts the subject is standard topics of trigonometry, the last one is devoted to spherical trigonometry. The book contains the following chapters: *Introduction, Goniometry, Plane Trigonometry and Spherical Trigonometry.*

It is interesting that Chapter of *Plane Trigonometry* contains the application of trigonometry to solving problems in stereometry, as well as that concerning problems of practical geometry and astronomy. This can be especially seen in Problems No 7–15. Let Problem No 13, p. 110, be selected. Its subject is the procedure of the distance determination Moon-to-Earth.

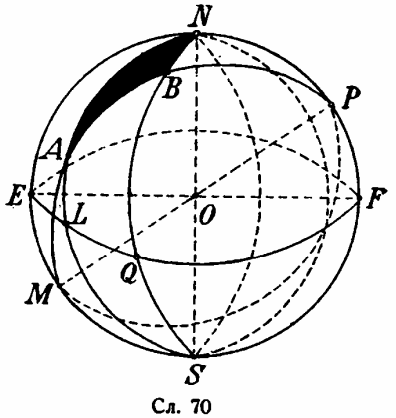
Бројни пример. **Зенитни угао Месеца измерен у Берлину (северна географска ширина $\varphi = 52^0\,31'\,33''$) је $z = 32^0\,3'\,51''$, а зенитни угао месеца, измерен у истом тренутку на гребену Добре Наде (јужна географска ширина $\varphi' = 33^0\,56'\,3''$), је $z' = 55^0\,42'\,48''$; наћи одстојање месеца до центра наше Земље када је њен полупречник $R = 6370{,}308$ *km*.**

Note. Aristarchus as early as in the III century BC found a geometric method for the purpose of determining the distance to the Moon, but the value obtained by him then was too crude. The first accurate distance determination for a celestial body using the Moon as the example was done by Lalande and La Caille in the XVIII century just measuring the coordinates of the Moon from the ends of a baseline connecting the Cape of Good Hope and Berlin. It is seen that namely this numerical example was used by Karljiković.

We want to pay attention to Chapter *Spherical Trigonometry* in which spherical triangles with their types and properties are described. Further on one gives the procedures how to find elements of a rectangular spherical triangle, also of a general spherical triangle, followed by applications in astronomy and geodesy. So on page 140 of *Тригонометрија* we find the following example:

III. Сферна раздаљина између два места на земљи

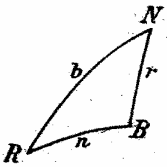
Да бисмо израчунали сферну раздаљину места A и B на земљи, треба да знамо географске ширине и географске дужине тих места. Ако су φ_1 и λ_1 географска ширина и дужина места A, φ_2 и λ_2 географска ширина и дужина места B, N и S полови земље, круг $NESF$ главни меридијан, а круг $ELQF$ екватор, кругови $NALS$ и $NBQS$ меридијани места A и B, онда је: $\varphi_1 = AL$, $\lambda_1 = EL$, $\varphi_2 = BQ$ и $\lambda_2 = EQ$, тада је тражена сферна раздаљина AB страна сферног троугла ABN у коме знамо две стране NA и NB ($NA = 90^0 - \varphi_1$ и $NB = 90^0 - \varphi_2$) и захваћени угао $N = \lambda_2 - \lambda_1$. Задатак се, дакле, своди на пети случај решавања сферног троугла из претходног параграфа.



Сл. 70

In addition to the general solution Karljiković gives also the calculation example for solving a spherical triangle. The example concerns the distance determination in kilometers between Rome and Vienna. Here it is assumed that the geographic latitudes of Rome and Vienna are positive. Thus already in 1930, much earlier than this convention was adopted by the International Union (in 1977).

Пример. Наћи сферну раздаљину између Рима и Беча, ако је геогр. ширина Рима $\varphi_1 = 41^0\,53'\,54''$ *а Беча* $\varphi_2 = 48^0\,12'\,35''$, *геогр. дужина Рима* $\lambda_1 = 12^0\,28'\,48''$, *а Беча* $\lambda_2 = 16^0\,22'\,42''$, *рачунајући од гриничког меридијана.*



Сл. 71

Ако сферни пол N, Рим (R) и Беч (B) дају сферни троугао NRB, онда је страна $NR = 90^0 - \varphi_1 = 48^0\,6'\,16''$, страна $NB = 90^0 - \varphi_2 = 41^0\,47'\,25''$, а угао код $N = \lambda_2 - \lambda_1 = 3^0\,53'\,54''$.

Тада је:

$$tg\frac{B+R}{2} = \frac{\cos\frac{b-r}{2}}{\cos\frac{b+r}{2}}\,cotg\frac{N}{2} \text{ и } tg\frac{B-R}{2} = \frac{\sin\frac{b-r}{2}}{\sin\frac{b+r}{2}}\,cotg\frac{N}{2} \text{ или}$$

$$tg\frac{B+R}{2} = \frac{\cos 3^0\,9'\,25{,}5''\cdot cotg\,1^0\,56'\,57''}{\sin 44^0\,56'\,50{,}5''} \text{ и}$$

$$tg\frac{B-R}{2} = \frac{\sin 3^0\,9'\,25{,}5''\cdot cotg\,1^0\,56'\,57''}{\sin 45^0\,56'\,50{,}5''};$$

$$\log tg\frac{B+R}{2} = 1{,}48340 \text{ и } \log tg\frac{B-R}{2} = 0{,}79754;$$

$$\frac{B+R}{2} = 88^0\,7'\,6''; \frac{B-R}{2} = 80^0\,56'\,38''; \quad B+R = 176^0\,14'\,12'',$$

$$B-R = 161^0\,53'\,16''; \; B = 169^0\,3'\,44'', \; R = 7^0\,10'\,28''.$$

$$tg\frac{n}{2} = \frac{\sin\frac{B+R}{2}}{\sin\frac{B-R}{2}}tg\frac{b-r}{2} = \frac{\sin 88^0\,7'\,6''\cdot tg\,3^0\,9'\,25{,}5''}{\sin 80^0\,56'\,38''}; \; \log tg\frac{n}{2} =$$

$= 2{,}74682; \frac{n}{2} = 3^0\,11'\,42''; n = 6^0\,23'\,24''$. Дакле, сферна раздаљина Беч—Рим износи $6^0\,23'\,24''$, или у дужинској јединици:

$$RB = \frac{40000000\cdot 6^0\,23'\,34''}{360^0} = \frac{40000000\cdot 23004}{1\,296\,000} =$$

$= 710\,000\,m = 710\,km.$

In an analogous way in astronomy the spherical distance between any two celestial bodies on the celestial sphere is determined.

Karljiković in his *Тригонометрија* in addition to interesting examples gives a large number of problems. In the case of some of them, especially difficult ones, he cites the source and the year. These are largely the problems which were given at the mature exams in the Yugoslav Kingdom (Belgrade, Užice, Karlovac, Priština, Šibenik, etc) and France (Sorbonne, Caen, Marseille, etc). Some well-known theorems he cites after the author's name: cosine theorem – Carnot's theorem, the Gauss formulae – Mollweide's equations. It is curious that Heron's formula yielding the area of a triangle he refers to also as Brahmagupta's formula. In the book one finds words almost lost today, for instance *cosmography* and *goniometry*.

The spherical trigonometry presented in Karljiković's book was taught in real gymnasiums only which would correspond to the scientific direction in the modern gymnasiums. Today these topics are not taught in secondary schools.

Алгебра by Rista Karljiković was used as a textbook in the final form of secondary schools before the Second World War. It was adopted by the Main Education Council and approved by the Education Minister, Decision IV No 5470 of April 17, 1938. The textbook

was published by the Đurković Publishing House in Belgrade. It had several editions and the third one was digitized. Karljiković's *Алгебра* contains the following chapters: *Theory of Derivatives and Notions of Maximum and Minimum, Fundaments of Integral Calculus, Complex Numbers, de Moivre's formula, Third-Power Equations* and

Appendix. The most interesting chapter in this book is *Complex Numbers, de Moivre's formula, Third-Power Equations* which was taught in real gymnasiums only. Here one presents the trigonometric form of a complex number and the operations with complex numbers. To find a power of a complex number he used de Moivre's formula and he gave the application of this formula to the case of calculating the functions of the *n*-th multiple of an angle by using its sine and cosine.

Из свега овога види се да једначина $x^3+px+q=0$ има један корен стваран а два имагинарна, ако је испуњен услов $27q^2+4p^3>0$. Образац:

$$x=\sqrt[3]{\frac{-27q+\sqrt{729q^2+108p^3}}{54}}+\sqrt[3]{\frac{-27q-\sqrt{729q^2+108p^3}}{54}},\ \text{или}$$

$$x=\sqrt[3]{\frac{-9q+\sqrt{81q^2+12p^3}}{18}}+\sqrt[3]{\frac{-9q-\sqrt{81q^2+12p^3}}{18}},\ \text{или}$$

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}\quad(11)$$

зове се ***Кардан-ов.*** Овај образац употребљава се при добивању корена оне једначине трећег степена облика $x^3+px+q=0$, која има имагинарне корене, за шта је потребан услов $\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3>0$ или $27q^2+4p^3>0$. Овај се образац не примењује при решавању једначине $x^3+px+q=0$, који има само

In this book Karljiković presents the procedure of solving a cube equation, a topic not learnt today in secondary schools. He cites various forms of third-power equations and their properties, types of roots (all three real or one real and a pair of a complex number and its conjugate) and the relationship between the roots and coefficients. The derivation of Cardano's formula is presented. This formula was used to obtain all the three roots of the third-power equation.

Note. Karljiković in *Appendix* says that the name of the formula after Cardano is an injustice since its real inventor is Tartaglia.

In the same chapter Karljiković presents a historical development of arithmetics and algebra, beginning with Phoenicians, Egyptians, Indians, Babylonians and Chaldeans which is unusual in more recent textbooks for secondary schools. So we can learn that Phoenicians created arithmetics, Egyptians geometry and Chaldeans and Babylonians astronomy. Further on Karljiković writes that the oldest arithmetical manuscripts found have their origin in Egypt and that they were written in the hieroglyph digits on papyrus between the XX and XVII centuries BC. He writes that the first elements of algebra are met by Euclid in his work *The Elements* and that the first work in algebra was published in the IV century AD by Diophant of Alexandria (325–409). Further on Karljiković presents the development of algebra in the Middle Ages and cites that the Arab and Indian arithmetics and algebra were brought to Western Europe in the XII century by Italian Leonardo of Pisa, whereas the creator of modern algebra is François Viète (1540–1603) who introduced algebraic symbolism which contributed to a significant simplification of mathematical operations and put up the level of algebra. Also, he presents the contributions of other mathematicians of that time saying that

Descartes (1596–1650) is the creator of analytic geometry and that he improved the application of algebra in geometry. He describes the contributions of later mathematicians saying that the theory of infinitesimal calculus was created in the XVII century by Leibniz and Newton, and that in the XVIII and XIX centuries there is a cluster of famous mathematicians: Euler, D'Alembert, Bézout, Laplace, Lagrange, Monge, Carnot, Argand?, Moore?, Gauss, Cauchy and Jacobi.

Karljiković at the end of the book gives a short review of introducing mathematical symbols. In this way we learn that the digits of nowadays originate from India and that they reached Europe through Arabs and for this reason are referred to as Arab digits. Then the symbols for addition and subtraction were introduced by Leonardo of Pisa, to appear for the first time in a printed form in Germany in 1489, the equality symbol was introduced by Recorde in 1552, the inequality symbols, greater and less (>, <), were introduced by Harriot in 1600 and the symbol "different from"(≠) by Christofen?

Аналитичка геометрија by Rista Karljiković was used as a textbook for higher forms of secondary schools. It was recommended as a textbook by the Main Education Council and approved by the Education Minister, Decision 23325 of August 6, 1931. This book was published by the House of Rajković and Đurković in Belgrade. It had several editions, the second one, corrected and extended, was digitized .

The book has the following chapters: *Introduction, Point, Straight Line, Circle, Ellipse, Hyperbola, Parabola* and *Tangents and Perpendicular Lines to Curves* .

Some sections in the book have an asterisk because they were foreseen for pupils of real gymnasiums only; examples are: Polar Equation of an Ellipse, Polar Equation of a Hyperbola, Polar Equation of a Parabola, Poles and Polar Lines of Curves, General Form of Equations of Second-Order Curves.

Геометрија by Karljiković was used as a textbook in lower forms of secondary schools before the Second World War. It was recommended as a textbook by the Main Education Council and approved by the Education Minister, Decision 24650 of July 17, 1929. This book was published by the Printing House of Jovanović in Belgrade. This textbook had many editions, the seventh one was digitized . It covers standard topics from geometry for the age of pupils.

Karljiković's books were written nicely from the methodical point of view and the material in them is introduced gradually. Karljiković gives a lot of treated examples so that any handbook containing problems which would follow the main book is not needed. A special accent the present author wants to stress on the simplicity in the derivation of fundamental theorems of spherical trigonometry in his *Тригонометрија*, also the procedure of obtaining the solution of cube equation in *Алгебра*. This part of mathematics was available in any case to the pupils of real gymnasiums of that time. Printing errors in his books are almost absent. Bearing in mind the methodical way of presentation, the contents, examples and problems the present author would recommend these books also to the pupils, teachers in secondary schools, even university students, of our time.

5. Conclusion

The oldest Serbian mathematical books *Аритметика* by Damjanović from the XVIII century and *Числениџа* by Došenović from the XIX one have been digitized in the framework of the Virtual Library of the National Digitization Centre and in this way preserved as an important cultural inheritance. Though in their contents elementary, these old books are, certainly, of importance and of interest to the history of the Serbian people, especially to the studies of the Serbian language because they were written before Vuk Karadžić's reform of

language and orthography. In them one can also find how and in what way elementary mathematics was learnt in the XVIII century and in the early XIX one, not only in Serbia, but also in other European countries. Of course, these first books cannot be matched in their contents and level with later Serbian books which appeared in the early XX century. Among them four textbooks by gymnasium teacher Rista Karljiković used between the two World Wars have a special place – *Тригонометрија, Алгебра, Аналитичка геометрија* and *Геометрија*. They are methodically nicely composed textbooks for secondary schools with many examples treated completely and explanations. With regard to their contents, methodical way of presentation, examples and problems they can be recommended to the pupils, teachers of secondary schools, even university students, of our time. In addition to Damjanović's *Аритметика* and Došenović's *Числениця*. these four books by Karljiković have been also digitized. All the six digitized copies are available on the Internet site of the Faculty of Mathematics of Belgrade University in the framework of the Virtual Library.[1]

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51 Astronomical Society *Magellanic Cloud*

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Astronomical society "Magellanic cloud" was founded in 2001 in the town of Prokuplje, Serbia, by A. Simonović, A. Valjarević, then students of geography at the University of Priština, Ž. Mijajlović, mathematician, and S. Šegan, astronomer, professors of the University of Belgrade. This article presents the activities of the Society in the popularization of astronomy and mathematics in Prokuplje and its vicinity. Also, the role of the Society in building of the Astronomical station of the Astronomical observatory of Belgrade at the mountain Vidojevica near Prokuplje is pointed out. At the moment of preparation of this book, Simonović is the graduated geographer and still is working on the popularization of astronomy, Valjarević is a prominent professor of geography at the University of Belgrade, but still he is keeping an eye on astronomy, while Šegan and Mijajlović are retired professors of the Belgrade University. The astronomical society "Magellanic Cloud" does not exist since 2015.

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ASTRONOMICAL SOCIETY "MAGELLANIC CLOUD"

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Abstract. Astronomical society "Magellanic cloud" was founded on May 16, 2001 in the town of Prokuplje, Serbia, by A. Simonović, A. Valjarević, then students of geography at the University of Priština, Ž. Mijajlović professor of mathematics, S. Šegan, professor of astronomy, both of the University of Belgrade. In this paper we present the activities of the Society in the popularization of astronomy and mathematics in Prokuplje and its vicinity. Also, the role of the Society in building the Astronomical station of the Astronomical observatory of Belgrade at the mountain Vidojevica close to Prokuplje is described.

1. FROM THE HISTORY OF THE SOCIETY

The astronomical society "Magellanic cloud" is founded in the town of Prokuplje on May 16, 2001. The Society is recorded in the Register of civil associations of Serbia under the registration number 2735, page no. 983. The founders of the Society are A. Simonović, A. Valjarević, then students of geography at the University of Priština, Ž. Mijajlović professor of mathematics, S. Šegan, professor of astronomy, both of the University of Belgrade. Apart from the founders, the members of the Board of the Society are professor Dušan Ćirić from the University of Niš, professor Nadežda Pejović from the Faculty of Mathematics, University of Belgrade and Dr. Stojan Obradović, at the time professor of physics in the Grammar School in Prokuplje. At this moment there are about twenty members of the Society, mostly students and professors from the Grammar School.

The Society was born as follows. Žarko Mijajlović built two telescopes of Newton type at the end of 1970's. The first telescope has an 18 cm primary mirror and a focal length of 145 cm. This telescope is still in good condition. In 2006 it was donated to the Grammar school in Prokuplje. The second telescope has a 23 cm primary mirror and a relative aperture f/8. The owner donated this telescope to the Society when it was founded, in 2001. Members of the Society, other people and scholars from Prokuplje had many opportunities to observe various celestial objects and phenomena through these telescopes. The observations included the last transition of Halley's

Comet in 1986, dark spots on Jupiter left after fragments of the Shoemaker-Levy (SL9) comet collided with Jupiter in 1994, the Great Red Spot and shadows of Jovian satellites on Jupiter, Cassini Division in Saturn rings, the Alpine Valley on the Moon, geostationary satellites and most of Messier objects. Aleksandar Valjarević, already as a pupil showed great interest in astronomy and observations of the sky. In the summer of 2000, Valjarević and Simonović, already students of geography, asked Mijajlović how to build or buy a telescope. Mijajlović gave them his 23 cm telescope, mentioning that this borrowing may last long if they could justify this somehow. Valjarević and Simonović did not waste time. During the next winter they organized sky observations through the telescope for the general public in Prokuplje. Many people gathered on this occasion, even the representatives of local authorities took part in the event. In the part of the town where the observations were organized, the street lights were turned off. That was probably the first successful action in our country against light pollution, the main enemy of astronomers. However, someone ironically noted that municipal officials welcomed this light shutdown with delight since at that time there was the shortage of electric power. After that, Valjarević and Simonović had two appearances on local TV station dedicated to astronomy. These were received with interest and the shows were repeated several times. Thereafter the Society was founded.

In the work of the Society other instruments were used as well. For example, in 2001 an expedition to the mountain Vidojevica was organized. Participants observed the Sun using one small automatized telescope, Mead ETX 60AT, of the 60 mm aperture. At the time the Sun was very active. Four large sunspots were observed near the equator and a group of smaller sunspots at the southern hemisphere. They were of various intensity and many details could be seen such as the shapes, umbra and penumbra.

Besides sky observing, members of the Society take photographs of the cosmic objects. In this, Simonović and Mijajlović are particularly active. They are using digital cameras Canon EOS 300D and 400D and a WEB camera. They made several hundreds of photos up to now and some of them are displayed at the Society Internet site, http://www.dorado.org.yu.

2. PROMOTION OF ASTRONOMY AND MATHEMATICS IN TOPLICA REGION

Since its foundation, the Society organizes about ten lectures per year on astronomy and mathematics for general public and students of the Grammar school in Prokuplje. The lecturers are professors of astronomy and mathematics from the Belgrade University, University of Niš and researchers from the Astronomical Observatory in Belgrade. Among others, the following prominent astronomers and mathematicians gave the lectures: Stevo Šegan, Zoran Knežević, Milan Dimitrijević, Slobodan Ninković, Jelena Milogradov- Turin, Nadežda Pejović, Olga Atanacković, Dušan Ćirić, Miodrag Rašković, Aleksandar Lipkovski, Luka Popović and Žarko Mijajlović. Since 2005, Nadežda Pejović organizes these activities of the Society. Due to her efforts, the facultative teaching of astronomy in the Grammar school in Prokuplje was introduced and preparation of pupils for astronomical competitions was initiated. These activities are supported by the teachers of physiscs and mathematics of the Grammar School in Prokuplje, particularly by Dragan Krstić, director of the Grammar school,

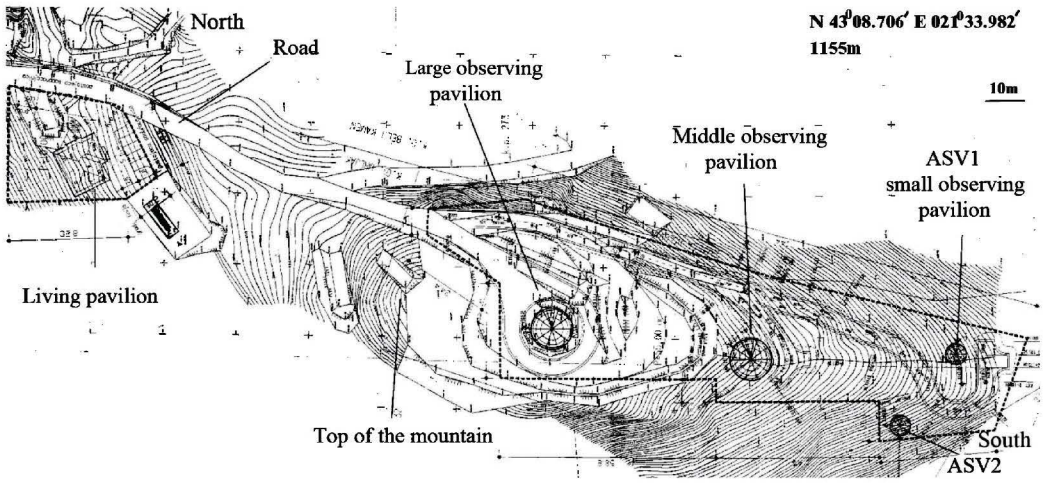

Figure 1: Situation plan of the Astronomical station at the mountain Vidojevica.

and Branislav Rosić, professor of physics. Several graduate students of astronomy of the Faculty of Mathematics took part in these activities, particularly Tatjana Jakšić and Sonja Vidojević. The members of the Society are also working on building of the Public Observatory on the Hissar hill in Prokuplje.

3. ASTRONOMICAL STATION AT THE MOUNTAIN VIDOJEVICA

In the period from 2000 to 2002, the Society played an important role in pursuing the idea of building the new astronomical station at the mountain Vidojevica (ASV) of the Astronomical Observatory in Belgrade (AOB). Namely, at that time Mijajlović, Valjarević and Simonović were searching for a suitable location for the observatory of the Society itself. They undertook several expeditions in Prokuplje countryside, and once, during such an expedition at the mountain Vidojevica in the summer of 2001 they have been joined by Stevo Šegan. They were inspecting the top of the mountain and instantaneously concluded that this place would be appropriate for their intention.

But Šegan said the key sentence, that not the observatory for amateur purpose, but a professional observatory should be built there. He also reminded us about the project, over twenty years old, of building an astronomical observatory exactly at this location. Soon we came to the idea of starting the initiative for renewing this old project. And later, several years after this, the Society was working on this professional project. The members of the Society were very active in many respects concerning the building of the ASV. For example, they supplied most of the documents from the local administration of Prokuplje necessary to build such an object. They also wrote several papers concerning the future function of the ASV and on astro-climate conditions on the mountain Vidojevica. We can say that the founding of the Society was the first step in building the Astronomical Station at the mountain Vidojevica.



Prokuplje



Valjarević and Simonović at Vidojevica





Solar eclipse 31.05.2003 (Simonović)





Newton telescope of the Society, apert. 23 cm



Starry sky (Simonović)



Seal of the Society



ASV living pavilion at Vidojevica



ASV observing pavilion under construction

Figure 2

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52 Development of astronomy in the Toplica Region

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This article presents activities connected with astronomy in the Toplica region. Until the end of the 20th century there were not significant endeavors in this area in Toplica, except some amateurish attempts. However, since the year 2000 considerable development started. They are linked mostly to the founding of the Astronomical Society "Magellanic Cloud" and, at the professional level, the rising of Astronomical station at the mountain Vidojevica nearby the town of Prokuplje.

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DEVELOPMENT OF ASTRONOMY IN TOPLICA REGION

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Abstract. In this article we present the undertakings connected with astronomy in the Toplica region. Until the end of the 20th century there were not significant endeavours in this area in Toplica, except some amateurish attempts. However, since the year 2000 considerable development started. They are linked mostly to the founding of the Astronomical Society *Magellanic Clouds* and, at the professional level, the rising of Astronomical station at the mountain Vidojevica nearby the town of Prokuplje.

1. INTRODUCTION

Until recently all activities in astronomy in the Toplica district were linked to the popularization and education of astronomy and other mathematical sciences. First attempts in this area rose at the beginning of the 20th century when the Gymnasium in Prokuplje was founded (1908), but they were not so successful. Here should be mentioned attempts in the twenties of Aleksa Savić, the prominent medical doctor, humanist and donator who lived in Prokuplje. First amateur telescopes were brought in Prokuplje in the beginning of seventies, and at the same time started the regular education of astronomy in high-schools. The turning point was 2001 when the amateur astronomical society *Magellanic Clouds* was founded by a group of university professors from Belgrade (Ž. Mijajlović and S. Šegan) and Niš (D. Ćirić) and two students from Prokuplje (A. Valjarević and A. Simonović), enthusiasts in astronomy. Immediately the old idea of rising an astronomical station of the Astronomical observatory in Belgrade (AOB) at the mountain Vidojevica nearby Prokuplje was renewed. Soon the building of the station started due to the efforts of the staff of AOB, first of all Milan Dimitrijević (the previous director of AOB) and Zoran Knežević (the actual director of AOB). The astronomers of the

Chair for astronomy of the Faculty of mathematics of the University of Belgrade contributed much in the beginning of the project, primarily professor Stevo Šegan. The significant role in this mission played the astronomical society ***Magellanic clouds*** and the administration of the Prokuplje County. Since then, popularization of astronomy was intensified and most prominent Serbian astronomers delivered public lectures there, more than ten every year. Now days, astronomy is popular science in Toplica region. For example, the Gymnasium in Prokuplje is a rare high school in Serbia where astronomy is regularly taught as a separate subject. It also has in it's possession a telescope of Newton type of the aperture 18 cm, while the society ***Magellanic clouds*** has a telescope (reflector) of the aperture of 23cm. There are also plans to build a small public observatory at the hill Hisar, located in the South of Prokuplje.

2. TOPLICA DISTRICT

Toplica region is located in the southern-central part of Serbia and it occupies about 2230 km^2 with population about 100.000. Its southwest border is with Serbian province of Kosovo, currently under UN. There are only two mayor towns: Prokuplje (population 30.000) and Kuršumlija (population 14.000). There are a lot of archeological sites, some of them dated 6000 B.C, but also many medieval monasteries and remains.



Figure 1: Pottery old 8000 years found near Prokuplje.



Figure 2: Remains of the Monastery Sv. Bogorodica (St. Madonna), Kuršumlija.

There are also many springs and natural rarities and curiosities. We shall mention two of them. The first one is Đavolja Varoš, meaning "Devil's Town", a geological formation, located by the Radan mountains near Kuršumlija. This geological rarity features exotic formations described as earth pyramids. They have been formed by erosion. Djavolja Varoš is under protection of the state and by a 1995 decision of the Serbian Government, it was proclaimed a natural good of outstanding importance and put under the first-category level of protection.



Figure 3: Đavolja varoš (Devils Town).



Figure 4: Meanders of the river Toplica, satelite view.

The second one is the double-twist of the river Toplica around the town of Prokuplje. At the distance of only several tens of meters, Toplica flows in two opposite directions, making the hill Hisar almost an island.

What make the Toplica region interesting for professional astronomy are astroclimate conditions. According to meteorological measurements, some of them dating from 1900, most of them since 1950, there are on the average about 120 clear nights a year. This region is one of the driest in Serbia, as it only gets 580 mm of accretion an annual basic. In particular, the Vidojevica Mountain nearby of the town of Prokuplje even belongs to semi-arid rain with the total precipitation amount of 533 mm, based on the annual measurement. Measurements in the last two decades show even decreasing of rainfall (the rainfall was above 600 mm until 1980). More details on climate conditions in the Toplica region including temperature, rainfall, relative humidity, insolation, haze, cloudiness and number of days with clear sky can be found in (Valjarević at al., 2008). These data show that this site is advantageous for astronomical observations.

3. DEVELOPMENT OF ASTRONOMY IN TOPLICA REGION SINCE 2000

Almost all activities and projects in astronomy in Toplica district are connected with the town of Prokuplje. The first mayor event was the foundation of the astronomical Society *Magellanic Cloud* in the year 2001. Since it was established, the Society works under supervision of the Faculty of Mathematics (University of Belgrade) and the Astronomical Observatory in Belgrade (AOB). Due to these circumstances, astronomy became rather popular among people of Toplica and many events popularizing astronomy were organized in the town.



Figure 5: First photo of Prokuplje (around 1870).

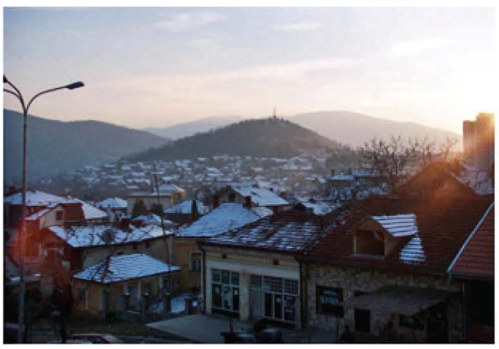


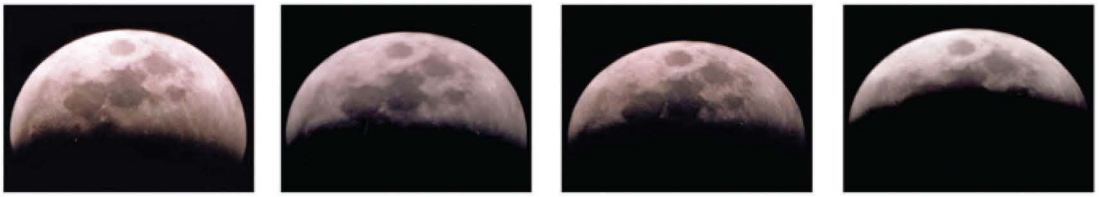
Figure 6: Prokuplje today - the hill Hisar dominates the town.

The Society *Magellanic Cloud* has in possession the reflector of the aperture 23 cm (the present to the Society by Ž. Mijajlović). For many astronomical events were organized watching the sky, not only for the members of the Society, but also for the general public too. The Society has now the rich library of photos of many of these astronomical occurrences. The county of Prokuplje helped much the work of the Society financially, but in other ways, too. For example, in one occasion (2002) during the public gazing of the Sky, the administrators of the County agreed to turn off the street light in part of the town in order to lower the light pollution. However, this decision was accompanied among the people with the somewhat cynic witticism that the County in reality wanted to save the electric power. Namely, at that time was the severe shortage of the electric energy not only in Toplica, but in whole Serbia as well.

Here are photos of partial eclipses of Sun and Moon that occurred recently. Photos are made in Prokuplje and nearby locations using the telescope of the Society *Magellanic Cloud* (Simonović, Valjarević, Mijajlović).



Sun eclipse, May 31, 2003



Moon eclipse, October 28. 2004

Figure 7:

Members of the Society organized many other activities as well. They arranged many public lectures; nearly 50 lectures on astronomy and mathematics were delivered in Prokuplje since the year 2000 up to now. The lecturers were the most prominent Serbian astronomers and mathematicians.



Figure 8: Left photo: Olga Atanacković delivers a lecture in Prokuplje (spring 2004). Right photo: The audience at the lecture.

Members of the Society were often interviewed by local news papers and also participated in many programs of local radio and television stations explaining astronomical phenomena and current astronomical occurrences. Simply saying, astronomy became popular science in Prokuplje and Toplica region. When the Society joined the astronomers Nadežda Pejović (in 2005) and Milan Dimitrijević (in 2006), the activites of the Society even intensified. Mrs. Pejović organised teaching of astronomy in the Gymnasium (the Grammar school) of Prokuplje and started the project of popularization of astronomy and mathematical sciences in Toplica Region, supported by the Ministry of Science of Serbia. Dr. Dimitrijević donated more than 1000 books to the Public library of Prokuplje, to the Grammar school and Museum in Prokuplje. Mijajlović donated the telescop (Newton reflector, 18 cm aperture) to the Gymnasium. The Society organized the expedition (M. S. Dimitrijević, N. Pejović, S. Ninković and Ž. Mijajlović) to Antalya, Turkey, to watch the total Solar eclipse that happened in March 29. 2006.



Figure 9: Left photo: N. Pejović, M. Dimitrijević and S. Ninković in Side near Antalya, during Sun eclipse, March 29, 2006. Right photo: Sun eclipse, March 29, 2006.

The Society is planning to build a small public observatory at the hill Hisar in near future. However, probably the most important activity of the Society *Magellanic Cloud* was its engagement in rising of the Astronomical station at the mountain Vidojevica nearby Prokuplje.

4. ASTRONOMICAL STATION ASV

There are already several papers written about the project of building the astronomical station at the mountain Vidojevica nearby the town of Prokuplje (see Ninković et al., 2006; Pejović and Mijajlović, 2006, Mijajlović, 2006). Therefore, we shall present here just few facts about this project.

The idea of building an astronomical station at Vidojevica is 30 years old. The project was postponed, almost forgotten, due to the circumstances that appeared in the former Yugoslavia during the last two decades. Two institutions, Faculty of Mathematics and Astronomical Observatory in Belgrade (AOB) renewed the project in 2002. and helped by the Serbian Ministry of Science began the building of ASV in 2003. The project is carried out by AOB. The site was chosen according to good astronomical criteria. The peak of the mountain of Vidojevica was the best observing site even in the former Yugoslavia, with the clearest sky and very good atmospheric conditions, as it was already explained in this paper (see also Valjarević et al., 2008 and Mijajlović et al., 2006).

ASV is located at the peak (1155m, N 43^0 08.706', E 21^0 33.982') of the mountain of Vidojevica and it is about 20 km far by road from the town of Prokuplje. The peak is almost flat and occupies about 1000 m^2. The infrastructure at the site is good. There are no significant obstacles on the horizon. The sky is open, particularly to the south. The light pollution is negligible. There are only some traces of light pollution to the north, coming from the town of Prokuplje.





Figure 10: Many expeditions were organized to Vidojevica before and during building it. Left photo: Yang astronomers from AOB at Vidojevica (2003). Right photo: N. Pejović, Z. Knežević, local man and A. Valjarević (2005).

The mayor equipment for ASV is already obtained. It includes one telescope and the stellar spectrograph.



Figure 11: Photo of M 104 (Sombrero galaxy) made with the telescope that will be situated at ASV.

Telescope. Here are some technical details of the telescope. This reflector is produced by the German company ASTROOPTIK, it has Lomo Sitall optics, primary focal ratio f/3 and system focal ratio f/10. The servomotor encoders are with <0".08 resolution, while the maximum slow rate is 3^0 per second. The software and the controlling system of the telescope are based on a PC WIN XP operating system with standard functions for computerized telescopes. It admits also remote control. The accessories include a reducer (0.6x for an effective focal length of 2500mm), field corrector, of axis guider and high-accuracy encoders (Heidenhain ERA 780 C Encoders with a very large diameter [473mm] and a very high resolution in both axes [0.2" accuracy per axis]).

Stellar spectrograph SpectraPro 750. This spectrograph is intended for use primarily at ASV telescope. The spectrograph is planed for the recording of low resolution spectra of relatively faint stars and asteroids, medium-resolution spectra of relatively bright stars and studies of the variations in highly broadened spectral line profiles. It can be used also for determination of gas movements in circumstellar disks and radial velocity measurements of close binary stars. In short, the spectrograph is intended for spectral recording and analysis in spectral regions from violet to infrared. The spectrograph is of the Czerny-Turner type and it is produced by the Action Research Company. It has the focal length 750 mm, the aperture ratio f/9.7, resolution 0.023 nm at 435.8 nm, 10 μm slits, dispersion 1.1 nm/mm (nom.) and accuracy 0.1 nm.



Figure 12: Left photo: Dome ASV1. Right photo: The living pavilion.

More details on planned scientific programs at the ASV the reader can find in Ninković et al. (2006); Pejović and Mijajlović (2006); Mijajlović (2006) and Pejović and Mijajlović (2007). It is expected that the Astronomical station at the mountain Vidojevica will be finished in near future.

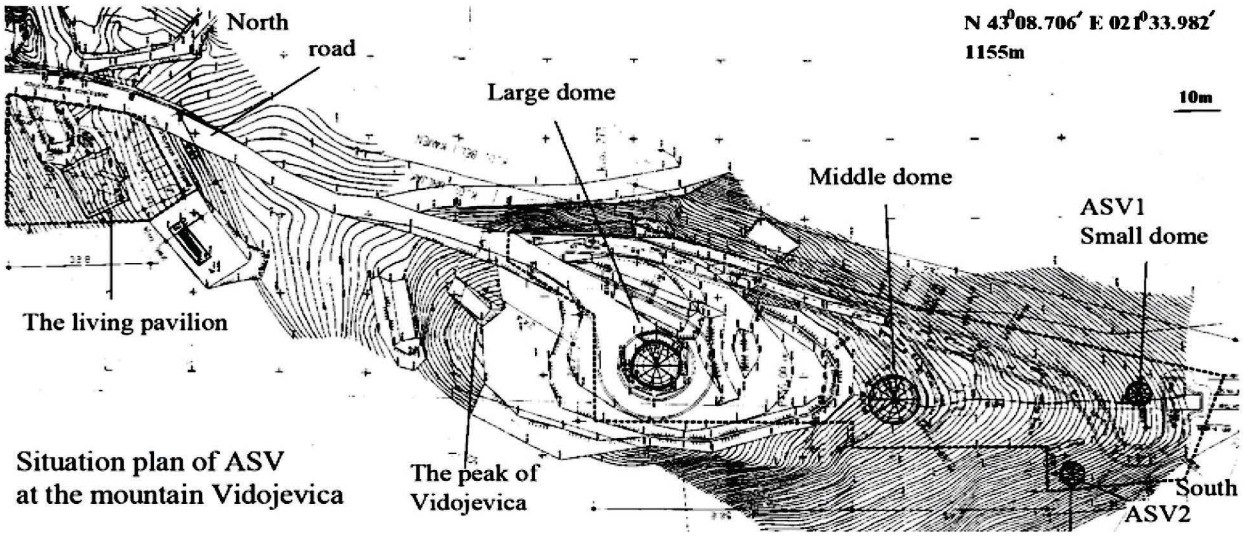


Figure 13:

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History of Astronomy in Prokuplje
N. Pejović, A. Valjarević, Ž. Mijajlović,
Publ. Astron. Druš. Rudjer Bošković, Beograd
Vol. 8, pp. 291 - 300, 2009.
Zb. Konf. "Razvoj astronomije kod Srba",
Beograd, 18-22 april, 2008, (M. Dimitrijević, ed).

This article is a supplement to the previous review "Development of astronomy in the Toplica Region". It is noted that the first major amateur attempts at astronomy have emerged after the founding of Gymnasium in Prokuplje in 1908. In that a significant place had Aleksa Savić, a reputable physician and benefactor. In the last two decades of the XX century, the work of the astronomical section of the elementary school Ratko Pavlović Ćićko was particularly notable. After the year 2000, astronomy was far more prevalent not only in the popular but also at the scientific level, thanks to the founding of the amateur society "Magellan Cloud" and the build of Astronomical Station on the Vidojevica mountain whose owner is the Astronomical Observatory in Belgrade. A list of talks of prominent Serbian astronomers to the general public in Prokuplje in the period 2002-2008 is attached.

Зборник радова конференције "Развој астрономије код Срба V"
Београд, 18-22. април 2008,
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 8, 2009, 291-300

ИСТОРИЈА АСТРОНОМИЈЕ У ПРОКУПЉУ

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Резиме. У чланку дајемо приказ историје астрономије у Прокупљу. До краја 20. века све активности на овом плану односиле су се на популаризацију и образовање у овој науци. Први значајнији аматерски покушаји у астрономији појавили су се оснивањем Гимназије у Прокупљу 1908 године. У томе значајно место имао је Алекса Савић, угледни прокупачки лекар и добротвор. У последње две деценије нарочито је запажен рад астрономске секције основне школе Ратко Павловић Ћићко. После 2000. године астрономија је далеко више присутна не само на популарном већ и на научном плану захваљујући оснивању аматерског друштва *Магеланов облак* као и изградњи Астрономске станице на Видојевици, Астрономске опсерваторије у Београду.

УВОД

Развој астрономије, боље речено упознавање са овом науком у Прокупљу не иде упоредо са развојем астрономије код Срба. Овај крај је, уосталом, један од последњих од Турака ослобођених делова Србије. Топлица је тек 1878. припојена српским територијама. Ова чињеница сигурно је утицала да се у народу касно разуме важност не само астрономије, већ целокупне науке. Најстарија научна и културна институција у Прокупљу је Гимназија, која управо ове, 2008 године, слави 100 година постојања. Преко 80 њених бивших ученика постали су доктори наука и преко 100 магистри.

Било је покушаја да се организује астрономско друштво, како у прошлом, тако и у 19. веку, али неког успеха није било. Најзад, 2001. основано је аматерско астрономско друштво ***Магеланов облак*** у Прокупљу. Споменимо да је пре оснивања Друштва постојала *Секција из астрономије* која је радила

више од 15 година у Основној школи *Ратко Павловић Ћићко*, под руководством наставнице физике Трајке Стојановић. Наравно, оснивање Астрономског друштва *Магеланов облак* је допринело да се Катедра за астрономију Математичког факултета Универзитета у Београду (МФ) поново заинтересује за подизање посматрачке Опсерваторије ван Београда. Наиме, у овом крају постоје одлични астроклиматски услови за постављање једног таквог објекта. Отуда је још седамдесетих година прошлог века постојао пројекат Астрономске опсерваторије у Београду (АОБ) да се у Топличком крају изграде посматрачки павиљони. Место је изабрано на планини Видојевици, врх Бандера (1154 м н.в). Изградња Опсерваторије је у току. Такође, споменимо да је 2005. формирано прво факултативно одељење астрономије у Гимназији у Прокупљу, које данас ради као астрономска секција. Популарност астрономије у Топличком крају доживела је процват последњих неколико година, с почетком изградње нове Опсерваторије.

ДЕВЕТНАЕСТИ ВЕК

Мада за овај период не можемо рећи да је астрономија као наука била присутна у Топлици, занимљиво је да се овде, као и у другим крајевима Србије, у народним епским песмама и предањима спомињу космички објекти (Историјски архив града Прокупља свеска 4-5, 1964.). Ево једне такве песме, *Вилина Топлица* народног гуслара Јанка Булатовића из Прокупља:

Када ме са планине потераше вуци
По муци се видеше ајдуци, гле тамо
гле вамо, у Топлицу стигоше унуци.

Ја погледах ***звезду Даницу*** *, и видех*
своју одступницу.....

Када ноћу погледам ка небу , ***Влашићи*** *ми*
оком поиграше.....

ДВАДЕСЕТИ ВЕК

После првог светског рата, Прокупље све више добија изглед праве вароши напуштајући изглед турске касабе. Прокупље у периоду 1905-1921. има већ 15000 становника. У културном и образовном миљеу града велики значај има Гимназија и њени професори. У томе посебно место имају шумарство и ботаника. Ове две дисциплине у овом периоду везане су за име лекара, др Алексе Савића. Он је први започео важне пројекте на нивоу града Прокупља. Извршено је пошумљавање брда, које данас Прокупчани зову Боровњак, по белом бору који је засађен. Затим, др Савић је планирао изградњу задужбине

на Хисару, која је требало да постане природњачки музеј. Такође је желео да постави телескоп на тераси овог музеја. У ту сврху издвојио је, како се тада говорило, 20 старих динара. Наводимо цитат из Архива града Прокупља (свеска 4-5, 1964.): *Телескоп би био величине 10 цм а имао би жижну даљину 1,5 м*. Из овога закључујемо да се ради о телескопу-рефрактору. Нажалост, телескоп на Хисару никада није постављен. Задужбина је завршена тек после његове смрти, али није имала научну намену, већ је коришћена у друге сврхе. Данас се историја некако понавља. Др Надежда Пејовић, професор МФ-а у Београду и члан Астрономског друштва *Магеланов облак*, дала је идеју да се сагради *Народна опсерваторија* на остацима Цитаделе на Хисару. Пројекат се припрема под покровитељством Општине Прокупље, Астрономског друштва *Магеланов Облак*, Математичког факултета у Београду и Министарства културе Републике Србије. Изградња Народне опсерваторије у Прокупљу, поред Астрономске станице на Видојевици, била би вредан допринос обележавању Међународне године астрономије 2009. у нашој земљи.

После Другог светског рата долази до развоја читавог друштва, па и Прокупља. По попису из 1953. Прокупље је имало 19000 становника. У Топличком крају до 1971. године није било покушаја формирања астрономског друшва. Тек 1971. при реформи средњошколске наставе и увођењу усмереног школовања уводи се и предмет Астрономија. По неки предметни наставници су на секцији из Физике обрађивали и теме из астрономије. У периоду од 1972-1978. предметни професор астрономије Лаза, покушава да заинтересује гимназијалце да се удруже и направе астрономско друштво на нивоу града Прокупља. Друштво је основано. Иако је ово Астрономско друштво трајало свега 2 године и 3 месеца, успело је да организује један семинар од 20 часова са научно-популарним темама из астрономије и физике. Занимљиво је да се ово друштво никада није регистровало па се подаци о његовом постојању веома тешко налазе. У то време појединци у Прокупљу праве своје телескопе. Поменућемо само неке од њих : Павловић Миодраг (1965.), Стојковић Жарко (1971.), Мицић Милован (1972.). Жарко Нешовић (1961.). Можемо рећи да Топлицу тада полако али сигурно освајају астрономи аматери..

Табела 1. Телескопи астронома аматера у Прокупљу

Павловић Миодраг	Рефрактор	r=12 cm	L=1100mm
Стојковић Жарко	Рефрактор	r=15 cm	L=1350mm
Мицић Милован	Рефлектор Њутн	r=16 cm	L=1600mm
Жарко Нешовић	Рефрактор	r=10 cm	L=1000mm

НАДЕЖДА ПЕЈОВИЋ *и др.*

АСТРОНОМСКА СЕКЦИЈА У ОСНОВНОЈ ШКОЛИ *РАТКО ПАВЛОВИЋ ЋИЋКО*

Астрономска секција у Основној школи *"Ћићко"* постоји од 1982, и представља важно место у представљању астрономије у Топличком крају. Рад ове секције везан је за име наставника Трајке Стојановић која је вредно радила на изучавању астрономије и њеној популаризацији у Прокупљу и Топличкој регији. Трајка Стојановић је у то време била наставник физике и сваке године је имала двадесетак ђака који су похађали Астрономску секцију. Највећи успех Секција је имала 80-тих и 90-тих година прошлог века, када су њени ученици освајали прва места на републичким и савезним такмичењима из астрономије. Најбољи радови ове Секције су се односили на две веома лепе и интересантне астрономске појаве. Једна тема је била *"Кратко Фукоово клатно"*, док је друга била практичан рад на моделу помрачења Сунца, гласила је *"Вештачко помрачење Сунца"*. За успешан дугогодишњи рад наставница Трајка Стојановић је награђена 1997. године Плакетом *"Наука младима"*, а 2002. године је добила *Захвалницу* и пригодан поклон од астрономског друштва *Магеланов облак*.

АСТРОНОМСКО ДРУШТВО *МАГЕЛАНОВ ОБЛАК*

У Прокупљу од 2000. делује астрономско друштво (АД) *Магеланов облак* под покровитељством Математичког факултета у Београду, АОБ и Општине Прокупље. Жарко Мијајловић поклонио је Друштву телескоп (Њутновог типа, пречник огледала 23 цм), рачунар, скенер и 50 књига. Друштво је имало значајну улогу у почетку изградње Астрономске станице АОБ. На самом почетку, током 2000-2002, значајно место у настанку идеје за изградњу Астроноске станице Астрономске опсерваторије у Београду (АСВ) на планини Видојевица, имало је Друштво.

ФАКУЛТАТИВНА НАСТАВА АСТРОНОМИЈЕ У ГИМНАЗИЈИ У ПРОКУПЉУ

Од 2002. постоји интензивна сарадња Гимназије у Прокупљу и Математичког факултета у Београду, у оквиру које су професори Факултета држали јавна предавања из области астрономије, математике и рачунарства. У Гимназији је 2005. формирано одељење које има факултативну наставу из астрономије. Оснивањем овог одељења сарадња Гимназије и Математичког факултета је повећана. Током претходне три године одржано је на десетине предавања за грађанство и школе у Прокупљу. Предавачи су били: а) Астрономија: Н. Пејовић, С. Шеган, Ј. Милоградов-Турин, О. Атанацковић (професори МФ), С. Нинковић, З. Кнежевић, М. Димитријевић, Л. Поповић (научни саветници АОБ). б) Математика: Ж. Мијајловић, А. Липковски (професори МФ), М. Рашковић (научни саветник САНУ), Д. Ћирић

(професор Природноматематичког факултета у Нишу). Ови предавачи су такође више пута наступали на локалној телевизији и учествовали у програмима радио станица у Прокупљу. Сви ови догађаји забележени су у локалним новинама, такође и у дневној штампи (*Политика, Вечерње новосзи, Блиц)*. Део ових активности суфинансирало је Министарство науке Републике Србије кроз *Програм подстицања, промоције и популаризације науке*, преко Гимназије у Прокупљу или преко Математичког факултета у Београду. Поменимо да је проф. Жарко Мијајловић, не само Друштву већ, и Гимназији поклонио телескоп (Њутновог типа, пречник огледала 18 цм), а др Милан Димитријевић, научни саветник АОБ, поклонио је Градској библиотеци 1000, а Гимназији неколико десетина књига.

Литература

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Ninković, S., Pejović, N., Mijajlović, Ž.: 2006, "Astronomical Station Vidojevica", Proc. SOLAR and STELLAR PHYSICS THROUGH ECLIPSES 29-29 March, 2006, Side-Antalya, Turkey, 2006, ed. O. Demircan, S. O. Selam, Ankara University Observatory, Ankara.

Pejović, N., Mijajlović, Ž.: 2006, "The Project ASV", The 5th Bulgarian-Serbian Conference on Astronomy, May 9-12, 2006, Sofia, ed. M. Tsvetkov, L. Filipov, M. Dimitrijević, L. Popović, BAS, Heron Press, Sofia.

ПРИЛОГ 1

Наводимо делимичан списак предавања из астрономије и математике за општу публику и ученике Гимназије која су одржана у Прокупљу од 2002. до 2008. До 2005. ова предавања оргаганизовало је АД *Магеланов облак*. Од 2005. ове активности водила је Надежда Пејовић у сарадњи са директором Гимназије у Прокупљу, Драганом Крстићем. Предавачи су били професори Математичког факултета (МФ) и Математичког института САНУ (МИ) у Београду, Природноматематичког факултета у Нишу (ПМФН) и истраживачи Астрономске опсерваторије у Београду (АОБ).

2002

1. Стево Шеган, МФ, *Открића астрономије.*
2. Жарко Мијајловић, МФ, *Доприноси астронома аматера.*
3. Душан Ћирић, ПМФН, *Непознати светови математике.*

2003

1. Зоран Кнежевић, АОБ, *Астероиди.*
2. Александар Липковски, МФ, *Геометријски и тополошки ликови.*
3. Стево Шеган, МФ, *Астрономска станица на Видојевици.*

2004

1. Олга Атанацковић, МФ, *Структура звезда.*
2. Миодраг Рашковић, МИ, *Како је Ератостен из Сирене измерио обим Земље.*
3. Надежда Пејовић, МФ, *Астрономски калејдоскоп.*
4. Жарко Мијајловић, МФ, *Математичка открића.*

2005

1. Јелена Милоградов -Турин, МФ, *Метеори и метеорити.*
2. Слободан Нинковић, АОБ, *Глобуларна јата.*
3. Надежда Пејовић, МФ, *Сферна тригонометрија и небеска сфера.*
4. Душан Ћирић, ПМФН, *Математика у природним наукама.*
5. Стево Шеган, МФ, *Календари.*
6. Жарко Мијајловић, МФ, *Геометријска својства параболе и трајекторије комета.*

2006

1. Милан Димитријевић, АОБ, *Астрономија на почетку трећег миленијума.*
2. Милан Димитријевић, АОБ, *Зашто је ноћно небо тамно - Нобелова награда за космологију (физику)2006.*
3. Милан Димитријевић, АОБ, *Аматери и астрономија.*
4. Слободан Нинковић, АОБ, *Глобуларна јата.*
5. Надежда Пејовић, МФ, Тотално *помрачење Сунца 29.03.2006*

6. Надежда Пејовић, МФ, *Месијеови објекти.*
7. Надежда Пејовић, МФ, *Сунчани часовник.*
8. Жарко Мијајловић, МФ, *Оптичке особине кривих и површи другог реда.*
9. Жарко Мијајловић, МФ, *Фибоначијеви бројеви.*

2007

1. Милан Димитријевић, АОБ, *Сунце – наша звезда.*
2. Слободан Нинковић, АОБ, *Објекти дубоког неба.*
3. Слободан Нинковић, АОБ, *Двојне и вишеструке звезде.*
4. Лука Поповић, АОБ, *Релативистички објекти и појаве у васиони.*
5. Надежда Пејовић, МФ, *Сазвежђа.*
6. Надежда Пејовић, МФ, *Сунчев систем.*
7. Душан Ћирић, ПМФН, *О математичком начину размишљања.*
8. Жарко Мијајловић, МФ, *Делски проблеми и квадратура круга.*
9. Жарко Мијајловић, МФ, *Шта је математичка логика.*

2008

1. Соња Видојевић, МФ, *Структура и еволуција звезда.*
2. Милан Димитријевић, АОБ, *Тајна великог ћутања ванземаљских цивилизација.*
3. Татјана Јакшић, МФ, *Највећи телескопи света.*

ПРИЛОГ 2: АСТРОНОМСКА СЕКЦИЈА У ОСНОВНОЈ ШКОЛИ *РАТКО ПАВЛОВИЋ ЋИЋКО*

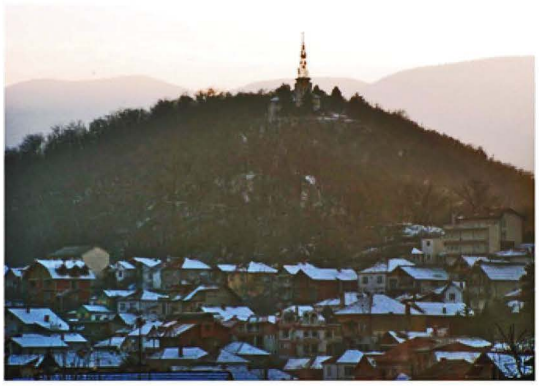
Астрономска секција у основној школи "Ратко Павловић Ћићко" у Прокупљу основана је 1982, и њена делатност представља важан део развоја астрономије у Прокупљу. Од оснивања секцијом је руководила Трајка Стојановић, наставник физике у Школи. Ученици школе, чланови секције, постигли су завидне резултате на савезним и републичким такмичењима из астрономије.

Табела Такмичења из астрономије на којима су учествовали ученици Секције за астрономију О.Ш. *Ратко Павловић Ћићко*.

Година	Ученик	Такмичење	Освојено место
1984	Зоран Петровић	Републичко	I
1986/87	Данијела Милошевић Снежана Ивезић	Републичко Републичко	II III
1987/88	Бојан Милић	Републичко Савезно	I III
1988/89	Бојан Милић	Републичко Савезно	I I
1989/90	Ненад Станковић	Републичко Савезно	II II

1990	Бранислав Станковић	Републичко	I
1991	Иван Вељковић	Републичко	II
	Бранислав Страхинић	Републичко	II
1992	Иван Вељковић	Републичко	I
1993	Вера Виријевић	Републичко	I
1994	Марко Лопичић	Републичко	I
1995	Славко Кнежевић	Републичко	II
	Владимир Красић	Републичко	III
	Дамјан Филиповић	Републичко	III
	Раде Раденовић	Републичко	IV
1997	Ана Виријевић	Републичко	II
1998	Никола Ђорђевић	Републичко	I

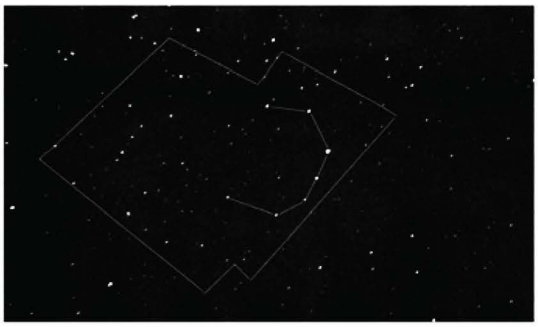
За успешан дугогодишњи рад наставница Трајка Стојановић је награђена 1997. Плакетом *"Наука младима"*, а 2002. године је добила *Захвалницу* и пригодан поклон од астрономског друштва *Магеланов облак*. (Извор: архива О.Ш. *Ратко Павловић Ћићко*).



Брдо Хисар код Прокупља



Др Обрадовић и А. Симоновић са ученицима испред Гимназије у Прокупљу



Снимак сазвежђа Corona Borealis помоћу телескопа друштва Магеланов облак



А. Ваљаревић и А. Симоновић на Видојевици

Додељене захвалнице за допринос у изградњи Астрономске опсерваторије на Видојевици

На лето прва посматрања

Припремила:
Љиљана Мировић

Поводом наставка радова на изградњи опсерваторије на Видојевици, Општина Прокупље доделила је захвалнице др Зорану Кнежевићу, директору Астрономске опсерваторије у Београду, др Жарку Мијајловићу, професору Математичког факултета и др Нади Пејовић, професору на катедри за астрономију.

Захвалнице је у име СО Прокупље доделио директор прокупачке Гимназије и члан Општинског већа за науку и технологију Драган Крстић. Он је овом приликом истакао, поред изградње Опсерваторије, значај још једног важног пројекта који се успешно реализује у сарадњи са Математичким факултетом из Београда. Ради се о увођењу факултативне наставе из астрономије у Гимназији коју спроводи проф. др Стојан Обрадовић. Према његовим речима, интересовање ученика је велико, чак и оних који не похађају наставу, као и других грађана Прокупља.



"У реализацији овог пројекта имали смо разумевање Министарства науке и заштите животне средине које је обезбедило средства за финансирање, помоћ људи из локалне самоуправе, координатора Александра Ваљаревића и Александра Симоновића који су упорношћу и љубављу према астрономији помогли да се ова дугогодишња идеја и реализује", рекао је др Зоран Кнежевић. Опсерваторија ће бити завршена најкасније до почетка јесени када ће бити обављена и прва пробна посматрања. Према његовим речима уколико се ово место покаже као право, у плану је инсталирање већег телескопа, што представља другу фазу овог пројекта која би била реализована у наредних 5 - 10 година.

Због изузетног значаја овог пројекта и напора који је локална самоуправа уложила да се он и реализује, награду за заслуге у развоју астрономије, Опсерваторија из Београда ће ове године доделити СО Прокупље.

Најсавременији телескоп

Министарство за науку и заштиту животне средине обезбедило је средства за изградњу опсерваторије у вредности од око 8 милиона динара. До сада је завршено око 70 одсто грађевинских радова, а ускоро ће бити инсталиран и најсавременији телескоп чија је вредност око 90 хиљада евра. Према процени стручњака, опсерваторија ће бити завршена најкасније августа месеца, када ће и званичноНо почети прва астрономска посматрања.

Чланак у локалним новинама (Топличке новине) поводом изградње астрономске станице на Видојевици. На фотографији виде се др Зоран Кнежевић, Др Надежда Пејовић и Др Жарко Мијајловић на састанку са функционерима Општине Прокупље.



Снимак Месеца помоћу телескопа друштва Магеланов облак

HISTORY OF ASTRONOMY IN PROKUPLJE

In this article we present history of astronomy in the town of Prokuplje. Until the end of the 20th Century, all activities were in the area of popularization and education of astronomy. The first amateur's attempts in astronomy occurred when the Gymnasium in Prokuplje was founded in the year 1908. The important place in these activities had Aleksa Savić, the recognized medical doctor and charitable man from Prokuplje. In the last two decades, the work of the astronomical section of the Elementary school *Ratko Pavlović Ćićko* has been particularly recognized. After the year 2000, astronomy has been much represented, not only at the popular but at the scientific level, too. This is due to the activities of the amateur's society *Magellanic cloud* and the rising of the *Astronomical station* at the mountain Vidojevica of the Astronomical Observatory in Belgrade, situated near Prokuplje.

54 Astronomija u udžbeniku *Osnovi Matematične i fizičke geografije* **Pavla Vujevića**

Astronomy in the university text-book
Fundamentals of Mathematical and Physical Geography of Pavle Vujević
N. Pejović and Ž. Mijajlović
Publ. Astron. Druš. Rudjer Bošković, Beograd
Vol. 8, pp. 391-402, 2009.
Conf Proc. "Razvoj astronomije kod Srba",
(Development of Astronomy in Serbs),
Beograd, 18-22 april, 2008, (M. Dimitrijević, ed).

The book "Fundamentals of mathematical and physical geography" is one of the first university textbooks on geography written in Serbian. The author of the book is Pavle Vujević (1881-1966), the notorious Serbian climatologist, the professor of the Belgrade University and the member of the Serbian Academy of Science and Art. The book consists of two volumes. The first volume is printed in 1923, while the second one is printed in 1926. For many generations of students of geography, even today, this textbook was an entering point into mathematical and physical geography and astronomy. The first volume is made up of two sections, Mathematical geography and Physical geography, while the second volume covers the atmosphere and oceans. The first volume is in fact an introduction to astronomy. This article is a review of the content of this volume. Also some interesting facts related to this book are presented, including changes in terminology and definitions since the book was published.

Зборник радова конференције "Развој астрономије код Срба V"
Београд, 18-22. април 2008,
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 8, 2009, 391-402

АСТРОНОМИЈА У УЏБЕНИКУ *ОСНОВИ МАТЕМАТИЧНЕ И ФИЗИЧКЕ ГЕОГРАФИЈЕ* ПАВЛА ВУЈЕВИЋА

НАДЕЖДА ПЕЈОВИЋ и ЖАРКО МИЈАЈЛОВИЋ

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e-mail: nada@matf.bg.ac.yu, zarkom@matf.bg.ac.yu

Резиме. У овом чланку представљамо део једног од првих универзитетских уџбеника из географије писаних на нашем језику. Написан је после Првог светског рата. Наиме, реч је о уџбенику ***Основи математичне и физичке географије*** професора Павла Вујевића који је штампан редом у две књиге 1923. и 1926. у Београду. Уџбеник се састоји од четири поглавља: *Математична геофрафија, Физичка географија, Атмосфера и Океани* у обиму од 815 страница. Иако веома велики, уџбеник је методички лепо написан, попут монографије. У овом чланку представићемо посебно занимљиво прво поглавље *Математична географија* које је посвећено астрономији. Енциклопедијски су побројани и објашњени до тада уведени астрономски појмови са детаљним и јасним описима. Да је прво поглавље штампано као посебна књига, био би то један од првих астрономских универзитетских уџбеника на нашем језику. Уџбеник *Основи математичне и физичке географије* Павла Вујевића је дигитализован и налази се у електронском облику у Виртуелној библиотеци Националног центра за дигитализацију (Virtual library, http://elib.matf.bg.ac.yu:8080/virlib/). Дигитализација ових књига део је пројекта електронског архивирања српских књига са математичким и астрономским садржајем штампаних у прошлости. У чланку су поменути и други разлози зашто је овај уџбеник изабран за укључење у Виртуелну библиотеку. Укратко представљамо садржаје ових књига као и њихове занимљивости.

УВОД

Основи математичне и физичке географије проф. Павла Вујевића је један од првих универзитетских уџбеника на нашем језику који је штампан после Првог светског рата у новонасталој држави Краљевини Срба, Хрвата и Словенаца. У ратом осиромашеној и уништеној земљи до страних уџбеника се тада тешко долазило. Било је неопходно писати домаће књиге. Овај уџбеник Павла Вујевића испуњава тај циљ и штампан је, како аутор пише у предговору, захваљујући великој предузимљивости тадашњег ректора Универзитета, Проф. Јована Цвијића. Уџбеник *Основи математичне и*

физичке географије састоји се од 4 поглавља: *Матматична географија, Физичка географија, Атмосфера и Океани* и штампан је у две књиге на 815 страница. У I делу налазе се прво и друго поглавље а у другој књизи је II део са трећим и четвртим. Прву књигу *Математична географија и Геофизика* штампала је Државна штампарија Краљевине Срба, Хрвата и Словенаца у Београду 1923, док је другу књигу *Атмосфера и Океани* штампала иста штампарија 1926. Како је Павле Вујевић био европски ђак и добар познавалац страних језика, уџбеник је написан уз коришћење тада најсавременије уџбеничке и научне литературе.

САДРЖАЈ КЊИГА


ВУЈЕВИЋ

ОСНОВИ

МАТЕМАТИЧНЕ И ФИЗИЧКЕ

ГЕОГРАФИЈЕ

I. ДЕО

ТЕМАТИЧНА ГЕОГРАФИЈА — ГЕОФИЗИКА

ДРЖАВНА ШТАМПАРИЈА
КРАЉЕВИНЕ СРБА, ХРВАТА И СЛОВЕНАЦА
БЕОГРАД — 1923.

Прва књига, ***Математична географија и Геофизика,*** написана је на 348 страница и састоји се од увода, првог и другог поглавља. Прво поглавље *Математична географија* има 15 глава, а друго поглавље *Физичка географија* састоји се од 6 глава. Прво поглавље *Математичну географију* чине главе: *Оријентација на хоризонту, Оријентација на небу, Облик и величина Земље, Оријентација на Земљиној површини, Географско одређење места на Земљи, Појмови о светском систему, Закони планетарног кретања, Карактеристика планетарних путања, Одређивање звезданих удаљења, Земљина кретања, Положај Земље и Земљине осовине према еклиптици, Гравитациони утицај небеских тела на Земљина кретања, Време и одређивање времена, Апсолутно кретање Сунчевог система* и *непомичних звезда* и *Положај Земље у васиони.* Друго поглавље *Физичка геграфија* садржи главе: *Земљина густина и маса, Земљина кора и језгро, Подела копна и мора, Земљотреси, Земљина магнетичност* и *Електрицитет у ваздуху и земљи.*

Друга књига, ***Атмосфера и Океани,*** написана је на 467 страница, нумерисаних као наставак прве књиге бројевима од 349 до 815 и састоји од

трећег и четвртог поглавља. Треће поглавље *Атмосфера* има 11 глава, а четврто поглавље *Океани* састоји се од 7 глава. Треће поглавље *Атмосфера* чине главе: *Општи појмови о ваздуху, Топлота, Температуре, Вертикална и хоризонтална подела температура, Ваздушни притисак, Ветрови, Кружење воде, Поремећаји у ваздуху, Климатски типови, Периодске промене климата на Земљи* и *Географска подела животиња и биљака на копну.* Четврто поглавље *Океани* састоји се од глава: *Топографија океана, Хемијски састав и физичке особине океанске воде, Живот у океанима, Састав океанског дна, Таласи, Океанске струје* и *Плима и осека.*

Већ по наведеним насловима глава види се свеобухватност и обимност Вујевићевог уџбеника. Заједно са географском науком доста детаљно обрађује и друге науке са којима се она преклапа. То су астрономија, метеорологија, климатологија, океанографија и геофизика. Код већине појмова дато је њихово историјско порекло, што је реткост код савремених уџбеника. По обимности, обиљу информација и уз коришћење тада најсавременије уџбеничке и научне литературе, Вујевићев уџбеник има све карактеристике монографије.

БИОГРАФСКИ ПОДАЦИ АКАДЕМИКА ПАВЛА ВУЈЕВИЋА



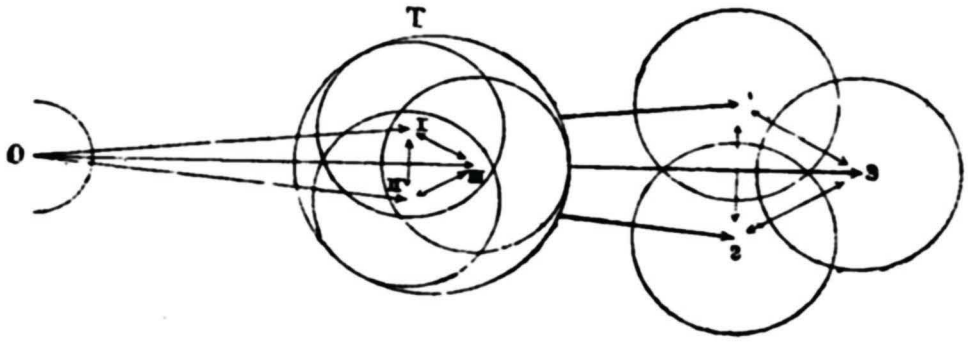
Pavle VUJEVIĆ
(1881–1966)

Павле Вујевић (1881-1966) је рођен у Руми. Школовање је започео у Новом Саду где је 1899. завршио Српску православну гимназију. Студирао је географију у Бечу, на тада чувеној *Бечкој географској школи.* Током боравка у Бечу упознао се и дружио са Милутином Миланковићем, који је у то доба и сам био студент. По завршетку студија пише докторску дисертацију из географије. Докторирао је 1904. у 23. години. Усавршавао се у Берлину и Потсдаму у области климатологије. На позив Јована Цвијића долази 1907. у Београд за доцента за предмет Климатологија и метеорологија на Филозофском факултету тада тек основаног Универзитета у Београду. По пензионисању управника Астрономско-метеоролошке опсерваторије Милана Недељковића, Опсерваторија се дели на две посебне установе. За управника Астрономске опсерваторије постављен је Војислав Мишковић а управник Метеоролошке опсерваторије постаје Павле Вујевић. Уџбеником *Основи математичне и физичке*

географије знатно је унапредио наставу географије и метеорологије на Београдском универзитету. По овом уџбенику училе су генерације студената не само географију и метеорологију, већ и климатологију, океанографију, геофизику и геодезију. Детаљнији подаци из Вујовићеве биографије могу се наћи у реф. Ракићевић (1998). За редовног члана САНУ изабран је 1958.

АСТРОНОМИЈА У ВУЈЕВИЋЕВОМ УЏБЕНИКУ

У ***Уводу*** Вујевић излаже Лапласову хипотезу о настанку Сунчевог система, самим тим и настанку планете Земље, (сл. 1 у Вујевић, 1923) (Референце које се не налазе у овом чланку, читалац може наћи у дигиталној копији Вујевићеве књиге на адреси http://elib.matf.bg.ac.yu:8080/virlib/). Укратко је изложио развитак природних наука као и повезаност *Географије* са тим наукама. На другој слици даје леп графички приказ са којег се види да је колевка природних наука *Астрономија*. Земља као небеско тело у томе има главно место и уз велики број других небеских тела представља предмет изучавања астрономије. С друге стране, у географији Земљине сфере су основни предмет изучавања. У овој науци, Земљини спољашњи омотачи, копно, вода и ваздух, представљени су сферама (скица 2) које се међусобно секу, дакле представљају место где се преплићу научне дисциплине које их изучавају. То су *Геологија, Хидрологија* и *Метеорологија*. Из њихових међусобних утицаја развиле су се нове гране науке: *Геоморфологија, Океанографија* и *Климатилогија*, такође на слици 2 представљене круговима. Управо о овим повезаностима и аспектима Вујевић пише у својој књизи. Стога је овај уџбеик био незаменљиво штиво генерацијама студената многих струка чије се интересовање односило не само на географију већ и на астрономију, метеорологију, климатологију, океанографију, геофизику и геодезију.



O = Астрономија = Физика = Хемија
T = Земља
I. Копно; Геологија;
II. Воде; Хидрологија:
III. Ваздух; Метеорологија
1. Геоморфологија (Физиогеографија)
2. Океанографија
3. Климатологија.

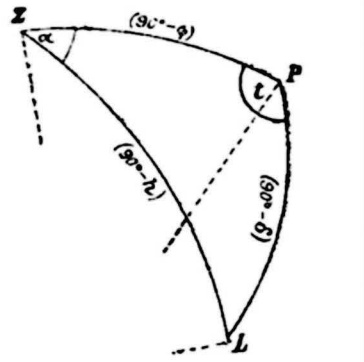
Скица 2.

АСТРОНОМИЈА У УЏБЕНИКУ *ОСНОВИ МАТЕМАТИЧНЕ И ФИЗИЧКЕ ГЕОГРАФИЈЕ* ПАВЛА ВУЈЕВИЋА

У првом поглављу, ***Математична географија,*** *Вујевић је* систематски изложо већину знања из астрономије тог времена. Детаљно је описао Земљу као планету Сунчевог система, њен настанак, облик, димензије, кретања, оријентацију, масу, густину, смену дана и ноћи, сумраке, смену годишњих доба, гравитацију, сеизмичност, магнетизам, њено место међу планетама као и место Сунца међу звездама. Стога је ово поглавље комотно могло да носи наслов *Астрономија* или као код Карљиковића (1931, 1935) *Космографија.* Под тим називом астрономи би ову књигу далеко више користили јер у називу *Математична географија* није препозната астрономија.

У првој глави овог поглавља, ***Оријентација на хоризонту,*** уведени су и описани основни појмови: видик, хоризонт, основне тачке хоризонта, гномон и одређивање даљина и правца на хоризонту.

У другој глави, ***Оријентација на небу,*** представљени су астрономски сферни координатни системи: хоризонтски, екваторски и еклиптички. Описане су координате у овим системима као и везе између хоризонтских и екваторских координата и екваторских и еклиптичких. У књизи пише о кретању звезда, кулминацији, затим о циркумполарним звездама. Такође је описано привидно кретање Сунца и зодијачка сазвежђа кроз која Сунце привидно пролази током године описујући велики круг на небу, еклиптику. Описи су чешћи од формула. Тако на пример на (сл. 9 у Вујевић, 1923) су графички представљене путање циркумполарних звезда за три карактеристична случаја уместо математички написаног услова:
$\delta \geq 90\text{-}\varphi$.



Скица 12.

Аутор је вероватно претпостављао да је описни начин прихватљивији за већину студената који имају скромно математичко знање. Занимљиво је да сферни троугао на сл. 12 назива *астрономски троугао*, што и јесте бољи назив, јер су тај појам астрономи увели. Вујевић често даје порекло многих речи, односно појмова, као на пример *тропски појас*. Наиме, у књизи пише да је *повратнике* Сунца први користио Хомер у својој Одисеји, где наводи да острво Сириа, у Кикладским острвима, лежи северно од *тропа*, дакле северно од повратника Сунца. Отуда је појас између повратника назван *тропски појас.*

У трећој глави, ***Облик и величина Земље,*** за модел Земље узета је лопта. Успут представља историјски развој овог модела, приказујући Ератостенову методу, затим мерење Земље код Арабљана, Снелиусову методу триангулације и Кеплерову методу. Потом наводи модел Земље у облику сфероида, приказујући успут доказе ове апроксимације: Хајгенсов закон о центрифугалој сили, Њутнови доказ о сфероидном облику Земље,

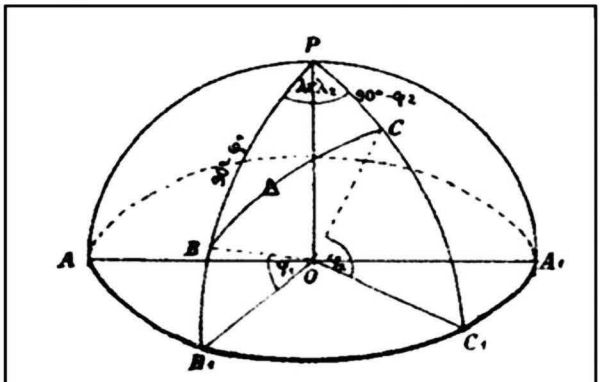
Касинијева мерења у Француској, Клерову теорему о Земљиној спљоштености и геодетска мерења у 19. веку. Најзад, наводи модел Земље у облику геоида. Приказује онда савремене доказе за овај модел наводећи поступак за мерење поремећаја силе Земљине теже. У књизи даје мерња Париског меридијана на основу којих је дефинсана основна јединица за дужину, *метар.*

У књизи су наведене многе значајне историјске занимљивости. Таква је на пример *Ератостенов (276-194. г. пре н.е..)* премер Земље. Растојања између места су у оно време исказивана у *стадијима* (1 стадиј=158м) и одређивана ходом камила. Ератостену је била позната раздаљина између Александрије и Асуана. Знао је да у Асуану постоји дубок бунар у којем се Сунце огледа само у једном дану у години. Врло оштроумно Ератостен је закључио да је то дан летњег солстиција и користећи ту астрономску чињеницу, као и сличност одговарајућих троуглова, налази да је обим Земље 39815 км уместо 40000 км. То је био генијалан резултат. Тек у XVIII веку тачнијим мерењима добијен је обим Париског меридијана од 40000 км и отуда је мера за дужину, *метар,* изведена као 40000000-ти део обима Париског меридијана.

Споменимо да се у књизи користи већ заборављен назив *секундна шеталица* за часовнике са клатном. Мерења Земљине спљоштености помоћу секундне шеталице су представљена у доњој табели (стр 46).

У четвртој глави, ***Оријентација на Земљиној површини,*** приказује мрежу паралела и меридијана, координантни систем преузет из астрономије. У књизи Вујевић даље пише о подели Земљине површине на климатске зоне као и о сферном растојању између места на Земљи, дакле присутни су елементи сферне тригонометије.

посматрач	место	географска ширина	дужина секун шеталице
Bouguer	на екватору	0° 0′ 0″	0,99669
„	Порто Бело (Ј. Бразилија)	9° 32′ 56″	0,99689
„	Гоава (Острва Хаити)	18° 27′ 0″	0,99828
La Caille	Рт Добре Наде (Ј. Африка)	33° 55′ 16″	0,99877
Bouguer	Париз	48 50′ 2″	1,00000
Mallet	Петроград	59° 56′ 24″	1.00101
„	Понои (Лапланд)	67° 4′ 37″	1,00148



Скица 35.

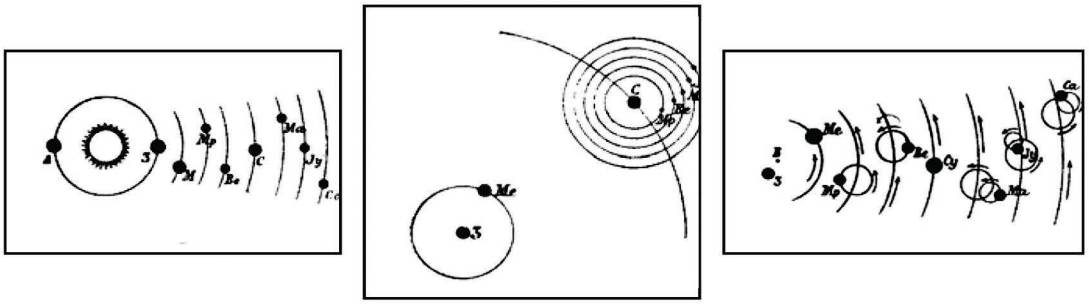
Занимљива је историја настанка почетног меридијана на Земљи коју је Вујевић укратко навео на стр. 55, као и порекло назива географска дужина и ширина, сл. 31, затим већ заборављене речи у астрономији, *ортодрома* за правац, и *локсодрома* за косину (стрмину), које се користе у морепловству, види сл. 34 у (Вујевић, 1923). На сл. 35 излаже одређивање најкраћег растојања на Земљиној сфери између два места коришћењем *астрономског троугла*, који су астрономи увели за одређивање растојања између небеских тела на небеској сфери, а касније је овај троугао нашао важну примену на Земљи.

У петој глави, ***Географско одређивање места на Земљи,*** наводи методе за одређивање географске ширине као што су Хоребо-Талкотова метода, методе помоћу кулминација звезда, циркумполарних звезда, мерења сенки и мерења времена. Такође наводи методе за одређивање географске дужине: помоћу светлосних сигнала, телеграфа, хронометра, Месечевих и Сунчевих помрачења, Месечевих кулминација и помрачења сателита других планета. Посебно се осврће на методе за геодетско одређивање географских координата: триангулација, сферни ексцеси и фотограметрија. На крају излаже поступке за одређивање надморске висине: помоћу нивелмана, тригонометријску методу, барометарску и термометарску.

Занимљиво је да у овој глави поред појма *астрономска рефракција* (сл. 38 у Вујевић, 1923), наводи и појам *терестријална рефракција* (сл. 39 у Вујевић, 1923), која се јавља при одређивању положаја места на Земљи. Подсетимо да се та појава односи на светлосне зраке који се преламају у одређеним слојевима ваздуха и не пролазе кроз целу Земљину атмосферу. На (сл. 46 у Вујевић, 1923) илустрован је леп пример *Хоребов-Талкотове методе* за одређивење географске ширине из посматрања две звезде у пару.

Шеста глава, ***Појмови о светском систему,*** односи се на историјски приказ поимања система света. Редом наводи Питагорин, Еудоксов, Хипархов, Птолемејев, Хераклидов, Коперников, Тихо Брахеов и Галилејев систем.

Издвајамо неколико занимљивих слика са графичким приказима Питагориног (сл. 58), Хипарх-Птолемејевог (сл. 65) и Тихо Брахеовог (сл. 68) геоцентричног система света. Коперников хелиоцентрични систем света детаљно је описан као и Галилејеви егзактни докази који су знатно допринели учвршћивању овог система. Споменимо да се у част Галилеја, 2009. година слави као Међународна година астрономије. Јер тада се навршило 400 година (1609) од како је по први пут Галилеј дурбином погледао у небо.



У седмој глави, ***Закони планетарног кретања,*** представљени су Кеплерови закони до којих је Кеплер дошао емпиријски, затим Њутнов закон гравитације и опис како је Њутн из свог закона гравитације теоријски извео Кеплерове законе. Такође спомиње рачун поремећаја и Тицијус-Бодеову прогресију по којој су у *астрономским јединицама* (растојање Земља-Сунце) дата растојања планета од Сунца. С тим у вези наводи откриће мале планете Церес (1801) и нове планете Нептун (1846), која је окривена *врхом пера.*

Напоменимо да је ово поглавље посвећено астрономији писано за студенте географије. Отуда је извођење Кеплерових закона из Њутновог закона гравитације дато описно и без сложеног математичког рачуна. Подсетимо да је Њутн за потребе овог извођења увео у математику *диференцијални рачун* (сл. 73 у Вујевић, 1923) .

Осма глава, ***Карактеристика планетарних путања,*** односи се на опште карактеристке планетских кретања, кретање планета у хелиоцентричном систему, затим право кретање Месеца, његове фазе као и привидно кретање Месеца и на крају помрачења Месеца и Сунца.

Права кретања планета су систематски и јасно изложена али су објашњена и њихова привидна кретања. Занимљиво је објашњење привидних кретања горњих планета које је дао Коперник и које је било најјачи доказ његове хелиоцентричне теорије. Ово објашњење у Вујевићевом уџбенику је представљено на сл. 75. Код помрачења Вујевић користи речи *умбра* и *пенумбра* за сенку и полусенку (сл. 77 у Вујевић, 1923).

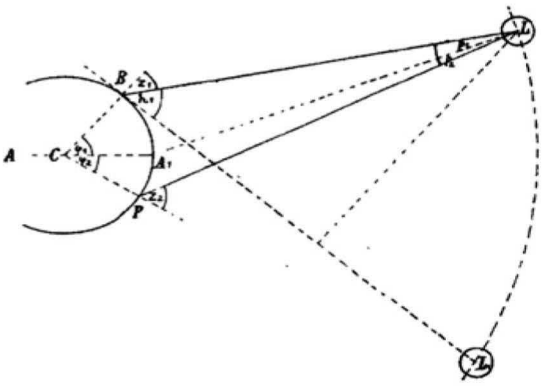
У деветој глави, ***Одређење звезданих удаљења,*** представљене су методе за одређивање Сунчеве паралаксе: Аристархова, Хипарова и метода планетарних пролаза. Такође излаже тригонометријску методу за одређивање Месечеве паралаксе и методе за одређивање звезданих паралакса. Подсетимо да одредити паралаксу небеског тела заправо значи одредити његову даљину.

У једном занимљивом пасусу Вујевић пише како Кеплер позива астрономе да посматрају пролазе доњих планета преко Сунчеве плоче. Одговарајући на овај позив, Халеј је прорачунао да ће пролази Венере бити 1761. и 1769, док Руђер Бошковић на позив Халеја врши израчунавања места са којих је посматрање ове појаве најпогодније. На сл. 82 Вујевић графички приказује ову методу. У књизи је на сл. 83 представљена и тригонометријска метода којом су Лаланд и Лакај 1751. одредили даљину Месеца. За

одређивање звезданих даљина поред годишње паралаксе (сл. 84 у Вујевић, 1923) дата је и метода одређивања даљина звезда релативном паралаксом (сл. 85 у Вујевић, 1923).

La Caille-Lalande-ова метода. — Опат *Ла Кај* се 1751 године бавио у Јужној Африци на Рту Добре Наде ради проучавања месечевог кретања, па је том приликом хтео да одреди и месечеву паралаксу. Пошто су и за то одређење потребна два посматрача на разним географским ширинама, и пошто је Рт Добре Наде скоро на истом меридијану Берлина, *Лаланд* је истовремено предузео посматрања у Берлину.

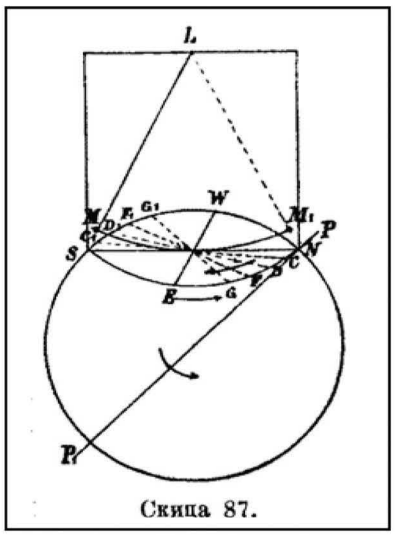
Метода с помоћу које су одредили месечеву паралаксу није сувише тешка и може се објаснити с помоћу графичког приказа. У скици 83. означује AA_1 земљин екватор, B место посматрача у Берлину, а P место посматрача на Рту Добре Наде. Географска ширина првог места је φ_1, а другог места φ_2. У тачци L лежи средиште Месеца, а у тачци C средиште Земље. Зенитално удаљење Месеца на месту B је z_1, на месту P је z_2, док је висинска паралакса на првом месту p_1, а на другом p_2.



Скица 83.

Према овим ознакама добиће се следећи односи:

У десетој глави, ***Земљина кретања***, историјским приступом Вујевић наводи доказе за Земљино дневно кретање приказујући експерименте за Земљину тежу, Земљину спљоштеност, слободни пад и девијације хоризонталних кретања. Наводи доказе и за Земљино годишње кретање помоћу одређивања годишње паралаксе звезда, аберације светлости, помрачења Јупитерових сателита и "звезда падалица" или метеора.



Скица 87.

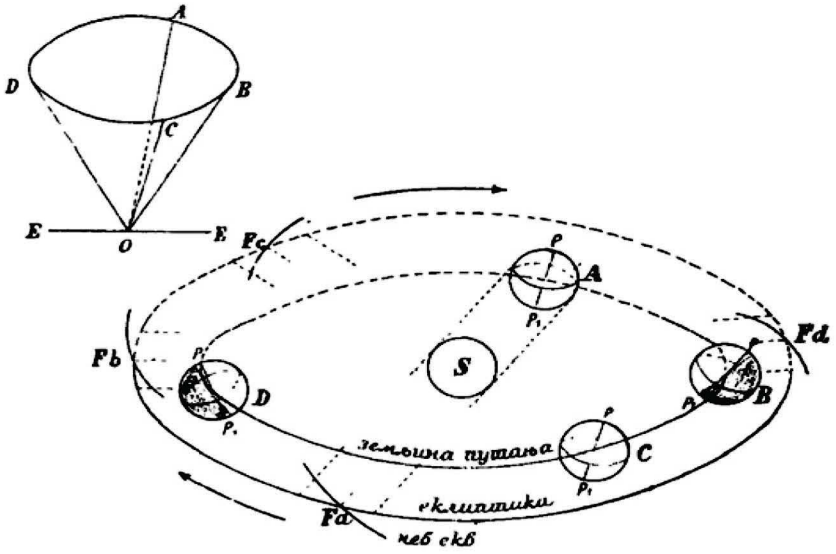
Занимљив је графички приказ (сл. 87) Фукоовог клатна за Земљину ротацију и аберацијска елипса (сл. 98 у Вујевић, 1923) за Земљину револуцију.

У једанаестој глави ***Положај Земље и Земљине осовине према еклиптици***, описане су последице Земљиног дневног и годишњег кретања: смена дана и ноћи и смена годишњих доба. Занимљиво је да Вујевић користи по мало заборављене термине *трајање осветљења и мрака* за обданицу и ноћ, *светле ноћи* за одомаћен термин беле ноћи, затим *астрономски сутон* за астрономски сумрак. Подсетимо да астрономи данас астрономски сумрак деле на јутарњи (зора) и вечерњи

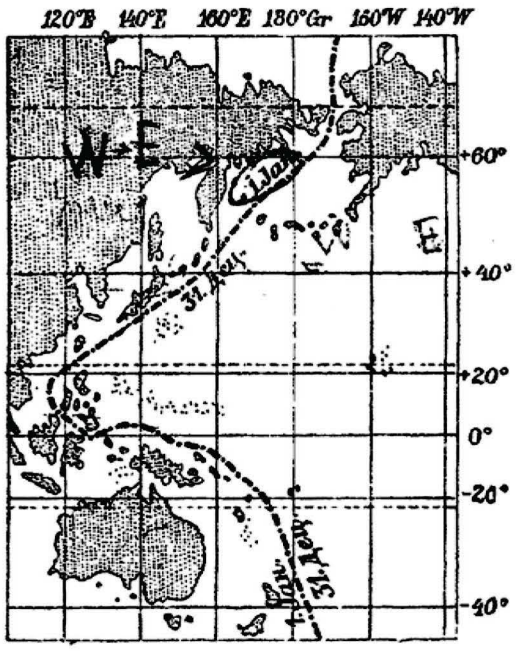
(сутон). На страници 159 је изложено различито трајање годишњих доба (сл. 106 у Вујевић, 1923).

Дванаеста глава, ***Гравитацијони утицај небеских тела на Земљина кретања,*** даје периодичне пертурбације, прецесију, нутацију и кретање Земљиних полова, као и секуларне пертурбације: колебање нагиба еклиптике, померање апсидне линије, промене у ексцентричности Земљине путање.

У овој глави јасно и сликовито је описано сложено прецесионо и нутационо кретање Земњине обртне осовине (сл. 113) и (сл. 114 у Вујевић, 1923) као кретање Земљиних полова (сл. 117 у Вујевић, 1923).



Слика 113.



Слика 125. — Датумска граница до 1845. године.

У тринаестој глави, ***Време и одређивање времена,*** представља систем звезданог времена и систем средњег времена, затим описује астрономски појам године (тропска, сидеричка, лунарна, грађанска), појам месеца (сидерички, синодички, тропски, драконистички, календарски), и два календара, Јулијански и Грегоријански. Затим представља системе локалног, међународног и зонског времена као и појам датумске границе.

Занимљиво је да датумску границу назива и *морнарска граница.* На сл. 125 представљена је датумска граница коришћена до 1845. године. Од 1845. за нову датскум границу усвојено је да се користи меридијан супротан Гриничком меридијану.

Четрнаеста глава, ***Апсолутно кретање Сунчевог система и непомичних звезда,*** за разлику од привидних, говори о правом кретању Сунчевог система где наводи координате апекса тог кретања. Такође пише о правим кретањима звезда, у астрономији познатим као сопствена кретања звезда.

Исто је у књизи представљена занимљива Хершелова графичка метода (сл. 128 у Вујевић, 1923) одређивања апекса и антиапекса кретања Сунчевог система.

У петнаестој глави, ***Положај Земље у васиони,*** укратко је дата представа о растојањима како унутар Сунчевог система тако и о огромним растојањима унутар наше Галаксије. Затим је представљена структура наше галаксије, типови звезда, њихов сјај, звездана јата и маглине.

Занимљив је начин на који Вујевић покушава да читаоцу докучи огромна васионска пространства. Растојања у Сунчевом систему даје преко времена које је потребно путнику (брзина 30 км/дан) и брзом возу (брзина 60 км/сат) да их пређу. Тако налази да је пешаку до Месеца потребно 36 година а брзом возу 9 месеци. У другом примеру пешак би до Сунца ходао 13600 година а брзи воз би стигао за 285 година. Простор Сунчевог система пореди са огромном пустињом по којој су као оазе тек по где-где расуте неколицине масе. Затим наводи да се огромна пустош види тек у Млечном путу јер Сунцу најближа звезда је 200000 пута даља од растојања између Сунца и Земље.

ЗАКЉУЧАК

У првом поглављу, *Математична географија,* универзитетског уџбеника *Основи математичне и физичке географије* Павла Вујевића *Астрономија* је лепо и прегледно представљена. Ово поглавље о астрономији је илустровано сликама, табелама, картама и дијаграмима.. Посебно истичемо Вујевићеву једноставност и лакоћу писања при излагању сложених астро-номских појава. Неке формуле и обрасци су изведени а неки су само наве-дени. Наравно, има у астрономији веома сложених формула чије извођење тражи доста простора и знања из математике. Вујевић астрономске појаве најчешће описно излаже, тако да и студенти који нису математичари могу тај текст са лакоћом да прате. Астрономи су навикли на коришћење математичког језика за представљање проблема који за познаваоца математичког апарата на једноставан начин даје сва могућа решења. Уместо тога у књизи налазимо пасусе описа појединих астрономских појава. Ипак, ови описи су толико питко и читко написани да их и они који нису астрономи могу разумети. У том погледу Вујевић је остварио свој циљ, да општем студенту географије приближи основна знања из астрономије.

Имајући у виду методички начин писања астрономије у уџбенику за географију, њен садржај, јасноћу и лакоћу излагања, препоручујемо ову књигу и данашњим студентима и средњошколским професорима не само георафије већ математике, астрономије, физике, геофизике и геодезије.

Универзитетски уџбеник *Основи математичне и физичке географије* Павла Вујевића је дигитализован и у електронском облику се налази у Виртуелниј библиотеци Националног центра за дигитализацију http://elib.matf.bg.ac.yu:8080/virlib/.

Захвалница

Користимо прилику да се захвалимо организатору др Милану С. Димитријевићу на позиву за учешће на конференцији *Развој астрономије код срба V*. Захваљујући томе, овај текст је написан.

Литература

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ASTRONOMY IN THE *TEXTBOOK FUNDAMENTALS OF MATHEMATICAL AND PHYSICAL GEOGRAPHY* OF PAVLE VUJEVIĆ

The book ***Fundamentals of mathematical and physical geography*** (in Serbian ***Основи математичне и физичке географије***) is one of the first university textbooks on geography written in Serbian. The author of the book is Pavle Vujević (1881-1966), the notorious Serbian climatologist, the professor of the Belgrade University and the member of the Serbian Academy of Science and Art. The book consists of two volumes. The first volume is printed in 1923, while the second one is printed in 1926. For many generations of students of geography, even today, this textbook was an entering point into mathematical and physical geography and astronomy. This book is rather comprehensive; it consists of 815 pages and four sections. The first volume is made up of two sections, ***Mathematical geography*** and ***Physical geography***, while the second volume covers ***The atmosphere*** and ***Oceans***. The first book is in fact an introduction to astronomy. It covers in details all basic notions and facts of this science known at the time when the book was written. In the second volume there are explanations of the chemical composition of the Earth atmosphere, climate types and their secular changes, then composition and physical characteristics of World oceans, the ebb-tide phenomena and the characteristics of the seabed. The book is digitized and it is included into the NCD ***Virtual library*** (http://elib.matf.bg.ac.yu:8080/virlib/). In this article we present the content of the first book, also some interesting facts related to this book, including changes in terminology and definitions in this area since the book was published.

55 Digitized works of academician Vojislav Mišković

N. Pejović and Ž. Mijajlović
NCD Review, vol. 15, pp. 818, 2009.

Professor Vojislav Mišković (1892-1976) was the leading Serbian astronomer in the twentieth Century. He wrote a large number of books and textbooks on astronomy. Eleven of these books, one chart and one scientific work are digitized and they are included into the Virtual Library of the Faculty of mathematics of the University of Belgrade. This article gives short reviews of these works and biography of professor Mišković as well.

Преглед НЦД 15 (2009), 8–18

Nadežda Pejović, Žarko Mijajlović
Faculty of Mathematics
University of Belgrade,

DIGITIZED WORKS OF ACADEMICIAN VOJISLAV MIŠKOVIĆ

Abstract Professor Vojislav Mišković (1892–1976) was the leading Serbian astronomer in the twentieth Century. He wrote a large number of books and textbooks on astronomy. Eleven of these books, one chart and one scientific work are digitized and they are included into the Virtual Library of the Faculty of mathematics of the University of Belgrade, http://elib.matf.bg.ac.yu:8080/virlib/. In this article we give short reviews of these works and his biography as well.

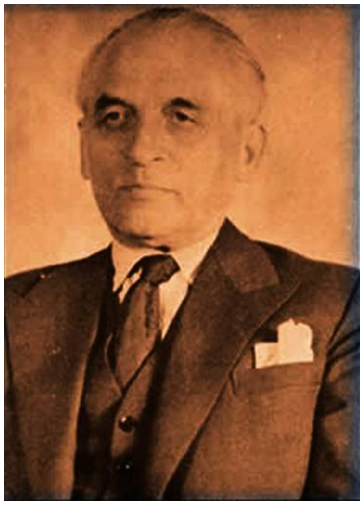
Key words: digitized books, Vojislav Mišković, astronomy

Introduction

In this paper we present eleven books and two more works of Professor Vojislav Mišković (1892–1976), a prominent Serbian astronomer. All these books and works have been digitized and are included in the Virtual Library of the Faculty of mathematics of the University of Belgrade at the Internet address http://elib.matf.bg.ac.yu:8080/virlib/. We would like to mention particularly his book *A Collection of Solved Problems of General Astronomy*, one of the first university collections concerning astronomy written in Serbian. The second part of this collection exists in a hand-written form only and has been never published. The digitized form of this book is the first appearance and presentation of this book (in any form) to the general audience.

Biography of Vojislav Mišković

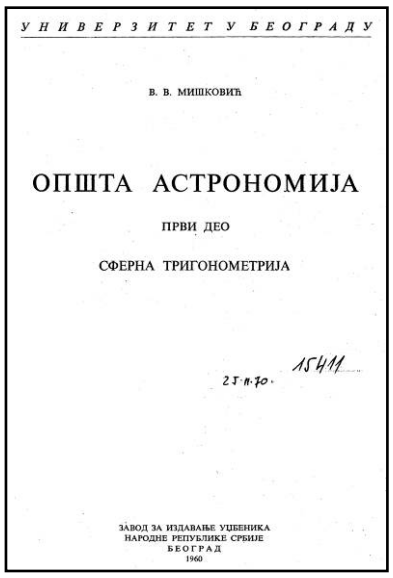
Vojislav Mišković (1892–1976) was born at Fužine in Croatia. He started to study astronomy at the Universities of Budapest and Goettingen shortly before the First World War. When the war began, he came illegally to Serbia, became a member of the volunteer platoon and took an active part in the war till the defeat of the Serbian Army. After the demobilization he went to France to finish his studies. He took degree in 1919 and became assistant at the Astronomical Observatory in Marseille. From 1922 he was engaged as an astronomer at the Observatory of Nice. He got his PhD degree in 1924 at the University of Montpellier. In France he was very active as an astronomer. He organized new services at observatories, was editor of a journal in astronomy, organized and performed the astronomical measurement for of triangulation connecting Corse with the French Eastern Alps, presented the first variant of his original astronomical instrument. For his contributions he was awarded by the French Academy of Sciences. He came to Belgrade in 1926, following an invitation, as an already affirmed as-



tronomer. In the same year he was appointed a Reader (Associate Professor) at the Faculty of Philosophy in Belgrade in the framework of the just founded Chair of Theoretical and Practical Astronomy. At that time he became Director of the newly founded Astronomical Observatory in Belgrade. It may be said that with Mišković's arrival from France to Belgrade an intensive development of astronomy began. In 1929 Mišković was elected a corresponding member of the Serbian Royal Academy and in 1939 a full member. In his honor a minor planet was named Mišković.

◄ Vojislav Mišković

1. General Astronomy



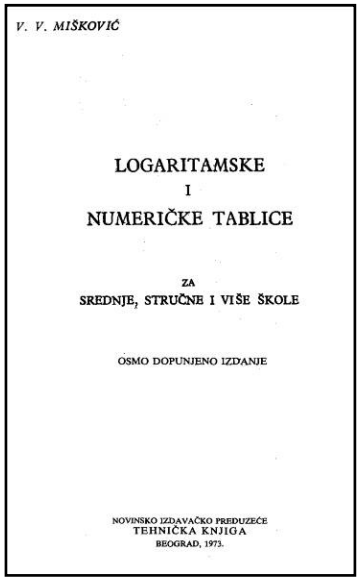
This book, as a non-temporary university textbook, was approved of using by the Commission for textbooks of the Belgrade University in its decision No 1500/2 of June 9, 1960. The introductory part of the textbook on *General Astronomy* became a separate book, entitled *Spherical Trigonometry,* was printed by Beogradski grafički zavod and published by Zavod za izdavanje udžbenika Narodne republike Srbije in Belgrade in 1960. The book contains 95 pages and is composed of *Introduction*, three chapters, *Appendix, List of Formulae and Tables.* In the first chapter *Geometry of a Sphere* the basic notions and definitions are introduced: sphere, great and small circles, poles, spherical distance, crescent, spherical triangle, spherical excess, polar spherical triangle, quadrant spherical triangle, measures for angles. The second chapter *Spherical Trigonometry* presents the basic formulae of spherical trigonometry, five-element formulae, as well as special formulae, then the areas of crescent and spherical triangle, square degree and also the transition from the spherical to the plane trigonometry, calculation of the values for trigonometric functions, geodetic triangle and Legendre's theorem. The third chapter *Applications* contains solved numerical examples concerning spherical triangles, as well as examples of using the differential formulae of spherical trigonometry. In *Appendix* some paragraphs are completed and explained in more details. *List of Formulae* is a small mathematical handbook containing various numerical values and special trigonometric formulae.

The textbook *General Astronomy,* the first part *Spherical Trigonometry* appears as a separate unit because it contains, as written in the preface, more mathematics than a student attending the subject of General Astronomy needs. In it a special attention is paid to numerical applications. Mišković explains that numerical applications in an astronomer's work are very important, but there are not many sources where a reader can find and learn them. The same numerical examples are often solved in several different ways in order to indicate the

dependence of the result quality on the method which is applied. In the numerical examples the last digit is not rounded, instead the result is written with one more digit which is given by using smaller characters. In order to save the space the usual designation $\log(x)$ is replaced by $[x]$.

The first part of *General Astronomy,* entitled *Spherical Trigonometry,* is written very nicely and illustratively. It contains many plots, drawings and solved examples so a collection of problems is not necessary. Here one finds some terms already forgotten in the Serbian of now-a-days, such as *goniometric functions* (trigonometric functions), *grad* (1/400 of a full circle), *centesimal minute* (1/100 of a grad, denoted with the sign for sexagesimal minute directed from the left to the right), *centesimal second* (1/100 of centesimal minute, denoted with the sign for sexagesimal second directed from the left to the right, p. 17). On page 88 a list of foreign textbooks on general astronomy is given. Unfortunately, except the first one, the other parts of the university textbook *General Astronomy* by Vojislav Mišković have been never published. Nevertheless, we recommend to our students even today the first part, which has been published, and from which they can learn spherical trigonometry very easily.

2. Logarithmic and Numerical Tables


V. V. MIŠKOVIĆ

LOGARITAMSKE
I
NUMERIČKE TABLICE

ZA
SREDNJE, STRUČNE I VIŠE ŠKOLE

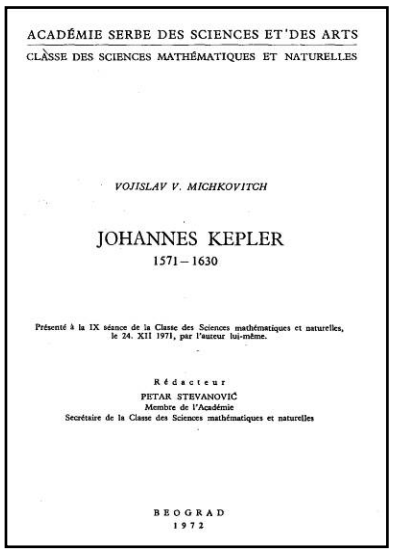
OSMO DOPUNJENO IZDANJE

NOVINSKO IZDAVAČKO PREDUZEĆE
TEHNIČKA KNJIGA
BEOGRAD, 1973.

The logarithmic and numerical tables have been a valuable handbook for complicated and large trigonometric calculations at the time when no calculators and computers existed. The tables have been used not only by professional astronomers, but also by students. The first edition of Mišković's *Logarithmic and Numerical Tables* for secondary and advanced schools was published in 1952. We have digitized the second edition printed by *Tehnička knjiga* in Belgrade in 1973. The book contains 265 pages and the tables in it are given separately and in different colors depending on the type: Briggs logarithms, values and logarithms of trigonometric functions, then the same for the hyperbolic trigonometric functions. All of this is followed by special numerical tables: for conversion of the Napierian into Briggsian logarithms, conversion of radians into degrees, minutes and seconds of arc and vice versa, then tables containing the values of the function of observational errors, values of factors for determining observational errors, list of Pythagorean numbers, factorial values for natural numbers between 1–20, as well as the values of binomial-coefficient degrees between 0–20. Each table is preceded by a plot of the functions presented in it and followed by a detailed instruction for use with a few examples.

3. Johannes Kepler

The book *Johannes Kepler 1571–1630* written by Mišković in 1971 is a Memory dedicated to the 400-anniversary from the birth of renowned astronomer Johannes Kepler. It is interesting that we digitized this book in 2009, just at the 400-anniversary of the publication of Kepler's first two laws in 1609. The book contains 48 pages and is composed of two parts. In the first part one finds the biographies of the first four founders of new astronomy as Mišković calls them in the preface: Nicolaus Copernicus, Tycho Brahe, Galileo Galilei and Sir Isaac Newton. The second part is devoted to the fifth, most meritorious, founder of new astronomy, Johannes Kepler. In this part one describes Kepler's difficult life and his brilliant contributions to the


ACADÉMIE SERBE DES SCIENCES ET DES ARTS
CLASSE DES SCIENCES MATHÉMATIQUES ET NATURELLES

VOJISLAV V. MICHKOVITCH

JOHANNES KEPLER
1571 – 1630

Présenté à la IX séance de la Classe des Sciences mathématiques et naturelles, le 24. XII 1971, par l'auteur lui-même.

Rédacteur
PETAR STEVANOVIĆ
Membre de l'Académie
Secrétaire de la Classe des Sciences mathématiques et naturelles

BEOGRAD
1972

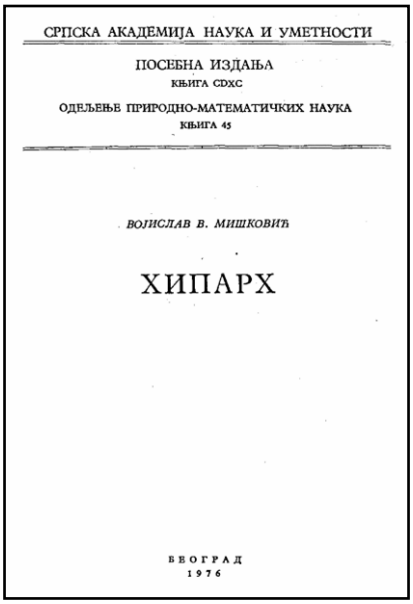
astronomical science. At the end of the book there are important dates from Kepler's life. The book was published by the Serbian Academy of Sciences and Arts and printed by "Naučno delo" in 1972.

At the very beginning of the book there is a quotation of Delambre: *Let glorious be the astronomer who was tenacious enough and had acuteness to, by calculating only, discover the laws of celestial motion at the time when any other way could not lead to them.*

The book *Johannes Kepler 1571–1630* is very interesting, historically valuable and is to be recommended for reading to anyone. From the book one can learn many details from the life and activity of this great man. For instance, Mišković writes about Kepler's "lodestar", more precisely about "harmony of the worlds" searched for by Kepler and found in the discovery of his third law. The book, certainly, deserves to be read by as many people as possible.

4. Hipparchus (190 ? – 125 ? BC)

The book *Hipparchus* Mišković devoted to the life and activity of the greatest astronomer of Classical Antiquity, the first observer and, it might be said, the first founder of space science. It contains IX+59 pages. It was published by the Serbian Academy of Sciences and Arts and printed by "Naučno delo" in Belgrade in 1976. The book is composed of seven chapters: *Preface, Hipparchus-Introductory Word, Pre-Hipparchus Period, Life and Activity of Hipparchus, Instruments from the times before Hipparchus and during its Activity, Post-Hipparchus Period and Summary* (in French). About Hipparchus Mišković writes: *What he did for astronomy of his times is so enormous that it is even hard to believe that a single man could do so much.*


СРПСКА АКАДЕМИЈА НАУКА И УМЕТНОСТИ

ПОСЕБНА ИЗДАЊА
КЊИГА CDXC

ОДЕЉЕЊЕ ПРИРОДНО-МАТЕМАТИЧКИХ НАУКА
КЊИГА 45

ВОЈИСЛАВ В. МИШКОВИЋ

ХИПАРХ

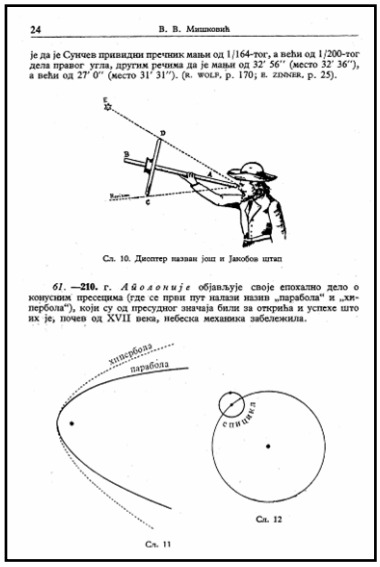
БЕОГРАД
1976

This book can be recommended to a wide circle of readers. In it one finds that Hipparchus determined the lengths of the tropical year and sidereal years, discovered the precession phenomenon, determined the inclination of the ecliptic to the equator, formed the first star catalogue, was the first to distribute stars in six groups according their apparent magnitudes, introduced the notions of geographic latitude (called *climate* by himself) and geographic longitude, determined the prime meridian which passed through his observatory on the island of Rhodes and all of this using a very primitive equipment. He also wrote twelve books about the chords and the corresponding central angles of circles which are predecessors of the trigonometry tables. For this reason geographers began to call Hipparchus "father of geography", whereas to astronomers he has been "founder of mathematical astronomy".

The book *Hipparchus* appears as an important contribution to the literature on history of astronomy written in Serbian, but also to philosophy and general history. This is the first book written in Serbian about this great man of Classical An-

tiquity. In it Mišković, *as the first in the world*, presented the bibliography of many books and works written by Hipparchus. The scripts of Hipparchus have largely not reached our era and about them, as well as about his life and activity, one obtains indirect information, from the records of his contemporaries and also from works of philosophers, historians and astronomers who have written about this topic. Some dates, events, even the origin of the results are, nevertheless, unreliable.

5. Chronology of Astronomical Achievements


24 В. В. Мишковић

је да је Сунчев привидни пречник мањи од 1/164-тог, а већи од 1/200-тог дела правог угла, другим речима да је мањи од 32′ 56″ (место 32′ 36″), а већи од 27′ 0″ (место 31′ 31″). (R. WOLF, p. 170; B. ZINNER, p. 25).

Сл. 10. Диоптер назван још и Јакобов штап

61. —**210.** г. *Айолоније* објављује своје епохално дело о конусним пресецима (где се први пут налази назив „парабола" и „хипербола"), који су од пресудног значаја били за открића и успехе што их је, почев од XVII века, небеска механика забележила.

Сл. 11 Сл. 12

◄ **Page 24 from Chronology**

Books *The Chronology of Astronomical Achievements I* and *II* are in fact a chronological encyclopedia of astronomical achievements realized by the end of the XIX century. The events and innovations are presented as entries and their total number is about 2000. In the first book Mišković included astronomical achievements from prehistory till 1700, whereas the second book comprises the period between 1700 and 1900. Both volumes were published by the Serbian Academy of Sciences and Arts in 1975 and 1976, respectively. They were printed by printing house *Naučno delo* from Belgrade. The first volume has 133 pages, the second one 148.

Chronology lists achievements ordered chronologically and in it all events, discoveries and innovations of importance in astronomy are described, i. e. all important astronomical achievements are included. That this is an encyclopedia can be seen in the fact that not only all data desired can be found easily and quickly, but also that the data are correct. Namely, at one place (volume II, p. 2) Mišković writes: *All data which should have been included in "Chronology" were searched for specially, regularly, in one of the known histories of astronomy; if necessary in a major textbook, or in an astronomical journal, or in the article where the given phenomenon or subject was the topic. If the search was successful, which was the most frequent case, at the end of the entry the complete reference was given.*

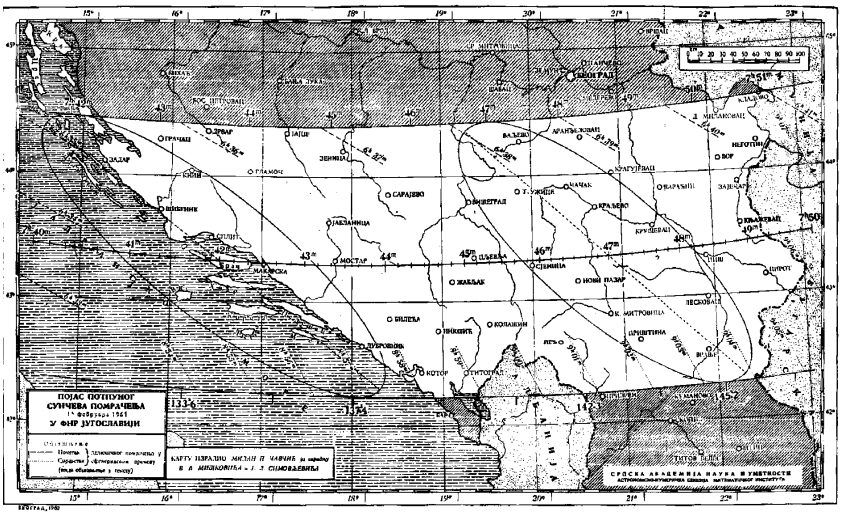
The value of these books can be seen from the following fact. In each entry one finds the name of at least one scientist, usually astronomer, or the name of her/his work. The foreign names are adapted to the Serbian language and written in Cyrillic, whereas the original is given within the parentheses in Latin characters. Some entries contain figures and schemes. At the end of each volume there is an index with the names of the scientists resulting in a more rapid and easy search.

We may say that *Chronology* appears as an important work from the history of astronomical science, not only because it is the first book of this kind in our midst, but also because it is comprehensive and substantially complete. Any reader will be surprised by the number and type of the discoveries invented, noticed and anticipated by ancient thinkers.

6. Solar Eclipse of February 15, 1961

The book *Solar Eclipse of February 15, 1961* was published by the Serbian Academy of Sciences and Arts within the series *Special Editions* in 1960. Mišković wrote this book on the occasion of the total solar eclipse seen from our territory on February 15, 1961. It con-

tains 5+75 pages and consists of *Preface,* four chapters and *Overview.* In the first chapter *Appearance and Composition of the Sun* one describes in details the solar photosphere, sunspots, flares, chromosphere, spectrum, prominences and the corona. The second chapter *Solar eclipses* deals with the eclipse types, cone size for the lunar shadow, eclipse duration, phases of a partial and those of a total eclipse, as well as the moments of the beginning and end of a total eclipse. In the third chapter *Organizing Programme of Eclipse Observation* Mišković describes the instruments and equipment for astronomical observations, instruments for photographing the corona, the first contact – beginning of the partial eclipse, the observation of arriving of the lunar shadow, the second contact – the beginning of the total eclipse, spectrum taking, taking of the corona spectrum, measuring of corona polarization, measuring of Einstein's effect, the appearance of the sky during the eclipse, the third contact – the end of the total eclipse, the fourth contact – the end of the partial-eclipse phase, then radio observations and observations in the conditions of a cloudy sky (on board of a balloon or airplane). In the fourth chapter *Course of the Eclipse as observed from Yugoslavia* Mišković presents a description of the general course of the approaching eclipse, then that of the course for the observations from Yugoslavia and finally the approximate determination of the moments of the phases for a given place in Yugoslavia. In *Overview* one finds the data about the main phases of the February eclipse in 1961 given for 29 Yugoslav biggest cities (towns), in particular, in addition to the latitude, longitude and altitude above the sea level also the moments of the beginning and end of both totality and partial eclipse at the given place (if inside the totality band), i. e. the moments of the beginning and end of the partial eclipse if the place is outside the totality band. These data were calculated by Jovan Simovljević by using a hand calculator who was then assistant at the Faculty of Sciences (University of Belgrade).



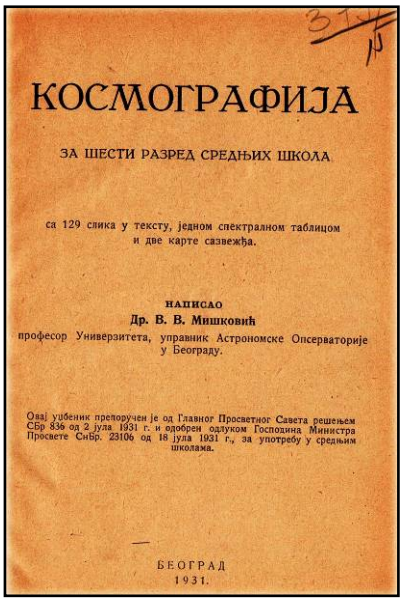
The totality band of the Solar eclipse 1961 represented on the map of Yugoslavia. which is enclosed at the end of the book (made by M. Čavčić)

This book was a valuable reading to all expeditions which observed the total solar eclipse on February 15, 1961. Since the solar eclipses for the same place are rare (once in three centuries), this is more in favour of the importance of the book. This short monograph is important because it was written and printed at the right moment, immediately before the most spectacular phenomenon which can be seen in the sky. The previous total solar eclipse for our territory took place in 1666, and the next, as written by Mišković, was expected on October 7, 2135. However, Mišković in this book mentions that the solar eclipse of August 11,

1999 will be partial as it largely was for our country. But, a little bit later on J. Simovljević performed precise calculations and found that for a small part of our country, northeastern Vojvodina, this eclipse would be total. Simovljević's results were correct. In one of the expeditions one of us (N. Pejović) took part and observed on August 11, 1999 this magnificent phenomenon from Đala, at the extreme north of our country. Note that this spectacular phenomenon took place in the sky only two months after the end of the heavy bombardment of our country by the NATO forces.

7. Cosmography

Mišković's *Cosmography* is an astronomy textbook for the pupils of the sixth form of secondary school at the time of its publication (today second form of secondary school). The textbook was recommended by Main Education Council (Decision No Sbr 836) on July 2, 1931 and was approved of using in secondary schools by the Education Minister (SnBr 23106, July 18, 1931). This is the first official astronomy textbook for secondary schools written in our country. It contains 186 pages with 129 figures in the text. In Appendix it contains two maps of the sky and one table with spectral lines. The book is informative and clearly written with many schemes and drawings giving nice illustrations of the phenomena and notions described in it. The apparent and true motions of the Sun, Moon, planets, comets and stars are explained. Time systems, the sidereal time and the solar (apparent, mean and universal) ones are defined. Celestial bodies, stars, planets, planet satellites, minor planets, comets and meteors, as well as celestial systems, double, triple, multiple stars, star clusters, nebulae and galaxies, are described.


КОСМОГРАФИЈА

ЗА ШЕСТИ РАЗРЕД СРЕДЊИХ ШКОЛА

са 129 слика у тексту, једном спектралном таблицом и две карте сазвежђа.

НАПИСАО
Др. В. В. Мишковић
професор Универзитета, управник Астрономске Опсерваторије у Београду.

Овај уџбеник препоручен је од Главног Просветног Савета решењем СБр 836 од 2 јула 1931 г. и одобрен одлуком Господина Министра Просвете СнБр. 23106 од 18 јула 1931 г., за употребу у средњим школама.

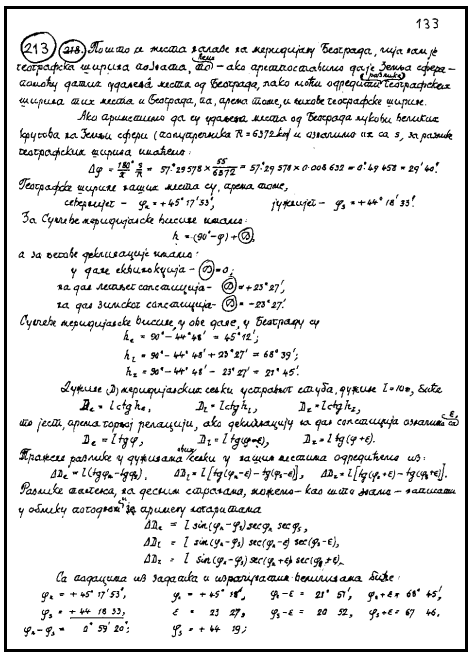
БЕОГРАД
1931.

Thanks to both the contents and the clarity of style we are glad to recommend this book also to the pupils of secondary schools of our epoch.

8. Collection of Solved Problems of General Astronomy

The first part of *Zbirka rešenih zadataka iz opšte astronomije* (A Collection of Solved Problems of General Astronomy) by Prof. Vojislav Mišković (hereafter referred to as the Collection) is the first university collection of astronomical problems printed in Serbian. It appeared in the middle of the last century. The need to publish this book arose, as written in the Preface by Mišković, on one hand due to an increase in the number of students who studied astronomy, and, on the other hand, due to the insufficient number of solved problems, not only in the Serbian literature, but also abroad. Whereas the number of astronomical textbooks was sufficient in all European languages, the lack of collections containing solved problems was evident. The Collection was approved of printing through an act of the Textbook Commission of Belgrade University No 896 of August 10, 1956 as a textbook for students of the Faculty of Sciences. Though the approval concerned the Collection as a whole, for technical reasons, as written by Mišković, it had to be divided into two parts. The first one was published by *Naučna knjiga* in Belgrade in the year of approval with a circulation of 2000 copies, whereas the second part should have been published in the following year. Unfortunately, the second part of the Collection is still in the manuscript form. The manuscript was given to the first co-

author of this article in 1995 by Professor of Astronomy Jovan Simovljević (1929–2007) when he got retired. From that year many problems contained in the manuscript have been solved by the students as exercises and during exams concerning the teaching subject of General Astronomy at the Astronomy Department of the Faculty of Mathematics in Belgrade. Since Mišković was a European scholar and knew foreign languages very well, he wrote the Collection using the most contemporaneous textbook and scientific literature of that time. In its *Introduction* one finds about ten references - foreign textbooks, mostly French, English and German. The problems taken from other authors have a designation which indicates their origin. The digitization comprising the whole Collection has been just finished and in this way Mišković's original manuscript became available to the public, after half a century.



133

213 218. Тошто су висина хоризонта на меридијану Београда, чија је географска ширина позната, (као) – ако астрономским путем дође Земља сферна – односно дужина удаљена места од Београда, такође одређена географских ширина тих места и Београда, па, према томе, и њихове географске ширине.

Ако астрономским путем дође удаљена места од Београда мањих великих кругова на Земљиној сфери (сферичарска $R = 6372\,km$) и означимо их са s, за разлике географских ширина имаћемо:

$$\Delta\varphi = \frac{180^\circ}{\pi}\cdot\frac{s}{R} = 57^\circ.29\,578 \times \frac{s}{6372} = 57^\circ.29\,578 \times 0.000\,632 = 0^\circ.49\,452 = 29'.40.$$

Географске ширине наших места су, према томе,

северњег – $\varphi_2 = +45^\circ\,17'.53$, јужњег – $\varphi_3 = +44^\circ\,18'.33$.

За Сунчеве меридијанске висине имаћемо:

$$h = (90^\circ - \varphi) + \delta,$$

а за висине девијације чланка:

у фазе еквиноксија – $\delta = 0$,

за фазе летњег солстиција – $\delta = +23^\circ.27'$,

за фазе зимског солстиција – $\delta = -23^\circ.27'$.

Сунчеве меридијанске висине, у обе фазе, у Београду су

$$h_e = 90^\circ - 44^\circ.48' = 45^\circ.12';$$
$$h_l = 90^\circ - 44^\circ.48' + 23^\circ.27' = 68^\circ.39';$$
$$h_z = 90^\circ - 44^\circ.48' - 23^\circ.27' = 21^\circ.45'.$$

Дужине (D) меридијанских сенки усправног штапа, дужине $l = 10\,m$, биће

$$D_e = l\,\mathrm{ctg}\,h_e, \qquad D_l = l\,\mathrm{ctg}\,h_l, \qquad D_z = l\,\mathrm{ctg}\,h_z,$$

што јесте, према горњој релацији, ако девијацију δ за фазе солстиција означимо са ε

$$D_e = l\,\mathrm{tg}\,\varphi, \qquad D_l = l\,\mathrm{tg}(\varphi - \varepsilon), \qquad D_z = l\,\mathrm{tg}(\varphi + \varepsilon).$$

Тражене разлике у дужинама сенки у нашим местима одређићемо из:

$$\Delta D_e = l(\mathrm{tg}\varphi_2 - \mathrm{tg}\varphi_3), \qquad \Delta D_l = l\,[\mathrm{tg}(\varphi_2 - \varepsilon) - \mathrm{tg}(\varphi_3 - \varepsilon)], \qquad \Delta D_z = l\,[\mathrm{tg}(\varphi_2 + \varepsilon) - \mathrm{tg}(\varphi_3 + \varepsilon)].$$

Разлике штапова, на десним странама, можемо – као што знамо – израчунати у облику погодном за аритметичко логаритмовање

$$\Delta D_e = l\,\sin(\varphi_2 - \varphi_3)\sec\varphi_2\sec\varphi_3,$$
$$\Delta D_l = l\,\sin(\varphi_2 - \varphi_3)\sec(\varphi_2 - \varepsilon)\sec(\varphi_3 - \varepsilon),$$
$$\Delta D_z = l\,\sin(\varphi_2 - \varphi_3)\sec(\varphi_2 + \varepsilon)\sec(\varphi_3 + \varepsilon).$$

Са подацима из задатка и израчунатим вредностима биће:

$\varphi_2 = +45^\circ\,17'.53'', \quad \varphi_3 = +44^\circ\,18'.., \quad \varphi_3 - \varepsilon = 21^\circ\,51', \quad \varphi_2 + \varepsilon = 68^\circ\,45',$
$\varphi_3 = +44\;\;18.33; \quad \varepsilon = 23\;\;27.., \quad \varphi_3 - \varepsilon = 20\;\;52., \quad \varphi_2 + \varepsilon = 67\;\;46.,$
$\varphi_2 - \varphi_3 = 0^\circ\,59'\,20'': \quad \varphi_3 = +44\;\;19;$

Mišković'ć handwritings (Collection, page 133)

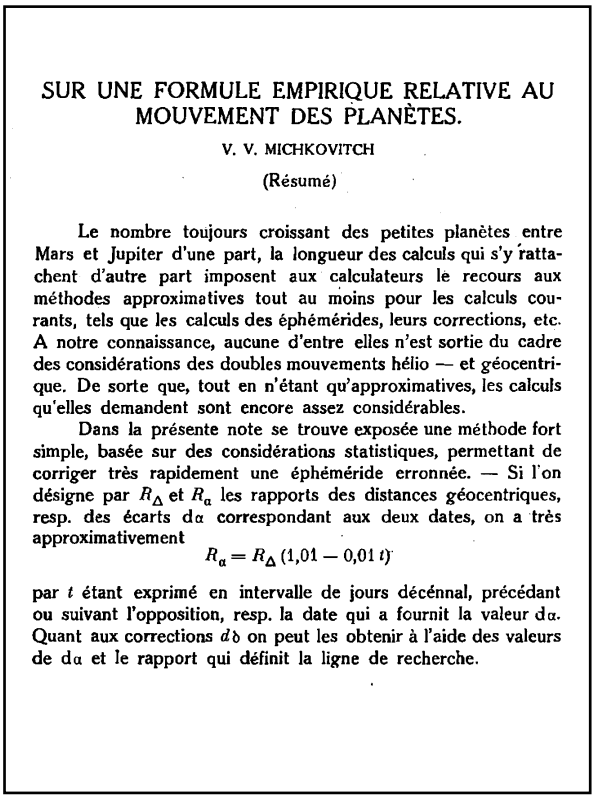
The book printed is the first part of the Collection. It has 150 pages and consists of *Preface, Introduction* and two chapters. *Introduction* consists of three parts: *Review of Formulae of Spherical Trigonometry, Series* and *Review of Formulae for Transforming Astronomical Coordinate Systems*. The first chapter *Problems* concerns three fields: *Spherical Trigonometry, Earth as a Celestial Body* and *Apparent Diurnal Motion of Celestial Sphere*. The second, much larger, chapter *Solutions* contains the instructions and solutions for 126 problems of the Collection.

The manuscript, the second part of the Collection, consists of *Introduction* and two chapters. The introduction to this part concerns three fields: *Astronomical Refraction*, *Elements of Theory of Motion for Planets and Comets* and *Apparent Annual Motion of the Sun*. The first chapter *Problems* contains problems, whereas the second *Solutions*, much larger, contain methods, explanations and solutions concerning 136 problems of the manuscript. The *Manuscript* also contains an instruction how to form a single volume of the first part (already printed) and second one (manuscript). According to the enclosed plan, Mišković planned a different arrangement of the whole collection.

Based on the contents presented above it is seen that the whole Collection should have covered six fields: spherical trigonometry, Earth as a celestial body, apparent diurnal motion of celestial sphere, astronomical refraction, elements of theory of motion for planets and com-

ets and apparent annual motion of the Sun. The importance of 262 problems explained and completely solved from these fields is very great even today because there are no other collections containing solved astronomical problems written in Serbian. There are textbooks, both for secondary schools and universities, but often without any solved problem or example. For this reason the Collection appears as a real jewel in the Serbian astronomical literature.

9. On an Empirical Formula in the Determination of the Planetary Motion



SUR UNE FORMULE EMPIRIQUE RELATIVE AU MOUVEMENT DES PLANÈTES.

V. V. MICHKOVITCH

(Résumé)

Le nombre toujours croissant des petites planètes entre Mars et Jupiter d'une part, la longueur des calculs qui s'y rattachent d'autre part imposent aux calculateurs le recours aux méthodes approximatives tout au moins pour les calculs courants, tels que les calculs des éphémérides, leurs corrections, etc. A notre connaissance, aucune d'entre elles n'est sortie du cadre des considérations des doubles mouvements hélio — et géocentrique. De sorte que, tout en n'étant qu'approximatives, les calculs qu'elles demandent sont encore assez considérables.

Dans la présente note se trouve exposée une méthode fort simple, basée sur des considérations statistiques, permettant de corriger très rapidement une éphéméride erronnée. — Si l'on désigne par R_Δ et R_α les rapports des distances géocentriques, resp. des écarts $d\alpha$ correspondant aux deux dates, on a très approximativement

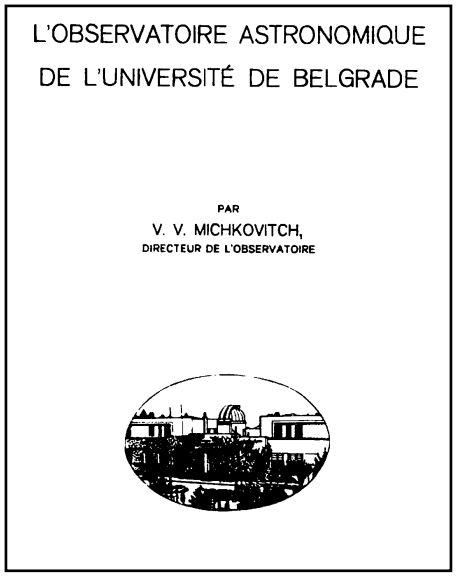
$$R_\alpha = R_\Delta (1{,}01 - 0{,}01\, t)$$

par t étant exprimé en intervalle de jours décénnal, précédant ou suivant l'opposition, resp. la date qui a fournit la valeur $d\alpha$. Quant aux corrections $d\delta$ on peut les obtenir à l'aide des valeurs de $d\alpha$ et le rapport qui définit la ligne de recherche.

Abstract in French of the paper *Empirical formula*

This work comprising about ten pages is in fact a covered reprint of a scientific paper published in "Glas CXXX SKA" (Serbian Royal Academy) printed by company "Makarije" in Belgrade in 1928. With regard that this paper is available in the Library of the Serbian Academy of Sciences and Arts as a special bibliographical unit, it is justified to suppose that that was Mišković's personal choice. For this reason we have, nevertheless, selected this manuscript to be digitized and included in the Virtual Library. Mišković's contribution in this paper is an empirical formula for correcting the ephemeris of a minor planet on the basis of an observation which showed that the planet was not at the place obtained in the calculation.

10. L'Observatoire Astronomique de L'Université de Belgrade



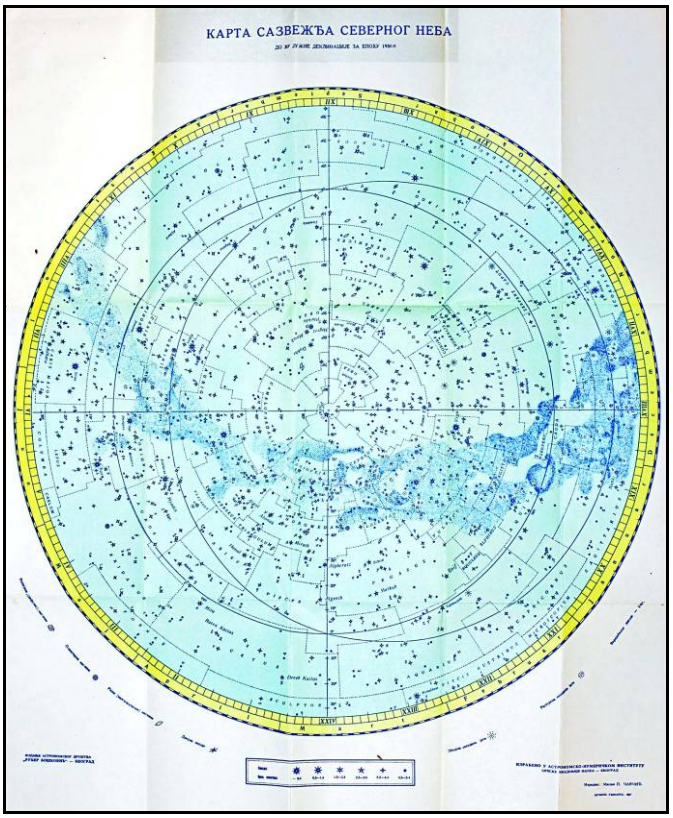
L'OBSERVATOIRE ASTRONOMIQUE
DE L'UNIVERSITÉ DE BELGRADE

PAR
V. V. MICHKOVITCH,
DIRECTEUR DE L'OBSERVATOIRE

This small booklet (contains only 19 pages) Mišković published anent the Balkan Congress of mathematicians which was held in Belgrade in September 1939. The booklet described the history of foundation and work of the newly founded Astronomical Observatory in Belgrade (in 1924). We remind that Mišković was appointed the Director of the Observatory almost immediately after its foundation, in 1926. Before him the director was Milan Nedeljković. The booklet also describes the local geographic position, activities of the Observatory, publications and instruments. Probably the greatest value of the booklet is the relatively large number (15) of photos for such a small edition. Besides the pictures of Observa-

tory there are photos of its instruments as well. The reader can see that at that time the Observatory was relatively well equipped and very active. As seen from Mišković's biography, this is mainly due to his efforts and engagement.

11. Map of the constellations of the Northern hemisphere



Mišković made this chart in collaboration with Milan Čavčić, the member of the technical staff of the Mathematical Institute of the Serbian Academy of Science and Arts. The map is published by the Astronomical Society *Rudjer Bošković* in 1957 and covers all constellations up to 30^0 of the South declination for the epoch 1950.0. The chart is suplemented with the small manual (six pages) for using the map, which also explains the basic astronomical notions. The supplement was written by P. Djurković, the astronomer from the Observatory in Belgrade. In addition, the map is supplemented with the transparent sheet of paper with charted grid for reading positions of the celestial bodies on the map.

Acknowledgement

We are grateful to the Ministry of Science of the Republic of Serbia which finances the Project *Application of Information Technologies in Digitization of the Scientific and Cultural Heritage, RP 13017,* as well as to the Library of the Serbian Academy of Sciences and Arts for lending us all these books for the digitization, except two of them *Cosmography and L'Observatoire Astronomique de L'Université de Belgrade* borrowed from the private library of Prof. Dr Jelena Milogradov-Turin.

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ДИГИТАЛИЗОВАНА ДЕЛА АКАДЕМИКА ВОЈИСЛАВА МИШКОВИЋА

Сажетак. Представљамо прве универзитетске уџбенике из астрономије писане на српском језику. Наиме, реч је о уџбеницима, збиркама и књигама професора Војислава Мишковића писаних средином XX века: *Општа астрономија*, *Хронологија астрономских тековина I* и *II*, *Хипарх*, *Јоханес Кеплер, Космографија, Сунчево помрачење од 15 фебруара 1961,* и *Логаритамске и нумеричке таблице*. У ову дигиталну колекцију такође су укључени први део збирке који је објављен 1956. под називом *Збирка решених задатака из Опште астрономије - Први део,* затим рукопис другог дела *Збирка решених задатака из Опште астрономије - Други део* који потпуно припремљен али никад није штампан. Ова књига је не само прва збирка задатака из астрономије на српском језику, већ и једина збирка ове врсте до сада написана. Стога је дигитализација, посебно њеног другог дела, од великог значаја за студије астрономије, јер је овим електронска копија рукописа постала доступна студентима. *Збирка* је методички занимљиво и лепо писана. Сви поменути астрономски појмови и наведени обрасци су детаљно и јасно представљени. Нарочита пажња је посвећена нумеричком решавању задатака, што се данас прилично занемарује. Свих дванаест наведених књига је дигитализовано и оне се налазе у Виртуелној библиотеци Националног центра за дигитализацију (Virtual library, http://elib.matf.bg.ac.yu;8080/virlib/). Дигитализација ових књига део је пројекта електронског архивирања српских књига из математичких наука штампаних у прошлости. Укратко представљамо садржаје ових књига као и њихове занимљивости.

Кључне речи: Дигитализоване књиге**,** Војислав Мишковић, астрономија

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zarkom@matf.bg.ac.rs

56 Manuscript collection of solved problems of general astronomy by Vojislav Mišković

N. Pejović
Publ. Astron. Obs. Belgrade
Vol. 86, pp. 211-216, 2009.

This review presents the first university problems book in astronomy written in Serbian. The book is written by Prof. Vojislav Mišković. The first part of this collection was published in 1956 in Belgrade. The second one is remind in the form of a manuscript. From the methodical point of view the collection was interestingly and nicely arranged. All astronomical notions and formulae are treated in details and well explained so that no complementary textbook is needed. A particular attention is devoted to the numerical solution of the problems, which is not only missing in Serbian schools, but also, as said by Mišković in the Preface, is not properly valued. There is only one copy of the second part which was inherited from one generation of professors of the Astronomy Department to the next generation. By means of the manuscript digitization this part of the Collection has become available to public use.

Publ. Astron. Obs. Belgrade No. 86 (2009), 211 - 216 *Contributed paper*

MANUSCRIPT *COLLECTION OF SOLVED PROBLEMS OF GENERAL ASTRONOMY* BY VOJISLAV MIŠKOVIĆ

N. PEJOVIĆ

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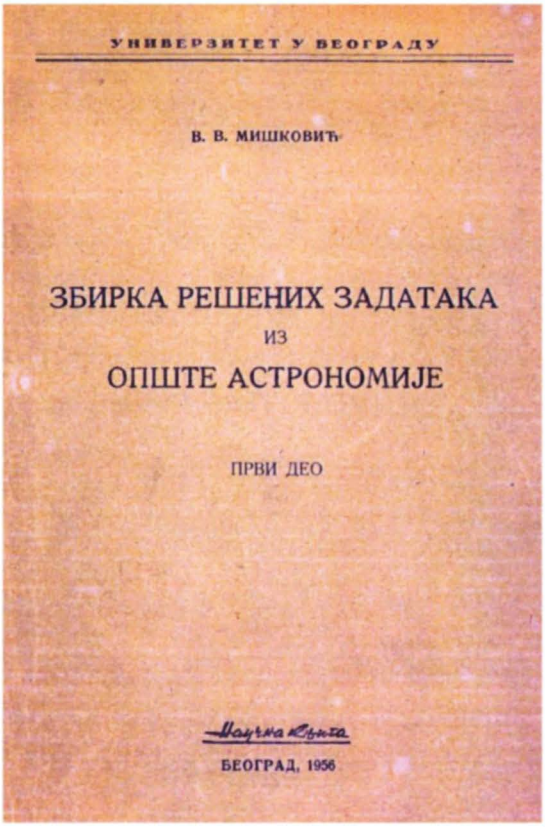
Abstract. In this paper we present the first university problems book concerning astronomy written in Serbian. The topic is **"Zbirka rešenih zadataka iz opšte astronomije"** (A Collection of Solved Problems of General Astronomy) by Prof. Vojislav Mišković. The first part of this collection was published in 1956 in Belgrade. The second one is still in the form of a manuscript. Though completely prepared for publishing, it has never been published. From the methodical point of view the collection was interestingly and nicely arranged. All astronomical notions and formulae are treated in details and well explained so that no complementary textbook is needed. A special attention is devoted to the numerical solution of the problems, which is not only missing in Serbian schools, but also, as said by Mišković in the Preface, is not properly valued. This paper treats both parts of this Collection which have been digitized and are available in the Virtual Library of the National Digitization Centre (Virtual Library, http://elib.matf.bg.ac.yu:8080/virlib/). By means of the manuscript digitization the second part of the Collection has become available to public use.

1. INTRODUCTION

The first part of **"Zbirka rešenih zadataka iz opšte astronomije"** (A Collection of Solved Problems of General Astronomy) by Prof. Vojislav Mišković (hereafter referred to as the Collection) is the first university collection of astronomical problems printed in Serbian. It appeared in the middle of the last century. The need to publish this book arose, as written in the Preface by Mišković, on one hand due to an increase in the number of students who studied astronomy, and, on the other hand, due to the insufficient number of solved problems, not only in Serbian literature, but also abroad. Whereas the number of astronomical textbooks was sufficient in all European languages, the lack of collections containing solved problems was evident. The Collection was approved of printing through an act of the Textbook Commission of Belgrade University No 896 of August 10, 1956 as a textbook for students of the Faculty of Sciences. Though the approval concerned the Collection as a whole, for technical reasons, as written by Mišković, it had to be divided into two parts. The first one was published by *Naučna knjiga* in Belgrade in 1956 in 2000 copies, whereas the second part should have been published in the following year. Unfortunately, the second part of the Collection is still in the manuscript form. The manuscript was

given to the author of this paper in 1995 by Professor of Astronomy Jovan Simovljević (1929-2007) when he got retired. Since 1995 many problems contained in the manuscript have been solved by the students as exercises or exams of the subject General Astronomy at the Astronomy Department of the Faculty of Mathematics in Belgrade. Since Mišković was a European scholar and knew several foreign languages very well, he wrote the Collection using the most contemporaneous textbooks and scientific literature of that time. In its *Introduction* one finds about ten references – foreign textbooks, mostly in French, English and German. The problems taken from other authors have a designation, which indicates their origin. The digitization comprising the whole Collection has just been completed and in this way Mišković's original manuscript became available to the public, after half a century.

2. CONTENTS OF THE BOOKS


УНИВЕРЗИТЕТ У БЕОГРАДУ

В. В. МИШКОВИЋ

ЗБИРКА РЕШЕНИХ ЗАДАТАКА
ИЗ
ОПШТЕ АСТРОНОМИЈЕ

ПРВИ ДЕО

Научна књига

БЕОГРАД, 1956

Figure 1: Cover page of the first part of the Collection.

The book printed is the first part of the Collection. It has 150 pages and consists of *Preface, Introduction* and two chapters. *Introduction* consists of three parts: *Review of Formulae of Spherical Trigonometry, Series* and *Review of Formulae for Transforming Astronomical Coordinate Systems.* The first chapter *Problems* concerns three fields: *Spherical Trigonometry, Earth as a Celestial Body* and *Apparent Diurnal Motion of Celestial Sphere.* The second, much larger, chapter *Solutions* contains the instructions and solutions for 126 problems of the Collection.

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Based on the contents presented above it is seen that the whole Collection should have covered six fields: spherical trigonometry, Earth as a celestial body, apparent diurnal motion of celestial sphere, astronomical refraction, elements of theory of motion of planets and comets and apparent annual motion of the Sun. The importance

of 262 problems from these fields, which are explained and completely solved, is very great even today because there are no other collections containing solved astronomical problems written in Serbian. There are textbooks, both for secondary schools and universities, but often without any solved problem or example. For this reason the Collection appears as a real jewel in Serbian astronomical literature.

3. BIOGRAPHY OF PROFESSOR VOJISLAV MIŠKOVIĆ



Figure 2: Vojislav Mišković.

Vojislav Mišković (1892-1976) was born at Fužine in Croatia. He started to study astronomy at the Universities of Budapest and Goettingen shortly before the First World War. When the war began, he came illegally to Serbia, became a member of the volunteer squad and took an active part in the war till the defeat of the Serbian Army. After the demobilization he went to France to finish his studies. He graduated in 1919 and became assistant at the Astronomical Observatory in Marseille. From 1922 he was engaged as an astronomer at the Observatory of Nice. He got his PhD degree in 1924 at the University of Montpellier. In France he was very active as an astronomer. He organized new services at observatories, was editor of a journal in astronomy, organized and performed the astronomical measurement for the purpose of triangulation connecting Corse with the French Eastern Alps, presented the first variant of his original astronomical instrument. For his contributions to stellar statistics he was awarded Valz prize by the French Academy of Sciences. Following an invitation he was elected in 1925 as Associate Professor of Practical Astronomy at the Faculty of Philosophy in Belgrade. He came to Belgrade in 1926 as an already recognized astronomer. At the same time he was appointed Director of the newly founded Astronomical Observatory in Belgrade. It may be said that an intensive development of astronomy began with Mišković's arrival from France to Belgrade. He became a full Professor in 1936. In 1929 Mišković was elected a corresponding member of the Serbian Royal Academy and in 1939 a full member. In his honour a minor planet was named Mišković.

4. FIRST PART OF THE COLLECTION - PRINTED VOLUME

The first part of the Collection is well known to astronomers because they used it during the exercises in the framework of subject General Astronomy when they were students. Since there are no other collections of astronomical problems in Serbian even today, this book is still in use. Naturally, today, students utilize calculators and computers for solving the problems. In the collection the use of logarithmic tables (Mišković wrote a well known book containing logarithmic tables) is recommended. For this reason, the Collection appears as an excellent base for controlling the solutions in the case that a new collection appeared where modern computing software, such as MATHEMATICA and MATLAB, would be used.

It should be emphasized that in *Preface* Mišković writes words which could be written also today: *include again astronomy in the plans for secondary schools.* Namely, he says that astronomy was excluded from schools shortly before and that its topics

are unjustifiably joined the framework of a descriptive-narrative discipline, geography. Unfortunately, today also, in the space era, in the time of the Hubble Space Telescope, astronomy is not present in secondary schools because it was excluded 19 years ago. Its topics are now scattered not only in the framework of geography, but also in that of physics. Of course, efforts are made to reintroduce astronomy as a separate subject, in the framework of the education-system reform which is taking place now. The chances are encouraging because the proposal to include astronomy with one lesson a week in real gymnasiums has been accepted.

In *Preface* it is seen that Mišković for the assistance in the formation of the Collection thanks his colleague Prof. R. Kašanin, for the manuscript redaction his assistants J. Simovljević and R. Djordjević for verifying the large numerical work (then there were neither calculators nor computers), as well as M. Čavčić from the Mathematical Institute of the Serbian Academy of Science and Art for making all the drawings in the Collection.

Introduction in the Collection has a review character. Necessary formulae of spherical trigonometry are given, both for oblique spherical triangle and for right one, also special formulae and differential ones for polar spherical triangle. Series (of trigonometric and other special functions) used in astronomy are also given, as well as a review of standard formulae for transforming astronomical coordinate systems.

There is a common property in the solutions of the problems contained in the Collection. Namely, the problems are explained in details, their general solutions are given and the numerical part is presented. Most frequently the same problems are solved numerically by using alternative methods or by applying alternative formulae so that in this way their verification is carried out. The problems are illustrated by the following examples:

A Problem from Spherical Trigonometry. This field contains 33 problems. For instance, Problem 15 is: *The sides of a spherical triangle* $b = 37^o47'18"$ *and* $c = 74^o51'50"$ *and the angle between them* $A = 44^o10'40"$ *are given. One should determine: 1) third side a and one of the angles along it, say C; 2) all the three unknown elements.*

A Problem from Earth as a Celestial Body. Here 42 problems are treated. Problem 39 on page 66 is: *The observer is aboard a balloon at an altitude* $h = 12$ *km above a topographical feature with geographic coordinates* $L = 0^o$, $\phi = +60^o$. *Assuming that the Earth is a sphere one should find the coordinates of the points along the meridian and the parallel reached by the observer's seeing.*

A problem from Apparent Diurnal Motion of Celestial Sphere. Here 51 problems are treated. For instance, Problem 114 is: *At what moment of sidereal time for an observer at latitude* $\phi = +36^o47'50"$ *will the stars* $\sigma_1(\alpha_1 = 1^h35^m57^s.2, \delta_1 = +67^o36'31")$ *and* $\sigma_2(\alpha_2 = 3^h39^m20^s.8, \delta_2 = +42^o18'28")$ *reach the same elevation simultaneously? What is the interval of sidereal time beginning with this moment after which the difference of their zenith distances attains 1"?* (Sorbonne 1922).

5. SECOND PART OF THE COLLECTION - MANUSCRIPT

This part of the Collection has never been published. The Manuscript in its original form was published recently for the first time in an electronic form. This part contains solved problems from the following fields: *astronomical refraction, elements of theory of motion for planets and comets* and *apparent annual motion of the Sun.* Like the

preceding part the problems are explained in details, they contain necessary formulae and numerical examples. The importance of these fully treated problems to the teaching of astronomy in our midst at the university level is great because we cannot find them elsewhere in the literature.

To mention that in the electronic form of *Manuscript* in the solution to every problem at the place, where the figure should have been, there is a hyperlink opening this figure.

A problem from Astronomical Refraction. In addition to the definitions here one finds formulae for calculating astronomical refraction, the tables, then the expressions treating the refraction influence on the change of coordinates of a celestial body, as well as that treating the horizon depression. In this field there are 23 problems treated fully. Here Problem 134 is presented: *A circumpolar star was observed in both culminations at apparent zenith distances* $z_s = 17°18'48".2$ *and* $z_i = 73°01'11".6$ *at a topographical feature of latitude* $\phi = +44°48'13",2$. *Determine:*

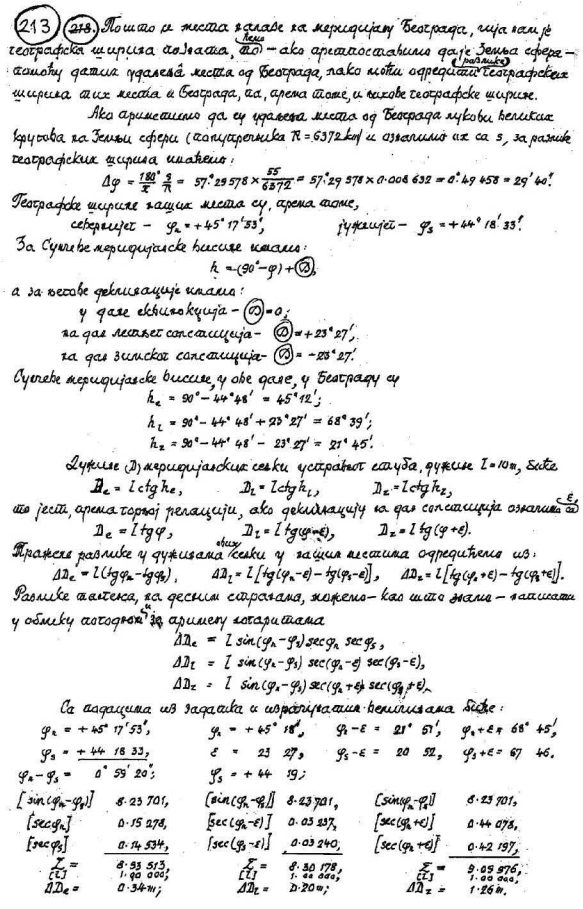

Figure 3: Mišković's solution of the Problem: *Apparent Annual Motion of the Sun.*

1) the value for constant k *in the expression for refraction* $R = k \tan z$,

2) the values for constants A *and* B *in the expression for refraction* $R = A \tan z - B \tan z$ *if it is known that the declination of the star was* $\delta = +62°07'20".2$.

A problem from Elements of Theory of Motion for Planets and Comets. Here Mišković gives the theoretical bases, including the mathematical apparatus, for solving problems from this part of astronomy, for instance Kepler's laws, law of universal gravitation, as well as the more correct form of Kepler's third law. Kepler's equation is presented, together with the procedures for its solving, to be followed by expanding functions in series of true, mean and eccentric anomalies and radius vector of a planet. The number of solved problems is 36. As an example Problem 158 is given: *Solve Kepler's equation if* $e = 0.147$ *and* $M = 136°25'32".4$.

A problem from Apparent Annual Motion of the Sun. In the theoretical part of this section of the manuscript one describes the apparent motion of the Sun, its apparent annual orbit across the celestial sphere, details concerning the apparent diurnal motion of the Sun, as well as the inequalities in the duration of the daylight and night and in that of twilights. The total of treated problems is 74. The example is No 213: *What is the difference in the lengths of the meridian shadows in the horizontal plane of a vertical pillar, 10 m high,*

on the days of equinoxes and solstices at topographical features along the meridian of Belgrade, of which one is 55 km north and the other one is south of Belgrade by the same distance?

Both parts of the Collection are characterized by an exceptional visibility and a systematic way of presentation. A larger part of the second part was written by hand, but the handwriting is very nice and readable. This was the reason to publish the original form of the second part of the Collection in the electronic form.

6. CONCLUSION

The collection of solved problems of general astronomy by Vojislav Mišković has, no doubt, had an important influence on the process of teaching astronomy at the Belgrade University. The first part was published and for this reason it has been available to the students. The second one has never been published, it has remained in the form of a manuscript only. Its straightforward use in the process of teaching began only when Prof. Simovljević gave the original copy to the author. Bearing in mind the fact that problems of this kind have been missing in the textbook literature in Serbian, my decision has been that the complete collection should be published in an electronic form. The digitized version can be found in the Virtual Library of the Faculty of Mathematics at http://elib.matf.bg.ac.yu:8080/virlib/.

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57 Život i delo profesora Dr Jovana Simovljevića

Life and work of Professor Jovan Simovljević
N. Pejović
Publ. Astron. Druš. Rudjer Bošković, Beograd
Vol. 8, pp. 377-389, 2009
Proc. Conf. "Razvoj astronomije kod Srba"
Beograd, 18-22 april, 2008
(M. Dimitrijevićc, ed).

This article presents scientific and personal biography of professor Jovan Simovljević (1929-2007), one of the most prominent Serbian astronomers. Professor Simovljević contributed significantly to the mathematical and computing methods for the ephemeris computation, as well as for computing total Solar eclipses. As the professor of the Department for astronomy of the Faculty of mathematics of the Belgrade University, he introduced the new subject in undergraduate studies, the theoretical astronomy. Professor Simovljevic was a very educated person, and he possessed encyclopedic knowledge in many areas. He was speaking several languages. Many generations of Serbian astronomers in the second half of the twentieth century were his students.

Зборник радова конференције "Развој астрономије код Срба V"
Београд, 18-22. април 2008,
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 8, 2009, 377-389

ЖИВОТ И ДЕЛО ПРОФЕСОРА ДР ЈОВАНА СИМОВЉЕВИЋА (1929-2007)

НАДЕЖДА ПЕЈОВИЋ

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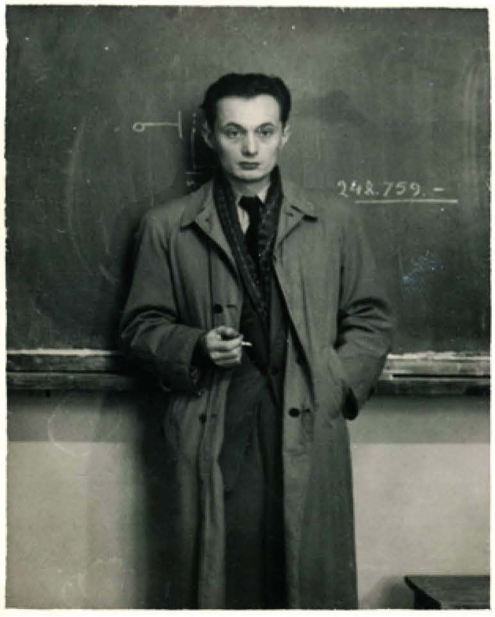
Резиме. Професор Јован Симовљевић припада најужем кругу истакнутих српских астронома. Значајно је унапредио теоријске и рачунске методе ефемеридског рачуна као и елемената тоталног помрачења Сунца. Као професор Београдског универзитета и Катедре за астрономију Математичког факултета увео је нов предмет *Теоријска астрономија*. Симовљевић је био врхунски интелектуалац и поседовао је енциклопедијско знање из многих других области. Генерације наших астронома из друге половине 20. века су биле његови студенти. Професор Симовљевић преминуо је у Београду, 19. октобра 2007. У овом чланку изложићемо укратко живот и дела професора Смовљевића.

БИОГРАФИЈА ПРОФЕСОРА СИМОВЉЕВИЋА



Професор др Јован Симовљевић је рођен 26. јула 1929. године у Шиду, од оца Лазара и мајке Данице, рођене Ковјанић. Основну школу је учио у Земуну и Новом Саду, а гимназију у Новом Саду где је матурирао 1948. Исте године се уписао на астрономску групу Природно-математичког факултета у Београду, где је дипломирао 1952. године. За професора приправника средње школе постављен је 1954. и истовремено додељен на рад тадашњој Катедри за механику и астрономију Природно-математичког факултета. За асистента за астрономију изабран је 1956. при истој Катедри. Три месеца током 1961/62. године боравио је у Варшави на

специјализацији код професора Фелицјана Кемпинског. Докторску дисертацију *Генерализација векторских елемената Кеплерова кретања* одбранио је 1963. на Природно-математичком факултету у Београду. У звање доцента за предмет *Теоријска астрономија* изабран је на истом Факултету 1964, за ванредног професора 1971, а за редовног професора 1980. године. Пензионисао се 1. јануара 1995. године.

Професор Симовљевић био је наш угледан астроном и изванредан професор. Научно-истраживачки рад др Јована Симовљевића је из области класичне теорије планетског кретања, а нарочито кретања планетоида и комета. Објавио је мноштво научних и стручних радова. Предавао је *Теоријску астрономију* студентима 4. године. Био је изврстан предавач, излагао је занимљиво и систематски и јасно уводио нове појмове. На занимљив и прегледан начин написао је универзитетски уџбеник *Основе теоријске астрономије* (Грађевинска књига, Београд, 1977). Генерације астронома училе су по овом уџбенику нумеричко интегрaљење диференцијалних једначина Кеплерова кретања, рачун ефемерида малих планета и комета, израчунавање орбита малих планета и комета, прорачун поправки орбита и одређивање специјалних поремећаја.

Поред знатног ангажовања у научно-истраживачком раду и настави, професор Симовљевић доста времена посветио је и другим активностима. Био је члан Управе, Већа и Скупштине Природно-математичког факултета (ПМФ), члан Савета Одсека за математичке, механичке и астрономске науке истог Факултта, затим члан Савета и члан Научног већа Астрономске опсерваторије у Београду и члан Савета Сеизмолошког завода СР Србије. Био је шеф Катедре за астрономију и небеску механику ПМФ-а и члан Међународне астрономске уније. Такође, био је члан Уређивачког одбора и Издавачког савета часописа *Publications of the Department of Astronomy*, који је издавала Катедра за астрономију ПМФ-а. Један је од оснивача Астрономског друштва *Руђер Бошковић*.

Поред свега, професор Симовљевић био је врхунски интелектуалац широког интересовања који се поред астрономије занимао и за многе друге области. Имао је енциклопедијско знање не само из астрономије, већ и из математике, историје и археологије. Био је познавалац више страних и старих језика, грчког и латинског. Посебно је неговао српски језик, и српско ћирилично писмо. Професор Симовљевић остаће запамћен у српској астрономији као драг професор који је значајно допринео развоју наше астрономије у сваком погледу, научном, стручном и наставном.

ПРОФЕСОР СИМОВЉЕВИЋ И КАТЕДРА ЗА АСТРОНОМИЈУ

Симовљевић је у књигама *Тридесет година Природно-математичког факултета Универзитета у Београду* (Београд, 1980) и у *Споменици 125 година Математичког факултета* (Београд, 1998) дао историјски осврт на развој астрономије код Срба у периоду до 1977. Период 1978-1998 у истој

Споменици представила је проф. јелена Милоградов-Турин. Ту налазимо низ занимљивих података.

Из периода до 1947. сазнајемо да је Коста Алковић (1836-1909), професор физике и механике Велике школе своје ђаке упознавао и са астрономијом. Његов ученик Милан Недељковић (1857-1950) постао је први професор астрономије и метеорологије на Катедри за астрономију и метеорологију основане Законом о изменама и допунама устројства Велике школе 1880. Милан Недељковић је оснивач прве привремене Астрономско-метеоролошке опсерваторије Велике школе која је почела са радом 1887. на Врачару. Још један наставник Велике школе, потом Универзитета је Ђорђе Станојевић (1858.-1921), који је написао прве научне радове из астрономије код Срба као и једну од првих књига из астрономије *Звездано небо независне Србије* (Београд, 1882). Универзитет је основан 1905. а 1909. је позивом изабран за професора примењене математике на Филозофском факултету др Милутин Миланковић (1879-1958), који је радио као грађ. инжењер у Бечу. Он је први започео наставу из небеске механике. Академик Милутин Миланковић је био научник светског гласа. О њему се много писало. Штампана су његова сабрана дела (Завод за издавање уџбеника у Београду, 1997). По Миланковићу име носи једна мала планета, кратер на Месецу и кратер на Марсу. После пензионисања проф. Недељковића за професора астрономије позивом је изабран 1926. др Војислав Мишковић (1892-1976), који је радио као астроном у Ници. Предавао је астрономију на новооснованој Катедри за теоријску и практичну астрономију на Филозофском факултету и био оснивач и дугогодишњи управник Астрономске опсерваторије на Звездари. Са доласком Мишковића почиње развој астрономије код Срба и у наставном и у научном погледу.

Други период 1947-1977 био је период 30 година Природно-математичког факултета. На развој астрономије у овом периоду свој "печат је оставио" проф. Симовљевић. Наиме, 1947. основан је Природно-математички факултет издвајањем из заједничког Филозофског факултета у посебан. Астрономија је постала друга студијска група, прва је била математика а трећа физика. Убрзо се небеска механика и механика издвајају из примењене математике и настаје Катедра за механику и астрономију. На ову Катедру су 1954. постављени први асистенти за механику и астрономију: Радмило Ђорђевић, Јован Симовљевић и Јован Лазовић. Тако су Симовљевић и Лазовић започели прве вежбе из астрономских предмета као асистенти проф. Мишковића. Професори Катедре за механику и астрономију тада су били М. Миланковић, В. Мишковић, А. Билимовић и Т. Анђелић.

Катедра за механику и астрономију се 1962. поделила на две: Катедру за астрономију и Катедру за механику. Симовљевић је цео свој радни век провео на Катедри за астрономију, до пензионисања 1995. На Катедри за астрономију са Симовљевићем су радили следећи професори: Јован Лазовић, Захарије Бркић, Иван Атанасијевић, Бранислав Шеварлић, Василије Оскањан, Јелена Милоградов-Турин, Мирјана Вукићевић-Карабин, Драгутин

Ђуровић и краће време Миливоје Ракић. Од асистената на Катедри су дуже или краће радили: Златко Ћатовић, Милан Вулетић, Предраг Пуношевац, Јелена Петровић, Катарина Ковач и Никола Витас. Године 1994. основан је Математички факултет издвајањем из Природно-математичког факултета. Катедру за астрономију Математичког факултета сада 2008. чине пет професора (Трајче Ангелов, Мике Кузманоски, Стево Шеган, Надежда Пејовић и Олга Атанацковић), два доцента (Дејан Урошевић и Анђелка Ковачевић) и три асистента (Драгана Илић, Бојан Арбутина и Душан Онић). Нажалост, Катедра за механику од 2007. нема ни професора ни студената.



Слика 1: Професор Симовљевић са колегама (с десна): Јован Лазовић, Јован Симовљевић, Љубиша Митић.

За оне који су га ближе познавали, нарочито за астрономе и механичаре, професор Симовљевић био је предусретљив, непосредан и добронамеран. Колико нам је познато, имао је миран живот и за већину колега са Факултета деловао је повучено, скромно и наизглед одсутно. Ипак, понекад се емотивно односио према неким догађајима, па је умео и да плане. Али разлоге за своје жестоке наступе умео је брзо да заборави. Остала је запамћена следећа Симовљевићева реченица. Једном приликом, 80-их година, један наставник је на НН Већу Математичког факултета упутио јавну критику свим редовним професорима, која се, пре свега, односила на математичаре. Професор Симовљевић је из неког разлога, само њему знаном, устао и кратко рекао: "Ко мисли да ја лажем нека са упаљеном свећом три пута обиђе Саборну цркву" (дакле, нека се покаје). На Већу је

настао тајац, а професор Симовљевић је изашао. На Катедри за астрономију највише је сарађивао са проф. Јованом Лазовићем, дружио се са професорима Катедре за механику. Заједно је студирао и највише се, до последњег дана, дружио са професором математике на Рударско-геолошком факултету Драгомиром Симеуновићем - Питагором.

ДОКТОРСКА ДИСЕРТАЦИЈА ЈОВАНА СИМОВЉЕВИЋА

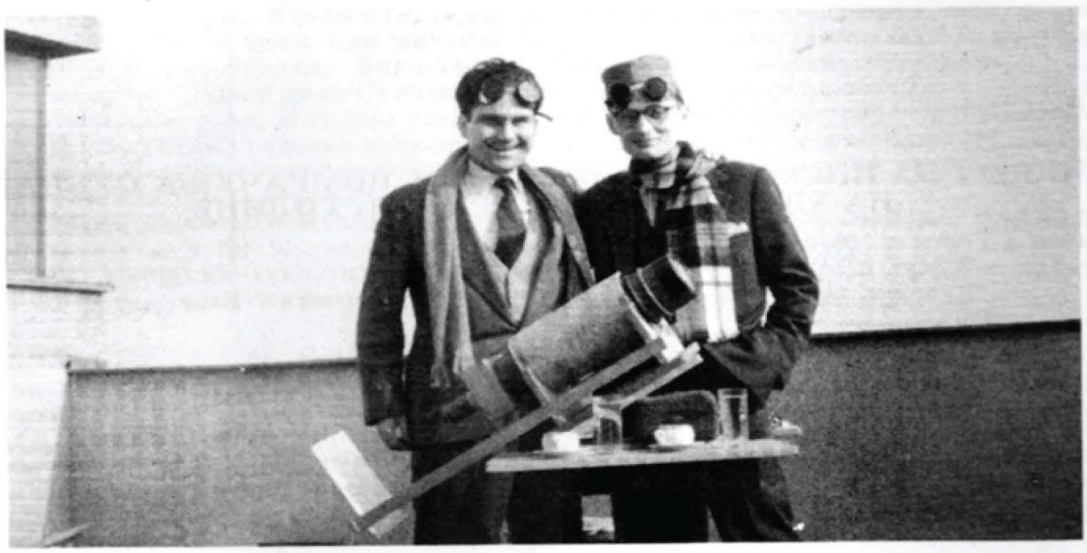
Јован Симовљевић је у својој докторској дисертацији *Генерализација векторских елемената Кеплерова кретања* увео појам векторског елемента планетског кретања, без обзира на њихово геометријско или кинематичко тумачење. Главна анализа овог појма односи се на случај поремећеног кретања, користећи диференцијалне једначине општег оскулационог елемента. Испитује дејство појединих компонената поремећајног убрзања у зависности од жељеног облика диференцијалне једначине елемента кретања. Тиме су на јединствен начин испитани сви до тада коришћени векторски елементи и показан је поступак формирања нових, који само у изузетним случајевима могу да буду погоднији за неке сврхе од већ познатих. Руководилац ове дисертације био је професор Варшавске политехнике Фелицјан Кемпински. Докторску дисертацију Симовљевић је одбранио 1963 на Приодно-математичком факултету у Београду. Била је то у оно време по обиму најмања докторска дисертација на Београдском универзитету. Симовљевићева докторска дисертација је дигитализована као део пројекта архивирања свих српских докторских дисертација са матемтичким, и астрономским садржајем штампаних у прошлости. Дигитална копија ове дисертације налази се на интернет страници Математичког факултета у оквиру Виртуелне библиотеке Националног центра за дигитализацију на адреси: http://elib.matf.bg.ac.yu:8080/virlib/.

НАУЧНИ РАД ПРОФЕСОРА СИМОВЉЕВИЋА

Радови Јована Симовљевића могу се разврстати у три групе. У првој групи су радови посвећени општим питањима оскулационих елемената планетског кретања и функција времена. Симовљевић је дао општи начин приказивања и њихове особине, чиме је искључио могућност увођења неких нових елемената или параметара, који би изгледали бољи од већ познатих у неким применама, како је то раније било. У радовима из ове групе Симовљевић је увек водио рачуна о стварним применама теоријске астрономије, о што бољим и применљивијим решењима конкретних задатака, најчешће у сложеним рачунима специјалних поремећаја.

Другу групу Симовљевићевих радова сачињавају они који се односе на различита питања проблематике проксимитета планетоида. За то су му давали подстрек бројни радови Ј. П. Лазовића и М. Кузманоског из ове области. Симовљевић је веома детаљно испитао поремећаје које изазива мало тело на малом растојању, но остајући стално у реалним границама

групе познатих планетоида, као и тадашњих могућности технике посматрања ових небеских тела. Последњим радом дао је и нов прилаз рачуну поремећаја при планетоидским проксимитетима.



Слика 4. Сима и Јоца на посматралишту задовољни после обављеног посла и изванредног доживљаја. Ту је и кафица. Сима је са поносном српском шајкачом, што је тада била реткост и храброст.

Слика 2: (Фотографија са коментаром преузета из *Васионе*, бр. 2-3, 2005).

Радови о Сунчевим помрачењима чине трећу групу. Ова колекција радова представља својеврсну монографију о појавама помрачења за последњих хиљаду година у нашим крајевима. Такви спискови и коментари о централним Сунчевим помрачењима, са основним астрономским подацима о њима, одавно су већ урађени за многе земље, за неке још у 19. веку. Симовљевић је то урадио у својим радовима и за нашу земљу у прошлом веку.

Занимљив је рад бр. 5 у којем је приказан рад чехословачког археолога Емануела Прохаске (Prochazka, 1954) везан за датирање. Полазећи од овог рада, Симовљевић користи астрономске методе у датирању гробова преисторијских култура. Гробови су у овом случају оријентисани према главним тачкама хоризонта, одређеним Сунчевим излазима или залазима. Датирање је засновано на чињеници да се услед прецесије мења изглед звезданог неба у току векова над хоризонтом једног одређеног места.

У раду бр. 4 проф. Симовљевић је дао оригиналан рачун за основне елементе потпуног помрачења Сунца у Југославији 15.02.1961. Одредио је географске координате трију главних линија појаса тоталитета у нашој земљи, као и трајање појаве и висине Сунца на средњој линији појаса тоталитета. Затим је већ 1965. у раду бр. 9, израчунао астрономске податке о другом, истовремено и последњем тоталном помрачењу Сунца 11.08.1999. у



Слика 3:

нашој земљи у претходном столећу. Раније се постављало питање да ли ће се ова појава моћи посматрати из наших крајева. Симовљевић је израчунао да ће се помрачење видети као тотално из мале области североисточне Војводине. Нашао је да ће најдуже трајање тоталитета бити око 100 секунди. Вредност овог прорачуна је велика, јер се у то време рачунари још увек нису користили у ове сврхе, њихова примена је тек долазила. Сведоци смо да је Симовљевићев прорачун био одличан, и да су експедиције из целе земље похрлиле 11.08.1999. да из североисточне Војводине посматрају ову појаву, један од најлепших призора који се може видети на небу. Напоменимо да се ова лепа појава дешава само два месеца након престанка бесомучног бомбардовања наше земље од стране НАТО алијансе. Сви смо већ били навикнути да гледамо у небо које су немилосрдно парали непријатељски бомбардери. Овога пута очи су биле упрте на прелеп и величанствен призор. У једној од експедиција са С. Шеганом била је и ауторка овог чланка у Ђали на крајњем северу земље. С. Шеган је за ово помрачење поново израчунао све неопходне податке [5], али коришћењем рачунара. Сви астрономи фасцинирани су сваким помрачењем Сунца попут проф. Симовљевића. Тако

је ауторка овог чланка поново посматрала ову велелепну појаву. 29. марта 2006. из турског града Сиде-а.

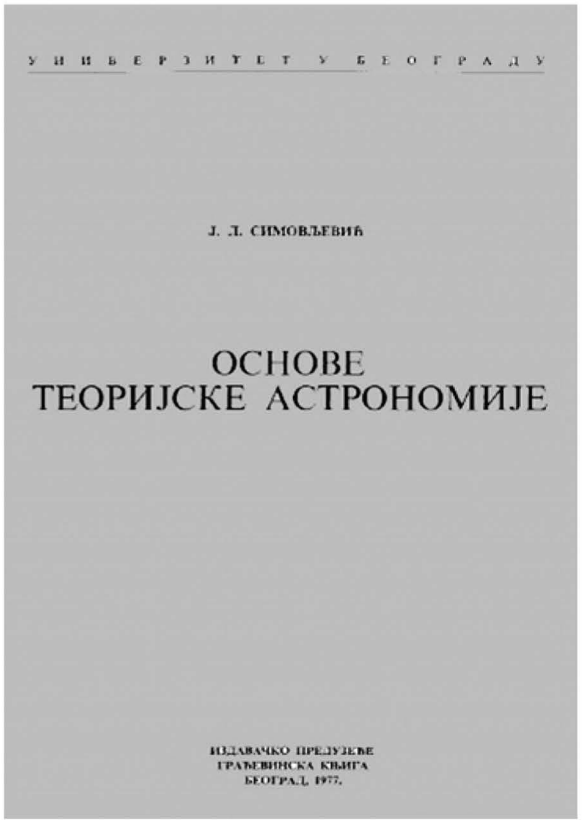
Даље у раду бр. 10 проф. Симовљевић поставио је циљ да што детаљније испита садржај две кратке вести о помрачењима Сунца, записане у нашем познатом историјском извору из средине 13. века, хроници сплитског архиђакона Томе. Симовљевић је израчунао све потребне астрономске елементе за оба помрачења, она су се догодила 3. јуна 1239. и 6. октобра 1241. Његов прорачун био је са већом тачношћу од рачуна његових претходника. На пример, дошао је до закључка да се друго помрачење у Сплиту видело само као делимично, истина веома велике фазе.

Што се тиче стручних радова Јована Симовљевића они су углавном популарне природе. Ипак из те групе издвајамо рад под бројем 3. у којем има оригинални допринос, као и рад број 5. са врло детаљним подацима о току потпуног помрачења Сунца у нашој земљи 1961. Најпотпуније податке о овој појави израчунао је Симовљевић и објављени су као прилог монографији В.В. Мишковића "*Сунчево помрачење од 15 фебруара 1961.*" (Посебно издање САН CCCXLIII, Прир.-мат. 27, 1960). Поред ових стручних радова Симовљевић је објавио и низ научно-популарних чланака. Писао је чланке за дневну штампу, држао предавања на Коларчевом народном универзитету, радију и телевизији.

ОСНОВИ ТЕОРИЈСКЕ АСТРОНОМИЈЕ ЈОВАНА СИМОВЉЕВИЋА

Уџбеник *Основи теоријске астрономије* штампан 1977. посвећен је успомени професора Војислава Мишковића (1892-1976). Написан је на 200 страница и састоји се од *Увода*, пет глава и *Додатка*. У *Уводу* је дат приказ одређивања путања малих планета и комета, јединице за масу, дужину и време, Гаусов дан, координатни системи и координате небеских тела. У првој глави *Кеплерово кретање* наведени су интеграли диференцијалне једначине Келерова кретања, Кеплерове путање (круг, елипса, парабола), Елементи кретања (класични астрономски и векторски као и прелаз са једних на друге), почетни услови, систем почетних услова, однос површине сектора и троугла (Гаусов и Хансенов поступак) и тетива параболичке путање. Друга глава *Рачун ефемерида* даје ефемериде малих планета и комета, поступак састављања ефемерида, одређивање датума опозиције, израчунавање положаја, састављање ефемериде нумеричким интеграњем диференцијалне једначине кретања и поређење рачунатих и посматраних положаја. У трећој глави *Рачун орбита* изложено је израчунавање непоремећених путања малих планета и комета, израчунавање кружне путање, израчунавање елиптичке путање, Гаус-Енкеова метода, Laplace-Leuschner-ова метода, Väisälä-ова метода и израчунавање параболичке путање. Четврта глава *Рачун поправки путања* даје поправљање Кеплерових путања, варијацију путањских елемената и поправку почетних услова кретања. У петој глави *Рачун специјалних поремећаја* дато је стварно

кретање малих планета и комета, нумеричко интеграљење, рачун специјалних поремећаја правоуглих координата, рачун специјалних поремећаја векторских елемената, диференцијалне једначине, рачун специјалних поремећаја скаларних елемената, посредне диференцијалне једначине и поремећајно убрзање. Додатак *Нумеричко интеграљење диференцијалних једначина кретања небеских тела* даје интерполацију, нумеричко диференцирање и нумеричко интеграљење.





Слика 4:

Овај Уџбеник је намењен студентима астрономије Математичког факултета за предмет *Теоријска астрономија*. Књига је методички лепо написана, градиво се постепено и јасно уводи. Занимљива је и прегледна. Посебно се истиче једноставност извођења сложених образаца ефемеридског рачуна. Књига је пуна математичких израза, једначина и метода. На први поглед подсећа на математичке уџбенике пуне симбола, вектора, диференцијалних једначина и са по мало пратећег текста. Уствари, теоријска астрономија представља примену математике за решавање конкретних сложених проблема планетског кретања. Занимљиво је да у књизи нема ни једне слике, на пример о координатним системима је писано без графичког представљања. Тамо где је по нека слика требала да буде, толико су сликовито и јасно описани појмови, да се могло и без слика. У књизи нема урађених нумеричких примера и задатака, с тога јој је неопходна пратећа

збирка. Али, на крају већине одељака дат је *Преглед образаца* за ефективно нумеричко рачунање, што је од велике помоћи при решавању појединих проблема теоријске астрономије. Генерације студената астрономије училе су по овом занимљивом уџбенику. Како су штампани примерци одавно распродати, одлучили смо се да дигитализујемо ову изврсну књигу и учинимо је доступном следећим генерацијама. Дигитална копија универзитетског уџбеник Јована Симовљевића *Основи теоријске астрономије* налази се у Виртуелној библиотеци Националног центра за дигитализацију (Virtual library, http://elib.matf.bg.ac.yu:8080/virlib/)

КЊИЖИЦА *ПОТПУНО ПОМРАЧЕЊЕ СУНЦА 15.2.1961*

Књижицу *Потпуно помрачење Сунца 15.2.1961, Упутство за посматрање* издало је астрономско друштво "Руђер Бошковић" 1960. Књижица има 24 странице и два прилога:

1) карту ФНР Југославије са зоном тоталитета,

2) картицу коју посматрачи треба да попуне и пошаљу на адресу Друштва.

Књижица је посвећена потпуном помрачењу Сунца 15.2.1961. Њу чине чланци Радована Данића, Ненада Јанковића, Пере Ђурковића и Јована Симовљевића. Јован Симовљевић је у овој књижици представио таблично сређене податке о тренуцима и положајним угловима контаката за петнаест градова ФНР Југославије. Књижица је била драгоцено упутство за посматрање једне од најлепших појава на небу, тотално помрачење Сунца.

ЗАКЉУЧАК

Професор Симвљевић волео је свој позив и био је посвећен астрономској науци и педагошком раду на Универзитету. О томе сведоче његови бројни научни и стручни радови. Био је човек широких интересовања и богатог општег образовања. Поред свега, био је свеобухватан, темељан и систематичан. Занимао се за многе теме и поред своје науке био је изврстан познавалац историје и археологије, религије, старих језика, нумизматике и многих других необичних и другим људима мало познатих тема. Значајно је унапредио српску астрономију. Генерације његових студената сачуваће са пијететом сећање на професора Јована Симовљевића.

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Захвалница

Користим прилику да се захвалим организатору др Милану С. Димитријевићу на позиву за учешће на конференцији *Развој астрономије код Срба V*. Захваљујући томе, овај текст је написан.

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LIFE AND WORK OF PROFESSOR JOVAN SIMOVLJEVIĆ (1929-2007)

Professor Jovan Simovljević belongs to the narrow circle of the most prominent Serbian astronomers. He contributed significantly to the mathematical and computing methods for the ephemeris computation, as well as for computing total Solar eclipses. As the professor of the Department for astronomy of the Faculty of mathematics of the Belgrade University, he introduced the new subject in undergraduate studies, the theoretical astronomy. Professor Simovljević was a very educated person, he possessed encyclopedic knowledge in many areas, and he was speaking several languages. Many generations of Serbian astronomers in the second half of the twentieth century were his students. Professor Simovljević died in Belgrade on 19. October 2007. In this article, his scientific and personal biography is presented.

58 Digitization of collected works of Jovan Simovljević

N. Pejović
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This article is a translation of of the review "Život i delo profesora Dr Jovana Simovljevića" (Life and work of Professor Jovan Simovljević) with some additions.

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DIGITIZATION OF COLLECTED WORKS OF JOVAN SIMOVLJEVIĆ

Abstract. Jovan Simovljević (1929–2007) was Professor at the University of Belgrade, at the Astronomy Department of the Faculty of Mathematics. Prof. Simovljević belongs to the closest circle of distinguished Serbian astronomers. He gave a significant improvement in the theoretical computational methods for the complicated calculations of ephemerides for the Solar-System bodies.. In the sixties of the last century he introduced at the Belgrade University a new subject of *Theoretical Astronomy.* To this subject he wrote textbook *Fundaments of Theoretical Astronomy (Основи теоријске астрономије)*. The digitized version of this book is available in the Virtual library of the National Digitisation Centre (http://elib.matf.bg.ac.yu:8080/virlib). In this article the author also writes about other works of Prof. Simovljević: PhD thesis, collected scientific papers and coauthor booklet *Total Solar Eclipse of February 15, 1961* (*Потпуно помрачење Сунца, 15. фебруар 196*). All these works are also available at the Virtual Library.

Key words: Digitization, Jovan Simovljević, theoretical astronomy, ephemerides

Introduction

In this article collected works of Jovan Simovljević, a distinguished Serbian astronomer from the second half of the XX century, are presented. All the works have been digitized and are available at the Virtual Library of the National Digitisation Centre (http://elib.matf.bg.ac.yu:8080/virlib). In addition, his biography is also shortly presented.

Biography of Professor Simovljević

Prof. Dr Jovan Simovljević was born on July 26, 1929 at Šid. His father's name is Lazar and his mother's name is Danica, née Kovjanić. His primary-school education took place in Zemun and Novi Sad, that in the secondary school (gymnasium) in Novi Sad where he took degree in 1948. In that same year he enrolled at the Group of Astronomy of the Belgrade Faculty of Sciences from which he took degree in 1952. In 1954 he obtained a teacher's position at a secondary school and at the same time joined the Department of Mechanics and Astronomy of the Faculty of Sciences. In 1956 he became assistant at the same Department. During the school year of 1961/1962 he spent three months in Warsaw working under the supervision of Prof. Felician Kempinski. He obtained his PhD in 1963 at the Faculty of Sciences in Belgrade, the title is A *Generalization of Vector Elements for the Keplerian Motion(Генерализација векторских елемената Кеплерова кретања).* He became assistant-professor at the same Faculty in 1964, the subject was *Theoretical Astronomy (Теоријска астрономија);* associate professor he became in 1971 and full professor in 1980. He was retired on January 1, 1995.

Professor Simovljević was our astronomer of high reputation and an excellent teacher. His research field concerned the classical theory of planetary motion, especially the motion of minor planets and comets. He published many papers. He taught *Theoretical Astronomy* to the



university students of the fourth year. He was an excellent teacher, he gave interesting presentations systematically and introduced clearly new notions. In an interesting and illustrative way he wrote the university textbook *Fundaments of Theoretical Astronomy* (*Основе теоријске астрономије) published* by Gradjevinska knjiga, Belgrade 1977. Many generations of astronomers learnt in this textbook how to integrate the differential equations of the Keplerian motion numerically, to calculate the ephemerides for minor planets and comets, to calculate the orbits of minor planets and comets, calculation of corrections to orbits and determination of special perturbations.

◄ **Professor Jovan Simovljević**

In addition to his engagement in the research and teaching Prof. Simovljević devoted a lot of his time to other activities. He was a member of the Leadership, Council and Assembly of the Faculty of Sciences, member in the Council of the Department of Mathematics, Mechanics and Astronomy at the same Faculty, then member of the Council and Scientific Council of the Astronomical Observatory in Belgrade and member in the Council of the Seismological Institute of Serbia. He was the head of the Chair of Astronomy and Celestial Mechanics at the Faculty of Sciences and a member of the International Astronomical Union. He was also member of the Editorial Board and Publishing Council of journal Publications *of the Department of Astronomy* published by the Astronomy Department of the Faculty of Sciences. He is among the founders of Astronomical Society *Ruđer Bošković.*

In addition to all of this Prof. Simovljević was a nice intellectual with an ample interest including except astronomy also many other fields. He possessed a knowledge of encyclopaedic level, not only in astronomy, but also in mathematics, history and archeology. He knew several foreign and ancient languages, Greek and Latin. He was especially keen in the use of the Serbian language where he clearly preferred the Cyrillic letters. Prof. Simovljević will be remembered in Serbian astronomy as a beloved teacher who gave a significant contribution to the development of our astronomy in every respect, scientific, technical and pedagogical.

Professor Simovljević and Astronomy Department

In the books *Thirty Years of the Faculty of Sciences of the Belgrade University (Тридесет година Природно-математичког факултета Универзитета у Београду)*, published in Belgrade in 1980, and *Memory 125 Years of the Faculty of Mathematics* (*Споменица 125 година Математичког факултета*), published in Belgrade in 1998, Simovljević gave a short historical review concerning the development of astronomy among Serbs by 1977. It is curious to note that these articles, each having about thirty pages, were written without a single error. The period between 1978 and 1998 in the same *Memory* was treated by Prof. Jelena Milogradov-Turin. There one finds many interesting data.

For the case of the period before 1947 we can learn that Kosta Alković (1836–1909), who taught physics and mechanics at the Grand School, also presented astronomical topics to

his students. One of them, Milan Nedeljković (1857–1950), was the first to teach astronomy and meteorology in the framework of the Chair of Astronomy and Meteorology founded according to the Law of Amendments to the Statute of the Grand School from 1880. Milan Nedeljković is the founder of the first temporary Observatory for astronomy and meteorology of the Grand School which started its activity in 1887 in Belgrade (its part Vračar). Đorđe Stanojević (1858–1921), the first among Serbs who wrote scientific papers in astronomy, as well as one of the first astronomical books *Starry Sky of the independent Serbia (Звездано небо независне Србије)* published in Belgrade in 1882, also taught at the Grand School, and at the University afterwards. The University was founded in 1905 and in 1909 Dr Milutin Milanković (1879–1958), who had worked as a civil engineer in Vienna, was invited to join the Faculty of Philosophy to teach applied mathematics. He was the first to teach celestial mechanics. Academician Milutin Milanković was a scientist of world reputation. A lot of papers have been written about him. His collected works have been published (publisher Zavod za izdavanje udžbenika Beograd, in 1997). After Milanković have been named a minor planet, a Moon crater and a Mars crater. After Prof. Nedeljković had been retired, Dr Vojislav Mišković (1892–1976), who had spent some time in Nice working as an astronomer, was invited in 1926 to teach astronomy. He taught astronomy in the framework of the newly founded Chair of Theoretical and Practical Astronomy at the Faculty of Philosophy and was the founder and Director of the new Astronomical Observatory in Belgrade (situated in Zvezdara) for many years. With his advent the development of astronomy among Serbs begins in the aspects of teaching and science.

The second period 1947–1977 was the period of thirty years of the Faculty of Sciences. Prof. Simovljević contributed to the development of astronomy in this period. Namely in 1947 the Faculty of Sciences was separated from that of Philosophy: astronomy became the second group of subjects, the first was mathematics and the third physics. Soon celestial mechanics and mechanics were separated from applied mathematics and the Department of Mechanics and Astronomy was founded. At this department the first assistants for mechanics and astronomy in 1954 became: Radmilo Đorđević, Jovan Simovljević and Jovan Lazović. So the first exercises in astronomical subjects were started under Simovljević who was assistant of Professor Mišković. Then at the Department of Mechanics and Astronomy the lectures were presented by M. Milanković, V. Mišković, A. Bilimović and T. Anđelić.

The Department of Mechanics and Astronomy was split in 1962 in two departments: that of astronomy and that of mechanics. Simovljević spent the rest of his career at the Astronomy Department, till the retirement in 1995. At the Astronomy Department in addition to Simovljević the lectures were given by the following persons: Jovan Lazović, Zaharije Brkić, Ivan Atanasijević, Branislav Ševarlić, Vasilije Oskanjan, Jelena Milogradov-Turin, Mirjana Vukićević-Karabin, Dragutin Đurović and during a short time Milivoje Rakić. As assistants at the Department during a longer or shorter time were engaged: Zlatko Ćatović, Milan Vuletić, Predrag Punoševac, Jelena Petrović, Katarina Kovač and Nikola Vitas. In 1994 the Faculty of Sciences ceased to exist, to be succeeded by several faculties, among them the Faculty of Mathematics. At the Department of Astronomy of the Faculty of Mathematics there are now (in 2008) two full professors (Trajko Angelov and Mike Kuzmanoski), three associate professors (Stevo Šegan, Nadežda Pejović and Olga Atanacković), two assistant-professors (Dejan Urošević and Anđelka Kovačević) and three assistants (Dragana Ilić, Bojan Arbutina and Dušan Onić).

Unfortunately, The Department of Mechanics is from 2007 without both teaching staff and students.

To those who knew him more closely, especially to astronomers and specialists in mechanics, Professor Simovljević was kind, straightforward and well-meaning. As far as known to us, his life was silent and for a majority of colleagues the impression was a reserved, modest and, at first glance, isolated person. Nevertheless, sometimes he reacted emotively knowing to say rather strong words. The reasons for his strong reactions he knew to forget quickly. The following sentence of his has been remembered. Once, in the eighties of the XX century during the Session of the Scientific and Teaching Council of the Faculty of Mathematics, one of the professors criticized all full professors which was, above all, directed against mathematicians. For a reason, known to him only, Professor Simovljević asked to speak saying just this: Let anyone, who thinks that I am lying, go with a burnt candle three times around the Cathedral" (thus to ask penitence). A silence followed and Prof. Simovljević went out. At the Astronomy Department he was in the closest relationship with Jovan Lazović and among his companions were professors from the Mechanics Department. He had a close contact till the very last day with Dragomir Simeunović - Pitagora who taught mathematics at the Faculty of Mining and Geology and who was together with Simovljević in their student days.

Jovan Simovljević's PhD Thesis

Jovan Simovljević in his PhD thesis *A Generalization of the Vector Elements of the Keplerian Motion* (*Генерализација векторских елемената Кеплерова кретања*) introduced the notion of the vector element of the planetary motion, without regard to its geometric or kinematical interpretation. The main analysis of this notion concerns the case of the perturbed motion using the differential equations of the general osculating element. The author examines the action of some components of the perturbational acceleration depending on the desired form of the differential equation of the motion element. Through this in a unique way one examines all vector elements used by that time and a procedure how to form the new ones was demonstrated. These new elements can be more suitable for some purposes than the already known ones in exceptional cases only. The supervisor of the thesis was Felician Kempinski, a professor at the Warsaw Polytechnic. The thesis was defended in 1963 in Belgrade at the Faculty of Sciences. At that time this thesis according to its size was the smallest one at the University of Belgrade. Simovljević's PhD thesis has been digitised as part of the project aimed at archiving all Serbian PhD theses with mathematical or astronomical contents printed in the past.

The Scientific Activity of Prof. Simovljević

Professor Simovljević's legacy contains reprints of his scientific papers published by 1980 in the form of a book. The book has about 250 pages with 22 reprints ordered chronologically. The list of these, as well as of other papers, is enclosed to this article. It is curious to note that, except for the case of one paper, he is always the only author.

Jovan Simovljević's papers can be divided into three groups. The first group comprises the papers devoted to the general questions of osculating elements in the planet motion and the function of time. Simovljević gave a general way of presentation and their properties by which he excluded the possibility of introducing new elements or parameters which would seem as better than those already known in some applications, as had been the case before. In the papers of this group Simovljević bore in mind the real applications of theoretical astronomy, the solutions of particular problems being as good and as applicable as possible, most frequently in the complicated calculations of special perturbations.



Слика 4. *Сима и Јоца на посматралишту задовољни после обављеног посла и изванредног доживљаја. Ту је и кафица. Сима је са поносном српском шајкачом, што је тада била реткост и храброст.*

Figure 1: The photograph with the caption is from journal "Vasiona" , No 2-3, 2005

The second group of Simovljević's papers is composed by those concerning various questions in the problems of proximity among minor planets. Here an impetus was given by numerous papers of J. P. Lazović and M. Kuzmanoski in this field. Simovljević examined in details the perturbations produced by a small body at a short distance, but always remaining within realistic limits containing the group of known minor planets and bearing in mind the capabilities of observational techniques of that time concerning these celestial bodies. In his last paper he also gave a new approach to the perturbation calculation for the case of the proximities between minor planets.

The papers dealing with solar eclipses belong to the third group. This collection of papers appears as a special monograph concerning the eclipse phenomenon for our territory during the last thousand years. Such lists and commentaries concerning the central solar eclipses containing the main data about them were formed long ago for many countries, in the case of some of them as early as in the XIX century. Simovljević did this for our country in the last century.

In what follows a short review concerning a few scientific papers written by Professor Simovljević will be given.

Of interest is Paper No 5 in which the work of archeologist Emanuel Prochazka [8] from Czechoslovakia concerning the dating is presented. Starting from this paper Simovljević uses astronomical methods for the purpose of dating the tombs of prehistoric cultures. The tombs in this case were oriented according to the main points of the horizon determined by the sunrise and sunset. The dating is based on the fact that due to the precession the appearance of the starry sky changes over centuries above the horizon of a given place.

In Paper No 4 Simovljević gave his original calculation for the basic elements of the total solar eclipse in Yugoslavia on February 15, 1961. He determined the geographic coordinates for the three main lines of the totality band in our country and also the duration of the phenomenon and the elevations of the Sun on the middle line of the totality band. Then as early as in 1965 in Paper No 9 he calculated the astronomical data concerning the second, also the last, total solar eclipse of August 11, 1999, in our country in the preceding century. This

calculation was preceded by the question of if this phenomenon would be visible from our territory. Simovljević determined that the eclipse would be total for a small area in northeastern Vojvodina. He found that the longest duration of the totality would be about 100 seconds. The importance of this calculation is high because at that time computers were still not used for such purposes, their application had to come only. We are witnesses that Simovljević's computation was excellent and that expeditions from the whole of the country clustered on August 11, 1999 in order to observe from the northeast of Vojvodina one of the most beautiful sights which can be seen in the sky. It should be noted that this nice phenomenon took place only two months after the cease of the heavy bombardment by the NATO Alliance. We all had already had the habit of gazing at the sky which had been ripped by the enemy bombers. This time our eyes saw a wonderful and magnificent sight. The present author participated in one of these expeditions with S. Šegan to Đala in the extreme north of Serbia. S. Šegan had calculated for this eclipse all the necessary data again [5] using a computer. All astronomers are fascinated by every solar eclipse like Professor Simovljević. So the present author observed this wonderful phenomenon again on March 29, 2006 from Turkish town of Side.

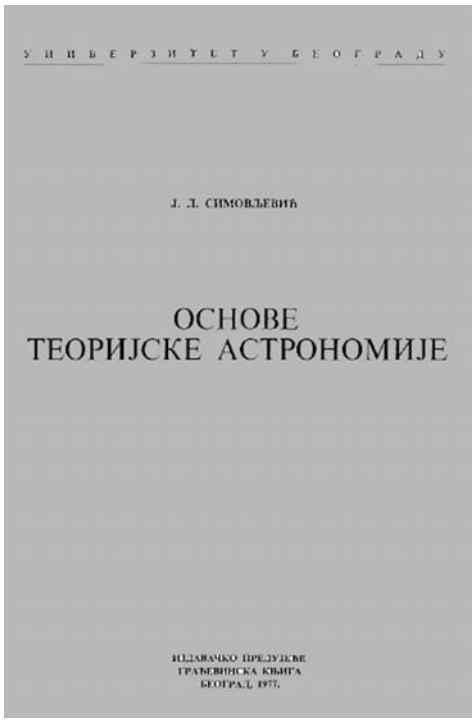
Further on in Paper No 10 Simovljević's intention was to examine in as many details as possible two short notes concerning solar eclipses which were registered in a known historical source of ours from the middle of the XIII century, the chronicle of Archdeacon Toma from Split. Simovljević calculated all the necessary astronomical elements for both eclipses. They took place on June 3, 1239 and October 6, 1241. His calculation was more accurate than those of his predecessors. For example, he concluded that the second eclipse was seen from Split as a partial one only, but having a very large phase.

As for other Simovljević's technical papers they are largely popular. Nevertheless, from this group we want to distinguish Paper No 3 in which he has an original contribution, as well as Paper No 5 with very detailed data concerning the course of the total eclipse in our country in 1961. The most complete data concerning this phenomenon were calculated by Simovljević and they were published as Appendix to monograph *The Total Solar Eclipse of February 15, 1961* ("*Сунчево помрачење од 15 фебруара 1961*") by V. V. Mišković (printed as Посебно издање САН CCCXLIII, Прир.-мат. 27, 1960) which has also been digitised. In addition to these technical papers Simovljević also published a number of articles in the field of science popularisation. He wrote articles for daily papers, delivered public lectures at the Popular University of Kolarac, popularised astronomy from radio and television.

Textbook *Fundaments of Theoretical Astronomy*

Printed in 1977 was dedicated to the memory of Prof. Vojislav Mišković (1892–1976). It occupies 200 pages and consists of *Introduction,* five chapters and *Appendix.* In *Introduction* the orbit determination for minor planets and comets is presented followed by units of mass, distance and time, Gaussian day, coordinate systems and coordinates of celestial bodies. In the first chapter *Keplerian Motion* (*Кеплерово кретање*) are given the integrals of the differential equation for the Keplerian motion, the Keplerian orbits (circle, ellipse, parabola), elements of motion (classical astronomical and vector) as well as the corresponding transformations), initial conditions, system of initial conditions, the ratio of the areas of sector and triangle (procedures of Gauss and Hansen) and the chord of a parabolic orbit. The second chapter *Ephemerides Calculation* (*Рачун ефемерида*) gives the ephemerides for minor planets and comets, the procedure of composing ephemerides, the determination of the

opposition date, position calculation and composing ephemerides by numerical integration of the differential equation of motion and the comparison of the calculated and observed positions. In the third chapter *Orbit Calculation* (*Рачун орбита*) one presents the calculation of unperturbed orbits for minor planets and comets, the calculation of a circular orbit, calculation of an elliptical orbit, the Gauss-Encke method, the Laplace-Leuschner method, Väisälä's method and the calculation of a parabolic orbit. The fourth chapter *Calculation of Orbit corrections* (*Рачун поправки путања*) gives the correcting of the Keplerian orbits, variation of orbital elements and the correction of the initial conditions of motion. In the fifth chapter *Calculation of special perturbations (Рачун специјалних поремећаја*) one presents the real motion of minor planets and comets, numerical integration, the calculation of special perturbations for the rectangular coordinates, calculation of special perturbations for vector elements, the differential equations, calculation of special perturbations for scalar elements, indirect differential equations and the perturbational acceleration. Appendix *Numerical Integration of Differential Equations for Motion of Celestial Bodies (Нумеричко интеграљење диференцијалних једначина кретања небеских тела*) deals with interpolation, numerical differentiation and numerical integration.



This textbook is provided for students who study astronomy at the Faculty of Mathematics for subject *Theoretical Astronomy*. From the methodological point of view the book was nicely written, the material is gradually and clearly introduced. It is interesting and illustrative. The simplicity of derivation of complicated formulae of the ephemeris calculus is specially underlined. The book is full of mathematical expressions, equations and methods. At first glance it resembles mathematical textbooks, full with symbols, vectors, differential equations followed by short texts. In fact theoretical astronomy is an application of mathematics for the purpose of solving particular complicated problems of planetary motion. It is curious to note that in the book there are no figures, for instance the coordinate systems are described without a single plot. At places where a figure should have been, the notions are described so picturesquely and clearly that a presentation without figures was possible. The book contains no examples or problems for solving so that it needs a collection of problems as its supplement. But at the end of the majority of chapters there is a *List of Formulae* (*Преглед образаца*) which serve for the purpose of effective numerical calculations and which is very useful in the solving of some problems of theoretical astronomy. Many generations of students of astronomy have learnt using this interesting textbook. Since all printed copies had been sold a long time ago, we decided to digitize this excellent book and make it available to the generations to come.

Booklet *Total Solar Eclipse of February 15, 1961*

The booklet *Total Solar Eclipse of February 15, 1961, Instruction how to observe (Потпуно помрачење Сунца 15.2.1961, Упутство за посматрање*) was published by astronomical society "Ruđer Bošković" in 1960. It has 24 pages and two appendices:

a map of the FPR of Yugoslavia with the zone of totality,
a small map which should be completed and sent to the address of the Society by observers.



The booklet was devoted to the total eclipse of the Sun of February 15, 1961. It is composed of articles written by Radovan Danić, Nenad Janković, Pero Đurković and Jovan Simovljević. Jovan Simovljević in this booklet presented the data concerning the moments and position angles of the contacts ordered as a table for 15 cities and towns of the FPR of Yugoslavia. The booklet appeared as a valuable instruction for observing one of the most beautiful celestial phenomena, total eclipse of the Sun.

Conclusion

Professor Simovljević liked his profession and was devoted to the science of astronomy and pedagogical work at the University. This is confirmed through numerous papers published by himself. He was a person of ample interests and rich general education. In addition to all, he was comprehensive, careful and systematic. He was interested in many themes and in addition to his science he knew excellently history and archeology, religion, ancient languages, numismatics and many other not usual and to other people poorly known themes. He improved significantly the Serbian astronomy. Generations of his students will preserve with respect the reminiscence of Prof. Jovan Simovljević.

Appendix: Selected Bibliography of Jovan Simovljević

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ДИГИТАЛИЗАЦИЈА ИЗАБРАНИХ РАДОВА ЈОВАНА СИМОВЉЕВИЋА

Професор Јован Симовљевић припада кругу најугледнијих српских астронома. Имао је значајне доприносе у математичким и рачунским методама за прорачун ефемерида, као и у прорачунима тоталних помрачења Сунца. Као професор Катедре sa астрономију Математичког факултета Универзитета у Београду увео је нови предмет на редовним студијама, *Теоријска астрономија.* Професор Симовљевић био је веома образована особа и поседовао је енциклопедијско знање из многих области и говорио је неколико језика. Професор Симовљевић умро је 19. Октобра 2007. У овом чланку представља се биографија професора Симовљевића и пишемо о књигама и научним радовима који су дигитализовани и налазе се у Виртуелној библиотеци Математичког факултета и Националног центра за дигитализацију, http://elib.matf.bg.ac.yu:8080/virlib. Посебно су представљене његове књиге *Основи теоријске астрономије* и *Потпуно помрачење Сунца 15-II-1961*.

Кључне речи: Дигитализација, Јован Симовљевић, теоријска астрономија, ефемериде

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59 Digitized books of Djordje Stanojević in the Virtual library of Faculty of mathematics

N. Pejović
NCD Review, vol. 17, pp. 32-38, 2010.

Djordje Stanojević was a respected electrical engineer, physicist and astronomer of the second half of the nineteenth and the beginning of the twentieth century. He taught applied physics at the Grand School and at the University of Belgrade after its foundation in 1905. He is responsible for the electrification of Belgrade after that Belgrade has become one of the first cities in Europe lightened by electricity. This article presents digital collection which consists of seven Stanojević books: Starry sky of independent of Serbia, From the science of light, Walk through the clouds, Incorrect celebration of the Resurrection in the Orthodox Church and the reform of the calendar, The universal energy and modern physics, Nikola Tesla and his discoveries and Industry of cold.

Преглед НЦД 17 (2010), 32–38

Nadežda Pejović
Faculty of Mathematics, Belgrade

DIGITIZED BOOKS OF ĐORĐE STANOJEVIĆ IN THE VIRTUAL LIBRARY OF FACULTY OF MATHEMATICS

Abstract. We present the books by Đorđe Stanojević that are digitized and are in the Virtual Library of the Faculty of Mathematics, University of Belgrade. Đorđe Stanojević was a respected electrical engineer, physicist and astronomer of the second half of the nineteenth and the beginning of the twentieth century. He taught applied physics at the Grand School and at the University of Belgrade after its foundation in 1905. He is responsible for the electrification of Belgrade after that Belgrade has become one of the first cities in Europe lightened by electricity.

This digital collection contains seven his books: *Starry sky of independent of Serbia, From the science of light, Walk through the clouds, Incorrect celebration of the Resurrection in the Orthodox Church and the reform of the calendar, The universal energy and modern physics, Nikola Tesla and his discoveries, Industry of cold.*

Virtual Library

Virtual Library of the Faculty of Mathematics http://elibrary.matf.bg.ac.rs has the largest database of digitized texts in Serbia which is open for public use. At this moment, it contains almost 800 books. Since April 2008 seven hundred books were uploaded into the Library. Important collections in this library are a collection of 360 doctoral theses in mathematical sciences from the great majority defended at the Faculty of Mathematics of the University of Belgrade and rare books from the 18th and 19 century. Most of these books are rare, some of them exist only in one copy and practically inaccessible to the public.

Short biography of Đorđe Stanojević

Đorđe Stanojević (1858–1921) was born in Negotin a town in Serbia. He finished Grand School in Belgrade, the forerunner of the Belgrade University. He then served as Grand School assistant to Professor Kosta Alković (1836–1909). After that, in 1883, he was a teacher at the First Belgrade Gymnasium. As a cadet of the Military Ministry during the period 1883-1887 he was sent to study and work in the most famous European astronomical and meteorological observatories (Potsdam, Hamburg, Meudon Observatory near Paris, Greenwich Observatory and Pulkovo Observatory near Saint-Petersburg). Due to his results he was invited by the Paris Observatory to participate in scientific research expedition to examine the Sun and in Petrovski where he participated in the study of total solar eclipses. Two years later he joined a France research expedition to Sahara in order to study the Sun and its thermal spectrum. At this time he published several scientific papers on astronomy in the editions of the Paris Academy of Sciences. These papers are the first scientific works in astrophysics among Serbs.



Đorđe Stanojević

Stanojević was a versatile, educated man with active interest in many fields of science and technology. His interest in physics was rather great. Upon return home he devoted himself to the study of modern physics. He became the lecturer in physics and engineering at the Military Academy. Also he taught applied physics and mechanics in High School and at the Belgrade University after establishing it in 1905. He is responsible for the construction of many hydropower plants in Serbia (Vučje, Negotin, Užice, Ivanjica and other places) and the electrification of Belgrade. Due to this effort of his, Belgrade became one of the first European capitals with full electric lighting. He died in Paris in 1921.

The Virtual Library contains seven digitized Stanojević' books: *Starry sky of independent Serbia*, 1882, Belgrade; *Walk through the clouds*, 1884, Belgrade; *The universal energy and modern physics* (introductory lecture on the occasion of his entrance to the Department of Physics at the Military Academy), 1887, Belgrade; *Nikola Tesla and his discoveries*, in 1894, Belgrade; *From the science of light*, 1895, Belgrade; *Industry of cold*, 1909, Belgrade; *Incorrect celebration of the Resurrection in the Orthodox Church and the reform of the calendar*, 1908, Belgrade

On this occasion we shall present four of these books: *Starry sky of independent of Serbia, From the science of light*, *Walk through the clouds* and *Incorrect celebration of the Resurrection in the Orthodox Church and the reform of the calendar*.

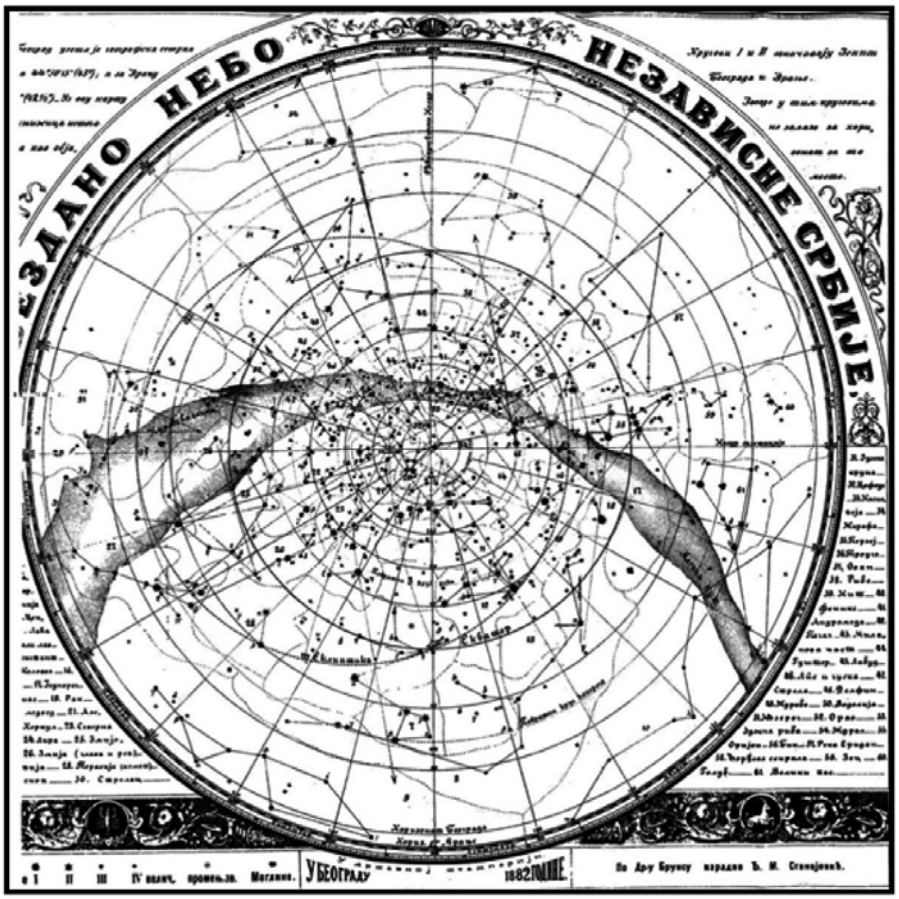
Starry sky of independent Serbia

This book is one of the first books on astronomy written in Serbian. It was published by the Royal-Serbian printing shop in 1882 in Belgrade. The hard copy of this book is located in the University Library in Belgrade, registered under the inventory number. 13006. The book is small in volume, but the content is a true jewel of our scientific literature of the nineteenth century. Stanojević studied in fine schools in European centers and he was well acquainted with foreign languages. So he wrote the book using then available literature written in several European languages. There are two French and one German book in the bibliography.

This is a small book, it has only 70 pages. The book consists of a preface, ten chapters and one star chart and has 22 pictures. It has the following chapters: *Stars in general, Astronomical division of the sky, Number, range and distribution of stars in size, Stellar maps, Star clusters and their stars, Starry skies change due to precession, Nutation, Aberration and refraction, Proper motion of stars, Variable stars and novae, Double stars, Star clusters and nebulae* and *Milky Way*.

Although this book is not great in the volume, from the titles of the chapters we can conclude that the reader of that time could gain a solid insight into astronomy. The book clarifies astronomical concepts in a simple and transparent way with the beautiful language. There is also a lot of the accompanying drawings and illustrations. Stanojević gave special attention to the description of the starry sky, particularly to the constellations description, orientation in the sky by constellations and by use a star maps. We especially emphasize Stanojević's writing skill in exposition of often complex astronomical phenomena. His

explanation of these phenomena is mostly descriptive and without mathematical expressions, but using fine graphics. Due to the methodical way in which this book was written, its content, clarity and easiness of presentation, it can be still recommended to today's high school students and teachers, not only of astronomy but also of mathematics, physics and geography.



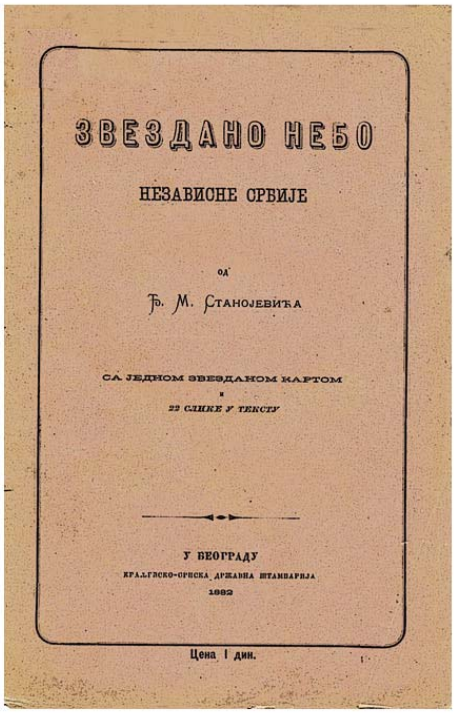
From the science of light

This book has 257 pages with the addendum of the 22 pages. There are 158 pictures scattered throughout the book. The book was published by the State Printing House of the Kingdom of Serbia in Belgrade in 1895. The hard copy of the book can be found in the library of the Serbian Academy of Sciences and Arts. It has five chapters: *Introduction, The science of light, Light sources, Propagation of light, Speed of light and reflections.*

Stanojević explains in the Preface that he wrote the book by the invitation of the Serbian Literary Management Co-operatives to prepare one book in the area in which he was working. He also writes that the book is aimed "*to explain by the scientific truth about light a part of incomprehensible and strange light phenomena which we encountered either by ourselves or were told us by others*. *How I did it, it remains to the readers to judge*".

This book is written in clear and simple language. It is easy to read and understand. It has no equations, patterns and formulas, but there are many illustrations and sketches for better understanding of light phenomena and effects. Special attention is paid to the description of light phenomena that occur in nature and brought the fear to at that time in general illiterate people. Stanojević describes these phenomena and explains all mysteries

attached to them. In each chapter for all of these phenomena he is trying to explain and eliminate prejudice against them, to free a view of the general reader of blunders and superstition. The book is well written, it is methodologically sound and we believe that it is interesting for the contemporary reader, particularly for students and teachers.


ЗВЕЗДАНО НЕБО

НЕЗАВИСНЕ СРВИЈЕ

ОД

Ђ. М. СТАНОЈЕВИЋА

СА ЈЕДНОМ ЗВЕЗДАНОМ КАРТОМ
И
22 СЛИКЕ У ТЕКСТУ

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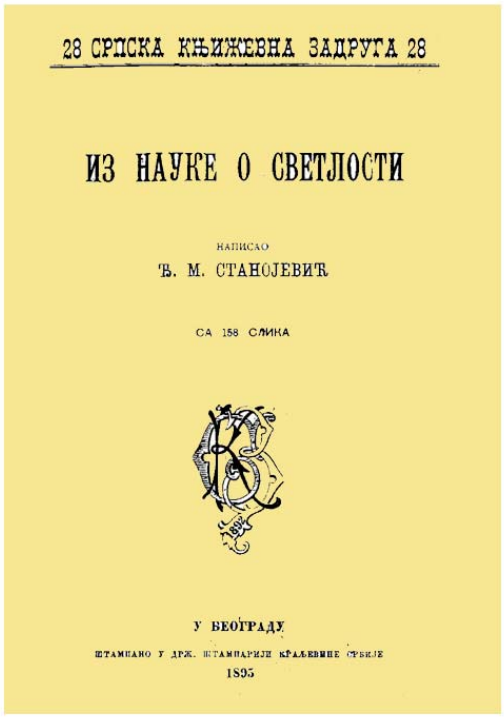
28 СРПСКА КЊИЖЕВНА ЗАДРУГА 28

ИЗ НАУКЕ О СВЕТЛОСТИ

НАПИСАО
Ђ. М. СТАНОЈЕВИЋ

СА 158 СЛИКА

У БЕОГРАДУ
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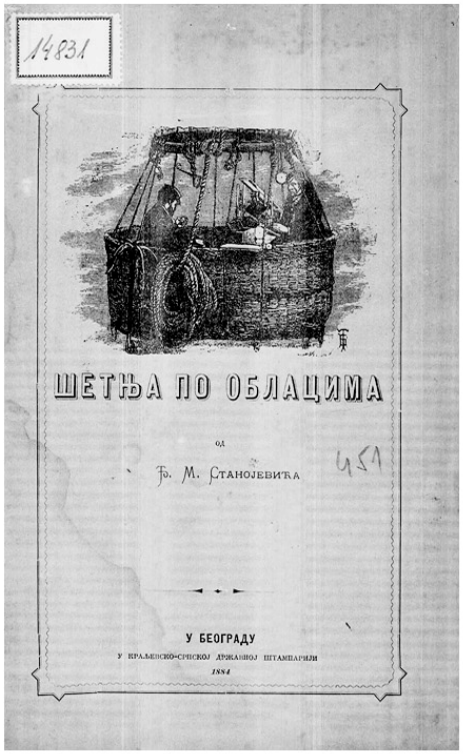
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ШЕТЊА ПО ОБЛАЦИМА

ОД

Ђ. М. СТАНОЈЕВИЋА

451

У БЕОГРАДУ
У КРАЉЕВСКО-СРПСКОЈ ДРЖАВНОЈ ШТАМПАРИЈИ
1884


НЕТАЧНО ПРАЗНОВАЊЕ
ВАСКРСЕЊА
у православној цркви
И
РЕФОРМА КАЛЕНДАРА

од
Ђ. М. Станојевића
проф. универз.

Прештампано из „Весника Српске Цркве"

БЕОГРАД
Штампарија Андре Петровића Кнез Михајлова ул. 24.
1908.

Walk through the clouds

This is the first book on aeronautics written in Serbian. In fact, the greatest part of the book is devoted to the balloons flying. Book was published by the Royal-Serbian state printing in 1884 in Belgrade. The hard copy of the book is located in the Library of the Serbian Academy of Sciences and Arts. It is short book; it has only 102 pages and is divided into seven chapters: *How to walk through the clouds, Preparation for travel, On the road, Clouds, Descent* and *Several air travel.*

Before he started to write this book, Stanojević wanted to translate *Promenades dans les Nuages* by French writer Charles Delon, a book on balloons flying. The reason was that Stanojević wanted to introduce Serbian readers with balloons and principles of balloons flying (air ballooning). But as he observed in the introduction of his book, Delon's book was written for the French audience and a simple translation of Delon's book would surpass the understandings of an average Serbian reader. Therefore he wrote a new book where he arranged the themes differently and focused more on individual episodes and history on balloons flying. In writing this book it was handy a book *Les ballons* (*A History of Balloons and Balloon Voyages*, printed in 1870) by F. Marion, as Stanojević explained in the preface. Let us mention that both books of French authors are digitized and can be freely download from Internet.

Stanojević describes in his book in details the first (Nov 21, 1783, in Paris, France) untethered manned air balloon flight that was made by Jean-François Pilâtre de Rozier and François Laurent d'Arlandes in a hot air balloon created in December 1782 by the Montgolfier brothers (Joseph and Etienne). The balloon was launched from the centre of Paris and flew for a period of 20 minutes. This event was the birth of hot air ballooning.

However, it should be mentioned that on the 19th September 1783 Pilâtre De Rozier, a scientist, launched the first hot air balloon called *Aerostat Reveillon.* The passengers were a sheep, a duck and a rooster and the balloon stayed in the air for a grand total of 15 minutes before crashing back to the ground. We can also learn from the Stanojević's book that already in the year to come (1794) there were 52 air balloons flights. Stanojević describes several similar historical events in air ballooning, but also describes physical principles of balloon flights and related notions such as aerostatic buoyancy which enables the flight of air balls, the term Stanojević used for air balloons.

Stanojević obviously wanted with his book to contribute to the scientific enlightening and education of his people, to learn more about then the latest technological achievements. In the case of this book, the focus was on balloons and principles of air ballooning as a part of then the modern scientific progress. The book is written clearly and **it is** easy for reading in spite of many scientific and technical details. There are no graphical illustrations or drawings as in the other Stanojević' books.

Incorrect celebration of the Resurrection in the Orthodox Church and the reform of the calendar

Stanojević wrote this book in the beginning of the XX century and printed it (print shop of Andra Petrović) in 1908 in Belgrade. This book was in fact the first issue of the *Messenger of Serbian Church*. The hard copy of the book can be found in the Library of the Serbian Academy of Sciences and Arts.

This book is also small, it has only 83 pages. It is devoted to the study of the calendar and it points out to some errors found in the calculation of dates that are celebrated by the

Serbian Orthodox Church. Besides, Stanojević discusses the reasons for the reform of the Calendar and proposes how to do that.

The book consists of an introduction and nine chapters. The first six chapters are devoted to the determination of the date of Easter with a detailed historical overview, while the last three chapters are related to the reform of the Julian calendar that was used by the Serbian and other Orthodox churches. He explains that there are the errors and irregularities in the determination of the date of Easter and that they should be corrected. In the Introduction, he says: *It can be said that the preparation and celebration of Easter starts at the beginning of Lent, especially eight days before Easter, Lazarus Saturday. These activities should remind Christians of Christ arrival in Jerusalem, the place of his suffering, death and resurrection*. Stanojević further emphasizes the importance of Easter as the greatest Christian holiday, and compares this holiday with the Solar System: *Easter with their holidays and rituals stands as the Sun stands with its planet system in which all the planets are dependent of it - the Sun. The planets are moving following him while the Sun is steering them.*

Stanojević also writes that in the second half of the second century Christians split into two groups. Some of them celebrate Easter as the Jews, whose Passover falls 14 Nisan of their calendar, and they are called *četrnaestnici* (fourteeners) and others, most of the Christians, who celebrate Easter on the first Sunday after that date. Stanojević explained the further history of division related to the date for celebrating Easter. Those who celebrated Easter on the first Sunday after that date but were tied for the week after the full moon after the equinox he called *ravnodnevičari (*equinoxers). The divisions were still occurring. In the fourth century, the Romans took for the beginning of spring and the vernal equinox March 18th, while the Alexandrians, having at that time in Egypt developed astronomy, for the beginning of spring took March 21st.

In more details, originally, the early Christians had followed the Jewish calendar and celebrated the resurrection on the Passover, which was the fourteenth day of Nisan, the first month of the Jewish year (*luna 14*, the fourteenth day of the first lunar month of spring). In time, Easter became separated from the Jewish festival. The Council of Nicaea, convened by Constantine the Great in AD 325, declared that Easter was to be celebrated on the Sunday after Passover, that is, between Nisan fifteen and twenty-first. And, if Easter did fall on the fourteen, even though that day was a Sunday, the Roman church still celebrated the following week, on the twenty-first. Stanojević warns that sometimes incorrectly it is stated that the Council of Nicaea corrected the calendar. The Council had not corrected the calendar, but only at the Council passed the proposal, that all Christians should celebrate Easter on the same day and the Alexandrian Church, which the vernal equinox tied for March 21st, should tell the date of the Easter.

In order to eliminate the errors in the Julian calendar used by the Serbian Church, Stanojević proposed the following three simple rules for reforming this calendar:

a. The year that is not divisible by 4 is simple (old rule).
b. The year that is divisible is leap year (old rule).
c. The year that is divisible by 128 is simple (new rule).

So, in the respect to the year when Stanojević wrote this book, the years 1920, 2048, 2176 will be the first simple years according to the third, new rule.

It should be mentioned that there are other Serbian scientists who also proposed the reform of the Julian calendar which would be then more accurate than the Gregorian calendar. The most notable proposal of the reform is that one by Milutin Milanković. It was accepted in general at the congress of Orthodox churches in Istanbul in 1923. However, it was not accepted by the Serbian Church, this Church is still using the old, Julian calendar. Probably one

reason for non-acceptance is that Milanković' calendar almost coincides with the Gregorian calendar, the first difference will occur in the year 2800.

Conclusion

We presented four books written by Đorđe Stanojević from the collection of seven books that are digitized and uploaded into the Virtual Library of the Faculty of mathematics in Belgrade (see [9]). The presented books are: *Starry sky of independent Serbia, From the science of light, Walk through the clouds, Incorrect celebration of the Resurrection in the Orthodox Church and the reform of the calendar.* A short biography of this prominent Serbian scientist and engineer is also given. From his work we see that he had broad interest in many branches of science, particularly in physics and astronomy. Some of his books were written in a popular manner with the aim to explain the physical phenomena to the general public. Due to his very good and engaging style of writing, his books should be also interesting to the today's readers.

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ДИГИТАЛИЗОВАНЕ КЊИГЕ ЂОРЂА СТАНОЈЕВИЋА У ВИРТУЕЛНОЈ БИБЛИОТЕЦИ МАТЕМАТИЧКОГ ФАКУЛТЕТА

Представљамо књиге Ђорђа Станојевића које су дигитализоване и налазе се у Виртуелној библиотеци Математичког факултета Универзитета у Београду. Ђорђе Станојевић био је угледан електроинжењер, физичар и астроном из друге половине XIX и с почетка XX века. Предавао је примењену физику на Великој школи, а по оснивању и на Београдском универзитету. Заслужан је за електрификацију Београда, чиме је Београд постао један од првих градова у Европи осветљен електричном струјом.

Ова дигитална колекција садржи седам књига: *Звездано небо независне Србије, Из науке о светлости, Шетња по облацима, Нетачно празновање Васкрсења у православној цркви и реформа календара, Васионска енергија и модерна физика, Никола Тесла и његова открића, Индустрија хладноће.*

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60 Digitization of textbook *Nebeska mehanika* by Milutin Milanković

Digitization of the textbook *Celestial mechanics*
by Milutin Milanković,
N. Pejović
NCD Review, vol. 19, pp. 63-68, 2011.

In this article the book "Celestial Mechanics", written by Milutin Milanković, a renowned Serbian scientist, is reviewed. This book was the first university textbook of celestial mechanics at the Belgrade University. In it motion of celestial bodies was for the first time treated by applying vector analysis.

Преглед НЦД 19 (2011), 63–68

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DIGITISATION OF TEXTBOOK *НЕБЕСКА МЕХАНИКА* BY MILUTIN MILANKOVIĆ

Abstract. We give a description of the book *Небеска механика (Celestial Mechanics)*, written by Milutin Milanković, a renowned Serbian scientist . This book was the first university textbook of celestial mechanics at the Belgrade University. In it motion of celestial bodies was for the first time treated by applying vector analysis. A digitised copy of the book has become accessible through the Virtual Library of the Faculty of Mathematics at http://elib.matf.bg.ac.rs.

1. Introduction

Milutin Milanković in this book uses the most up-to-date mathematical apparatus of that time. About this in *Предговор* (*Foreword*) the author says: *The science of motion of celestial bodies is treated by using the modern tools of vector analysis. This first, consistent, application of vector calculus to the problems of classical celestial mechanics shows all advantages compared to the tools used by this science up to now. To me it has been also useful in creating the theory of motion of the terrestrial poles built by Belgrade scholars of applied mathematics.*

The university textbook *Небеска механика* (*Celestial Mechanics*) by Milutin Milanković introduces a reader in a very interesting way in the very complex problems of motion of celestial bodies giving them clear mathematical explanations and solutions. Although written, in Serbian, as long ago as in 1935, as the first textbook of celestial mechanics, it has remained, till nowadays, the only one. Using this textbook many generations of students have been learning.

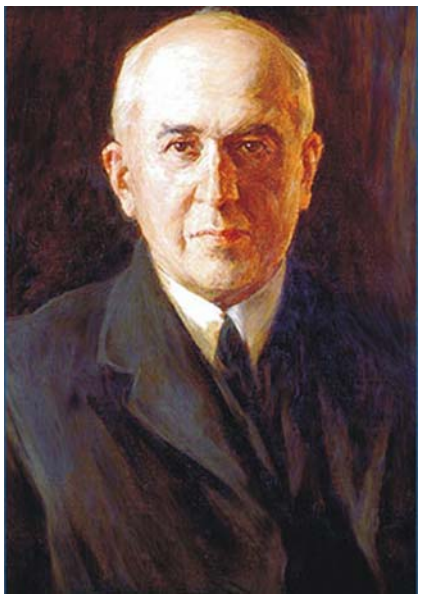
2. Milutin Milanković's biography

Milutin Milanković was born on May 28, 1879 in Dalj (Slavonia, nowadays part of Republic of Croatia). The real gymnasium he finished in Osijek, to become a student of civil engineering at the Technische Hochschule in Vienna. There he took the first degree in 1903 and the PhD one in 1904. He was the first Serb who acquired the PhD title in engineering.

Following an invitation Milutin Milanković came from Vienna in 1909, after the University of Belgrade had been founded in 1905, to teach applied mathematics at the Faculty of Philosophy of this University. His coming was a merit of Bogdan Gavrilović and Mihailo Petrović Alas who both taught at the University of Belgrade. As a staff member Milanković was at the Belgrade University till his retirement in 1955. He taught applied mathematics (theoretical physics, mechanics and astronomy). He was the first in starting lectures in

celestial mechanics. Among other subjects he taught history of astronomy and theory of relativity.

Milanković was a full member, and near the end of his life also Vice President of the Serbian Academy of Sciences, member of Deutsche Akademie der Naturforscher Leopoldina in Halle and corresponding member of a number of other academies and scientific societies in the world.



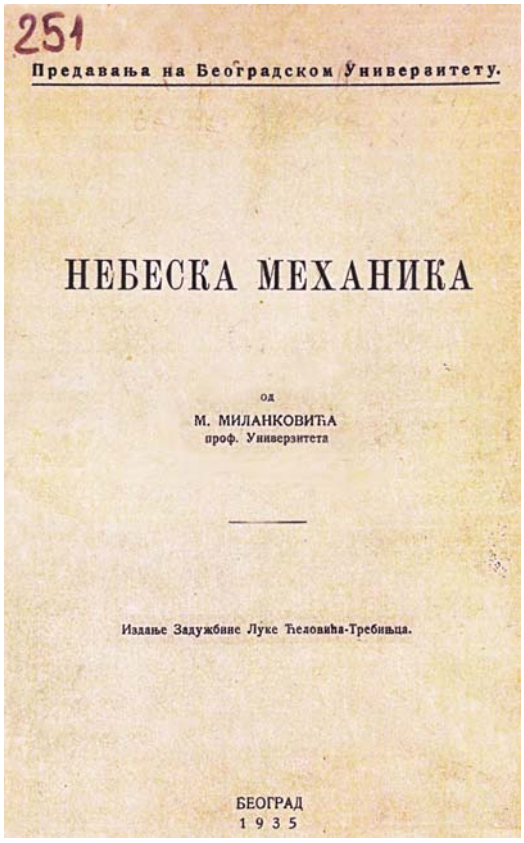
Milanković died on December 12, 1958 in Belgrade. His activity resulted in about ten books and more than hundred papers in mathematics, celestial mechanics, astronomy and geophysics.

Milutin Milanković gave to the world science community his well known theory of glacial ages. The results achieved by him taking into account the complex secular computation of perturbations in planet motions were published in *Théorie mathématique des phénomènes thermiques produits par la radiation solaire* in 1922; the publishers were Yugoslav Academy of Sciences and Arts in Zagreb and publishing house Gauthier Villars, Paris. Due to these results he became well known in the world scientific community, so that great German climatologist W. Köppen invited him to cooperate in building a great work *Handbuch der Klimatologie*. For this purpose a part, which was published in 1930, entitled *Mathematische Klimalehre und Astronomische Theorie der Klimaschwankungen* was written by Milanković. Here his theory of planet heating based on insolation was extended with a special reference to the Earth. By applying this theory to the run of glacial ages it was shown that Milanković had given an excellent model of Earth, i. e. that he had created a good and mathematically exact theory of terrestrial climate. This work was translated into Russian in 1939.

Already as a well known scientist he started the cooperation on creation of work *Handbuch der Geophysik* prepared by B. Gutenberg. For this work Milanković wrote four sections where he returned to an old and very difficult problem: that of motion of the terrestrial poles. In these contributions of his Milanković created a theory of motion of terrestrial poles and succession of glacial ages.

His main work is *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* the printing of which was finished shortly before the beginning of the Second World War. It should be mentioned that the printed sheets were damaged by a bomb which hit the printing office on April 6, 1941 during the Nazi air raid of Belgrade. Fortunately the original typography of the book was preserved. By using it the book was reprinted during the war on a sufficiently low-quality paper.

During the four war years Milanković could send abroad just a few copies. So this important scientific work was printed at the most unfavorable moment, in war time. Due to this the scientific world community learnt about Milanković's theory too late. As a consequence the world recognition arrived only after that. This work appears as a synthesis of his many earlier works which concern his research activity within boundary fields between many natural sciences and mathematics.



Milanković was successful in the field of calendar reform. During a Congress devoted to the calendar question organised by the Orthodox Churches in Constantinople in 1923 he proposed an improvement of the Gregorian time reckoning. The proposal contained in Milanković's calendar-reform project was accepted by the Orthodox Christian Community, but some Churches present at the Congress (for instance, the Serbian one) have not started its application.

After great Serbian scientist Milutin Milanković a minor planet, a lunar crater and a Mars crater have been named. In Belgrade one street and one gymnasium have been named after him.

2.The Textbook

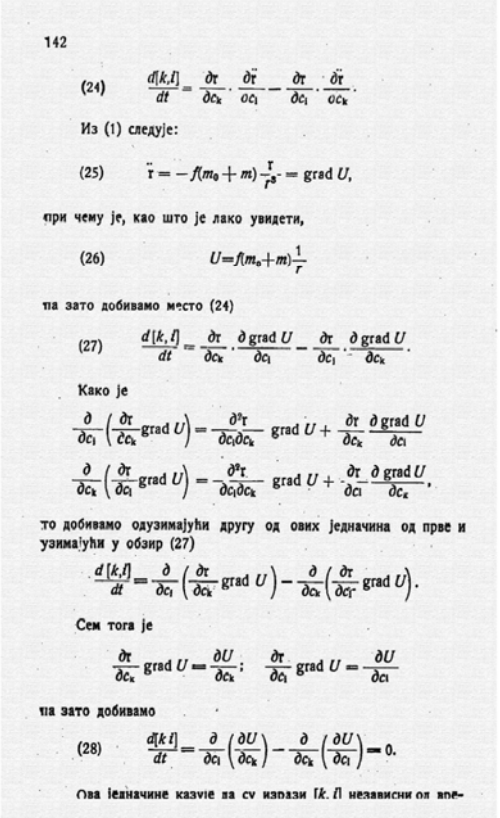
Milanković wrote two versions of his university textbook *Небеска механика.* The book was published due to the sponsorship of "Luka Ćelović-Trebinjac" in Belgrade in 1935. The base for this book was the lectures delivered by Milanković at the Belgrade University within the framework of subject Celestial Mechanics, Chair of Applied Mathematics. The main characteristic of this textbook is that in it vector calculus is introduced and used systematically in solving problems of celestial mechanics. Professor Milanković used the vector-scalar system of elements of planetary orbits. For this reason connecting his name with this system is fully justified.

Небеска механика was written on 333 pages with 22 figures in its text. The book contains two parts presented in 15 chapters. The first part *Транслаторно кретање небеских тела* (*Translatory Motion of Celestial Bodies*) contains nine chapters; the second one *Ротационо кретање небеских тела* (*Rotational Motion of Celestial Bodies*) is written on 130 pages containing six chapters. At the end of both parts there is a list of references in foreign languages.

2.1 First Part – *Translatory Motion of Celestial Bodies.* In the first chapter Milanković in an illustrative and very interesting way presented the historical development of the science concerning the motion of celestial bodies from the Chaldeans and Egyptians, through Greeks and Alexandrians, then through the Middle Ages, Copernicus, Galileo and Kepler till Newton. Concisely, chronologically and very illustratively he presented the Ptolemaic geocentric system of the world. Afterwards he explains Galileo's discoveries and Kepler's laws of planetary motion towards Newton's gravitation law. The first chapter offers to the reader an excellently clear and illustrative picture about the discoveries and development of astronomical science through centuries.

In the other chapters of this part Milanković in studying translatory motion of celestial bodies meets a number of problems: at first two-body problem, after this general ***n*-**body problem, three-body problem, asteroid-motion problem, perturbation force, variation-constant method in equations of motion of celestial bodies, as well as perturbation calculation for motion of celestial bodies. At the beginning of each chapter he clearly formulated the problem which was the subject. Besides, he gave definitions, mathematical expressions, theorems with

proofs, as well as methods necessary in solving these problems. The notions were introduced gradually and systematically, from simple towards the most complicated ones. In solving exceptionally complicated problems of translatory motion of celestial bodies he used, as in other cases, the vector-scalar system of elements of planetary orbits.



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(24) $$\frac{d[k,l]}{dt}=\frac{\partial\tau}{\partial c_k}\cdot\frac{\partial\ddot{\tau}}{\partial c_i}-\frac{\partial\tau}{\partial c_i}\cdot\frac{\partial\ddot{\tau}}{\partial c_k}.$$

Из (1) следује:

(25) $$\ddot{\tau}=-f(m_0+m)\frac{\tau}{r^3}=\operatorname{grad}U,$$

при чему је, као што је лако увидети,

(26) $$U=f(m_0+m)\frac{1}{r}$$

па зато добивамо место (24)

(27) $$\frac{d[k,l]}{dt}=\frac{\partial\tau}{\partial c_k}\cdot\frac{\partial\operatorname{grad}U}{\partial c_i}-\frac{\partial\tau}{\partial c_i}\cdot\frac{\partial\operatorname{grad}U}{\partial c_k}.$$

Како је

$$\frac{\partial}{\partial c_i}\left(\frac{\partial\tau}{\partial c_k}\operatorname{grad}U\right)=\frac{\partial^2\tau}{\partial c_i\partial c_k}\operatorname{grad}U+\frac{\partial\tau}{\partial c_k}\frac{\partial\operatorname{grad}U}{\partial c_i}$$

$$\frac{\partial}{\partial c_k}\left(\frac{\partial\tau}{\partial c_i}\operatorname{grad}U\right)=\frac{\partial^2\tau}{\partial c_i\partial c_k}\operatorname{grad}U+\frac{\partial\tau}{\partial c_i}\frac{\partial\operatorname{grad}U}{\partial c_k},$$

то добивамо одузимајући другу од ових једначина од прве и узимајући у обзир (27)

$$\frac{d[k,l]}{dt}=\frac{\partial}{\partial c_i}\left(\frac{\partial\tau}{\partial c_k}\operatorname{grad}U\right)-\frac{\partial}{\partial c_k}\left(\frac{\partial\tau}{\partial c_i}\operatorname{grad}U\right).$$

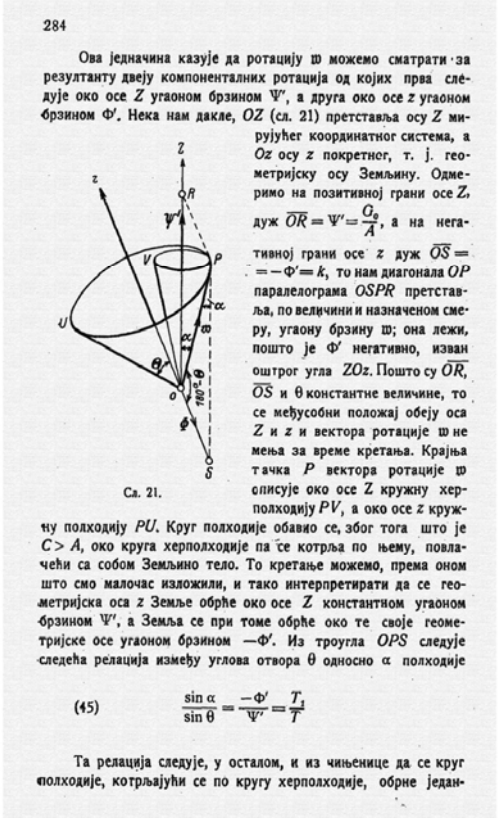
Сем тога је

$$\frac{\partial\tau}{\partial c_k}\operatorname{grad}U=\frac{\partial U}{\partial c_k};\quad\frac{\partial\tau}{\partial c_i}\operatorname{grad}U=\frac{\partial U}{\partial c_i}$$

па зато добивамо

(28) $$\frac{d[k,l]}{dt}=\frac{\partial}{\partial c_i}\left(\frac{\partial U}{\partial c_k}\right)-\frac{\partial}{\partial c_k}\left(\frac{\partial U}{\partial c_i}\right)=0.$$

Ова једначине казују да су изрази $[k,l]$ независни од вре-



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Ова једначина казује да ротацију ω можемо сматрати за резултанту двеју компонената ротација од којих прва следује око осе Z угаоном брзином Ψ', а друга око осе z угаоном брзином Φ'. Нека нам дакле, OZ (сл. 21) претставља осу Z мирујућег координатног система, а Oz осу z покретног, т. ј. геометријску осу Земљину. Одмеримо на позитивној грани осе Z, дуж $\overline{OR}=\Psi'=\frac{G_0}{A}$, а на негативној грани осе z дуж $\overline{OS}=-\Phi'=k$, то нам диагонала OP паралелограма $OSPR$ претставља, по величини и назначеном смеру, угаону брзину ω; она лежи, пошто је Φ' негативно, изван оштрог угла ZOz. Пошто су $\overline{OR}$, $\overline{OS}$ и θ константне величине, то се међусобни положај обеју оса Z и z и вектора ротације ω не мења за време кретања. Крајња тачка P вектора ротације ω описује око осе Z кружну херполходију PV, а око осе z кружну полходију PU. Круг полходије обавио се, због тога што је $C>A$, око круга херполходије па се котрља по њему, повлачећи са собом Земљино тело. То кретање можемо, према оном што смо малочас изложили, и тако интерпретирати да се геометријска оса z Земље обрће око осе Z константном угаоном брзином Ψ', а Земља се при томе обрће око те своје геометријске осе угаоном брзином $-\Phi'$. Из троугла OPS следује следећа релација између углова отвора θ односно α полходије

Сл. 21.

(45) $$\frac{\sin\alpha}{\sin\theta}=\frac{-\Phi'}{\Psi'}=\frac{T_1}{T}$$

Та релација следује, у осталом, и из чињенице да се круг полходије, котрљајући се по кругу херполходије, обрне један-

Note. On page 142, Milanković's inference of mathematical formulas involving Langrage's brackets by use of vector analysis is displayed. On page 284, the cones of polhode and herpolhode are depicted. In fact, this picture represents the mechanism of moving of the Earth's poles.

2.2 Second Part – Rotational Motion of Celestial Bodies. At the beginning of this part, in Chapter 10, Milanković presented the theorems and formulae of rational mechanics which are necessary in studying rotational motions of celestial bodies. Among them are the impulse theorem, the one concerning the centre-of-mass motion and the Eulerian equations, followed by descriptions of Euler's angles, of polhode and herpolhode, as well as other formulae.

In the next chapter he described the rotation of a fluid body, presented Appell's theorem, equilibrium conditions and Clairaut' theorem. In the other chapters he mainly focused on the planet Earth. He described the precession of the Earth's axis, astronomical nutation of the Earth's axis, free nutation of the Earth and secular motion of the terrestrial poles. The Earth as a whole he considered as a fluid body, which in the case of short-duration forces behaves as a solid body, but under an influence behaves as an elastic body. Milanković possessed an extraordinary ability of describing natural phenomena by using the language of mathematics. For instance, using vector analysis he made an excellent mathematical model of the Earth which served him in creating a theory of secular motion of the terrestrial poles. In Section 69 he derived the equation of secular trajectory of a terrestrial pole and also the

equation of pole motion along this trajectory. He found that the secular pole trajectory depends only on the configuration of the terrestrial outer shell and the instantaneous pole position on it, more precisely on geometry of the Earth mass. On this basis he could calculate the secular pole trajectory. In the next section (**70**) he gives a short presentation of another theory concerning the same problem that of Anton Bilimovitch, also a representative of the Belgrade school of applied mathematics.

Milanković's results have been confirmed by the International Latitude Service on the basis of measurements. The results obtained on the basis of his theory of secular motion of the terrestrial poles are concordant with those based on the measurements organised by the Service. For instance, both Milanković's model and the measurements show that this motion is very slow and its value does not exceed 160 mm per year. Such a value for the motion of the terrestrial poles has enabled us to explain, for instance, the formation of big coal reserves on the Spitsbergen Islands, which could not form at the present latitude of these islands. Also, based on Milanković's model one can determine the space orientation of the terrestrial axis depending on the polar motion. The mechanism of the phenomenon is given in Fig. 21.

The number of figures given in the textbook is small. In most cases instead of a figure Milanković gave its description, but exceptionally well, clearly and illustratively. So, a figure arose before reader's eyes.

The list of references shows that Milanković used a lot of references in foreign languages, both books and the most recent scientific results of his time. He also cites works of Ptolemy, Copernicus, Galileo, Kepler, Newton, as well as of Laplace, Poincaré and other great names of world science.

3. Conclusion

Milanković in his textbook *Небеска механика* presented systematically the use of vector calculus in solving problems of classical celestial mechanics. This is an important application of vector analysis in solving the complex problems of motion of celestial bodies. By using these modern tools the mathematical expressions and differential equations describing the translatory and rotational motions of celestial bodies were written in a clear and elegant way. Vector analysis was used by Milanković also in creating the theory of secular motion of the terrestrial poles presented in Chapter 15, the last one of this book. The theory of secular motion of the terrestrial poles is an important contribution of Milanković to the world science.

This textbook was used by many generations of students of the Belgrade University for the purpose of learning celestial mechanics. It is still used. Time has not made this textbook out-of-date, so it has had many editions. It is still the only textbook of celestial mechanics written in the Serbian language.

To meet the needs of students of the Faculty of Mathematics Dr S. Šegan, Reader at the Department of Astronomy, has prepared an edition of this textbook [3]. In the Virtual Library of the Faculty of Mathematics [5] there are digital copies of ten books written by Milanković, among them are all books mentioned here.

References

[1] М. Миланковић, *Небеска Механика*, Издање Задужбине Луке Ћеловића-Требиња, Штампарија Графички институт, Београд, 1935.

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[3] М. Миланковић, *Небеска механика*, едитор С. Шеган, Vesta Company, Београд, 1995.

[4] J. Симовљевић, *Астрономија*, Тридесет година Природно-математичког факултета Универзитета у Београду 1947–1977, Београд, 1980, 165–195.
[5] Ж. Мијајловић, Виртуелна библиотека: http://elib.matf.bg.ac.rs

Сажетак. Представљамо књигу *Небеска механика*, српског научника светског гласа Милутина Миланковића. Књига је била први универзитетски уџбеник из Небеске механике на Београдском универзитету. У њој су по први пут обрађена кретања небеских тела применом векторске анализе. Дигитализован примерак књиге постављен је у Виртелну библиотеку Математичког факултета на адреси http://elib.matf.bg.ac.rs.

nada@matf.bg.ac.rs

61 Early astronomical heritage in Virtual Library of Faculty of Mathematics in Belgrade

N. Pejovićand Ž. Mijajlović,
NCD Review, vol. 19, pp. 11-25, 2011.

The Virtual Library of the Faculty of Mathematics, http://elibrary.matf.bg.ac.rs, contains about 100 digitized books on astronomy written mostly by Serbian scientists. This comprehensive article presents a selection of books from this collection written up to the first half of the XX century by the following authors: Rudjer Bošković, Zaharije Orfelin, Atanasije Stojković, Djordje Stanojević, Milan Andonović, Kosta Stojanović, Nikolai Tsinger and Milutin Milanković. For each of these authors a short biography is included.

Преглед НЦД 19 (2011), 11–25

Nadežda Pejović, Žarko Mijajlović
Faculty of Mathematics, Univ. of Belgrade

EARLY ASTRONOMICAL HERITAGE IN VIRTUAL LIBRARY OF FACULTY OF MATHEMATICS IN BELGRADE

Abstract. The Virtual Library of the Faculty of Mathematics, http://elibrary.matf.bg.ac.rs, contains about 80 digitized books on astronomy written mostly by Serbian scientists. This article presents a selection of books written up to the first half of the XX century by the following authors: Ruđer Bošković, Zaharije Orfelin, Atanasije Stojković, Đorđe Stanojević, Milan Andonović, Kosta Stojanović, Nikolai Tsinger and Milutin Milanković. For each of these authors a short biography is included.

Keywords: Virtual library, astronomical books, history of astronomy.

Introduction

The Virtual Library of the Faculty of Mathematics is the largest database of digitized texts in Serbia which is open for public use. At the time of writing this article, it contains nearly 1500 works. Important collections in this library are a collection of 400 doctoral dissertations in mathematical sciences, most of them obtained at the Faculty of Mathematics of the University of Belgrade and rare books from the 18th and 19th century. Collection of astronomical books numbers about 80 digital copies and includes the oldest books on astronomy written by Serbian scientists. Some of these books exist only in one copy and in fact are unavailable to the general public.

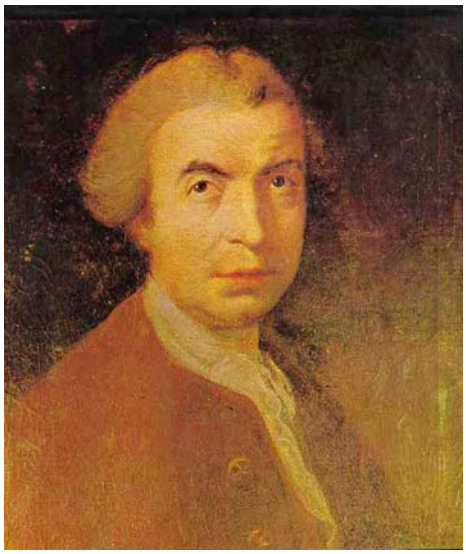
In this paper we present a collection of books on astronomy written mostly by Serbian authors and printed up to the XX century that are digitized and uploaded into the Virtual library. These books are the first Serbian astronomical books, and we can freely say that they are the first scientific works of Serbian authors. Let us note here a general feature of these first Serbian scientists, particularly those from the XVIII and XIX century. They were universal in their research interest and in other respects as well, so astronomy was not their only specialty. They were successful in other sciences, too: in mathematics, physics or philosophy, for example. For them, science mostly was the view of a unique being - Nature. The language of science was used by them to describe the nature of this being. Besides, they were often successful politicians, artists, travelers and writers of novels and poems. Their importance for the development of Serbian culture, language and science is enormous. Therefore, in addition to their manuscripts from astronomy we present here their brief biographies and other works as well. In this article we present the short biographies and a selection of their astronomical works of the following authors:

- Ruđer Bošković (1711–1787), *Elementorum Universal Matheseos*, T. I–III, 1757, Venetis.
- Zaharije Orfelin Stefanović (1726–1785), *Perpetual Calendar*, 1783, Vienna.
- Atanasije Stojković (1773–1832), *Physics*, 1810, Buda.
- Đorđe Stanojevic (1858–1921), most of his works (8) is digitized, including: *The starry sky of independent of Serbia*, 1882, Belgrade.
- Milan Andonović (1849–1926), *Cosmography* (1888), *On the Cosmos* (1889), Belgrade.

- Kosta Stojanović (1867–1921), *Mechanics*, 1892, Niš.
- Nikolai J. Zinger, (1842–1918), *Course on Astronomy,* 1925, Belgrade (published in 1898 in Russian and translated by Stevan Bošković, military general and professor of geodesy).
- Milutin Milanković (1879–1958), most of his books, 10 books, are digitized, including *Through Space and Centuries*, Belgrade, 1935 and *Kanon Der Erdbestrahlung*, 1941.

In the Virtual Library there are 15 digitized books of Serbian scientists printed in the XVIII and XIX centuries. The authors of these books are R. Bošković, Orfelin, A. Stojković, Đ. Stanojević, M. Andonović and K. Stojanović.

Ruđer Bošković

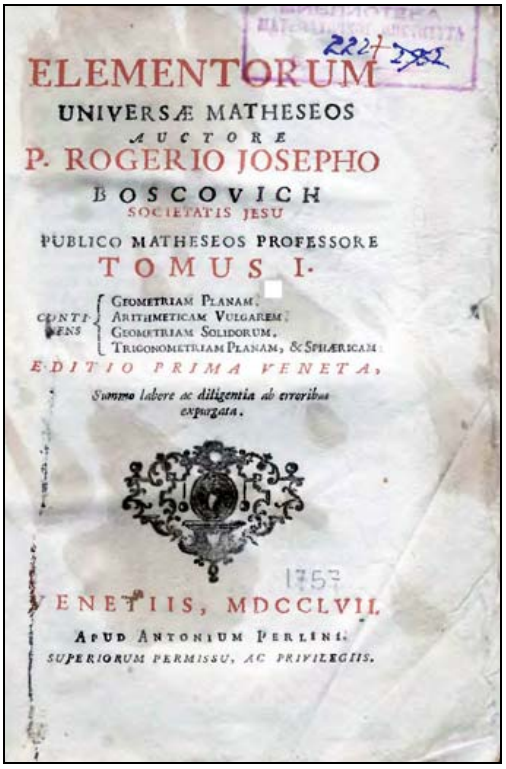


Ruđer Bošković (1711–1787) is a famous mathematician and astronomer of Dubrovnik and one of the most important scientists of his time. He is included among one hundred most prominent Serbs of all time. He was a university professor, founder of the Milano Observatory and the director of the French Navy Optical Institute. He was a universal creator: philosopher, mathematician, astronomer, physicist, geologist, architect, archaeologist, diplomat, writer, professor, poet and polyglot. He was born on 18 May 1711 as the seventh child of Nikola Bošković (a Serb from Orahova Dole, a village near Trebinje in Herzegovina) and mother Paola (of Italian descent, from the family of Baro Bettera, a notable poet from Dubrovnik). He lived mostly in Italy, where he gained world fame. Bošković died on February 13, 1787.

Bošković formulated the Unified Law of Forces, assuming that there is not only an attracting force (as Newton's law states), but that there is also repulsion which alternately changes with attraction at small distances between bodies. According to him, an elementary dimensionless particle is the source of the force, and time and space are relative, unlike in Newtonian theory. Hence, he is often considered as the forerunner of Albert Einstein.

Ruđer Bošković had numerous contributions to astronomy, among others: he introduced two geometric methods for determining the elements of the rotation of the Sun by observing the positions of three bodies, calculated the dimensions and the flatness of the Earth and discovered the comets. He was the first (1782) who accurately determined the trajectory of Uranus based on the measurement of coordinates of Uranus obtained by Messier and Méchain. It should be said that at the time of discovery it was thought that Uranus was a comet. By calculating the trajectory of Uranus, he perfected the method for determining the orbits of comets on the basis of four measured positions. In the 1800th it was one of the most accurate methods. He also studied the atmosphere of Jupiter and the nature of the Aurora.

Ruđer Bošković published numerous numbers of paper sheets in mathematics, astronomy and physics. His well-known books are *Theoria philosophiae naturalis redakta ad unicam legem virium in natura existentium*, *Opera partinentia ad opticam et astronomia* and *Elementorum universae matheseos*.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UNIVERSÆ MATHESEOS
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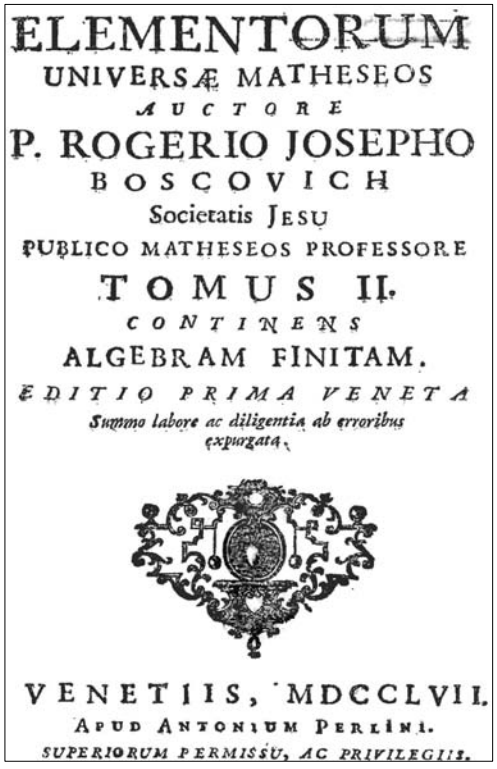
P. ROGERIO JOSEPHO
BOSCOVICH
SOCIETATIS JESU
PUBLICO MATHESEOS PROFESSORE
TOMUS I.
CONTINENS
GEOMETRIAM PLANAM,
ARITHMETICAM VULGAREM,
GEOMETRIAM SOLIDORUM,
TRIGONOMETRIAM PLANAM, & SPHÆRICAM
EDITIO PRIMA VENETA,
Summo labore ac diligentia ab erroribus expurgata.
VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.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BOSCOVICH
Societatis JESU
PUBLICO MATHESEOS PROFESSORE
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CONTINENS
ALGEBRAM FINITAM.
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Summo labore ac diligentia ab erroribus expurgata.
VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.

Pages from Bošković's *Elementorum Universal Matheseos*

In the Virtual library there are the following books by him: *Elements of Mathematics* (*Elementorum Universal Matheseos* - Tomus I-III, 1757, Venetis), *Diary on the Jjourney from Constantinople to Poland* (1762), translated by D. Nedeljeković, 1937, Belgrade, *On the law of continuity and its consequences in respect to the basic elements of matter and their forces*, printed in 1754, translated into Serbian by E. Stipanić, 1975, Belgrade.

Zaharije Stefanović Orfelin

Zaharije Stefanović Orfelin (1726–1785) was a prominent Serbian poet, historian, engraver, baroque educator, calligrapher and writer of textbooks. He was born into a Serbian family in Vukovar (Slavonia).

Among his most important works is *Slavenoserbski magazine* printed in Venice in 1768. This is the first South Slavic magazine. Although it was printed just in one volume, its importance is great. In the preface he presented the idea of civil enlightenment, and he aslo said that science, literature and philosophy should leave the narrow circle of educated people and that it must become available to everyone. In 1768 Orfelin introduced into Serbian literature a language which was a mixture of Church and common language. He also included many Russian words. In this way he practically founded the Slavoserbian language. Because of his artistic works, primarily in the copperplate, he was elected a member of the Art Academy in Vienna.

Orfelin is the author of the first Serbian primer (*bukvar*) which was used since 1767 in teaching many generations of students in writing and reading. He is also the author of the first Serbian-Latin textbook. His most comprehensive work was *The Life of Peter the Great* (Venice, 1772), whom he had seen as the enlightened monarch. He also wrote *Perpetual* (eternal) *calendar.* This book includes a comprehensive chapter on astronomy.



The cover pages from the Orfelin's *Perpetual Calendar*

Orfelin's *Perpetual Calendar* was printed in Vienna in 1783. The book has 366 pages and 9 astronomical drawings at the end of the book. Its content concerns mainly natural sciences and astronomical phenomena and contains, as well, description of historical events since the creation of the World in a chronological order. Chapters on astronomy bear the names: *Space, The Moon and the Planets, Comets, Solar Eclipse and Moon Eclipse*. It includes information on weather cycles, tables with information about the Sun and the Moon (needed for the calculation of the date of Easter according to the Big indicium), tables on the length of day and night, moon phase tables, and more. Therefore *Perpetual calendar* is the first book on astronomy written in the Slavic language, how Orfelin said "to the benefit of the Slavic-Serbian (*slovenoserbskim*) people". For full review of this book see article [4].

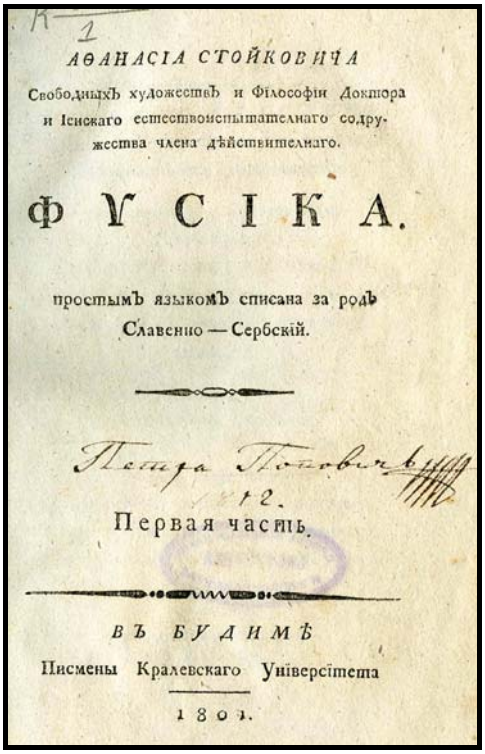
Atanasije Stojković

Atanasije Stojković was born in 1773 in Ruma, Vojvodina. In this town he finished Ascension school (Serbian Grammar school). He went soon to Sopron where in 1794 he completed the first six forms of high school and then enrolled in a secondary school in Szeged, where in 1796 graduated philosophy. In Vienna, in July 1797, he met Dositej Obradović and introduce himself with Dositej's enlightenment ideas. In the fall of this year he managed to obtain support from Metropolitan Stratimirović for studies in Göttingen, at that time one of the most famous universities. There he obtained PhD degree in philosophy (1799), and began intensive studies in various sciences (physics, mathematics, astronomy, history). After a brief stay in Ruma (1799) he moved to Budapest, where he published his main work *Physics* (1801–1803), printed in 3 volumes.



In the Habsburg monarchy it was difficult to obtain a civil job, so after the call of Severin Osipovič Potocki, the future Minister of Education of Russia, Atanasije accepted the position of professor of physics at newly established Kharkov University. He soon became the dean of the Department of Physics and Mathematics and on two occasions (1807–1808 and 1811–1813) he was the Rector of the University of Kharkov. He is the founder of the Kharkov Learned Society and he also received many honors and decorations. He became a member of the Imperial Academy of Sciences, obtained many material privileges, became a national consultant and Tsar Alexander I gave him the Medal of St. Vladimir of the third degree.

Atanasije Stojković was a hardworking and capable man, of wild spirit, very successful and was appreciated in his surroundings. He spoke several languages: German, Latin, French, Italian, English, Greek, Hungarian, and almost all Slavic languages. He had an interesting and exciting life that next to science was marked by an affair that ended his professorial career. He was also engaged in a secret mission and diplomacy. He died in Kharkov in 1832.


АѲАНАСІА СТОЙКОВИЧА

Свободныхъ художествъ и Філософіи Доктора и Ієнскаго естествоиспытателнаго содружества члена дѣйствителнаго.

ФУСІКА.

простымъ языкомъ списана за родъ Славено — Сербскій.

Петра Поповића

Часть 2.

Первая часть.

ВЪ БУДИМѢ

Писмены Кралевскаго Університета

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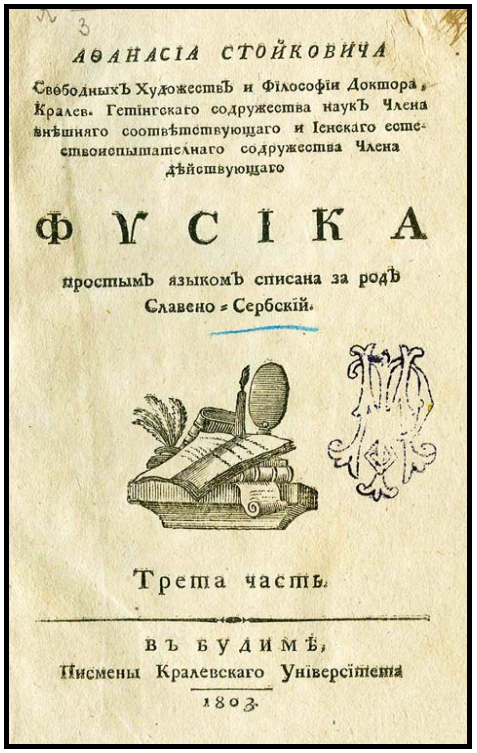

АѲАНАСІА СТОЙКОВИЧА

Свободныхъ Художествъ и Філософіи Доктора, Кралев. Гетінгскаго содружества наукъ Члена внѣшняго соотвѣтствующаго и Ієнскаго естествоиспытателнаго содружества Члена дѣйствующаго

ФУСІКА

простымъ языкомъ списана за родъ Славено-Сербскій.

Трета часть.

ВЪ БУДИМѢ,

Писмены Кралевскаго Університета

1803.

The cover pages from the Stojković's *Physics*

Stojkvić's book *Фисика* (Physics) was printed in 3 volumes. The first two volumes have 320 pages while the third one was written on 284 pages. The book presents an overview of the knowledge of that time of nature and natural phenomena. At the beginning of the first volume there is a chapter on mechanics (the properties and movements of the body). The most of the first volume (the second and the third chapter) is devoted to astronomy. There celestial

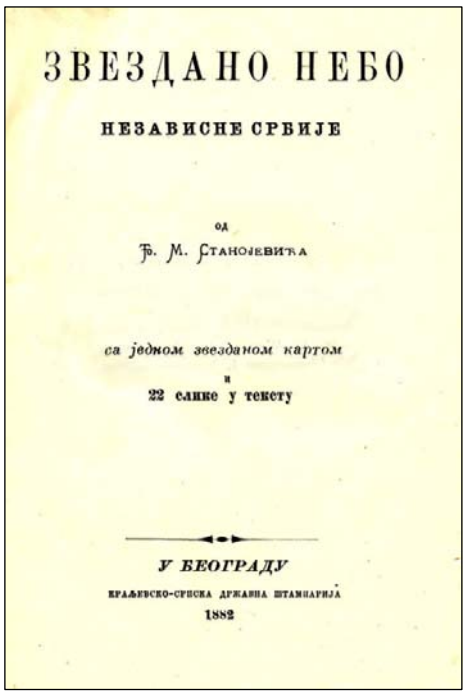
bodies, apparently moving spheres of the sky, stars, solar system, the Sun, planets, comets, the Moon, the eclipses are described. The third chapter describes our planet Earth. Also, the concepts of celestial equator, horizon, zenith, and nadir are introduced. He also explains the geographical coordinates, the size of the Earth, the Earth's movement, the change of the day and night on the Earth and the Earth's interior. The rest of the book is devoted to the description of geographical notions and meteorological phenomena. The book is written in the common language, without complex mathematical formulas and equations, having the aim to present knowledge about astronomy and nature to ordinary people.

Đorđe Stanojević



Đorđe Stanojević (1858–1921) was born in Negotin. He completed the Grand School in Belgrade. After that he became an assistant at Grand School, working with his teacher Kosta Alkovića (1836–1909) and then as a teacher at the First Belgrade Gymnasium (1883). Having the scholarship of the Ministry of Defense (1883–1887), he studied in the most famous astronomical and meteorological observatories in Europe (Potsdam, Hamburg, Medon, Greenwich, Pulkovo).

On behalf of his results he received an invitation from the Paris Observatory to participate in a scientific research expedition to study the Sun and in Petrovsky where he participated in the study of the total eclipse of the Sun.


ЗВЕЗДАНО НЕБО

НЕЗАВИСНЕ СРБИЈЕ

ОД

Ђ. М. СТАНОЈЕВИЋА

са једном звезданом картом

И

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У БЕОГРАДУ

КРАЉЕВСКО-СРПСКА ДРЖАВНА ШТАМПАРИЈА

1882

Two years later he joined another scientific-research expedition to Sahara to study the thermal spectrum of the Sun. At that time and later, he published several research papers in astronomy in the publications of the Paris Academy of Sciences. These are the first scientific works in astronomy among Serbs.

Stanojević was a versatile, knowledgeable man with a lot of active interest in many fields of science and technology. His interest in physics was great. Upon his return to Serbia he devoted himself to the study of modern physics. In the same time, he became the lecturer in physics and mechanics at the Military Academy. Also, he taught applied physics and mechanics at the High School and at the Belgrade University after establishing it in 1905. He was responsible for the construction of hydroelectric power plants in Serbia (Vučje, Negotin, Užice, Ivanjica and other places) and for the electrification of Belgrade. This put Belgrade among the first European capitals with full electric lighting. He died in Paris in 1921.

In Negotin there is a "Đorđe Stanojević" Square and a museum is dedicated to him. In New Belgrade there is a "Đorđe Stanojević" street.

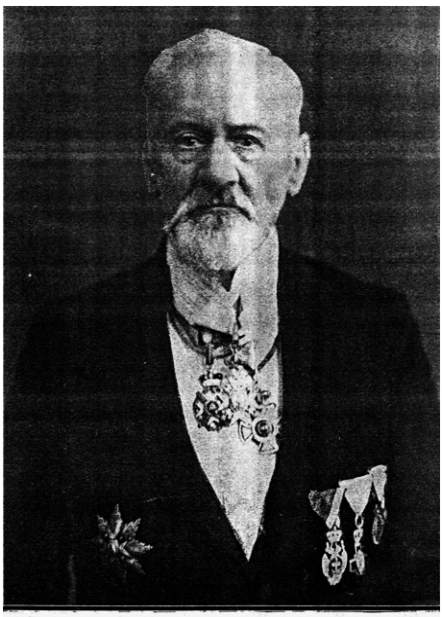
The Electric Power Industry of Serbia on the occasion of its day, 6th October, gives recognition "Đorđe Stanojević" for contributions to the development of Serbian electric power industry.

The starry sky of independent Serbia. The Virtual Library contains eight digitized books of Đorđe Stanojević: *The starry sky of independent of Serbia*, 1882, Belgrade; *A walk through the clouds*, 1884, Belgrade; *Cosmic energy of modern physics* (introductory lecture, 22nd September, 1887, Department of Physics at the Military Academy in Belgrade); *Nikola Tesla and his discovery*, 1894, Belgrade; *Science of light*, 1895, Belgrade; *Central forces in nature*, 1906, Belgrade; *Industry of coldness*, 1909, Belgrade; *Inaccurate knowledge of the Resurrection in the Orthodox Church and the reform of the calendar*, 1908, Belgrade.

Stanojević works are characterized by the simplicity and easiness of writing when exposed to often complex astronomical phenomena. These phenomena, however complex, are mostly descriptively presented, without mathematical expressions and using very fine graphics. Therefore, this book is primarily intended for general education and general readers. In this regard, Stanojević achieved his goal, to bring the basic knowledge of astronomy to the general reader. It is our great pleasure to have this precious book out of the darkness of library storage into the light through the Virtual Library and made it easily accessible to the future readers.

More details on Stanojević's biography and his books can be found in [6], [11] and [13].

Milan Andonović


МИЛАН Ј. АНДОНОВИЋ

Milan Andonović (1849–1926) was born in Požarevac where he finished primarz and seconddaz schools. He was educated in Belgrade at the engineering departments of the Grand School in Belgrade and at the German Universities in Karlsruhe, Aachen and Munich under the supervision of famous professors Ritter, Jordan, and Helmert. There he specialized in mechanics, geodesy and astronomy. Upon his return to Serbia in 1880, he was appointed as teacher of geodesy at the Grand School in Belgrade. He was responsible for the introduction and enforcement of the cadastre in Serbia in the late 19th century. He founded the Geodetic Institute under whose auspices were surveyed many towns in Serbia. In 1907 he founded the Surveying and Building Academy where many Serbian surveyors and civil engineers were trained. Together with Jovan Dragašević, Milan Nedeljković and Đorđe Stanojević he is bearing the credit for introducing the teaching of astronomy into Serbian schools and promotion of science in Serbia. He died in Vienna in 1926.

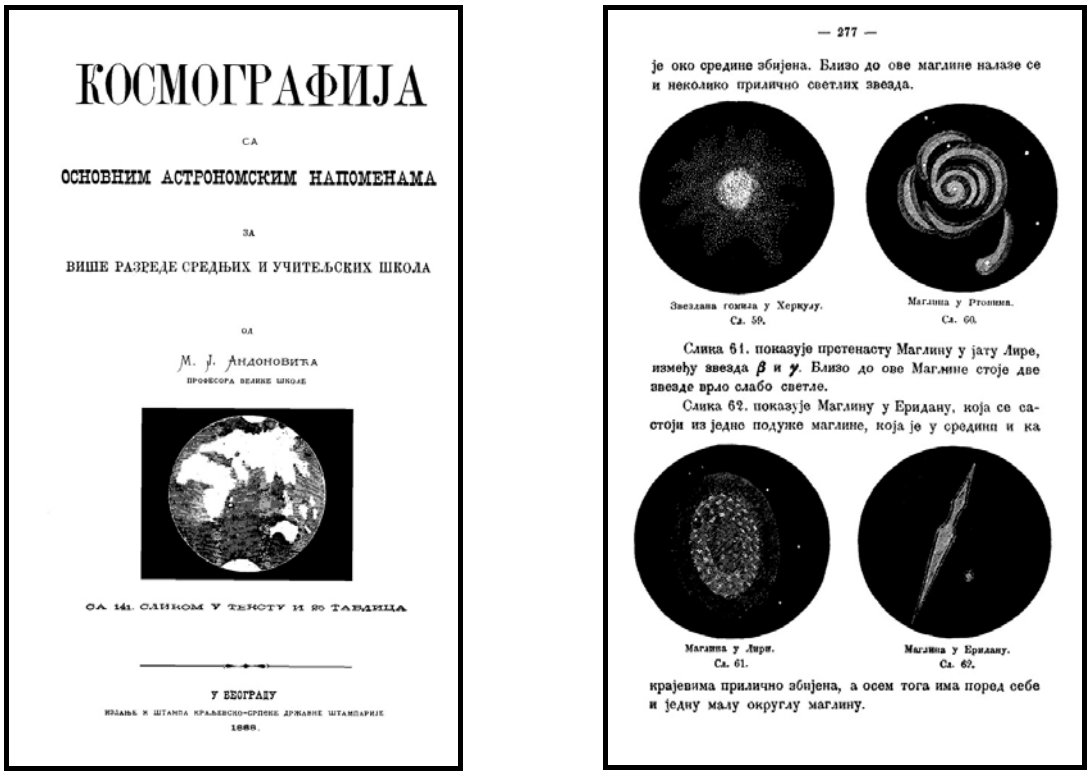
Andonović published a number of scientific papers, books (all printed in Belgrade), and discussions related to surveying, astronomy, shape and size of the Earth, the theory of least squares, survey and cadastre: *Basics probability and the theory of least squares*, 1886; *The shape and size of the Earth*, 1886; *Cosmography with basic astronomical notes for high*

school teachers, 1888; *The volume and size of our Earth*, 1889; *The Universe*, 1889; *Basic geodesy with distinctive view of the cadastral question*, 1890–1897 (1st part 1890, p.368, 2nd part 1897 vol. 1, p. 369–1390; vol. 2, p.1391–1641); *The cadastre*, 1889.

He also wrote during the First World War, political discussions and articles (in German and French) in favor of Serbia. There are the following digital copies of his books in the Virtual Library: *Cosmography*, 1888, and *The Universe*, 1889, both printed in Belgrade.

Cosmography was intended for gzmnasium students and pupils of other secondary schools. Probably, it was used as a secondary textbook at the Military Academy and the Grand School, the forerunner of the Belgrade University. In addition to the foreign literature, Andonović mentioned in the bibliography two Serbian authors: *Trigonometry* of Dimitrije Nešić and *Cosmometry* by Jovan Dragašević. The book has many illustrations: 141 photos and drawings, 17 tables and a large star map. He took most of these drawings, with permission, from a book of E. Weiss, then the director of the Vienna Observatory.

Cosmography is an extensive (533 pages) and quite comprehensive book. It describes celestial sphere, constellations, the shape and size of the Earth, the Earth's rotation and revolution, the apparent movement of the Sun, coordinate systems, time, calendars, Solar system, stars, tides, precession, Cosmogony and among other things, Kant-Laplace's theory about the origin of the World (i.e. the Solar system). Besides all that, there are twenty fully solved astronomical problems with detailed explanations.



КОСМОГРАФИЈА

СА

ОСНОВНИМ АСТРОНОМСКИМ НАПОМЕНАМА

ЗА

ВИШЕ РАЗРЕДЕ СРЕДЊИХ И УЧИТЕЉСКИХ ШКОЛА

ОД

М. Ј. Андоновића

ПРОФЕСОРА ВЕЛИКЕ ШКОЛЕ

СА 141. СЛИКОМ У ТЕКСТУ И 20 ТАБЛИЦА

У БЕОГРАДУ

ИЗДАЊЕ И ШТАМПА КРАЉЕВСКО-СРПСКЕ ДРЖАВНЕ ШТАМПАРИЈЕ

1888.

— 277 —

је око средине збијена. Близо до ове маглине налазе се и неколико прилично светлих звезда.

Звездана гомила у Херкулу. Сл. 59.

Маглина у Ртовима. Сл. 60.

Слика 61. показује протенасту Маглину у јату Лире, између звезда β и γ. Близо до ове Маглине стоје две звезде врло слабо светле.

Слика 62. показује Маглину у Еридану, која се састоји из једне подуже маглине, која је у средини и ка

Маглина у Лири. Сл. 61.

Маглина у Еридану. Сл. 62.

крајевима прилично збијена, а осем тога има поред себе и једну малу округлу маглину.

Pages from the Andonović's *Cosmography*

The book is written in a very beautiful style, almost as a literary work, and is worth reading also from that point of view. There we find many forgotten words and terms whose meaning were forgotten or has changed till today. For example, the cluster (Serb. *zvezdano jato*) there means constellation, stardust (Serb. *ozvezdina*) – meteorite, comet star (Serb. *zvezda repatica*) – comet, the inflow and outflow (Serb. *priliv i odliv*) – the tide. Also, for

each constellation he introduces the *alinjman* – having the meaning similar to asterism. For example the *alinjman* of Big Dipper (in Serbian also called *Velika kola* - Big Chariot) has four stars – the asterism trapeze of the constellation Big Dipper, while the *alinjman* of North Star is the line that passes through the rear wheels of the Big Chariot towards to the Polar Star. The Virtual Library contains digital copies of his books: *Cosmography* (1888) and *The Universe* (1889), both published in Belgrade.

Kosta Stojanović



Kosta Stojanović (1867–1921) was born in Aleksinac. There he finished elementary school and lower secondary school, while the high school he completed in Niš. He graduated in mathematical sciences at the Grand School in Belgrade. For some time he was working as a high school teacher in Niš and Belgrade. He went to Paris in 1893 where he studied mathematics, astronomy, mechanics and physics under the supervision of famous professors Poincaré, Picard and Appell. There he became a member of the Historical Society of France and the French Astronomical Society. He went to Leipzig in 1897 where he entered doctoral studies in mathematics. Unfortunately, he returned to Serbia after three months because of his illness.

Kosta Stojanović was a university professor, but he had a very rich political career, too. At the beginning of 1900th he became a deputy as the representative of the Niš district.


КОСТА СТОЈАНОВИЋ

ПРЕДАВАЊА НА УНИВЕРЗИТЕТУ
ИЗ
ПРИМЕЊЕНЕ МАТЕМАТИКЕ

МЕХАНИКА

БЕОГРАД
ШТАМПАНО У ДРЖАВНОЈ ШТАМПАРИЈИ КРАЉЕВИНЕ СРБИЈЕ
1912.

Since 1903 he taught applied mathematics at the Grand School and at the Belgrade University after it was established in 1905. Whereupon he abruptly interrupted academic career and 1906 he became Minister of Commerce, at the time of the custom war between Belgrade and Vienna.

Around that time, Kosta Stojanović wrote the book *Fundamentals of economy value*. This book is a work in mathematical economics and Stojanović there used very advanced mathematical apparatus, for example, the theory of partial differential equations to describe economic models. Even today there is an interest for this book, not only in Serbia but also in the other parts of former Yugoslavia. This is probably the most advanced book in economics ever printed in Serbia due to the mathematical tools that were used in it.

He was Minister of Agriculture (1919–1920) and Minister of Finance in 1921 in the government of Nikola Pašić. A few days after the appointment as the minister of finance, he suddenly died.

The book *Mechanics* is actually a university textbook in applied mathematics. Let us mention that at that time, under the term applied mathematics it was assumed mechanics, mathematical

physics and celestial mechanics. The most of the book is related to the vector calculus, then the kinematics, statics, dynamics, analytical mechanics and the theory of differential equations. Chapter Ten of this book is devoted to astronomy. There he gave the basics of celestial mechanics and the theory of differential equations of the motion of planets, comets and satellites. Also, the theory of elliptical movement, the method of variation of constants and parabolic motion of comets are presented.

Nikolai Yakovlevich Tsinger (Николай Яковлевич Цингер**)**



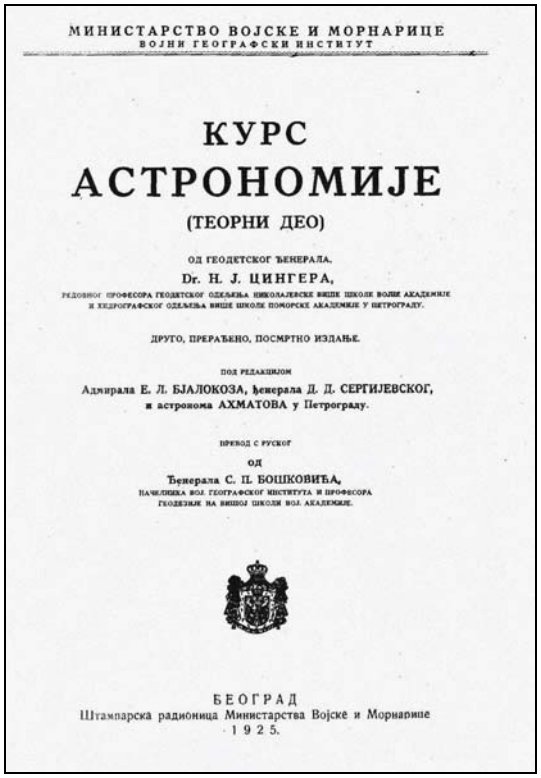
Tsinger was Russian astronomer, but we decided to include his book *Course of Astronomy* into this collection since the translation of this book was an important contribution to the scientific astronomical literature in Serbian. The appearance of this book foreran the new generation of Serbian university textbooks and monographs in astronomy.

Nikolai Tsinger (1842–1918) was a Russian astronomer, the prominent geodesist and cartographer. He is considered as the founder of Russian Geodetic School. He graduated in 1863 Artillery Academy and worked at the Pulkovo Observatory (1872–1883) near St. Petersburg. After that he was the professor at Nicholas Naval Academy and became the lieutenant general (1896). He was also the member of the Imperial St. Petersburg Academy of Sciences. Since 1905 and until 1917 Tsinger was one of the leaders of the Russian Geographical Society (the chairman of the Department of Mathematical Geography). He had a great teaching career and did much to spread in Russia geodetic and astronomical knowledge.

In 1874 Tsinger proposed a method of determining clock corrections from observations of two stars at equal altitudes. This procedure is called “method Tsinger”. Since this method is based on simple observation and produces high precision, it was widespread. Since 1926, Astronomical Institute in Leningrad published “Ephemerides pairs Tsinger" for the practical application of this method. The method is described, for example, in the book *General Astronomy* by B. Ševarlić and Z. Brkić (Belgrade 1971, in Serbian; digital copy of the book is in the Virtual Library).

A cape on Bolshevik Island in the Severnaya Zemlya archipelago is named Tsinger, as well as a crater on the far side of the Moon. Nikolai Tsinger was the younger brother of Vasily Yakovlevich Tsinger (1836–1907), mathematician, botanist, professor at Moscow University and the president of the Moscow Mathematical Society. It is interesting that Nikolai’s name is spelled in the Internet in several different ways (for example Nikolaj Jakovljevič Zinger).

The Tsinger textbook on astronomy *Course on Astronomy* (Russ. *Kurs astronomii - Chast’ teoreticheskaia,* Serb. *Kurs Astronomije*), was translated from Russian to Serbian by academician, general Stevan Bošković (1868–1957), geodesists and professor at Military Academy in Belgrade. The Russian edition of the book was published in 1899, while the Serbian edition appeared in 1925. Bošković translated this book not by chance. He used Tsinger’s method for the purpose of time determination for the points of all geographic latitudes in Serbia (see [**3**], [12]) during his very extensive geodetic measurements in Serbia in the first decade of XX century. Tsinger’s book without doubt was his valuable companion during this exploration. Secondly, he spent some time in the late XIX century studying astronomy and geodesy at the Pulkovo Observatory. Bošković certainly met Tsinger there who was then the leading Russian geodesists and professor in St. Petersburg.


МИНИСТАРСТВО ВОЈСКЕ И МОРНАРИЦЕ
ВОЈНИ ГЕОГРАФСКИ ИНСТИТУТ

КУРС
АСТРОНОМИЈЕ
(ТЕОРНИ ДЕО)

ОД ГЕОДЕТСКОГ ЂЕНЕРАЛА,
Dr. Н. Ј. ЦИНГЕРА,
РЕДОВНОГ ПРОФЕСОРА ГЕОДЕТСКОГ ОДЕЉЕЊА НИКОЛАЈЕВСКЕ ВИШЕ ШКОЛЕ ВОЈНЕ АКАДЕМИЈЕ И ХИДРОГРАФСКОГ ОДЕЉЕЊА ВИШЕ ШКОЛЕ ПОМОРСКЕ АКАДЕМИЈЕ У ПЕТРОГРАДУ.

ДРУГО, ПРЕРАЂЕНО, ПОСМРТНО ИЗДАЊЕ.

ПОД РЕДАКЦИЈОМ
Адмирала Е. Л. БЈАЛОКОЗА, ђенерала Д. Д. СЕРГИЈЕВСКОГ,
и астронома АХМАТОВА у Петрограду.

ПРЕВОД С РУСКОГ
ОД
Ђенерала С. П. БОШКОВИЋА,
НАЧЕЛНИКА ВОЈ. ГЕОГРАФСКОГ ИНСТИТУТА И ПРОФЕСОРА ГЕОДЕЗИЈЕ НА ВИШОЈ ШКОЛИ ВОЈ. АКАДЕМИЈЕ.

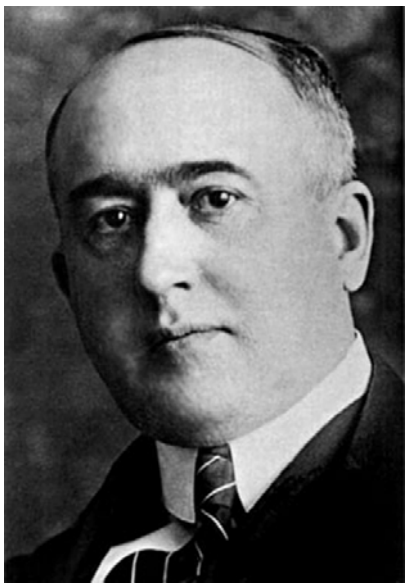
БЕОГРАД
Штампарска радионица Министарства Војске и Морнарице
1925.

The cover page of *Course of astronomy*

This book was far the most advanced book comparing to the existing astronomical books in Serbian at the time of appearance. It covers all main topics in general astronomy, astrometry and celestial mechanics. All together there are 27 chapters that spread out on 330 pages which explain in great details spherical trigonometry, all main astronomical notions, formulas and astronomical measurements and computations. Meticulous care is given to the mathematical apparatus, particularly to the theory of errors and related topics such as the least squares method. The book was intended for students of geodesy and hydrography and the students at military academies. Tsinger book significantly influenced development of astronomy in Serbia, particularly astronomical applications in geodesy. The book should be searched in the Virtual Library under the author's name *Cinger*.

Milutin Milanković

Milanković was not the first Serbian astronomer. However he was certainly the most recognized and most important Serbian astronomer and scientist as well. His work is enormous and so important that it is not possible in a short article as this one to present even in a part all his scientific achievements. However, the collection of Milanković's works in the Virtual Library is the richest one, so we decided to include this short note. For more details on his the reader is directed to more complete articles on his biography.



Milutin Milanković was born in 1879 in Dalj (Slavonia, nowadays part of Republic of Croatia). He becomes a student of civil engineering at the Technische Hochschule in Vienna. There he took the first degree in 1903 and the PhD one in 1904. He was the first Serb who acquired the PhD title in engineering. Following an invitation Milanković came to Belgrade from Vienna in 1909 to teach applied mathematics at the Faculty of Philosophy of Belgrade University. His coming was a merit of Bogdan Gavrilović and Mihailo Petrović Alas who both taught mathematics at the University. Milanković was at the Belgrade University till his retirement in 1955. He taught applied mathematics (theoretical physics, mechanics and astronomy). He was the first in starting lectures in celestial mechanics.

Milanković was a full member of the Serbian Academy of Sciences, member of Deutsche Akademie der Naturforscher Leopoldina in Halle and corresponding member of a number of other academies and scientific societies in the world. Milanković died in 1958 in

Belgrade. His activity resulted in about ten books and more than hundred papers in mathematics, celestial mechanics, astronomy and geophysics.

Milutin Milanković is best known for his theory of glacial ages. The theory takes into account the complex secular computation of perturbations in planet motions and it were published in *Théorie mathématique des phénomènes thermiques produits par la radiation solaire* in 1922. Due to these results he became well known in the world scientific community, so that great German climatologist W. Köppen invited him to cooperate in building a great work *Handbuch der Klimatologie*.



Љ. Ивановић

I

На дунавском пароброду „Сатурнус"

Баш пред сам полазак из Беча, примио сам Ваше мило писмо. Ево Вам одговарам још успут, на белом дунавском броду који ме носи отаџбини. Седећи на његовим широким лећима, гледам како пролазе поред мене подунавски предели: равнице и брежуљци, старе рушевине и модерне вароши; не хитно и дрхтаво, као што то изгледа са брзога воза, него лагано и достојанствено. Тако могу у исти мах да пишем и да се дивим лепотама природе, у које с правом убрајам и једну витку сапутницу, која се, са Бедекером и лорњоном у руци, грациозно шета по пространој палуби брода.

Ви ми предлажете, поштована пријатељице, да наше никад недовршене разговоре о васиони, њеним чудесима, загонеткама и судбинама, продужимо писменим путем. Ја врло радо прихватам тај Ваш предлог, нако са зебњом да Вас наша преписка, од које очекујете толико задовољства, не разочара. Преписка није разговор; њој ће, пре свега, недо-

9

A page from *Through Space and Centuries*

For this purpose a part, which was published in 1930, entitled *Mathematische Klimalehre und Astronomische Theorie der Klimaschwankungen* was written by Milanković. Here his theory of planet heating based on insolation was extended with a special reference to the Earth. By applying this theory to the run of glacial ages it was shown that Milanković had created a good and mathematically exact theory of terrestrial climate. Milanković also created a theory of motion of terrestrial poles and succession of glacial ages. His main work is *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* printed shortly before the beginning of the Second World War. Due to this the scientific world community learnt about Milanković's theory with some delay. As a consequence the world recognition arrived with postponement. This work appears as a synthesis of his earlier works which concern his research activity within boundary fields between many natural sciences and mathematics.

Milanković also proposed an improvement of the Gregorian time reckoning. It was accepted by the Orthodox Christian Community during a Congress of Orthodox Churches in Constantinople in 1923. Milanković's calendar-reform was accepted by the Orthodox Christian Community, but not by all Churches, including the Serbian one (see the article [**5**]).

Milanković also wrote several books on history and popularization of science and autobiography books, for example *Through Space and Centuries*, *Through the Realm of Science* and *Recollection, Experiences and Vision*.

After Milanković a minor planet, a lunar crater and a Mars crater have been named. In Belgrade one street and one gymnasium have been named after him. European Geosciences Union established in 2002 an award *Milutin Milankovic Medal* for achievement in the area of climate research.

We shall shortly review his book *Through Space and Centuries*. The book fictionalized the development of astronomy. Milanković highly cherished the written word. The novel explains the scientific history of astronomy written as a travelogue prose. It consists of a collection of letters sent by Milanković to the loved one describing the long imaginative journey through the past of millions of years and travel through the deep Universe. Letters represent a treasury of scientific explanations and portraits of famous scientist: Eratosthenes, Ptolemy,

Pythagoras, Copernicus, Kepler, Newton, Einstein and Tesla. The book is a collection of records about the cradle of wisdom, scientific knowledge and philosophical views of ancient people of Babylon, China, India, old Greece and Rome up to the modern times. This book of stories about the ancient beauties and thoughts written in the splendor language has become his brand name. Five editions of the book were published in Serbian and two editions in German.

Pavle Vujević and Vojislav Mišković

Pavle Vujević (1881–1966) is known for his work in mathematical geography. His major work is the university textbook *Fundamentals of the mathematical and physical geography* published in 1924 in Belgrade. A review of this book and details on Vujević's biography can be found in the article [**8**].

Vojislav Mišković (1892–1976) is a prominent Serbian astronomer from the first half of the XX century. Most of his books (10) are digitized and included into Virtual Library. This collection includes *Cosmography* (1931, Belgrade) and *Collection solved problems in general astronomy* (Part I, 1956, Belgrade; Part II, electronic edition, Virtual Library, 2009). Mišković's biography and review of this collection is given in article [**9**].

Conclusion

The Virtual Library of the Faculty of Mathematics has a significant place in the presentation of old and rare books to the Serbian scientific and professional community and general public wide-world. This fact we can convince, for example, by the choice of books from the collection of astronomical works of the Serbian scholars presented in this article.

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Сажетак. Виртуелна библиотека, http://elibrary.matf.bg.ac.rs, Математичког факултета у Београду садржи око 80 дигитализованих књига из астрономије које су написали углавном Српски научници. У овом раду представља се избор књига написаних до педесетих година XX века следећих аутора: Руђер Бошковић, Захарије Орфелин Стефановић, Атанасије Стојковић, Ђорђе Станојевић, Милан Андоновић, Коста Стојановић, Николай Яковлевич Цингер и Милутин Миланковић.

Кључне речи: Виртуелна библиотека, астрономске књиге, историја астрономије.

Supplement: Early astronomical books and works in Virtual Library

(with active links in electronic version of this paper)

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Faculty of mathematics written in
the XVIII and XIX century
N. Pejović and Ž. Mijajlović
Publ. Astron. Druš. Rudjer Bošković, Beograd
Vol. 10, pp. 785-800, 2011
Proc. Conf. "Razvoj astronomije kod Srba VI"
Beograd, 22-26 april, 2010
(M. Dimitrijević, ed).

This article in the most extent is a translation of the previous review "Early astronomical heritage in Virtual Library of Faculty of Mathematics in Belgrade" into Serbian.

Зборник радова конференције "Развој астрономије код Срба VI"
Београд, 22-26. април 2010,
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 10, 2011, 785-800

АСТРОНОМСКЕ КЊИГЕ ИЗ 18. И 19. ВЕКА У ВИРТУЕЛНОЈ БИБЛИОТЕЦИ МАТЕМАТИЧКОГ ФАКУЛТЕТА

НАДЕЖДА ПЕЈОВИЋ и ЖАРКО МИЈАЈЛОВИЋ

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Резиме: У Виртуелној библиотеци Математичког факултета, http://elibrary.matf.bg.ac.rs, има више од 50 дигитализованих књига из астрономије српских писаца. У овом чланку представљамо избор књига из ове колекције аутора из 18. и 19. века: Руђера Бошковића, Захарија Орфелина, Атанасија Стојковића, Ђорђа Станојевића, Милана Андоновића и Косте Стојановића. За сваког од ових аутора укључена је кратка биографија.

УВОД

Виртуелна библиотека Математичког факултета је највећа база дигитализованих текстова у Србији отворена за јавно коришћење. У овом тренутку садржи близу 800 књига. Важне колекције у оквиру ове библиотеке су збирка од 360 докторских дисертација из математичких наука од којих је највећи део одбрањен на Математиком факултету Универзитета у Београду и ретке књиге из 18. и 19. века. Већи део ових књига су раритети, неке од њих постоје само у једном примерку и фактички су недоступне јавности.

У овом раду представљамо колекцију књига српских аутора из астрономије штампаних у 18. и 19. веку које се налазе у дигитализованом облику у Виртуелној библитеци. То су прве српске астрономске књиге, па се може рећи и прва дела из науке српских писаца. Споменимо овде једну општу карактеристику ових првих српских научника. Њих је красила универзалност, дакле астрономија није била њихова једина специјалност. Исто тако били су успешни и у другим наукама, математици, физици или филозофији, на пример. За њих је наука по највише био поглед на јединствено биће – природу. Језиком науке служили су се да опишу природу тог бића. Поред свега, често су били успешни политичари, уметници, путници, писци романа и песама. Њихов значај за развој српске културе,

језика и науке је огроман. Стога поред њихових рукописа из астрономије укратко представљамо њихов животопис и друга дела.

КЊИГЕ ИЗ АСТРОНОМИЈЕ ИЗ XVIII И XIX ВЕКА

У Виртуелној библиотеци налазе се 15 књига следећих аутора:

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Захарије Стефановић Орфелин (1726-1785), *Вечни календар,* 1783, Беч.

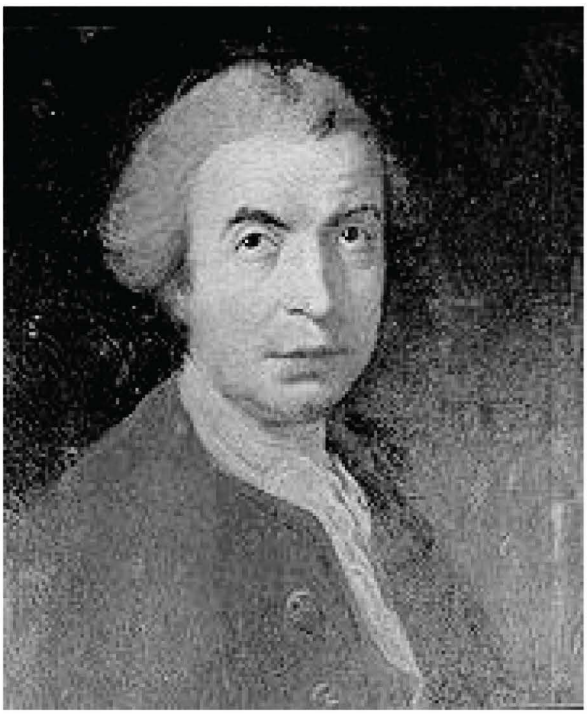
Атанасије Стојковић (1773-1832), *Фисика,* 1810, Будим.

Ђорђе Станојевић (1858-1921), највећи део његових дела; дигитализовано је 8 књига укључујући: *Звездано небо независне Србије,* 1882, Београд.

Милан Андоновић (1849-1926), *Космографија* (1888), *О Космосу* (1889), Београд.

Коста Стојановић (1867 - 1921), *Атомистика Руђера Бошковића,* 1892, Ниш.

КЊИГЕ РУЂЕРА БОШКОВИЋА



Слика 1.

Руђер Бошковић (1711-1787) је највећи математичар и астроном Дубровника, један од најзначајнијих научника свога времена, уврштен међу 100 најзначајнијих Срба свих времена. Био је професор универзитета, оснивач Миланске опсерваторије и директор Оптичког института француске морнарице. Био је универзалан стваралац: филозоф, математичар, астроном, физичар, геолог, архитекта, археолог, дипломата, путописац, професор, песник и полиглота.

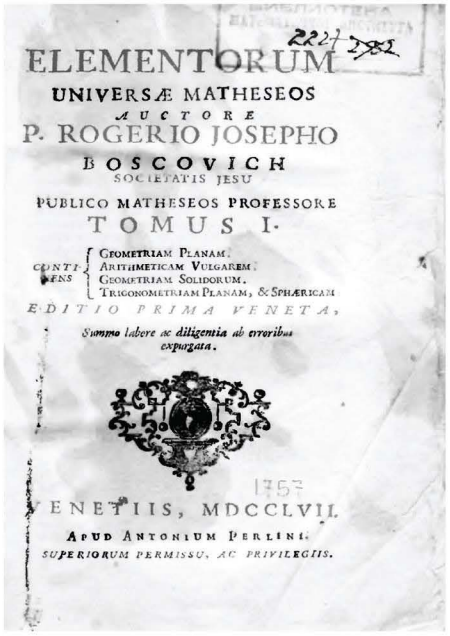
Рођен је 18. маја 1711. као седмо дете трговца Николе Бошковића (Србина из Орахова Дола код Требиња у Херцеговини) и мајке Павле (италијанског порекла, из породице Бара Бетере, познатог дубровачког песника). Живео је углавном у Италији, где је стекао и светску славу. Бошковић је умро 13. фебруара 1787.

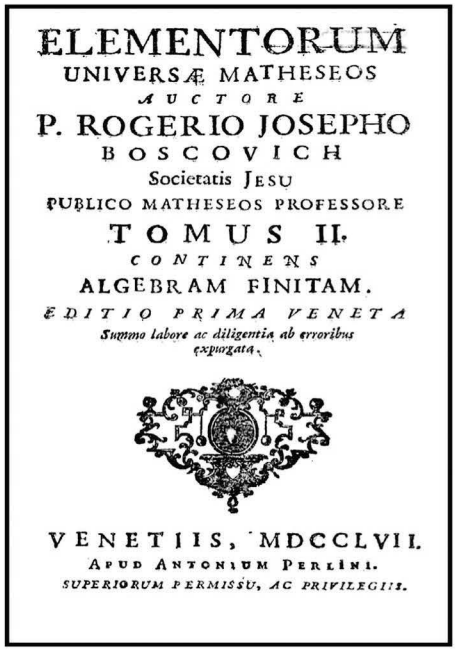
Формулисао је јединствен закон силе, узимајући да постоји не само привлачење (Њутнов закон), већ и одбијање у наизменичном мењању на малим растојањима међу телима. По њему је елементарна честица без димензија извор силе, а време и простор је, насупрот Њутну, узимао релативним. Отуда се Бошковић сматра претечом Алберта Ајнштајна.

Руђер Бошковић имао је у астрономији многобројне доприносе, између осталог: увео је две геометријске методе за одређивање елемената Сунчеве ротације на основу посматрања положаја три тела; израчунао је димензије и спљоштеност Земље; откривао је комете. Први је (1782) тачно одредио путању Урана на основу мерења координата Урана која је добио од Месјеа и Мешена. Споменимо да се у то време за Уран сматрало да је комета. Израчунавањем путање Урана усавршио је методу за одређивање орбита комета на основу 4 мерена положаја, која је око 1800. важила за једну од најтачнијих метода. Такође је изучавао атмосферу Јупитера и природу ауроре (поларне светлости).

Руђер Бошковић објавио је велики број радова из математике, астрономије и физике. Његове познате књиге су *Theoria philosophiae naturalis redakta ad unicam legem virium in natura existentium*, *Opera partinentia ad opticam et astronomia* и *Elementorum universae matheseos*.

У Виртуелној библитеци налазе се његова књиге: *Елементи математике* (*Elementorum Universae Matheseos - Tomus I-III*, 1757, Venetis), *Дневник са пута из Цариграда у Пољску* (1762), превод Д. Нељековића, 1937, Београд, *О закону континуитета и његовим последицама у односу на основне елементе материје и њихове силе*, 1754, превео Е. Стипанић, 1975, Београд.


ELEMENTORUM
UNIVERSÆ MATHESEOS
AUCTORE
P. ROGERIO JOSEPHO
BOSCOVICH
SOCIETATIS JESU
PUBLICO MATHESEOS PROFESSORE
TOMUS I.
CONTINENS
GEOMETRIAM PLANAM,
ARITHMETICAM VULGAREM,
GEOMETRIAM SOLIDORUM,
TRIGONOMETRIAM PLANAM, & SPHÆRICAM
EDITIO PRIMA VENETA,
Summo labore ac diligentia ab erroribus expurgata.
VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.


ELEMENTORUM
UNIVERSÆ MATHESEOS
AUCTORE
P. ROGERIO JOSEPHO
BOSCOVICH
Societatis JESU
PUBLICO MATHESEOS PROFESSORE
TOMUS II.
CONTINENS
ALGEBRAM FINITAM.
EDITIO PRIMA VENETA
Summo labore ac diligentia ab erroribus expurgata.
VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.

Слика 2.

НАДЕЖДА ПЕЈОВИЋ и ЖАРКО МИЈАЈЛОВИЋ

ЗАХАРИЈЕ СТЕФАНОВИЋ ОРФЕЛИН

Орфелин (1726 – 1785) је био истакнути српски песник, историчар, бакрорезац, барокни просветитељ, гравер, калиграф и писац уџбеника. Рођен је у српској породици у Вуковару 1726. од оца Јована.

Међу његова најзначајнија дела убраја се *Славеносербски магазин* штампан у Венецији 1768. То је први јужнословенски часопис. Мада је изашао само један број, његов значај је велики. У предговору износи идеју грађанске просвећености, да наука, књижевност и филозофија треба да изађу из уског круга учених људи и да постану доступни свима. Године 1768. Захарије Орфелин је први увео у српски књижевни језик - мешавину црквенословенског и народног језика, у којој су се налазиле и многобројне руске речи. Тиме је практично основао славеносрпски језик.

Орфелин је аутор првог српског буквара из 1767. по којем су училе бројне генарације ђака. Аутор је првих уџбеника латинског језика. Његово најопсежније дело је *Житије Петра Великог* (Венеција, 1772) у коме је видео узор просвећеног монарха, што је иначе био филозовски идеал 18. века. Написао је и *Вечни календар*, штампан 1783. у Бечу, у којем се уз календарске податке налази и обимно поглавље о астрономији.

Због његових уметничких радова, пре свега у бакрорезу, изабран је седамдесетих година 18. века за члана Уметничке академије у Бечу.

Орфелинов *Вечни календар* штампан је у Бечу 1783 у штампарији Јосифа Курцбека. О садржају књиге и намерама с којима је писана најбоље говори пуни наслов који гласи:

Вечни, то јест од почетка до краја света трајући календар који у себи садржи свецелослов и кратке, по рачунању источне цркве, годишње кругове и друге природне законе; уз то и физичке о небеским телима, и о воденим и ваздушним додатним расуђивањима; прибављеним из свештених и светских хронологија сада први пут на словенском језику на корист словеносербским народима.

Књига *Вечни календар* Захарија Орфелина написана је на 366 страница са 9 цртежа из астрономије на крају књиге. Садржи углавном астрономске и природњачке научне појмове, као и историјске догађаје од настанка света који су хронолошки изложени. Поглавља из астрономије носе називе: *О Свету* (свемиру*), О Месецу и планетама, О кометама, О помрачењу Сунца, О помрачењу Месеца*. Ту се налазе подаци о временским циклусима, таблице са подацима о Сунцу и Месецу (за рачунање датума Ускрса по Великом индикту), таблице о дужини трајања дана и ноћи, таблице Месечевих мена и друге. Стога *Вечни календар* представља прву књигу из астрономије написану на словеносрпском језику.



Слика 3.

АТАНАСИЈЕ СТОЈКОВИЋ



Слика 4.

Стојковић је рођен у Руми 1773, где је учио Вазнесењску школу (Српску гимназију). Убрзо одлази у Шопрон, где 1793/94. завршава прво шест разреда гимназије, а потом се уписује у Гимназију у Сегедину где 1796. завршава филозофију. Покушава да пронађе добротворе за наставак школовања у Немачкој. У Бечу, у јулу 1797. среће Доситеја Обрадовића и упознаје се са његовим просветитељским идејама. У јесен исте године од Митрополита Стратимировића добија помоћ за школовање у Гетингену, у то доба један од најпознатијих универзитета. Тамо докторира филозофију (1799), и почиње ин-

тензивно да се бави различитим наукама (физика, математика, астрономија, историја). После краћег боравка у Руми 1799. прелази у Будим где завршава и објављује своје главно дело ***Фисика*** (1801-1803), написано у 3 тома.

У Хабзбуршкој монархији било је тешко добити државну службу, па на позив Северина Осиповича Потоцког, будућег министра просвете Русије, Атанасије прихвата избор за редовног професора физике новооснованог Харковског Универзитета. Убрзо постаје декан одељења физичких и математичких наука, а у два наврата (1807-1808 и 1811-1813) и ректор Универзитета у Харкову. Оснивач је харковског Ученог друштва и добија многе почасти. Постао је члан Царске академије наука, добија од цара Александра I орден светог Владимира трећег степена, материјалне привилегије и постаје државни саветник.

У Харкову Атанасије Стојковић је објавио велики број дела писаних на руском језику, углавном намењених студентима. Посебно се истиче књига из астрономије *О воздушных камнях и их происхождении* (О метеоритима и њиховом пореклу) објављена 1807. а настала када је Стојковићу у руке доспео комад "ваздушног камена" који је 1787. пао код села Жигајловка у Харковској губернији. Сматра се да је ова књига прво дело о метеоритима у свету и њено објављивање је знатно утицало на развој науке о метеорима. У Тунгуској области, где је 30. јуна 1908. године експлодирао чувени велики метеорит, постоји брежуљак висок 150 метара који је назван по овом научнику – Стојковићево брдо. Планирано је да се дигитална копија ове књига постави у Виртуелну библиотеку. Примерак те књиге налази се у Музеју Српске православне цркве у Патријаршији у Београду. Поред поменутих књига Стојковић је такође написао и ове књиге, обе објављене 1813:

Начальныя основания физической географии (Почетне основе физичке географије).

Начальныя основания физической астрономии (Почетне основе физичке астрономије).

Атанасије Стојковић је био вредан и способан човек, необузданог духа, веома успешан и цењен. Говорио је више језика: немачки, латински, француски, италијански, енглески, грчки, мађарски и скоро све словенске језике. Имао је занимљив и узбудљив живот који је поред науке био обележен једном афером која је окончала његову професорску каријеру. Умро је у Харкову 1832.

Својом свестраношћу, амбициозошћу и интелигенцијом, вероватно под утицајем просветитељских идеја Доситеја Обрадовића, Стојковић је писањем књига покушавао да просвети свој народ, да унапреди његову науку и културу, у време када се тек назирао настанак нове Српске државе.

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Стојковићева *Фисика* састоји се из 3 тома. Прва два тома имају по 320 страница док је трећи том написан на 284 странице. Књига представља пресек дотадашњих сазнања о природи и природним појавама. У првом тому на почетку је изложено поглавље о механици (о својствима и кретањима тела). Највећи део првог тома (друго и треће поглавље) посвећен је астрономији. Описана су небеска тела, привидно кретање небеске сфере, звезде, Сунчев систем, Сунце, планете, комете, Месец, помрачења. У трећем поглављу описана је Земља. Уведени су појмови: небески екватор, хоризонт, зенит и надир, затим ширина и дужина места на Земљи, величина Земље, Земљина кретања, неједнакост дана и ноћи на Земљи, Земљина унутрашњост. Такође су описане геофизичке карактеристике планина, река и потока. У другом тому пише о морима, језерима и ветровима. Једно поглавље је посвећено светлости. У трећем тому описани су атмосфера (ваздух), и атмосферске појаве: облаци, магла, роса, снег.



АѲАНАСІА СТОЙКОВИЧА

Свободныхъ художествъ и Філософіи Доктора и Ієнскаго естествоиспытателнаго содружества члена дѣйствителнаго.

ФѴСІКА.

простымъ языкомъ списана за родъ Славенно — Сербскій.

Петра Поповића

12.

Первая часть.

ВЪ БУДИМѢ

Писмены Кралевскаго Університета

1801.

АѲАНАСІА СТОЙКОВИЧА

Свободныхъ Художествъ и Філософіи Доктора, Кралев. Гетінгскаго содружества наукъ Члена внѣшняго соотвѣтствующаго и Ієнскаго естествоиспытателнаго содружества Члена дѣйствующаго

ФѴСІКА

простымъ языкомъ списана за родъ Славено = Сербскій.

Трета часть.

ВЪ БУДИМѢ,

Писмены Кралевскаго Університета

1803.

Слика 5.

Књига је, како је Стојковић просветитељски надахнуто испод наслова написао, *простим језиком списана за род Словено-Сербски*. Писао је народним језиком, без формула и сложених математичких једначина, с циљем да знања о астрономији и природи приближи што више народу у својој домовини.

НАДЕЖДА ПЕЈОВИЋ и ЖАРКО МИЈАЈЛОВИЋ

ЂОРЂЕ СТАНОЈЕВИЋ



Слика 6.

Ђорђе Станојевић (1858-1921) рођен је у Неготину. Завршио је Велику школу у Београду. Потом је био асистент Велике школе код свог професора Косте Алковића (1836-1909) а затим суплент у Првој београдској гимназији 1883. Као питомац војног Министарства био је 1883-1887. на студијама и раду у најпознатијим астрономским и метеоролошким опсерваторијама Европе (Потсдам, Хамбург, Медон, Гринич, Кју, Пулково). Због својих резултата добио је позив од стране Париске опсерваторије да учествује у научно-истраживачкој експедицији ради проучавања Сунца и то у Петровску где је посматрао и проучавао потпуно помрачења Сунца. Две године касније учествује у другој научно-истраживачкој експедицији ради проучавања Сунца и термичког спектра у Сахари. У то време, и нешто касније, објавио је неколико научних радова из астрономије у издањима Париске академије наука. То су први научни радови из астрономије код Срба.

Станојевић је био свестран, образован и са пуно активног интересовања за многе области науке и технике. Његово занимање за физику било је велико. По повратку у Србију посветио се изучавању модерне физике. Предаво је физику и механику на Војној академији, примењену физику и механику на Великој школи, а по оснивању 1905 Београдског универзитета и на Универзитету. Заслужан је за градњу хидроелектрана по Србији (Вучје, Неготин, Ужице, Ивањица и друга места) и електрификацију Београда. То је увело Београд у прве престонице Европе са потпуним електричним осветљењем. Умро је у Паризу 1921.

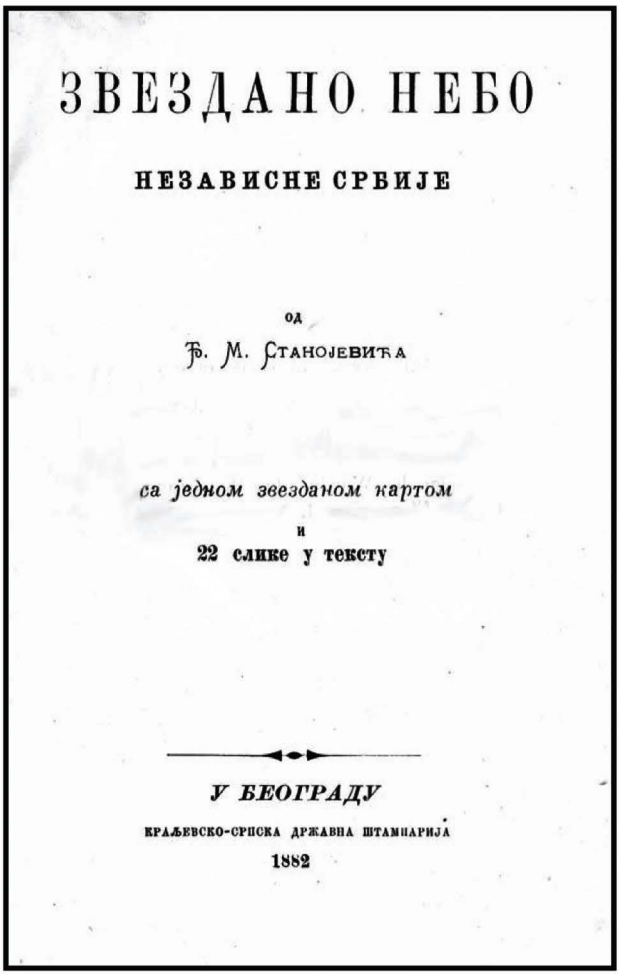
У Неготину постоји Трг Ђорђа Станојевића и у Електродистрибуцији Неготина музеј посвећен њему, а на Новом Београду од 2005. постоји улица *Ђорђа Станојевића*. Електропривреда Србије поводом свог дана, 6. октобра, додељује признање *Ђорђе Станојевић* за допринос у развоју српске електропривреде.

У Виртуелној библиотеци налазе се осам дигитализованих књига Ђорђа Станојевића:

Звездано небо независне Србије, 1882, Београд; *Шетња по облацима,* 1884, Београд; *Васионска енергија и модерна* физика (Уводно предавање, професора Ђ. М. Станојевића, држано 22. септембра 1887. год. приликом, ступања на Катедру физике на Војној академији у Београду), 1887, Београд;

Никола Тесла и његова открића, 1894, Београд; *Из науке о светлости*, 1895, Београд; *Централне силе у природи,* 1906, Београд; *Индустрија хладноће*, 1909, Београд; *Нетачно познавање Васкрсења у православној цркви и реформа календара,* 1908, Београд

Звездано небо независне Србије професора Ђорђа Станојевића је једна од првих књига из астрономије написана на српском језику. Књигу је издала Краљевско-српска државна штампарија 1882. у Београду. Дигитализован је примерак књиге који се налази у Универзитетској библиотеци у Београду, заведен под инвентарским бр. 13006. Књига је по обиму мала али садржајна и представља прави бисер наше научне литературе деветнаестог века. Ђорђе Станојевић учио је школе у европским центрима и био је добар познавалац страних језика, тако да је књигу написао коришћењем онда доступне стране литературе. Станојевић као референце наводи две француске и једну немачку књигу.



ЗВЕЗДАНО НЕБО

НЕЗАВИСНЕ СРБИЈЕ

ОД

Ђ. М. СТАНОЈЕВИЋА

са једном звезданом картом

И

22 слике у тексту

У БЕОГРАДУ

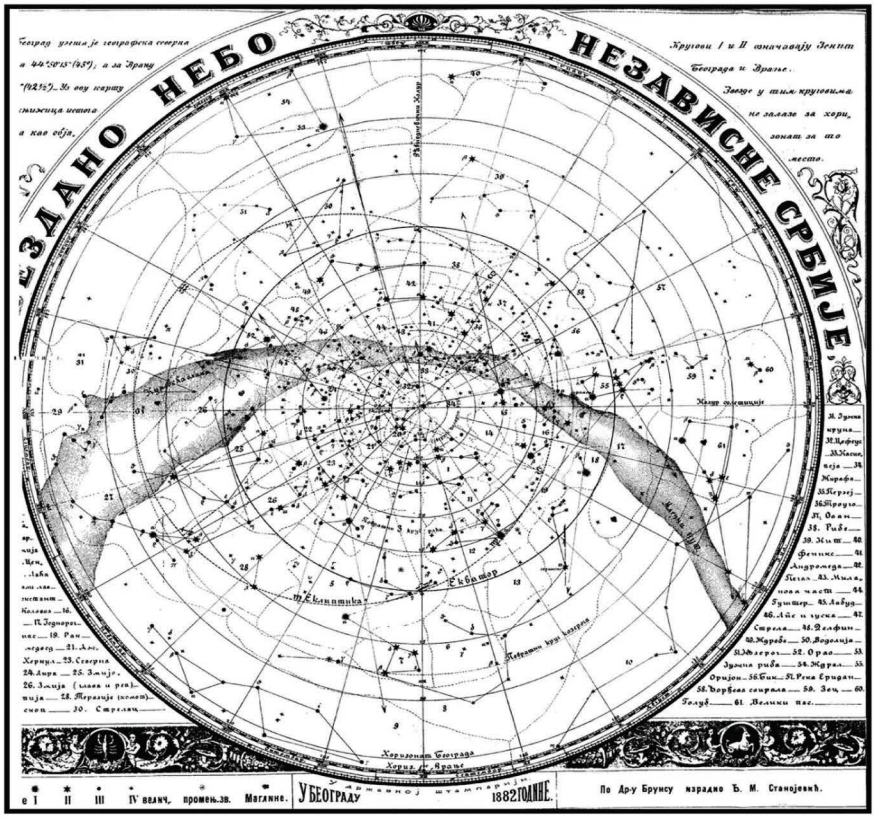
КРАЉЕВСКО-СРПСКА ДРЖАВНА ШТАМПАРИЈА

1882

Слика 7.

Књига је написана на 70 страница, састоји се од предговора, десет поглавља, једне звездане карте и има 22 слике. Књигу чине поглавља: *Звезде уопште, Астрономска подела неба, Број, даљина и подела звезда по величини; звездане карте, Звездана јата и њихове звезде, Промена звезданог неба услед прецесије, нутације, аберације и рефракције, Право кретање звезда, Променљиве и нове звезде, Двогубе звезде, Звездане гомиле и маглине* и *Млечни пут*.

Мада по обиму није велика, по називима поглавља можемо закључити да је читалац оног времена могао да стекне солидан увид у астрономију. У књизи су астрономски појмови објашњени на једноставан и прегледан начин лепим језиком уз пратеће цртеже и илустрације, Станојевић је посебну пажњу посветио звезданом небу, описима сазвежђа, оријентацији на небу помоћу сазвежђа и коришћењу звездане карте.

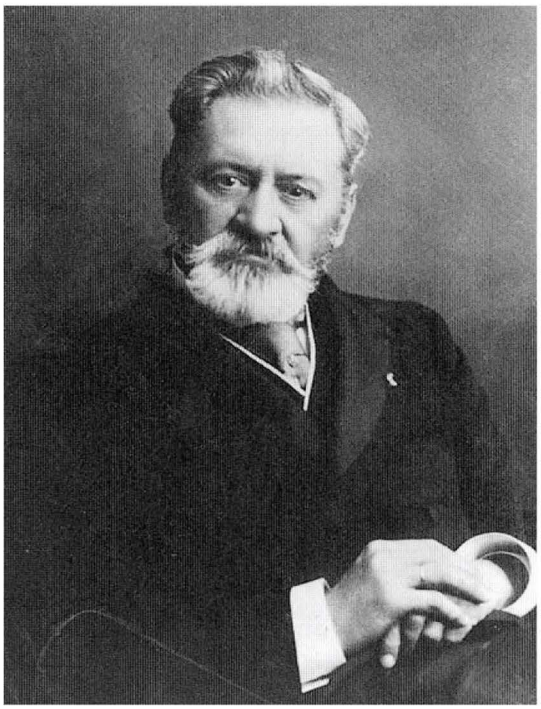


Слика 8.

Посебно истичемо Станојевићеву једоставност и лакоћу писања при излагању често сложених астрономских појава. Ове појаве, мада сложене, углавном описно представља без математичких израза уз коришћење веома лепих графичких приказа. Стога је ова књига пре свега опште образовна и намењена широј читалачкој публици. У том погледу Станојевић је постигао свој циљ, да што већем броју читалаца приближи основна знања из астрономије. Велико нам је задовољство што смо ову драгоцену књигу извукли из таме библиотечког магацина на светлост дана Виртуелне библиотеке и учинила је лако доступном будућим читаоцима

МИЛАН АНДОНОВИЋ

Милан Андоновић (1849-1926), рођен је у Пожаревцу где је завршио гимназију. Школовао се у Београду на техничким факултетима Велике школе у Београду и немачких универзитета у Карлсруе-у, Ахену и Минхену код познатих професора Ритера, Јордана и Хелмерта. Тамо се специјализирао у механици, геодезији и астрономији. По повратку у Србију 1880. постављен је за професора геодезије на Великој школи. Заслужан је за увођење и извршење катастра у Србији крајем 19. века. Основао је Геодетски завод под чијим су окриљем у то време премерене многе вароши у Србији, а 1907. Геодетску и Грађевинску Академију где су се школовали многи српски



Слика 9: *Милан Ј. Андоновић.*

геометри и грађевинари. Заједно са Јованом Драгашевићем, Миланом Недељковићем и Ђорђем Станојевићем заслужан је за увођење наставе из астрономије у српским школама и афирмацију ове науке у Србији. Умро је у Бечу 1926.

Андоновић је објавио већи број научних и стручних радова, књига и расправа везаних за геодезију, астрономију, облик и величину Земље, теорију најмањих квадрата, премер, катастар и комасацију земљишта:

Основе рачуна вероватноће и теорије најмањих квадрата, 1886, Београд.

Облик и величина Земље, 1886, Београд.

Космографија са основним астрономским напоменама за више разреде средњих и учитељских школа, 1888, Београд.

О обиму и величини наше Земље, 1889, Београд.

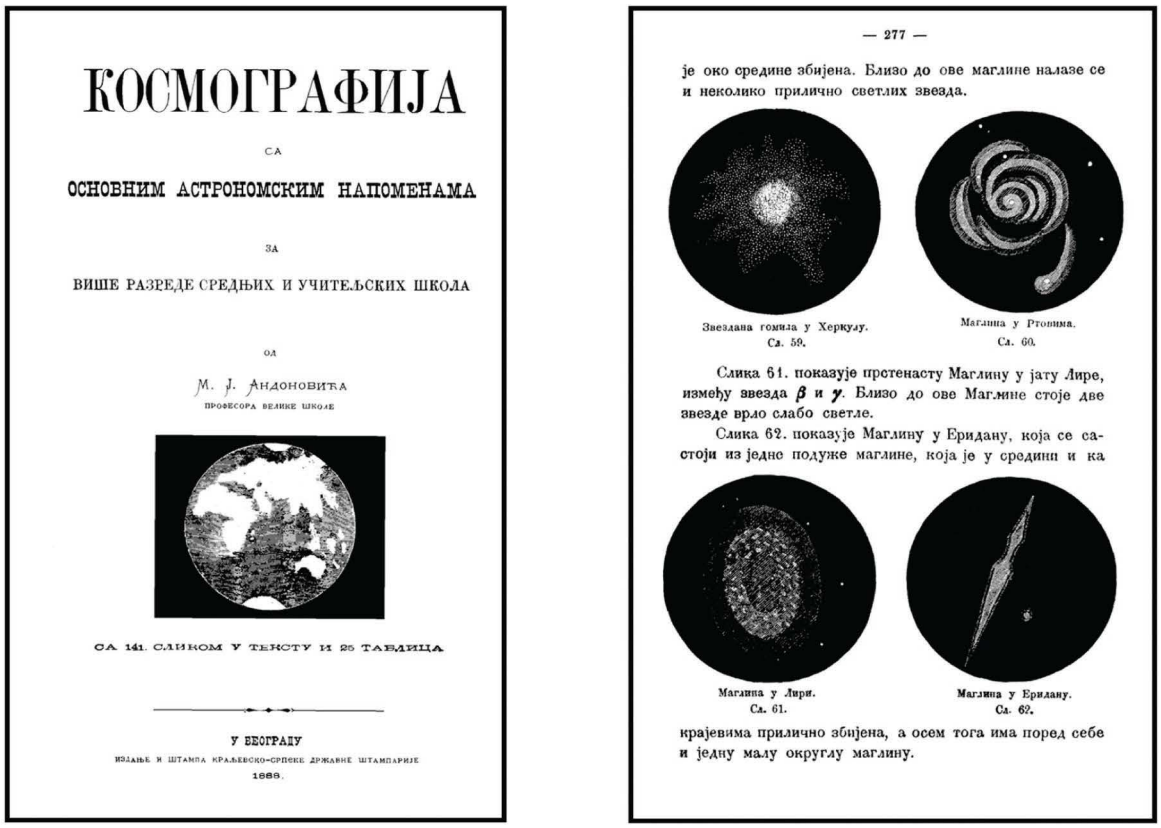
О космосу, 1889, Београд.

Нижа геодезија са особитим погледом на катастарско питање, 1890-1897 (1. део 1890, 1-368; 2. део 1897, свеска 1, 369-1390; свеска 2, 1391-1641), Београд.

О катастру, 1889, Београд.

Такође је за време Првог светског рата писао политичке расправе и чланке (на немачком и француском) у одбрану Србије.

У *Виртуелној библиотеци* налазе се дигиталне копија његових књига: *Космографија* (1888) и *О космосу* (1889), обе објављене у Београду. Ове књиге налазе се у библиотеци Астрономске опсерваторије у Београду. Књиге су дигитализоване захваљујући љубазности Војиславе Протић-Бенишек.



КОСМОГРАФИЈА

СА

ОСНОВНИМ АСТРОНОМСКИМ НАПОМЕНАМА

ЗА

ВИШЕ РАЗРЕДЕ СРЕДЊИХ И УЧИТЕЉСКИХ ШКОЛА

ОД

М. Ј. Андоновића

ПРОФЕСОРА ВЕЛИКЕ ШКОЛЕ

СА 141. СЛИКОМ У ТЕКСТУ И 25 ТАБЛИЦА

У БЕОГРАДУ

ИЗДАЊЕ И ШТАМПА КРАЉЕВСКО-СРПСКЕ ДРЖАВНЕ ШТАМПАРИЈЕ

1888.

— 277 —

је око средине збијена. Близо до ове маглине налазе се и неколико прилично светлих звезда.

Звездана гомила у Херкулу. Сл. 59.

Маглина у Ртопима. Сл. 60.

Слика 61. показује пртенасту Маглину у јату Лире, између звезда β и γ. Близо до ове Маглине стоје две звезде врло слабо светле.

Слика 62. показује Маглину у Еридану, која се састоји из једне подуже маглине, која је у средини и ка

Маглина у Лири. Сл. 61.

Маглина у Еридану. Сл. 62.

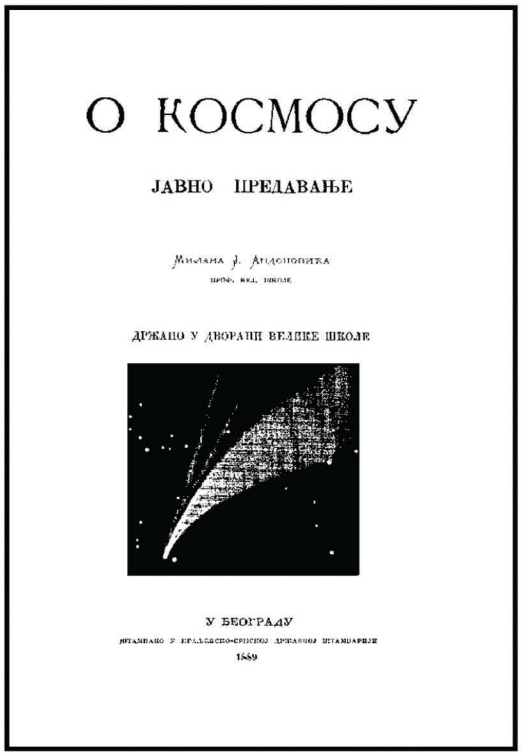
крајевима прилично збијена, а осем тога има поред себе и једну малу округлу маглину.

Слика 10.

Књига *Космографија* била је намењена ђацима гимназија и других средњих школа. Ипак се користила, вероватно као помоћни уџбеник, на Војној академији и на Великој школи. Споменимо да у списку литературе Андоновић поред страних дела наводи и два српска аутора: *Тригонометрију* од Димитрија Нешића и *Космометрију* од Јована Драгашевића. У књизи има велики број илустрација: 141 слика и цртежа, 17 таблица и једна велика звездана карта. Већину слика и цртежа преузео је из књиге Е. Вајса, директора бечке Звездарнице, уз његово одобрење.

Космографија је обимна (533 сраница) и доста свеобухватна књига. У њој се описују небеска сфера, сазвежђа, облик и величина Земље, Земљина ротација и револуција, привидно кретање Сунца, координатни системи, системи времена, календари, Сунчев систем, звезде, плима и осека, прецесија, космогонија, између осталог Кант-Лапласова претпоставка о настанку Света (тј. Сунчевог система). Уз све то у књизи има двадесетак потпуно урађених задатака са детаљним објашњењима.

Књига је написана веома лепим стилом, скоро као књижевно дело, те је вреди читати и у том погледу. Тамо можемо наћи многе заборављене речи па и термине чије је значење данас промењено. На пример: *звездана јата* код њега значе сазвежђа, *озвездине* - метеорити, *звезде репатице* – комете, *прилив* и *одлив* – плима и осека, такође за свако сазвежђе уводи појам


О КОСМОСУ

ЈАВНО ПРЕДАВАЊЕ

Милана Ј. Атанопитка

ДРЖАНО У ДВОРАНИ ВЕЛИКЕ ШКОЛЕ

У БЕОГРАДУ

1859

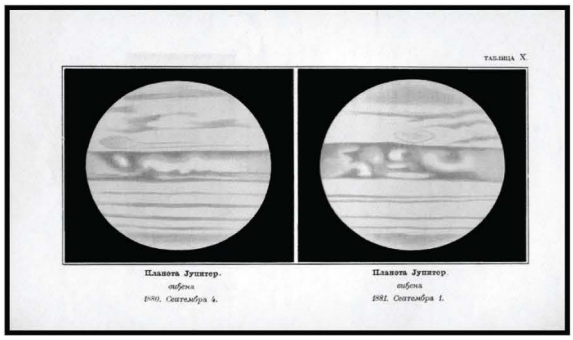


ТАБЛИЦА X

Планета Јупитер.

Планета Јупитер.

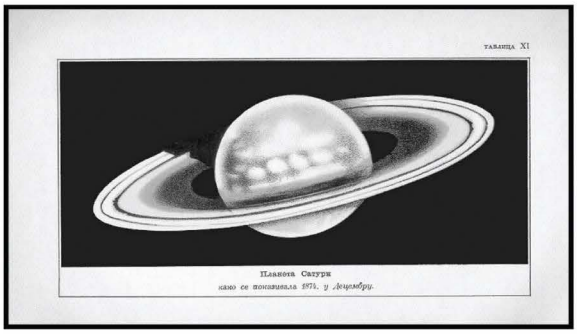


ТАБЛИЦА XI

Планета Сатурн

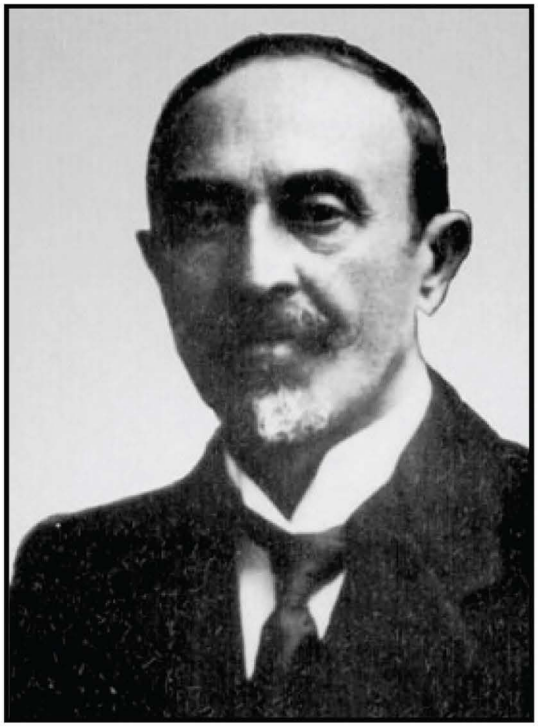
Слика 11.

алињман – са значењем слично астеризму. На пример *алињман Великог медведа* чине четири звезде – трапез астеризма Великиз кола, *алињман Северњаче* представља линију која пролази кроз задње точкове Великих кола и води ка Северњачи.

КОСТА СТОЈАНОВИЋ

Коста Стојановић (1867-1921) рођен је у Алексинцу. Ту је завршио основну школу и нижу гимназију, више разреде завршио је у Нишу. Дипломирао је математичке науке на Великој школи у Београду. Једно време радио је као гимназијски професор у Нишу и Београду. У Париз одлази 1893. где учи математику, астрономију, механику и физику код познатих професора Поенкареа, Пикара и Апела. Тамо постаје члан Историјског друштва Француске и Астрономског друштва Француске. У Лајпциг одлази 1897. на докторске студије из математике. На жалост враћа се у Србију после три месеца болестан.

Коста Стојановић је поред универзитетске имао богату политичку каријеру. Почетком 1900. постаје народни посланик као представник Нишког округа. Од 1903. предаје примењену математику на Великој школи а по оснивању 1905. на Београдском универзитету. Академску каријеру нагло прекида и 1906. постаје министар народне привреде, у време царинског рата



Слика 12.



Слика 13.

између Београда и Беча. У то време на Катедру примењене математике по позиву Богдана Гавриловића и Михаила Петровића, професора математике Београдског универзитета, долази 1909. Милутин Миланковић из Беча.

Негде у то време Коста Стојановић написао је *Основе теорије економских вредности*. Та књига представља дело из математичке економије и у њој Стојановић користи врло напредан математички апарат, на пример теорију парцијалних диференцијалних једначина за описивање економских модела. И данас постоји занимање за њу не само у Србији већ и у деловима претходне Југославије. То је вероватно најнапредније дело из економских наука икада штампано у Србији с обзиром на математички апарат у њему.

У влади Николе Пашића био је министар пољопривреде (1919-1920) и министар финансија 1921. Неколико дана по постављењу за министра финансија изненада је умро.

Коста Стојановић је такође написао ове књиге: *Атомистика* (Руђера Бошковића), 1891, Ниш, *Механика*, 1912, Београд, *Расправе и чланци из науке и филозофије* (постхумно), 1922, Београд. Све три књиге дигитализоване су и налазе се у Виртуелној библиотеци.

Књига ***Механика*** био је заправо универзитетски уџбеник из примењене математике. Споменимо да су се под примењеном математиком сматрале механика, математичка физика и небеска механика. Највећи део књиге односио се на векторски рачун, затим кинематику, статику, динамику, аналитичку механику и

теорију диференцијалних једначина. Глава десет ове књиге посвећена је астрономији. Овде се дају основе небеске механике, затим се излажу диференцијалне једначине кретања планета, комета и сателита. Такође се даје теорија елиптичког кретања, метод варијације константа и параболичко кретање комета.

ЗАКЉУЧАК

Виртуелна библиотека Математичког факултета има значајно место у представљању старих и ретких српских књига научној и стручној јавности и општој публици у свету и код нас. У ову чињеницу можемо се уверити на примеру избора књига из колекције астрономских дела српских научника представљених у овом чланку.

Захвалница

Захваљујемо се Министарству за науку и технолошки развој Републике Србије које финансира Пројекат *Примена информационих технологија у дигтализацији научне и културне баштине, ИТ 13017,* као и Библиотеци САНУ на позајмљивању књига за потребе дигитализације.

Такође се захваљујемо организатору др Милану С. Димитријевићу на позиву за учешће на конференцији *Развој астрономије код Срба VI.* Захваљујући томе, овај текст је написан.

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НАДЕЖДА ПЕЈОВИЋ и ЖАРКО МИЈАЈЛОВИЋ

ASTRONOMICAL BOOKS FROM THE VIRTUAL LIBRARY OF THE FACULTY OF MATHEMATICS WRITTEN IN THE XVIII AND XIX CENTURY

Virtual library, http://elibrary.matf.bg.ac.rs, of the Faculty of Mathematics of the University of Belgrade, contains more than 50 retro-digitized astronomical books written by Serbian authors. We present in this article a selection of books from this collection of Serbian authors from the XVIII and XIX century: Ruđer Bošković, Zaharije Orfelin, Atanasije Stojković, Đorđe Stanojević, Milan Andonović and Kosta Stojanović. For each of this author we present his short biography.

63 Knjige Vojislava Mišковića u Virtuelnoj biblioteci

Books of Vojislav Mišković in Virtual Library,
N. Pejović and Ž. Mijajlović
Publ. Astron. Druš. Rudjer Bošković, Beograd
Vol. 10, pp. 451-465, 2011
Proc. Conf. "Razvoj astronomije kod Srba VI"
Beograd, 22-26 april, 2010
(M. Dimitrijević, ed).

Professor Vojislav Miskovic (1892-1976) was one of the leading Serbian astronomers in the twentieth Century and he has great merits for development of the Astronomical Observatory in Belgrade and the scientific research in astronomy in Serbia as well in the period 1925-1960. He wrote large number of books and textbooks on astronomy. Eleven of these books, one chart and one scientific work are included into the Virtual Library of the Faculty of mathematics of the University of Belgrade, This article gives review of this digital collection and a short biography of Professor Mišković as well.

Зборник радова конференције "Развој астрономије код Срба VI"
Београд, 22-26. април 2010,
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 10, 2011, 451-465

КЊИГЕ ВОЈИСЛАВА МИШКОВИЋА У ВИРТУЕЛНОЈ БИБЛИОТЕЦИ

НАДЕЖДА ПЕЈОВИЋ и ЖАРКО МИЈАЈЛОВИЋ

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E-mail: nada@matf.bg.ac.rs, zarkom@matf.bg.ac.rs

Резиме: У овом чланку представљамо прве универзитетске уџбенике из астрономије писане на српском језику. Наиме, реч је о уџбеницима, збиркама и књигама професора **Војислава Мишковића** писаним средином XX века: ***Општа астрономија***, ***Хронологија астрономских тековина I* и *II***, ***Хипарх***, ***Јоханес Кеплер***, ***Космографија***, ***Сунчево помрачење од 15 фебруара 1961,* и *Логаритамске и нумеричке таблице***. У ову дигиталну колекцију такође су укључени први део збирке који је објављен 1956. под називом ***Збирка решених задатака из Опште астрономије***, затим рукопис другог дела исте збирке који је потпуно припремљен за штампу али није био објављен. Ове књиге налазе се у Виртуелној библиотеци Математичког факултета, http://elibrary.matf.bg.ac.rs.

1. УВОД



Слика 1: *Војислав Мишковић.*

Професор Војислав Мишковић (1892-1976) био је водећи српски астроном у првој половини XX века. Написао је већи број књига и уџбеника из астрономије. Једанаест његових књига, једна звездана карта и један његов научни рад су дигитализовани и постављени у Виртуелну библиотеку Математичког факултета Универзитета у Београду на Интернет адреси http://elibrary.matf.bg.ac.rs. Међу овим делима налази се други део књиге *Збирка решених задатака из Опште астрономије* која никад није штампана. Дигитализацијом, ова књига доживела је своје прво прегледе Мишковићевих дела и његову кратку биографију.

2. БИОГРАФИЈА ВОЈИСЛАВА МИШКОВИЋА

Војислав Мишковић (1892-1976) рођен је у Фужинама у Хрватској. Студије астрономије започео је у Будимпешти и Гетингену пред Први светски рат. По избијању рата прелази илегално у Србију, ступа у добровољачки одред и активно учествује у рату све до слома српске војске. После демобилисања одлази у Француску на довршетак студија. Дипломирао је 1919. и постављен је за асистента Астрономске опсерваторије у Марсеју. Од 1922. ради као астроном Опсерваторије у Ници. Докторирао је 1924. на Универзитету у Монпељеу. У Француској је радио у струци веома активно. Организовао је нове службе по опсерваторијама, био је уредник једног астрономског часописа, организовао је и извео астрономски премер триангулационог повезивања Корзике са француским источним Алпима, приказао прву варијанту свог оригиналног астрономског инструмента. За своје доприносе добио је награду Француске академије наука. У Београд је дошао по позиву 1926. као већ афирмисан астроном. Исте године изабран је за ванредног професора Филозофског факултета у Београду на тек основаној Катедри теоријске и практичне астрономије и постављен је за управника нове Астрономске опсерваторије на Звездари. Може се рећи да доласком Војислава Мишковића из Француске у Београд, почиње интензиван развој астрономије у Србији. Мишковић је 1929. изабран за дописног члана Српске краљевске академије, а за редовног 1939. године. Њему у част једна мала планета добила је име Мишковић.

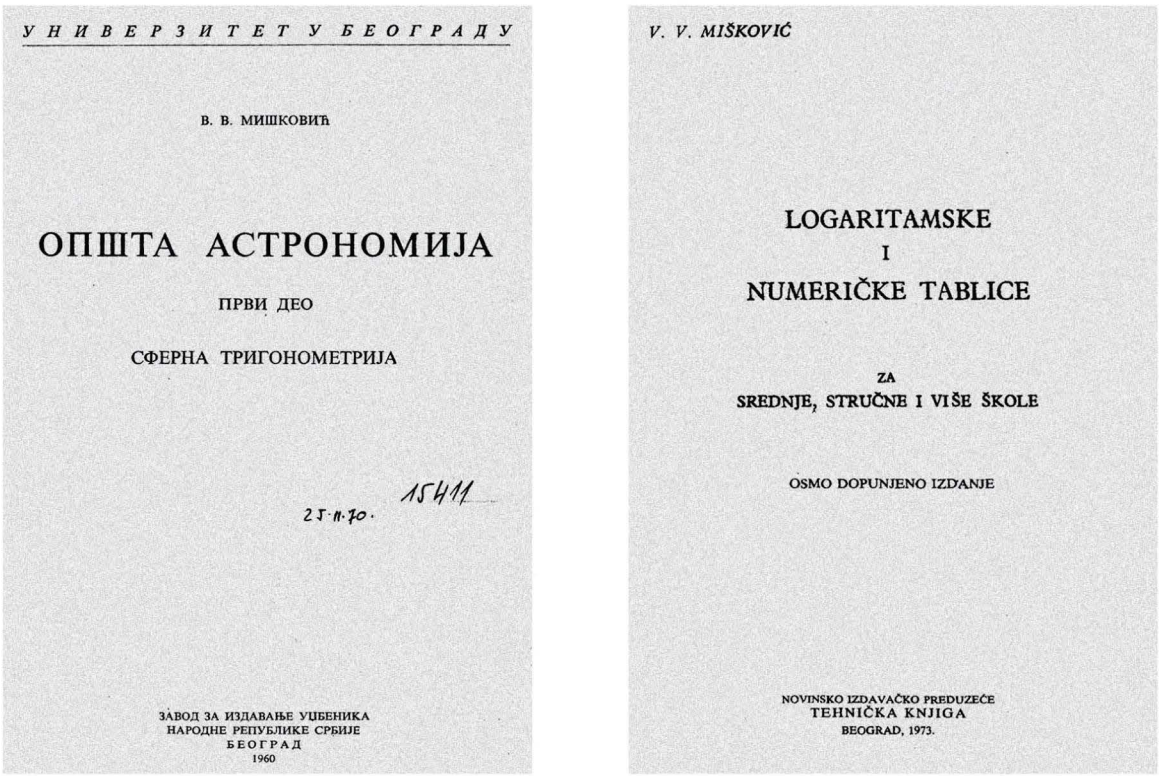
3. УЏБЕНИК *ОПШТА АСТРОНОМИЈА*

Ову књигу, као стални универзитетски уџбеник, одобрила је за употребу Комисија за уџбенике Универзитета у Београду својим решењем број 1500/2 од 9. јуна 1960. године. Први део уџбеника из *Опште астрономије* издвојен у засебну књигу, под насловом *Сферна тригонометрија*, штампао је Београдски графички завод а издао Завод за издавање уџбеника Народне републике Србије у Београду 1960. године. Књига је написана на 95 страница и садржи *Увод*, три главе, *Додатак*, *Преглед образаца* и *Таблице*. У првој глави *Геометрија сфере* уведени су основни појмови и дефиниције: сфера, велики и мали кругови, полови, сферна даљина, сферни угао, сферни двоугао, сферни троугао, сферни ексцес, поларни сферни троугао, квадратни сферни троугао, мере за углове. Друга глава *Сферна тригонометрија* даје основне обрасце сферне тригонометрије, петоелементне обрасце као и специјалне обрасце, затим су наведени површина сферног двоугла и сферног троугла, квадратни степен, као и прелаз са Сферне на Равну тригонометрију, израчунавање вредности тригонометријских функција, геодезијски троугао и Лежандрова теорема. У трећој глави *Примене* урађени су нумерички примери решавања сферних троуглова као и примери коришћења диференцијалних образаца сферне тригонометрије. У *Додатку* су поједини

параграфи детаљније допуњени и објашњени. *Преглед образаца* је мали приручник из математике са разним бројним вредностима и специјалним тригонометријским обрасцима.

Универзитетски уџбеник *Општа астрономија,* Први део *Сферна тригонометрија,* издвојен је у посебну целину јер обухвата, како аутор у предговору пише, више математике него што је слушаоцима предмета Општа астрономија потребно. У њему су обрађене нарочито нумеричке примене. Мишковић објашњава да су нумеричке примене у астрономском раду од основног значаја, па и да нема много места где их читалац може наћи и научити. Често су исти бројни примери урађени на више начина, да би се читаоцу илустровало колико тачност решења зависи од изабране методе. У нумеричким примерима последња децимала није заокруживана већ је резултат писан са децималом више у ситнијој величини. Због скраћеног и једноставнијег писања уместо $\log(x)$ користио је ознаку $[x]$.

Први део *Опште астрономије*, под називом *Сферна тригонометрија* изузетно је лепо и прегледно написан. Садржи мноштво скица, цртежа и урађених примера тако да пратећа збирка није потребна. Овде налазимо већ заборављене изразе као што су *гониометријске функције* (тригонометријске



Слика 2.

функције), *град* (400-ти део пуног угла), *центезимална минута* (100-ти део града, означава се знаком сексагезималне минуте усмерене с лева удесно), *центезимална секунда* (100-ти део центезималне минуте, означава се знаком сексагезималне секунде усмерене с лева удесно, стр. 17). На страници 88 дат је списак страних уџбеника из Опште астрономије. Нажалост, осим првог дела, остали делови универзитетског уџбеника *Општа астрономија* Војислава Мишковића никада нису објављени. Али, својим студентима и данас препоручујемо објављени *Први део* из којег са лакоћом могу да науче сферну тригонометрију.

4. ЛОГАРИТАМСКЕ И НУМЕРИЧКЕ ТАБЛИЦЕ

Логаритамске и нумеричке таблице су биле драгоцен приручник за сложене и обимне нумеричке рачуне у време када дигитрона и рачунара није било. Таблице су користили не само професионални астрономи већ и студенти. Прво издање Мишковићевих *Логаритамских и нумеричких таблица* за средње, стручне и више школе штампано је 1952. године. Дигитализовали смо осмо допуњено издање које је штампала *Техничка књига* у Београду 1973. Књига је штампана на 265 страница и садржи, посебним бојама, одвојене таблице за декадне логаритме, вредности и декадне логаритме тригонометријских функција, затим таблице вредности и њихове логаритме хиперболичких тригонометријских функција. Овде су и специјалне нумеричке таблице: за претварање Неперових у Бригсове логаритме, за претварање радијана у степене, минуте и секунде, и обрнуто, затим таблице вредности функције грешака посматрања, вредности фактора за одређивање грешака посматрања, преглед Питагориних бројева, вредности факторијела бројева 1-20, као и вредности биномних коефицијената степена 0-20. Пре сваке таблице дат је графички приказ утабличених функција. После сваке таблице дато је детаљно упутство за употребу као и неколико урађених примера.

5. КЊИГА *JOHANNES KEPLER*

Књига *Johannes Kepler 1571-1630* коју је Мишковић написао 1971. представља Споменицу, посвећену четрстогодошњици од рођења прослављеног астронома Јохана Кеплера. Занимљиво је да смо ову књигу дигитализовали 2009. на четирстогодишњицу од објављивања прва два Кеплерова закона 1609. Књига има 48 страница и састоји се из два дела. У првом делу су дате биографије прва четири, како их Мишковић у предговору назива, оснивача нове астрономије: Николе Коперника, Тихо Брахеа, Галилео Галилеја и Исака Њутна. Други део је посвећен петом, најзаслужнијем, оснивачу нове астрономије Јохану Кеплеру. У овом делу је описан тежак Кеплеров живот и његови бриљантни доприноси астрономској науци. На крају књиге су издвојени значајни датуми из Кеплеровог живота.

Књигу је издала Српска академија наука и уметности и штампало "Научно дело" 1972. На самом почетку књиге стоји Деламбреов цитат: *Нека је слава астроному који је смогао истрајности и имао оштрине духа да, једино рачуном, открије законе небеских кретања, и то у време када се до њих ниједним другим путем није стићи могло.*

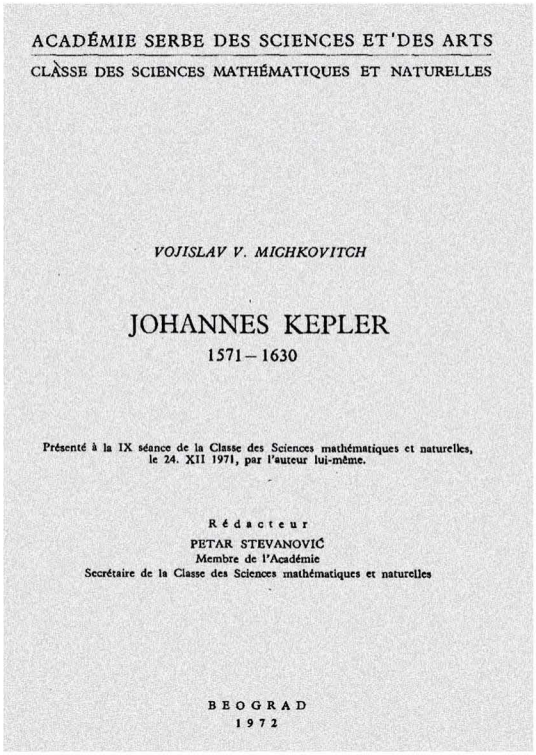
Књига *Johannes Kepler 1571-1630* је веома занимљива, има историјску вредност и представља опште образовно штиво. Из ње се могу сазнати многе појединости из живота и рада овог великана. На пример, Мишковић пише о Кеплеровој "звезди водиљи", заправо та звезда је "хармонија света" за којом је Кеплер трагао и коју је пронашао открићем свог трећег закона. Књигу свакако треба прочитати.

6. КЊИГА *ХИПАРХ*

Књигу *Хипарх* Мишковић је посветио животу и раду највећег астронома Старог века, првог посматрача, а могло би се рећи и првог оснивача науке о Васиони. Написана је на IX+59 страница. Књигу је издала Српска академија наука и уметности и штампало "Научно дело" у Београду 1976. Књига садржи седам поглавља: *Предговор, Хипарх-Уводна реч, Предхипарховски период, Хипархов живот и рад, Инструменти из предхипархова и Хипархова времена, Постхипарховски период и Резиме* (на француском). За Хипарха Мишковић пише: *Што је за астрономију свога доба урадио, толико је огромно, да је тешко скоро и поверовати да је један човек у стању да толико уради.*

Књига је опште образовног карактера. Овде налазимо како је Хипарх помоћу "штапа и канапа" одредио дужину тропске године, дужину сидеричке године, открио појаву прецесије, одредио нагиб еклиптике према екватору, направио први каталог звезда, први поделио звезде по сјају у шест привидних величина, увео појмове географска ширина (коју је назвао *климом*) и географска дужина, као и почетни меридијан који је ишао преко његове опсерваторије на острву Родосу. Написао је и дванаест књига о тетивама круга над њиховим централним угловима, претече тригонометријских таблица. Ове таблице користили су стари астрономи за своја израчунавања. Зато су Хипарха географи назвали "оцем географије", а астрономи "творцем математичке астрономије".

Књига *Хипарх* представља значајан допринос српској литератури из историје астрономије, али и филозофији и општој историји. Ово је прва књига написана на нашем језику о овом великану Старог века. Мишковић је у њој дао *први у свету* библиографију великог броја Хипархових књига и дела. Хипархова дела углавном нису сачувана, о њима, као и његовом живо ту и раду, се посредно сазнаје из записа његових савременика, као и филозофа, историчара и астронома који су касније о њему расправљали и писали. Неки датуми, догађаји па и порекло резултата ипак су непоуздани.



ACADÉMIE SERBE DES SCIENCES ET DES ARTS

CLASSE DES SCIENCES MATHÉMATIQUES ET NATURELLES

VOJISLAV V. MICHKOVITCH

JOHANNES KEPLER

1571 – 1630

Présenté à la IX séance de la Classe des Sciences mathématiques et naturelles, le 24. XII 1971, par l'auteur lui-même.

Rédacteur

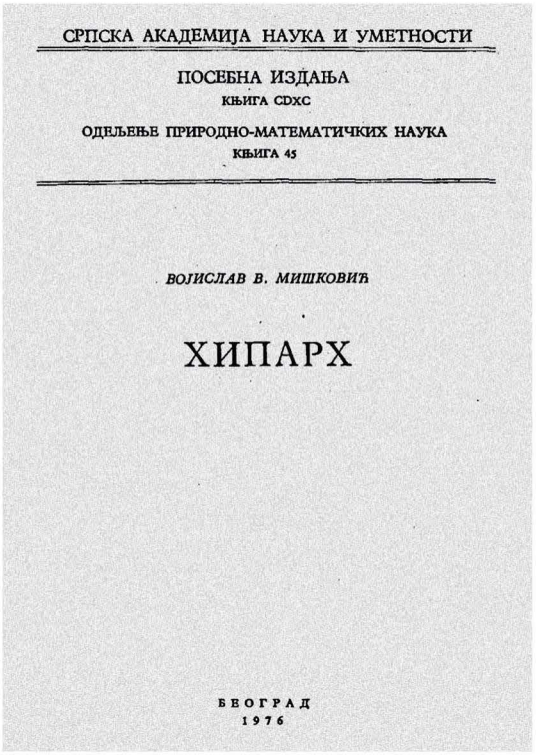
PETAR STEVANOVIĆ

Membre de l'Académie

Secrétaire de la Classe des Sciences mathématiques et naturelles

BEOGRAD

1972



СРПСКА АКАДЕМИЈА НАУКА И УМЕТНОСТИ

ПОСЕБНА ИЗДАЊА

КЊИГА CDXC

ОДЕЉЕЊЕ ПРИРОДНО-МАТЕМАТИЧКИХ НАУКА

КЊИГА 45

ВОЈИСЛАВ В. МИШКОВИЋ

ХИПАРХ

БЕОГРАД

1976

Слика 3.

7. ХРОНОЛОГИЈА АСТРОНОМСКИХ ТЕКОВИНА I И II

Књиге *Хронологија астрономских тековина I* и *II* представљају заправо хронолошку енциклопедију астрономских тековина остварених до краја XIX века. Догађаји и проналасци описани су у виду одредница којих има око 2000. У првој књизи Мишковић је обухватио астрономске тековине у периоду од прастарих времена до 1700, а у другој књизи период 1700-1900. Књиге је издала Српска академија наука и уметности редом 1975. и 1976. године, штампала и технички припремила штампарија *Научно дело* у Београду. Прва књига има 133 странице, а друга 148.

Хронологија је зборник достигнућа поређаних по годинама и у њој се описују сви важнији астрономски догађаја, открића, проналасци, заправо све значајне астрономске тековине. Енциклопедијски карактер Хронологије проистиче не само из тога да се брзо и лако може доћи до сваког жељеног податка, већ из чињенице да су ови подаци тачни. Наиме, на једном месту (књига II, стр.2) Мишковић пише:

Сваки податак, који је требало да буде унесен у "Хронологију", тражен је накнадно, редовно, у некој од познатих историја астрономије; ако је требало и у неком од већих уџбеника, или у неком астрономском часопису,

или у чланку, у којима је о дотичној појави, или предмету, било говора. Ако је налажено – што је најчешће и био случај – назначавани су, на завршетку текста података: име писца астрономије, односно име писца и назив дела, или часописа, уопште извора, где је податак нађен, са страном.

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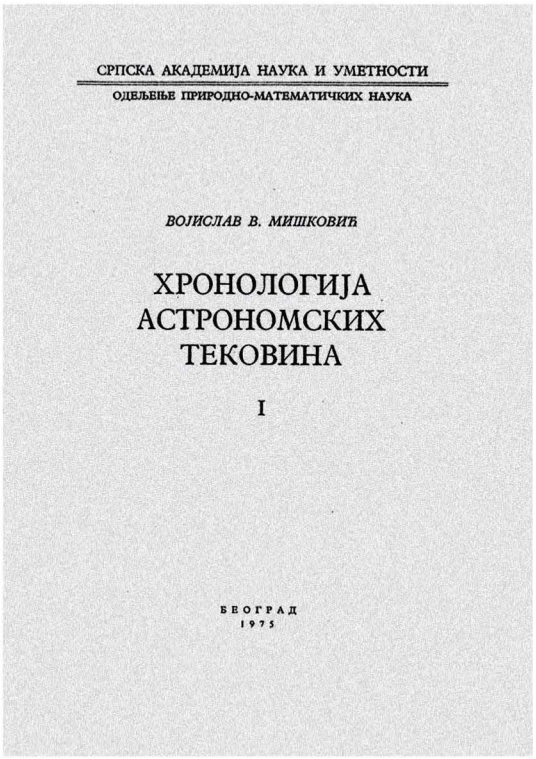
Вредност ових књига лежи такође у следећој чињеници. У свакој одредници појављује се име бар једног научника, махом астронома, или назив његовог дела. Страна имена и називи су транскрибовани, писани ћирилицом, док је у загради написано оригинално име или назив латиницом. Неке одреднице су илустроване сликама и скицама. На крају сваке књиге налази се регистар имена научника који претрагу убрзава и олакшава.

Можемо рећи да *Хронологија* представља значајно дело из историје астрономских наука, не само зато што је прва књига ове врсте код нас написана, већ зато што је свеобухватна и прилично потпуна. Читалац ће бити изненађен и бројем и врстом открића које су стари мислиоци проналазили, уочавали и наслућивали.



СРПСКА АКАДЕМИЈА НАУКА И УМЕТНОСТИ

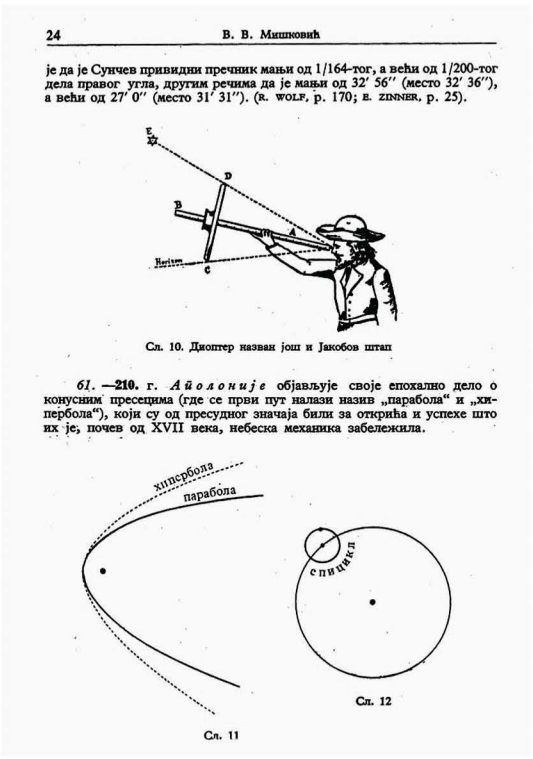
ОДЕЉЕЊЕ ПРИРОДНО-МАТЕМАТИЧКИХ НАУКА

ВОЈИСЛАВ В. МИШКОВИЋ

ХРОНОЛОГИЈА
АСТРОНОМСКИХ
ТЕКОВИНА

I

БЕОГРАД
1975



24 В. В. Мишковић

је да је Сунчев привидни пречник мањи од 1/164-тог, а већи од 1/200-тог дела правог угла, другим речима да је мањи од 32′ 56″ (место 32′ 36″), а већи од 27′ 0″ (место 31′ 31″). (R. WOLF, p. 170; E. ZINNER, p. 25).

Сл. 10. Диоптер назван још и Јакобов штап

61. —**210.** г. *Аполоније* објављује своје епохално дело о конусним пресецима (где се први пут налази назив „парабола“ и „хипербола“), који су од пресудног значаја били за открића и успехе што их је, почев од XVII века, небеска механика забележила.

Сл. 11

Сл. 12

Страница из *Хронологије*

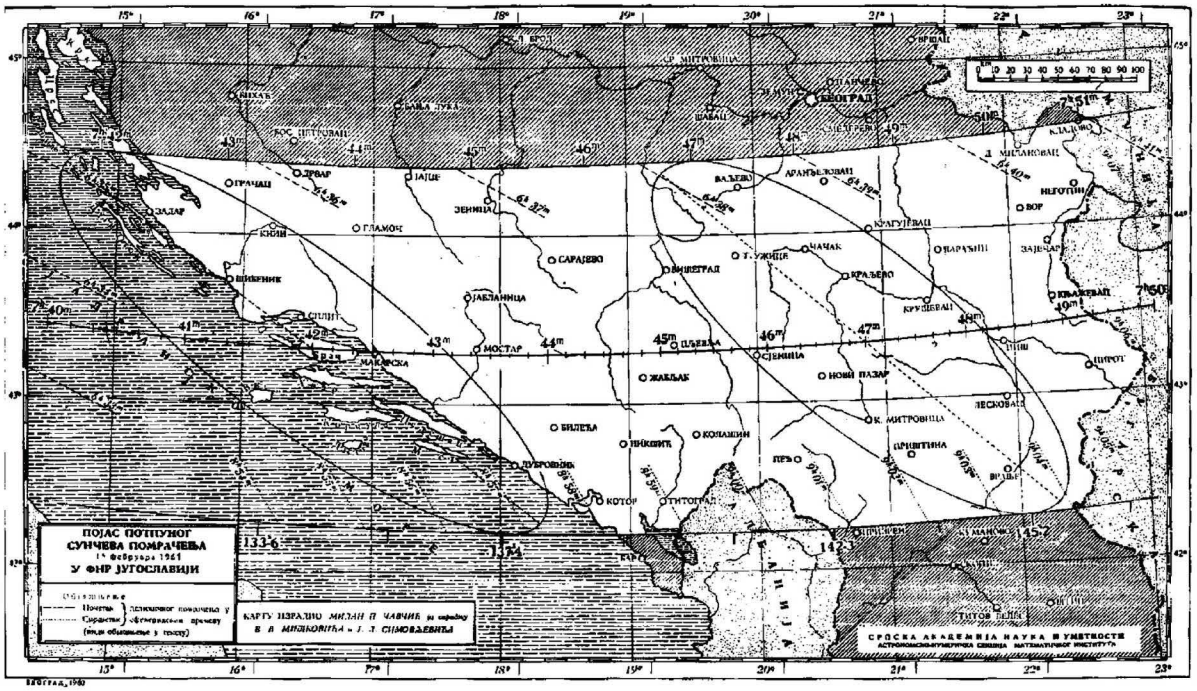
Слика 4.

НАДЕЖДА ПЕЈОВИЋ и ЖАРКО МИЈАЈЛОВИЋ

8. СУНЧЕВО ПОМРАЧЕЊЕ ОД 15. ФЕБРУАРА 1961.

Књигу *Сунчево помрачење од 15. фебруара 1961* издала је Српска академија наука и уметности у серији *Посебна издања* 1960. Мишковић је ову књигу написао поводом тоталног помрачења Сунца које се видело из наших крајева 15. фебруара 1961. године. Књига је написана на 5+75 страница и састоји се од *Предговора*, четири главе и *Прегледа*. У првој глави *Изглед и састав Сунчев* детаљно су описани Сунчева фотосфера, пеге, факуле, хромосфера, спектар, протуберанце, корона. Друга глава *Сунчева помрачења* описује врсте помрачења, димензије конуса Месечеве сенке, трајање помрачења, фазе делимичног и фазе потпуног помрачења као и тренутке почетка и завршетка потпуног помрачења. У трећој глави *Програм рада. Посматрање помрачења* Мишковић описује инструменте и опрему за астрономска посматрања, инструменте за снимање короне, први додир-почетак делимичног помрачења, посматрање наиласка Месечеве сенке, други додир-почетак потпуног помречења, снимање спектра, снимање корониног спектра, мерење коронине поларизованости, мерење Ајнштајновог ефекта, изглед неба за време помрачења, трећи додир-свршетак потпуног помрачења, четврти додир-завршетак фазе делимичног помрачења, затим радио-телескопска посматрања помрачења као и програм посматрања при облачном времену (из балона и авиона). У четвртој глави *Ток помрачења посматран из Југославије* Мишковић даје опис општег тока предстојећег помрачења, затим тока појаве посматраног из наше земље, као и приближно одређивање тренутака појединих фаза за изабрано место посматрања. У *Прегледу* су дати подаци о главним фазама фебруарског помрачења 1961. за 29 највећих градова Југославије, и то, поред географских координата и надморске висине, дати су тренуци почетка и завршетка делимичног и тоталног помрачења у том месту (за места у појасу тоталитета), као и тренуци почетка и завршетка делимичног помрачења за места изван појаса тоталитета. Ове податке је ручно израчунао Јован Симовљевић, тада асистент Природно-математичког факултета. На основу ових података уцртан је појас тоталитета на карти Југославије, коју је израдио М. Чавчић и која је приложена на крају књиге.

Ова књига је била драгоцено штиво за све експедиције које су посматрале тотално помрачење Сунца 15. фебруара 1961. Како су помрачења Сунца за једно место ретка (једном у три века) то још више говори о значају ове књиге. Ова кратка монографија је важна јер је написана и штампана у правом тренутку, непосредно пред најспектакуларнију појаву која се може видети на небу. Претходно тотално помрачење Сунца за наше крајеве било је 1666. а следеће, како пише Мишковић, се очекује 7. октобра 2135. Међутим, Мишковић у овој књизи спомиње да ће помрачење Сунца 11. августа 1999. бити делимично, што је оно за нашу земљу углавном и било. Али, нешто касније Ј. Симовљевић је урадио прецизне прорачуне и нашао да ће за мали

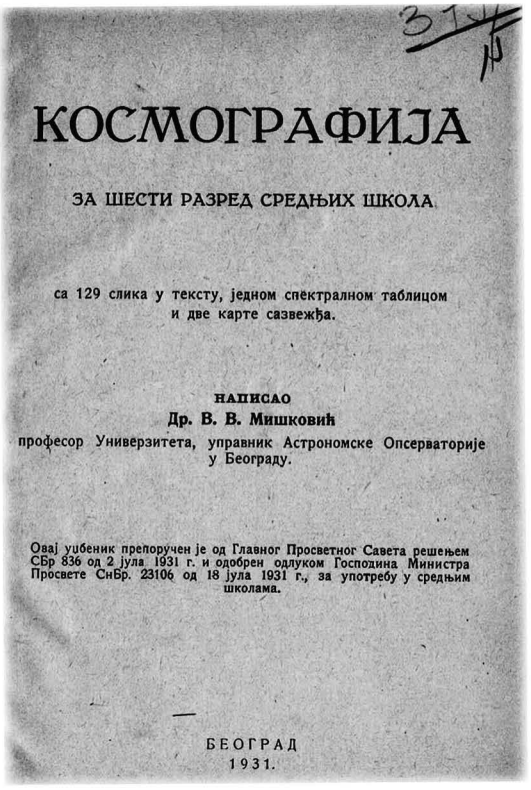


Слика 5: *Појас тоталног помрачења 1961. представљен на карти Југославије (цртеж из књиге, урадио М. Чавчић).*

део наше земље, североисточне крајеве Војводине, ово помрачење бити тотално. Симовљевићеви рачуни били су тачни. Напоменимо да се ова величанствена појава дешавала на небу само два месеца након престанка бомбардовања наше земље од стране НАТО алијансе.

9. КОСМОГРАФИЈА

Мишковићева *Космографија* је уџбеник из астрономије за ученике шестог разреда ондашњих средњих школа (данашњи други разред). Овај уџбеник препоручен је од Главног Просветног Савета решењем СБр 836 од 2. јула 1931. и одобрен одлуком Господина Министра Просвете СнБр 23106 од 18. јула 1931. за употребу у средњим школама. Ово је први званичан средњошколски уџбеник из астрономије који је код нас написан. Штампан је у Београду 1931. Уџбеник је написан на 186 страница са 129 слика у тексту. У прилогу има две звездане карте и једну табелу спектралних линија. Књига је садржајна и јасно написана са пуно скица и цртежа које лепо илуструју приказане појаве и појмове. Описана су привидна и права кретања Сунца, Месеца, планета, комета и звезда. Уведени су сферни координатни системи: хоризонтски, екваторски и еклиптички. Дефинисани су системи времена: звездано, право, средње, грађанско и светско време. Описана су небеска тела: звезде, планете, сателити планета, мале планете, комете и метеори, затим небески системи: двојне звезде, тројне, вишеструке, звездана јата, маглине и галаксије.



КОСМОГРАФИЈА

ЗА ШЕСТИ РАЗРЕД СРЕДЊИХ ШКОЛА

са 129 слика у тексту, једном спектралном таблицом и две карте сазвежђа.

НАПИСАО

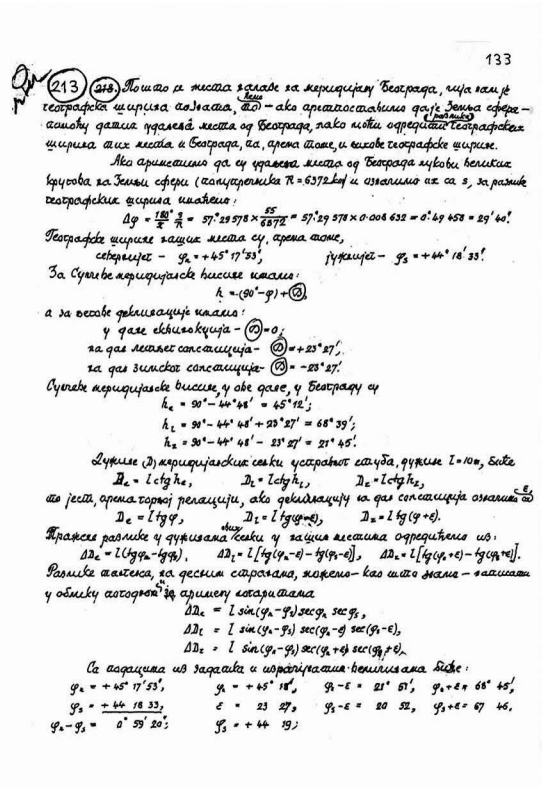
Др. В. В. Мишковић

професор Универзитета, управник Астрономске Опсерваторије у Београду.

Овај уџбеник препоручен је од Главног Просветног Савета решењем СБр 836 од 2 јула 1931 г. и одобрен одлуком Господина Министра Просвете СнБр. 23106 од 18 јула 1931 г., за употребу у средњим школама.

БЕОГРАД

1931.



133

Мишковићев рукопис (*Збирка*)

Слика 6.

10. ЗБИРКА РЕШЕНИХ ЗАДАТАКА ИЗ ОПШТЕ АСТРОНОМИЈЕ

Књига, први део *Збирке решених задатака из Опште астрономије* професора Војислава Мишковића, прва је универзитетска збирка из астрономије штампана на српском. Збирка се појавила половином прошлог века. Потреба да се ова књига штампа осетила се, како Мишковић у предговору пише, повећањем броја студената астрономије с једне стране и с друге стране недостатаком урађених задатака не само у домаћој већ и у страној литератури. Док је уџбеника из астрономије било довољно на свим европским језицима, недостатак збирки урађених задатака био је евидентан. *Збирка* је актом Комисије за уџбенике Универзитета у Београду бр. 896 од 10.08.1956. године добила одобрење да се штампа као сталан уџбеник за студенте Природно-математичког факултета. Мада одобрена у целини, из техничких разлога, како Мишковић пише, *Збирка* је морала да буде подељена на два дела.

Први део *Збирке* издала је *Научна књига* у Београду исте године када је и одобрена, у тиражу од 2000 примерака, док је други део требало да буде штампан следеће године. На жалост, други део *Збирке* још увек је у рукопису. Коауторка овог чланка рукопис је добила 1995. од професора

астрономије Јована Симовљевића (1929-2007) по његовом одласку у пензију. Од те године многе задатке из рукописа студенти су решавали на вежбама и испитима из предмета Општа астрономија на Катедри за астрономију Математичког факултета у Београду. Како је Војислав Мишковић био европски ђак и добар познавалац страних језика, збирку је написао уз коришћење тада најсавременије уџбеничке и научне литературе. У Уводу Збирке наведено је десетак референци – страних уџбеника, углавном француских, енглеских и немачких. Задаци преузети од других аутора носе ознаку порекла. Дигитализација оба дела *Збирке* је завршена 2009. и на тај начин је оригинални Мишковићев рукопис, други део *Збирке*, после пола века, постао доступан јавности.

Штампана књига је први део *Збирке*. Има 150 страница и састоји се од *Предговора, Увода* и два поглавља. *Увод* се састоји од три дела: *Преглед образаца сферне тригонометрије, Редови* и *Преглед образаца за трансформацију астрономских координатних система.* Прво поглавље *Задаци* односи се на три области: Сферну тригонометрију, Земљу као небеско тело и Привидно дневно кретање небеске сфере. Друго, много веће поглавље *Решења* састоји се од упутстава и решења 126 задатака.

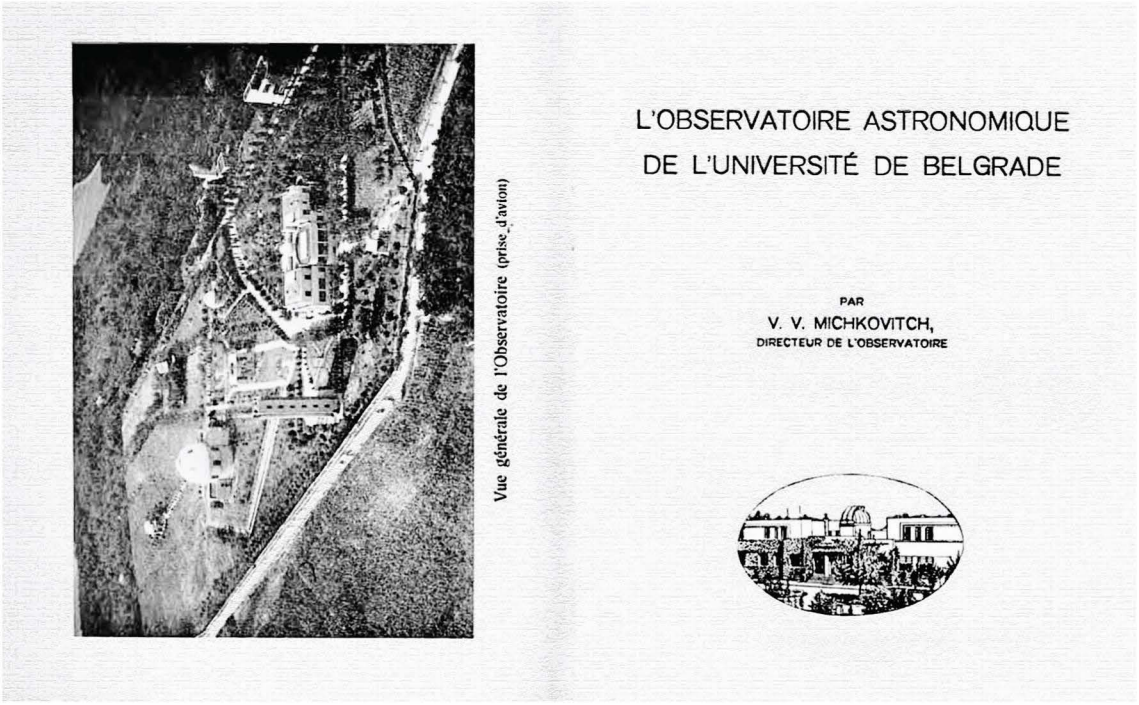
Рукопис садржи други део *Збирке* који се састоји од *Увода* и два поглавља. Увод овог дела односи се на три области: *Астрономска рефракција*, *Елементи теорије кретања планета и комета* и *Сунчево привидно годишње кретање*. Прво поглавље *Задаци* садржи задатке, док друго много обимније поглавље *Решења* садржи методе, упутства, објашњења и решења 136 задатака из рукописа.

Значај објашњених и комплетно урађених 262 задатка из ових области и данас је велики јер других збирки са решеним задацима из астрономије на нашем језику нема. Уџбеника, како средњошколских тако и универзитетских има, али често без и једног урађеног задатка или примера. Стога је Мишковићева *Збирка решених задатака из Опште астрономије* прави драгуљ у нашој астрономској литератури.

11. О ЈЕДНОМ ЕМПИРИЧКОМ ОБРАСЦУ ПРИ ОДРЕЂИВАЊУ ПЛАНЕТСКОГ КРЕТАЊА

Ово дело од десетак страница заправо је укоричен сепарат научног рада објављеног у Гласу CXXX Српске Краљевске Академије који је штампао Графички завод "Макарије" у Београду 1928. С обзиром да се овај научни рад налази као посебна библиографска јединица у Библиотеци САНУ, с разлогом можемо претпоставити да је то био лични Мишковићев избор. Отуда смо ипак овај рукопис уврстили у избор дела за дигитализацију и укључење у Виртуелну библиотеку. Иначе, у овом раду Мишковић даје

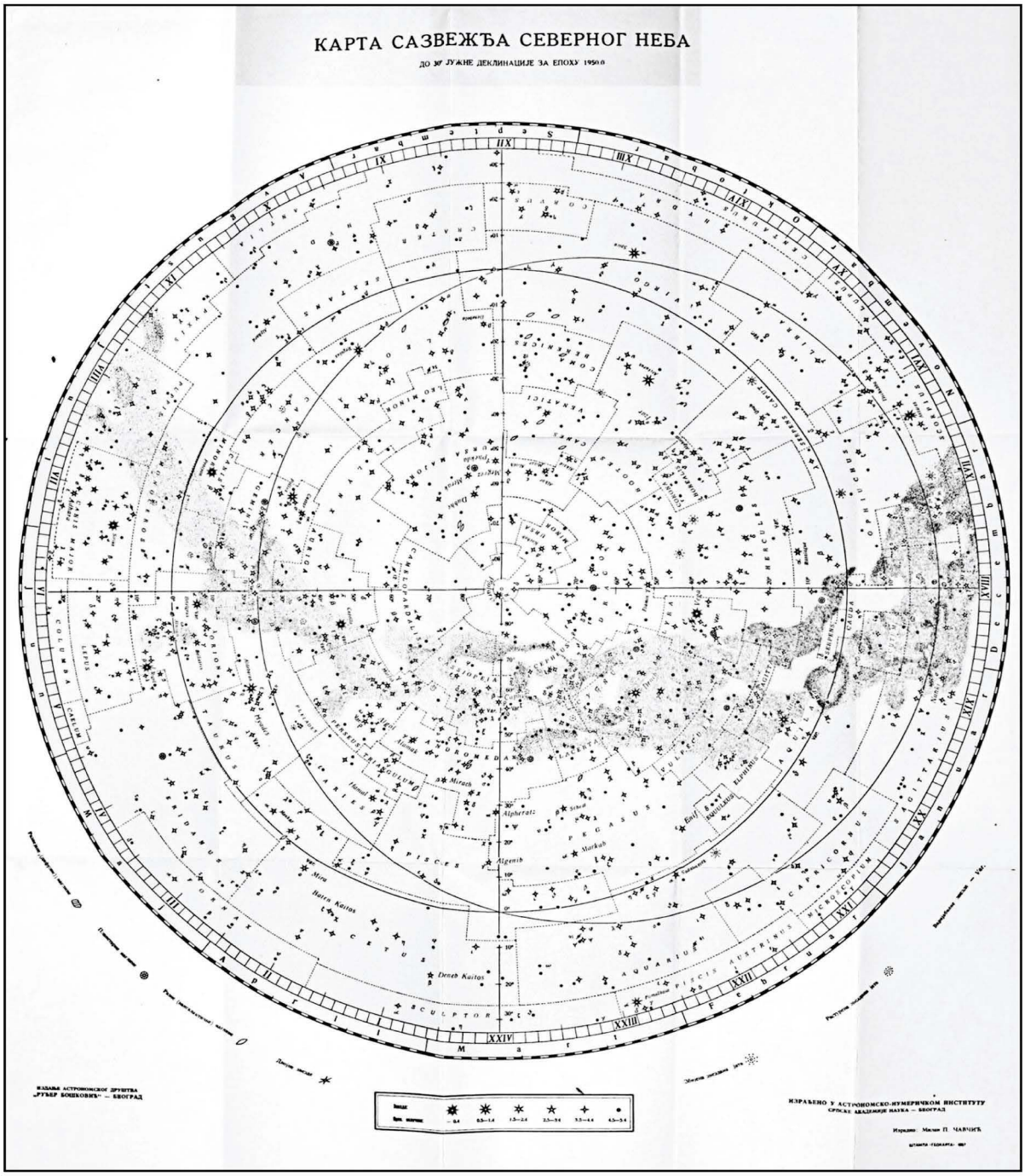
емпиријски образац за поправку првобитне ефемериде мале планете на основу једног посматрања које је показало да мале планете нема на већ израчунатом месту.



Слика 7.

12. L'OBSERVATOIRE ASTRONOMIQUE DE L'UNIVERSITÉ DE BELGRADE

Ову малу књижицу (има свега 19 страница) Мишковић је публиковао поводом Балканског конгреса математичара који се одржао у Београду септембра 1939. У буклету се описује историја оснивања и рада нове Астрономске опсерваторије у Београду (основана 1924). Мишковић је иначе постављен за директора Опсерваторије 1926, одмах по њеном оснивању. Пре њега, директор је био Милан Недељковић. У књижици се такође наводе локалне географсе координате и активност Опсерваторије, публикације и њени инструменти. Највећу вредност буклета чине 15 фотографија. У овој колекцији, поред слика Опсерваторије, налазе се и фотографије инструмената. Читалац овде може сазнати да је Опсерваторија у то време била добро опремљена и да се у њој одвијао напредан научни рад. Мишковић је у свим овим пословима био веома ангажован и свакако има велике заслуге за напредак и развој Опсерваторије и научног рада у њеним оквирима.


КАРТА САЗВЕЖЂА СЕВЕРНОГ НЕБА
ИЗРАЂЕНО У АСТРОНОМСКО-НУМЕРИЧКОМ ИНСТИТУТУ
СРПСКЕ АКАДЕМИЈЕ НАУКА — БЕОГРАД

Слика 8.

13. КАРТА САЗВЕЖЂА СЕВЕРНЕ ХЕМИСФЕРЕ

Мишковић је направио ову карту у сарадњи са Миланом Чавчићем, чланом групе за техничку подршку Математичког института САНУ. Мапу је публиковало 1957. године Астрономско друштво "Руђер Бошковић". Карта приказује сазвежђа до 30^0 јужне деклинације за епоху 1950.0. Уз мапу се налази суплемент од 6 страница у којем се објашњавају основни асрономски појмови као и сама карта. Додатак је написао П. Ђурковић, астроном који је радио на Опсерваторији. Уз карту се такође налази провидан лист папира са уцртаном мрежом за исчитавање позиција небеских тела на карти.

14. ЗАКЉУЧАК

Професор Војислав Мишковић, угледни српски астроном из прве половине XX века, имао је значајан научни опус и заслужан је за развој научног и стручног рада на Астрономској опсерваторији у Београду. Написао је већи број књига и уџбеника из астрономије. Једанаест његових књига, једна звездана карта и један његов научни рад су дигитализовани и постављени у Виртуелну библиотеку Математичког факултета на Интернет адреси http://elibrary.matf.bg.ac.rs. Дигитализацијом, његове књиге постале су доступне широј научној и стручној јавности и општој публици.

Захвалница

Захваљујемо се Министарству за науку и технолошки развој Републике Србије које финансира Пројекат *Примена информационих технологија у дигтализацији научне и културне баштине, ИТ 13017,* као и Библиотеци САНУ на позајмљивању свих ових књига за дигитализацију, осим две, *Космографија и L'Observatoire Astronomique de L'Université de Belgrade*, која је позајмљена из приватне библиотеке проф. Јелене Милоградов-Турин.

Такође се захваљујемо организатору др Милану С. Димитријевићу на позиву за учешће на конференцији *Развој астрономије код Срба VI.* Захваљујући томе, овај текст је написан.

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BOOKS OF VOJISLAV MIŠKOVIĆ IN VIRTUEL LIBRARY

Professor Vojislav Mišković (1892-1976) was one of the leading Serbian astronomers in the twentieth Century and he has great merits for development of the Astronomical Observatory in Belgrade and the scientific research in astronomy in Serbia as well in the period 1925-1960. He wrote large number of books and text-books on astronomy. Eleven of these books, one chart and one scientific work are digitized and they are included into the Virtual Library of the Faculty of mathematics of the University of Belgrade, http://elib.matf.bg.ac.yu:8080/virlib. In this article we give short reviews of these works and his short biography as well.

64 Doctoral dissertations in NCD Virtual Library

Ž. Mijajlović, N. Pejović, Z. Ognjanović,
NCD Review, vol. 18, pp. 15-18, 2011.

This article presents the collection of digital copies of doctoral dissertations in mathematics and related sciences, mechanic and astronomy. This collection is deposited in the Virtual Library of the faculty of mathematics in belgrade. Metadata, about forty fields, accompanying each dissertation are presented, too. These data are needed for library processing and for exchange with other digital depositories of the similar nature.

Преглед НЦД 18 (2011), 15–18

Žarko Mijajlović, Nadežda Pejović
Faculty of Mathematics, Univ. of Belgrade
Zoran Ognjanović
Mathematical Institute,
Serb. Academy of Science and Art

DOCTORAL DISSERTATIONS IN NCD VIRTUAL LIBRARY

Abstract: We present the collection of digital copies of doctoral dissertations which is a part of the Virtual Library (http://elibrary.matf.bg.ac.rs) of the Faculty of Mathematics of the University in Belgrade. The library is operated under the supervision of NCD (National Center for Digitization). Metadata associated to this collection are also proposed.

Introduction

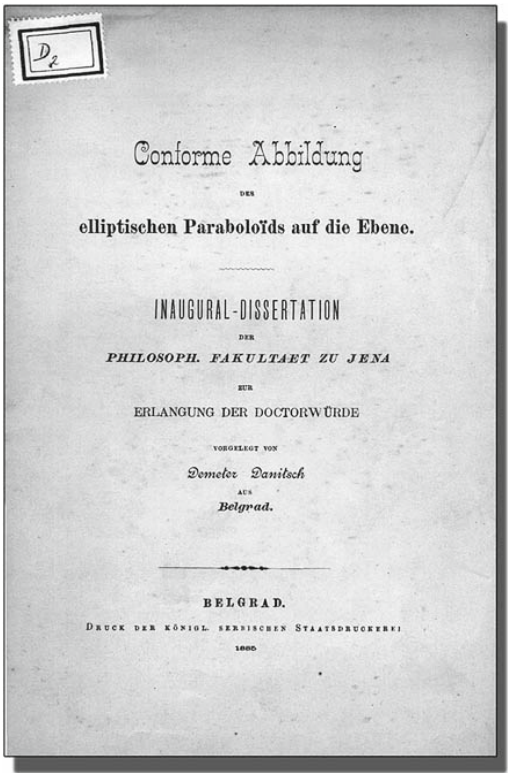
One of the aims of the current digitization project *Applications of information technologies in digitization of the scientific and cultural heritage* is to build the digital depositary of all Serbian doctoral dissertations in mathematical sciences. The project is financed by the Serbian Ministry of Science and Technological Development. It started in April 2008, and since then, about 360 doctoral dissertations were digitized. The dissertations are coming from all areas of the mathematical sciences: mathematics, astronomy, mechanics and computer science. Most of the digitized dissertations were obtained at the Belgrade University and the collection contains most of these dissertations defended since the University was founded in 1905. There are a few dissertations obtained at the other Serbian universities and several theses of Serbian mathematicians who live and work in Diaspora. The collection is a part of the Virtual Library http://elibrary.matf.bg.ac.rs. Dissertations, as well as all digitized books contained in this library, can be freely downloaded.

Collection of doctoral dissertations

The collection contains all doctoral dissertations of Serbian mathematicians written before the First World War (8), including the oldest one (see figure): Dimitrije Danić, *Conforme Abbildung des Elliptischen Paraboloids auf die Ebene*, Inauguration dissertation der Philosoph. Fakultaet zu Jena, printed in Belgrade, 1885.

Some of these theses were translated and whenever it was possible, the related documentation (biographies, archive materials, etc.) was also digitized and included in the presentation. The Internet presentation of retro-digitized books, theses and manuscripts can be also found at http://elib.matf.bg.ac.rs:8080/virlib/. At the same address, part of the presentation/database contains more or less recent (published and unpublished) doctoral and master theses defended at the Faculty of Mathematics in Belgrade. The Council of the Faculty of mathematics decided in 2007 that all submitted theses must be given also in the digital

form, so we expect that this part of the database will grow significantly in the next period. Our future plans are:



Conforme Abbildung

DES

elliptischen Paraboloïds auf die Ebene.

INAUGURAL-DISSERTATION

DER

PHILOSOPH. FAKULTAET ZU JENA

ZUR

ERLANGUNG DER DOCTORWÜRDE

VORGELEGT VON

Demeter Danitsch

AUS

Belgrad.

BELGRAD.

DRUCK DER KÖNIGL. SERBISCHEN STAATSDRUCKEREI

1885

- To digitize and include into the existing collection all Serbian doctoral dissertations. This is not an easy task, as the hard copies of certain theses have been lost. Secondly, the number of Serbian doctoral dissertations is large. Only at the University of Belgrade more than 18000 dissertations were defended. Our estimation is that at this moment the number of all Serbian doctoral dissertations is about 25000. Thirdly, this project is expensive. Our estimation is that the cost of whole process of digitization, including labor, equipment and software development, is about 30-40 EUR per one thesis.
- On the SEEDI (*South-Eastern European Digitization Initiative*) meeting held in 2007 in Cetinje, Montenegro, the idea to establish cooperation in this project between partners involved in SEEDI was supported. The existing database of Virtual Library gives a good opportunity to start the realization of this idea. The Virtual library may serve as a digital depository for this aim. The meeting of SEEDI organized in Sarajevo (2010) was the good place for final modification and the acceptance of metadata for this collection.
- We are planning also to interconnect our collection with similar collections and database in the world. In particular, it would be very interesting to connect the collection of mathematical doctoral dissertations contained in the Virtual Library with the Mathematics Genealogy Project, http://genealogy.math.ndsu.nodak.edu.

Of course, there are many issues to resolve such as metadata for such collections and copy-right questions related to our database and the use of doctoral dissertations deposited there. In this paper we propose the list of metadata for digitized doctoral dissertations.

Metadata

The new version of Virtual Library is operating under DSpace, open-source software released by MIT for archiving e-prints and other kinds of academic content. This software sustains Dublin Core standard, for overview of included elements see the site on the Internet address: http://www.scribd.com/doc/15284184/Dspace-Dublin-Core-elements-Overview.

Doctoral dissertation metadata

The elements prefixed by **dc** are predefined in DSpace. However, we included a set of new elements, prefixed by **mf** in order give more information about the theses and the authors. The set of elements is compatible with the recommendation for the national standard for describing digitised heritage in Serbia (for more information see: Zoran Ognjanović, Tamara Butigan, Bojan Marinković, NCD Recommendation for the National Standard for Describing Digitised Heritage in Serbia, in: Metadata and Semantics, eds: M.-A. Sicilia, M. D. Lytras, Springer, 45-54, 2009. ISBN: 978-0-387-77744-3). Our proposal of metadata is as follows.

Data that should be supplied by the author of the thesis

Field	Description
dc.contributor.advisor:	The name of the thesis adviser.
dc.contributor.author:	The name of the author of the dissertation.
dc.date.issued:	The year when the thesis was defended.
dc.description.abstract:	The abstract of the thesis.
dc.publisher:	The (geographical) place where the thesis was obtained.
dc.title:	The title of the thesis (translated in English).
dc.title.alternative:	The alternative title of the thesis.
mf.author.middle:	The middle name of the author.
mf.author.birth-date:	The birth year of the author.
mf.author.birth-place:	The birth place of the author.
mf.author.birth-country:	The country of birth of the author.
mf.author.residance-state:	The residence of the author at the time when the thesis was obtained.
mf.author.citizenship:	The author's citizenship.
mf.author.nationality:	The author's nationality.
mf.title.original:	The title of the thesis in the original language (in which it was written).
mf.abstract.original	The original abstract of the thesis
mf.subject.area:	The scientific field of the thesis.
mf.subject.keywords:	Keywords related to the thesis.
mf.subject.subarea:	The first scientific subfield of the thesis.
mf.subject.subarea:	The second scientific subfield of the thesis.
mf.subject.msc:	Mathematical Subject Classification.
mf.subject.udc.:	Universal Decimal Classification.
mf.contributor.committee:	Thesis committee
mf.university.original:	University where the thesis was obtained (original name).
mf.university.en:	University where the thesis was obtained (in English).
mf.faculty.original:	Faculty (department) where the thesis was obtained (original name).
mf.faculty.en:	Faculty (department) where the thesis was obtained (in English).
mf.document.references:	Number of references (bibliography) in the thesis.
mf.document.pages:	Number of pages in the thesis.
mf.document.location:	Location (library) where the hard copy of the thesis can be found.
mf.document.location:	The second location where the hard copy of the thesis can be found.
mf.document.genealogy-project:	YES if the thesis is presented in the Mathematics Genealogy Project, www.genealogy.ams.org.
mf.format.resolution:	scanning resolution (DPI).
mf.decription.original	Review of the thesis (original language)
mf.decription.en	Review of the thesis (in English)
mf.histrory.original	History of the thesis (original language)
mf.histrory.en	History of the thesis (in English)
mf.related_links	Links related to the thesis
mf.note.original	Note (in the original language)
mf.note.en	Note (in English)

Machine generated data:

dc.date.accessioned:	Date when the thesis was uploaded into the database.
dc.date.available:	Date since the thesis was available to the general public.

dc.identifier.uri:	Uri address of the digital copy of the thesis.
dc.description.provenance:	A description of any changes in ownership and custody of the resource that are significant for its authenticity, integrity, and interpretation.
dc.format.extent:	The size or duration of the resource.

Example

The next excerption from Virtual Library illustrates metadata and data associated to a doctoral dissertation in mathematics.

dc.contributor.advisor	Prešić, Slaviša	
dc.contributor.author	Marković, Marko	en_US
dc.date.accessioned	2009-12-03T12:14:50Z	
dc.date.available	2009-12-03T12:14:50Z	
dc.date.issued	1977	en_US
dc.identifier.uri	http://hdl.handle.net/123456789/196	
dc.description.abstract	Part1. Basic notions of model theory are given. Part2. Dual notions in categories of Boolean algebras and Stone spaces are studied in respect to natural contra-variant functor. The cellularity number of a Boolean algebra B, celB is studied, certain cardinal properties are proved, e.g. it is consistent with ZFC that celB is attained for every Boolean algebra B.	en
dc.description.provenance	Made available in DSpace on 2009-12-03T12:14:50Z (GMT). No. of bitstreams: 1 phdZarkoMijajlovic.pdf: 27390423 bytes, checksum: 5b828c388c5ebc07e988e80dce9212c9 (MD5) Previous issue date: 1977	en
dc.format.extent	158	
dc.publisher	Belgrade	en_US
dc.title	A Contribution to Model Theory and Boolean Algebras	en_US
dc.title.alternative	Prilog teoriji modela i Booleovih algebri	sr
mf.author.middle	Dušan	
mf.author.birth-date	1948	
mf.author.birth-place	Prokuplje	
mf.author.birth-country	Yugoslavia	
mf.author.residance-state	Serbia	
mf.author.citizenship	Serbian	
mf.author.nationality	Serbian	
mf.title.original	Prilog teoriji modela i Booleovih algebri	
mf.subject.area	Mathematics	
mf.subject.keywords	model theory, Boolean algebras, Lindenbaum algebras, type, elimination of quantifiers, model completion, atomless, absolutness, Levy hierarchy	
mf.subject.subarea	Algebra - Boolean algebras	
mf.subject.subarea	Mathematical logic - Model theory	
mf.subject.msc	03C10, 03C52, 03G05	
mf.contributor.committee	Kurepa, Đuro; Prešić, Slaviša; Alimpić, Branka	
mf.university.en	University of Belgrade	
mf.faculty.en	Faculty of Science and mathematics	
mf.document.references	43	
mf.document.pages	VI + 158	
mf.document.location	Faculty of Mathematics, University of Belgrade, Serbia	
mf.document.location	Library of the University of Belgrade, Serbia	
mf.document.genealogy-project	Yes	
mf.format.resolution	300 DPI	

zarkom@matf.bg.ac.rs nada@matf.bg.ac.rs zorano@mi.sanu.ac.rs

65 Digitized works of B. Ševarlić in Virtual Library of Faculty of Mathematics

N. Pejović and M. Dačić
Publ. Astron. Obs. Belgrade
vol. 91, pp. 357-364, 2012.

In the Virtual Library of the Faculty of Mathematics there are eleven digitized works of Branislav Sevarlic, professor of astronomy of the Belgrade University. These works include his doctoral dissertation, four university textbooks on astronomy, three translations of books written by Russian authors, two textbooks for secondary schools and an astronomical atlas. A particularly important work is his book General Astronomy (1971) as it was used for generations for teaching of astronomy at the University of Belgrade. This paper is presents all these works as well as the short biography of Professor Ševarlić.

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DIGITIZED WORKS OF B. ŠEVARLIĆ IN VIRTUAL LIBRARY OF FACULTY OF MATHEMATICS

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Abstract. In the Virtual Library of the Faculty of Mathematics http://elibrary.matf.bg.ac.rs, there are eleven digitized works of Branislav Ševarlić, professor of the Belgrade University. These works include his doctoral dissertation, four university text-books on astronomy, three translations of books written by Russian authors, two textbooks for secondary schools, and the last one is the Astronomical Atlas. A particularly important work is his book General Astronomy (1971) as it was used for generations for teaching of astronomy at the University of Belgrade. The aim of this paper is to present all these works as well as the short biography of Professor Branislav Ševarlić.

1. BIOGRAPHICAL NOTE

Professor Dr Branislav Ševarlić was born on June 28, 1914 in Belgrade where he finished all pre-university education. From 1932 he was a student at the Faculty of Philosophy of Belgrade University. He obtained the university degree in 1936 as an excellent student in a study group defined as a) Theoretical Mathematics, b) Rational and Celestial Mechanics and c) Astronomy. At the age of 23 in 1937, after having took the degree and doing the military service, Branislav Ševarlić came to the Astronomical Observatory in Belgrade to become an astronomical observer. As a member of the staff, led by Prof. Mišković who was the Director at that time, he had various activities. Mišković recognised a vigorous and smart young man and made him his helper in administration. The young Ševarlić, in addition to the calculation and astronomical activity, was during some time also the secretary of the Observatory.

In the middle of 1950 he became the chief of the Service of Latitude Variations. Thanks to his efforts and results achieved with the coworkers the Observatory was included in the International Polar-Motion Service. Through his monographs and PhD thesis defended in 1960 he gave a significant contribution to the study of latitude variations. The chief in the Group of Relative Coordinates he officially became in 1963 and, therefore, he started his activity in the field of fundamental astrometry. He organised the examination of the parts and measuring devices of the Belgrade Large Meridian Circle (LMC) and took part in the first observations carried out with



Professor Branislav Ševarlić

this instrument. Later on the observations with LMC resulted in seven observational catalogues, out of which some appear as a valuable astronomical contribution.

As for Prof. Ševarlić's university career, he became assistant as early as in 1939, at the Faculty of Philosophy for the subject of astronomy. After the Second World War, when at the Faculty of Civil Engineering the Geodesy Department was founded, Branislav Ševarlić applied to teach positional astronomy, a subject in the fourth year of studying. In 1948 he began with the lectures, from 1953 as assistant-professor, from 1957 as associate professor, to become full professor in 1961; the subject was geodetic astronomy at the Faculty of Civil Engineering. In 1964 he became full professor also at the Faculty of Sciences of the same university, the subject was astronomy and he was retired there in 1980. Many generations of astronomers passed Prof. Ševarlić's school, his name is well known to numerous students, pupils and amateurs oriented towards astronomy for diverse reasons. In addition to technical and scientific papers Prof. Ševarlić also wrote a few textbooks, several books and many papers which popularise the science of astronomy. From 1937, when his first paper, written by several coauthors, appeared, till 1997, when "*Astronomski atlas 2*" (Astronomical Atlas 2) was published, there is a period of roundly sixty years of his creativity.

Prof. Branislav M. Ševarlić is remembered as a versatile person, erudite and an unexceedable narrator. In his texts there is an easily noticeable intention to achieve a good Serbian, to indicate its beauty, to present a thought as precisely as possible, to represent the science of astronomy as successfully as possible.

The subject of the present paper is his PhD thesis, seven books and three translations. The digital copies of the books can be found in the Virtual Library of the Faculty of Mathematics at http://elibrary.matf.bg.ac.rs.

2. PROF. ŠEVARLIĆ'S WORKS

Branislav M. Ševarlić's PhD thesis *Prilog ispitivanju promena geografske širine Beograda* (A Contribution to the Examination of Variations in the Latitude of Belgrade) resulted in a few important issues: local non-polar influences were established, a distinction among the periods of latitude variations was found, for instance, Chandler's period, semi-annual and annual ones, and the influence of the wind speed and direction was specially treated. All of this tells us about the problems and difficulties met by astronomers in the middle of the XX century in

the determination of motion of the terrestrial poles. Prof. Ševarlić gave here a procedure for quick determining the pole coordinates from observations performed at one latitude station. We want specially to indicate the value for the aberration constant obtained by him based on the Belgrade latitude observations. At that time this constant was assumed to be equal to 20.47 arcseconds. With regard that he from the Belgrade observations had found a value of 20.52 arcseconds, he insisted on enlarging the aberration-constant value to at least 20. 50. It is well known that the modern value for this constant in the System of Astronomical Constants of IAU (1976) is equal to 20. 49552.

Geodeska astronomija **(Geodetic Astronomy)** is one of the Serbian first university textbooks. It was written by B. Ševarlić and Z. Brkić in 1963 for the students at the Geodesy Department of the Faculty of Civil Engineering in Belgrade. This textbook was preceded by a translation from Russian of *Kurs astronomije* (Astronomy Course) by N. F. Tsinger (translated by S. P. Bošković, Belgrade, 1925). This book of a clear and illustrative style has 266 pages. It contains many illustrations (92) and tables (33). There are also 63 solved problems very useful in learning and understanding the material. At the end of the book there is a copious literature including the lists of used books, problem collections, astronomical tables, handbooks, monographs, stellar catalogues and star charts and, at the end, a list of important scientific papers. These special lists were very important to students who continued their education in the field of astronomy.

Opšta astronomija **(General Astronomy)** by Prof. Ševarlić and Prof. Brkić appears as a significant augmentation of the university textbook *Geodeska astronomija* (Geodetic Astronomy) by the same authors. In it one treats the fundaments of classical astronomy disciplines, such as: spherical, practical and theoretical astronomy, celestial mechanics and fundaments of astronomical and physical geodesy. The book was to be used by the second-year astronomy students at faculties of sciences, as well as by future secondary-school teachers of astronomy, offering them a good basic knowledge in this field. Even today an astronomer should have this university textbook at the disposal. The purpose is not only reminding of a definition or checking a forgotten formula, but also solving a dilemma concerning the language.

Astronomski atlas **(Astronomical Atlas)** was prepared by Prof. Ševarlić together with Sofija Sadžakov. Its was an auxiliary textbook of astronomy for the primary and secondary school pupils. Assuming that the basic notions must be enlightened by using pictures, because in this way they would be understood and learnt more quickly and easily, this atlas contains numerous illustrations followed by short explanations. Clearly, such a book appears as a successful addition to the textbook material expected to be easy enough and interesting to the readers. It consists of 14 chapters which mainly follow the syllabus of that time and the official textbook. After the Introduction one finds chapters about astronomical instruments, astronomical observatories and famous astronomers. The whole material contains many figures and photographs so that it appears to be of interest even to those who are not specially fond of astronomical science.

The university textbook ***Istorija astronomske nauke - od Njutnova doba, do naših dana*** **(History of Science of Astronomy - from Newton's Age to Our Days)** is provided first of all for students of astronomy, but can be used as

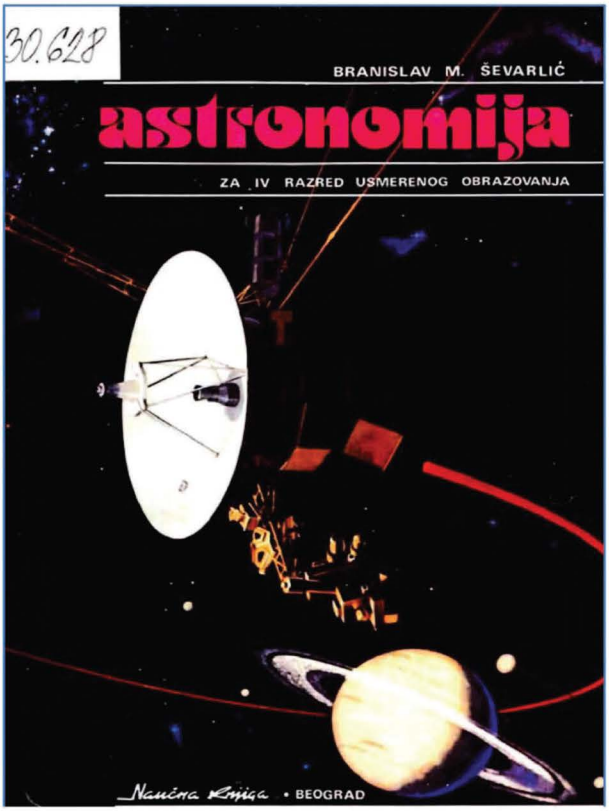
well by all whose field astronomy is, or they are only interested in it. It appears as a continuation of Milanković's *Istorija astronomske nauke od najstarijih vremena do 1727* (History of Science of Astronomy from the Earliest Times till 1727). The title of the book indicates the time interval covered by it. Naturally, all results and minor discoveries, which gave way to later more important, even epochal, discoveries, are not presented in the book. As the author says "the book contains a selection as careful as possible, but only a selection among many discoveries and theories and, also, it is free of all technical details and formulae being taught elsewhere."

The book *Putevi saznanja o Vasioni* (The roads of knowing about the Universe) written by Branislav Ševarlić was edited by *Mlado pokolenje* in Belgrade in 1967. It has 178 pages with 54 illustrations within the text. Its purpose concerns general education. In it one describes astronomical observations from the earliest ones towards very modern, to use the author's own words, "from stick and rope towards a radio telescope". The questions of origin for the Solar System and for our stellar system, the Milky Way, are described using legend and science. The diversities of our stellar home are also described, as well the scientific facts concerning space and the Universe. This book is very interesting and pedagogically useful. We are glad to recommend it to be read, not only by pupils and students, but also by an extended reader population.

The gymnasium textbook *Astronomija* (Astronomy) was written by B. Ševarlić, M. Vukićević-Karabin and S. Sadžakov. It was published by the publishing house "Zavod za udžbenike i nastavna stedstva" in Belgrade in 1975. *Astronomija* was a textbook for the last (IV) gymnasium form. Before in Serbian there was a textbook entitled *Kosmografija* (Cosmography) by Vojislav Mišković printed in 1953 in Belgrade and also the translation from Russian of the secondary-school textbook *Astronomija* (Astronomy), the author of which is B. A. Vorontsov-Vel'yaminov. *Astronomija* has 195 pages with 128 figures within the text and a star chart at its end. After every chapter there are lists of questions and problems. These questions and problems are suitable as exercises for learning. A good supplement to this textbook is *Astronomski atlas*, already mentioned above.

The secondary-school textbook *Astronomija* (Astronomy) was written by B. Ševarlić and published by "Naučna knjiga" in Belgrade in 1980. This textbook was for the last (IV) form of the reformed secondary school, mathematico-technical specialty. It contains eleven chapters, on 167 pages with 125 figures within the text. The astronomical notions and phenomena are explained in an understandable and simple way. The large number of figures contributes to a higher clarity of the material. At the end of each chapter there are problems to be solved and also practical examples concerning observations. The problems and instructions how to observe made that the learning could become more evident and interesting to the pupils.

***Praktična astronomija* (Practical Astronomy)** is a university textbook written by Prof. S. N. Blazhko in Russian in 1940, translated into Serbian by Branislav Ševarlić in 1952. With regard to lack of textbooks written by Serbian authors this translation was very important to the students of astronomy and geodesy in studying the observational methods and instruments. It was written on 323 pages with 104 figures within the text. These figures contribute to have a more informative way of describing astronomical instruments. Unlike the original the translation was provided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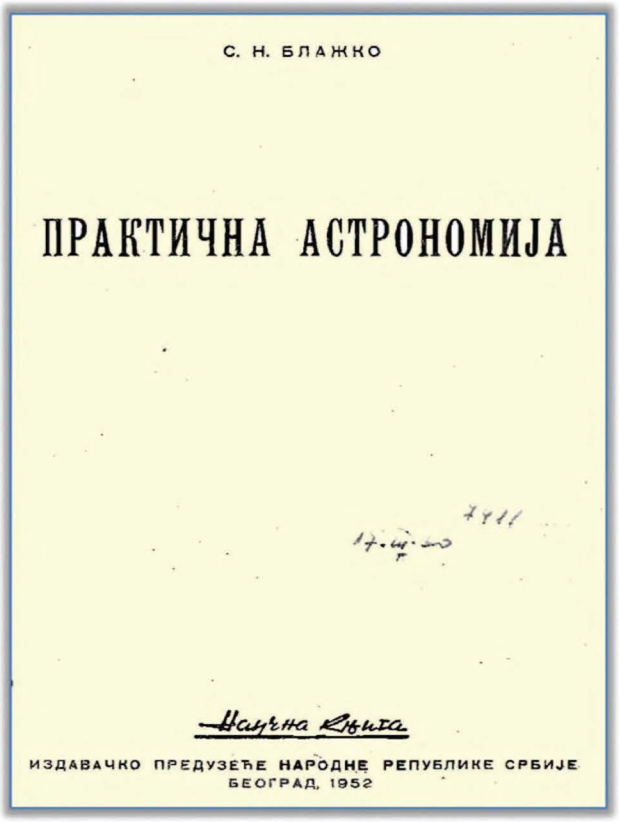
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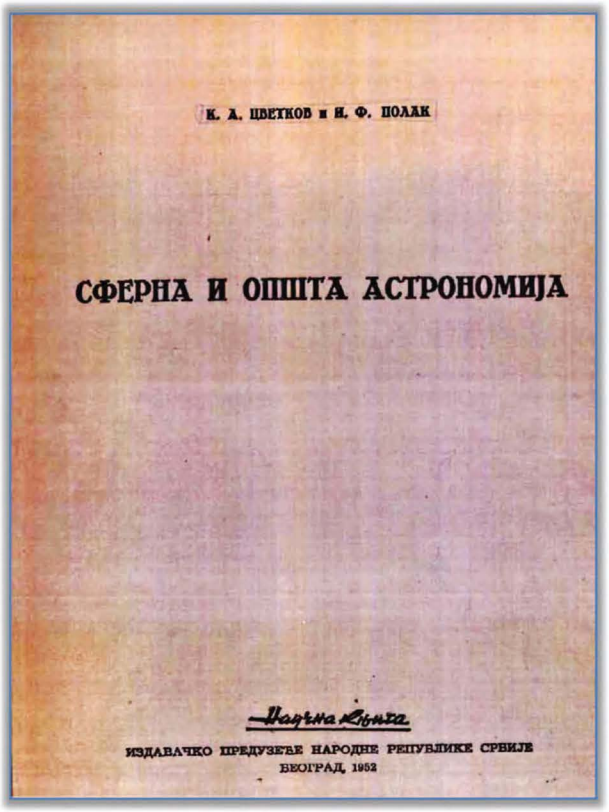
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СФЕРНА И ОПШТА АСТРОНОМИЈА
Научна књига
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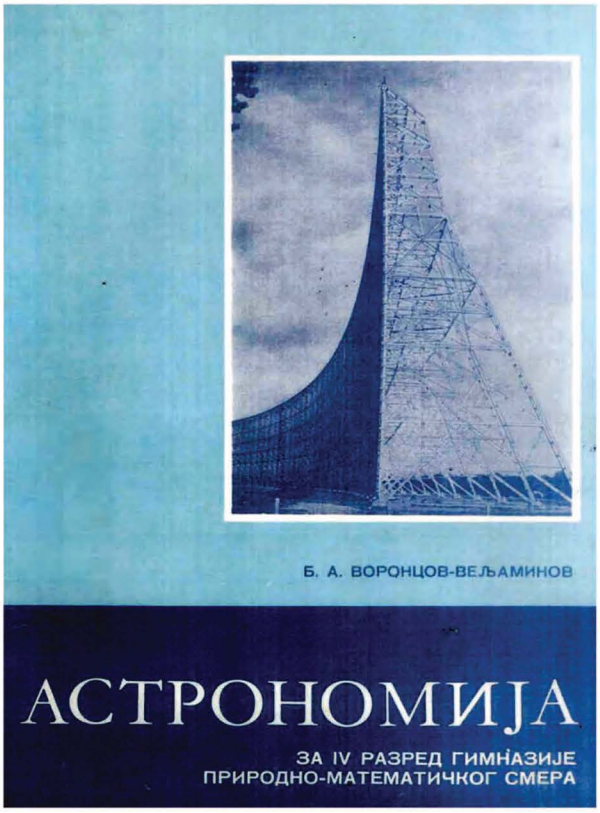
БЕОГРАД, 1952


Б. А. ВОРОНЦОВ-ВЕЉАМИНОВ
АСТРОНОМИЈА
ЗА IV РАЗРЕД ГИМНАЗИЈЕ
ПРИРОДНО-МАТЕМАТИЧКОГ СМЕРА


4.550
BRANISLAV ŠEVARLIĆ
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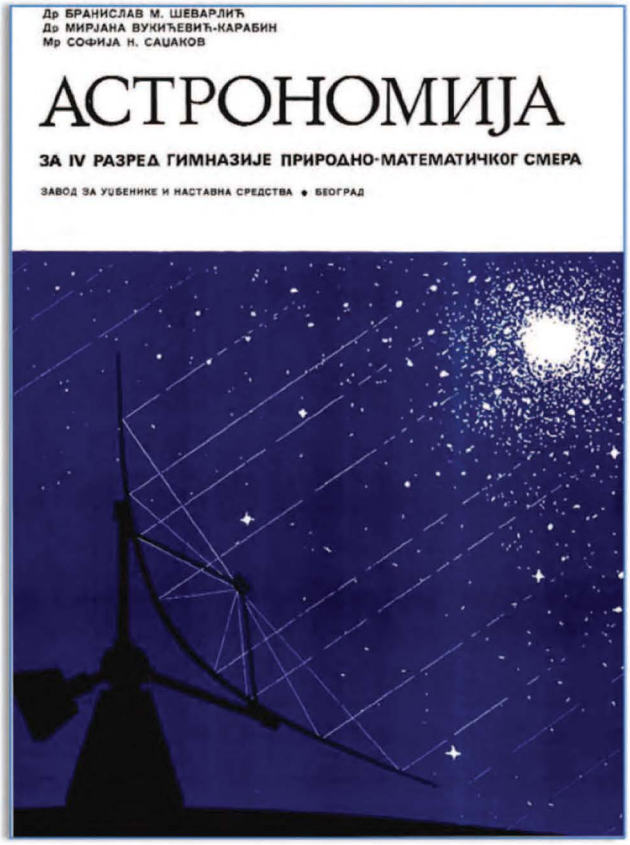
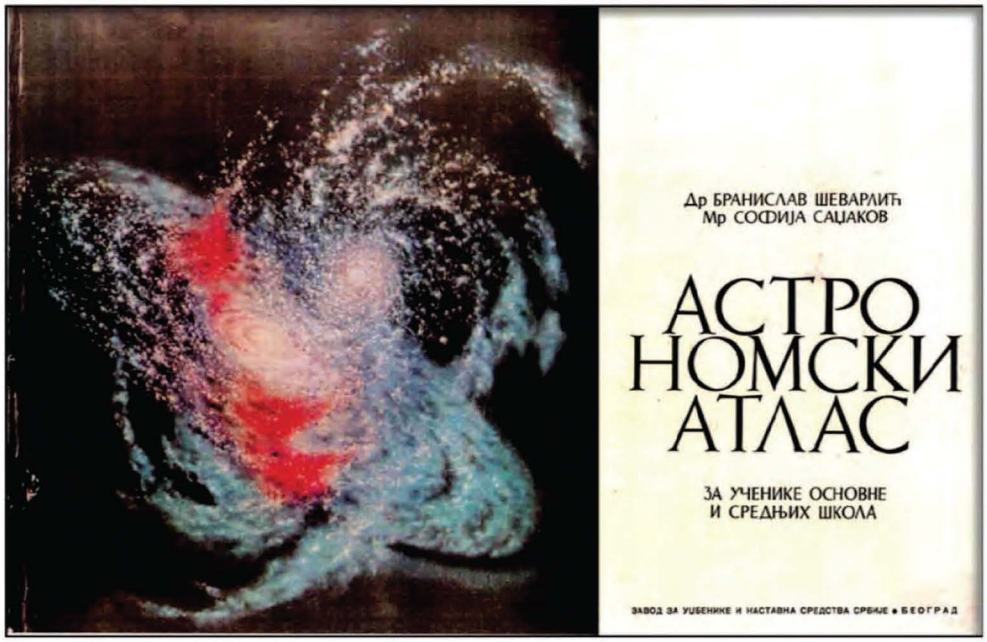
O VASIONI


Др БРАНИСЛАВ М. ШЕВАРЛИЋ
Др МИРЈАНА ВУКИЋЕВИЋ-КАРАБИН
Мр СОФИЈА Н. САЦАКОВ
АСТРОНОМИЈА
ЗА IV РАЗРЕД ГИМНАЗИЈЕ ПРИРОДНО-МАТЕМАТИЧКОГ СМЕРА
ЗАВОД ЗА УЏБЕНИКЕ И НАСТАВНА СРЕДСТВА • БЕОГРАД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Мр СОФИЈА САЦАКОВ
АСТРО
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АТЛАС
ЗА УЧЕНИКЕ ОСНОВНЕ
И СРЕДЊИХ ШКОЛА
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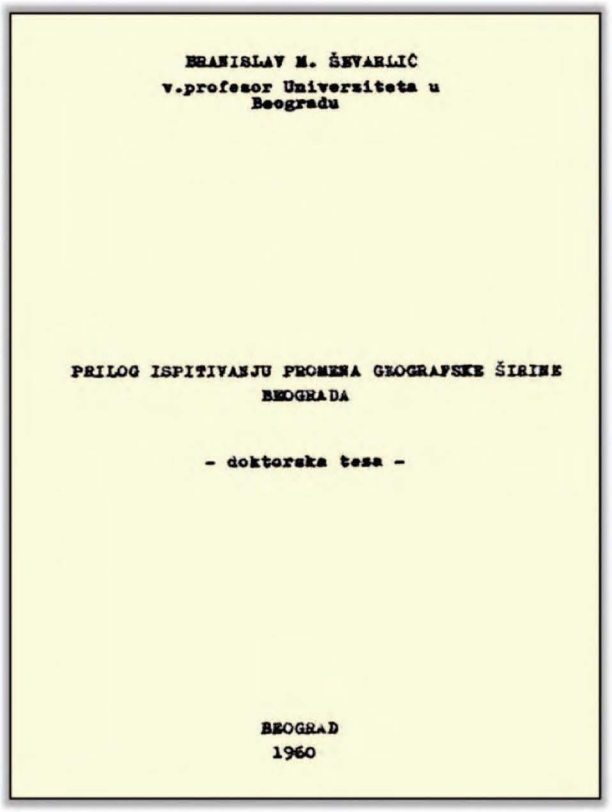
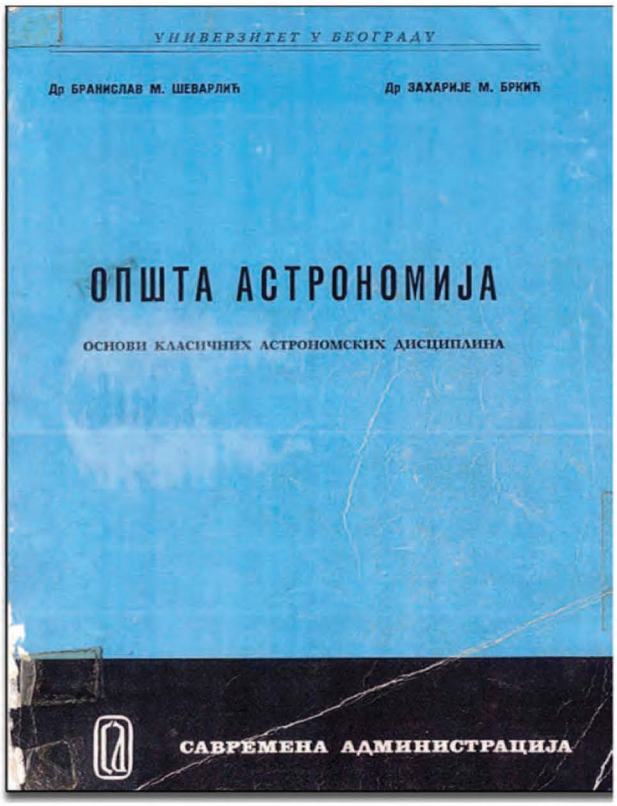
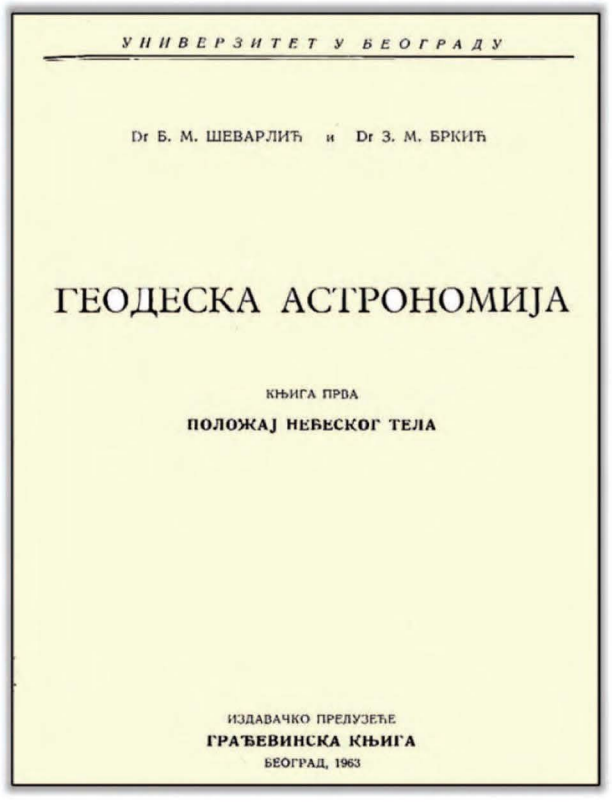

BRANISLAV M. ŠEVARLIĆ
v.profesor Univerziteta u Beogradu

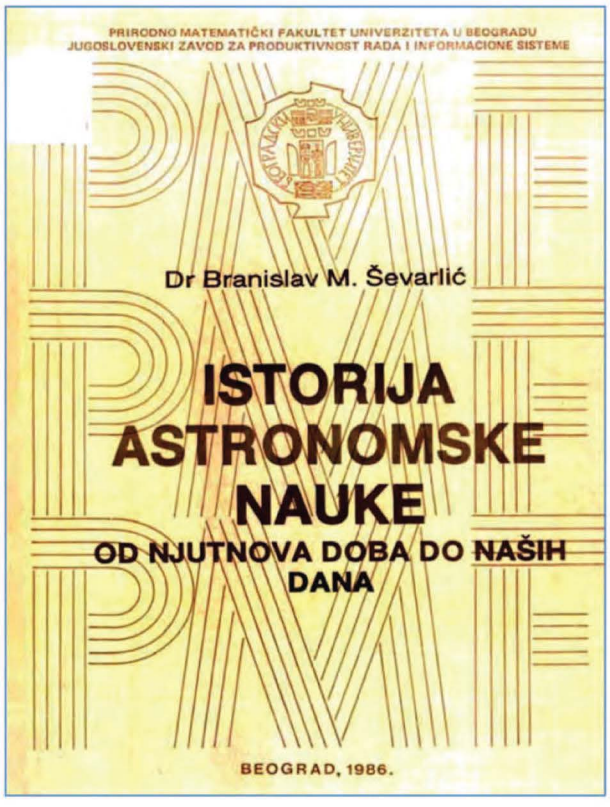
PRILOG ISPITIVANJU PROMENA GEOGRAFSKE ŠIRINE BEOGRADA

- doktorska teza -

BEOGRAD
1960







with a star chart in stereographical projection made by an engineer of geodesy, D. Šaletić. Unfortunately, the chart has not been preserved and, therefore, we have no possibility of digitising it.

***Sferna i opšta astronomija* (Spherical and General Astronomy)** is a university textbook written by Profs. K. A. Tsvetkov and I. F. Polak in Russian in 1945, and translated into Serbian by Branislav Ševarlić in 1952. This translation was very useful to the students of astronomy and geodesy because of lack of textbooks written by Serbian authors. It had a significant importance for the purpose of creating the astronomical terminology in Serbian. A great contribution in creating this terminology is due to Prof. Ševarlić. The book contains 362 pages, 119 figures and 19 telescope photographs of celestial bodies. It involves the part of spherical astronomy which concerns the Earth, the Solar System and stars. The main objective is qualifying astronomers to utilise annuals, ephemerides and catalogues.

***Astronomija* (Astronomy)** is a secondary-school textbook written in Russian by B. A. Vorontsov-Vel'yaminov (publication: Moscow, 1968), and translated into Serbian by B. Ševarlić and S. Sadžakov in 1969. The original of this book has been popular and utilised very much in the Russian-speaking area so that it has had several editions. It was written exceptionally nicely and systematically and it contains many illustrations, examples and problems. For this reason its translating appeared as a significant contribution to the secondary-school-textbook literature in Serbian.

3. CONCLUSION

The Virtual Library is a place where, via Internet, one can find the works of Serbian scientists written sufficiently long ago. By including the digital copies of the books written by Prof. Ševarlić we have made them easily available to an extended public.

References

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66 National Serbian digitization project: its achievements and activities

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This paper presents the efforts for the period 1990-2010 in the area of digitization and digital preservation of scientific and cultural heritage of a group of Serbian scientists from the Faculty of Mathematics of the University of Belgrade and the Mathematical Institute of the Serbian Academy of Science and Art. Virtual library, the repository of digitized books and other works of Serbian authors and E-Library of Serbian mathematical journals are portrayed.

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NATIONAL SERBIAN DIGITIZATION PROJECT: ITS ACHIEVEMENTS AND ACTIVITIES

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Abstract. The aim of this paper is to present the efforts in the area of digitization and digital preservation of scientific and cultural heritage of a group of Serbian scientists from the Faculty of Mathematics of the University of Belgrade and the Mathematical Institute of the Serbian Academy of Science and Art. Virtual library, the repository of digitized books and other works of Serbian authors and E-Library of Serbian mathematical journals are presented.

1. INTRODUCTION

For almost fifteen years there is a group of mathematicians in Belgrade which is active in the area of digitization. The members of the group are mainly from the Faculty of Mathematics of the Belgrade University and the Mathematical Institute of the Serbian Academy of Science and Art. The starting point of this group is that digitized scientific works are one of the most helpful resource and tool for scientific works and fast exchange of scientific information. Also, we understand that every scientific work becomes after some time the part of history and the part of cultural heritage as well. So, our activities are oriented in building of electronic repositories of digitized books, journals and other scientific works.

This group is also active in other areas related to digitization. They built several databases containing digitized materials. Some of these Internet oriented databases are the largest ones in Serbia and probably in the whole Balkan area. We founded an informal organization, the National Center for Digitization (NCD) which gathered several Serbian leading institutions in science and culture (Faculty of Mathematics of the University of Belgrade, Mathematical Institute of the Serbian Academy of Science, National Library, National Museum and Serbian Archive). In spite of the fact that NCD is an informal organization, NCD is the main driver of many activities in the area of digitization in Serbia. It started in 2003 the journal *NCD Review*. Until now 16 volumes of the journal were published. Under the auspices of NCD nine national and three international

conferences were organized since 2000. It is one of the founders of SEEDI, the South East European Digitization Initiative, and the seat of the messenger of SEEDI, *SEEDI communications*, is in Belgrade. In fact, *SEEDI Communications* is the second name of *NCD Review*. Finally, more than fifty papers were written by this group on the technical issues appearing in digitization and on digitized works.

The activities of this group have been supported since 1995 by the Serbian Ministry of Science through the technological projects. Up to now there were four such projects, the last one, *Applications of information technologies in digitization of the scientific and cultural heritage* is still running (up to end of 2010). In the next sections we shall describe two subprojects related to digitization of the scientific heritage.

2. VIRTUAL LIBRARY - DATA BASE OF TEXTUAL DATA, http://elibrary.matf.bg.ac.rs


Новая
Сербская
АРИѲМЕТИКА
или
Простое наставление къ Хитрому
Изъ Германскихъ книгъ собрано, новыми примѣрами
краткими правилами сокращено, и на
употребленіе Сербскаго учащагося
юности издано
отъ
ВАСИЛІЯ ДАМЯНОВИЧА.
Въ Маткахъ 1767.

Aritmetika, 1767
Vasilije Damjanović

There are a large number of rare books in Serbian public and private libraries. For the purpose of our project, we made an electronic catalog according to librarian standards of almost 500 books (published until the beginning of the 20th century) in possession of the Faculty of Mathematics in Belgrade. Just a few of them can be found in the joint catalogue of the network of the largest Serbian libraries (National Library, all university libraries, etc.). This was one of the reasons why we started the project of retro-digitization of mathematics-related works.

Around 1995, we digitized the collected works of the prominent Serbian mathematician Bogdan Gavrilović (1864–1947). Gavrilović obtained doctor's degree in mathematics in 1887 at the Philosophical Faculty of the University in Budapest. He was appointed professor at the University of Belgrade, two times elected president of the Serbian Academy of Sciences (1931–1937), a member of Circolo matematico di Palermo, and doctor honoris causa of the University of Athens. He published two voluminous university textbooks which had the character of monographs: *Analytical Geometry* (1896) and *Theory of Determinants* (1899). Those books, together with his theses (in Hungarian), other papers and archive were digitized (1996–2001), put first in TEX frame, then in PDF, and published as a compact disk. Even if he introduced several mathematical disciplines in the studying at the university level, particularly algebra and geometry, he was almost forgotten by the modern Serbian

mathematicians. This is the reason why we decided to digitize his works. This collection was the first retro-digitized corpus of the books in Serbia.

Besides the collected works of Bogdan Gavrilović, there are also collections devoted to other prominent Serbian scientist. Let us mention in this context the astronomers Đorđe Stanojević, Vojislav Mišković and Milutin Milanković.

The further steps in our work were related to the development of a digital library as a comprehensive and semantically interconnected collection of retro-digitized materials satisfying some of the following criteria:

- Books and manuscripts selected for digitization should be related with mathematical sciences: mathematics, mechanics, astronomy, physics, mathematical geography etc.

- Books considered for digitization had to be published before certain date in the past. We have chosen for this date the beginning of the World War II. (1941).

- Preference is given to Serbian authors, or to written works related to the area of Balkan.

So far, about 800 books, theses and manuscripts have been digitized, including first two books on mathematics written in Serbian language:

- Vasilije Damjanović, *Aritmetika* (*Arithmetics*), Venice, 1767, and
- Jovan Došenović, *Čislenica* (*Arithmetics*), Budim, 1809.


KÖNIGLICH SERBISCHE AKADEMIE

KANON DER ERDBESTRAHLUNG

UND

SEINE ANWENDUNG

AUF

DAS EISZEITENPROBLEM

VON

M. MILANKOVITCH

BELGRAD 1941

These two textbooks are very elementary, and their authors did not pretend to be original. They are, however, important for the cultural history of the Serbian people. For example *Čislenica* was the only printed textbook used in the Big School in Belgrade, in the first state created by the Serbian Revolution at the beginning of the 19th century. Thus, it could be considered the first Serbian university textbook, because the Big School is the forerunner of the University of Belgrade. Other digitized materials are works of Ruđer Bošković (1711–1787), two books of the famous Serbian scientist (mechanic, mathematician and astronomer), Milutin Milanković, including his celebrated work *Kanon Der Erdbestrahlung* (1935, 1941), all doctoral dissertations of old Serbian mathematicians (8), including the oldest one: Dimitrije Danić, *Conforme Abbildung des Elliptischen Paraboloids auf die Ebene*, Inauguration dissertation der Philosoph. Fakultaet zu Jena, printed in Belgrade, 1885.

Some of these theses were translated and whenever it was possible, the related documentation (biographies, archive materials, etc.) was also included. The

Internet presentation of this material can be found in the Virtual library. At the same address, part of the presentation and database contain more or less recent (published and unpublished) doctoral and master theses earned at the Faculty of Mathematics in Belgrade. The council of the Faculty decided in 2007 that all submitted theses must be given also in the digital form, so we expect that this part of the database will grow significantly in the next period. Now, there are in the Virtual library 360 doctoral dissertations of Serbian mathematicians of about 500.

An important collection of the Virtual library contains more than fifty Serbian books on astronomy. The oldest books from these collection were written in the XVIII century (books written by Zacharie Orfelin Stefanović (1726-1785), and Ruđer Bošković). The authors of other books are Atanasije Stojković (1773 - 1832), Đorđe Stanojević (1858 -1921), Milan Andonović (1849 -1926), Kosta Stojanović (1867 - 1921), Vojislav Mišković (1892–1976), Pavle Vujević (1881–1966), Tatomir Anđelić (1903-1993) and Jovan Simovljević (1929–2007). More details on this collection the reader can find in Pejović (2011).

The access to the Virtual library is free and open via Internet to the general public. There are about fifty visits to the Virtual Libray daily from all parts of the World.

3. E-LIBRARY OF SERBIAN MATHEMATICAL JOURNALS, http://elib.mi.sanu.ac.rs

Digitization of mathematical journals printed in Serbia started in 1995 with the oldest (founded in 1932) and most important one - *Publications de l'Institut Mathématique*. Until the Second World War it was published by the Belgrade University, and afterwards by the Mathematical Institute. More than 2000 articles appeared in about 100 volumes. The scope of the journal in the beginning was broader, including not only mathematics, but also papers from mechanics and astronomy. Almost every Serbian mathematician published there at least one paper, as well as many world leading mathematicians (Henri Lebesgue, Paul Montel, Waclaw Sierpinski, Paul Erdős, Saharon Shelah, Johan van Benthem, etc.). The first archiving technique was retyping articles using TeX. About 25 volumes (published between 1982 and 1995) were electronically archived in that way. The archive was very compact, having less than 100 MB. It included source (TeX) and output (device independent – DVI, and PDF) files. The archive has been permanently enlarged by adding all new volumes until these days. This archive is included in the Electronic Library of Mathematics (ELibM) offered through EMIS (European Mathematical Information Service, Mijajlović, 2003; Mijajlović et al., 2010).

However, since archiving using TEX system was expensive, it was decided that the remaining old volumes would be scanned. So, in 2007, the rest (since 1932) of *Publications* were completely retro-digitized. In the same period another journal, *Publications of the Faculty of Electrical Engineering, Series Mathematics and Physics* (today called *Applicable Analysis) and Discrete Mathematics*, was

also retro-digitized (about 1000 papers). It was founded in 1956. In the beginning, each contribution appeared separately bound and numbered consecutively, several times a year. Since 1959, the issues have been appearing collected in one or more volumes per year. In the first years, the journal had contributions from different fields apart from mathematics: physics, mechanics, and electrical engineering, but in the course of time, the journal focused almost exclusively on mathematics, especially convexity, functional equations and differential equations. It repository currently contains 285 issues with 3762 articles.

As Internet became more and more popular, in the year 2002 we decided to create an Internet database and the corresponding presentation of freely accessible full-text mathematical journals. The following journals have been involved so far:

- *Publications de l'Institut Mathématique*, Mathematical Institute in Belgrade (since 1932),
- *Bulletin, Classe des Sciences Mathématiques et Naturelles, Sciences mathématiques*,
  Serbian Academy of Sciences and Arts (since 2001),
- *Kragujevac Journal of Mathematics*, Faculty of Sciences Kragujevac (since 2000),
- *Matematički Vesnik*, Mathematical Society of Serbia (35 volumes, since 1993),
- *Nastava Matematike*, Mathematical Society of Serbia (since 1992, in Serbian),
- *Review of the National Center for Digitization*, Faculty of Mathematics in Belgrade and
  National center for digitization (since 2002),
- *Teaching of Mathematics*, Mathematical Society of Serbia (since 1998),
- *Publications of Department of Astronomy,* Faculty of Mathematics in Belgrade (the whole
  period of publishing the journal1969-1990),
- *Zbornik radova*, Mathematical Institute in Belgrade (since the first volume, 1952).

The corresponding presentations of journals are dynamically generated from the database and can be searched (both in English and Serbian) by: authors' names, titles, titles of special sections within the journals, key words and words contained in abstracts, classification numbers, and downloaded and printed. The last version of the database and presentation of mathematical journals is given at the address

http://elib.mi.sanu.ac.rs, while the retro-digitized journal can be found on

http://pefmath2.etf.bg.ac.rs.

Since 1999 Mathematical Institute publishes the journal *Visual Mathematics* (Ognjanović, 2003) with the goal to show the beauty of mathematics in a broad artistic-scientific context. It was one of the first journals appearing in digital form

only. All 39 volumes of the journal are available at http://www.mi.sanu.ac.rs/vismath.

Other activities

Our activities are not limited only to retro-digitization of printed matter, books and journals. We shall mention shortly other undertakings of our group.

Electronic catalog of cultural monuments in Serbia, http://spomenicikulture.mi.sanu.ac.rs. site includes various data, including GIS data and tracks (electronic paths) for more than 1500 most important monuments in Serbia (monasteries, archeological sites, historical sites). This is the most comprehensive Serbian Internet site of this kind and many there presented cultural monuments includes several hundred descriptions, data and other items. In recent years members of the Faculty of Natural Sciences of the University in Priština, now situated in Kosovska Mitrovica joined our project. They gave valuable contributions concernig digtization of cultural values in Kosovo and Metohia and Toplica region.

Clippings of Nikola Tesla, http://virlib.matf.bg.ac.rs/tesla. Nikola Tesla (1856-1943) is the great electrical engineer and inventor, world famous for his discoveries and innovations in the everyday use of electric power and electromagnetism. In Belgrade there is a museum devoted to Nikola Tesla. In the possession of the museum there are 57 books of clippings from news papers and professional journals related to science and collected by Tesla himself. Several members of the groups developed Internet oriented software and data base for the presentation of these clippings.

4. MEMBERS OF THE PROJECT

The members of the group are the specialists in the various areas of science. Here are the names of the professors of the Belgrade University and the senior researchers from the institutes of the Serbian Academy of Science, the members of the group: Žarko Mijajlović (mathematician, project leader), Zoran Ognjanović (mathematician), Nadežda Pejović (astronomer), Miomir Korać (archeologist), Dragan Blagojević (mathematician), Dragi Radojević (mechanics), Dragan Radovanović (geographist), Vesna Vučković, Nenad Mitić and Saša Malkov (computer scientists). The members of the project are also several graduate students and programmers: Aleksandar Pejović, Nadica Đorđević, Tijana Zečević and Aleksandar Valjarević.

5. CONCLUSION

The presented subprojects are oriented towards *Digital mathematical library*, a World project on which works many World institutions. The final aim of this project is the fulfillment of a mathematical dream of a digital archive containing

all peer reviewed mathematical literature ever published, properly linked and validated and verified.

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Pejović, N., et al.: 2011, this Proceedings.

Related Internet sites

Virtual library: http://elibrary.matf.bg.ac.rs
eLibrary of mathematical journals: http://elib.mi.sanu.ac.rs
NCD Review: http://elib.mi.sanu.ac.rs/pages/browse_publication.php?db=ncd
NCD: http://www.ncd.org.rs
SEEDI: http://seedi.ncd.org.rs

67 Teaching of Astroinformatics at the University of Belgrade

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The studies of astroinformatics at the Faculty of Mathematics of the Belgrade University were started in 2009 and they are organized according to the Bologna declaration. These studies were introduced with the aim that graduated students of this subject should be able to apply the knowledge of informatics and information technologies, necessary in the space research, also in other situations with all rights as any student taking degree in Informatics. This paper presents these studies and reforms of studies of astronomy induced by Bologna declaration at the Faculty of Mathematics.

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TEACHING OF ASTROINFORMATICS AT THE UNIVERSITY OF BELGRADE

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Abstract. The aim of this paper is to present the studies of astroinformatics at the Faculty of Mathematics of the Belgrade University. The studies of astroinformatics were started in 2009 and they are organized according to the Bologna declaration.

1. INTRODUCTION

The Bologna Declaration is the main document which has conditioned the education reorganization throughout Europe. This declaration was signed in 1999 by the education Ministers of the EU member-countries. Till the present moment this document has been signed by a majority of countries including ours as well. The Bologna process is aimed at the formation of a unique European system of university teaching and scientific research till 2010. In this way one tends to form a more efficient system of advanced education in Europe which is in accordance with the world knowledge market.

From the school year 2006/07 at the Faculty of Mathematics the teaching in mathematics, informatics and astronomy has been reformed in accordance with the Bologna process. According to the Law of the Republic of Serbia concerning the post-secondary school education, which follows the Bologna Declaration, there are three levels of university education:

Basic academic studies lasting 3 or 4years. The condition to finish them is to collect 180 or 240 EPTS points. In this way a student acquires the bachelor degree and they may continue toward the master studies.

Diploma academic studies lasting 5 years and to finish them one needs 300 EPTS points. A student after finishing the basic studies may, possibly having also a working position, continue during one or two years in order to gather additional 60 or120 EPTS points (up to 300), through which one acquires the master diploma.

PhD academic studies last 3 years and to finish them one needs 180 EPTS points, provided that the diploma master studies have already been finished. By finishing these studies and passing the PhD examination one acquires the PhD degree..

EPTS (European Point-Transfer System) is a unique system of evaluating a student's effort in knowledge acquiring for each subject included in a curriculum. The number of points per subject is different. It is not directly dependent of the number of lessons (lectures and exercises), but it also involves the estimate of the total time which a student needs to master the subject. The points are not marks. Every student gets an equal number of points for the same subject, independently of the mark obtained at the examination. The points are assigned only after successful examination.

At the Faculty of Mathematics the Basic Academic Studies of mathematics and astronomy last 4 years (240 EPTS), i. e. 3 years in the case of informatics (180 EPTS). After finishing the Basic Academic Studies a student becomes: mathematician, informatician or astronomer. The novelty is that after finishing the studies in addition to the certificate a student also gets an official document named Supplement to Certificate. This document contains a number of details: curriculum, list of subjects, number of EPTS points for each subject, subject contents, as well as the mark obtained at the exam. The Certificate and Supplement are delivered in both Serbian and English and they are in accordance with the documents of European universities.

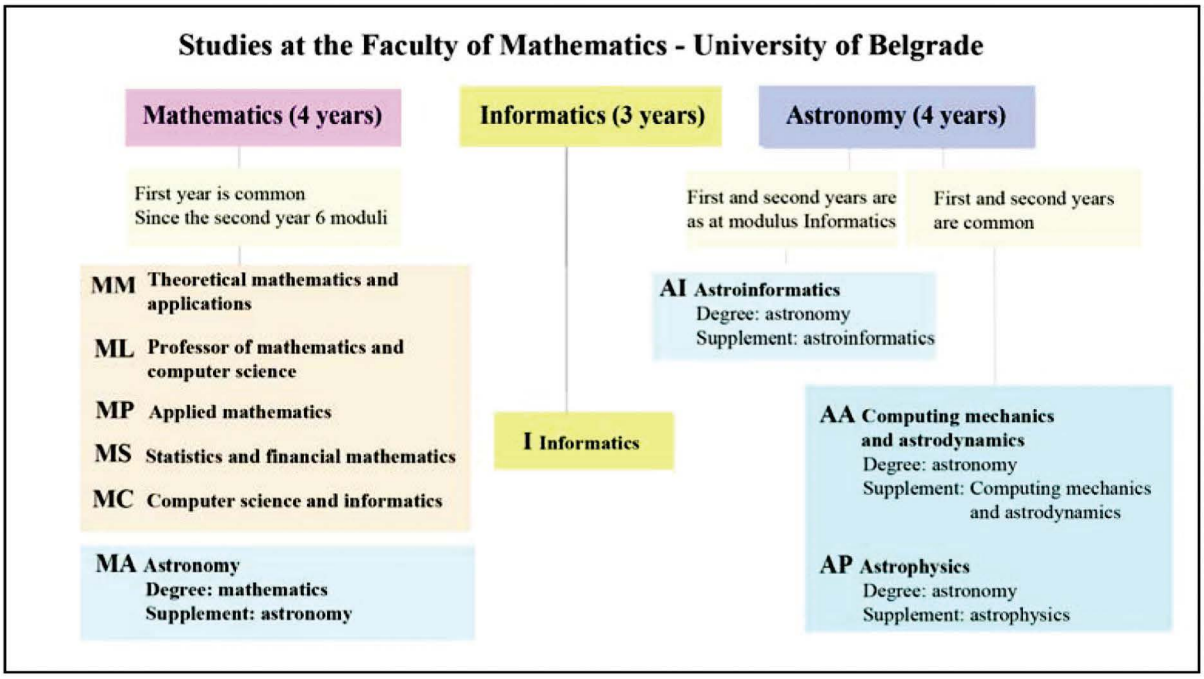
In the year 2009 the Faculty of Mathematics finished its accreditation for three directions of studies (mathematics, informatics and astronomy) which are subject to a further ramification of several moduli. The studies following new curricula also started in the autumn of 2009. The basic academic studies in mathematics after the first year have six moduli, in astronomy three moduli (two at the beginning and one after the second year), whereas in informatics there is only one modulus. The Certificate will contain the name, mathematician, informatician or astronomer, whereas the Supplement will give a detailed modulus description.

More detailed information concerning the studies within the directions of mathematics and informatics and the corresponding moduli can be found on the Internet Site of the Faculty of Mathematics. Here more detailed information will be given for the direction of astronomy only.

2. ASTRONOMY

From the scheme bellow one can see that astronomy is studied in the framework of four moduli. One of them belongs to direction of mathematics, the other three to direction of astronomy:

1. **MA** - direction **Mathematics** – modulus **Astronomy**
2. **AP** - modulus **Astrophysics**
3. **AA** - modulus **Computer Mechanics and Astrodynamics**
4. **AI** – modulus **Astroinformatics**



3. MODULI WITH TRADITION

Modulus MA – Astronomy has been present at the Faculty of Mathematics for decades. It is studied in the framework of the Direction of Mathematics and after the first year a student has the option of choosing the modulus of astronomy. According to the earlier curriculum a student after finishing the studies became a mathematician-astronomer; according to the new one in the Certificate it will be written mathematician and in the Supplement astronomy. As in the case of the earlier curriculum, students taking degree in the framework of this modulus may teach mathematics, informatics and astronomy in primary and secondary schools. This modulus offers the possibility of learning high-class mathematics which has many applications in astronomy. Only with such a high knowledge level in mathematics any person taking degree, in addition to the study of classical problems of astronomy, can be active in the field of the advanced research like studying cosmological models.

Modulus AP - Astrophysics has been also present at the Faculty of Mathematics for decades. According to the earlier curriculum a student after finishing the studies became an astrophysicist; according to the new one in the Certificate it will be written astronomer and in the Supplement astrophysics. Students studying in the framework of this modulus can learn high-class astronomy and physics so that with great success they can later study complex astrophysical processes on distant and not well-known celestial bodies and systems, such as stars, star clusters, nebulae, galaxies, active galaxies, black holes, quasars and blasars. The data about these distant worlds are collected at all wavelengths of the electromagnetic spectrum, from gamma to radio ones. After taking the degree astrophysicists will have the opportunity to get positions in addition to those concerning the astrophysical research, also as physicists at scientific institutes, universities and schools (primary or secondary).

4. NEW MODULI

Modulus AA – **Computer Mechanics and Astrodynamics** contains computer science dominant in the third and fourth years of studying. To this modulus and to that of Astrophysics the first two years are in common (see the scheme). After two years students have two options: towards physics by choosing the Astrophysics modulus or towards computer science by choosing the modulus of Computer Mechanics and Astrodynamics. Taking degree in Computer Mechanics and Astrodynamics students will get a certificate with name astronomer, whereas the Appendix will contain Computer Mechanics and Astrodynamics followed with the list of all subjects covering computer science, mechanics and astronomy. The contents of subjects from computer science will show what kind of new information technologies are mastered by students; they are applicable not only in astronomy and mechanics, but in economy, financial institutions and software houses, as well.

Modulus AI – Astroinformatics has two years in common with the direction of informatics. As the studies of informatics last 3 years, and those of astronomy 4 years, it follows that by taking degree at the four-year modulus of astroinformatics one also finishes the complete triennial studies of informatics. In the Certificate it will be written astronomer, in the Supplement astroinformatics. Similarly to the case of the preceding modulus, students will be able to apply the knowledge of informatics and information technologies, necessary in the space research, also in other situations with all rights as any student taking degree in Informatics.

5. PROGRAM OF STUDIES OF ASTROINFORMATICS

Among the products of our time is the informatic revolution. The data transfer becomes more and more. In this new world the role of computer science and informatics has grown enormously. There is practically no activity field deprived

of applying computer science and informatics. On the other hand, in the world enormous means are invested into astronomy and space research. For instance, at the moment a few observatories orbit the Earth, such as the Hubble Space Telescope and Chandra, sending continually the data of high importance to the space research. In addition to them there are many cosmic vehicles sent to the most remote parts of the Solar System, then many artificial Earth satellites, as well as a large number of telescopes situated at observatories throughout the surface of the Earth. An enormous body of data about our cosmic neighbours, the Moon, the Sun, the planets, their satellites, asteroids, comets, meteors, trans-neptunian objects in the Kuiper belt, small icy bodies of the Oort cloud, but also about distant worlds, like stars, extrasolar planets, star clusters, nebulae, galaxies, black holes and quasars, are continually collected. The majority of instruments are computerized and the observations are automatic. Without mathematics, computer science and informatics any use of these world databases and data treatment is unthinkable. Astronomers after finishing these new directions can successfully apply their acquired knowledge of mathematics except in astronomy, also in all other activities where programmers and informaticians find their job. It should be mentioned that on lists of unemployed persons there are neither programmers nor informaticians.

Now we shall present in more details the program and the content of the subjects that are taught at the modulus astroinformatics. First, we note that the interest of students in studying astronomy considerably raised since the modulus of astroinformatics was introduced. For example, the number of the enrolled students in astronomy doubled in 2010 in respect to the previous school year. Most of them are in fact the students of astroinformatics.

The courses at the modulus of astroinformatics are divided into three almost equal in size groups: astronomy, computer science and mathematics. Mathematics and informatics are mostly taught in the first two years of studies. In the third year of studies, the number of courses in astronomy and informatics is same. In the fourth year of studies, students are specialized in astronomy. All main branches of mathematics are taught through the specialized courses such as discrete structures (mathematical logic and combinatorics), mathematical analysis (calculus), algebra, geometry, computational mathematics and probability and statistics. Courses in informatics are the modern subjects in computer science and they cover algorithms, data bases, programming languages and Internet oriented courses. Finally, courses in astronomy are the standard basic courses in astronomy and they give the sufficient knowledge to the student to continue further specialization in astronomy and graduate studies. There are several courses in astronomy with heavily use of computers. Such a course is for example *Processing of astronomical observations*. Every student who graduated astroinformatics also has the diploma of informatics (the modulus **I,** *Informatics*) and in fact he can further specialize in three professions: astronomy, programming and teaching astronomy

and astroinformatics in the elementary and secondary schools. Even if the studies of astroinformatics just started (in 2009), our experience with students and teaching of astroinformatics is fine.

6. CONCLUSION

Anyone fond of astronomy can study it together with mathematics, physics, computer science or informatics by choosing one of the moduli named Astronomy, Astrophysics, Computer Mechanics and Astrodynamics or Astroinformatics.

A Certificate of the Faculty of Mathematics in Belgrade has been recognised abroad. Persons having it, in addition to positions in primary or secondary schools, universities and scientific institutes, can find positions within a wide range of various activities including financial institutions, economy, software houses. The internet site of the Faculty of Mathematics offers the possibility of more detailed informing not only concerning undergraduate studies, but the diploma and PhD ones as well, for the case of mathematics, informatics and astronomy.

The curricula with subject names and contents for each subject for all three directions of the Basic Academic Studies at the Faculty of Mathematics can be found at http://www.matf.bg.ac.rs.

The list of courses in Astroinformatics

First year – first semester			
	Course	classes	EPTS
1	Programming 1	3+3+0	8
2	Introduction to the organization of computers	3+2+0	6
3	Discrete structures 1	3+2+0	6
4	Linear algebra	3+2+0	6
5	General astronomy A	2+2+1	5

First year – second semester			
	Course	classes	EPTS
1	Programming 2	3+3+0	8
2	Introduction to the architecture of computers	3+2+0	6
3	Discrete structures 2	3+2+0	6
4	Analysis 1	3+2+0	6
5	General astronomy B	2+2+1	5

Second year – first semester			
	Course	classes	EPTS
1	Algorithms and data structures	3+3+0	7
2	Architecture and operating systems	3+2+0	7
3	Geometry	3+2+0	6
4	Analysis 2	3+2+0	6
5	General astrophysics A	2+2+1	6

Second year – second semester			
	Course	classes	EPTS
1	Analysis and construction of algorithms	3+3+0	6
2	Introduction WEB and Internet technologies	3+2+0	6
3	Object oriented programming	3+2+0	6
4	Analysis 3	3+2+0	6
5	Algebra	2+2+1	6

Third year – first semester			
	Course	classes	EPTS
1	Relational data bases	3+3+0	8
2	Compiling of programming languages	3+2+0	6
3	Positional astronomy A	3+2+0	6
4	Processing of astronomical observations A	3+2+0	6
5	Probability and statistics	2+2+1	5

Third year – second semester			
	Course	classes	EPTS
1	Artificial intelligence	3+3+0	8
2	Celestial mechanics	3+2+0	6
3	Practical astronomy A	3+2+0	6
4	Introduction to numerical mathematics	3+2+0	6
5	Selected topics	2+2+1	5

Fourth year – first semester			
	Course	classes	EPTS
1	Selected topics	3+3+0	23
2	Selected topics	2+2+0	4

Fourth year – second semester			
	Course	classes	EPTS
1	Operating systems	3+3+0	6
2	Theoretical astronomy A	3+2+0	5
3	Selected topics	3+2+0	5
4	Selected topics	3+2+0	5
5	Selected topics	2+2+1	5

Further details on the courses (the content, professors, etc) and selected topics (special courses) can be seen at the address http://www.matf.bg.ac.rs.

Note: 2+2+1 in the column **classes** in the table means that there is one lecture lasting two hours (2 X 45 minutes), one exercise lecture (oriented to problem solving) lasing three hours and one hour of practice (usually devoted to the student seminar works).

68 Serbian Astronomical Works in the Virtual Library of the Faculty of Mathematics in Belgrade

N. Pejović, Ž. Mijajlović, A. Valjarević
and G. Damljanović
Publ. Astron. Druš. Rudjer Bošković, Beograd
Vol. 11, pp. 311-323, 2012
Proc. VII Bulgarian-Serbian
Astronomical Conference (VII BSAC)
1-4 June 2010, Chepelare-Rozhen, Bulgaria
(M. Tsvetkov et al, ed).

This article presents a selection of books from the Virtual Library of the Faculty of mathematics in Belgrade, written in the 18th and 19th century by the authors: Ruder Bošković, Zaharije Orfelin, Atanasije Stojković, Djordje Stanojević, Milan Andonović and Kosta Stojanović. This review is similar by content to the articles "Early astronomical heritage in Virtual Library of Faculty of Mathematics in Belgrade" and "Astronomske knjige iz XVIII i XIX veka u Virtuelnoj biblioteci Matematičkog fakulteta".

Proceedings of the VII Bulgarian-Serbian Astronomical Conference (VII BSAC)
Chepelare, Bulgaria, June 1-4, 2010,
Editors: M. K. Tsvetkov, M. S. Dimitrijević, K. Tsvetkova, O. Kounchev, Ž. Mijajlović
Publ. Astron. Soc. "Rudjer Bošković" No 11, 2012, 311-323

SERBIAN ASTRONOMICAL WORKS IN THE VIRTUAL LIBRARY OF THE FACULTY OF MATHEMATICS IN BELGRADE

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ALEKSANDAR VALJAREVIĆ[2], GORAN DAMLJANOVIĆ[3]

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valjarkosmos@yahoo.com, gdamljanovic@aob.bg.ac.rs

Abstract. The Virtual Library of the Faculty of Mathematics, http://elibrary.matf.bg.ac.rs, contains more than 50 digitized books on astronomy written by Serbian scientists. This article presents a selection of books written in the 18th and 19th century by the authors: Ruđer Bošković, Zaharije Orfelin, Atanasije Stojković, Đorđe Stanojević, Milan Andonović and Kosta Stojanović. For each of these authors a short biography is included.

INTRODUCTION

The Virtual Library of the Faculty of Mathematics is the largest database of digitized texts in Serbia which is open for public use. At the time of writing this article, it contains nearly 1500 books mostly in mathematical sciences of old Serbian scientists (e.g. N. Pejović, 2009). Important collections in this library are a collection of 360 doctoral dissertations in mathematical science, most of them defended at the Faculty of Mathematics of the University of Belgrade and rare books from the 18th and 19th century. Most of these books are rather rare; some of them exist only in one copy and in fact are unavailable to the general public. First steps in building of the Virtual Library were done in 2004 (see Mijajlović at al., 2004).

In this paper we present a collection of books on astronomy written by Serbian authors and printed in the 18th and 19th century that are digitized and uploaded into the Virtual library. These books are the first Serbian astronomical books, and we can also say that they are the first scientific works of Serbian writers. Let us note here a general feature of these first Serbian scientists. They were universal in

their research interest and in other respects as well, so astronomy was not their only specialty. They were successful in other sciences, too: in mathematics, physics or philosophy, for example. For them, science mostly was the view of a unique being - Nature. The language of science was used by them to describe the nature of this being. Besides, they were often successful politicians, artists, travelers and writers of novels and poems. Their importance for the development of Serbian culture, language and science is enormous. Therefore, in addition to their manuscripts from astronomy we present here their brief biographies and other works as well. The good source for the history of astronomy in Serbia is Simovljević's (1980) article *Astronomy*.

BOOKS IN ASTRONOMY FROM THE EIGHTEENTH AND NINETEENTH CENTURIES.

In the Virtual Library there are 15 digitized books from this period by the following authors:

Ruđer Bošković (1711-1787), *Elementorum Universal Matheseos*, T. I-III, 1757, Venetis.

Zechariah Orfelin Stefanović (1726-1785), *Perpetual Calendar*, 1783, Vienna.

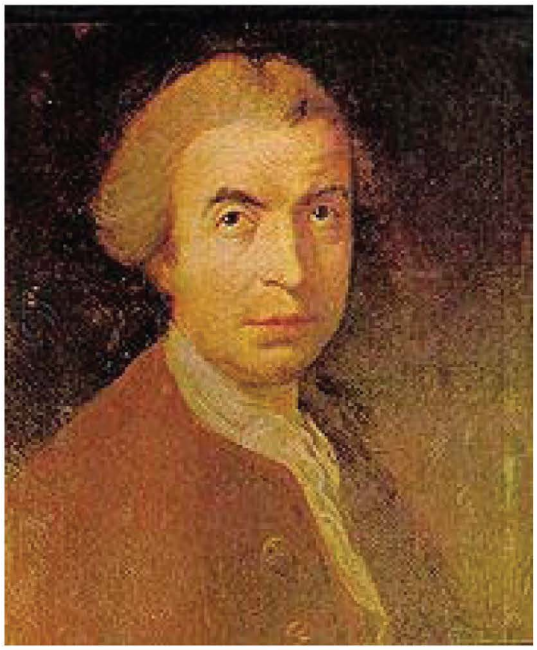
Atanasije Stojković (1773 -1832), *Physics*, 1810, Buda.

Đorđe Stanojevic (1858 -1921), most of his works (8) is digitized, including: *The starry sky of independent of Serbia*, 1882, Belgrade.

Milan Andonović (1849 -1926), *Cosmography* (1888), *On the Cosmos* (1889), Belgrade.

Kosta Stojanović (1867 - 1921), *Atomistics by Ruđer Bošković*, 1892, Niš.

Ruđer Bošković



Ruđer Bošković (1711-1787) is a famous mathematician and astronomer of Dubrovnik and one of the most important scientists of his time. He is included among one hundred most prominent Serbs of all time. He was a university professor, founder of the Milano Observatory and the director of the French Navy Optical Institute. He was an universal creator: philosopher, mathematician, astronomer, scolar, geologist, architect, archaeologist, diplomat, writer, professor, poet and polyglot.

He was born on 18th May 1711 as the seventh child of Nikola Bošković (a Serb from Orahov Do, a village near Trebinje in Herzegovina) and mother Paola (of

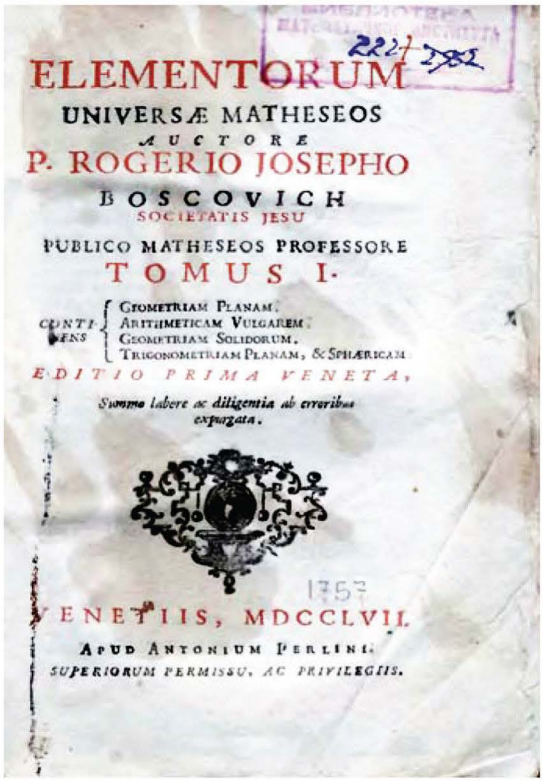
Italian descent, from the family of Baro Bettera, a notable poet from Dubrovnik). He lived mostly in Italy, where he gained world fame. Bošković died on February 13, 1787.

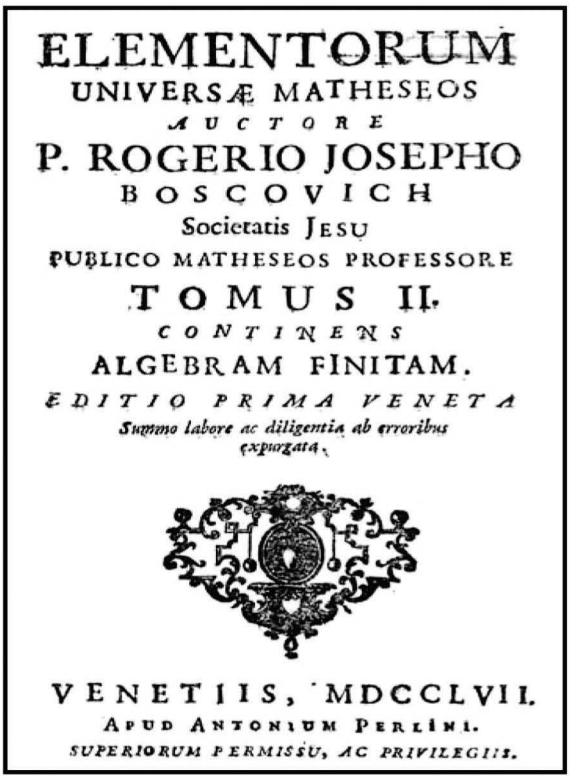
Bošković formulated the Unified Law of Forces, assuming that there is not only an attracting force (as Newton's law states), but that there is also repulsion which alternately changes with attraction at small distances between bodies. According to him, an elementary dimensionless particle is the source of the force, and time and space are relative, unlike in Newtonian theory. Hence, he is often considered as the forerunner of Albert Einstein.

Ruđer Bošković had numerous contributions to astronomy, among others: he introduced two geometric methods for determining the elements of the rotation of the Sun by observing the positions of three bodies, calculated the dimensions and the flatness of the Earth and discovered the comets. He was the first (1782) who accurately determined the trajectory of Uranus based on the measurement of coordinates of Uranus obtained by Messier and Méchain. It should be said that at the time of discovery it was thought that Uranus was a comet. By calculating the trajectory of Uranus, he perfected the method for determining the orbits of comets on the basis of four measured positions. In the 1800th it was one of the most accurate methods. He also studied the atmosphere of Jupiter and the nature of the Aurora.

Ruđer Bošković published numerous numbers of paper sheets in mathematics, astronomy and physics. His well-known books are *Theoria philosophiae naturalis redacta ad unicam legem virium in natura existentium*, *Opera pertinentia ad opticam et astronomia* and *Elementorum universae matheseos*.

In the Virtual library there are the following books by him: *Elements of Mathematics* (*Elementorum Universal Matheseos* - Tomus I-III, 1757, Venetis), *Diary on the Journey from Constantinople to Poland* (1762), translated by D. Neljeković, 1937, Belgrade, *On the law of continuity and its consequences in respect to the basic elements of matter and their forces*, printed in 1754, translated into Serbian by E. Stipanić, 1975, Belgrade.


ELEMENTORUM
UNIVERSÆ MATHESEOS
AUCTORE
P. ROGERIO JOSEPHO
BOSCOVICH
SOCIETATIS JESU
PUBLICO MATHESEOS PROFESSORE
TOMUS I.
CONTINENS
GEOMETRIAM PLANAM,
ARITHMETICAM VULGAREM,
GEOMETRIAM SOLIDORUM,
TRIGONOMETRIAM PLANAM, & SPHÆRICAM.
EDITIO PRIMA VENETA,
Summo labore ac diligentia ab erroribus expurgata.
VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.


ELEMENTORUM
UNIVERSÆ MATHESEOS
AUCTORE
P. ROGERIO JOSEPHO
BOSCOVICH
Societatis JESU
PUBLICO MATHESEOS PROFESSORE
TOMUS II.
CONTINENS
ALGEBRAM FINITAM.
EDITIO PRIMA VENETA
Summo labore ac diligentia ab erroribus expurgata.
VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.

Zechariah Stefanović Orfelin

Zechariah Stefanović Orfelin (1726 - 1785) was a prominent Serbian poet, historian, engraver, baroque educator, calligrapher and writer of textbooks. He was born into a Serbian family in Vukovar (Slavonia).

Among his most important works is *Slavenoserbski magazine* printed in Venice in 1768. This is the first South Slavic magazine. Although it was printed just in one volume, its importance is great. In the preface he presented the idea of civil enlightenment, and he aslo said that science, literature and philosophy should leave the narrow circle of educated people and that it must become available to everyone. In 1768 Zechariah Orfelin introduced into Serbian literature a language which was a mixture of Church and common language. He also included many Russian words. In this way he practically founded the Slavoserbian language. Because of his artistic works, primarily in the copper plate, he was elected a member of the Art Academy in Vienna.

Orfelin is the author of the first Serbian primer (*bukvar*) which was used since 1767 in teaching many generations of students in writing and reading. He is also the author of the first Serbian-Latin textbook. His most comprehensive work was *The Life of Peter the Great* (Venice, 1772), which he had seen as the enlightened monarch. He also wrote *Perpetual* (eternal) *calendar.* This book includes a comprehensive chapter on astronomy.

Orfelin's *Perpetual Calendar* was printed in Vienna in 1783. The book has 366 pages and 9 astronomical drawings at the end of the book. Its content concerns mainly natural sciences and astronomical phenomena and contains, as well, description of historical events since the creation of the World in a chronological order. Chapters on astronomy bear the names: *Space, The Moon and the Planets,Comets,Solar Eclipse and Moon Eclipse*. It includes information on weather cycles, tables with information about the Sun and the Moon (needed for the calculation of the date of Easter according to the Big indicium), tables on the length of day and night, moon phase tables, and more. Therefore *Perpetual calendar* is the first book on astronomy written in the Slavic language, how Orfelin said "to the benefit of the Slavic-Serbian (*slovenoserbskim*) people". More details about Orfelin's Calendar, the reader can find in Milovanović (2007).



The cover pages from the Orfelin's *Perpetual Calendar*

Atanasije Stojković



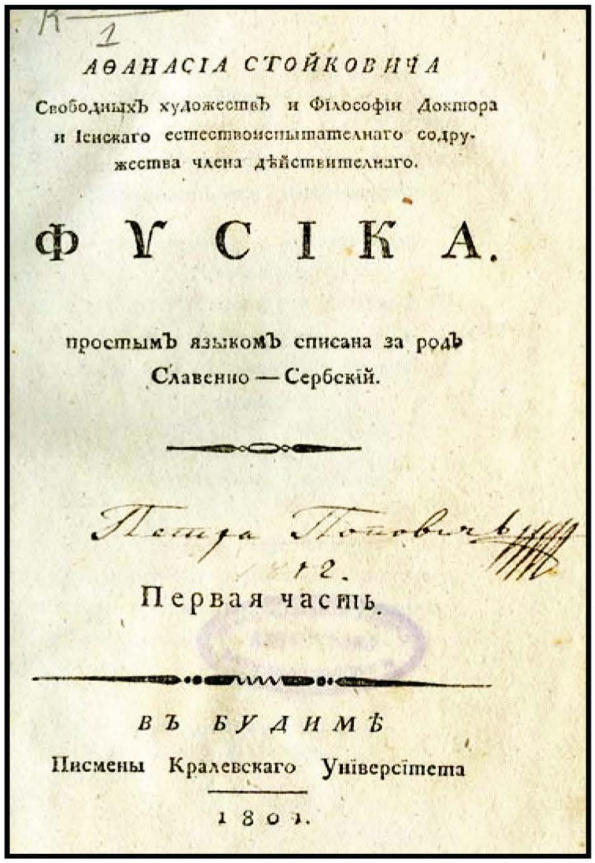
Atanasije Stojković was born in 1773 in Ruma, Vojvodina. In this town he finished Ascension school (Serbian Grammar school). He went soon to Šopron where in 1794 completed the first six grades of high school and then enrolled in a secondary school in Szeged, where in 1796 graduated philosophy. In Vienna, in July 1797th, he met Dositej Obradović and introduces himself with Dositej's enlightenment ideas. In the fall of this year he managed to obtain support from Metropolitan Stratimirović for studies in Göttingen, at that time one of the most famous universities. There he obtained PhD degree in philosophy (1799), and began intensive studies in various sciences (physics, mathematics, astronomy, history). After a brief stay in Ruma (1799) he moved to Budapest, where he published his main work *Physics* (1801-1803), printed in 3 volumes.

In the Habsburg monarchy it was difficult to obtain a civil job, so after the call of Severin Osipovič Potocki, the future Minister of Education of Russia, Atanasije accepted the position of regular professor of physics at newly established Kharkov University. He soon became the dean of the Department of Physics and Mathematics and on two occasions (1807-1808 and 1811-1813) he was the Rector of the University of Kharkov. He is the founder of the Kharkov Learned Society and he also received many honors and decorations. He became a member of the Imperial Academy of Sciences, obtained many material privileges, became a national consultant and Tsar Alexander I gave him the Medal of St. Vladimir of the third degree.

Atanasije Stojković was a hardworking and capable man, of wild spirit, very successful and was appreciated in his surroundings. He spoke several languages: German, Latin, French, Italian, English, Greek, Hungarian, and almost all Slavic languages. He had interesting and exciting life that next to science was marked by an affair that ended his professorial career. He was also engaged in a secret mission and diplomacy. He died in Kharkov in 1832.

With his versatility, ambition and intelligence, probably under the influence of enlightenment ideas of Dositej Obradović, Stojković was trying to educate his people, to improve their science and culture, at a time when barely was foreseen the emergence of a new Serbian state.

The Stojković's book *Фисика* (Physics) was printed in 3 volumes. The first two volumes have 320 pages while the third was written in 284 pages. The book presents an overview of the knowledge at that time of nature and natural phenomena. At the beginning of the first volume there is a chapter on mechanics (the properties and movements of the body). The most of the first volume (the second and the third chapter) is devoted to astronomy. There are described celestial bodies, apparently moving spheres of the sky, stars, solar system, the Sun, planets, comets, the Moon, the eclipses. The third chapter describes our planet Earth. Also, the concepts of celestial equator, horizon, zenith, and nadir are introduced. He also explains the geographical coordinates, the size of Earth, the Earth's movement, the change of the day and night on the Earth and the Earth's interior. The rest of the book is devoted to the description of geographical notions and meteorological phenomena.



АѲАНАСІА СТОЙКОВИЧА

Свободныхъ художествъ и Фїлософіи Доктора и Ієнскаго естествоиспытателнаго содружества члена дѣйствителнаго.

ФѴСІКА.

простымъ языкомъ списана за родъ Славено — Сербскій.

Петра Поповића

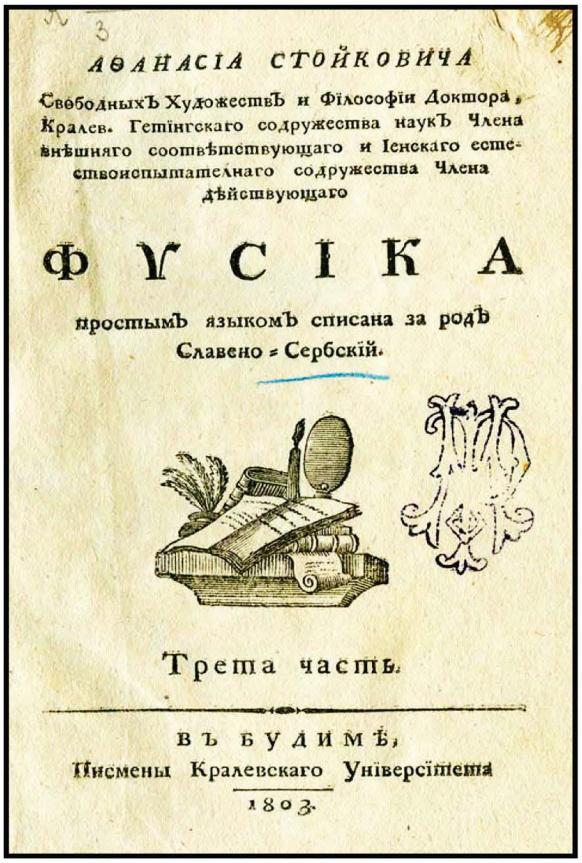
12.

Первая часть.

ВЪ БУДИМѢ

Писмены Кралевскаго Универсїтета

1801.



АѲАНАСІА СТОЙКОВИЧА

Свободныхъ Художествъ и Фїлософіи Доктора, Кралев. Гетінгскаго содружества наукъ Члена внѣшняго соотвѣтствующаго и Ієнскаго естествоиспытателнаго содружества Члена дѣйствующаго

ФѴСІКА

простымъ языкомъ списана за родъ Славено = Сербскій.

Трета часть.

ВЪ БУДИМѢ,

Писмены Кралевскаго Универсїтета

1803.

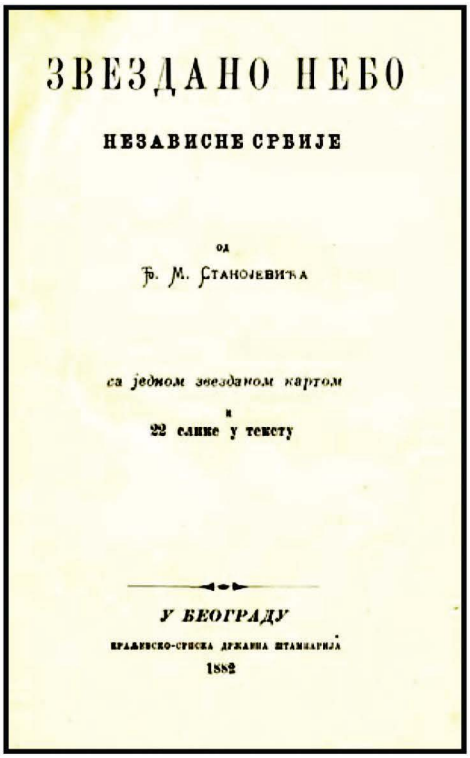
The book is written in the common language, without complex mathematical formulas and equations, having the aim to present knowledge about astronomy and nature to ordinary people.

Đorđe Stanojević



Đorđe Stanojević (1858-1921) was born in Negotin. He completed the Grand School in Belgrade. After that he became an assistant at High School, working with his teacher Kosta Alkovića (1836-1909) and then as the professor at the First Belgrade Gymnasium (1883). Having the scholarship of the Ministry of Defense (1883-1887), he studied in the most famous astronomical and meteorological observatories in Europe (Potsdam, Hamburg, Meudon, Greenwich, Pulkovo).

On behalf of his results he received a call from the Paris Observatory to participate in scientific research expedition to study the Sun and in the Petrovsk where he participated in the study of the full eclipse of the Sun. Two years later he joined another scientific-research expedition to the Sahara to study the thermal spectrum of the Sun. At that time and later, he published several research papers in astronomy in the publications of the Paris Academy of Sciences. These are the first scientific works in astronomy among the Serbs.



ЗВЕЗДАНО НЕБО

НЕЗАВИСНЕ СРБИЈЕ

од

Ђ. М. СТАНОЈЕВИЋА

са једном звезданом картом

и

22 слике у тексту

У БЕОГРАДУ

1882

Stanojević was a versatile, knowledgeable man with a lot of active interest in many fields of science and technology. His interest in physics was great. Upon his return to Serbia he devoted himself to the study of modern physics. In the same time, he became the lecturer in physics and mechanics at the Military Academy. Also, he taught applied physics and mechanics at the High School and at the Belgrade University after establishing it in 1905. He was responsible for the construction of hydroelectric power plants in Serbia (Vučje, Negotin, Užice, Ivanjica and other places) and for the electrification of Belgrade. This put Belgrade among the first European capitals with full electric lighting. He died in Paris in 1921.

In Negotin there is "Đorđe Stanojević" Square and the museum is dedicated to him. In New Belgrade there is "Đorđe Stanojević" street. The Electric Power Industry of Serbia on the occasion of its day, 6th October, gives recognition "Đorđe Stanojević" for contributions to the development of Serbian electric power industry.

The Virtual Library contains eight digitized books of Đorđe Stanojević: *The starry sky of independent of Serbia*, 1882, Belgrade; *A walk across the clouds*, 1884, Belgrade; *Cosmic energy of modern physics* (introductory lecture, 22nd September, 1887, Department of Physics at the Military Academy in Belgrade); *Nikola Tesla and his discoveries*, 1894, Belgrade; *Science of light*, 1895, Belgrade; *Central forces in nature*, 1906, Belgrade; *Industry of coldness*, 1909, Belgrade; *Inaccurate celebration of the Easter in the Orthodox Church and the reform of the calendar*, 1908, Belgrade.

Stanojević works are characterized by the simplicity and easiness of writing when exposed to often complex astronomical phenomena. These phenomena, however complex, are mostly descriptively presented, without mathematical expressions and using very fine drawings. Therefore, this book is primarily intended for general education and general readers. In this regard, Stanojević scored his goal, to bring the basic knowledge of astronomy to the general reader. It is our great pleasure to have this precious book out of the darkness of library storage into the light through the Virtual Library and made it easily accessible to the future readers.

More details about Stanojević life and works the reader can find in Pejović (2008ab, 2009).

Milan Andonović

Milan Andonović (1849-1926) was born in Požarevac where he finished high school. He was educated in Belgrade at the engineering departments of the Great School in Belgrade and at the German Universities in Karlsruhe, Aachen and Munich under the supervision of famous professors Ritter, Jordan, and Helmert. There he specialized in mechanics, geodesy and astronomy. Upon his return to Serbia in 1880, he was appointed as the professor of geodesy at the Great School in Belgrade. He was responsible for the introduction and enforcement of the cadastre in Serbia in the late 19th century. He founded the Geodetic Institute under whose auspices were surveyed many towns in Serbia. In 1907 he founded the Surveying and Building Academy where many Serbian surveyors and civil engineers were trained. Together with John Dragašević, Milan Nedeljković and Djordje Stanojević he is bearing the credit for introducing the teaching of astronomy in Serbian schools and promotion of science in Serbia. He died in Vienna in 1926.



МИЛАН Ј. АНДОНОВИЋ

Andonović had published a number of scientific papers, books (all printed in Belgrade), and discussions related to surveying, astronomy, shape and size of the Earth, the theory of least squares, survey and cadastre:

Basics probability and the theory of least squares, 1886; *The shape and size of the Earth*, 1886; *Cosmography with basic astronomical notes for high school teachers*, 1888; *The volume and size of our Earth*, 1889; *The Universe*, 1889; *Basic geodesy with distinctive view of the cadastral question*, 1890-1897 (1st part 1890, p.368, 2nd part 1897 vol. 1, p. 369-1390; vol. 2, p.1391-1641); *The cadastre*, 1889.

He also wrote during the First World War, political discussions and articles (in German and French) in favor of Serbia. There are the following digital copies of his books in the Virtual Library: *Cosmography*, 1888, and *The Universe*, 1889, both printed in Belgrade

Hard copies of both books are in the Library of Astronomical Observatory of Belgrade. Books are digitized by the courtesy of Vojislava Protić-Benišek.

Cosmography was intended for high school students and students of other secondary schools. Probably, it was used as a secondary textbook at the Military Academy and the Grand School, the forerunner of the Belgrade University. In addition to the foreign literature, Andonović mentioned in the bibliography two Serbian authors: *Trigonometry* of Dimitrije Nešić and *Cosmometry* by Jovan Dragašević. The book has many illustrations: 141 photos and drawings, 17 tables and a large star map. He took most of these drawings, with permission, from a book of E. Weiss, then the director of the Vienna Observatory.

Cosmography is an extensive (533 pages) and quite comprehensive book. It describes celestial sphere, constellations, the shape and size of the Earth, the Earth's rotation and revolution, the apparent movement of the Sun, coordinate systems, time, calendars, Solar system, stars, tides, precession, Cosmogony and among other things, Kant-Laplace's theory about the origin of the World (i.e. the Solar system). Besides all that, there are twenty fully solved astronomical problems with detailed explanations.

The book is written in a very beautiful style, almost as a literary work, and is worth reading also from that point of view. There we find many forgotten words and terms whose meaning were forgotten or had changed since today. For example, the cluster (Serb. *zvezdano jato*) there means constellation, stardust

(Serb. *ozvezdina*) - meteorite, comet star (Serb. *zvezda repatica*) - comet, the inflow and outflow (Serb. *priliv i odliv*) – the tide. Also, for each constellation he introduces the *alinjman* – having the meaning similar to asterism.



КОСМОГРАФИЈА

СА

ОСНОВНИМ АСТРОНОМСКИМ НАПОМЕНАМА

ЗА

ВИШЕ РАЗРЕДЕ СРЕДЊИХ И УЧИТЕЉСКИХ ШКОЛА

ОД

М. Ј. Андоновића

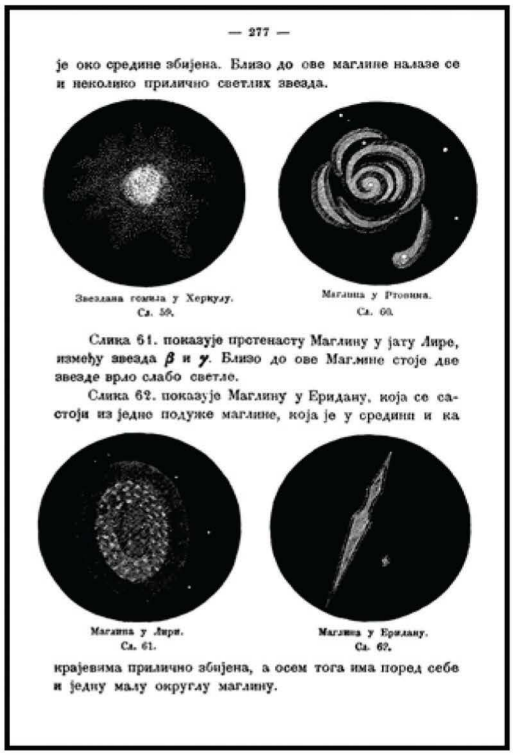
ПРОФЕСОРА ВЕЛИКЕ ШКОЛЕ

СА 141. СЛИКОМ У ТЕКСТУ И 26 ТАБЛИЦА

У БЕОГРАДУ

ИЗДАЊЕ И ШТАМПА КРАЉЕВСКО-СРПСКЕ ДРЖАВНЕ ШТАМПАРИЈЕ

1868.



— 277 —

је око средине збијена. Близо до ове маглине налазе се и неколико прилично светлих звезда.

Звездана гомиља у Херкулу.
Сл. 59.

Маглина у Ртоници.
Сл. 60.

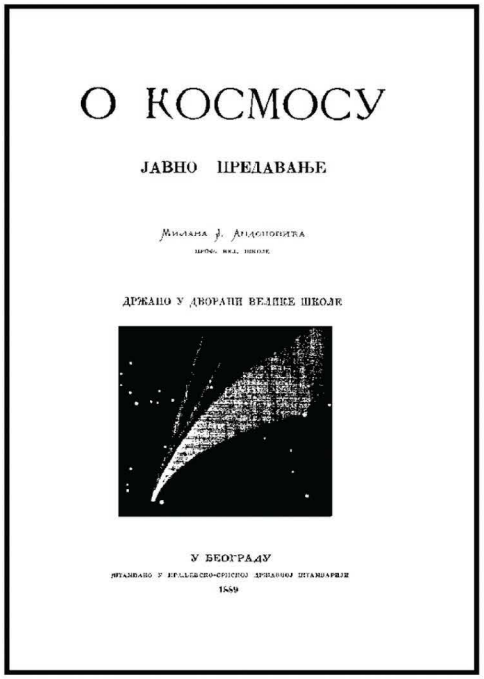
Слика 61. показује протенасту Маглину у јату Лире, између звезда β и γ. Близо до ове Маглине стоје две звезде врло слабо светле.

Слика 62. показује Маглину у Еридану, која се састоји из једне подуже маглине, која је у средини и ка

Маглина у Лири.
Сл. 61.

Маглина у Еридану.
Сл. 62.

крајевима прилично збијена, а осем тога има поред себе и једну малу округлу маглину.



О КОСМОСУ

ЈАВНО ПРЕДАВАЊЕ

Милана Ј. Андоновића

ПРОФ. ВЕЛ. ШКОЛЕ

ДРЖАНО У ДВОРАНИ ВЕЛИКЕ ШКОЛЕ

У БЕОГРАДУ

ИЗДАВАО И ШТАМПАО КРАЉЕВСКО-СРПСКО ДРЖАВНО ШТАМПАРСКО

1859



ТАБЛИЦА X.

Планета Јупитер.

Планета Јупитер.

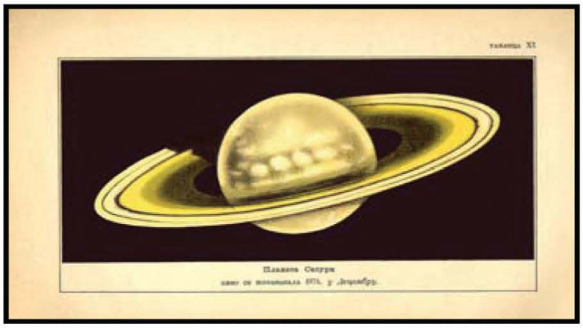
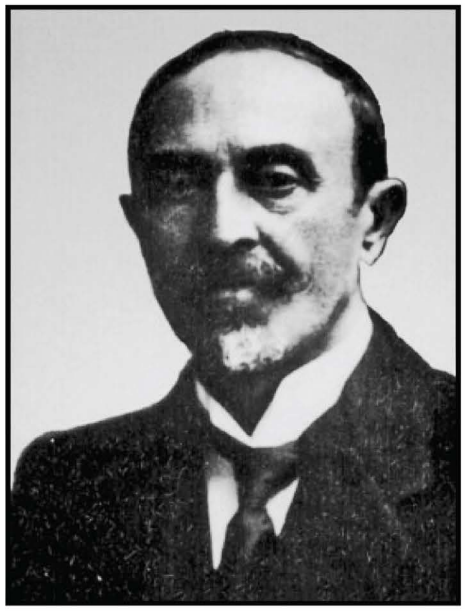


ТАБЛИЦА XI.

Планета Сатурн

For example the *alinjman* of Big Bear (in Serbian also called *Velika kola* - Big Chariot) has four stars – the asterism trapeze of the constellation Big Bear, while the *alinjman* of North Star is the line that passes through the rear wheels of the Big Chariot towards to the North Star. The Virtual Library contains digital copies of his books: *Cosmography* (1888) and *The Universe* (1889), both published in Belgrade.

Kosta Stojanović





Kosta Stojanović (1867-1921) was born in Aleksinac. There he finished elementary school and lower secondary school, while the high school he completed in Niš. He graduated in mathematical sciences at the Grand School in Belgrade. For some time he was working as a high school teacher in Niš and Belgrade. He went to Paris in 1893 where he studied mathematics, astronomy, mechanics and physics under the supervision of famous professors Poincaré, Picard and Appeal. There he became a member of the Historical Society of France and the French Astronomical Society. He went to Leipzig in 1897 where he entered doctoral studies in mathematics. Unfortunately, he returned to Serbia after three months because of his illness.

Kosta Stojanović was a university professor, but he had a very rich political career, too. At the beginning of 1900th he became a deputy as the representative of the Niš district.

Since the 1903rd he taught applied mathematics at the Grand School and at the Belgrade University after it was established in 1905. Whereupon he abruptly interrupted academic career and 1906 he became the Minister of Commerce, at the time of the custom war between Belgrade and Vienna.

Around that time, Kosta Stojanović wrote the book *Fundamentals of economy value*. This book is a work in mathematical economics and Stojanović there used very

advanced mathematical apparatus, for example, the theory of partial differential equations to describe economic models. Even today there is an interest for this book, not only in Serbia but also in the other parts of former Yugoslavia. This is probably the most advanced book in economics ever printed in Serbia due to the mathematical tools that were used in.

He was the Minister of Agriculture (1919-1920) and the Minister of Finance in 1921 in the government of Nikola Pašić. A few days after the appointment as the minister of finance, he suddenly died.

The book *Mechanics* is actually a university textbook in applied mathematics. Let us mention that at this time, under the term applied mathematics it was assumed mechanics, mathematical physics and celestial mechanics. The most of the book is related to the vector calculus, then the kinematics, statics, dynamics, analytical mechanics and the theory of differential equations. Chapter Ten of this book is dedicated to astronomy. There he gave the basics of celestial mechanics and the theory of differential equations of the motion of planets, comets and satellites. Also, the theory of elliptical movement, the method of variation of constants and parabolic motion of comets are presented.

CONCLUSION

The Virtual Library of the Faculty of Mathematics has a significant place in the presentation of old and rare books to the Serb scientific and professional community and general public wide-world. This fact we can convince, for example, by the choice of books from the collection of astronomical works of the Serbian scholars presented in this article.

Acknowledgements

This paper is written under the project "Application of information technologies in digitization of scientific and cultural heritage", IT 13 017, funded by the Serbian Ministry of Education and Science.

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Milovanović, M.: 2007, *NCD Review*, **11**, 43.
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Pejović, N.: 2008b, Proc. Conf. "Đorđe Stanojević – život i delo – povodom 150 godina od rođenja", Novi Sad, 10-11 april 2008, SANU Novi Sad, 163.
Pejović, A.: 2009, *NCD Review*, **15**, 27.
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69 Astronomical books in Virtual Library of Faculty of Mathematics in Belgrade

N. Pejovićand Ž. Mijajlović
Publ. Astron. Obs. Belgrade
Vol. 91, pp. 267-271, 2012.

This article gives more technical details such as metadata and semantical organization of astronomical works of Serbian scientists in the past in the Virtual Library of the Faculty of mathematics in Belgrade. Therefore, this review may be considered as a technical supplement to papers with the similar thematic already published by the same authors.

Publ. Astron. Obs. Belgrade No. 91 (2012), 267 - 271 *Contributed paper*

ASTRONOMICAL BOOKS IN VIRTUAL LIBRARY OF FACULTY OF MATHEMATICS IN BELGRADE

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Abstract. The Virtual Library of the Faculty of Mathematics in Belgrade, at the address http://elibrary.matf.bg.ac.rs, contains about 100 digitized books related to astronomy. These books are written mostly by Serbian scientists but there are also translations into Serbian by foreign authors. This article presents a list of books written by the following authors: Rudjer Bošković, Zaharije Orfelin, Atanasije Stojković, Djordje Stanojević, Milan Andonović, Kosta Stojanović, Nikolai Tsinger, Milutin Milanković, Milan Nedeljković, Pavle Vujević, Vojislav Miković, Zaharije Brkić, Branislav Ševarlić, Jovan Simovljević and Sergei Blažko. Some of these books had the great influence on the development of astronomy, geodesy and mathematical geography in Serbia. The full list of these books is given and a selection of them is presented and commented.

1. INTRODUCTION

In the last decade, mainly due to the development of the broad band Internet, many kinds of digital databases are freely available on the WWW. In particular, there are several important projects related to building digital repositories of retro-digitized books. This kind of digital databases are called digital, or virtual libraries. We found in Wikipedia probably the best explanation of the essence of the idea of digital library:

Virtual Library means library without walls. The resources are available in digital format, there is no paper, microforms etc. The resources are locally held or accessed through computer networks.

Some of the large world digital libraries are: Project Gutenberg, at the address http://www.gutenberg.org, Europeana, http://www.europeana.eu and arXiv at the address http://arxiv.org. We started our project a decade ago, but we intensified it in the last three years As a result we have now the fully functional Virtual Library of the Faculty of Mathematics which contains a collection of about 2000 digital copies of books.

2. VIRTUAL LIBRARY

The overall aim of our digital library is to create a comprehensive and interconnected collection of retro-digitized books and other digital documents. Objectives of the

project include electronic archiving first of all the old manuscripts and their publishing in electronic photo-type form and presentation to the general audience. Our proposal was at the beginning mathematically inclined, but since 2009 we started to include the books from other areas of science and literature. However, the preference will be still given to Serbian authors and works related to South-East European (Balkan) area.

Some of the books in the Virtual library are rather rare and it is known that there are left only few copies of them in the printed form. We think that it is very important to preserve them in some form. Not only as a cultural or scientific heritage important for our local community but also as a part of the World heritage. We decided to preserve and present them to the wide audience in the electronic - digitized form. Virtual library of the Mathematical Faculty of the University of Belgrade is the largest digital depository of digitized texts in Serbia available to the general public. At this moment it contains about 1600 digitized books. In the last two years more than 1000 digital copies of books were uploaded in the library. There are several important collections, including 360 doctoral dissertations in mathematical sciences (mathematics, astronomy, mechanics, theoretical physics and mathematical geography) most of them obtained at the University of Belgrade. There is also the collection of old Serbian books dating the XVIII and XIX century. Most of these books are rare and some of them are left only in one known printed copy and they are not easily accessible.

Digitization is heavily based on information technologies. On the other side information technologies are one of the fastest changing parts of the contemporary world. For example, since the appearance of disk and tape based digital storage in the 1960s, there occurred more than 200 different storage formats. Therefore digital libraries and information archives face a continuing challenge in maintaining files on currently supported storage hardware and media in supported file formats. We had in mind all these issues in considering the following aspects of the this project:

1. Rules for choosing preferential books and manuscripts to be digitized.
2. The mathematics of information storage (database architecture, data manipulation, data forms, etc.).
3. Information retrieval.
4. Metadata and rules for handling metadata.
5. Distribution of informations to the wide audience using Internet.
6. The ability to search electronic files efficiently and retrieve information quickly.
7. The ability to reuse information in other documents and other formats.
8. Fast retrieving of digitized material through electronic transactions and data downloading.
9. The proper choice of data format and resolution.

There are many standards for the database architecture, distributions of information and information retrieval. Common practice in Database Projecting is usage

of Data Base Management Systems (DBMS) and particularly Relational DBMS, for example ORDBMS or Oracle RDBMS. These systems include variety of tools which allow users to create, update, and extract information from their databases. However, some procedures for data description that are independent from hardware and software-platforms were proposed more or less recently. We also note that metadata for digitized documents may differ significantly from data for printed books obeying some of the existent standards (e.g. "Unimarc"). For example, metadata for a digitized document should include the following items, ordinary not found in the mentioned standards: name of the editor who made decision for digitization, then resolution, file type, revision number and date, etc. In developing our Virtual library we relied on the Dublin core metadata initiative which concerns developing metadata standards and disciplinary-specific metadata sets and frameworks for the interoperation of metadata sets. The Virtual Library is implemented in open source software Dspace, http://www.dspace.org.

3. AUTHORS

In this section we present the collection of astronomical books of Virtual Library printed through XVIII to the first half of XX century. The Virtual Library, contains about 100 digitized books related to astronomy. These books are written mostly by Serbian scientists but there are also translations into Serbian by foreign authors. Some of these books had the great influence on the development of astronomy, mechanics, geodesy and mathematical geography in Serbia. The full list of these books is given in the appendix.

It should be noted the following general feature of first Serbian scientists, particularly those from the XVIII and XIX century. They were universal in their research interest and in other respects as well, so astronomy was not their only specialty. They were successful in other sciences, too: in mathematics, physics or philosophy, for example. For them, science mostly was the view of a unique being - Nature. The language of science was used by them to describe the nature of this being. Besides, they were often successful politicians, artists, travellers and writers of novels and poems. Their importance for the development of Serbian culture, language and science is enormous. Therefore, in addition to their contributions to astronomy, they should be studied in other respects as well, their life and their contribution to science and history of Serbian people. The oldest books cited in this article are the first Serbian astronomical books, and we can freely say that they are the first scientific works of Serbian authors.

In this section we list the names of all authors whose books are retro-digitized and deposited in the Virtual Library. They are also presented by one or two of their most important books. There are many articles about these books and their authors. The bibliography at the end of this article is just a short choice of recent references (in most cases with self explained titles).

1. Rudjer Bošković(1711–1787), *Elementorum Universae Matheseos* - tomus I-III, 1757,Venetis (the hard copy of the book has the Mathematical Institute in Belgrade). Latin–English edition *Theoria Pholosophiae Naturalis – A theory of Natural Philosophy*, transl. of Venetian edition (1763), translated by James-Mark Child, with an introduction *Life of Roger Joseph Boscovich* by Branislav Petronijević, London 1922 (the printed copy of the book has the Astronomical Observatory in Belgrade).

2. Zaharije Orfelin (1726–1785), *Eternal Calendar*, 1783, Vienna (the printed copy of the book has the Astronomical Observatory in Belgrade).

3. Atanasije Stojković (1773–1832), *Physics*, three volumes, 1810, Budapest.

4. Djordje Stanojević(1858–1921), most of his books, eight of them are digitized including *The starry sky of independent Serbia.*

5. Milan Andonović, (1849–1926), *Cosmography*, 1888, Belgrade.

6. Kosta Stojanović, (1867–1921), *Atomistics by Rudjer Bošković*, 1892, Niš.

   Besides books from the XVIII and XIX century there are also the following astronomical books printed in the XX century:

7. Nikolai J. Zinger, (1842–1918), *Lectures on Astronomy*, 1925, Belgrade (translated by S. P. Bošković, the general and professor of geodesy at the Serbian Military Academy, edition 1899).

8. Milutin Milanković (1879–1958), most of his books, ten, are digitized, including *Celestial Mechanics*, Belgrade, 1935 and *Kanon Der Erdbestrahlung*, Belgrade, 1941.

9. Pavle Vujević (1881–1966), *Fundamentals of the mathematical and physical geography*, 1924, Belgrade.

10. Vojislav Mišković (1892–1976), most of his books, ten of them are digitized, including *Cosmography*, 1931, Belgrade and *Collection solved problems in general astronomy*, Part I, 1956, Belgrade; Part II, electronic edition, Virtual Library, 2009.

11. Tatomir Andjelić (1903–1993), *Astrodynamics*, 1983, Belgrade.

12. Zaharije Brkić (1910–1979), *General astronomy*, 1971, Belgrade.

13. Branislav Ševarlić (1914–2001), *General astronomy*, 1971, Belgrade.

14. Jovan Simovljević (1929–2007), *Collected works*, electronic edition, Virtual Library, 2010.

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Mijajlović,ŗ Z.: 2003, On certain undertakings in the area of digitization in the last decade (in Serbian), *NCD Review*, **1**, 12-27.
Mijajlović, Ž., Ognjanovic, Z., Djordjević, N., Zečević, T.: 2004, Virtual library - data base of textual data, *NCD Review*, **5**, 42-48.
Dačić, M.: Reduction of astrogeodetic determinations on the unique system, Proc. IV Serb.-Bulg. Astron. Conf., Belgrade 21-24 April 2004, eds. M. S. Dimitrijević et al.
Milovanović, M.: 2007, Digitization of the book the Eternal Calender written by Zacharius Orphelin, *NCD Review*, vol. **11**, 43-47.
Pejović, N, Mijajlović, Ž.: 2008, Fundamentals of mathematical and physical geography by Pavle Vujević, *NCD Review*, **13**, 74-90.
Djordjević, R., Milogradov-Turin, J.: 2008, Views of Djordje Stanojević on History and Progress of Science, Proc. Conf. Djordje Stanojević – Life and Work, Novi Sad, 10-11. Oct 2008, ed. V. Marić, 85-90.
Pejović, N.: The Starry Sky of Independent Serbia, Proc. Conf. Djordje Stanojević – Life and Work, Novi Sad, 10-11. Oct 2008, ed. V. Marić, 163-171.
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Pejović, N.: 2009, Manuscript Collection of solved problems of General astronomy by Vojislav Mišković, *Publ. Astron. Obs. Belgrade*, No **86**.
Pejović, A.: 2009, Digitization of Book Cosmic Energy and Modern Physics by Djordje Stanojević, *NCD Review*, **15**, 27-30.
Ogrizović: 2009, A construction of an advanced measuring system for astro-geodetic determinations, *Publ. Astron. Obs. Belgrade*, No. **86**, 145-150.
Mijajlović, Ž., Ognjanović, Z., Pejović, A.: 2010, Digitization of Mathematical Editions in Serbia, *Mathematics in Computer Science*, Vol. **3**, No. 3, 251-263.
Pejović N.: 2010, Digitized Books of Djordje Stanojević in the Virtual Library of Faculty of Mathematics, *NCD Review*, **17**, 32-38.
Pejović, N, Mijajlović, Ž.: 2011, Early Astronomical Heritage in Virtual Library of Faculty of Mathematics in Belgrade, *NCD Review*, **19**, 11-25.
Pejović, N.: 2011, Digitization of Textbook Celestial Mechanics by Milutin Milanković, *NCD Review*, **19**, 63-68.
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Virtual Library, http://elib.matf.bg.ac.rs, editor in chief Ž. Mijajlović.

70 Digital legacies and archives

N. Pejović, N. Mitić, S. Malkov and
Ž. Mijajlović
Zb. III Simp. *Matematika i primene*
Matematički fakultet u Beogradu
25-26. maj 2012, pp. 1-3
Matematički fakultet, Univ. u Beogradu, 2012.
(Z. Stanimirović et al, ed). ISBN 978-86-7589-097-3.

This paper presents the digital form of the legacies of the most important Serbian people from the past that had significant works in mathematical sciences. The project has started by developing a Digital legacy "Milutin Milanković", which contains digital copies of various items related to Milankovic's life and work: photos, letters, diplomas, patents, and also documents that witness about his education, research and recognitions. At the moment of the preparation of this book besides Milankoviś legacy there are also legacies of mathematicians Bogdan Gavrilović, Mihailo Petrović Alas, Djuro Kurepa and Slaviša Prešić, astronomer MIhailo Brkić and mechanics Anton Bilimovič.

DIGITAL LEGACIES AND ARCHIVES

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Abstract: In this paper we introduce the digital form the legacies of most important Serbian people from the earlier period that had significant works in mathematical sciences. The project has started by developing a Digital legacy "Milutin Milanković, which contains owns digital copies of various items related to Milanković's life and work: photos, letters, diplomas, patents, and also documents that witness about his education, research and recognitions.
Keywords: digital legacies, digital archives, Serbian scientists, national heritage

1.Introduction

There are important collections of books, photos, documents and other material related to the people from the past which had important contributions to science, or had significant roles in the development of our scientific institutions. We decided to present in digital form the legacies of most important Serbian people from the earlier period that had significant works in mathematical sciences: pure mathematics, mechanics, astronomy, theoretical physics and geosciences. Our plan is to organize these digital data in two levels.

The first level will be the digital archive, where all material will be classified, but kept in the original form. In this part of the project special attention is given to metadata and digitization standards. They have the key role in the organization of the entries in the database and they are crucial for search, retrieval and further use of this material. It is supposed that all accessible and collected material related to a selected person will be held there. Obviously, this work supposes collaboration that provides advice and support of individuals interested in this kind of project, not only of mathematicians. At this moment we are considering what the software platform for digital archive would be appropriate. One possibility is the open source software DSpace which is measured as a turnkey institutional repository application. It has already set up a preliminary version based on DSpace of the digital archive of the Faculty of Mathematics. We have just started uploading into digital archive more than 2000 items (books, papers, photos, etc) related to Milutin Milanković.

The second level will be the digital legacies related to each historical individual for which data are presented in the digital archive. While the digital archive is oriented mostly to specialists, digital legacy should address the general public. Only selected items from the digital archive will be placed there, but they will be commented and appropriately illustrated. We decided to separate digital legacies from the digital archive for several reasons. First, data in digital archive are in the row form, often not suitable for presentation, while in the legacy they are esthetically processed and adapted for Internet presentation. Further, data in the archive often overlap, for example there are often tens of photos of the same subject. Also, the archive contains copyrighted digital material, and the owner in most cases gives only the limited access to them. There are also technical reasons, such as quality, then the resolution (of the photo for example) and the size of the data that limits the browsing of the presentation.

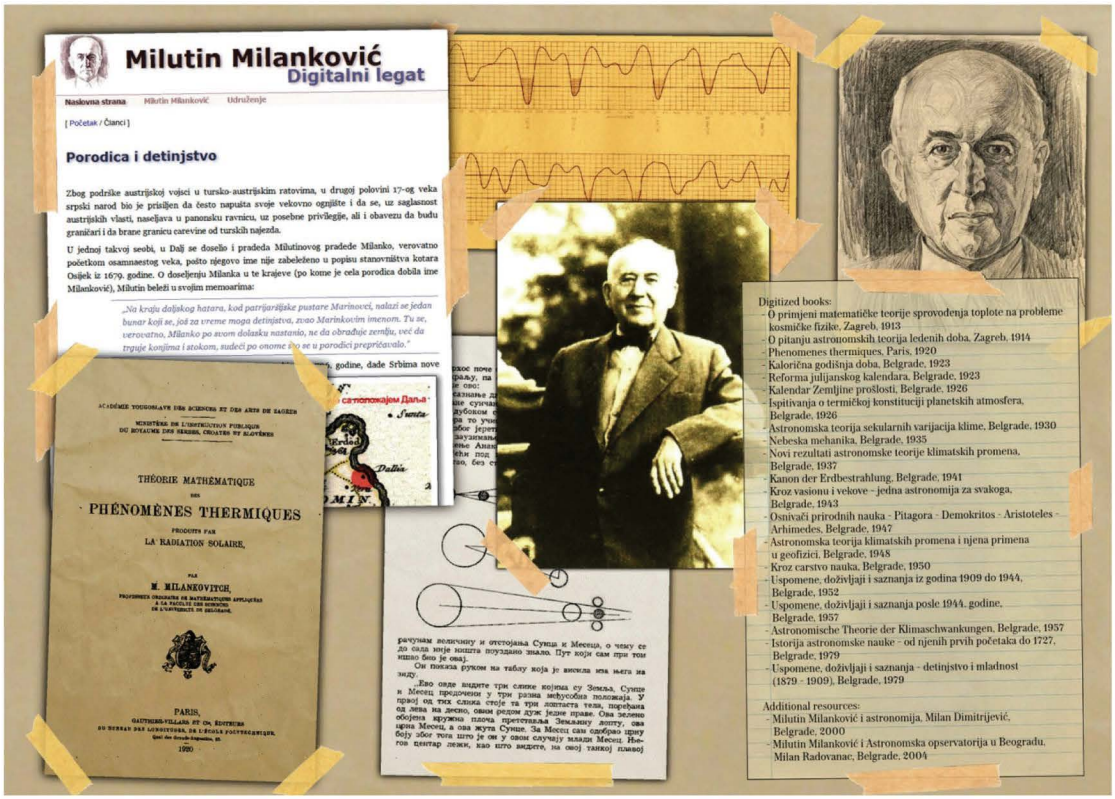
We plan to build digital legacies related to: Bogdan Gavrilović (1864-1948, mathematician, one of the founders of the University of Belgrade), Mihailo Petrović Alas (1868-1943, the founder of Belgrade mathematical school), Milutin Milanković (1879-1958, geophysicist, astronomer,

mechanist and civil engineer, best known for his theory of ice ages), Anton Bilimovič (1879-1970, mechanist, the founder of the leading Serbian mathematical journal *Publications de l'Institut Mathématique*, 1932), Jovan Karamata (1902-1967, famous for his theory of regularly varying functions) and Đuro Kurepa (1907-1993, mathematician, known for his important contributions to set theory). At this moment the digital legacy of Milutin Milanković is finished. This project is coordinated by Žarko Mijajlović.

A large part of the printed materials related to these scientists, as well as for some other, see Nikola Tesla - Clippings Library, especially books and scientific papers, has been already collected and partially digitized; see Virtual Library of the Faculty of Mathematics. All these scientists were professors at the Faculty of Mathematics of the University of Belgrade and members of the Serbian Academy of Sciences and Arts.

2. Digital Legacy Milutin Milanković

We started the project first developing a Digital legacy *Milutin Milanković*. We are working on this presentation together with the *Milutin Milanković* Society, which owns several thousand digital copies of various items related to Milanković's life and work: photos, letters, diplomas, patents, and also documents that witness about his education, research and recognitions. These data were mainly collected by Slavko Maksimović, the president of the *Milutin Milanković* Society.



This electronic archive includes all these data in its original form, while Digital legacy *Milutin Milanković* contains a selection from this material with detailed explanation and comments. The accompanying text is taken mainly from [1]. The archive will contain several thousand digital items of books, paper, photos, films, patents, correspondence, diplomas and papers of other authors on Milutin Milanković. In the Digital legacy *Milutin Milanković* almost all Milanković's books are

presented, including the first edition of his famous *Canon of the Earth's Insolation* (*Kanon der Erdbestrahlung*, Serbian Academy of Sciences and Arts, 1941). There are also several books on Milanković's work and life. All these books are deposited in the Virtual Library of the Faculty of Mathematics. There is also extensive but relatively popular explanation of his theory of ace ages. Finally, there is a very good biography of Milutin Milankovic written by academician Tatomir Anđelić, [2].

Finally we should mention all the people who worked on digital archive and digital legacies (in alphabetical order): Aleksandar Simonović, Mirjana Maljković, Nadežda Pejović, Nenad Mitić, Nikolina Vukša, Saša Malkov, Slaviša Milisavljević, Slavko Maksimović, Slobodan Stojanović and Žarko Mijajlović. Particular contributions of each member of the team can be found at the appropriate above mentioned Internet sites. We also had very generous supports of the Faculty of Mathematics of the University of Belgrade and the *Milutin Milanković* Society, Belgrade. This work was partially supported by the subproject 44006-4, *Digitization of scientific and cultural heritage* of the Serbian Ministry of Education and Science.

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71 Kalendari u Virtuelnoj biblioteci Matematičkog fakulteta

N. Pejović and Ž. Mijajlović,
Zb. Konf. Kalendarsko znanje i doprinos
Milutina Milankovića
14-15 sept. 2011, Gradj. fakultet, Beograd,
pp. 146-157
(S. Maksimović, ed.)
Udruženje Milutin Milanković, Beograd, 2012.
ISBN 987-86-910617-5-3.
Reprinted in "Srbi i kalendarsko pitanje",
publ. by Udr. Milutin Milanković, ed. D. Antić,
2017.
ISDN 978-86-910617-9-1.

The aim of this paper is to present three books related to calendars from the Virtual Library of the Faculty of mathematics in Belgrade of recognized Serbian scientists from the past. The first book is the Eternal calendar written by Zacharias Orfelin Stefanović (1726-1785), printed in Vienna in 1783. The second book is the incorrect celebration of the Resurrection in the Orthodox Church and the reform of the calendar by Djordje Stanojević (1858-1921). This book was printed in Belgrade in 1908. The third one is the reform of Julian calendar by Milutin Milanković (1879-1958), printed in Belgrade in 1923. Each of these books reflects the characteristics of the times in which they were written and therefore differ in content and alm. For example. the main purpose of Orfelin's Eternal Calendar is the Enlightenment, while the Milanković's book offers significant repair of the Ju-

lian calendar. These works are an important part of the Serbian scientific heritage, particularly in the calendar and astronomical literature.

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КАЛЕНДАРИ У ВИРТУЕЛНОЈ БИБЛИОТЕЦИ МАТЕМАТИЧКОГ ФАКУЛТЕТА



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Сажетак: Виртуелна библиотека Математичког факултета Универзитета у Београду, http://elibrary.matf.bg.ac.rs, садржи око 100 књига из астрономије српских аутора, укључујући и ретке књиге из 18. и 19. века. Већи део ових књига су раритети, неке од њих постоје само у једном примерку и практично су недоступне јавности. У овом раду приказујемо три књиге из ове астрономске колекције, из области календарске литературе. То су књиге Вечни календар Захарија Орфелина Стефановића, Нетачно празновање Васкрсења у православној цркви и реформа календара Ђорђа Станојевића и Реформа Јулијанског календара Милутина Миланковића.

Кључне речи: Виртуелна библиотека, календари, Орфелин, Ђ. Станојевић, М. Миланковић

УВОД

Виртуелна библиотека Математичког факултета, http://elibrary.matf.bg.ac.rs, највећа је база дигитализованих текстова у Србији отворена за јавно коришћење. Формирана је 2009. у оквиру пројекта *Примена информационих технологија у дигитализацији научног и културног наслеђа* Министарства за науку Србије. До сада је у Виртуелну библиотеку постављено око 1600 дигитализованих књига. Највећим делом ове књиге су из математичких наука које су написали старији српски научници (18. век – прва половина 20. века). Само колекција астрономских књига садржи око 100 дела. Важне колекције у оквиру ове библиотеке су збирка од 400 докторских дисертација из математичких наука од којих је највећи део одбрањен на Математичком факултету Универзитета у Београду и ретке књиге из 18. и 19. века. Већи део ових књига су раритети, неке од њих постоје само у једном примерку и фактички су недоступне јавности.

Циљ овог рада је да представи три књиге значајних српских аутора из астрономске колекције ове библиотеке, а које се односе на календаре. Прва књига је *Вечни календар*, штампана у Бечу 1783, Захарија Орфелина Стефановића (1726–1785). Друга књига је *Нетачно празновање Васкрсења у православној цркви и реформа календара* Ђорђа Станојевића (1858–1921). Ова књига штампана је у Београду 1908. Трећа књига је *Реформа Јулијанског календара* Милутина Миланковића (1879–1958), штампана у Београду 1923. Свака од ових књига одсликава карактеристике времена у којем су писане и сходно томе разликују се по садржају и намени. На пример, Орфелинов *Вечни календар* је пре свега просветитељског карактера, док Миланковићево дело утврђује значајну поправку Јулијанског календара. Ова дела представљају важан део српског научног наслеђа, посебно у области календарске и астрономске литературе.

ЗАХАРИЈЕ СТЕФАНОВИЋ ОРФЕЛИН

Орфелин (1726–1785) био је истакнути српски песник, писац, историчар, барокни просветитељ, гравер, бакрорезац и калиграф. Рођен је у српској породици у Вуковару 1726. од оца Јована.

Међу његова најзначајнија дела убраја се *Славеносербски магазин* штампан у Венецији 1768. То је први јужнословенски часопис. Мада је изашао само један број, његов значај је велики. У предговору износи идеју грађанске просвећености, да наука, књижевност и филозофија треба да изађу из уског круга учених људи и да постану доступни свима. Године 1768. Захарије Орфелин први уводи српски књижевни језик – мешавину црквенословенског и народног језика, у којој су се налазиле и многобројне руске речи. Тиме је практично основао славеносрпски језик.

Он је аутор првог српског буквара из 1767. према којем су училе бројне генарације ђака, као и првих уџбеника латинског језика. Његово најопсежније дело је *Житије Петра Великог* (Венеција, 1772) у коме је видео узор просвећеног монарха, што је иначе био филозовски идеал 18. века. Аутор је прве књиге о календарима на словеносрпском језику *Вечни календар* из 1783. која садржи обимно поглавље о астрономији.

Поводом његових уметничких радова, пре свега у бакрорезу, изабран је седамдесетих година 18. века за члана Уметничке академије у Бечу. Мотив из бакрореза *Стварање Света* налази се на првој страници књиге Вечни календар.

Орфелинов ***Вечни календар*** [7] штампан је у Бечу 1783 у штампарији Јосифа Курцбека. О садржају ове ретке књиге и намерама с којима је писана најбоље говори пуни наслов који гласи: *Вечни, то јест од почетка до краја света трајући календар који у себи садржи свецелослов и кратке, по рачунању источне цркве, годишње кругове и друге природне законе; уз то и физичке о небеским телима, и о воденим и ваздушним додатним расуђивањима; прибављеним из свештених и светских хронологија сада први пут на словенском језику на корист словеносербским народима.*

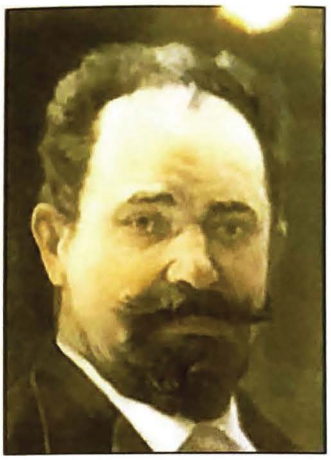


Вечни календар Захарија Орфелина написана је на 366 страница и садржи 9 цртежа из астрономије на крају књиге. Уз календарске податке садржи обимно поглавље о астрономији. Такође, има поглавље о природним појавама, као и поглавње с хронологијом историјских догађаја, од настанка света све до друге половине 18. века. Наслови из астрономије су: О Свету (свемиру), *О Месецу и планетама, О кометама, О помрачењу Сунца, О помрачењу Месеца*. Ту се налазе подаци о временским циклусима, таблице са подацима о Сунцу и Месецу (за рачунање датума Ускрса по Великом индикту), таблице о дужини трајања дана и ноћи, таблице Месечевих мена и друге. Вечни календар је свеообухватна књига, првенствено просветитељског карактера.

ЂОРЂЕ СТАНОЈЕВИЋ

Ђорђе Станојевић (1858–1921) рођен је у Неготину. Завршио је Велику школу у Београду. Потом је био асистент Велике школе код свог професора Косте Алковића (1836–1909), а затим суплент у Првој београдској гимназији 1883. Као питомац војног Министарства био је 1883–1887. на студијама и раду у најпознатијим астрономским и метеоролошким опсерваторијама Европе (Потсдам, Хамбург, Медон, Гринич, Кју, Пулково). Због својих резултата добио је позив од стране Париске опсерваторије да учествује у научноистраживачкој експедицији

ради проучавања Сунца и то у Петровску где је посматрао и проучавао потпуно помрачења Сунца. Две године касније учествује у другој научноистраживачкој експедицији ради проучавања Сунца и термичког спектра у Сахари. У то време, и нешто касније, објавио је неколико научних радова из астрономије у издањима Париске академије наука.



Ђорђе Станојевић

Станојевић је био свестран, образован и са пуно активног интересовања за многе области науке и технике. Његово занимање за физику било је велико. По повратку у Србију посветио се изучавању модерне физике. Био је професор физике и механике на Војној академији у Београду, професор физике на Великој школи у Београду, продекан и декан Филозофског факултета у Београду, професор експерименталне физике и ректор Универзитета у Београду. Заслужан је за градњу хидроелектрана по Србији (Вучје, Неготин, Ужице, Ивањица и друга места) и електрификацију Београда. То је увело Београд у прве престонице Европе са потпуним електричним осветљењем. Утемељивач је Електропривреде Србије, астрономске науке и наставе експерименталне физике на Београдском универзитету. Дао је велики допринос развоју науке у Србији. Написао је петнаестак књига и око 150 научних и стручних радова и популарних чланака. Умро је у Паризу 1921.

У Неготину постоји Трг Ђорђа Станојевића и у Електродистрибуцији Неготина музеј посвећен њему. На Новом Београд постоји улица *Ђорђа Станојевића*. Електропривреда Србије поводом свог дана, 6. октобра, додељује признање *Ђорђе Станојевић* за допринос у развоју српске електропривреде.

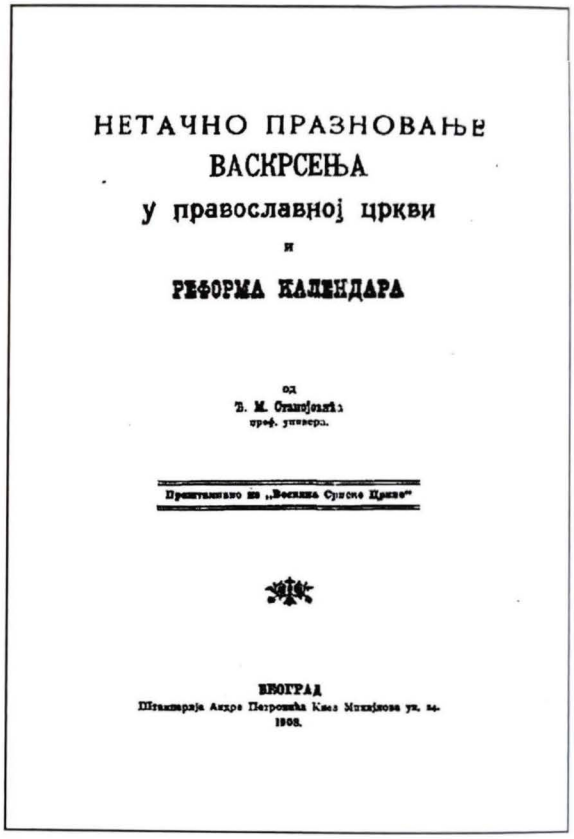
Дело **Нетачно празновање Васкрсења у православној цркви и реформа календара** [8] Ђорђе Станојевић написао је крајем 19, а штампана је почетком 20. века. Књигу је издала Штампарија Андре Петровића 1908. у Београду и представља прештампано издање из *Весника Српске Цркве*.

Књига има 83 странице, и представља студију која детаљно указује, по мишљењу Станојевића, на нетачности око одређивања датума празновања Ускрса у православној цркви. Главно место у књизи су образложени разлози за неопходну реформу календара. Поред тога, Станојевић у књизи излаже свој предлог реформе Јулијанског календара који је 1892. упутио митрополиту Михаилу. Дигитализован је примерак књиге који се налази у Библиотеци Српске академије наука и уметности.

Књига се састоји од ***Увода*** и девет поглавља. Првих шест поглавља посвећено је одређивању датума празновања Ускрса са детаљним историјским прегледом, док се последња три поглавља односе на реформу календара. Реформа календара би, по Станојевићу, требала да исправи грешке и неправилности око празновања Ускрса, великог хришћанског празника.

У ***Уводу*** Станојевић пише: *Може се рећи да припрема и празновање Ускрса почиње почетком великог поста, а нарочито осам дана пре Ускрса, Лазаревом Суботом, којим се чином опомињу хришћани на улазак Христов у Јерусалим, место Његова страдања, смрти и васкрсења.* На занимљив начин Станојевић истиче значај Ускрса као највећег хришћанског празника, упоређујући *Ускрс са Сунчевим системом: Ускрс са својим празницима и обредима стоји као сунце са својим планетским системом у коме све планете од њега – сунца – зависе, с њим се крећу и по њему се управљају.*

Станојевић у књизи наводи честе поделе хришћана око празновања Ускрса. Прва подела датира још из 2. века, када настају две групе. Једни празнују Ускрс заједно са Јеврејима, чија Пасха пада 14. нисана по њиховом календару, њих назива *четрнаестници* и други, већи део хришћана, који Ускрс славе у прву недељу после овог датума, и назива их *недељници*. Потом наводи историјат даљих, показало се несрећних, подела око датума празновања Ускрса. Недељнике који су се везивали за недељу после пуног месеца после равнодневице назива *равнодневичарима*.


НЕТАЧНО ПРАЗНОВАЊЕ
ВАСКРСЕЊА
у православној цркви
и
РЕФОРМА КАЛЕНДАРА

од

Ђ. М. Станојевића
проф. универз.

Штампано из „Весника Српске Цркве"

БЕОГРАД
Штампарија Андре Петровића Краљ Михајлова ул. бр.
1908.

У 4. веку долази до нове поделе због различитог одређивања датума пролећне равнодневице. Римљани су за почетак пролећа узимали 18. март, док су Александријци, имајући у то време у Египту развијенију астрономију, за почетак пролећа узимали 21. март. Због ове, нове поделе хришћана, сазван је Никејски сабор. Наиме, у Никеји 325. године одржан је васељенски сабор који је сазвао цар Константин и који се бавио овим важним питањем: датумом празновања Ускрса. Станојевић наводи да се понекад погрешно пише да је *Никејски сабор исправио календар. Није!* На Сабору је донет предлог, пре би се рекло молба, да сви хришћани славе Ускрс истог дана и да Александријска црква, која је пролећну равнодневицу везивала за 21. март, саопшти датум Ускрса.

Станојевић у књизи на стр. 55 објашњава разлоге због којих сматра да је неопходна реформа Јулијанског календара и пише: ако би смо задржали у важности досадашње правило за празновање Ускрса, изведено из решења Никејског Сабора, *ако би смо задржали у исти мах и садашњи погрешан Цезаров (Јулијански) календар, Ускрс би био правилно празнован по стварној равнодневици, али би се са равнодневицом заједно премештао кроз поједине месеце годишње, те би у довољном низу година прошао кроз све датуме годишње.*

Споменимо, да је питање реформе Јулијанског календара било веома актуелно крајем 19. века, јер је година 1900. била преступна по Јулијанском календару, а није била преступна по Грегоријанском календару. Тада је разлика између ова два календара са 12 нарасла на 13 дана. Како је 2000. преступна по оба календара то је разлика остала непромењена, али ће до повећања разлике на 14 дана доћи 2100. Због ове разлике која се временом повећава, Станојевић каже: *Ако источна православна црква хоће, а она то мора хтети, да празновање Ускрса држи у смислу црквених прописа, она мора усвојити други начин за уклањање погрешног празновања Ускрса, она мора поправити Цезаров календар.*

Станојевић је такође, описао реформу календара коју је спровео Јулије Цезар. Да би уредио дотадашњи римски календар, Јулије Цезар је из Египта у Рим довео астронома Сосигена који је извршио реформу 46. године пре нове ере. Цезар је прво у дотадашњи римски календар у *последњу годину забуне* додао 90 дана, и тако је ту годину довео у сагласност

са природом. По предлогу Сосигена Цезар је увео:
1) Јулијанску годину од 365.25 дана.
2) Да од сваке четири године, прве три буду *просте* од по 365 дана, а четврта (дељива са 4 без остатка) буде *преступна* и да има 366 дана. И данас се део хришћена, припадника Православне цркве, управља по овом календару.

Даље, код Станојевићева налазимо историјске околности које су довеле до увођења Грегоријанског календара. Папа Гргур XIII придржавајући се договора Никејског сабора и правила које су поставили Александријци, да Ускрс треба празновати по стварној равнодневици, решио је да равнодневицу, која је у 16. веку била 11. марта врати на 21. март, како је било у доба Никејског сабора. На предлог италијанског астронома Лилија, Папа Гргур је извршио реформу Јулијанског календара тако што је *булом* од 24. фебруара 1582. наредио да после четвртка 4. октобра буде петак 15. октобар. На тај начин Папа Гргур је *избацио 10 дана* и тиме поправио грешке прошлости, како пише Станојевић. За грешке будућности, да се не би јављале, Папа Гргур је прописао правило: *Од година којима се завршавају векови биће преступна само она чији је број векова дељив са 4 без остатка.* Тако су године којима се завршавају векови 1600, 2000, 2400, ... постале преступне. За Грегоријанску реформу календара Станојевић у књизи пише:

Прву поправку, која се тиче прошлости, извршио је Гргур неправилно; другу, која се односи на будућност извршио је нетачно. Са тих разлога држим да не би било упутно усвојити тако поправљени Гргуров календар.

За прву поправку, Станојевић замера Гргуру што је као почетну тачку, за датум пролећне равнодневице узео годину Никејског сабора, а није узео годину Христовог рођења, јер се по Христу називамо хришћанима а не по Никејском сабору. Што се тиче друге поправке Станојевић замера Гргуру што разлика између природне и Гргурове године брзо нарасте на један дан.

Станојевићев предлог реформе Јулијанског календра који гласи:

1) Године чији се бројеви не деле са 4 без остатка биће просте од по 365 дана.

2) Године чији се бројеви деле са 4 без остатка, биће преступне од по 366 дана.

3) Године чији се бројеви деле са 128 без остатка биће просте (*ново правило*).

Прва таква година била би 1920, затим 2048, па 2176. итд. За основу реформе Јулијанског календара узима трајање *тропске године*, за коју се тада знало да износи 365 дана 5ч 48м 45.00сек. За ту годину Станојевић предлаже назив *календарска* или *хришћанска* година, па и назив *Николајевска* година у част руског цара Николаја II, како каже, највећег представника источне православне цркве. Станојевић нарочито истиче једноставност овакве реформе. Сличан предлог за реформу календара (стр. 63) пре Станојевића дао је професор Медлер 1864, тада директор опсерваторије у Дорпату. Средње трајање године по Медлерову предлогу износило би 365 дана 5ч 48м 52.13 сек. Руска Комисија за реформу календара усвојила је 1899. Менделејев–Медлеров предлог, који је остао непримењен. Станојевић у својој књизи описује и Трпковићев предлог реформе Јулијанског календара из 1900. Трпковић, за трећу ставку Станојевићевог предлога, предлаже формулу да се вековне године деле са 9, па да оне, које при деоби са 9 дају остатак 0 и 4 буду преступне, а све остале просте. Средње трајање године по Трпковићевом предлогу износило би 365 дана 5ч 48м 48 сек. Споменимо да је сличан предлог за реформу календара пре Трпковића дао Барнаба Ориани 1785. Проблемом одређивања трајања тропске године иначе бавили су се највећи светски астрономи: Бесел, Њукомб, Леверије, Ханзен и други. С друге стране, многи аутори, на пример, Ориани, Медлер, Станојевић и Трпковић, радили су на рефори календара с циљем да се календарска година што тачније усклади са тропском годином. Станојевићев предлог реформе календара није прихваћен.

МИЛУТИН МИЛАНКОВИЋ



Миланковић је великан светске науке, најзначајнији српски астроном, небески механичар, геолог и геофизичар. Његов допринос је од великог значаја за српску и светску науку. Написао је петнаестак књига и више од стотину научних и стручних радова из математике, небеске механике, астрономије и геофизике. Збирка Миланковићевих књига у Виртуелној библиотеци Математичког факултета је најбогатија. У овом чланку биће представљена његова књига *Реформа Јулијанског календара*.

Милутин Миланковић је рођен у 1879 у Дању (Славонија, данас део Републике Хрватске). Студије грађевине завршио је на Техничкој високој школи (Technische Hochschule) у Бечу. Ту је дипломирао 1903. и докторирао 1904. Био је први српски доктор техничких наука. Миланковић 1909. године по позиву долази из Беча на Београдски универзитет за професора примењене математике на Филозофском факултету. За његов долазак заслужни су Богдан Гавриловић и Михаило Петровић Алас који су тада били професори математике на Универзитету. Остаје на Београдском универзитету све до одласка у пензију 1955. године. Предавао је примењену математику (теоријску физику, механику и астрономију). Био је први професор који је држао предавања из небеске механике и космологије. Миланковић је 1924. постао редовни члан Српске Краљевске академије наука, од 1948. потпредседник Српске академије наука у три мандата, изабран је 1955. за члана Академије природних наука (Deutsche Akademie der Naturforscher Leopoldina) у Халеу и био дописни члан низа других академија и научних друштава у свету. За реформу календара 1923, на предлог Министарства просвете, српски краљ Александар Карађорђевић одликује га Орденом Светог Саве трећег реда. Академски сенат Велике техничке школе у Бечу 1954, додељује му златну докторску диплому за успешан рад на развитку техничких наука и подизање угледа те велике школе. Миланковић је умро 1958. у Београду. За више детаља из његове биографије погледати [6].

Миланковић је светској науци постао познат по математичкој терији ледених доба, која га је прославила. Прво његово потпуније дело о осунчавању Земље била је књига *Математичка теорија топлотних појава изазваних Сунчевим зрачењем (Théorie mathématique des phénomènes thermiques produits par la radiation solaire)*, објављена у Паризу 1922. Како у проблему Земљиних климатских промена, атмосфера игра велику улогу, то су се геофизичка питања повезала са чисто астрономским: положајем Земље према Сунцу. Са веома израженом способношћу да природну појаву опише језиком математике, Миланковић је кренуо у решавање овог веома сложеног проблема. Данашњи истраживач би овом проблему приступио са тимом интердисциплинарних стручњака за разне области астрономије, математике, информатике и геофизике; Миланковић је приступио сам (видети [1] и [6]). Успео је да климатске промене у Земљиној прошлости објасни секуларним (вековним) променама у кретању Земље око Сунца. У сагледавању овог комплексног проблема, пошао је од претпоставке да промена положаја Земље према Сунцу није строго периодична појава, са периодом од једне године. Показао је да се због поремећајног дејства осталих чланова Сунчевог система, првенствено великих планета, Земљина годишња путања споро, али непрекидно мења са временом. Показао је да дугопериодични, вековни поремећаји могу осетно да промене положај Земље према Сунцу, по истеку времена које се мери десетинама хиљада година. Вековни циклуси: промена оријентације Земљине осе ротације (прецесија), промена нагиба Земљине осе према равни еклиптике и промена

ексцентрицитета Земљине путање око Сунца узрок су климатским променама у давној, геолошкој прошлости Земље. Ови циклуси познати су у палеоклиматологији под назвом *Миланковићеви циклуси*. Захваљујући развоју моћног математичког апарата, векторског рачуна, и његовог увођења у небеску механику, решио је Миланковић још један тежак проблем: проблем померања Земљиних полова. Дао је математичку теорију секуларног (вековног) кретања Земљиних полова, која се односи на давну геолошку прошлост, али и на далеку будућност. Теорија је заснована на математичко-механичком моделу Земље. Дошао је до основне теореме о кретању Земљиних полова, која је названа *Миланковићева теорема*. Миланковићева терија Земљиних климатских промена као и теорија померања Земљиних полова су сјајани примери математичких модела и решења изузетно тешких астрономских проблема.

Његово главно дело је *Канон осунчавања Земље и његова примена на проблем ледених доба (Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem)*, у издању Српске Краљевске Академије. Дело представља синтезу његових научних радова насталих истраживањима и изучавањима на границама многих природних наука и математике. Штампање *Канона* је завршено 1941. пред почетак Другог светског рата. За време бомбардовања Београда 6. априла, бомба је погодила штампарију, и тако су пропали последњи штампани табаци дела. Срећом био је сачуван још нерастурен слог, па је њиме у току рата одштампан завршетак књиге на жутој неквалитетној хартији. Тада је слање књиге у иностранство било немогуће. За време четири ратне године Миланковић је успео да пошаље ван земље свега неколико примерака. С обзиром на ову околност, Миланковићу је светско признање стигло са закашњењем.

Светска научна заједница одала је дужно поштовање Миланковићевом научном делу. Једна мала планета, један кратер на Месецу и један кратер на Марсу носе његово име. Европско геофизичко друштво 1993. установило је медаљу *Милутин Миланковић*, за успешна достигнућа у области климатских истраживања. Од 2003. ову медаљу додељује Европска унија за геонауке (Geosciences). НАСА је уврстила Миланковића у 5 највећих научника 20-ог века и 15 највећих научника свих времена који су се бавили истраживањима посвећеним планети Земљи. У Београду једна улица и две школе носе име *Милутин Миланковић*.

Књигу ***Реформа Јулијанског календара [9]*** Милутина Миланковића издала је Српска Краљевска академија наука и уметности као Посебна издања (књига XLVII), серија *Науке природне и математичке* (књига 11), у Београду 1923. Књига садржи девет прилога, међу којима су занимљиви документи и табеле, чије називе наводимо: *1) Трпковићев пројекат реформе календара, 2) Предлог Српске делегације, 3) Додаци и тумачења српском пројекту (на француском), 4) Прва редакција одлука о реформи календара (на француском), 5) Дефинитивна одлука о реформи календара (превод са грчког), 6) Одлука о учествовању на светској конференцији за изједначење хришћанских календара (превод са грчког), 7) Захвално писмо патријарха Мелетија (превод са грчког), 8) Први пун месец после пролетне равнодневнице у нашем столећу (табела) и 9) Датум Ускрса у наредним (педесет) годинама у оба хришћанска календара.*

Миланковић на самом почетку књиге (стр. 6) даје дужно поштовање Трпковићу за његов допринос реформи Јулијанског календара 1900. године. Назива Трпковића нашим великим календарографом. Његови радови као и сакупљен богат хронолшки материјал послужили су Миланковићу као одлична почетна основа за рад на српском предлогу реформе Јулијанског календара. Трпковићев допринос реформи календара налази се у првом прилогу Миланковићеве књиге.


СРПСКА КРАЉЕВСКА АКАДЕМИЈА НАУКА И УМЕТНОСТИ

РЕФОРМА ЈУЛИЈАНСКОГ КАЛЕНДАРА

ОД

МИЛУТИНА МИЛАНКОВИЋА

ред. професора Универзитета у Београду

БЕОГРАД

ШТАМПАРИЈА „С В. С А В А". — БРАНКОВА 16. ТЕЛЕФОН 249.

1923.

У Цариграду је 1923. одржан Свеправославни Конгрес који се бавио важним питањем реформе Јулијанског календара. На том конгресу Миланковић је учествовао као делегат владе наше земље, али и као делегат Српске Патријаршије поред митрополита црногорскоприморског, господина Гаврила Дожића.

Предлог за реформу Јулијанског календара који је Српска делегација поднела Конгресу урадио је Милутин Миланковић. У основи тог предлога налазе се најновија достигнућа астрономске науке оног времена као и тачнији математички рачун са остацима (Гаусов конгруенцијски рачун). Предлог је представљен на стр. 13 ове књиге и гласи:
1. Нека се прескоче 13 дана и нови календар доведе на исти датум са Грегоријанским. За будућност нека се утврди ово правило о интеркалацији преступних година:
2. Преступне године биће оне које се могу поделити са четири без остатка (као што је и до сада било). Изнимку чине секуларне године за које важи следеће:
3. Секуларне године (тј. оне које се свршавају са две нуле) биће само онда преступне ако број њихових векова (секулара), подељен са 9, даде остатак 2 или 6. Све остале секуларне године биће просте. (Ново правило).

Према овом правилу од наредних секуларних година биће преступне: 2000, 2400, 2900, 3300, ..., док су остале године којима се завршавају векови просте.

За дужину тропске године у данима Миланковић користи следећи израз:

$$365{,}24219878117 - 0{,}000006138 \cdot t$$

где је време у јулијанским столећима (од по 36525 дана) протекло од почетка 1900. Овај израз за дужину тропске године, као и средњу дужину године предложеног новог календара која износи 365,24222222 дана, Миланковић користи за рачун разлике Δ која ће се после t векова иза 1900-те нагомилавати између датума новог календара и астрономског времена. Разлику Δ рачуна по изразу

$$\Delta = 0{,}002575 \cdot t + \tfrac{1}{2} \cdot 0{,}00000524 \cdot t^2$$

Ова разлика код новог календара достићи ће износ од једног дана тек после 28800 година, док код Грегоријанског календара разлика између тропске и грегоријанске године достиже износ од једног дана за 3280 година. На овај начин Миланковић је добио тачнији нови календар, принципијелно различит од Грегоријанског, а који се са њим поклапа дуги низ година, до 2800-те.

Српска делегација имала је веома тежак задатак на Конгресу 1923. Како пише Миланковић на стр. 17: *пре, но што је дошло до одлуке пленума, почели су се појављивати нови пројекти*

и отпочео је закулисани рад против српског пројекта, јер су се одједном разбуктале личне и националне амбиције. Отуда је настао трећи прилог *Додаци и тумачења српском пројекту*, који је Миланковић накнадно припремио на француском, изнео и на научним основама бранио на Конгресу. Тек после добро припремљене јавне одбране успео је да придобије делегате Конгреса да гласају за Српски предлог. У Додацима је епактни рачун (који је служио за одређивање Месечевих мена) замењен тачним астрономским рачуном. Усвојено је да се Месечеве мене и датум Ускрса одређују у сарадњи са астрономским опсерваторијама и катедрама Небеске механике у Атини, Београду, Букурешту и Петрограду (Пулковска опсерваторија). Ово је био велики Миланковићев успех, значајно признање астрономској науци и велики допринос календарском питању. За научни допринос решењу календарског питања, Миланковић добија писмо захвалности од Његове Светости васељенског патријарха Мелетија.

Српски краљ Александар Карађорђевић, 1923. године, одликовао је Миланковића Орденом Светог Саве трећег реда, за допринос реформи календара.

Прилог бр. 7

Захвално писмо патријарха Мелетија

(превод с грчког г. универз. проф. Д. Анастасијевића)

(М. П.) † Мелетије, милошћу Божјом архиепископ Константинопоља — Новог Рима и васељенски патријарх.

Број Протокола
3128

Велеучени Господине М. Миланковићу, професоре Београдског Универзитета, чедо у Господу љубљено наше Смерности, благодат нека је с Вашом Велеучености и мир од Бога.

Пошто је у нашем Светом и Часном Синоду прочитана одлука Свеправославног Конгреса о Календару ради правилног усвајања њеног, које је усвајање већ и следовало, као што то јављамо Пресветим Православним Црквама; Часни Синод је с особитим уважењем примио к знању најозбиљнију сарадњу коју је дала Ваша дубока Велеученост као члан Свеправославног Конгреса при састављању те одлуке, којом је тако срећно и потпуно решен један од првих задатака Свеправославног Конгреса, и у опште тако важно календарско питање.

С тога, по једногласној синодалној одлуци, изјављујемо драге воље овим нашим честитим писмом нарочиту похвалу и захвалност Вашој Велеучености за такву просвећену и корисну сарадњу њену.

Шаљући пак уз то и наше очинске поздраве и благослове, молимо се за све што је најбоље Богу, чија благодат нека буде са Вашом дубоком Велеученошћу.

1923. г. јуна 26.

† Константинопољски у Христу богомољац.

Споменимо да је Миланковић у књизи описао неповољне прилике и догађаје који су пратили Свеправославни Конгрес. Ови догађаји везани су за историјске и верске околности које су владале 1821, али и 1923. Чланови Конгреса били су сведоци и учесници немилог догађаја у многим детаљима сличном оном од пре једног века.

На основу изложеног у књизи *Реформа Јулијанског календара* налазимо да је Миланковићев календар тачнији од Грегоријанског и да се са њим поклапа до 2800. када ће доћи до разилажења од једног дана. Датум празновања Ускрса Миланковић је одредио применом астрономског израчунавања Месечевих мена, тако да се Миланковићев календар

у празновању Ускрса и других покретних празника разилази са Грегоријанским календаром само у случајевима када пасхални рачун Западне цркве даје погрешне резултате. Миланковићев нови календар православних цркава једногласно је усвојен на Свеправославном конгресу у Цариграду 1923. Миланковићев календар примениле су Грчка, Румунска, Цариградска и Александријска православна црква, док су Руска, Српска, Јерусалимска црква и Света Гора остале на старом календару.

ЗАКЉУЧАК

Виртуелна библиотека Математичког факултета има значајно место у представљању старих и ретких српких књига научној и стручној јавности и општој публици у свету и код нас. У ову чињеницу можемо се уверити на примеру избора књига из колекције календарских дела Орфелина, Станојевића и Миланковића представљених у овом чланку.

ЗАХВАЛНИЦА

Захваљујемо се Министарству за науку Републике Србије које финансира Пројекат Примена информационих технологија у дигитализацији научне и културне баштине, III 44006–4, као и Библиотеци САНУ на позајмици књига за потребе дигитализације.

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CALENDARS IN VIRTUAL LIBRARY OF THE MATHEMATICAL FACULTY

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Abstract: Virtual Library, http://elibrary.matf.bg.ac.rs, of the Faculty of Mathematics in Belgrade, was established in 2009. So far, about 1600 digitized books were uploaded in the Virtual library. Most of these books are in the mathematical sciences that are written by old Serbian scientists (18th century – the first half of the 20th century). The collection of astronomical books contains about 100 works. The aim of this paper is to present three books of recognized Serbian authors in this collection which are related to calendars. The first book is the *Eternal calendar* written by Zacharias Orfelin Stefanović (17261785), printed in Vienna in 1783. The second book is *The incorrect celebration of the Resurrection in the Orthodox Church and the reform of the calendar* by Đorđe Stanojević (18581921). This book was printed in Belgrade in 1908. The third one is The reform of Julian calendar by Milutin Milanković (18791958), printed in Belgrade in 1923. Each of these books reflects the characteristics of the times in which they were written, and therefore differ in content and aim. For example, the main purpose of Orfelin's *Eternal Calendar* is the Enlightenment, while the Milanković's book offers significant repair of the Julian calendar. These works are an important part of the Serbian scientific heritage, particularly in the calendar and astronomical literature.

Key words: Virtual library, astronomical books, calendars.

72 Works of Rudjer Bošković in Virtual Library of the Faculty of Mathematics in Belgrade

Ž. Mijajlović and N. Pejović,
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This article presents four books by Rudjer Bošković deposited in the Virtual library of the Faculty of Mathematics in Belgrade. Contents of this works are analyzed and reviewed, particularly of book Elementorum Universale Matheseos (in latin, eng. *Elements of a universal mathematics*).

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WORKS OF RUĐER BOŠKOVIĆ IN VIRTUAL LIBRARY OF THE FACULTY OF MATHEMATICS IN BELGRADE

Abstract. Four books written by Ruđer Bošković deposited in the *Virtual library* of the Faculty of Mathematics in Belgrade are presented. Contents of this works are analyzed and reviewed, particularly of book *Elementorum Universale Matheseos.* An overview of the Virtual library itself is given and its objectives, collections and related projects are explained.

Keywords: virtual library, Ruđer Bošković, Faculty of Mathematics

1. Virtual Library

The overall objective of the *Virtual Library of the Faculty of Mathematics*, University of Belgrade,[1] is to create a comprehensive collection of retro-digitized books and other digital documents from the past, see [5]. The main part of this project relates to an electronic archive which contains primarily all old manuscripts electronic form, and to present them to the general public.

The project initially aimed at mathematics, but since 2009 the books from other areas are also deposited in the Library. The priority will be Serbian authors, and works related to scientific and cultural works of the Southeast Europe. Some of the books in the Library are rare; only a few copies of them are available in printed form, and are practically inaccessible to the general public. We considered that they should be somehow preserved, not only as a Serbian cultural and scientific heritage important, but also as part of the World Heritage. We decided to preserve these books, and present them to the general public in an electronic, digitized form.

Several volunteers started the project in 2007. Now, the Library has strong support from the Faculty of Mathematics, University of Belgrade, the Mathematical Institute of the Serbian Academy of Sciences and Arts (SASA), the National Center for Digitization and the Ministry of Science of Serbia.

Virtual Library of the Faculty of Mathematics is the largest Internet oriented database in Serbia of digitized texts with free access. The library also keeps 2,000 books, and several important collections, for example, an important collection of 400 doctoral dissertations in mathematical sciences (most of them defended at the Faculty of Mathematics). Another important collection consists of rare books from the 18th and 19th century. There are also small collections of digitized books from all of the former Yugoslav republics.

The important part of the Library are collected works of some leading Serbian scientists from the past: Atanasije Stojković, Bogdan Gavrilović, Milutin Milanković, Đuro Kurepa, Đorđe Stanojević and several others.

Works related to the archive of the Virtual Library are published in the journal NCD Review (SEEDI Communication), issued by the Faculty of Mathematics in Belgrade.

[1] Virtual Library of the Faculty of Mathematics webpage: http://elibrary.matf.bg.ac.rs

We will present a collection of books written by Ruđer Bošković (1711-1787); works have been digitized and deposited in the Virtual Library. These are the first scientific works of an author of the Serbian origin (his father Nikola was a Serb, mother Paola Bettera was Italian).

The Virtual Library comprises books of other Serbian writers of that period, as well. In fact, shortly after the publication of Bošković's books, in the second half of the eighteenth century, the first scientific works written in Old Serbian were written (see [1] and [2]):

- **Vasilije Damjanović** (1734-1792), *Arithmetic*, 1767, Venice. This is the first Serbian book on mathematics. There is only one known printed copy left and it is kept in the Library of ***Matica Srpska*** in Novi Sad.
- **Zechariah Orfelin Stefanović** (1726-1785), *Eternal Calendar*, 1783, Vienna. Much of this book is devoted to descriptions and explanations of astronomical phenomena. Hence we can consider that this work is the first Serbian book on astronomy. Printed copy of this book is in the Library of the Astronomical Observatory in Belgrade.
- **Atanasije Stojković** (1773-1832), *Physics*, 1800, Budapest. This three-volume book is the first Serbian work in physics. Printed copy of this book is in the Library of the Serbian Academy of Science and Art.

Let us note here the general characteristic of the first Serbian scientists, particularly those from the XVIII and XIX century. They were universal in their research interests, and in other aspects as well, so mathematics, for example, was not their only activity field. They were successful in other sciences, too: in astronomy, physics and philosophy. For them, science was primarily an insight into a unique creature - Nature. Besides, they were often successful politicians, artists, travelers, writers and poets. These books have primary the enlightening character. The writers dedicated their works to the Serbian people, or as Orfelin says in his *Calendar*, "*to the benefit of the Slavo-Serbian people"*.

Their importance for the development of Serbian culture, language and science is enormous.

2. Bošković's works in Virtual Library

On this occasion, we will not discuss the biography of Ruđer Bošković (1711-1787). There are many extensive books on his biography. However, it should be stressed that Bošković was a great mathematician and astronomer, one of the most important scientists of his days; he is listed as one of the 100 most significant Serbs of all time.

He was a university professor, the founder and of the Milan Observatory and the director of the French Navy Optical Institute. He was a universal creator: theologian, philosopher, mathematician, astronomer, physicist, geologist, architect, archeologist, designer, optometrist, diplomat, traveler, teacher, poet and polyglot. Bošković wrote his books mainly in Latin and Italian.

In the history of the Catholic Church he is mentioned as an important member of the Jesuit order. Bošković was born on 18 May in the year 1711 as the seventh child of the merchant Nikola Bošković, a Serb from Herzegovina: His mother Paola was of Italian origin, a member of the family of Baro Bettera, a poet from Dubrovnik. He lived mainly in Italy, where he gained international fame. He died on 13 February 1787 in Milan.

In the Virtual Library there are four digitized works written by Bošković. In addition to the basic bibliographic information about these books, there is also a more detailed review of the book *Elementorum Universale Matheseos*. The reason that we moved into a rather professional discussion about the content of this book, beyond librarian domain and style, is best explained by Mícheál Mac

an Airchinnigh in his article [3]. At least, it is reasonable to assume that Bošković's books have nothing to do with computer technologies and standards that were used to produce their digital copies.

- *Elementorum Universale Matheseos* - Tomus I-III (*Elements of General Mathematics*), Venetis, 1757. The printed copy of this three-volume book is located in the Library of the Mathematical Institute SASA.
- *A dissertation on the law of continuity and its consequences pertaining to the first elements of matter and of its powers*, translated by Darinka Nevenić-Grabovac, the preface and comments written by Ernest Stipanić, 1975, Belgrade. This book is a translation from Latin *De lege continuitatis et eius consectariis pertinentibus ad receives materiae element eorumque vires Dissertatio*, Roma, 1754.
- *A Theory of Natural Philosophy*, Latin-English edition, London, translated by J. M. Child, London, 1922, including *Life of Rudjer Boscovich,* written by B. Petronijević. This book is a parallel of the Latin-English edition of *Theoria Philosophiae Naturalis*, Venetis, 2nd edition, 1763. The printed copy of the London edition of Boscovich theory is in the library of the Astronomical Observatory of Belgrade.
- *Diary of the journey from Constantinople to Poland*, translated by Dušan Neljeković, 1937, Belgrade, (translation of French edition *Journal D'Un Voyage De Constantinople En Pologne*, original *Giornale di un viaggio to Constantinople in Polonia* Dell'Abate Ruggiero Giuseppe Boscovich, 1762).
- The manuscript *Law or Principle of Continuity* is the main starting point and the guide to Bošković's development of the theory of forces that exists in nature; it is also related to the theory about the composition of matter. Therefore, the discussion on the *Law of continuity and its consequences with regard to the basic elements of matter and their forces* is closely related to his theory of natural philosophy.

In addition to the other discussions (*On living forces*, Rome, 1745; *A dissertation on light*, 1748; *A dissertation on the law of continuity and its consequences pertaining to the first elements of matter and of its powers*, Rome, 1754; *On the law of powers in the nature of existing things*, 1755), it is the basis from which emerged the main Bošković's work *A theory of natural philosophy* (*Theoria Phylosophiae naturalis redacta ad Unicam legem virium in natura existentium*, 1758). Hence, immediately in the second paragraph, Bošković said that the debate on "*analysis of the whole is carefully prepared to lead us from the simplest principles of nature and the necessary combination of spontaneous conclusions to the theory itself*". This speaks itself about the importance of this discussion for Boskovic's scientific and philosophical work. According to professor Ernest Stipanić, it was the reason that the Mathematical Institute decided to publish translation of this book into Serbian with comments.

Bošković's *magnum opus*, *Theory of Natural Philosophy*, is undoubtedly his most important work. The first edition of this book was printed in 1758th in Vienna. The Virtual Library has the digital copy of the English - Latin edition, printed in London 1922.

This book is edited and translated by Mark James Child according to the second edition of Bošković's book printed in Venice, 1763.

The second edition of *Theory of Natural Philosophy* is extended and completed version of the first edition. It was printed under Bošković's supervision. There is a short Bošković's biography written by a distinguished Serbian philosopher Branislav Petronijević.

The book *Elements of general mathematics* was obviously a textbook intended for Bošković's students. Many examples and a number of problems scattered through the book confirm this claim. It is interesting that in many papers and in many places on the Internet it is said that the book was printed in 1754 in Rome; however there is no such information in Bošković's bibliography. Digital copy of this book comes from the 1757 edition, printed in Venice (imprint: Editio Prima Veneta, MDCCLVII). The same is with digital copies that are available in the large digital library Europeana and Google books.

Part of the first volume of *Elements* is related to elementary geometry, but the exposition mostly follows the Euclid's elements. The other chapters are related to the basic properties of arithmetical operations, rational numbers, the properties of proportions, arithmetical and geometrical progressions, logarithms, and finally to the elements of plain and spherical trigonometry.

The second volume, *Algebra finite*, is completely devoted to then known procedure of solving algebraic equations. The properties of polynomials and the arithmetic of the radicals are also described. The Cardano (Gerolamo Cardano, 1501-1576) method for solving cubic equations and Ferrari (Lodovico Ferrari, 1522-1565) solution of algebraic equations of the fourth degree are presented. There he used complex numbers (*quantitates imaginarae*) what was rare at that time. Bošković also gave approximate, numerical methods for solving some algebraic equations of a higher degree for which it was later proved that they cannot be solved algebraically (using radicals, Niels Abell, Evaristo Galois, first half of the 19th century).

The third volume of the *Elements* is certainly the most interesting. Bošković's theory of second order curves, presented in this part of the *Elements* was the first complete and systematic theory of conics (conic sections). In the presentation of this subject in which purely geometrical arguments were used there were many new ideas. Although already the method of Decartes's system of coordinates had been developed, Bošković did not use this analytical approach. In particular, for the definition of the conic sections he chose the following one: *the conic is the locus of points in the plane for which the ratio of the distance to the fixed point (focus) and the given line* (*directrix*) *is a constant*. He called this quotient the conic's *ratio determinans*. Today, this ratio is usually called the eccentricity of the conic. Although this property had been known to ancient Greek mathematicians, particularly to Pappus of Alexandria, it had never been taken for a definition previously. In modern analytic geometry, the definition of second-order curves, in most cases exactly follows Bošković's approach. Before Bošković, the conics were intersections of the cone and the plane, therefore the definition of three-dimensional space must be assumed. Hence, his theory of conics becomes the theory of conic curves in the plane.

He introduced a very useful tool for proving various statements about conic sections. This is the wellknown Bošković's *eccentric circle*, i.e. circle with arbitrary center which has the property that the ratio of its radius and the distance of the center to the directrix is equal to the eccentricity. Using this notion, Bošković described various constructions and proved many properties of conic sections. He deduced them using the properties of circles and the correspondence between the eccentric circle and the conics. In the addendum *De transformatione locorom geometricorum* to the third volume of the *Elements*, Bošković presented an entirely new theory of geometric transformations which are essentially collinear, and therefore he can be considered as the forerunner of the development of projective geometry; it emerged much later, in the 19th century.

3. Other authors in Virtual Library on Bošković's work

The Virtual Library also contains books and manuscripts related to the life and work of Ruđer Bošković. The most interesting is the book written by Kosta Stojanović, *Atomism - a part of the philosophy of Joseph Ruđer Bošković*, Niš, 1891. The book is actually a comparative analysis of Bošković's natural philosophy.

In the Virtual Library there are also other books written by Kosta Stojanović. Stojanović was a versatile scholar and professor at the Great School in Belgrade. He founded theoretical mechanics in Serbia, and is the author of The *Basis of the theory of economic value,* probably the best book on economics in Serbian. He was a MP and a cabinet minister of the government Nikola Pašić.

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[3] Mícheál Mac an Airchinnigh, *The practical sense of philosophizing: why preserve anything at all, even digitally?* NCD Review, 4 (2004), 111—134.
[4] Ž. Mijajlović, Z. Ognjanović, A. Pejović, *Internet presentations of mathematical works in Serbia*, NCD Review, 12 (2008), 43–48.
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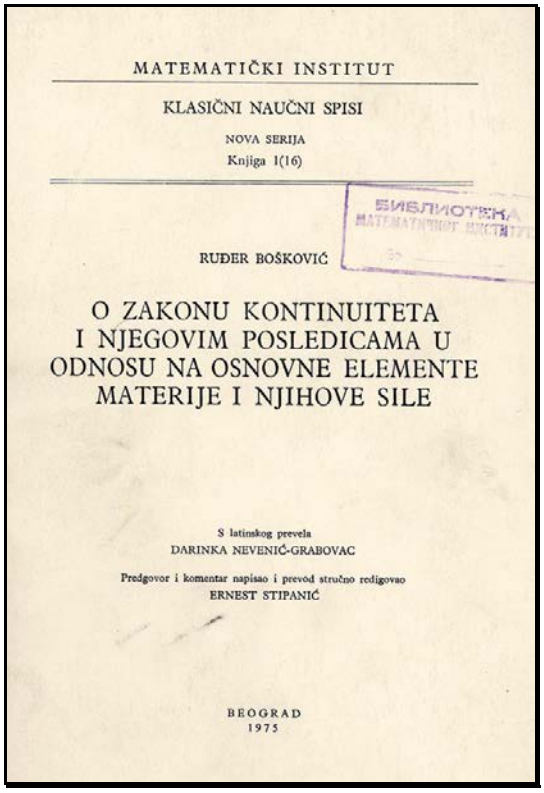
[6] F. M. Brueckler, *300th birthday of Ruđer Josip Bošković (Roger Joseph Boscovich),* 2011, http://www.mathematics-in-europe.eu
[7] Virtual Library, http://elibrary.matf.bg.ac.rs; NCD Review, http://elib.mi.sanu.ac.rs.

zarkom@matf.bg.ac.yu
nada@matf.bg.ac.yu


MATEMATIČKI INSTITUT

KLASIČNI NAUČNI SPISI

NOVA SERIJA

Knjiga 1(16)

БИБЛИОТЕКА МАТЕМАТИЧКОГ ИНСТИТУТА

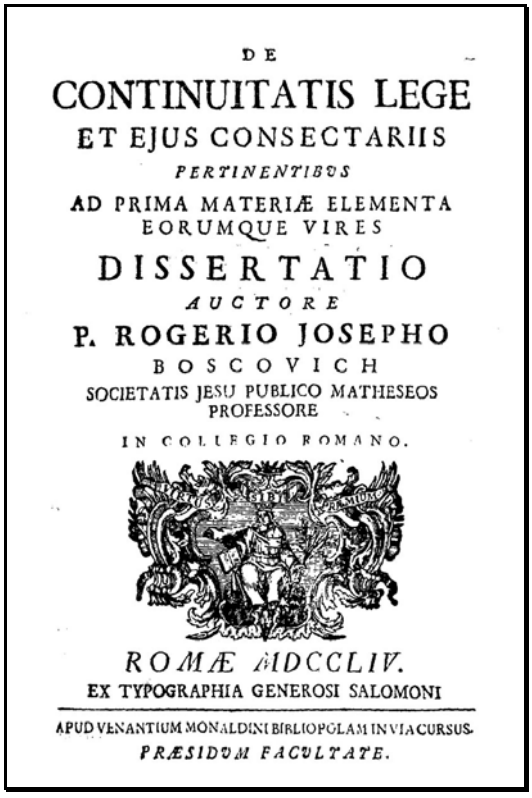
RUĐER BOŠKOVIĆ

O ZAKONU KONTINUITETA I NJEGOVIM POSLEDICAMA U ODNOSU NA OSNOVNE ELEMENTE MATERIJE I NJIHOVE SILE

S latinskog prevela
DARINKA NEVENIĆ-GRABOVAC

Predgovor i komentar napisao i prevod stručno redigovao
ERNEST STIPANIĆ

BEOGRAD
1975


DE

CONTINUITATIS LEGE

ET EJUS CONSECTARIIS

PERTINENTIBUS

AD PRIMA MATERIÆ ELEMENTA EORUMQUE VIRES

DISSERTATIO

AUCTORE

P. ROGERIO JOSEPHO BOSCOVICH

SOCIETATIS JESU PUBLICO MATHESEOS PROFESSORE

IN COLLEGIO ROMANO.

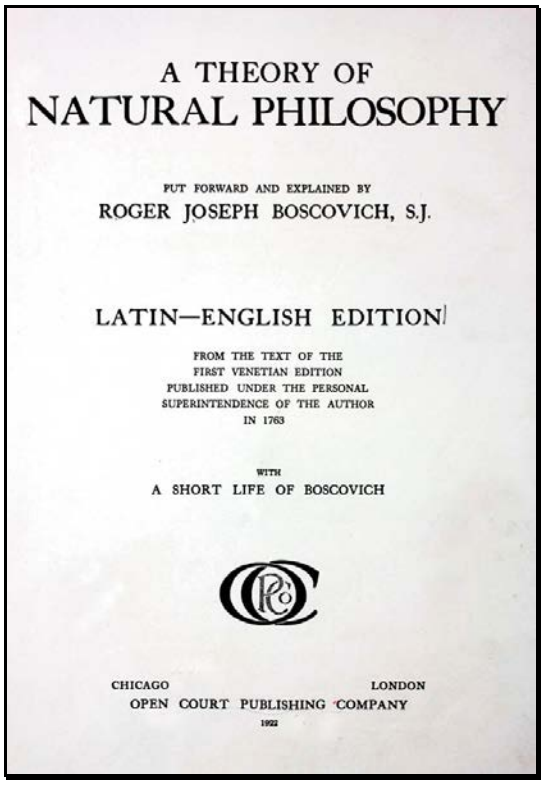
ROMÆ MDCCLIV.

EX TYPOGRAPHIA GENEROSI SALOMONI

APUD VENANTIUM MONALDINI BIBLIOPOLAM INVIA CURSUS.

PRÆSIDUM FACULTATE.

***О закону континуитета и његовим последицама у односу на основне елементе материје и њихове силе*, приредио Е.Стипанић, 1975, Београд**


A THEORY OF

NATURAL PHILOSOPHY

PUT FORWARD AND EXPLAINED BY

ROGER JOSEPH BOSCOVICH, S.J.

LATIN—ENGLISH EDITION

FROM THE TEXT OF THE FIRST VENETIAN EDITION PUBLISHED UNDER THE PERSONAL SUPERINTENDENCE OF THE AUTHOR IN 1763

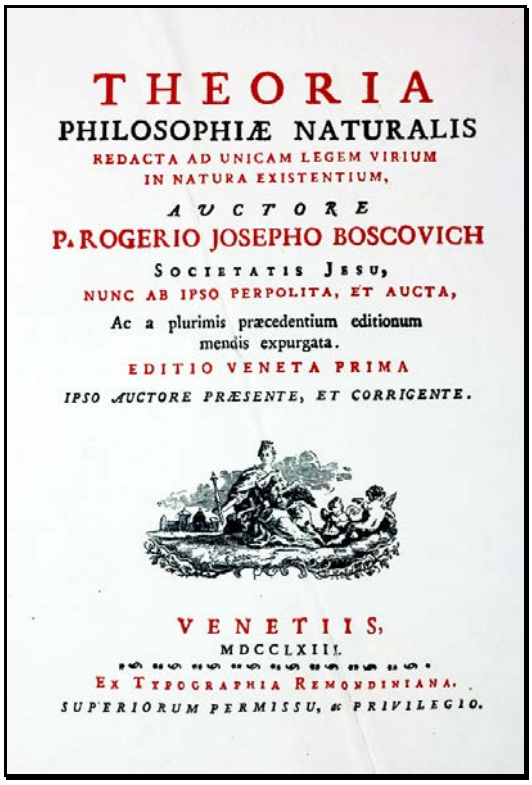
WITH

A SHORT LIFE OF BOSCOVICH

CHICAGO LONDON

OPEN COURT PUBLISHING COMPANY

1922


THEORIA

PHILOSOPHIÆ NATURALIS

REDACTA AD UNICAM LEGEM VIRIUM IN NATURA EXISTENTIUM,

AUCTORE

P. ROGERIO JOSEPHO BOSCOVICH

SOCIETATIS JESU,

NUNC AB IPSO PERPOLITA, ET AUCTA,

Ac a plurimis præcedentium editionum mendis expurgata.

EDITIO VENETA PRIMA

IPSO AUCTORE PRÆSENTE, ET CORRIGENTE.

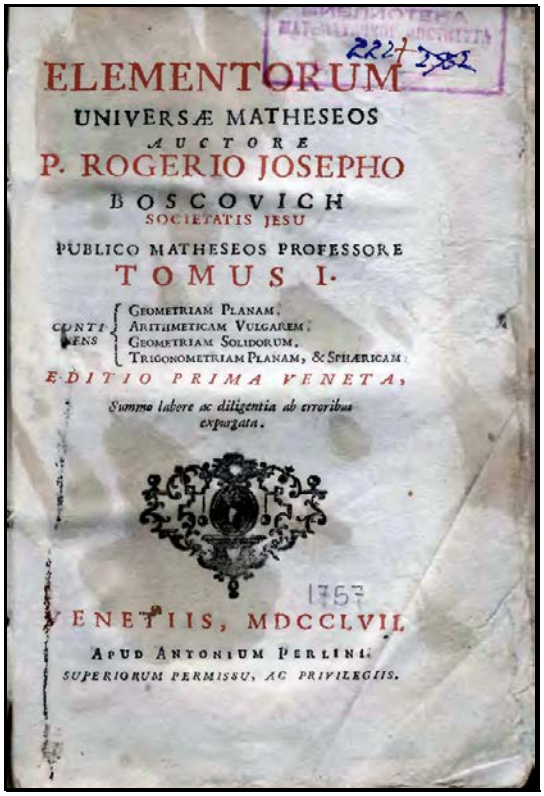
VENETIIS,

MDCCLXIII.

EX TYPOGRAPHIA REMONDINIANA.

SUPERIORUM PERMISSU, & PRIVILEGIO.

Упоредно латинско-енглеско издање Теорије природне филозофије приредио J. M. Child, Лондон, 1922

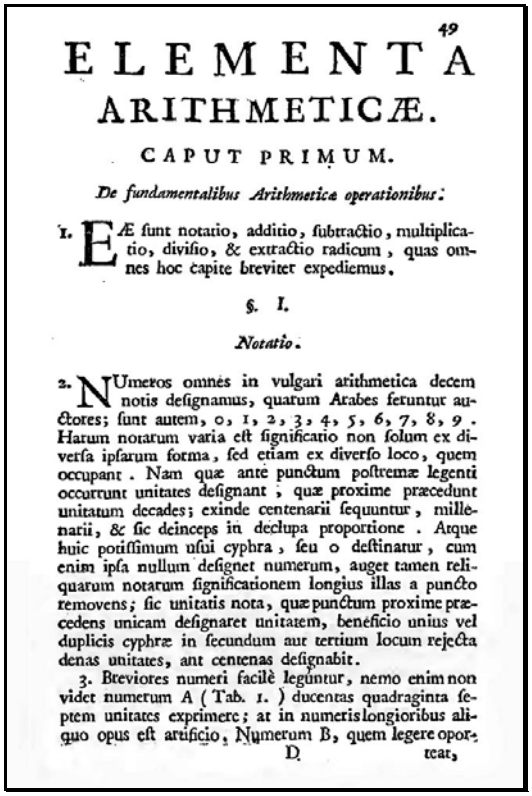


ELEMENTORUM
UNIVERSÆ MATHESEOS
AUCTORE
P. ROGERIO JOSEPHO
BOSCOVICH
SOCIETATIS JESU
PUBLICO MATHESEOS PROFESSORE
TOMUS I.

CONTINENS
GEOMETRIAM PLANAM,
ARITHMETICAM VULGAREM,
GEOMETRIAM SOLIDORUM,
TRIGONOMETRIAM PLANAM, & SPHÆRICAM.

EDITIO PRIMA VENETA,
Summo labore ac diligentia ab erroribus expurgata.

VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.



49

ELEMENTA
ARITHMETICÆ.

CAPUT PRIMUM.

De fundamentalibus Arithmeticæ operationibus.

1. EÆ sunt notatio, additio, subtractio, multiplicatio, divisio, & extractio radicum, quas omnes hoc capite breviter expediemus.

§. I.

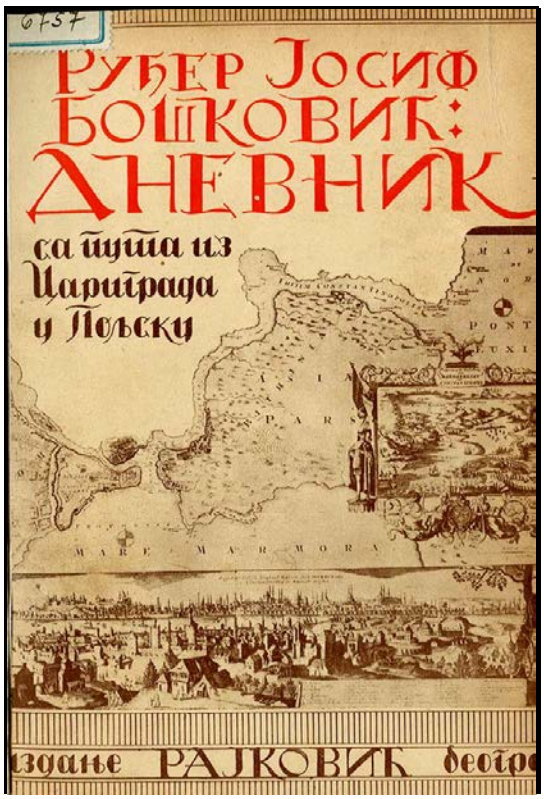
Notatio.

2. NUmeros omnes in vulgari arithmetica decem notis designamus, quarum Arabes ferunt auctores; sunt autem, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Harum notarum varia est significatio non solum ex diversa ipsarum forma, sed etiam ex diverso loco, quem occupant. Nam quæ ante punctum postremæ legenti occurrunt unitates designant; quæ proxime præcedunt unitatum decades; exinde centenarii sequuntur, millenarii, & sic deinceps in declupa proportione. Atque huic potissimum usui cyphra, seu 0 destinatur, cum enim ipsa nullum designet numerum, auget tamen reliquarum notarum significationem longius illas a puncto removens; sic unitatis nota, quæ punctum proxime præcedens unicam designaret unitatem, beneficio unius vel duplicis cyphræ in secundum aut tertium locum rejecta denas unitates, aut centenas designabit.

3. Breviores numeri facilè leguntur, nemo enim non videt numerum A (Tab. 1.) ducentas quadraginta septem unitates exprimere; at in numeris longioribus aliquo opus est artificio, Numerum B, quem legere oportet,

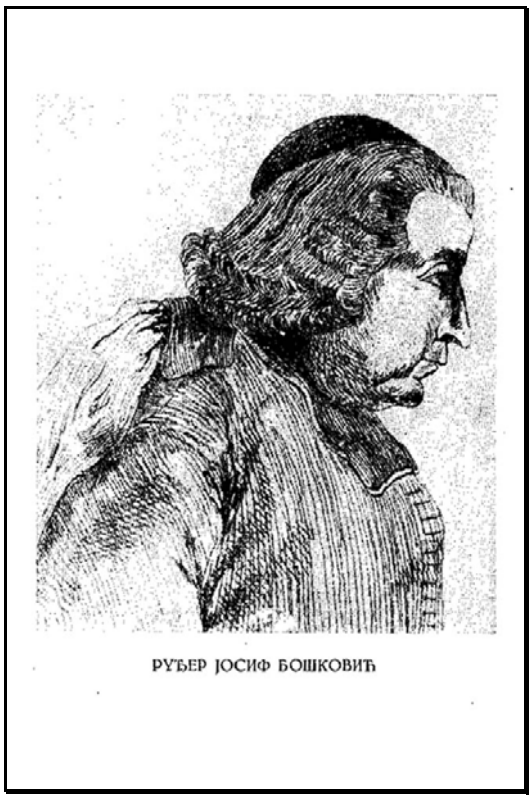
D

Елементи опште математике, Венеција, 1757



РУЂЕР ЈОСИФ
БОШКОВИЋ:
ДНЕВНИК
са пута из
Цариграда
у Пољску

издање РАЈКОВИЋ београ



РУЂЕР ЈОСИФ БОШКОВИЋ

Странице из Бошковићевог *Дневника са пута из Цариграда у Пољску*
превод Д. Недељковић, Београд, 1937

73 Twenty four manuscripts in the Virtual Library of the Faculty of Mathematics in Belgrade

Ž. Mijajlović and N. Pejović
NCD Review, vol. 25, pp. 29-35, 2014.

This interesting paper presents 24 almost forgotten handwritten manuscripts that were found in 2014 in the library of the Mathematical Institute of the Serbian Academy of Sciences and Arts. These writings belong to the professors of the Belgrade University who lectured pure and applied mathematics at the beginning of the XX century: Dimitrije Danić (1862-1932), two manuscripts on applied mathematics; Kosta Stojanović (1867-1921), two manuscripts on physics and geometry; Mihailo Petrović Alas (1868-1943), 14 manuscripts on mathematics, mostly on differential calculus and algebra; Milutin Milanković (1879-1958), five writings on applied mathematics, physics, and theoretical mechanics and one manuscript by an unknown author from the middle of the XIX century. Digital copies of these writings are deposited in the Virtual Library of the Faculty of Mathematics of the University of Belgrade.

Преглед НЦД 25 (2014), 29–35

Žarko Mijajlović, Nadežda Pejović
Faculty of Mathematics
University of Belgrade, Serbia

TWENTY FOUR MANUSCRIPTS IN THE VIRTUAL LIBRARY OF THE FACULTY OF MATHEMATICS IN BELGRADE

Abstract: The aim of this paper is to present 24 almost forgotten handwritten manuscripts that were recently found in the library of the Mathematical Institute of the Serbian Academy of Sciences and Arts. These writings belong to the prominent professors of the Belgrade University who lectured pure and applied mathematics at the beginning of the previous century: Dimitrije Danić (1862-1932), two manuscripts on applied mathematics; Kosta Stojanović (1867-1921), two manuscripts on physics and geometry; Mihailo Petrović (1868-1943), 14 manuscripts on mathematics, mostly on differential calculus and algebra; Milutin Milanković (1879-1958), five writings on applied mathematics, physics, and theoretical mechanics and one manuscript by an unknown author. Digital copies of these writings are deposited in the Virtual Library of the Faculty of Mathematics of the University of Belgrade.

Keywords: manuscripts, mathematical sciences, handwritings, lectures

1. Introduction

Almost ten years ago the Virtual Library of the Faculty of Mathematics in Belgrade was founded [12]. It is the largest database of digitized texts in Serbia which is open for public use. At the time of writing this article, it contained about 3000 digitized items and several important collections, including doctoral dissertations in mathematics and rare books from the 18th and 19th century. In this paper we present a collection of the recently digitized 24 manuscripts dating at the end of the 19th and the beginning of the 20th century. They are handwritten notes of lectures delivered by the most outstanding mathematicians and professors of the Belgrade University. Digital copies of manuscripts are deposited in the Virtual Library.

2. Manuscripts

The original manuscripts belong to the Library of Mathematical Institute in Belgrade. Their origin and existence is closely related to the emergence and the work of the Library. Therefore we believe that a short history of the Library and circumstances how manuscripts come to the existence and how they were discovered could be interesting to the reader.

Today the Library has the status of a specialized and central library in the field of mathematics and related disciplines such as mechanics, astronomy and computer science. Besides the library of the Faculty of Mathematics in Belgrade, it is the most important source of printed scientific literature for Serbian mathematical community. Therefore, its main goal is to collect and keep the necessary scientific literature, documentation and information related to mathematical sciences. Library holdings consist of books, journals, and data-bases, as well as various electronic editions. Even if the origin of the Library goes back to the end of the 19th century, a few books and journals in the Library are dated before the Second World War. This shall be explained later.

The library of the Mathematics Seminar, the forerunner of the Library of the Mathematical Institute, was established already in 1894. It provided generations of mathematicians of the Belgrade University with ample opportunities for scientific work. Until the First World War the library was led by outstanding university professors of mathematics, Bogdan Gavrilović and Mihailo Petrović, and later other mathematicians participated. The Library had a rich collection of books, sets of journals, monographs and other mathematical literature. Until the fifties of the 20th century, the Department of Mathematics was a part of the Faculty of Philosophy of the Belgrade University. All mathematical activities, studies and scientific work were conducted there. So, the Mathematics Seminar and its library as well, were placed there. When the scientists of the Faculty of Philosophy moved in 1938 to a new building built next to the old building of the Faculty in Captain Miša's building, the Library was moved to the same place. Unfortunately, only two days before the liberation of Belgrade, on October 18, 1944, the retreating enemy army set fire to the library and destroyed it. Only a few books that were borrowed at the time by individuals were saved. The saved first book of inventory up to the 1907, shows that Bogdan Gavrilović himself manually recorded the books, up to the number of 110, and Mihailo Petrović over that number up to 301. Due to these historical circumstances, now there are only a few old books and manuscripts in the library. Hence, we were quite astonished when we found almost fortuitously, in the autumn of 2012, twenty four bound manuscripts sitting lonely at one not so easily accessible rack. These writings were handwritten notes of lectures in mathematics, mechanics and theoretical physics which were delivered at the Belgrade University by prominent university professors and mathematicians Dimitrije Danić, Kosta Stojanović, Mihailo Petrović and Milutin Milanković. There is an additional manuscript in this collection, dated 1846. It covers lectures on astronomy and is written in French. The name of the author or the owner of the book is written on the first page, but it is illegible.

By inspection of the manuscripts assigned to Danić and Stojanović, we found that they were written by the authors. On the first page of the notes of Petrović' and Milanković' lectures Borivoj J. Pujić is written. As all signed copies were written in the same beautiful handwriting, one may suppose with certainty that Pujić took the notes himself. Searching old archives of the Faculty of Mathematics, we found that Pujić finished studies of mathematics at the Belgrade University in 1914. Therefore we can determine quite accurately that the notes of Petrović' and Milanković' lectures were created in the period 1910-1914. Studying the writing style, language and the used mathematical terminology, we concluded that Danić and Stojanović wrote their manuscripts at least a decade before, around the beginning of the 20th century. All these books were registered in the library catalog in the sixties of the 20th century. Hence, we may assume that the manuscripts, at least those signed by Pujić, were donated to the Mathematical Institute, most probably by Pujić himself. We do not know much about Pujić, neither had we found traces of his name in other historical or archival documents. He signed as a "Bor. J. Pujić, prof.", so most certainly he was a professor of mathematics in a secondary school. But despite the fact that Pujić was unbeknown, he left us important and very complete documents that witness the status and the level of mathematical sciences at the Belgrade University a century ago.

These lectures cover almost all courses of mathematics and mechanics taught at that time at the Belgrade University. They do not contain only the notes of lectures delivered by Bogdan Gavrilović, another outstanding university professor and academician. This is understandable, since his two university text-books on linear algebra and analytic geometry were already printed at the end of the 20th century. We do not want to enter here into description of the manuscripts. It should only be mentioned that they cover all main topics in mathematics and applied mathematics at the level of similar courses taught at the

leading European universities of that time. Our explanation is that all authors of the manuscripts were educated and they defended their doctoral dissertations at the leading European university centers: in Germany, Austro-Hungary and France. Also, their mentors were prominent mathematicians of their time. This is also noted by other authors, for example by Robson and Stedall [10].

We digitized all manuscripts and they are now deposited in the Virtual Library of the Faculty of Mathematics[1] and we invite the reader to browse them. The reader will find that some of the lectures were rather advanced, for example Petrović' lectures on differential equations and the theory of functions and Milanković' lectures on theoretical mechanics. Looking into the titles and contents of the courses, one can conclude that pure mathematics was not the only goal of the university studies of mathematics, but also applications of mathematics to other sciences.

3. Authors

We provide only short biographical notes on authors; much more complete biographies of these prominent professors and outstanding men one can find in references and supplied links. All manuscripts, except that one written by an unknown author are written in Serbian, but we also supplied translations of their titles into English. Digital copies of manuscripts can be found in the Virtual Library of the Faculty of Mathematics.



Dimitrije Danić

Dimitrije Danić (1862 – 1932) is the first Serbian mathematician with a Ph.D. degree (Jena University, 1885). He was professor of mathematics at the Military Academy, but he also lectured occasionally at the Belgrade University. Manuscripts were written by the author himself around the beginning of the 20th century.

1. Теорија конформног снимања и њена примена у Картографији и Вишој Геодезији (Theory of conformal mapping and its application to Cartography and Higher Geodesy), IV+147, Belgrade, end of XIX Century.
2. Теорни основи Методе најмањих квадрата (Foundation of the least square method), II+78, Belgrade, end of XIX Century.

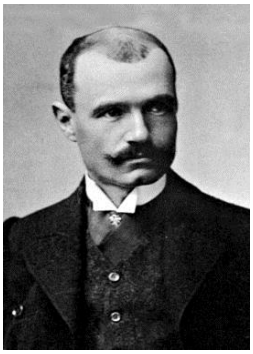


Kosta Stojanović

Kosta Stojanović (1867 – 1921) was the professor of mechanics appointed at the Belgrade University until 1905. After that he turned to politics and advanced to the position of the Minister of economics. He wrote probably the best Serbian book ever on mathematical economy (*Foundations of the Theory of Economic Values*, published in 1910). The book is inspired by phenomena appearing in mechanics and physics and involves high mathematical formalism. Manuscripts were written by the author himself around the beginning of the 20th century.

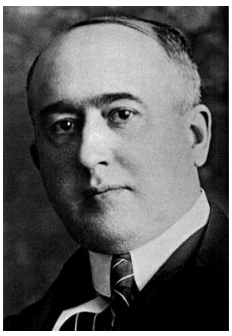
[1] http://elibrary.matf.bg.ac.rs. In the electronic version of this paper we supplied electronic hyperlinks to the manuscripts and so the reader will have the opportunity to browse and read them online.

1. О полу и полари код кривих линија (On poles and polars of curved lines), III+127, 8 tables, Belgrade, end of XIX Century.
2. Предавања на Универзитету из Математичке физике (University lectures on Mathematical Physics), 470p, Belgrade, beginning of XX Century.


Mihailo Petrović

Mihailo Petrović Alas (1868 – 1943) is probably the most famous Serbian mathematician, also known as the founder of the world-wide recognized Belgrade Mathematical School. Besides several hundred scientific papers in mathematics, he also wrote several books related to philosophy, as well as on fishing and water-life. He was a great traveler and a member of expeditions which visited and examined distant places in South and Nordic sees. His books on these travels are still popular in the general reading public. Petrović delivered lectures to students of mathematics at the Mathematical Department of the Belgrade University. Notes of all lectures, except of "Теорија алгебарских једначина" were taken by Petrovic' student Borivoj J. Pujić around 1910.

1. Основи теорије детерминаната (Basics of the theory of determinants), II+103.
2. Аналитична геометрија. I део: у равни (Analytic geometry. Part I: in plane), II+382.
3. Аналитична геометрија. II део: у простору (Analytic geometry. Part II: in space), II+384.
4. Теорија алгебарских једначина (Theory of algebraic equations), II+254.
5. Теорија извода са применама (Theory of [function] derivatives with applications), II+227.
6. Интегрални рачун (Integrals), II+384.
7. Диференцијални рачун (Differential calculus), II+278.
8. Геометријске примене диференцијалног рачуна (Geometrical applications of differential calculus), II+246.
9. Геометријске примене интегралног рачуна (Geometrical applications of integrals), II+110.
10. Линеарне диференцијалне једначине (Linear differential equations), II+116.
11. Обичне диференцијалне једначине (Ordinary differential equations), II+116.
12. Геометријске примене диференцијалних једначина (Geometrical applications of differential equations), II+82.
13. Парцијалне диференцијалне једначине (Partial differential equations), II+126.
14. Теорија функција (Theory of functions), II+126.


Milutin Milanković

Milutin Milanković (1879 – 1958) is one of the Serbian greatest and most cited scientists of all times. His theory on the Ice age that accurately explains the change of climate on a large time scale is accepted world-wide. In recognition of his scientific achievements, a crater on the Moon, another one on the Mars and a planetoid were named after him. Besides his the most famous work *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* (published in 1941) and many other scientific papers, he also wrote an excellent book on celestial mechanics and books that popularize the science. He is also known as a good civil engineer. Notes of all lectures were taken by Borivoj. J. Pujić, a student of mathematics, around 1910.

1. Модерне теорије о електрицитету и магнетизму (Modern theories on electricity and magnetism), II+280.
2. Општа теорија физикалних поља (General theory of physical fields), II+294.
3. Рационална механика, I deo (Rational mechanics, Part I), II+277.

4. Рационална механика, II deo (Rational mechanics, Part II), II+372.
5. Векторска анализа са применама из Теориске физике (Vector analysis with applications to Theoretical Physics), II+250.

4. Unknown author

Cours d'Astronomie et de Geodesie, (Course on Astronomy and Geodesy), Ecole Polytechnique, II+578, Paris, 1846. This is an interesting but unfortunately unsigned hand-written book on astronomy and geodesy. A copy of this manuscript can be found in several French university libraries (Lyon and Bordeaux) and in the Indiana University Library. The catalogue record of this book is available in OCLC[2] Worldcat (no. 25637690). The book is written in French and according to metadata the author is unknown, even the country of origin is unknown. However, there is a possibility that the book was written by a French astronomer and geodesist, later the minister of education, Hervé Auguste Etienne Léopold Alban Faye[3] (1814-1902). Namely, he published several books in the second half of the 19th century with similar titles and contents. Another argument in the favor of this thesis is that he was first a student and later a lecturer (1848-1854) at l'Ecole Polytechnique of a course of geodesy.

5. Conclusion

We presented an invaluable collection of digitized manuscripts belonging to the Mathematical Institute in Belgrade, dating from the end of the 19th and the beginning of the 20th century. The manuscripts are handwritten notes of university lectures which were held by prominent Serbian mathematicians at the Belgrade University. This collection is an important part of scientific heritage and also the testimony of how mathematical sciences were taught at the Belgrade University at the beginning of the 20th century.

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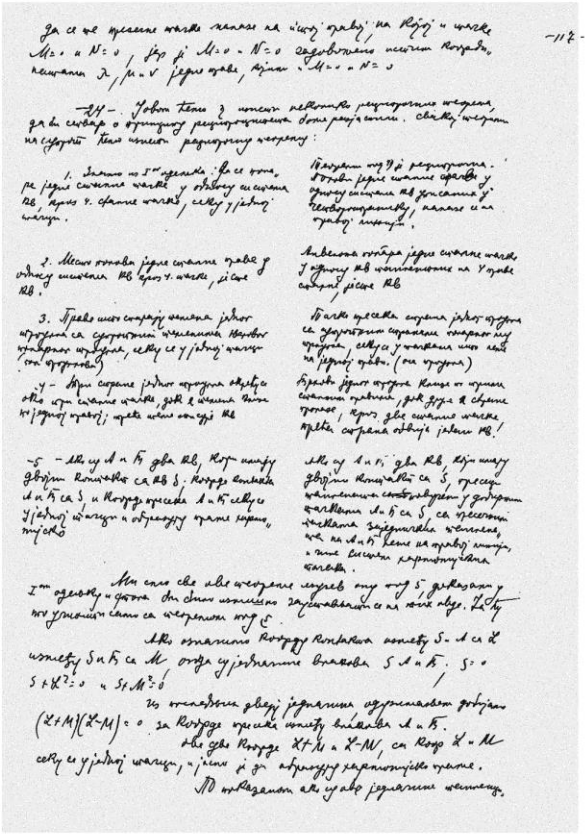
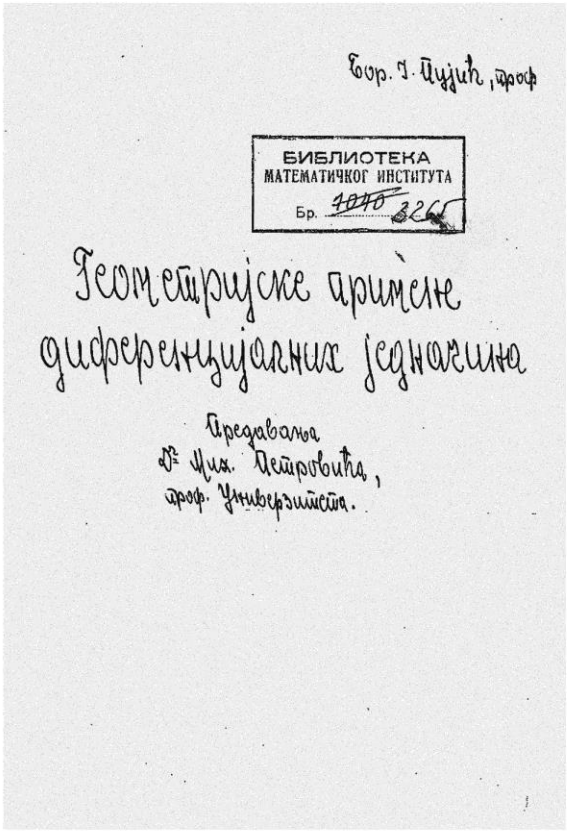
zarkom@matf.bg.ac.rs
nada@matf.bg.ac.rs

Pages from manuscripts

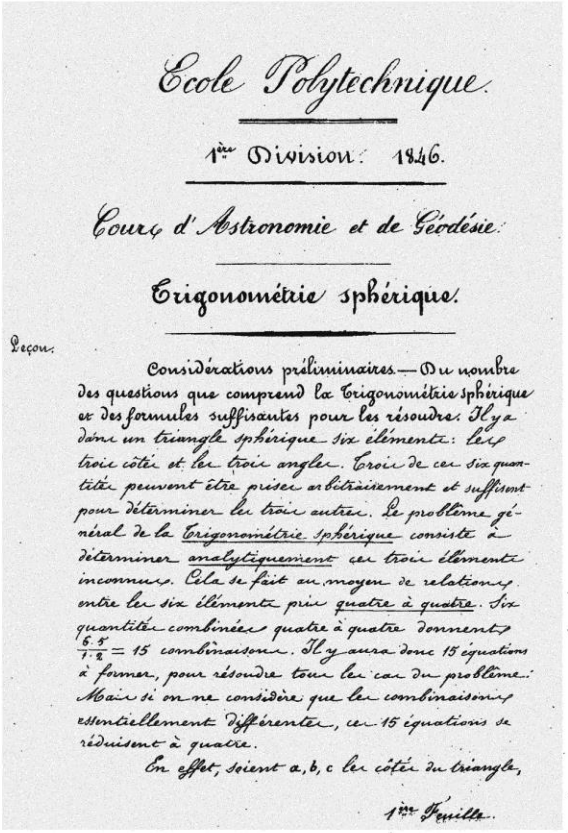


A page from Stojanović
On poles and polars of curved lines



First page from Petrović'
Geometrical applications of differential equations

Pages from manuscripts



Ecole Polytechnique.

1ère Division. 1846.

Cours d'Astronomie et de Géodésie.

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Leçon.

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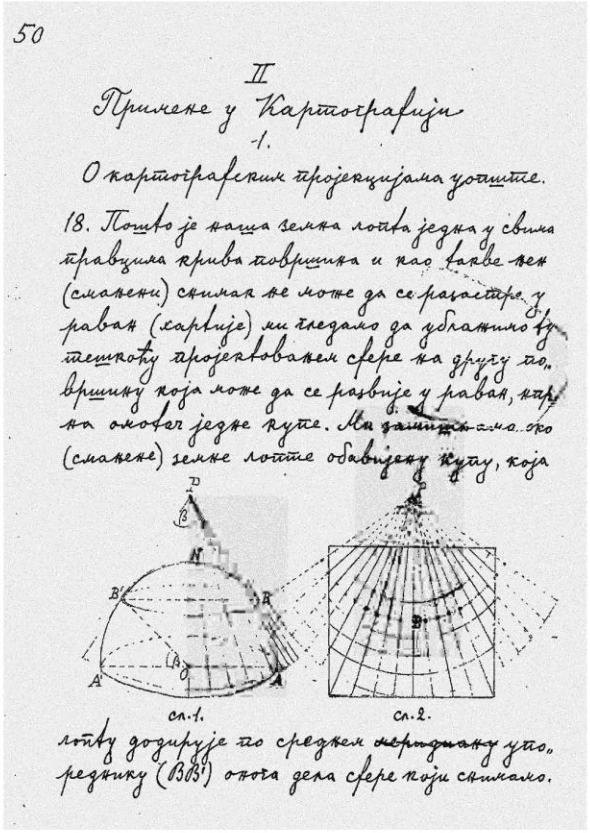
En effet, soient a, b, c les côtés du triangle,

1ère Feuille.

First page from *Cours d'Astronomie et de Geodesie*



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II

Примене у Картографији

1.

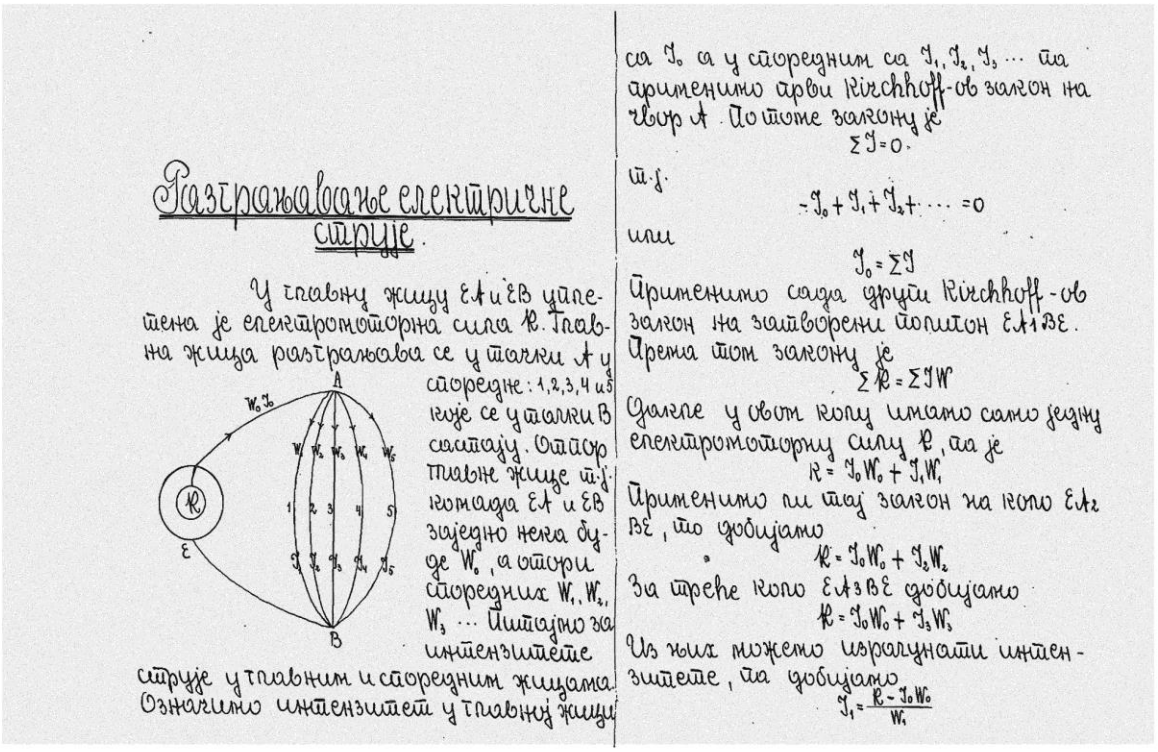
О картографским пројекцијама уопште.

18. Пошто је наша земна лопта једна у свима правцима крива површина и као такве њен (смањени) снимак не може да се разастре у раван (хартије) ми гледамо да ублажимо ту тешкоћу пројектовањем сфере на другу површину која може да се развије у раван, нпр. на омотач једне купе. Ми замишљамо око (смањене) земне лопте обавијену купу, која

сл. 1. сл. 2.

лопту додирује по средњем ~~меридијану~~ упореднику (BB') оноча дела сфере који снимамо.

A page from Danić' *Theory of conformal mapping*



Разграњавање електричне струје.

У главну жицу EA и EB уплетена је електромоторна сила R. Главна жица разграњава се у тачки A у споредне: 1, 2, 3, 4 и 5 које се у тачки B састају. Отпор главне жице т.ј. комада EA и EB заједно нека буде W_0, а отори споредних $W_1, W_2, W_3 \dots$ Питајмо за интензитете струје у главним и споредним жицама. Означимо интензитет у главној жици

са J_0 а у споредним са $J_1, J_2, J_3 \dots$ па применимо први Kirchhoff-ов закон на чвор A. По томе закону је

$$\Sigma J = 0.$$

т.ј.

$$-J_0 + J_1 + J_2 + \dots = 0$$

или

$$J_0 = \Sigma J$$

Применимо сада други Kirchhoff-ов закон на затворени полигон EA1BE. Према том закону је

$$\Sigma R = \Sigma J W$$

Дакле у овом колу имамо само једну електромоторну силу R, па је

$$R = J_0 W_0 + J_1 W_1$$

Применимо ли тај закон на коло EA2BE, то добијамо

$$R = J_0 W_0 + J_2 W_2$$

За треће коло EA3BE добијамо

$$R = J_0 W_0 + J_3 W_3$$

Из њих можемо израчунати интензитете, па добијамо

$$J_1 = \frac{R - J_0 W_0}{W_1}$$

Pages from Milanković' *Modern theories on Electricity and Magnetism*

74 A manuscript on astronomy and geodesy of an unknown author

N. Pejović and S. Ninković
NCD Review, vol. 26, pp. 27-36, 2015.

This article is a continuation of the review "Twenty four manuscripts in the Virtual Library of the Faculty of Mathematics in Belgrade". The authors try to track to origin of handwritten text in French from this collection and to find out who wrote it. The year of the manuscript is known, given on the first page - 1846. The reviwers believe that the manuscript was written by Herve Faye (Herve Auguste Etienne Uopold Alban Faye, 1814-1902), a French astronomer and geodesist. They concluded that by the style of writing and from the fact that Faye published a book with the similar title and similar content. It is interesting that Faye was Minister of Public Instruction in the Rochebouet cabinet in 1877.

Преглед НЦД 26 (2015), 27–36

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Astronomical Observatory, Belgrade, Serbia

A MANUSCRIPT ON ASTRONOMY AND GEODESY OF AN UNKNOWN AUTHOR

Summary (Abstract). The present contribution concerns a text in the form of handwriting, written in French. The year is known, given on the first page - 1846, but the author not. The present authors believe that the manuscript was written by Hervé Faye (Hervé Auguste Étienne Léopold Alban Faye, 1814-1902), a French astronomer and geodesist. The manuscript belongs to a collection of 24 manuscripts which is a property of the Mathematical Institute of Serbian Academy of Sciences and Arts in Belgrade. The entire collection has been digitised and it is deposited in the Virtual Library of the Faculty of Mathematics in Belgrade. The manuscript containing almost 600 pages is subjected to an analysis which is reported here.

Introduction

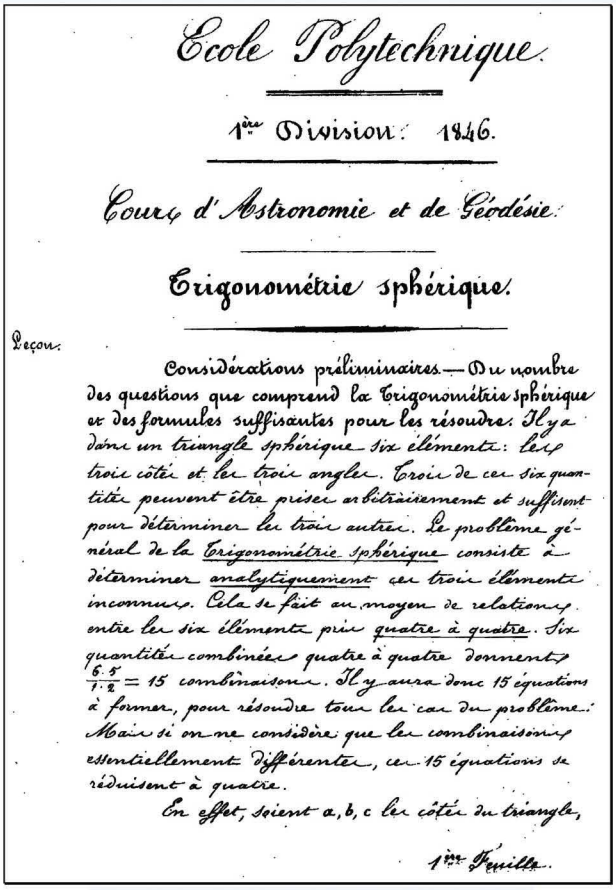
There exists a collection containing 24 manuscripts which concern mathematics and related sciences, see [3]. It is a property of the Mathematical Institute of Serbian Academy of Sciences and Arts. This collection has been digitised to be available at http://elibrary.matf.bg.ac.rs.

Among the manuscripts there is one existing in the form of handwriting only. From its title one can conclude that its field is astronomy and geodesy. The text was written in French, but the author is, unfortunately, unknown. More precisely, in the material we possess there is no signature, no indication who wrote this text. The information given on the first page (Fig. 1) contains the name of the institution (École polytechnique), the year and the general title (Cours d'astronomie et de géodésie, in English: A Course of Astronomy and Geodesy).

It is not known how this manuscript reached Serbia. It is possible that someone of the Serbs, who stayed in France as students in the XIX century, brought it. For instance, the two first Directors of the Belgrade Observatory, Milan Nedeljković (led the Observatory 1887-1899 and 1900-1924) and Đorđe Stanojević (1899-1900) both spent some time in France for the purpose of progressing their knowledge in astronomy, physics, geodesy and related sciences. The first page (Fig. 1) is preceded by two pages referred to as ex libris. In Fig. 2 one can see one of them. Both contain the signatures of persons thought to be the former owners of the Belgrade copy.

Though the manuscript has been never published, its copies do exist. For instance, several universities in France (e.g. Bordeaux and Lyon) possess a copy, also the Indiana State University in the USA. To the present authors it is also unknown if the text of the manuscript has been translated into any language. What is sure is that no translation into Serbian (Serbocroat) has been done.

We suppose that the author of the manuscript is Hervé Auguste Étienne Léopold Alban Faye (1814-1902). As it is usual, herein we shall use the name Hervé Faye.



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1re Division. 1846.

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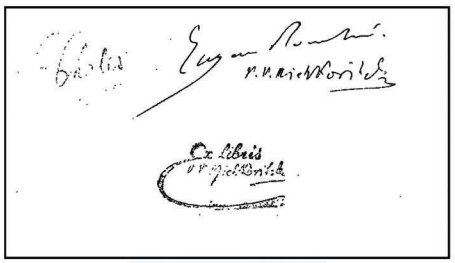
Fig. 1: The first page of the manuscript



Ex libris

Fig. 2: Ex libris

Who was Hervé Faye

Hervé Faye (Fig. 3) was born at Saint-Benoît-du-Sault, in the Department of Indre, 278 km southwest of Paris. He studied at École polytechnique. In 1832 he joined the Paris Observatory thanks to François Arago. His activity was fruitful, In 1843 Hervé Faye discovered a periodic comet named after him (4P/Faye). This discovery brought him the Lalande prize in 1844. In gravimetry Hervé Faye corrected the Bouger formula which is known as Faye's correction. It is known that Hervé Faye spent some time as a teacher at École polytechnique; according to the existing data (Larousse 1930, Wikipedia) it should be

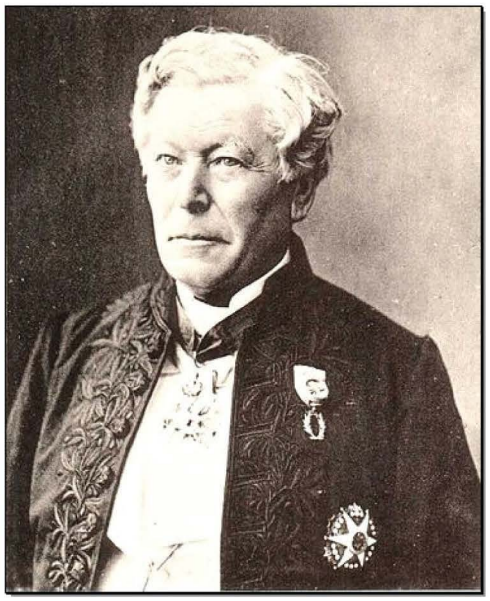


Fig. 3: Hervé Faye

between 1848 and 1854, but the year on the first page (Fig. 1) is 1846. Therefore, it seems probable enough that Hervé Faye taught the subjects of astronomy and geodesy at École polytechnique in the midforties of the XIX century and that then he wrote his course.

The authorship would, certainly, deserve a sufficiently large study. On the other hand, the present paper is merely a contribution article in a Proceedings, in other words it is not a thorough study concerning the authorship.

In 1847 Hervé Faye became member of the Academy of Sciences, to become its President in 1872; in 1876 he presided in Bureau des longitudes. In 1877 for a short time Hervé Faye was a Minister in (Public Instruction) in the French government. A lunar crater was named after him in 1935.

General analysis of the manuscript

The manuscript comprises 552 (main text) handwritten pages. After the main text there are another 25 pages. Most of these additional pages are to describe the text organization in lessons (leçons). Each lesson has a number, but without any title indicating the contents. The total number of lessons is 26. The information what a lesson contains is given in the organizational part. In this part there are titles containing the lesson number and the pages occupied by the lesson. After the description of the lessons there are two notes. The first note concerns the approximate calculation of angles; the second one is errata. It should be said that at the time when the manuscript appeared, radians were not used, at least as they are used in our time. The term radian was mentioned for the first time in a printed form in 1873 (Wikipedia). Though the manuscript was written as a book, there is no contents. The only information what it contains can be found by reading the description of the lessons.

The way in which the material is presented is somewhat different from the form expected from the point of view of nowadays. The first pages (about forty) are devoted to spherical trigonometry, which can be seen from Fig. 1. According to the lesson description spherical trigonometry appears within three lessons, only at the end of the third lesson one mentions the devices for time measuring.

Lessons from 4 to 6 are completely devoted to the equipment. One should be aware that in the late first half of the XIX century photography was at its mere beginnings only. Therefore, it is not surprising that all illustrations given in the manuscript are just drawings. The figures are not numbered. The same is true for the tables. Due to this if one wants to establish the total number of figures and tables, there is no choice, but to count them. There are many figures, but seldom more than one per page. A very rough estimate may be that on the average every five pages contain a figure. In this way one infers 110 figures in the manuscript as a whole. The total number of tables is much smaller, perhaps not more than about 15 tables at all. The equations are numbered, if they are referred to, but the numbers are repeated, even in the very same lesson one can find two equations with the same number.

There are footnotes in the text; they are not numbered, instead one can find a designation (*) behind a sentence. Then at the bottom of the page the same designation appears to be followed by the proper text. Sometimes on the same page there was no space enough, so the footnote had to be continued on the next page.

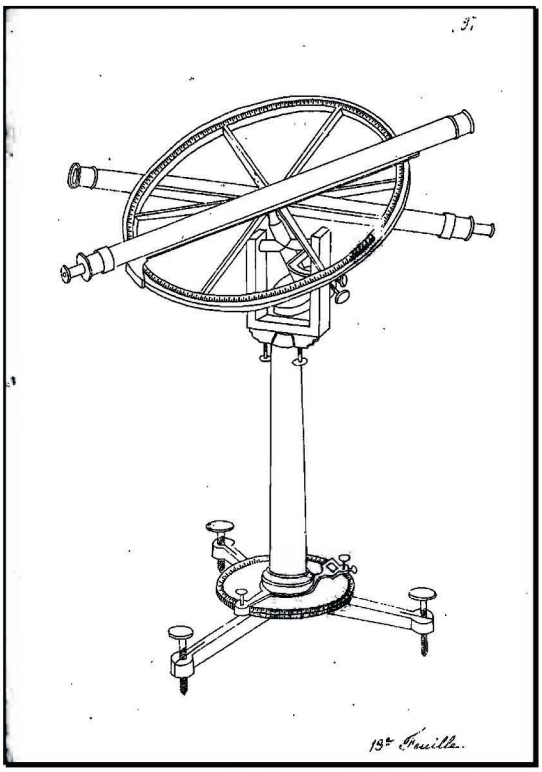
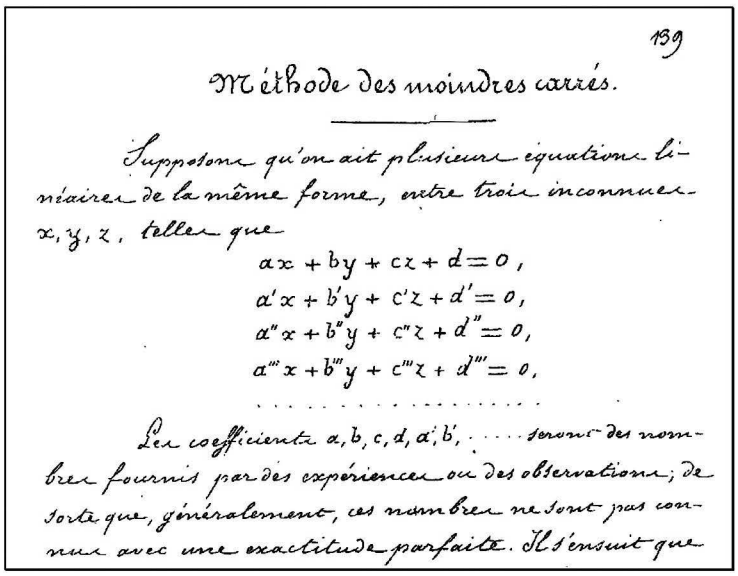


Fig. 4: Reflecting Circle

The comment concerning drawing becomes particularly interesting when the equipment is the topic. A nice example is Fig. 4 in which an instrument referred to as reflecting circle (cercle répétiteur) is presented. A substantial part of the instruments were (or are still) used mainly in geodesy. The theodolite was the only piece of equipment left for another lesson (lesson 7). In this lesson one starts with astronomical refraction. This phenomenon was paid a sufficient attention. It was explained that the refraction calculation is not simple. In this connection a table is given and also there is a part concerning the method of least squares (Fig. 5). The description of this method, still of much interest, occupies five pages. After a few pages dealing with some effects of refraction (de qulelque effets des réfractions) one finds within Lesson 8 a title named „Astronomy“ (astronomie).

It is of interest to note how astronomy was viewed by the manuscript author or, in general, in the time of the writing. It was defined as ”science of celestial bodies“ (science des astres) As its main subject one defines ”knowledge of their motion“ (connaissance de. leurs mouvements). Other characteristics are mentioned as ”also their distances, their size, their form, their physical constitution, etc“ (leurs distances, leur grandeur, leur forme, leur constitution physique). The author says that astronomy “is based on observations and calculations”, but the main calculation according to the author is spherical trigonometry (Son principal procédé de calcul est la trigonométrie sphérique). As the main instruments (les principaux instruments d’observation) the author mentions: chronometer, reflecting circle,

theodolite, equatorial, meridian circle and mural circle (chronomètre, cercle répétiteur, théodolite, équatorial, lunette méridienne, cercle mural). Unlike the descriptions of the first three, which have been already mentioned in the present paper, the remaining three are the subject in later lessons. The reason is that the first three instruments are also used in geodesy.



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Méthode des moindres carrés.

Supposons qu'on ait plusieurs équations linéaires de la même forme, entre trois inconnues x, y, z, telles que

$$ax + by + cz + d = 0,$$
$$a'x + b'y + c'z + d' = 0,$$
$$a''x + b''y + c''z + d'' = 0,$$
$$a'''x + b'''y + c'''z + d''' = 0,$$
$$\ldots\ldots\ldots\ldots\ldots$$

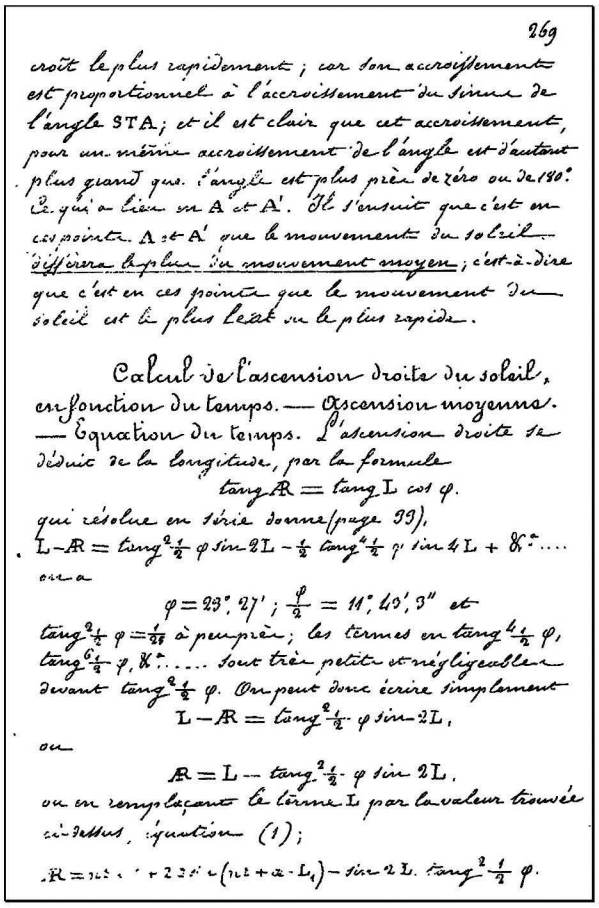
Les coefficients $a, b, c, d, a', b', \ldots$ seront des nombres fournis par des expériences ou des observations; de sorte que, généralement, ces nombres ne sont pas connus avec une exactitude parfaite. Il s'ensuit que

Fig. 5: The page where the presentation of the least-square method begins

The notions of celestial sphere, immovable stars, constellations, rotation of celestial sphere are treated only in Lessons 8 and 9, together with the "typical astronomical instruments". Then the author introduces right ascension and declination. It is curious to note that the definition of right ascension is not preceded by that of vernal-equinox point. The right-ascension designation (Fig. 6) is also unusual (of course, to a modern reader). In the entire text one does not find the notion of hour angle.

The very end of Lesson 9 belongs to stars. There one finds classification of stars, which is a survey of their magnitude classes, just following the classical scheme of Hipparchus. The author says that stars of the first size (grandeur) are the brightest ones, but the role of the distance is not clearly stated. As well known, the first successful annual-parallax determinations originate from about 1840, but they were not mentioned in the manuscript. There is a short article within Lesson 9 entitled "About the star distance from the Earth" (De la distance des Etoiles à la terre) where the author states that such a distance is "immensely large" offering a comparison to the two main distance scales: diameter of the Earth and diameter of its orbit around the Sun. Finally the author mentions the speed of light (rather correct value depending on the ratio *lieue* to kilometre) saying even the light of a first-size star needs at least three years to reach the Earth (dont la lumière nous parvienne en moins de trios ans).

After a list of the most important constellations, within a new lesson (number 10) the zodiac and ecliptic are explained. Then, in the very same lesson (10) geodesy begins, in the sense that not its equipment is the subject only. The lessons to come afterwards (11, 12, 13) deal completely with the Earth.



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croît le plus rapidement ; car son accroissement est proportionnel à l'accroissement du sinus de l'angle STA ; et il est clair que cet accroissement, pour un même accroissement de l'angle est d'autant plus grand que l'angle est plus près de zéro ou de 180°. Ce qui a lieu en A et A'. Il s'ensuit que c'est en ces points A et A' que le mouvement du soleil différera le plus du mouvement moyen ; c'est-à-dire que c'est en ces points que le mouvement du soleil est le plus lent ou le plus rapide.

Calcul de l'ascension droite du soleil, en fonction du temps. — Ascension moyenne. — Equation du temps. L'ascension droite se déduit de la longitude, par la formule

$$\operatorname{tang} \text{Æ} = \operatorname{tang} L \cos \varphi.$$

qui résolue en série donne (page 33),

$$L - \text{Æ} = \operatorname{tang}^2 \tfrac{1}{2} \varphi \sin 2L - \tfrac{1}{2} \operatorname{tang}^4 \tfrac{1}{2} \varphi \sin 4L + \&^{a} \ldots$$

on a

$$\varphi = 23°, 27' ; \frac{\varphi}{2} = 11°, 40', 3'' \text{ et}$$

$\operatorname{tang}^2 \frac{1}{2} \varphi = \frac{1}{25}$ à peu près ; les termes en $\operatorname{tang}^4 \frac{1}{2} \varphi$, $\operatorname{tang}^6 \frac{1}{2} \varphi$, &ª ..... sont très petits et négligeables devant $\operatorname{tang}^2 \frac{1}{2} \varphi$. On peut donc écrire simplement

$$L - \text{Æ} = \operatorname{tang}^2 \tfrac{1}{2} \varphi \sin 2L,$$

ou

$$\text{Æ} = L - \operatorname{tang}^2 \tfrac{1}{2} \varphi \sin 2L.$$

ou en remplaçant le terme L par la valeur trouvée ci-dessus, équation (1) ;

$$\text{Æ} = nt + 2\alpha \sin(nt + \alpha - L_1) - \sin 2L . \operatorname{tang}^2 \tfrac{1}{2} \varphi.$$

Fig. 6: Note the designation used for right ascension

Lessons from 14 to 26 (the last) have astronomical contents. At first one treats the apparent motion of the Sun, time measuring, precession and nutation are introduced, diurnal parallax is also the subject and in Lesson 17, to be finished in the next one, solar physics is treated, especially sunspots and rotation. In Lesson 18 most of space is devoted to the Moon. The only natural satellite of our planet is also the subject in Lessons 19 and 20, its motion, eclipses and, finally, in the first part of Lesson 21 lunar physics becomes the subject. After this the author explains the facts about the planets, at first inner and outer planets, their motion, physics of planets and Kepler's laws. Though Kepler's laws are introduced in Lesson 22, their interpretation is given in Lesson 24 only, but the phenomenon of gravitation is explained in more detail in Lesson 25. The reason is that this explanation is closely connected to the question of free fall near the terrestrial surface. Also, since the gravity appears as the cause of other phenomena on the Earth, precession, tidal action, the last lesson (26) is devoted to them. In this part of the manuscript astronomical aberration has much space, in accordance with it the speed of light is treated, its first successful determination was astronomical, took place in the XVII century.

Information of interest in the text - historical notes

The manuscript contains parts referred to as historical notes (notice historique). They are usually followed by a comment "not required" (non exigé). Most likely this "not required"

concerns the examination material; in other words the students were not expected to know the history of a problem on the same level as it was the case with the problem itself. Nevertheless, this matter deserves attention. Some facts concerning the history of a problem before the manuscript appeared (middle XIX century) may have faded into oblivion till our time. For this reason, any old manuscript can be a valuable source of information to historians of science. Of course, it is also possible that in the meantime, from the time when the manuscript appeared till nowadays, new facts have been discovered, in the light of which the history of a problem changes its character.

All historical notes treated in the manuscript cannot be the subject here. Since the manuscript concerns both geodesy and astronomy, each will have a problem presented here. The author (p. 212) emphasizes two questions related to the Earth, to determine its shape and to determine its size ("de déterminer la forme de la terre et de déterminer sa grandeur"). According to the author the size of the Earth seemed to be the subject of a great interest from early antiquity. The feat of Eratosthenes is well known to the manuscript author. On the beginning of the paragraph speaking about Eratosthenes one finds something, obviously added afterwards: "276 ans avant J. C.". If we cite modern data, say Wikipedia, this year appears as the birth year of Eratosthenes. However, in the preceding paragraph the author mentions Aristotle (384-322 BC), more precisely, Aristotle's treatise on the sky, Book 2, where the famous Greek philosopher says "they do not give it a circumference exceeding 400,000 stadia" (ne lui donnent guère que 400 000 stades de circonférence). Without regard to all uncertainties which concern the exact length of one stadia, say converted into kilometres, (e. g. Preti, 2002 p. 108), as well as the question if the stadia used by Aristotle is the same as that used by Eratosthenes, the value cited by Aristotle is quite different from that found by Eratosthenes (252,000 stadia). Since the latter value is recognised, even today, as a very good one, the former value (Aristotle's one) seems obsolete. Of course, later works are also mentioned, but the most exciting place in that part of the book is related to the determination of the shape of the Earth. The theories of Huygens and Newton predicted that the surface of the Earth cannot be a perfect sphere, more precisely the distance between the poles should be shorter than the equatorial diameter. However, the results of the first measurements were in favour of a prolate shape for the Earth. Finally, in the XVIII century, after new measurements, it became clear that the Earth is, nevertheless, oblate, as the two theories had predicted.

In the lesson treating the planetary motions there is a part devoted to the first measuring of the light speed. This is a well known event when Roemer observing a Jovian satellite established the value for the light speed. The text is followed by a footnote in which it is said that Descartes, as early as in 1634, had a similar idea. In one of his letters Descartes stated that in the case of a finite value for the speed of light one would observe a delay in the lunar eclipse. The author's comment is that Descartes failed to find a confirmation in the observational results because the Moon is too close to the Earth; if Descartes had applied this reasoning to a Jovian satellite, then astronomy would have owed him the nice discovery of Roemer ("l'astronomie lui eût été redevable de la belle découverte de Roemer").

Among other historical notes presented in the manuscript, of interest may be that concerning the discovery of attraction law ("découverte da la loi d'attraction"). This topic occupies four pages, several scientists are mentioned, some of them are very well known. The author says that Hook, like Kepler, Fermat and Roberval, expressed the principle of a mutual attraction comprising all celestial bodies ("le principe d'une attraction mutuelle de tous les coprps célèstes"). In the further text the author's words are that the law proposed by Hook to be found was exactly what Newton looked for, Newton who was fortunate to discover it

("Cette loi que le docteur Hook proposait de trouver, fut précisément celle que chercha Newton …, et qu'il eut le bonheur de découvrir").

State of the art at the time of manuscript appearing

When reading a text written sufficiently long ago, as it is the case with the manuscript under study, it becomes possible to view the state of the art in the given field (here astronomy and geodesy) at that time. As said above, the classical disciplines, spherical trigonometry, astrometry, celestial mechanics are presented in a way not very different from that usual nowadays. This is due to the fact that in these disciplines a substantial progress had taken place by the time when the manuscript was written. Therefore, it is not surprising that the Solar System is described with many details. In the physics of the Sun much was unknown at that time, but there is a footnote (p. 367) wher the author mentions "a third atmosphere of the Sun" (troisième atmosphère du solei). With regard that something quite new is borne in mind, we may speculate a little bit, an indication of the solar corona? Also interesting, on page 380, "the possibility that the lunar mountains have a volcanic character" (caractère volcanique des montagnes de la lune). The volcanic activity on the lunar surface has been the subject of many studies, but the modern point of view is that the eruptions belong to the distant past (more than billion years ago).

On pp. 382-383 the author mentions the absence of water and atmosphere on the Moon. Perhaps, the most intriguing concerning the Moon is the relationship between the lunar phases and weather. The author says that according to astronomers the Moon has no influence on weather (Les astronomes croyent que la lune n'influe pas sur le temps), but many people think the opposite, for instance the weather changes when the Moon changes its phases. Finally the author concludes that "it seems that an influence of the Moon (lunar phases) should be recognised" (Il semble donc qu'on doive reconnaître une certaine influence à la lune), "but the effect is not so strong as it has been usually thought from early antiquity" (mais qui n'est pas celle que la public lui attribute depuis le plus haute antiquité).

The basic facts about the Solar-System planets are given in the text. It would be of interest to comment them in the light of our time. On page 419 one finds a table containing the data on the orbital inclination (with respect to the plane of ecliptic). There are two columns, the left-hand one concerns the planets in the modern sense, of course without Neptune which was discovered as late as in 1846, approximately at the time when the manuscript was written. Neptune is nowhere mentioned. The orbital inclinations for the six planets (Earth not included) are just as we find them now. In other words, the inclination values were known rather precisely even long ago. The right-hand column concerns the objects referred to as minor planets (petites planètes) and it contains the data for Vesta, Juno, Ceres and Pallas, the only known minor planets at that time.

There is another table (p. 504) which contains data about the planets. This time the four minor planets and the Earth are included so that there are eleven rows in the table. Mercury occupies the place at the top, as closest to the Sun, Uranus is at the bottom, as the most distant. The columns (from left to right) give: the distance to the Sun, sidereal revolution, rotation period and diameter. As can be expected, the distances and periods are very accurate (we recognise the values we know today), also to note that the two quantities are related to each other (Kepler's third law). The rotation periods are not given for all bodies, they are missing for the four minor planets and Uranus. In the case of Mercury and Venus the values are completely erroneous from the modern point of view. As for Mars, Jupiter and Saturn, they are quite correct.

This table is preceded by a section dealing with the planet distances from the Sun in which the author mentions the rule, usually referred to as that of Titius and Bode. However, for the author it is Bode's rule, more precisely law (loi de Bode). It is said that at the time when the rule was formulated, no minor planet was known. The fact that soon after the discovery of the first among them - Ceres - the other three were also discovered, is connected with the hypothesis made by Olbers according to which "the four minor bodies could be the fragments of the same planet destroyed in an explosion" (ces quatre petits cops pouvaient être des fragments d'une meme planète brisée dans une explosion). The number of satellites for the Jovian planets could be mentioned - Jupiter 4, Saturn 7 and Uranus 2. It is, of course, not surprising that we know today that these planets have much more satellites.

Mentioning of Rudjer Bošković's name

Rudjer Bošković (1711-1787), the Southern Slav scientist whose three-hundredth anniversary from the birth was celebrated recently and who worked in France for a sufficiently long time, was mentioned in the text twice (p. 129 and p. 203). The first mentioning concerns a formula developed for the purpose of calculating the atmospheric refraction. In this case Bošković was mentioned together with Simpson and Duséjour. It is said that "several geometrists" (plusieurs géomètres) "have reached, through different considerations, a relation of the (same) form" (sont parvenus, par des considérations différentes, à une relation de la forme). The formula was written down and commented. It has two numerical coefficients to be determined observationally when the refraction is known. Also that its form is that of Bradley's formula. The present authors have been able to find this formula in the literature, but referred to as Simpson's formula. Perhaps, the fact that the other two scientists had obtained the same formula as Simpson, has faded into oblivion?

The second place where Bošković's name is mentioned concerns the geodetic measurements, in particular the meridian arc degree. A table is presented and in a row of its one finds Italy as the country where the measurements took place, then the mean latitude value, the grade length in metres and in the fourth (last) column giving the names of the observers two names - Bošković and Lemaire. On this topic one can also find in Battinelli's (2014) article. There is a slight difference in the family name of Bošković's coworker. According to Battinelli it is Maire, not Lemaire, as cited in the manuscript which is here under study. Battinelli also gives the first name and the years of birth and death. With these data a search via Internet (e. g. Oxford Dictionary of National Biography) leads to the Jesuit Christopher Maire born in England with exactly the same years of birth and death as given by Battinelli and for whom it is said that was engaged in the measurements on the terrestrial surface together with Bošković. Therefore, the family name given by the manuscript author is erroneous.

Conclusion

Old texts written sufficiently long ago can be very useful for history of science. The relevant facts about a discovery can always fade into oblivion. As a consequence, the information at our disposal concerning this discovery will be erroneous or, at least, incomplete. For this reason it is important to preserve our heritage. The digitisation is the best way to keep it in good condition. Then old manuscripts can be easily available.

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75 Scientific papers of Milutin Milanković in his digital legacy

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NCD Review, vol. 26, pp. 69-76, 2015.

This article presents scientific papers of Milutin Milanković, one of the greatest Serbian scientists of all time. The reviewed papers are contained in his Digital Legacy, a portal of the Faculty of Mathematics in Belgrade. Papers are classified in accordance with the areas to which they belong.

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SCIENTIFIC PAPERS OF MILUTIN MILANKOVIĆ IN HIS DIGITAL LEGACY

Abstract. In this paper we present scientific papers of Milutin Milanković, one of the greatest Serbian scientists, in his Digital Legacy. Papers are classified in accordance with the areas to which they belong.

1. Introduction

A group of participants of the Digitization project of the Faculty of Mathematics in Belgrade and the Mathematical Institute of the Serbian Academy of Sciences and Arts started in 2012 to build the *Digital Legacy*[1], an Internet site of the Faculty of Mathematics. The general idea of the Legacy was to present there biographies of the Serbian scientists from the past who worked in mathematical sciences: mathematics, mechanics and astronomy. Another goal was to have at a single place all their scientific and technical works and personal data such as photos, letters and other related documents in order to better understand their work and life. We also put there some writings of other people on these authors. This Internet site is designed having in mind the general audience, but the scientific community, too. At this moment, three scientists are presented there: Anton Bilimovič (1879–1970, mechanist, the founder of the leading Serbian mathematical journal *Publications de l'Institut Mathématique*, 1932), Bogdan Gavrilović (1864–1948, mathematician, one of the founders and rectors of the University of Belgrade) and the most cited Serbian scientist Milutin Milanković (1879–1958, geophysicist, astronomer, mechanist and civil engineer, best known for his theory of ice ages). We are planning to include there in the near future presentations of other prominent Serbian mathematicians: Mihalo Petrović Alas, Jovan Karamata and Đuro Kurepa.

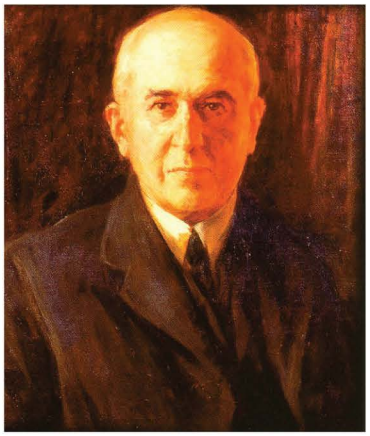
The current activities on the Legacy include the digitization and preparation of all scientific works of Milanković and this post is almost done. We recently finished digitization almost all of his scientific papers, books and lectures. His legacy includes a lot of additional materials: Milanković's photos and patents, but also books and papers of other authors on Milanković. This presentation is also the most complete in the Legacy. In this paper we present digitized scientific articles of Milutin Milanković posted in the Legacy.

2. Short biography of Milutin Milanković

Milutin Milanković (1879–1958) is one of the Serbian greatest and most cited scientists of all times. His theory of the Ice ages that accurately explains the change of climate

[1] http://legati.matf.bg.ac.rs

on a large time scale is accepted world-wide. In recognition of his scientific achievements, a crater on the Moon, another one on Mars and an asteroid were named after him. Besides his most famous work *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* (published in 1941) and many other scientific papers, he also wrote an excellent book on celestial mechanics and books that popularize tile science. He is also known as a good civil engineer. Milanković came to Belgrade from Vienna in 1909 to teach applied mathematics at the Faculty of Philosophy of Belgrade University. His coming was a merit of Bogdan Gavrilović and Mihailo Petrović Alas, who both taught mathematics at the University. Milanković was at the Belgrade University till his retirement in 1955. He was lecturing there applied mathematics. It should be mentioned that theoretical physics, mechanics and astronomy were considered then as the areas of applied mathematics. Milanković also was the first professor who started lecturing celestial mechanics at the University of Belgrade.



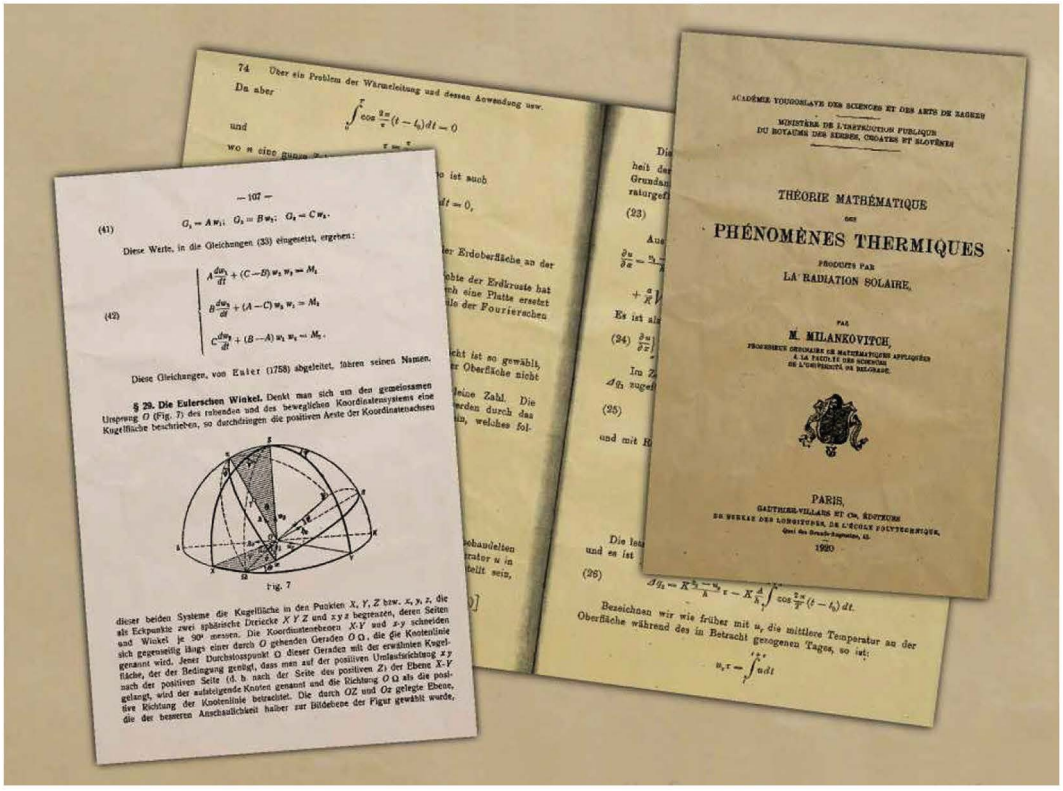
Portrait of Milutin Milanković (by Paja Jovanović)

3. Scientific papers of Milutin Milanković

Milanković was a universal scientist with wide interest in many areas of natural sciences. Besides that, he was a very educated mathematician. His understanding and ability to use mathematics in advanced areas of sciences, particularly in theoretical mechanics, exceeded in great extent an average knowledge of a civil engineer where his basic education laid. All these aspects of his scientific abilities are reflected in his scientific opus very well, not only in scientific papers but in his books and monographs as well.

We should mention a few other general characteristics of his scientific work. There is a number of recognized scientists and engineers who originated from Serbia. But their main work was done abroad, out of Serbia. Milanković was born in Dalj, then a village in Austro-Hungarian Empire (now in Croatia). His complete education was in Austro-Hungary, first in a secondary school in nearby Osijek then in Vienna where he moved to study civil engineering. He got his PhD there and started to work as civil engineer. However, his main scientific work and achievements were completely accomplished in Belgrade. Secondly, most of his scientific treatises were written in Germany, many of them in Serbian and just a few in French. A possible explanation why so many papers Milanković wrote in Serbian could be that he published them in the publications of the Serbian Academy of Sciences. Then the Academy had the rule that papers published there should be in Serbian. However, papers written in Serbian and even in French were almost always accompanied with long abstracts in German. Sometimes the abstracts were longer than the original papers! Finally, we should mention that

he was very talented as a writer. His writing gift particularly can be seen in his popular books on science. Here we present in short his scientific work and papers grouped into suitable subjects.



Samples of digitized publications

Astronomy. Milanković was not the first Serbian astronomer. However he was certainly the most recognized and most important Serbian astronomer. His work is enormous and so important that it is not possible in a short article as this one to present even in a part all his scientific achievements. His astronomical works were mostly concerned with perturbation of planet orbits, planets rotations and planets climates. His main stand point of view was that the planets climates are strongly influenced by these astronomical phenomena. So Milanković's papers in astronomy are closely related to his work in geosciences and climatology.

Calendars. Milanković's main contribution in this field was his proposal and an improvement of the Gregorian time reckoning. It was accepted by the Orthodox Christian Community during a Congress of Orthodox Churches in Constantinople in 1923. Milanković's calendar-reform was accepted by the Orthodox Christian Community, but not by all Churches, including the Serbian one.

Mechanics. Milanović's papers in mechanics are mainly in the area of dynamics and he was particularly interested in *n*-body problem. Milanković wrote these papers in his early period of scientific work.

Geosciences - Climatology. Milutin Milanković is best known for his theory of glacial ages. The theory takes into account the complex secular computation of perturbations in planet motions and it were published in *Théorie mathématique des phénomènes thermiques*

produits par la radiation solaire in 1922. Due to these results that were also published in scientific papers, he became well known in the world scientific community, so that great German climatologist W. Köppen invited him to cooperate in building a great work *Handbuch der Klimatologie*. For this purpose a part, which was published in 1930, entitled *Mathematische Klimalehre und Astronomische Theorie der Klimaschwankungen* was written by Milanković. Here his theory of planet heating based on insolation was extended with a special reference to the Earth. By applying this theory to the run of glacial ages it was shown that Milanković had created a good and mathematically exact theory of terrestrial climate. Milanković also created a theory of motion of terrestrial poles and succession of glacial ages. Results published in the scientific papers and mentioned works are sithetized into his main work *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* printed shortly before the beginning of the Second World War. Due to this, the scientific world community learnt about Milanković's theory with some delay. As a consequence the world recognition arrived with postponement. This work appears as a synthesis of his earlier works which concern his research activity within boundary fields between many natural sciences and mathematics.

Theory of relativity and cosmology. Milanković did two short excursions to these areas. In fact these papers were on special relativity and both are on Michelson experiment (now known as Michelson-Morley experiment) which gave the strong evidence against ether theory. He discussed there, in the light of the Michelson experiment, the validity of the second postulate of Special theory of relativity, that the speed of light is the same in all reference frames. T. Anđelić and A. Stojković wrote an extensive paper on Milanković's views expressed in theses papers.

History of Science. Milanković wrote several books on history and popularization of science and autobiography books, for example *Through Space and Centuries*, *Through the Realm of Science* and *Recollection, Experiences and Vision*. But he also wrote several papers on topics ranging from biographical to the discussion of some ancient problems related to geometry and mechanics. These papers often touched philosophical themes but also history of mechanics and astronomy.

Building and construction (civil engineering). Milanković wrote these papers in the beginning of his scientific work, in the first decade of XX century. Most of these papers were concerned with the theory and calculation of statics of girders and membranes used in construction. Mathematical apparatus used in some of these papers was rather advanced.

4. Conclusion

Milutin Milanković published about fifty scientific papers, most of them related to astronomy, geosciences and theoretical mechanics. All these papers are digitized and they are now a part of his digital legacy. We believe that the material presented in the legacy will serve to better understand Milanković's life and scientific work.

5. Acknowledgment

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Supplement: Scientific papers of Milutin Milanković in Virtual Library and his Digital Legacy - thematic division (with active links in electronic version of this paper)
Papers denoted by * can be found in Digital Legacy, http://legati.matf.bg.ac.rs/milankovic.
Papers denoted by † are published in *Publications Mathémat. Univers. de Belgrade* and can be found in the repository of this journal at the address http://elib.mi.sanu.ac.rs.

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76 Rukopisi prvih univerzitetskih predavanja Milutina Milankovića

N. Pejović, Ž. Mijajlović, S. Malkov
and N. Mitić,
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Zb. Konf. "Razvoj astronomije kod Srba VIII
Beograd, 22-26 april 2014",
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Scientific and professional heritage of Milutin Milanković is very rich and most of it is contained his Digital legacy, a portal of the Faculty of Mathematics in Belgrade. This article presents five manuscripts of his lectures at the University of Belgrade at the beginning of the XX century. Manuscripts belong to the library of the Mathematical Institute of Serbian Academy of Sciences and Arts and are part of a wider collection of notes from lectures, which were held in late nineteenth and early twentieth century, at the University of Belgrade. These lectures are shortly presented in the article "Twenty four manuscripts in the Virtual Library of the Faculty of Mathematics in Belgrade".

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РУКОПИСИ ПРВИХ УНИВЕРЗИТЕТСКИХ ПРЕДАВАЊА МИЛУТИНА МИЛАНКОВИЋА

НАДЕЖДА ПЕЈОВИЋ, ЖАРКО МИЈАЈЛОВИЋ,
САША МАЛКОВ и НЕНАД МИТИЋ

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Резиме: У богатој научној и стручној заоставштини Милутина Миланковића налазе се пет рукописа његових предавања на Београдском универзитету. Рукописи припадају библиотеци Математичког института САНУ и део су шире колекције белешки са предавања која су држана крајем XIX и почетком XX века на Београдском универзитету. У овом раду представљамо рукописе Миланковићевих предавања.

Кључне речи: Милутин Миланковић, дигитализација, историја астрономије, базе података

1. УВОД

Током 2012. започели смо пројекат за изградњу дигиталне архиве и дигиталних легата - Интернет презентација значајних српских научника који су радили и стварали у области математичких наука. Наша идеја је да се поред очувања српског научног и културног наслеђа, интернет оријентисаним базама представе на једном месту општој и стручној публици научна дела, али и други документи везани за живот и дело ових стваралаца. Најпотпунија колекција у оквиру ове дигиталне архиве је дигитални легат Милутина Миланковића (1879-1958), великог српског астронома, математичара и грађевинског инжењера. Базе се налазе на адресама: http://digitalnilegati.matf.bg.ac.rs и http://legati.matf.bg.ac.rs/milankovic.

Обе базе су новијег датума, формиране су отприлике пре две године. Њима је претходила Виртуелна библиотека Математичког факултета у Београду која је постављена пре више од једне деценије. Виртуелна

бибиотека, на адреси http://elibrary.matf.bg.ac.rs, је највећа база дигитализованих текстова у Србији са отвореним приступом. У тренутку писања овог текста, Виртуелна библиотека садржи преко 3500 дигитализованих књига и неколико важних колекција, укључујући све докторске дисертације из математичких наука које су одбрањене на Математичком факултету и ретке књиге из 18. и 19. века. Једну од колекција чине 24 рукописа универзитетских предавања која датирају с краја XIX и почетка XX века. Рукописи су заправо руком писане белешке са предавања која су држали најпознатији српски математичари и професори тог времена на Београдском универзитету: Димитрије Данић, Коста Стојановић, Михаило Петровић и Милутин Миланковић. Дигиталне копије свих рукописа постављене су у Виртуелну библиотеку. Шири текст о овој колекцији и о томе како су рукописи пронађени у библиотеци Математичког института написали су аутори овог рада, видети Мијајловић и други (2013). Рукописи Миланковићевих предавања о којима овде пишемо такође се налазе у његовом дигиталном легату и дигиталној архиви.

2. МЕХАНИКА У СРБИЈИ ПРЕ ДОЛАСКА МИЛУТИНА МИЛАНКОВИЋА

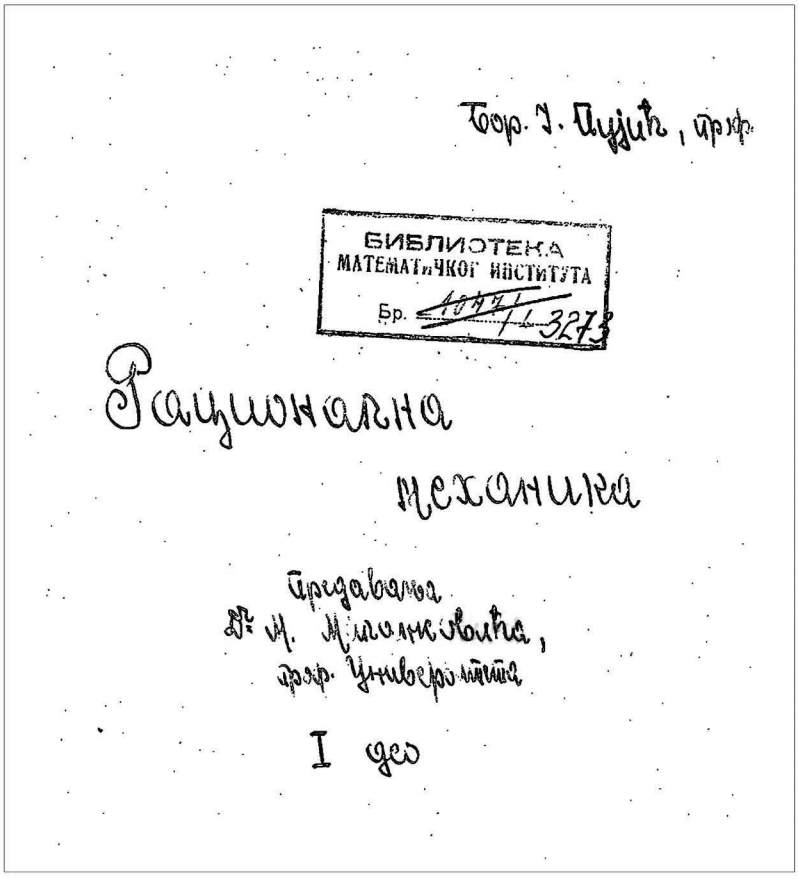
Миланковић је био универзалан научник широког интересовања. Ипак, механика је имала посебно место у његовом научном раду, можда не толико у теоријским основама колико у применама и објашњењима астрономских и планетарних појава. Ово његово интересовање за механику рефлектовало се у бројним универзитетским курсевима које је држао из те области, али и у великом броју његових књига и научних радова који су се дотицали механике. Његовим доласком у Србију, студије механике на Београдском универзитету приближиле су се оним у главним центрима у Европи. Београдски професори механике пре њега фактички нису објављивали научне радове и често су били разапети између науке и неких других активности, политике на пример. Инструктивно је видети статус механике на Београдском универзитету пре Миланковићевог доласка у Београд.

Теоријска механика се изучавала у Србији крајем XIX и почетком XX века једино у Београду, најпре на Великој школи, а затим на Београдском универзитету од оснивања 1905. Први наставник теоријске механике на Великој школи, изабран за професора 1875, био је Љубомир Клерић (1844-1910). Клерић је по образовању био рударски инжењер, па је природно био више наклоњен примењеној механици. Бавио се у оно време популарним математичким машинама и познат је његов изум *тракториографа* помоћу којег се механичким путем могу конструисати трансцедентни бројеви e и π. Споменимо да су се овим аналогним рачунским машинама бавили и други наши математичари тог времена или нешто касније, Михаило Петровић (1868-1943) и Сима Марковић (1888-1939) (видети Трифуновић, 1977, Мијајловић, 2013). Клерић је такође написао уџбеник *Теоријска механика за*

ученике Велике школе у три свеске које су се редом појављивале 1880, 1883 и 1888. Имао је важне функције у Србији тог времена, између осталог био је министар просвете и министар народне привреде. Такође је био члан Српске академије наука.

Професор рационалне механике на Великој школи у периоду 1889-1905 био је Мијалко Ћирић. Мада је скоро две деценије био професор Велике школе и ученик чувеног француског математичара Charles Hermite-a код кога је учио и Михаило Петровић (извор: *Oxford handbook*), Ћирић није оставио запажен траг у науци нити на Великој школи, осим што је написао збирку задатака из диференцијалног рачуна. На тек основаном Београдском универзитету, од 1905 до 1908 на новој катедри за примењену математику механику предаје Коста Стојановић (1867-1921). Поред механике, Коста Стојановић бавио се и математичком економијом. Објавио је једну од најзначајнијих књига у српској стручној литератури из математичке економије. Био је министар народне привреде. Више детаља из историје примењене математике, посебно из астрономије и механике читалац може наћи у Пејовић и Мијајловић (2011).

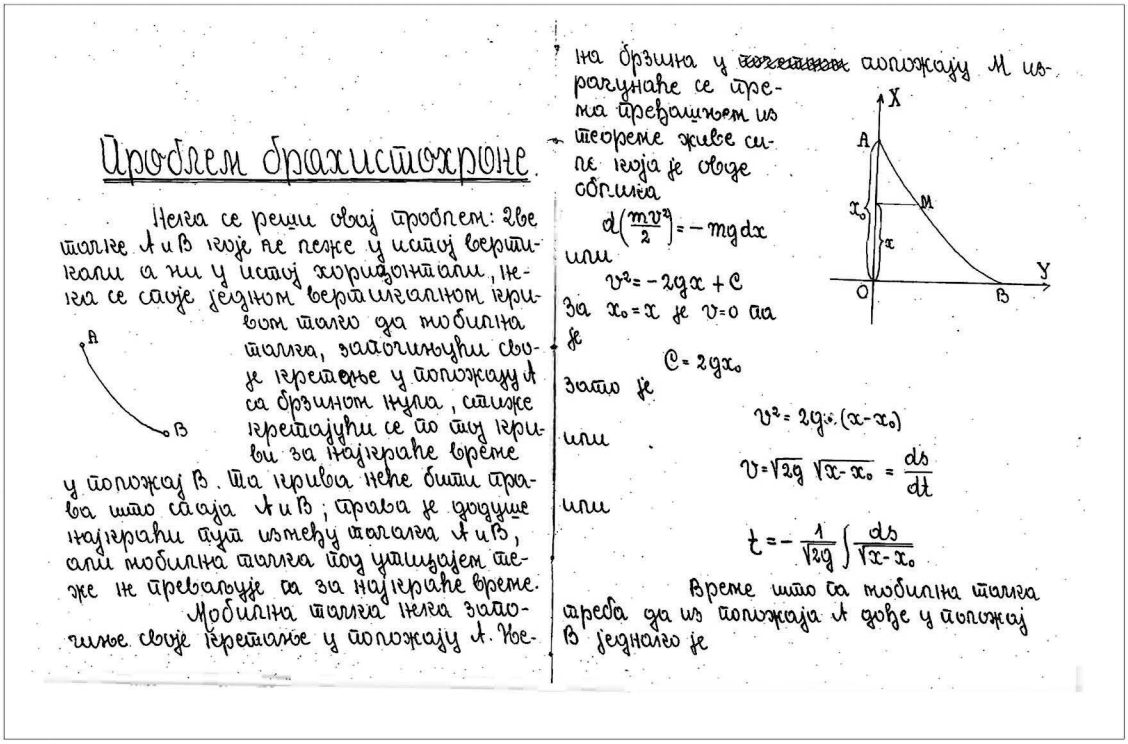
3. МИЛАНКОВИЋЕВИ РУКОПИСИ



Насловна страница рукописа *Рационална механика* I.

Милутин Миланковић долази на Београдски универзитет 1909. за професора примењене математике на Филозофском факултету по позиву Михаила Петровића и Богдана Гавриловића. Већ његово приступно предавање *Поглед на развитак механике и њен положај према осталим наукама* наговештава његово занимање у следећим деценијама, практично до краја живота. То ће бити механика и њене примене у астрономији где је Миланковић имао достигнућа од највећег светског ранга.

Курс примењене математике који је Миланковић одмах почео да предаје на Београдском универзитету био је веома обиман. Трајао је шест семестра, а недељни фонд износио је шест часова. Списак области на које се курс односио указује нам на велику научну ширину и велико знање Милутина Миланковића. У оквиру курса, Миланковић је предавао векторску анализу,



Проблем брахистохроне описан у рукопису *Рационална механика* I.

рационалну механику, хидромеханику, општу теорију поља, термодинамику, науку о електрицитету и магнетизму и небеску механику. Из ових предмета касније су се на Београдском универзитету развиле посебне дисциплине, односно студијске групе: рационална механика, небеска механика и теоријска физика. Миланковић је наставио да предаје небеску механику између два светска рата и 1935. године објавио је универзитетски уџбеник из ове области. Друге две области углавном су преузели руски научници који су се доселили у Србију после Октобарске револуције. Тако, Антон Билимовић

је предавао рационалну механику, док је Вјачеслав Жардецки предавао теоријску физику.

Миланковићева универзитетска предавања су записана у пет рукописа и сачувана су у библиотеци Математичког института САНУ. Рукописи су написани до Првог светског рата, тј. у периоду 1910-1914. На првој страни сваког рукописа у горњем десном углу пише "Бор. Ј. Пујић, проф". Поређењем са белешкама других предавања, можемо закључити да је белешке свих Миланковићевих предавања написао студент математике Боривој Ј. Пујић. О Пујићу не знамо много, нити има некох трагова о њему у документима који су нама били доступни, осим у архиви Математичког факултета. Тамо је Пујић као студент евидентиран под бројем 3236/1914, што значи да је започео студије пред Први светски рат и да је по свој прилици касније био гимназијски професор математике. Рукописи су заведени у библиотеци Математичког института САНУ шесдесетих година прошлог века и можемо претпоставити да је рукописе Институту поклонио сам Пујић. Рукописи представљају такође добро сведочанство за терминологију која се користила у то време у механици.

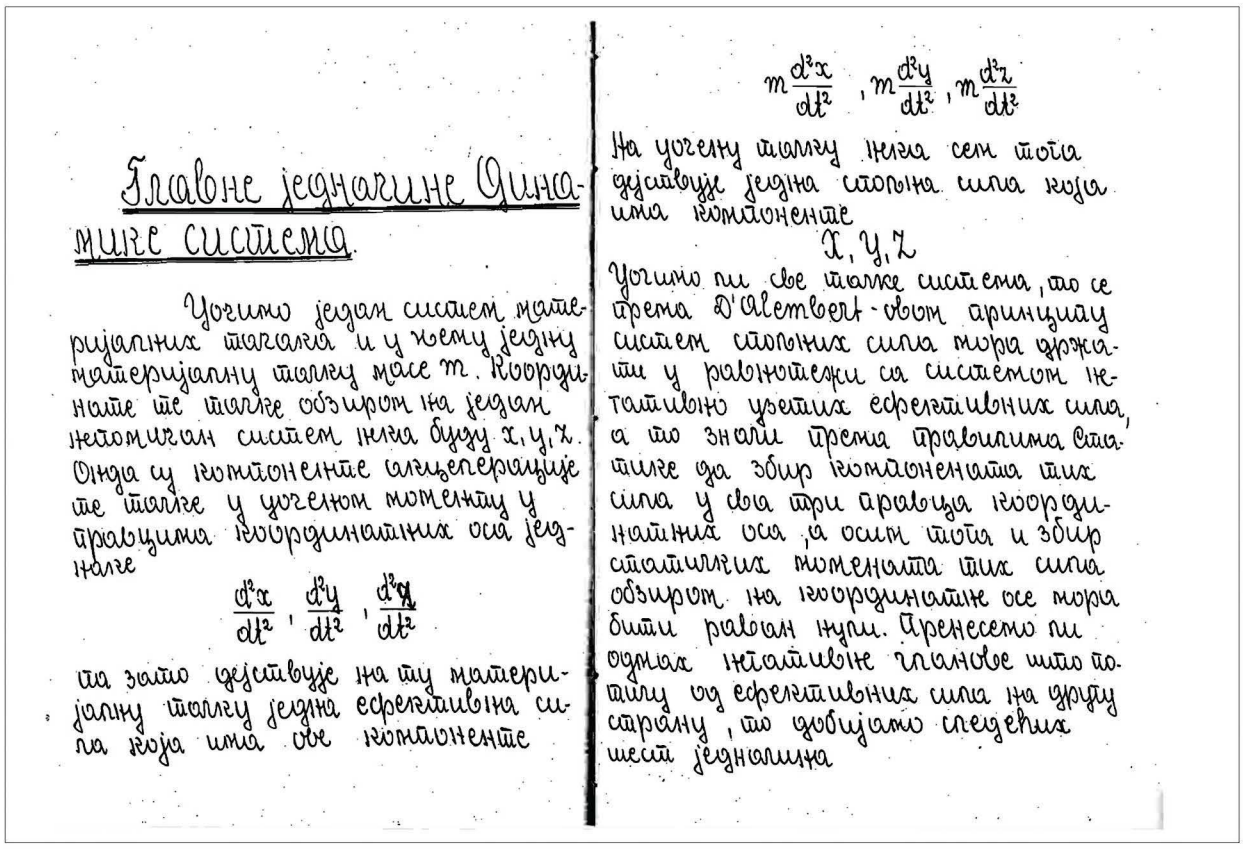
Мислимо да су рукописи и околности које су пратиле њихов настанак слабо познати не само широј јавности већ и стручним круговима. Зато смо се одлучили да у овој прилици рукописе представимо нешто детаљније.

4. РУКОПИС ИЗ РАЦИОНАЛНЕ МЕХАНИКЕ

Овај рукопис састоји се из два дела. У првом делу, *Рационалној механици* I, представљене су кинематика, статика и динамика материјалне тачке. У првом поглављу изложени су елементи векторског рачуна који ће потом у излагању механике послужити као математичка основа. Излагање је аксиоматске природе, тако да се вероватно први пут у српској математичкој литератури спомињу алгебарски термини као што су асоцијативни, комутативни и дистрибутивни закони. Колико нам је познато, ове појмове срешћемо поново у српској математичкој литератури тек код Јована Карамате у његовој књизи *Комплексан број* која је објављена тридесет година касније, одмах после Другог светског рата. Полазећи од поменутих основа, у оквиру поглавља *Кинематика* Миланковић уводи појмове вектора брзине и вектора акцелерације (убрзања). Треба споменути да је векторска анализа у то време и у свету представљала напредан математички апарат у излагању механике. Миланковић је заправо увео овај метод у српску научну и уџбеничку литературу. У поглављу *Статика* Миланковић излаже услове за равнотежу материјалне тачке везану за површ или криву. Теоријска излагања допуњена су примерима и цртежима. Најобимније је треће поглавље, *Динамика*. Доследно се држећи векторског приступа, на почетку се уводи вектор *квантитативног* кретања (количине кретања). Не улазећи у детаље, споменимо да се овде такође дају једначине кретања косог хица, као и

кретање материјалне тачке на ротационој површини. Овај рукопис има 380 страница текста писаног руком.

У другом делу, *Рационална механика* II, излажу се следеће области: Статика материјалне линије и крутих тела и Динамика крутог тела. Под материјалном линијом, аутор подразумева систем материјалних тачака поређаних на некој кривој од којих је свака у интеракцији са суседним тачкама. Дејство суседних тачака Миланковић назива *интерним* (унутрашњим) силама. У поглављу *Статика крутог тела* за силе отпора ослонца наводи пример мостова, што је сасвим разумљиво у светлу чињенице да је Миланковић у Бечу завршио студије за грађевинског инжињера и тамо одбранио докторат из ове области. По доласку у Србији пројектовао је неколико десетина грађевинских објеката, па и неколико мостова. У поглављу *Динамика крутог тела* између осталог Миланковић пише о моменту инерције, моментима девијације, елипсоиду инерције, транслаторном кретању, ротационом кретању крутог тела. Такође изводи главне једначине динамике система и даје мноштво примера сложеног кретања крутог тела. Овај рукопис има 376 страница текста писаног руком.



Главне једначине Динамике система.

Уочимо један систем материјалних тачака и у њему једну материјалну тачку масе m. Координате те тачке обзиром на један непомичан систем нека буду x, y, z. Онда су компоненте акцелерације те тачке у уоченом моменту у правцима координатних оса једнаке

$$\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}$$

па зато дејствује на ту материјалну тачку једна ефективна сила која има ове компоненте

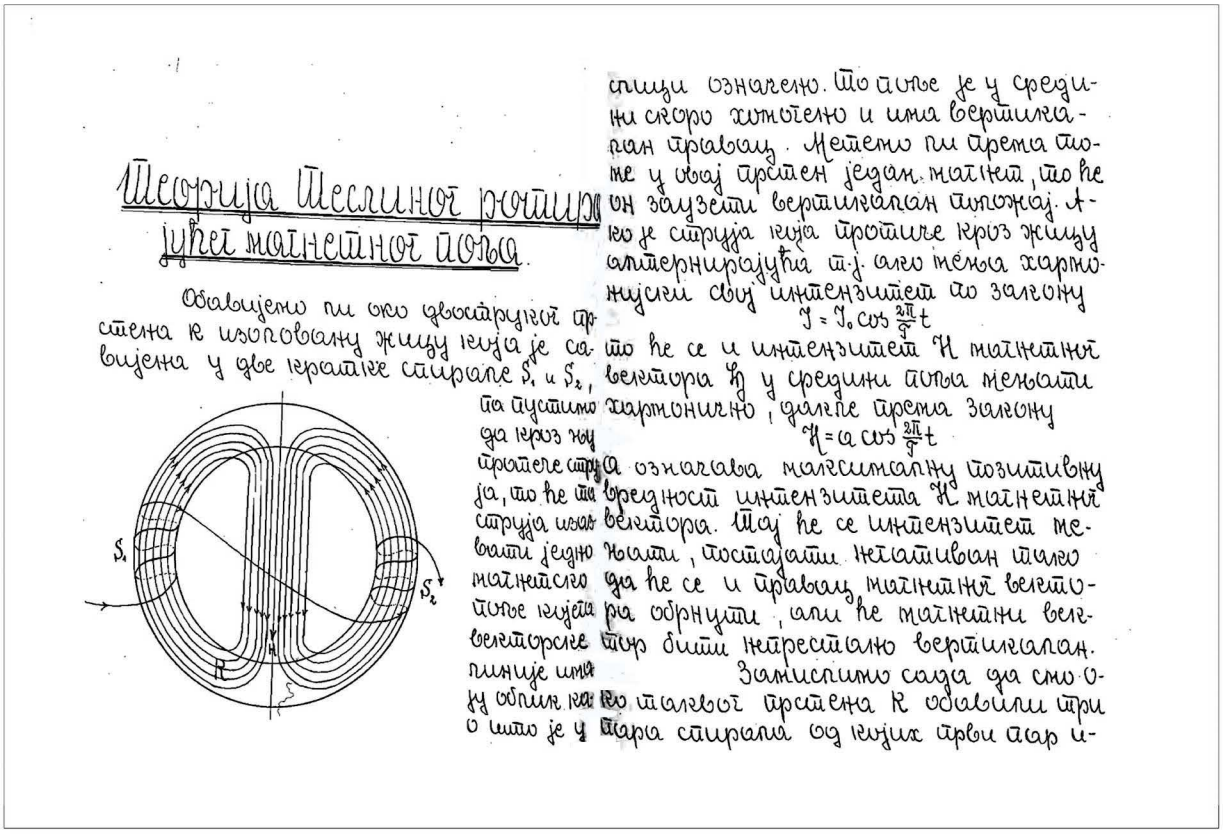
$$m\frac{d^2x}{dt^2}, m\frac{d^2y}{dt^2}, m\frac{d^2z}{dt^2}$$

На уочену тачку нека сем тога дејствује једна спољна сила која има компоненте

$$X, Y, Z$$

Уочимо ли све тачке система, то се према D'Alembert-овом принципу систем спољних сила мора држати у равнотежи са системом негативно узетих ефективних сила, а то значи према правилима Статике да збир компонената тих сила у сва три правца координатних оса, а осим тога и збир статичких момената тих сила обзиром на координатне осе мора бити раван нули. Пренесемо ли одмах негативне чланове што потичу од ефективних сила на другу страну, то добијамо следећих шест једначина

Рукопис *Рационална механика* II: Једначине динамичких система.



Теорија Теслиног ротирајућег магнетног поља.

Обавијемо ли око двоструког стакла K изоловану жицу која је савијена у две гвратиће спирале S_1 и S_2, па пустимо да кроз њу протиче струја, то ће струја изазвати једно магнетско поље које има вектореске линије облик као о што је у

спици означено. То поље је у средини скоро хомогено и има вертикалан правац. Метнемо ли према томе у овај простор један магнет, то ће он заузети вертикалан положај. Ако је струја која протиче кроз жицу алтернирајућа т.ј. ако мења хармонијски свој интензитет по закону

$$J = J_0 \cos \frac{2\pi}{T} t$$

то ће се и интензитет H магнетног вектора $\mathfrak{H}$ у средини поља мењати хармонично, дакле према закону

$$H = a \cos \frac{2\pi}{T} t$$

a означава максималну позитивну вредност интензитета H магнетног вектора. Тај ће се интензитет мењати, постајати негативан тако да ће се и правац магнетног вектора обрнути, али ће магнетни вектор бити непрестано вертикалан.

Замислимо сада да смо о- ко малога прстена K обавили три пара спирала од којих први пар и-

Рукопис *Модерне теорије електрицитета и магнетизма*: Објашњење Теслиног ротирајућег магнетног поља.

5. РУКОПИС ИЗ МОДЕРНЕ ТЕОРИЈЕ ЕЛЕКТРИЦИТЕТА И МАГНЕТИЗМА

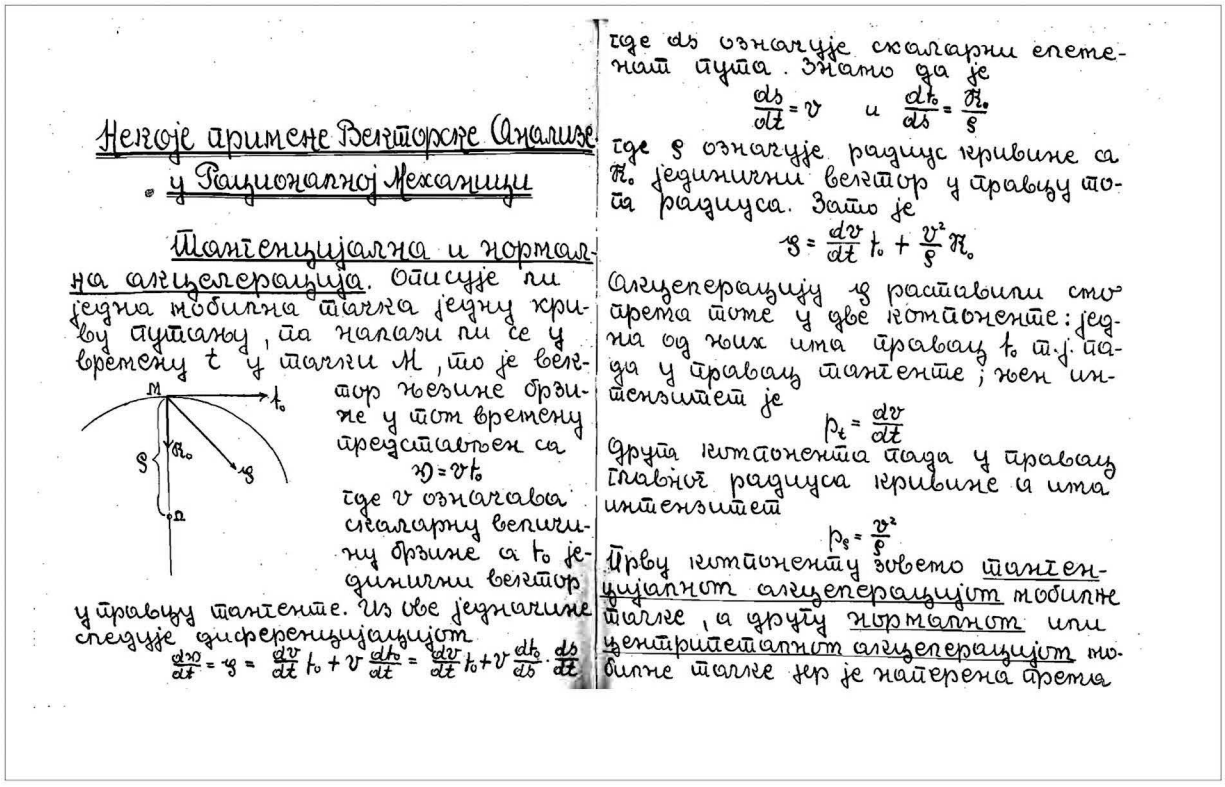
Читањем овог рукописа можемо се уверити да су предавања из овог предмета била на високом нивоу, равна оним у најбољим универзитетским центрима у свету почетком двадесетог века. У њима су била укључена најновија сазнања и открића из науке о електрицитету и магнетизму. Део рукописа посвећен је проналасцима и изумима нашег великана Николе Тесле. Тако, представљена је теорија Теслиних наизменичних струја, затим Теслино ротирајуће магнетно поље и објашњење рада Теслиног мотора. Од других важних тема, представљена је Максвелова електромагнетна теорија светлости и теорија електрона. У излагању Миланковић користи високи математички апарат, између осталог елементе диференцијалне геометрије, односно векторског диференцијалног рачуна (операторе rot, div и grad). Захваљујући примени овог математичког апарата, извођења и једначине које описују ове физичке феномене имају компактан и концизан облик.

Рукопис садржи шест поглавља са следећим називима: *Електростатика*, *Теорија магнетизма*, *Електрична струја и њено магнетно поље*, *Индукција*,

О електричним осцилацијама и таласима и *Теорија електрона*. Рукопис има 366 страница руком исписаног текста.

6. РУКОПИС ВЕКТОРСКА АНАЛИЗА СА ПРИМЕНАМА ИЗ ТЕОРИСКЕ ФИЗИКЕ

По начину излагања и високом степену ригорозности овај рукопис је математичког карактера и показује да је Миланковић такође био одличан математичар. У првом делу рукописа, отприлике у првих сто страна, излаже се елементарна теорија геометријских вектора. Данас би тај садржај описали као теорију тродимензионог еуклидског простора. Поред основног рачуна са векторима и скаларима, изведене су особине скаларног, векторског и мешовитог производа вектора. У остатку рукописа излаже се векторска анализа са применама у механици и у том делу доста се преклапа са рукопиом *Рационална механика* I. Изучавају се особине векторских функција где се независна реална променљива интерпретира као време и развија се диференцијални рачун над њима. Истовремено се даје природна физичка интерпретација за уведене математичке појмове. Тако се уз изводе векторских функција одмах уводе вектори брзине и убрзања материјалне



Некоје примене Векторске Анализе у Рационалној Механици

Тангенцијална и нормална акцелерација. Описује ли једна мобилна тачка једну криву путању, па налази ли се у времену t у тачки M, то је вектор њезине брзине у том времену представљен са

$$\mathfrak{v} = v t_0$$

где v означава скаларну величину брзине а t_0 јединични вектор у правцу тангенте. Из ове једначине следује диференцијацијом

$$\frac{d\mathfrak{v}}{dt} = \mathfrak{g} = \frac{dv}{dt} t_0 + v \frac{dt_0}{dt} = \frac{dv}{dt} t_0 + v \frac{dt_0}{ds} \cdot \frac{ds}{dt}$$

где ds означује скаларни елеменат пута. Знамо да је

$$\frac{ds}{dt} = v \quad \text{и} \quad \frac{dt_0}{ds} = \frac{\mathfrak{R}_0}{\rho}$$

где ρ означује радиус кривине а $\mathfrak{R}_0$ јединични вектор у правцу тога радиуса. Зато је

$$\mathfrak{g} = \frac{dv}{dt} t_0 + \frac{v^2}{\rho} \mathfrak{R}_0$$

Акцелерацију $\mathfrak{g}$ раставили смо према томе у две компоненте: једна од њих има правац t_0 т.ј. пада у правац тангенте; њен интензитет је

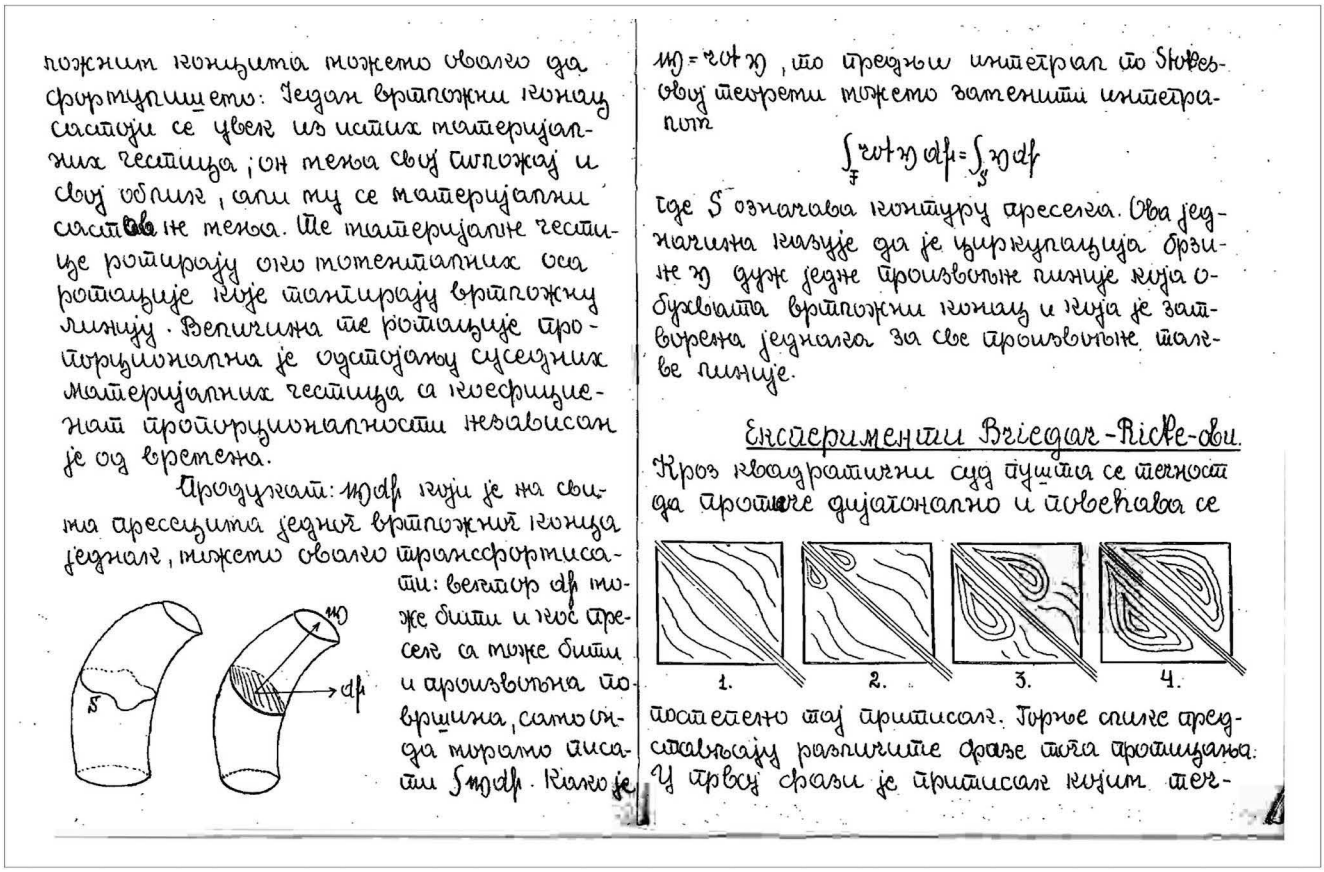
$$p_t = \frac{dv}{dt}$$

Друга компонента пада у правац главног радиуса кривине а има интензитет

$$p_\rho = \frac{v^2}{\rho}$$

Прву компоненту зовемо тангенцијалном акцелерацијом мобилне тачке, а другу нормалном или центрипеталном акцелерацијом мобилне тачке јер је наперена према

Рукопис *Векторска анализа са применама из теориске физике*: Примене векторске анализе у рационалној механици.



Рукопис *Општа теорија физикалних поља*: Опис вртложног кретања течности.

тачке, а потом и силе. Поред диференцирања по времену Миланковић уводи и диференцирање по луку, што му омогућава да уведе основне појмове диференцијалне геометрије као што је кривина и полупречник кривине код просторне кривуље. Све то на крају примењује на кретање крутог тела. Нарочито је занимљиво поглавље које се бави векторским и скаларним пољима. У том контексту уводи Хамилтонов набла оператор и на сасвим модеран начин заснива фундаменталне операторе векторске анализе, rot, div и grad. Поред особина ових оператора такође изводи основне теореме векторске анализе као што су Гаусова теорема (теорема о дивергенцији, позната и као теорема Остроградског) и Стоксова теорема. Рукопис има 254 страница текста писаног руком.

7. РУКОПИС ОПШТА ТЕОРИЈА ФИЗИКАЛНИХ ПОЉА

Овај рукопис су белешке са курса завидног нивоа, теоријске (математичке) физике. Прва половина рукописа посвећена је математичким основама које и у овом случају леже на апарату векторске анализе.

Миланковић најпре дефинише скаларна и векторска поља као физичке величине које у свакој тачки простора имају неку вредност. Та вредност је скаларног, односно векторског типа, већ у зависности од врсте поља. Полазећи од могућих вредности векторских оператора rot, div и grad, поља се класификују на безвртложна, потенцијална, ламерална, безизворна, соленоидна и Лапласова. Свако од ових поља изучава се у оквиру посебног одељка. Миланковић тамо изводи Гаусову теорему о потенцијалу, мада не спомиње њен назив. У другом делу рукописа примењује уведени математички апарат у објашњењу физичких феномена у термодинамици, хидродинамици и динамици система материјалних тачака. Тако, у оквиру термодинамике изводи једначине провођења топлоте кроз штап, жицу и плочу. У делу из хидродинамике изводи Ојлерове и Лангражове једначине хидродинамике. У делу из динамике система материјалних тачака изводи, на пример, Бернулијеву теорему о стационарним стањима коју потом користи у извођењу Торичелијеве теореме која описује истицање течности из резервоара. Можемо приметити да курс записан у овим белешкама претпоставља добро познавање апарата векторске анализе. Овај рукопис има 298 страница текста исписаног руком.

8. ЗАКЉУЧАК

Белешке са Миланковићевих предавања представљају значајно сведочанство о статусу механике и теоријске физике на Београдском универзитету пред Први светски рат. Прочитавањем ових рукописа сазнајемо да је Миланковић поставио темеље модерних универзитетских предавања у Србији у овим наукама, затим да је у излагању ових дисциплина користио онда врло напредан математички апарат који је највећим делом био смештен у оквире векторске анализе и диференцијалне геометрије. Ови списи такође сведоче о Миланковићевој научној универзалности, али и да је био одличан математичар. Имао је тачно 30 година када се преселио у Београд 1909, стога ови рукописи могу имати значајно место у изучавању Миланковићевог научног стваралаштва, нарочито у његовим раним тридесетим годинама.

Свих пет рукописа Миланковићевих предавања налазе се на адреси:
http://legati.matf.bg.ac.rs/milankovic/books.wafl?c=predavanja

Овај чланак је настао као резултат рада на Пројекту III 44006 који финансира Министарство просвете, науке и технолошког развоја Републике Србије.

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MANUSCRIPTS OF THE FIRST UNIVERSITY LECTURES OF MILUTIN MILANKOVIĆ

In the rich scientific and professional legacy of Milutin Milankovića there are five manuscripts of his lectures at the University of Belgrade. Manuscripts belong to the library of the Mathematical Institute of Serbian Academy of Sciences and Arts and are part of a wider collection of notes from lectures, which were held in late nineteenth and early twentieth century, on the University of Belgrade. In this paper we present the manuscripts of Milanković lectures.

Key words: Milutin Milanković, digitization, History of Astronomy, Databases

77 Serbian and international honors awarded to Milutin Milanković

N. Pejović, S. Malkov, N. Mitićand
Ž. Mijajlović
NCD Review, vol. 28, pp. 79-86, 2016.

This paper presents honors and awards granted to Milutin Milanković, one of the greatest Serbian scientists. Awards are classified in accordance with the areas in which Milanković had achievements. Most of this honors are also presented in his Digital Legacy, a portal of the Faculty of Mathematics in Belgrade. There are few eternal honors granted to Milanković. A crater on Moon and a crater on Mars are named after Milanković. Also, a small planet carries his name.

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SERBIAN AND INTERNATIONAL HONORS AWARDED TO MILUTIN MILANKOVIĆ

Abstract. In this paper we present honors and awards granted to Milutin Milanković, one of the greatest Serbian scientists. Awards are classified in accordance with the areas in which Milanković had achievements. Most of this honors are also presented in his Digital Legacy, an Internet site of the Faculty of Mathematics in Belgrade.

Introduction

This paper belongs to the series of articles related to the life and scientific work of Milutin Milanković and his Digital Legacy, http://legati.matf.bg.ac.rs/milankovic. A group of participants of the Digitization project of the Faculty of Mathematics in Belgrade and the Mathematical Institute of the Serbian Academy of Sciences and Arts started in 2012 to build the Digital Legacy, an Internet site of the Faculty of Mathematics. The general idea of the Legacy was to present biographies of the prominent Serbian scientists from the past who worked in mathematical sciences. We also wanted to have at one place all their scientific and technical works and personal data such us photos, letters and other related documents in order to better understand their work and life. We designed this Internet site having in mind the general audience, but the scientific community, too. Up to now, biographies of three scientists, academicians and the former professors of the Faculty of mathematics are offered there: Anton Bilimovič, Bogdan Gavrilović and Milutin Milanković. In this paper we present international and Serbian honors granted to Milutin Milanković, one of the greatest Serbian scientists. Most of these awards are documented and presented in his Digital Legacy.



Short biography

Milutin Milankovic was born on 28 May 1879 in Dalj (Slavonia, then part of Austro-Hungarian Empire, now part of the Croatian Republic). He finished high school in Osijek, so called "Gimnasia realka" and then he started studies in engineering construction at the Hochschule in Vienna. There he graduated in 1902 and defended his doctorate in 1904. Milanković was the first Serbian engineer with the title of Doctor of Science.

Belgrade University was founded in 1905. Soon, in 1909, Milanković was invited to come in Belgrade for professor of applied mathematics at the newly founded

university. This year started his scientific and university career in Belgrade. For his coming were responsible Bogdan Gavrilović and Mihailo Petrović who were the only university professors of mathematics in Belgrade before Milanković arrival.

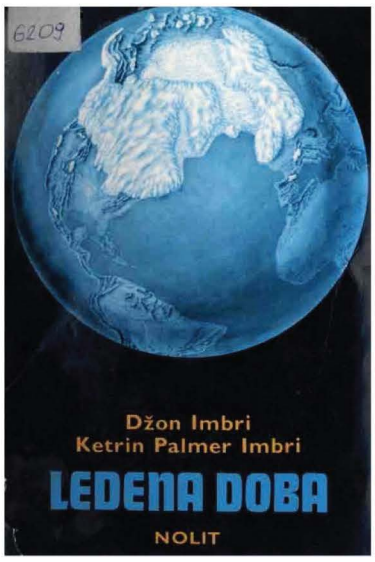
Milankovic remained at the university until his retirement in 1955. There he taught applied mathematics (mechanics, astronomy and theoretical physics). He was the first University of Belgrade professor who taught celestial mechanics. He also taught the history of astronomy and the Special Theory of Relativity. Milanković in 1924 becomes a full member of the Serbian Academy of Sciences and Arts. He was also a member of the "Deutsche Akademie der Naturforscher Leopoldina" in Halle and the corresponding member of other academies in the world. Professor Milankovic died on 12 December 1958 in Belgrade.

Milanković has written twenty books and fifty scientific papers on climatology, mathematics, celestial mechanics, astronomy and geophysics. Belgrade's publishing house "Zavod za udžbenike i nastavna sredstva" released in 1997. *Selected works of Milutin Milanković* in seven books. The most important and the most famous book from this collection is *Canon of Insolation and the ice-age problem* (Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem). This was the first edition of this book in Serbian.

Achievements and honors

The main Milanković's contribution to science is his famous theory of ice ages. This theory was developed starting from the complex calculation of secular perturbations in the movement of the planets. The first variant of his theory is published in the book ***Théorie mathématique des phénomènes thermiques produits par la radiation solaire*** (1922, publishers: *Yugoslav Academy of Science and Arts*, Zagreb and the publishing house *Gauthier Villars*, Paris). Thanks to this work he became known in the global scientific community, and soon receives a call for the cooperation with the famous German climatologist W. Köppen-a on the preparation of the capital work, the Handbuch der Klimatologie (Handbook of Climatology, printed in 1930).

Milanković wrote the part "Mathematische Klimalehre und Astronomische Theorie der Klimaschwankungen" (*Mathematical doctrine of climate and astronomical theory of climate fluctuations*) of this book. He develops there the theory of the solar heating and the planet insolation, with special reference to the Earth. Applying this theory, Milanković gives a very good model of evolution of ice ages on the Earth and the precise mathematical theory of terrestrial climate. This work has been translated into Russian in 1939. Now as a recognized scientist, he started cooperation on the part of "Handbuch der Geophysik" (Handbook of Geophysicsl) also a capital work prepared by B. Gutenberg. Milanković's wrote there four chapters where he studies the old and difficult problem, the movement of the Earth's poles. Here Milanković develops the theory of displacement of the Earth's poles and alternation of ice ages.



Cover of ***Ice ages: solving the mystery*** (Serbian translation)

As already mentioned, his main work is "Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem". Milanković finished the book just before the start of the Second World War. Already

printed copies of the book were destroyed in the Nazi bombing of Belgrade on 6 April 1941. However, typographic plates of the book were saved, and the book was quickly reprinted and finally published. This book is the synthesis of all Milankovic's previous work and research that were located on the border of several natural sciences: astronomy, celestial mechanics, geophysics and mathematics. The book appears at the worst time, during the war. At this time Milanković almost had no communication with colleagues and scientists in the world. Hence, the world scientific community was quite late familiar with the Milankovitch theory. John Imbrie and Katherine Palmer Imbrie wrote the book *Ice ages: solving the mystery*. There they explained in a popular way the mystery of ice ages, what they were like, why they occurred and when the next one will occur. A part of the book is devoted to Milutin Milanković and his astronomic theory and proofs that the earth's irregular orbital motions lead to the peculiar climatic changes which bring on ice ages. The book is translated into Serbian.

In recent years new proofs for Milanković theory are given. For example, in the cliffs of Kerguelen islands in the southern Indian Ocean have been found fossils which helped to determine changes in water temperature during the last 450 thousand years. The obtained results are in excellent agreement with the theory of ice ages.

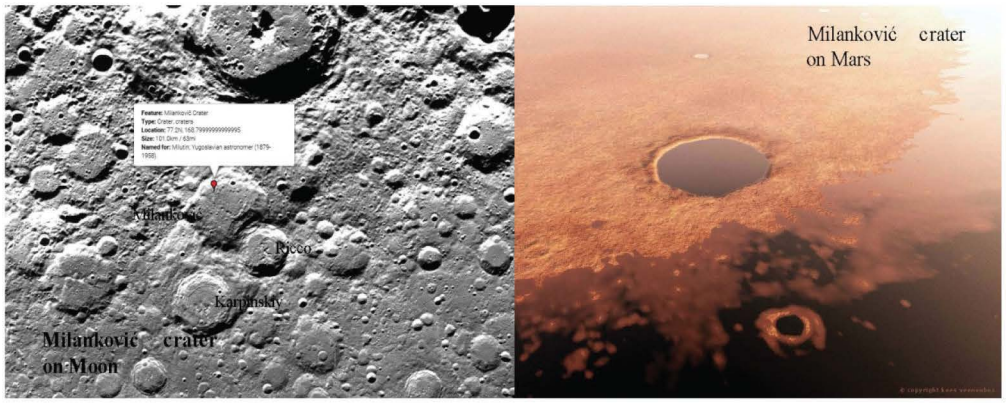
Milanković was also successful in the calendar reforms. Orthodox Church organized in the 1923 congress dedicated to the reform Julian calendar that was used in the Orthodox Church. Milanković, as a representative of the Serbian Orthodox Church gave an improvement in the reckoning of the Gregorian calendar. Milankovic's calendar reform proposal was accepted by the community of Orthodox churches. However, most of the Churches of the orthodox federation, including Serbian, although they were present at the Congress, did not start the implementation of the reformed calendar. In the same year, King Aleksandar Karadjordjevic awarded Milanković the Medal of St. Sava of the third order for his contribution to the calendar reform.



Milankovic's contribution to world science is honored in many ways. We list them in free order.

European Geosciences Union, Division on Climate, established in 1993 Milutin Milankovic medal in recognition of the scientific and editorial achievements of Milutin Milankovic. This medal is reserved for scientists for their outstanding research in long term climatic changes and modeling. Until 2016, this medal is awarded to 25 scientists, including the mentioned John Imbrie.

IAU (International Astronomical Union) named several celestial objects after Milanković. One minor planet is named after him: 1609 Milankovitch. One crater on Moon and one crater on Mars are also named after him.





Google doodle, the Google introductory icon, devoted to Milanković theory of ice ages was published in May 28, 2010, on the occasion of his birthday. The path of the Earth around the Sun is depicted.

Streets in 13 towns in Serbia are named after Milanković, including the great boulevard in Belgrade. Also, the name Milutin Milanković is given to several secondary schools in Belgrade, Kruševac in Serbia and one in Rebulika Srpska.

On two occasions, celebrating 100 years and 125 years of his birth, the Yugoslavien (FRY) Postal Service has issued a commemorative stamp and a first day envelope.

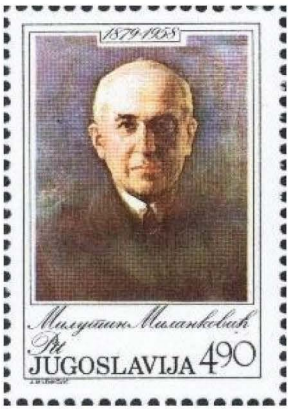




Image and work of Milutin Milanković are also depicted on Serbian coins and banknotes. National bank of Serbia issued in 2009 the coin having the value of 20 dinars and in 2011 issued the banknote having the value of 2000 dinars. The avers of the banknote is dominated by the Milanković's portrait. There is also shown the computation of the snow line for the past part of the quarter of 600,000 years. On the other hand, the revers depicts the Milanković's figure and fragment of stylized presentation of the solar disk. The central part is dominated by Milanković's work "The path of the northern celestial pole".









Many scientific and thematic meetings and conferences were devoted to Milutin Milanković. Here we shall mention here just few of them.

SASA (Serbian Academy of Sciences and Arts) is the centenary of 1979. organized Symposium "The Life and Work of Milutin Milankovic 1879-1979"

At Columbia University in New York in 1982 an international symposium entitled "Milankovic and Climate" was held.

SASA on the occasion of the 125th anniversary of his birth, in 2004, organized a symposium "Paleoklima Earth's climate system".

Association "Milutin Milanković" organized a conference on science and work of Milutin Milanković "Calendar knowledge and contribution of Milutin Milankovic" 14-15. September 2011 in Belgrade. The Association issued the Proceedings of the conference.

To mark 120 years of Meteorological Observatory in Belgrade, in 2007, it was established the National Center for Climate Change as a part of the Republic Hydro-meteorological Service of Serbia. Center, in accordance with the final documents of the Sixth Ministerial Conference of the UNECE "Environment for Europe", held in October 2007 in Belgrade, performs the functions of Sub-regional Climate Centre for South East Europe. The Centre has been given the name of Milutin Milankovic.

In recent years, a special role in popularizing the work of Milutin Milanković, has the Association "Milutin Milanković" (http://milutinmilankovic.rs). The association was founded in 2007 with the headquarter in Belgrade. Here are some of the activities of the Association:

The Association has made several films about Milankovic.

In the premises of the Association a permanent exhibition devoted to Milankovic life and work is organized. It also organized a series of thematic exhibitions and lectures on Milankovic in Belgrade and other cities in Serbia, Republica Srpska and Montenegro.

Association published an illustrated book: *Milutin Milanković - traveler through the Universe and the Ages.*

Faculty of mathematics in Belgrade and the Association "Milutin Milanković" made the digital legacy "Milutin Milanković" located at http://legati.matf.bg.ac.rs/milankovic. Legacy contains: 17 books of Milutin Milanković, five his manuscripts of lectures at the University, his 47 published scientific, eight books that other authors wrote on Milanković, two scientific papers on Milanković written by other authors.

Several authors wrote on Milanković: the academicians Tatomir Andjelić, Nikola Pantić and Zoran Knezević and a group of researchers working on the project Digitization of scientific and cultural Heritage: Nadežda Pejović, Žarko Mijajlović, Saša Malkov and Nenad Mitić.



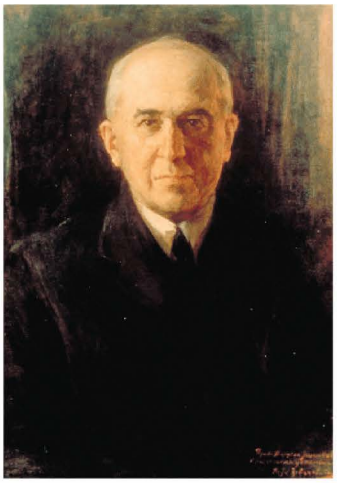


The original look of Milanković's birth house and his memorial center now

The Government of the Republic of Serbia together with the Croatian government restored in 2007 the birth house of Milutin Milanković in Dalj (Croatia). The house was built in 1820, and in 1979 became a cultural monument. The house was damaged and very ruined. After the reconstruction and renovation it turned into the educational memorial center, with a memorial room of the famous scientist, museum space, the Heritage collection of Milutin Milanković, lecture hall and the exhibition gallery.



Sreten Stojanović: Milanković's bust (1944)



Paja Jovanović: Milanković's portrait (1943)

There are also many artistic presentations of Milanković's image and work. Busts of Milutina Milankovića are posted at the following places: In front of the University of Novi Sad (sculptor Vladimir Jokanovic); iIn the lobby of the Mathematical Institute SASA, Belgrade; the Hydro-Meteorological and Geological School "Milutin Milanković" (Sculptor Sreten Stojanović); in front of the geomagnetic Institute Grockoj (sculptor Drinka Ivanović-Radovanović); SASA library (sculptor Sreten Stojanović); at the Museum of Contemporary Art in Belgrade (sculptor Sreten Stojanović) and the Association "Milutin Milanković" in Belgrade.

Conclusion

Milutin Milanković had great achievements and gave important contributions to geosciences and climatology. He published more than seventy scientific papers and books, most of them related to these fields. For his merits in science he obtained many

awards and his personality and his work are still honored and celebrated in many ways. All his scientific work is digitized and it is now a part of his digital legacy. There is also presented most of the awards granted to him. We believe that the material in the legacy will serve to better understand Milanković's life and scientific work.

Acknowledgment

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78 Digitized works of Emilijan Josimović

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NCD Review, vol. 29, pp. 815, 2016.

This paper presents digitized works of Emilijan Josimović, a respectable 19th century Serbian architect, engineer and professor of mathematics. He is considered as a first Serbian urbanist and in the mid of the sixties of the XIX century he gave a plan pursuant to which the center of Belgrade is reconstructed. He also was the university professor of mathematics, rector of the Grand School, the forerunner of the Belgrade university. Josimović wrote first Serbian university books in mathematics and architecture. It interesting that the books he wrote in the fifties of the XIX century were written in the old Serbian language and script. His later books were written in modern Serbian in accordance of Vuk Karadžić's reform. Four textbooks in mathematics, one textbook in physics and the proposal of the regulation plan of the city of Belgrade are reviewed in this article. The printed copies of these books belong to the Library of the Serbian Academy of Sciences and Arts. Their digital copies are deposited in the Virtual Library of the Faculty of mathematics.

Преглед НИД 29 (2016), 8–15

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DIGITIZED WORKS OF EMILIJAN JOSIMOVIĆ[1]

Abstract. In this paper we present digitized works of Emilijan Josimović, a respectable 19th century Serbian architect, engineer and professor of mathematics. He is considered as a first Serbian urbanist and in the mid of the sixties of the XIX century he gave a plan pursuant to which the center of Belgrade is reconstructed. He also was the university professor of mathematics, rector of the Grand School, the forerunner of the Belgrade university. He wrote first Serbian university books in mathematics and architecture. It interesting that the books he wrote in the fifties of the XIX century were written in the old-Serbian language and script. His later books were written in modern Serbian in accordance of Vuk Karadžić reform. We digitized six his works, four textbooks in mathematics, one textbook in physics and the proposal of the regulation plan of the city of Belgrade. The printed copies of these books belong to Serbian Academy of Sciences and Arts. Their digital copies are deposited in the Virtual Library of the Faculty of mathematics, elibrary.matf.bg.ac.rs.

Keywords: Emilijan Josimović, digitized works, XIX century.

Introduction

This paper belongs to the series of articles related to the digitization of distinguished former Serbian scientists and their digital legacies:

http://legati.matf.bg.ac.rs
http://elibrary.matf.bg.ac.rs

A group of participants of the Digitization project of the Faculty of Mathematics in Belgrade and the Mathematical Institute of the Serbian Academy of Sciences and Arts started in the first decade of this century to build a virtual library and a Digital Legacy, Internet sites of the Faculty of Mathematics. The general idea of these sites was to present biographies and works of the prominent Serbian scientists from the past whose opus was related in some way to mathematical sciences. We also wanted to have all their scientific and technical works and personal data in one place. These items include photos, letters and other related documents. We believe that these collections will help to better understand their work and life. We designed this Internet site bearing in mind the general audience, but the scientific community, too. Up to now, biographies of three scientists, academicians and the former professors of the Faculty of mathematics are offered there: Anton Bilimovič (1879-1970), Bogdan Gavrilović (1864-1947) and Milutin Milanković (1879-1958). Recently, a preparation of digital legacy dedicated to Slaviša Prešić (1933-2008) was started (Mirjana Maljković and Biljana Stojanović). In

[1] This work has been presented at the XIV Conference Digitization of Cultural Heritage, Old Records from the Natural and Social Sciences and Digital Humanities, Belgrade, September 29, 2016

this paper we present digitized works of Emilijan Josimović (1820? – 1897), who was a respectable 19th century Serbian architect, engineer and professor of mathematics.

Short biography of Emilijan Josimović

Emilijan Josimović (1820? – 1897) is considered as a first Serbian urbanist and in the mid of the sixties of the XIX century he gave a plan pursuant to which the center of Belgrade was reconstructed. He also was a university professor of mathematics and the rector of the Grand School, the high school that proceeded of the Belgrade University. It is not known for certain the year when he was born, but there are some evidences that he was born most probably in 1820 in Old Moldava. This settlement of now modern Romania lies on the left bank of the river Danube and it is known for a large Serbian minority in the past. He got his elementary and secondary education in Caransebeș and Lugoj (Romania). Afterwards he finished the Polytechnic school in Vienna. In the 1845 he came in Belgrade for a professor of architecture and mathematics at the Belgrade Liceum and other highest educational institutions in Serbia.

Not a known painted portrait or a photograph was left after him, even if he was a celebrity and photographing was widespread in his time. Most probably the reason for that is the following. After his coming to Serbia, he lived in a village of Poreč (now the town of Donji Milanovac). But his house was destroyed in 1942 during the great flood and most probably personal data regarding Emilijan Josimović were lost during this event. Even if he lived and worked in Belgrade, he did not have his own house there.

He had very important role in the urban planning and the reconstruction of Belgrade. In fact, he is most known for his contribution to the reconstruction of Belgrade in the mid of XIX century. He managed surveying Belgrade (1864-1867) and in 1867 published a proposal for the urban plan of Belgrade. This regulation plan of Belgrade was the most important Josimović work and a significant architectural and urban deed which laid in the foundation of urban planning in Serbia. For example, the famous and the most known Knez Mihailova street in the center of Belgrade got it's today look in accordance to his plan. At the beginning of this street, there is a memorial devoted to him.

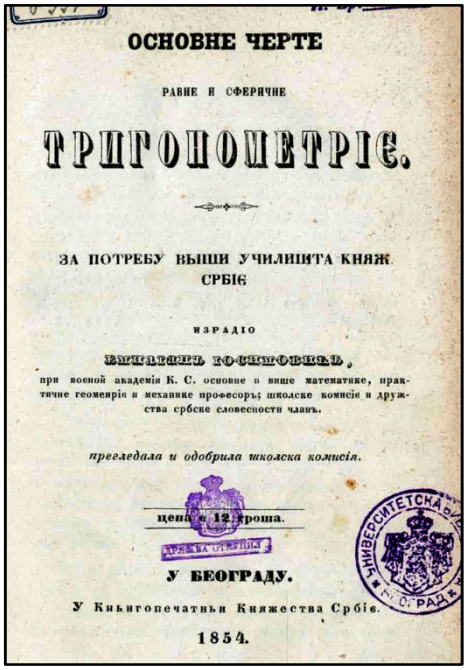
Since his coming to Serbia in 1845 until his retirement in 1878 he was the professor at the highest educational institutions in Serbia which were in this time concentrated in Belgrade. Firstly, he was appointed to the professor of mathematics at the Belgrade Liceum. After that, in 1850, he became a professor of mathematics, mechanics and geodesy at the Army artillery school (but he remained the honorary professor of Liceum). Finally, he was appointed to the professor of mathematics in Belgrade Grand School (1869) where he also was a rector (1876-1877). During his professorship he wrote first Serbian university books in mathematics and "applied geometry" how he used to call geodesy. It interesting that the books he wrote in the fifties of the XIX century were written in the old-Serbian language and script. His later books were written in modern Serbian in accordance of Vuk Karadžić linguistic reform.

He was also a member of the highest Serbian academic societies ("Društvo srbske slovesnosti" – Society of Serbian sciences, "Srpsko učeno društvo" – Serbian scientific society, etc) and the honorary member of the Serbian Academy of Science since its foundation (1886).

Digitized works of Emilijan Josimović

We digitized six works of Emilijan Josimović, four textbooks in mathematics, one textbook in physics and the proposal of the regulation plan of the city of Belgrade. The printed copies of these books belong to the Serbian Academy of Sciences and Arts and due to the courtesy of the Academy this digitization was possible. Digital copies of the books are deposited in the Virtual Library of the Faculty of mathematics, elibrary.matf.bg.ac.rs. We present shortly the contents of the books.

Trigonometrija (1854)
(Trigonometry)


ОСНОВНЕ ЧЕРТЕ
РАВНЕ И СФЕРИЧНЕ
ТРИГОНОМЕТРІЄ.
ЗА ПОТРЕБУ ВЫШИ УЧИЛИШТА КНЯЖ. СРБІЄ
ИЗРАДІО
ЕМИЛИЈАНЪ ІОСИМОВИЋЪ,
при воєной академіи К. С. основне и више математике, практичне геомеиріє и механике професоръ; школске комисіє и дружства србске словесности членъ.
прегледала и одобрила школска комисія.
цена є 12 гроша.
У БЕОГРАДУ.
У Кньигопечатньи Княжества Србіє.
1854.

Josimović wrote this book in old Serbian and published it in 1854. The book consists of three chapters and about 170 pages. At the end there are bibliography (German and French authors), a two page list of corrections and large table of figures.

The first part, *Goniometry*, explains the basic properties of trigonometric functions. The second one, *Plain trigonometry*, is devoted to the solving problems on triangles. There are also some applications in measuring in geodesy. The third chapter, *Spherical trigonometry*, is the largest, most advanced and the most interesting part of the book. Besides deriving and explaining notions and formulas from spherical trigonometry, he solves there a lot of problems related mainly to spherical triangles. There are also some applications to advanced problems of geodesy and astronomy.

This Josimović's book was not the first Serbian book on trigonometry. Atanasije Nikolić (1803-1882), a professor of mathematics who preceded Josimović at Liceum wrote *Geometry* (published in 1841). A part of this book is devoted to trigonometry. However, Josimović *Trigonometry* is much more advanced and complete work on trigonometry. In fact, Josimović was very proud about his book. He wrote there (in old – Serbian) in somewhat challenging way:

U ostalom eda li i u koliko ovo moe delce odgovara izjavljenoj nameris njim i potrebi naši biši škola, za koje je napisano, - kao i je li raspoređenje i izlaganje njegovi predmeta pedagogično i praktično shodno: neka presude oni, koi sve to, pa dakle i istu nauku poznaju bolje nego ja. Kritika u tom obziru i od takovi lica bit'će mi tim milija, što bi se njom osim mana, koje na svaki način ispraviti valja, odkrilo ujedno i sve ono, što je originalno, i koje mi se dakle u pravu auktorsku zaslugu brojati ima. (Let's other, who know better this science than me, decide if this work is suitable, pedagogically written and practical for use in our high schools. Any appropriate critiques which find not only shortcomings, but also my original contributions will be very valuable for me.)

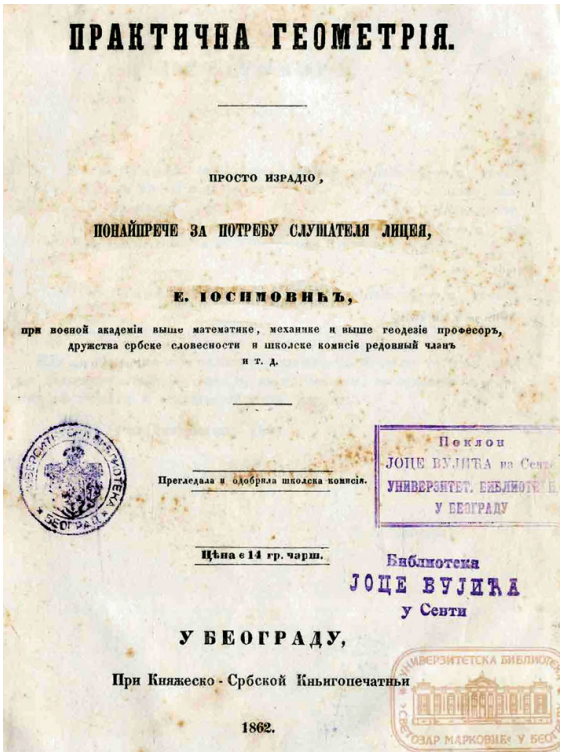
He also had to solve a lot of terminological problems. He studied mathematics in German, so he had to introduce many new words for mathematical notions not appearing until then in Serbian, e.g.: triugao (triangle - ms[2]: trougao), pruga (segment -

[2] in modern Serbian

ms: duž), osnovak (element - ms: element), dirka (tangent - ms: tangenta), prečnica (hypotenuses - ms: hipotenuza), upravnica (chatetus - ms: kateta).

Praktična geometrija (1862)
(Practical Geometry)

Josimović wrote this book for the students of Liceum, also in old Serbian. As we can anticipate from the title and his occupation in this time, the content of the book is manly on solving some practical problems in geodesy. The book itself consists of two chapters. The first part is devoted to the „elementary geodesy", while the second refer to measuring and use of various instruments (barometers, etc) in geodesy and leveling.


ПРАКТИЧНА ГЕОМЕТРІЯ.

ПРОСТО ИЗРАДІО,

ПОНАЈПРЕЧЕ ЗА ПОТРЕБУ СЛУШАТЕЛЯ ЛИЦЕЯ,

Е. ІОСИМОВИЋЪ,

при военой академіи више математике, механике и више геодезіе професоръ, дружства српске словесности и школске комисіе редовный члан и т. д.

Прегледала и одобрила школска комисія.

Цѣна с 14 гр. чарш.

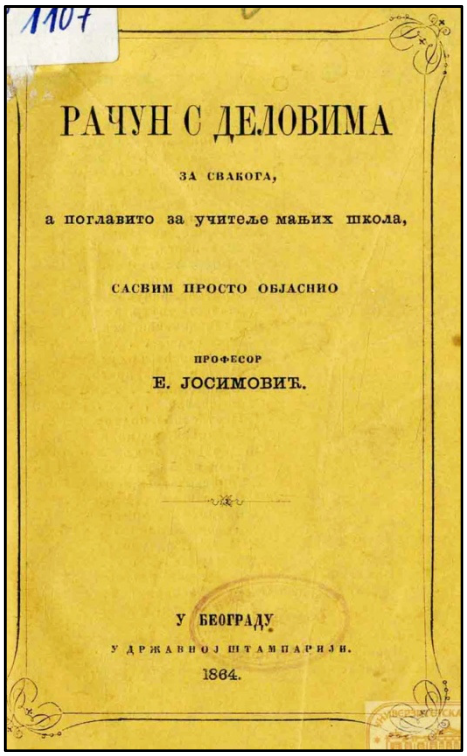
У БЕОГРАДУ,

При Княжеско-Српской Кньигопечатньи

1862.

Поклон ЈОЦЕ ВУЛИЋА из Сенте УНИВЕРЗИТЕТ. БИБЛИОТЕКА У БЕОГРАДУ

Библиотека ЈОЦЕ ВУЛИЋА у Сенти


1107

РАЧУН С ДЕЛОВИМА

ЗА СВАКОГА,

а поглавито за учитеље мањих школа,

САСВИМ ПРОСТО ОБЈАСНИО

ПРОФЕСОР
Е. ЈОСИМОВИЋ.

У БЕОГРАДУ
У ДРЖАВНОЈ ШТАМПАРИЈИ.
1864.

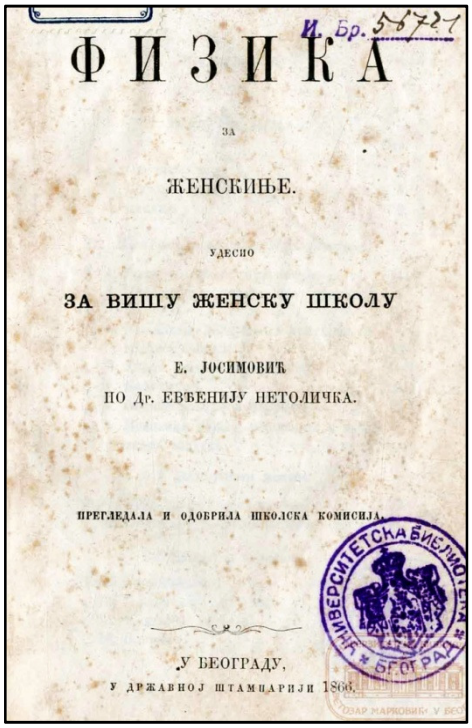
Račun s delovima (1864)
(Fractional calculus)

This small booklet (it has about 100 small format pages) is probably his the least interesting book on mathematics but give us an insight in the content of mathematics that was taught in elementary schools in Serbia in the middle of 19. century. It is devoted to the fractional calculus (rational numbers and basic arithmetical operation with these numbers) and it is written mainly for the elementary school teachers. In a sense this book belongs to the series of the books written in the past: first Serbian mathematical book *Aritmetika* (printed in 1767) by Vasilije Damjanović (1734 – 1792) and *Čislenica* (printed in 1809) by Jovan Došenović (1781- 1813).

Fizika (1866)
(Physics)

The proper name of this book is ***Fizika za ženskinje*** (Physics for ladies). As the title suggests this book is written for students of high lady school. The book is very

elementary but relatively large, it has 270 pages. There is no mathematical formula or expression which would represent any physical low. It could be said that he explained natural phenomena following the ancient Greek idea of four classical elements: air, water, earth and fire. Of course, he explained these entities from the modern point of view. The book also clarifies some other physical phenomena such as sound, light, magnetism and electricity. The construction and uses of some instruments (barometer, microscope and telescope) is explained. Finally, some applications of introduced notions are given such as steam machine, telegraphy and galvanization for gilding and silver-plating - "galvansko zlatenje i srebrenje". The book is written in modern Serbian in accordance to Vuk Karadžić linguistic reform which was officially adopted in 1868, even if its fundaments were laid down two decades before that. The book contains a lot of nice illustrations and vignettes. It is worth reading this book because of nice writing style and use of old Serbian words, now completely forgotten.


ФИЗИКА

ЗА

ЖЕНСКИЊЕ.

УДЕСНО

ЗА ВИШУ ЖЕНСКУ ШКОЛУ

Е. ЈОСИМОВИЋ

ПО Др. ЕВЂЕНИЈУ НЕТОЛИЧКА.

ПРЕГЛЕДАЛА И ОДОБРИЛА ШКОЛСКА КОМИСИЈА.

У БЕОГРАДУ,

У ДРЖАВНОЈ ШТАМПАРИЈИ 1866.


XVI. МИКРОСКОП И ТЕЛЕСКОП.

§. 59. Микроскоп.

Човечији ум и труд пронашао је и око подпомажућа оруђа. Помоћу таковиx видимо светове, који, милионима миља од нас удаљени, у неизмерном простору својим путем ходају; погледамо у невидно, огромно царство инфузорија, у тајни строј животиња и биља, и уверавамо се свуда о безграничној мудрости творца. Оруђа којима до свега тога долазимо, јесу микроскоп и телескоп.

Свака справа што онаке предмете, које само око, због незнатне њихове величине, на обичној видној даљини или нејасно или никако неопажа, показује увећане и јасно, зове се **микроскоп.**

Свако стакло дакле, кога је жижна даљина мања од разговетне видне даљине,

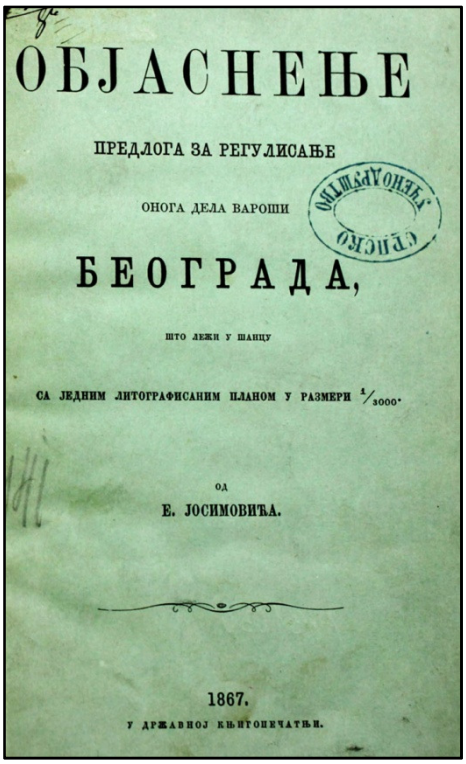
Objasnenje predloga za regulisanje onog dela varoši Beograda što leži u Šancu (1867)
(Explanation of the proposal for a regulation that part of the City of Belgrade which lies in Trench)

This book is certainly Emilijan's the most important and best known work. There he proposed the first architectural and urban plan of the city of Belgrade. Plan referred to the area of today the city center, then bounded by the so called trench. This trench were the remains of the fortification which was built by Austrian Empire in the beginning of the XVIII century. This fortification and the origin of the trench has the interesting history. Namely, in 1717 the Austrians, led by Prince Eugene of Savoy, after big battle for Belgrade against Ottomans took over Belgrade and held it until 1739. In that period the medieval fortified town definitely disappeared and a modern Baroque artillery fortress was constructed on the same site. The reconstruction project (authored by Austrian Colonel of Swiss origin Nicolaus Doxat de Demoret) was carried out during the next 15 years. For its time that was a robust and pretty extensive fortification of Vauban's type (according to Marquis de Vauban, French engineer of the 17. century). It

was built in the hope that it will help the defense of Europe against the Otoman Empire. However, after the Austrian army had suffered several defeats, Austrian Belgrade surrendered to the Ottomans without fighting. According to a clause of the armistice contract signed on that occasion, Austrians had the right to hand Belgrade over to Turkey under the condition that all new fortifications (and some other buildings in the town) built after 1717, be destroyed. The demolition of the newly built Austrian fortifications lasted for almost six months and so the trench arose.



Part of the city of Belgrade bounded by the trench: it lies between the rivers Sava and Danube and the trench depicted by the yellow line.


ОБЈАСНЕЊЕ

ПРЕДЛОГА ЗА РЕГУЛИСАЊЕ

ОНОГА ДЕЛА ВАРОШИ

БЕОГРАДА,

ШТО ЛЕЖИ У ШАНЦУ

СА ЈЕДНИМ ЛИТОГРАФИСАНИМ ПЛАНОМ У РАЗМЕРИ 1/3000.

ОД

Е. ЈОСИМОВИЋА.

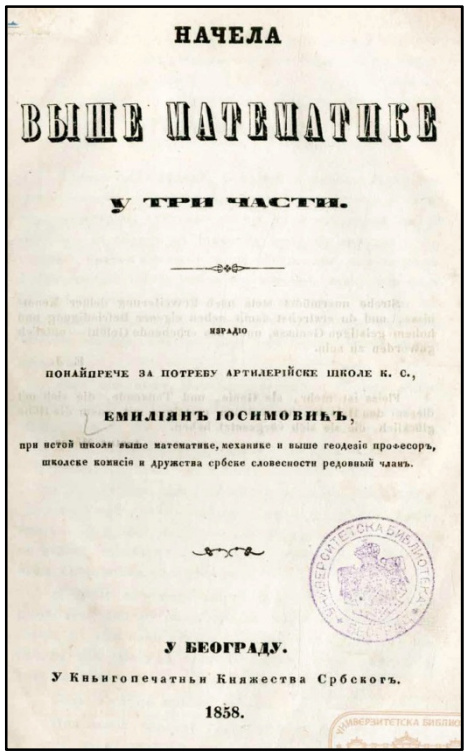
1867.

У ДРЖАВНОЈ КЊИГОПЕЧАТЊИ.

This short book, it has only 47 pages and the supplement consisting of several drawings and maps, is considered as the first urban plan of the city of Belgrade. This urban plan had the great influence on the reconstruction and further urban planning of Belgrade in the 19. century but later as well. Due to this plan the most important streets in the center of Belgrade obtained their today's look. It is considered that this book not only represented in the modern sense of the word urban-engineering study, but its domain encompassed virtually all aspects of life in the city, ranging from social and cultural to anthropogeographic. Here is an interesting detail concerning the publication of the book. Even if the urban plan was made by order of Prince Mihailo Obrenović, then the ruler of Serbia, we read in the Introduction that the author completely financed the printing of the book.

Načela više matematike
(Fundament of higher mathematics)

The book is composed of three volumes that were printed respectively in 1858, 1860 and 1872. The first two volumes were printed in old Serbian, while the last one in modern Serbian. The big delay between the second and the third volume can be explained that Josimović wrote the third volume first in old Serbian and then he had to "translate" it in modern Serbian according to the adopted Vuk Karadžić language reform. Even the short look in these books tell us that they are for this time the most advanced mathematical textbooks that far exceeded all previously published similar


НАЧЕЛА

ВЫШЕ МАТЕМАТИКЕ

У ТРИ ЧАСТИ.

ИЗРАДІО

ПОНАЈПРЕЧЕ ЗА ПОТРЕБУ АРТИЛЕРІЙСКЕ ШКОЛЕ К. С.,

ЕМИЛІЯНЪ ІОСИМОВИЋЪ,

при истой школи выше математике, механике и выше геодезіе професоръ, школске комисіи и дружства србске словесности редовный чланъ.

У БЕОГРАДУ.

У Кньигопечатньи Княжества Србскогъ.

1858.

books. For a modern Serbian reader it is very interesting to read in old Serbian such advanced topics as the convergence and summability of infinite series, or the properties of Bernoulli numbers. The first volume covers some algebraic concepts such as polynomials and rational expression, solving algebraic equations of the third and fourth degree and some general properties of functions. The second volume is mainly concerned with the differential calculus, while the third is on the integral calculus and the rather advanced topic, variational calculus. We will not enter here into the detailed analysis of these books, they deserve an article completely devoted to them. Here we mention that there is a rather fine analysis of this books written in Serbian by Đura Paunić [8].

Conclusion

Emilijan Josimović had great achievements and gave important contributions to the urban planning of the city of Belgrade, but also to the development of the teaching of mathematics and related sciences at the highest educational institutions in Belgrade in the middle of the 19. century. He published the first urban plan of Belgrade and several textbooks in mathematics, physics and geodesy. All his published works are digitized and they are now a part of his digital legacy in the Virtual library of the Mathematical Faculty in Belgrade. We believe that this material in the Library will serve to better understand Josimović's life and work.

Acknowledgment

This work is supported by a grant of the Serbian Ministry of education, science and technology under the project III 44006.

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8. Đura Paunić, *Emilijan Josimović*, Spomenica 130 godina Matematičkog fakulteta, Matematički fakultet, 2003.



A monument devoted to Emilijan Josimović. It is located at the beginning of the *Knez Mihailova*, the central Belgrade street.

79 Digitization of the book *Principles of Physics* of Vuk Marinković

Ž. Mijajlović and N. Pejović,
NCD Review, vol. 31, pp. 25-29, 2017.

This paper presents digitized version of a book "Principles of physics" (Serb. "Načela fizike", Belgrade, 1851) written by Vuk Marinković, a prominent Serbian medical doctor and physicist from the middle of the XIX century. This relatively large book is considered as a first Serbian textbook on physics. It is interesting that it was one of the last books published in the old-Serbian language and script. Later text-books were written in modern Serbian in accordance with Vuk Karadžić's reform. The printed copy of this book belongs to the Serbian Academy of Sciences and Arts. Digital copy is deposited in the Virtual Library of the Faculty of mathematics.

Преглед НЦД 31 (2017), 25–29

Žarko Mijajlović, Nadežda Pejović
Faculty of Mathematics, Belgrade, Serbia

DIGITIZATION OF THE BOOK "PRINCIPLES OF PHYSICS" OF VUK MARINKOVIĆ

Abstract. In this paper we present digitization of a book "Priciples of physics" ("Начела физике", Belgrade, 1851) written by Vuk Marinković, a prominent Serbian medical doctor and physicist from the middle of the XIX century. This relatively large book is considered as a first Serbian text-book on physics. It is interesting that it was one of the last books published in the old-Serbian language and script. Later text-books were written in modern Serbian in accordance with Vuk Karadžić's reform. The printed copy of this book belongs to the Serbian Academy of Sciences and Arts. It's digital copy is deposited in the Virtual Library of the Faculty of mathematics, elibrary.matf.bg.ac.rs.

Keywords: Vuk Marinković, digitized works, physics, XIX century.

Introduction

The Virtual Library of the Faculty of Mathematics in Belgrade (http://elibrary.matf.bg.ac.rs) was founded 15 years ago, see [2], [3] and [4], and it is one of the largest database of digitized texts in Serbia with free access. At the time of writing this article, it contains more than 4000 digitized items and several important collections, including digital copies of PhD theses of Serbian mathematicians and rare Serbian books - some of them dating the 18th century. This paper belongs to the series of articles on digitization of books contained in this library.

The general idea of this online database is to preserve in digital form the works in mathematical sciences and related subjects of Serbian scientists from the past. We consider it as an important activity in the area of the preservation of scientific and cultural heritage of the Serbian people. The other but not less important goal is to present these works to the general and scientific audience using the modern information technologies. This work is done by the support of the Faculty of Mathematics in Belgrade and projects - funds granted by the Serbian Ministry of Education, Science and Technological Development. It is recognized and supported in certain forms also by the other Serbian scientific and cultural institutions, particularly by the Mathematical Institute SASA, Serbian Academy of Sciences and Arts, University Library in Belgrade and the National Library. These institutions are in fact the main sources of the material: books, manuscripts and other writings that are digitized and placed in the Virtual Library. However, we consider as the most important the work of participants of the Digitization projects of the Faculty of Mathematics. Without their enthusiasm this library would not exist.

XV Conference Digitization of Cultural Heritage and Digital Humanities, Belgrade, September 26, 2017


Др Вук Маринковић
(1807-1859)

In this occasion we shall present digitization of the book "Principles of physics" ("Načela fizike", Belgrade, 1851) written by Vuk Marinković, an outstanding Serbian medical doctor and physicist from the middle of the XIX century. Even if there are books in Physics (Atanasije Stojković) published almost a half a century before it, this large book is considered as a first Serbian text-book on physics. Several authors, see [1], [2] and [6], have already written a comprehensive biography of Vuk Marinković and gave it's amply reviews. So, we shall not enter deeply into the biography of Vuk Marinković, neither in his book. The main purpose of this article is to mark and announce this event, the digitization of Marinković's *Physics*.

Short biography of Vuk Marinković

Vuk Marinković was born in Novi Sad on January 5, 1808 (December 24, 1807, by the Julian calendar). He finished elementary school in Novi Sad and studied at the Gymnasium in Novi Sad and Jegra (Eger, Hungary). After that he went on studies of medicine at the Pest and Viennese University from 1826 to 1830. He defended the doctoral dissertation: *Dissertatio inauguralis medica de epilepsia*, at the Royal University of Pest on May 23, 1830, at age 23. Marinković then returned to Novi Sad where he starts private medical practice and worked there until the European revolution in 1848 which caught up Austrian Empire, also. He has enjoyed a great reputation among the citizens of his town. Even if Novi Sad at that time was not in Serbia, he was also known in Serbia as a good medical doctor and educated man. For example, in 1831 Miloš Obrenović offered him to work in the rank of court (knjaževski) physician in the town of Kragujevac. At that time, he began to interest in physics and devoted himself to the study this subject. His seriousness in this field is witnessed by his visit to Berlin in 1844 and 1947 where he attended the lectures of the famous Alexander von Humboldt. Vuk Marinkovic was married with Sofia, the daughter of wealthy Jovan Pavlović.

After the destruction of Novi Sad in the aftermath of the bombing, in June 1849 during the European Revolution 1848-1849, Vuk became virtually a homeless person, without a personal library and personal files which he lost during these events. It was

rather difficult for him to restore private medical practice, so he moved to Serbia. He was immediately offered there the position of physics professor at Lyceum. a foreran of the Belgrade Higher School and the later Belgrade University. He accepted this position and remained there until August 1857 when he suddenly died.

He also taught chemistry until the school year 1853/54 when chemistry was given a status of an independent subject. Upon Marinković's arrival at Lyceum, the theoretical and experimental work is upgraded to a higher level, so Marinković is considered the founder of physics in Serbia. During his academic work in Belgrade he was twice elected for the rector of Lyceum.

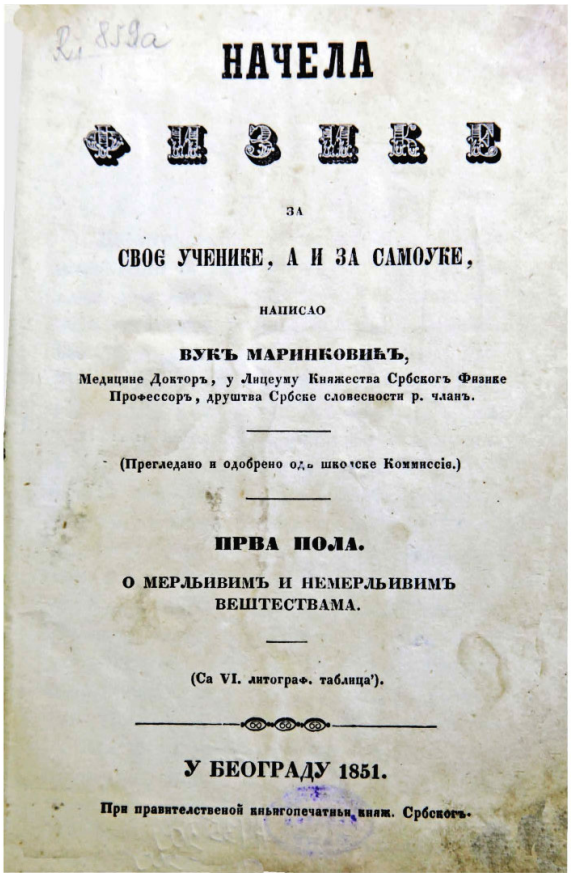
His main works are "Principles of Physics", Belgrade, 1851, 840 pages, "History of Nature" (*Јестествена повјесница*), Beograd, 1851. 493 pages and *Atlas* (1851). He also published his translation of Mochnik's geometry for the higher schools of the Principality of Serbia. He wrote all these books in Old Slavic-Serbian language and the script.

Начела физике (Principles of Physics)

Certainly the most important work of Vuk Marinković is his the two-volume textbook *Principles of Physics*. However this is not the book only on physics but also covers may topics in chemistry, astronomy, physical geography, and meteorology. This is a rather large book, altogether (in both volumes) there are XI + 801 pages and several tables with drawings. In the writing of his book Marinković used works of then leading German and French authors: Scholz, Neuman, Müller and Baumgartner, Puile, Eisenlor, Shchilov, Derlor, Marbach, Helmut, Frik, Shoedler, Dermart, Reno, Delfs, Jan, Litrov, Sporishl, Humbolt and Kempic. The first volume discuss phenomena on "measurable and non-measurable matter" ("мерьливимь и немерьливимь вештествама"), while the second part is mainly on the phenomenons "appearing on the sky and earth". The book is written in old Serbian. Vuk Karadžić linguistic reform came only a decade after the book was published.

The first volumes covers some rudiments of mechanics and gravitaion, then electricity, galvanism, light and magnetism. It is ineterssting that he considers and discusses "coldness" as a separate notion from heat. He also presents there in quite extensive manner many notions from chemistry, such as the basic physical and chemical properties of all then known chemical elements.

The second volume covers topics in astronomy, such as Keplers laws, Sun and Moon eclipses, and then known "17 planets". Namely, he included in planets the greatest asteroides such as Vesta and Ceres. In the continuation he also discused geomorpholgical features of the Earth's surface, such as mountaines, deserts, oceans, caves, volcanos etc. In the final section *Meteorology* Marinković puts together of all of the sky events such as the astronomical meteors and meteorities, but also the atmospheric precipitation, lightnings and winds.



НАЧЕЛА

ФИЗИКЕ

ЗА

СВОЄ УЧЕНИКЕ, А И ЗА САМОУКЕ,

НАПИСАО

ВУКЪ МАРИНКОВИЋЪ,

Медицине Докторъ, у Лицеуму Княжества Србскогъ Физике Професоръ, друштва Србске словесности р. членъ.

(Прегледано и одобрено одъ школске Комиссіе.)

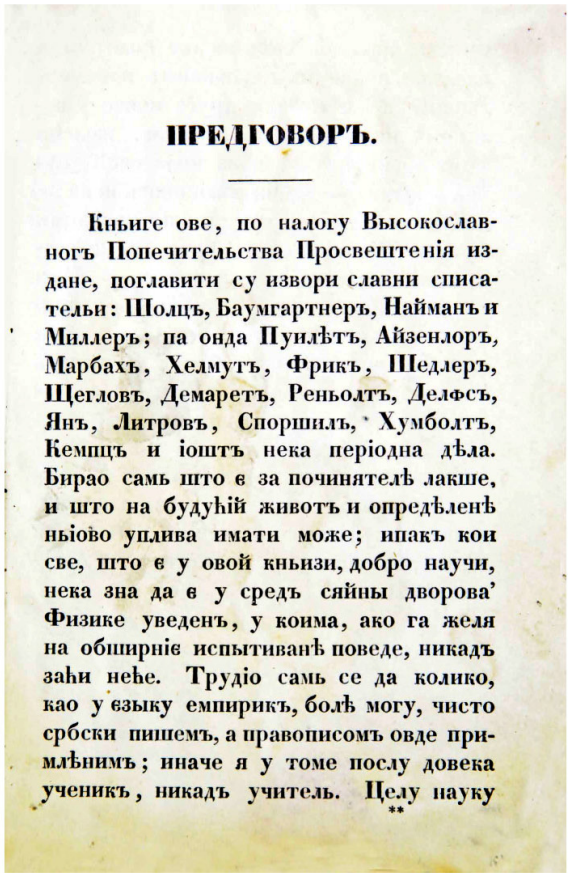
ПРВА ПОЛА.

О МЕРЉИВИМЪ И НЕМЕРЉИВИМЪ ВЕШТЕСТВАМА.

(Са VI. литограф. таблица').

У БЕОГРАДУ 1851.

При правителственой књигопечатњи књаж. Србскогъ.



ПРЕДГОВОРЪ.

Књиге ове, по налогу Високославногъ Попечительства Просвештенія издане, поглавити су извори славни списателји: Шолцъ, Баумгартнеръ, Најманъ и Миллеръ; па онда Пуилѣтъ, Айзенлоръ, Марбахъ, Хелмутъ, Фрикъ, Шедлеръ, Щегловъ, Демаретъ, Реньолтъ, Делфсъ, Янъ, Литровъ, Споршилъ, Хумболтъ, Кемпцъ и іоштъ нека періодна дѣла. Бирао самъ што є за починителѣ лакше, и што на будућій животъ и опредѣленѣ ньіово уплива имати може; ипакъ кои све, што є у овой књизи, добро научи, нека зна да є у средъ сяйны дворова' Физике уведенъ, у коима, ако га жеља на обширніе испитиванѣ поведе, никадъ заћи неће. Трудіо самъ се да колико, као у єзыку емпирикъ, болѣ могу, чисто србски пишемъ, а правописомъ овде примљнимъ; иначе я у томе послу довека ученикъ, никадъ учитель. Целу науку

Cover page and Introduction

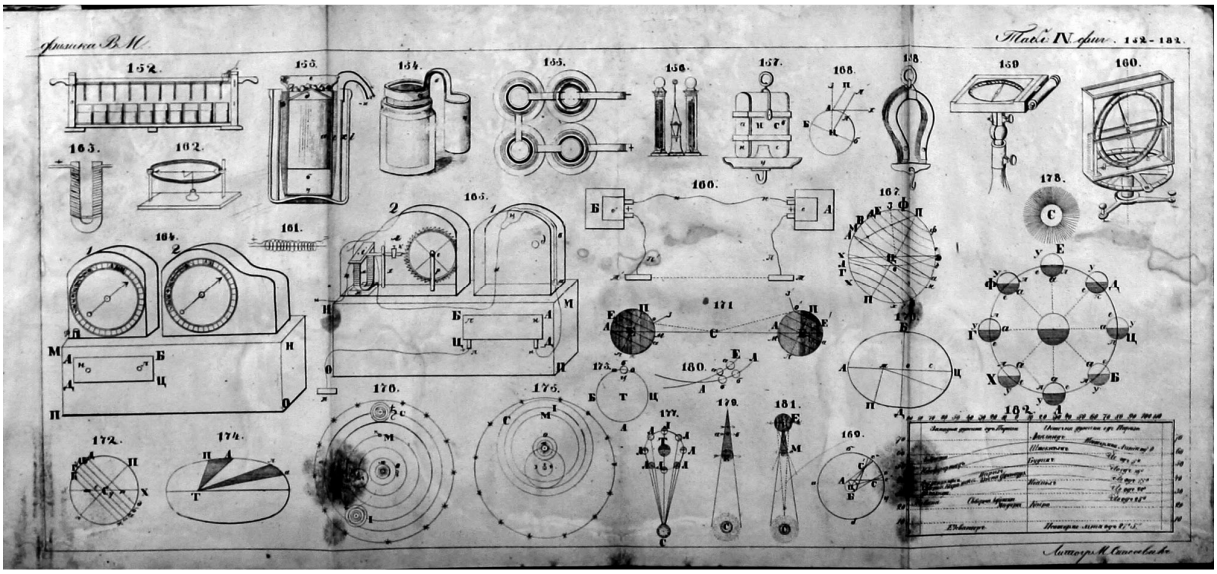


Table with drawings from the book

The book is written not only for students, but also for the general public. This is probably the reason why it does not rely much on mathematics. By style, it resembles an encyclopedia or a lexicon as it consists of numerous thematic short articles. These articles in most cases can be read independently. At the end of the book there are

several tables of drawings of some instruments or illustration of physical phenomena described or discussed in the book.

The book contains many interesting details for today's readers, for example, the explanation of the origin of terms of physics and chemistry (p. 5), but also the wider meaning of the word *meteor*. Here this word denotes any physical or meteorological phenomenon appearing in the atmosphere or on the sky.

It is interesting that he did not mention his predecessors and their books on physics, Orfelin and his Eternal Calendar (1783), nor three volume Physics of Atanasije Stojković published in the period 1800-1803.

Conclusion

Vuk Marinković is an important and an outstanding physician and the professor of Physics at the Belgrade Lyceum in the middle of the XIX century. It is considered that he wrote the first text-book on physics in Serbian. This book is recently digitized and now it is deposited in the Virtual Library of the Faculty of Mathematics in Belgrade. We believe that this book, now widely accessible in digital form will witness the level of studies of physics in Belgrade at that time.

Acknowledgment

This work is partially supported by the grant of the Serbian Ministry of education, science and technology development under the project number III 44006. We are very grateful to the anonymous Referee for the constructive critics and valuable suggestions.

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80 Digitalizovane knjige Rudjera Boškovića

Ž. Mijajlović and N. Pejović,
Publ. Astron. Druš. Rudjer Bošković, Beograd
Beograd, vol. 17, pp. 329-336, 2018.
Zb. Konf. "Razvoj astronomije kod Srba IX,

Beograd, 18-22 april 2017",
(M. Dimitrijević, ed.).
ISBN 978-86-89035-10-0 (ADRB).

This article is a translation into Serbian of the paper "Works of Rudjer Bošković in Virtual Library of the Faculty of Mathematics in Belgrade" with some additions.

Зборник радова конференције "Развој астрономије код Срба IX"
Београд, 18-22. април 2017,
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 17, 2018, 329-336

ДИГИТАЛИЗОВАНЕ КЊИГЕ РУЂЕРА БОШКОВИЋА

ЖАРКО МИЈАЈЛОВИЋ и НАДЕЖДА ПЕЈОВИЋ

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Резиме: Виртуелна библиотека, http://elibrary.matf.bg.ac.rs, Математичког факултета у Београду садржи око 100 дигитализованих књига из астрономије које су написали углавном српски научници. У овом рукопису представљене су четири књиге Руђера Бошковића које су дигитализоване и налазе се у Виртуелној библиотеци

Кључне речи: Руђер Бошковић, Виртуелна библиотека, историја астрономије

ВИРТУЕЛНА БИБЛИОТЕКА

Општи циљ Виртуелне библиотеке Математичког факултета Универзитета у Београду, http://elibrary.matf.bg.ac.rs, је да се имплементира потпуна и повезана колекција ретро-дигитализованих књига и других дигиталних докумената. Главни део овог пројекта односи се на електронско архивирање, пре свега, старих рукописа, њихово објављивање у електронском фото-типском облику и презентацију широкој публици. Пројекат је у почетку био наклоњен математици, али од 2009. у Библиотеку се такође постављају књиге из других наука и књижевности. Предност ипак имају књиге српских аутора и дела која се односе на научни и културни простор Југоисточне Европе.

Неке од књига у Библиотеци су ретке и зна се да их је преостало тек по неколико примерака у штампаном облику. Сматрали смо да је веома важно да се очува њихово постојање у неком облику. Не само као културно или научно наслеђе важно за нашу земљу, већ и као део Светске баштине. Одлучили смо да се оне сачувају и представе широкој публици у електронској, дигитализованој форми. Пројекат је започело 2007. неколико волонтера. Сада Библиотека има снажну подршку од стране Математичког факултета Универзитета у Београду, Математичког института, Српске

академије наука и уметности, Националног центра за дигитализацију и у пројектима Министарства за науку Србије.

Виртуелна библиотека Математичког факултета, највећа је база дигитализованих текстова у Србији са слободним приступом. У тренутку писања овог текста, Библиотека садржи близу 4000 књига. У оквиру Библиотеке постоје неколико важних колекција. На пример, значајна је збирка коју чине 400 докторских дисертација из математичких наука (од којих је највећи део одбрањен на Математичком факултету). Другу важну колекцију чине ретке књиге из 18. и 19. века. Већи део ових књига су раритети, неке од њих постоје само у једном примерку у штампаном облику и фактички су недоступне јавности. Важан део Библиотеке чине сабрана дела неких водећих српских научника из прошлости: Богдана Гавриловића, Милутина Миланковића, Ђуре Курепе, Ђорђа Станојевића и неколико других. Радови у вези са Виртуелном библиотеком објављују су у часопису НЦД Преглед који издаје Математички фаултет.

Овде ћемо представити колекцију књига Руђера Бошковића (1711-1787) које се налазе у дигитализованом облику у Виртуелној библиотеци. Ове књиге су прва дела из науке неког писца српског порекла. Напоменимо да се у Виртуелној библиотеци налазе књиге и других српских писаца из тог раздобља. Наиме, убрзо по објављивању Бошковићевих књига, у другој половини XVIII века, појавила су се прва дела из науке писана на старосрпском:

Василије Дамјановић (1734–1792), *Аритметика*, 1767, Венеција. Ово дело је прва српска књига из математике. Постоји само један познат штампан примерак ове књиге и он се налази у Библиотеци Матице српске.

Захарије Стефановић Орфелин (1726-1785), *Вечни календар,* 1783, Беч. Велики део ове књиге посвећен је описима и објашњењима астрономских феномена. Отуда можемо сматрати да је ово дело прва српска књига из астрономије. Штампани примерак ове књиге налази се у Библиотеци Астрономске опсерваторије у Београду.

Атанасије Стојковић (1773-1832), *Физика,* 1800, Будим. Ово тротомно дело је прва српска књига из физике. Штампани примерак ове књиге налази се у Библиотеци САНУ.

Дигиталне копије ових књига налазе се у Виртуелној библиотеци.

Споменимо овде једну општу карактеристику ових првих српских научника. Њих је красила универзалност, дакле астрономија или математика није била њихова једина специјалност. Исто тако били су успешни и у другим наукама, физици или филозофији, на пример. За њих је наука пре свега био поглед на јединствено биће – природу. Језиком науке служили су се да опишу природу тог бића. Поред свега, често су били успешни политичари, уметници, путници, писци романа и песама. Њихов значај за развој српске културе, језика и науке је огроман. Ове књиге имале су примарно просветитељски катактер. Писци су своја дела посвећивали

српском народу, или како то Орфелин на једном месту у свом *Календару* пише "на корист Славено-српског народа".

БОШКОВИЋЕВА ДЕЛА У ВИРТУЕЛНОЈ БИБЛИОТЕЦИ

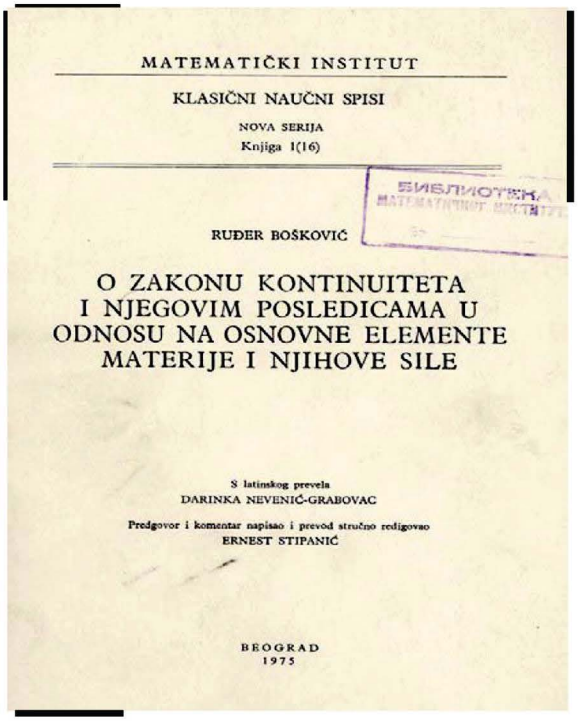
У овој прилици нећемо се упуштати у биографију Руђера Бошковића (1711-1787), о томе постоје многа опширна дела. Ипак споменимо да је Бошковић био највећи математичар и астроном Дубровника и један је од најзначајнијих научника свога доба, уврштен међу 100 најзначајнијих Срба свих времена. Био је професор универзитета, оснивач Миланске опсерваторије и директор Оптичког института француске морнарице. Био је универзалан стваралац: теолог, филозоф, математичар, астроном, физичар, геолог, архитекта, археолог, конструктор, оптичар, дипломата, путописац, професор, песник и полиглота. У историји Католичке цркве забележен је као значајан припадник језуитског реда. Бошковић је рођен 18. маја 1711. као седмо дете трговца Николе Бошковића, Србина из Орахова Дола код Требиња у Херцеговини, и мајке Павле, италијанског порекла, из породице Бара Бетере, познатог дубровачког песника. Живео је углавном у Италији, где је и стекао светску славу. Умро је 13. фебруара 1787 у Милану.

У Виртуелној библиотеци налазе se четири дигитализована Бошковићева дела. Поред основних библиографских података о тим књигама дајемо нешто шири осврт на књигу *Elementorum Universae Matheseos* . Напоменимо да је Руђер Бошковић писао своја дела углавном на латинском и италијанском.

Elementorum Universae Matheseos - Tomus I-III (*Елементи опште математике*), 1757, Venetis. Штампано издање ове тротомне књиге налази се у библиотеци Математичког института САНУ.

О закону континуитета и његовим последицама у односу на основне елементе материје и њихове силе, превод Даринке Невенић-Грабовац, предговор и коментаре написао Ернест Стипанић, 1975, Београд. Ова књига је превод са латинског дела *De continuitatis lege et eius consectariis pertinentibus ad prima materiae elementa eorumque vires dissertatio*, Romae, 1754.

A Theory of Natural Philosophy, Latin-English edition, London, tranls. J. M. Child, *London, 1922, Introduction*: B. Petronijević. Ова књига је упоредно латинско-енглеско издање дела *Theoria philosophiae naturalis*, Venetis, 1763. Штампани примерак Лондонског издања Бошковићеве *Теорије* налази се у библиотеци Астрономске опсерваторије у Београду.


MATEMATIČKI INSTITUT

KLASIČNI NAUČNI SPISI

NOVA SERIJA
Knjiga 1(16)

БИБЛИОТЕКА
МАТЕМАТИЧКОГ ИНСТИТУТА

RUĐER BOŠKOVIĆ

O ZAKONU KONTINUITETA
I NJEGOVIM POSLEDICAMA U
ODNOSU NA OSNOVNE ELEMENTE
MATERIJE I NJIHOVE SILE

S latinskog preveli
DARINKA NEVENIĆ-GRABOVAC

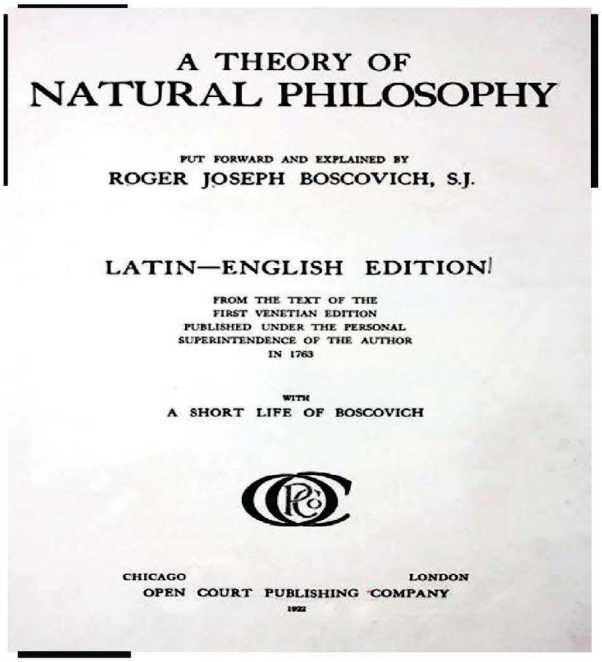
Predgovor i komentar napisao i prevod stručno redigovao
ERNEST STIPANIĆ

BEOGRAD
1975


DE
CONTINUITATIS LEGE
ET EJUS CONSECTARIIS
PERTINENTIBUS
AD PRIMA MATERIÆ ELEMENTA
EORUMQUE VIRES
DISSERTATIO
AUCTORE
P. ROGERIO JOSEPHO
BOSCOVICH
SOCIETATIS JESU PUBLICO MATHESEOS
PROFESSORE
IN COLLEGIO ROMANO.

ROMÆ MDCCLIV.
EX TYPOGRAPHIA GENEROSI SALOMONI
APUD VENANTIUM MONALDINI BIBLIOPOLAM IN VIA CURSUS.
PRÆSIDUM FACULTATE.

О закону континуитета и његовим последицама у односу на основне елементе материје и њихове силе, приредио Е. Стипанић, 1975, Београд.


A THEORY OF
NATURAL PHILOSOPHY

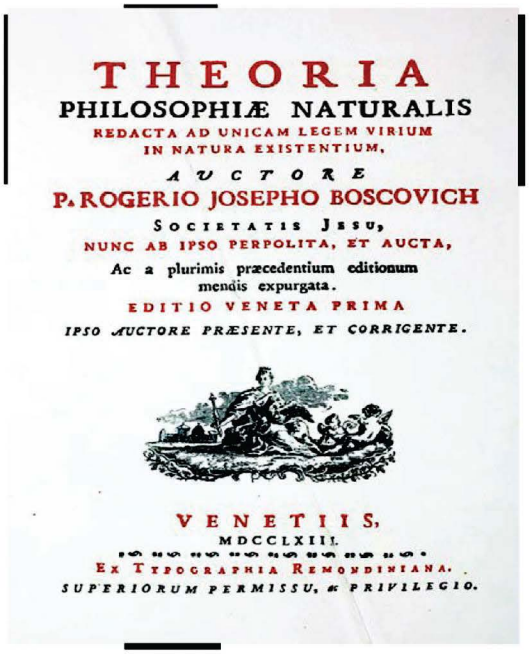
PUT FORWARD AND EXPLAINED BY
ROGER JOSEPH BOSCOVICH, S.J.

LATIN—ENGLISH EDITION

FROM THE TEXT OF THE
FIRST VENETIAN EDITION
PUBLISHED UNDER THE PERSONAL
SUPERINTENDENCE OF THE AUTHOR
IN 1763

WITH
A SHORT LIFE OF BOSCOVICH

CHICAGO LONDON
OPEN COURT PUBLISHING COMPANY
1922


THEORIA
PHILOSOPHIÆ NATURALIS
REDACTA AD UNICAM LEGEM VIRIUM
IN NATURA EXISTENTIUM,
AUCTORE
P. ROGERIO JOSEPHO BOSCOVICH
SOCIETATIS JESU,
NUNC AB IPSO PERPOLITA, ET AUCTA,
Ac a plurimis præcedentium editionum
mendis expurgata.
EDITIO VENETA PRIMA
IPSO AUCTORE PRÆSENTE, ET CORRIGENTE.

VENETIIS,
MDCCLXIII.
EX TYPOGRAPHIA REMONDINIANA.
SUPERIORUM PERMISSU, & PRIVILEGIO.

Упоредно латинско-енглеско издање *Теорије природне филозофије* приредио J. M. Child, Лондон, 1922.

Дневник са пута из Цариграда у Пољску (1762), превод Душана Нељековића, 1937, Београд, назив оригина *Giornale di un viaggio da Constantinopoli in Polonia dell'abate Ruggiero Giuseppe Boscovich, con una sua relazione delle rovine di Troia*, Venetis, 1784.

Рукопис *Закон или принцип континуитета* је Бошковићу главно полазиште и путоказ у његовој изградњи теорије сила које владају у природи и у вези са тим теорије о саставу материје. Зато је расправа *О закону континуитета и његовим последицама и односу на основне елементе материје и њихове силе* у најужој вези са Бошковићевом теоријом природне филозофије. Она, поред осталих расправа *(О живим силама,* Рим, 1745; *О светлости,* први и други део, Рим, 1748; *О дељивости материје и почелима тела,* написана 1748, објављена у Луки, 1757; *О закону сила што постоје и природи*, Рим, 1755), представља основу из којих је произашло главно Бошковићево дело *Теорија природне филозофије сведена на један једини закон сила што постоје у природи* (*Philosophiae natura1is theoria redacta ad unicam legem virium in natura existentium, Viennae*; 1758).

Отуда, одмах у другом параграфу, Бошковић истиче да је у расправи "читава анализа брижљиво припремљена да би нас од најједноставнијих принципа *природе нужним и спонтаним сплетом закључака довела до саме теорије*". То довољно говори од каквог је ова расправа значаја за Бошковићев научни и филозофски опус. Из тог разлога је, према речима приређивача професора Стипанића, Математички институт САНУ објавио превод ове књиге на наш језик са коментарима.

Бошковићев *magnum opus Теорија природне филозофије* свакако је његово најзначајније дело. Прво издање ове књиге штампано је 1758. у Бечу. У Виртуелној библиотеци налази се дигитална копија упоредног енглеско – латинско издања, штампаног у Лондону 1922. Књигу је приредио James Mark Child према другом издању Бошковићеве књиге штампане у Венецији 1763. Друго издање *Теорије природне филозофије* проширено и употпуњено је у односу на прво издање и штампано је под Бошковићевом супервизијом. У књизи се налази кратак Бошковићев животопис који је написао угледни српски филозоф Бранислав Петронијевић.

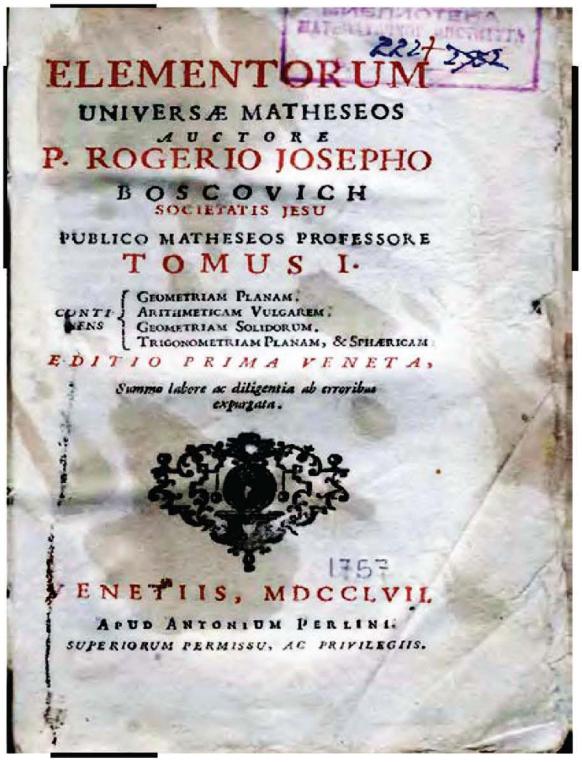
Књига *Елементи опште математике* по свој је прилици представљала уџбеник намењен Бошковићевим студентима. Да је то случај, уверавамо се на многобројним примерима у књизи и великом броју задатака. Занимљиво је да у многим рукописима и на многим местима на Интернету пише да је књига штампана 1754. у Риму, мада у Бошковићевој библиографији која се може наћи у поузданим изворима таквог податка нема. Дигитална копија ове књиге потиче од издања из 1757, штампаног у Венецији (импресум: *Editio Prima Veneta*, MDCCLVII). Исти је случај са дигиталним копијама које се налазе у великим дигиталним библиотекама *Google books* и *Europeana*.

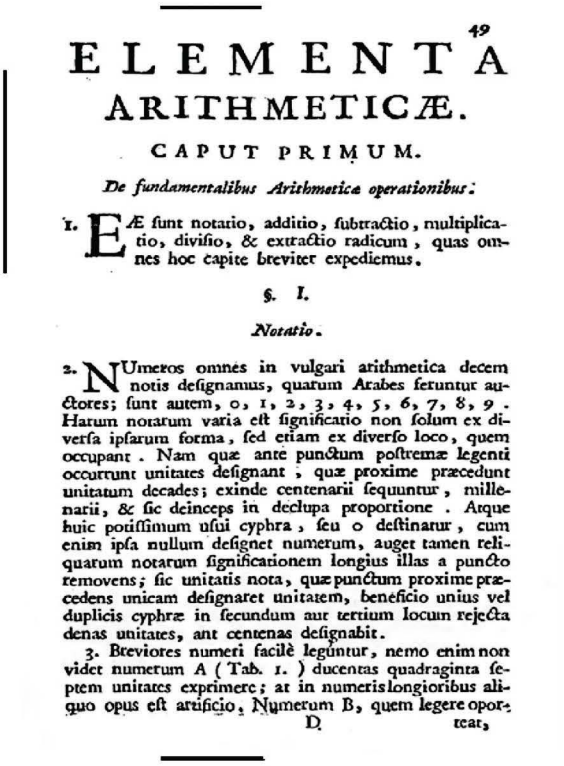
Део првог тома *Елемената* односи се на елементарну геометрију, с тим да излагање углавном прати *Еуклидове елементе*. У другим поглављима

описују се особине основних аритметичких операција: излаже се рачун са рационалним бројевима, особине пропорција, затим аритметичке и геометријске прогресије, логаритми и најзад елементи равне и сферне тригонометрије.

Други том, *Algebra Finitam*, у потпуности је посвећена решавању алгебарских једначина. Најпре се описују особине полинома и рачун са радикалима. Затим се излаже Карданов (Gerolamo Cardano, 1501-1576) метод за решавање кубне једначине и Фераријево (Lodovico Ferrari, 1522-1565) решење алгебарске једначине четвртог степена. У овом рачуну користи комплексне бројеве (*quantitates imaginarae*), што је било реткост у то време. Бошковић такође даје приближне, нумеричке, методе за решавање неких алгебарских једначина већег степена за које ће се касније испоставити да их и није могуће решити алгебарски, помоћу радикала (Niels Abell, Evariste Galois, прва половина 19. века)

Свакако је најзанимљивији трећи том *Елемената.* Бошковићева теорија кривих другог реда, коју излаже у овом делу *Елемената* била је прва потпуна и систематична теорија коника (конусних пресека). Овде се налазе многе нове идеје а у излагању користи се чисто геометријским аргументима. Мада је већ увелико био развијен метод Декартовог система координата, Бошковић у свом делу не користи овај аналитички приступ.


ELEMENTORUM
UNIVERSÆ MATHESEOS
AUCTORE
P. ROGERIO JOSEPHO
BOSCOVICH
SOCIETATIS JESU
PUBLICO MATHESEOS PROFESSORE
TOMUS I.
CONTINENS
GEOMETRIAM PLANAM,
ARITHMETICAM VULGAREM,
GEOMETRIAM SOLIDORUM,
TRIGONOMETRIAM PLANAM, & SPHÆRICAM
EDITIO PRIMA VENETA,
Summo labore ac diligentia ab erroribus expurgata.
1757
VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.


49
ELEMENTA
ARITHMETICÆ.
CAPUT PRIMUM.
De fundamentalibus Arithmeticæ operationibus:

1. EÆ sunt notatio, additio, subtractio, multiplicatio, divisio, & extractio radicum, quas omnes hoc capite breviter expediemus.

§. I.

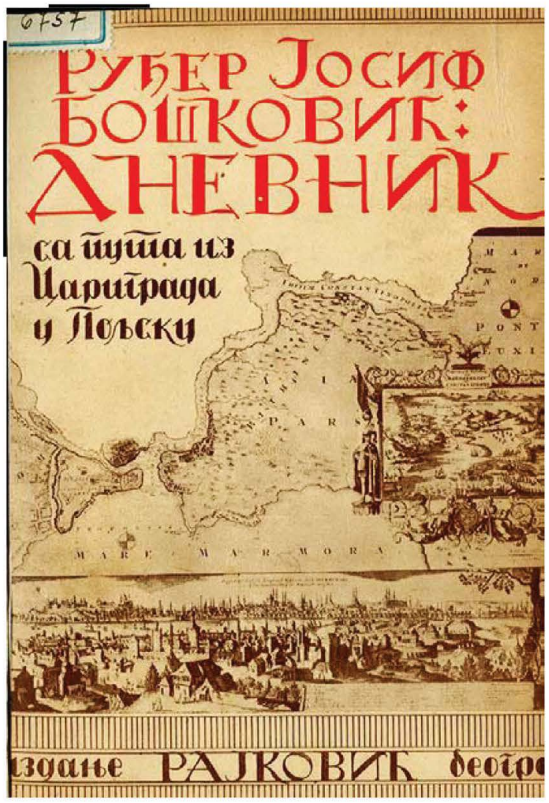
Notatio.

2. NUmeros omnes in vulgari arithmetica decem notis designamus, quarum Arabes feruntur auctores; sunt autem, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Harum notarum varia est significatio non solum ex diversa ipsarum forma, sed etiam ex diverso loco, quem occupant. Nam quæ ante punctum postremæ legenti occurrunt unitates designant, quæ proxime præcedunt unitatum decades; exinde centenarii sequuntur, millenarii, & sic deinceps in declupa proportione. Atque huic potissimum usui cyphra, seu 0 destinatur, cum enim ipsa nullum designet numerum, auget tamen reliquarum notarum significationem longius illas a puncto removens; sic unitatis nota, quæ punctum proxime præcedens unicam designaret unitatem, beneficio unius vel duplicis cyphræ in secundum aut tertium locum rejecta denas unitates, aut centenas designabit.

3. Breviores numeri facilè leguntur, nemo enim non videt numerum A (Tab. 1.) ducentas quadraginta septem unitates exprimere; at in numeris longioribus aliquo opus est artificio. Numerum B, quem legere oportet,

D

Елементи опште математике, Венеција, 1757.





РУЂЕР ЈОСИФ БОШКОВИЋ

Странице из Бошковићевог *Дневника са пута из Цариграда у Пољску* превод Д. Недељковић, Београд, 1937.

Специјално, као дефиницију коника из које изводи њихова својства изабрао је следећу: коника је геометријско место тачака у равни за које је однос удаљености до фиксиране тачке (фокуса) и дате праве (директрисе) константа. Тај количник назвао је *ratio determinans* конике, за који данас иначе користимо назив ексцентрицитет. Мада је ово својство било познато већ старогрчким математичарима, посебно Папусу Александријском, никад раније није узимано за дефиницију. У савременој аналитичкој геометрији, дефиниција кривих другог реда у највећем броју случајева прати управо Бошковићев приступ. Пре Бошковића, за конике су узимане равне криве које настају пресеком конуса и равни, дакле у дефиницији се претпоставља тродимензионални простор. Код њега теорија коника постаје теорија кривих *у равни*.

Увео је и врло користан метод за доказивање разних исказа о коникама. То је позната Бошковићева ексцентрична кружница, тј. кружница с произвољним центром која има својство да јој је однос полупречника и удаљености средишта до директрисе једнак ексцентритету. Користећи овај појам, Бошковић је описао разне конструкције и доказао многа својства коника тако што их је изводио из својстава кружница користећи кореспонденцију између тачака ексцентричне кружнице и конике. У прилогу

De transformatione locorom geometricorum трећем тому *Елемената*, Бошковић је представио потпуно нову теорију геометријских трансформација, које су у основи колинеације, па се стога може сматрати претечом развоја пројективне геометрије која настаје тек у 19. веку.

РУКОПИСИ ДРУГИХ АУТОРА О БОШКОВИЋЕВОМ ДЕЛУ

У Виртуелној библиотеци налазе се такође књиге и рукописи који се односе на живот и дело Руђера Бошковића. Свакако је најзанимљивија ова књига:

Коста Стојановић, *Атомистика – један део из философије Руђера Јосифа Бошковића*, Ниш, 1891. Књига је заправо једна компаративна анализа Бошковићеве природне филозофије.

Поред ове у Библиотеци се налазе и друге књиге Косте Стојановића. Стојановић је био свестрани интелектуалац и професор Велике школе у Београду. Поставио је темеље теоријске механике у Србији и написао је *Основе теорије економских вредности*, вероватно најбољу књигу на српском из економских наука. Био је народни посланик и министар у влади Николе Пашића.

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Mijajlović, Ž., Ognjanović, Z., Pejović, A.: 2008, "Internet presentations of mathematical works in Serbia", *Преглед НЦД* **12**, 43–48.

Mijajlović, Ž., Ognjanović, Z., Pejović, A.: 2010, "Digitization of Mathematical Editions in Serbia", *Mathematics in Computer Science*, **3(3)**, 251-263.

Pejović, N.: 2008, "Digitization of mathematical textbooks used in Serbia in the past", *Преглед НЦД* **12**, 55–64.

Преглед НЦД, http://elib.mi.sanu.ac.rs.

Virtual Library, http://elibrary.matf.bg.ac.rs.

WORKS OF RUĐER BOŠKOVIĆ IN VIRTUAL LIBRARY OF FACULTY OF MATHEMATICS

Virtual Library, http://elibrary.matf.bg.ac.rs, Faculty of Mathematics in Belgrade, contains about 100 digitized books on astronomy, written mostly by Serbian scientists. In this manuscript four books written by Ruđer Bošković that are digitized and are deposited in the Virtual Library are presented.

Keywords: Ruđer Bošković, Virtual Library, History of Astronomy

81 First Serbian works on theory of relativity

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It is usually considered that special theory of relativity originated in 1905 when Albert Einstein published his first paper to this subject. Very soon, already in the twenties of XX century several Serbian mathematicians and astronomers also wrote papers on special theory of relativity. Some of the papers were popular explanation of basics of this theory and they were intended to the general public, but there were also papers with original contributions. All papers were written in Serbian and certainly this is the main reason that these results were neglected. This article presents works in this field of a great Serbian astronomer and climatologist Milutin Milankovićc (1879-1958), a great Serbian mathematician and the founder of the Serbian mathematical school Mihailo Petrović Alas (1868-1943) and Sima Marković (1888-1939), mathematician, philosopher and politician, also known as one of the founder of the Communist Party of Yugoslavia.

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FIRST SERBIAN WORKS ON THEORY OF RELATIVITY

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Abstract. Even if A. Michelson, H. Lorentz, H. Poincaré and others performed experiments and wrote papers to the subject known later as the special theory of relativity more than two decades before Albert Einstein published his paper on this topic in 1905, it is usually considered that special theory of relativity originated in this year. This theory was soon accepted and many authors wrote papers on this matter. It is not so well known that several Serbian mathematicians and astronomers also wrote papers on special theory of relativity already in the twenties of XX century. Some of the papers were popular explanation of basics of this theory and they were intended to the general public, but there were also papers with original contributions. All papers were written in Serbian and certainly this is the main reason that these results were neglected. We shall present works in this field of a great Serbian astronomer and climatologist Milutin Milanković (1879-1958), a great Serbian mathematician and the founder of the Serbian mathematical school Mihailo Petrović Alas (1868-1943) and Sima Marković (1888-1939), mathematician, philosopher and politician, also known as one of the founder of the Communist Party of Yugoslavia.

1. INTRODUCTION

Albert Michelson, Hendrik Lorentz, Henri Poincaré and others performed experiments and wrote papers on the subject known later as the special theory of relativity in the interval ranging more than two decades before Albert Einstein published his first paper on this topic in 1905. However, it is usually considered that special theory of relativity originated in this year. A decade later the general theory of relativity was founded in the works of David Hilbert and Albert Einstein. These theories were soon accepted and many authors wrote papers on this matter. It is not so well known that several Serbian mathematicians and astronomers also wrote papers on the theory of relativity already in the

twenties of XX century. Some of the papers were popular explanation of basics of these theories and they were intended to the general public, but there were also papers with original contributions. All papers were written in Serbian and certainly this is the main reason that results offered there were neglected. We shall present works in this field of a great Serbian mathematician and the founder of the Serbian mathematical school Mihailo Petrović Alas (1868-1943), the great Serbian astronomer and climatologist Milutin Milanković (1879-1958), and Sima Marković (1888-1939), the mathematician, philosopher and politician (Figure 1).

There are some common details in personalities and writings of all three authors. Their first characteristic is the universality. In terms of scientific work, all three belong to a specific time. Due to the accumulated scope of scientific and mathematical knowledge, it was difficult, if not impossible, for an individual to know the whole body of natural sciences and mathematics. The time of universal mathematicians and scientists was slowly passing by. By understanding and the wideness of scientific work in mathematics, mechanics and philosophy, Henri Poincaré, French mechanist and mathematician, was certainly one of the last homo universal of sciences. And it was Poincaré who was one of the professors of Mihailo Petrović. After a later work in science, we can conclude that the spirit of universalism of his professor passed on to Petrović. Sima Marković was a Petrović's student and this universal determination and the wide scientific curiosity certainly was transferred from Professor to his student. A similar characteristic feature was presented in the work of Milutin Milanković. Hence all of them were not working only in their basic fields, but in many other areas as well, including theory of relativity. Their second characteristic is associated to the relationship of mathematics and physics. All three had a point of view that both disciplines are highly interconnected. Petrović even invented the whole new philosophical system, mathematical phenomenology, in which the main point in science should have establishing analogies, including physically disparate phenomena, by reducing to the same abstract essence. Of course, mathematics must have the main role in this reduction, in Petrović's case that were differential equations. The greatest scientific achievement of Milutin Milanković was insolation theory which characterizes the climates of all the planets of the Solar system and explanation of Earth's long-term climate changes caused by changes in position of the Earth in comparison to the Sun. The main tool in the development of his theories had advanced mathematical method and equations of celestial mechanics, the subject which was considered then as applied mathematics. It took several decades to find empirical evidences which led to recognition and the final acceptance of his theory. As we shall see, Marković had a similar, or even stronger attitude to the ties of physics and mathematics. All of them were very good popularizers of science and that was their third characteristic. They wrote many articles and books which appreciate science, ranging from history and philosophy of science to ichthyology and travelogues. They were very good writers and some of their books are still very popular and a part of obligatory school reading. Just to mention Milanković's Trough distant worlds and times (Kroz vasionu i vekove),

which consists of 37 letters on science addressed to an unnamed yang lady and the Petrović's books In the pirate's country (U zemlji gusara) and Eel's story (Roman jegulje).



Figure 1: From left to right: Mihailo Petrović, Sima Marković and Milutin Milanković.

Their writings on relativity appeared in the early stage of development of this theory. When their books appeared, there were discoveries that were still waiting to emerge. For example, Petrović's writing was printed in 1921, while Friedmann equations appeared in 1922. Marković's book was published in 1924, several years before Georges Lemaître and Edwin Hubble discovered the Universe expansion and the property of the universe what is now known as the Hubble law. They had a positive attitude to the theory of relativity and were very curious about it. But this not was the case in general. There were many opponents in science and even more in broad parts of general community, sometimes very hostile. To illustrate these circumstances, we mention the following extreme opinion of Justin Popović (1894-1979), Eastern Orthodox theologian, archimandrite and professor of Belgrade University (at the Faculty of Theology): "No doubt, relativism is also the logic, and the nature and soul of humanism. Einstein's theory of relativity is the final, aggregate resultant of humanism and all its philosophical, scientific technical and political branches. But not only this, nevertheless in the last line to its own, humanism is nothing else but - nihilism... Humanism inevitably evolves into atheism, passes through anarchism and ends with nihilism. If anyone today is an atheist, know, if he is consistent, tomorrow he will be an anarchist, and the day after tomorrow, a nihilist. And who is a nihilist, know, that it has come to him from humanism through atheism... Humanism is in fact the basic evil, the original evil of man." Justin Popović was proclaimed a saint in 2010.

2. MIHAILO PETROVIĆ ALAS

Mihailo Petrović was an outstanding mathematician and a colorful character of the public life of Belgrade in the first half of the twentieth century. He studied mathematics in Belgrade and Paris at Sorbonne (École normale supérieure) where he got diplomas in mathematics and physics. In the year 1894 he defended his doctoral dissertation on differential equations under supervision of the prominent mathematicians Charles Hermite and Charles Émile Picard. In the same year he returned to Belgrade and was elected for the professor of mathematics at the Grand School, the later University of Belgrade. He was scientifically very productive and soon became the leading Serbian mathematician. When he was only 32, he was elected as the full member of the Serbian Academy of Sciences. As the representative of French mathematical school, he brought in Belgrade the spirit of the contemporary mathematics, particularly in analysis and ordinary differential equations. He has the great merit for the development of mathematics in Serbia and by the general opinion, he is considered as the founder of Serbian mathematical school. For example he was the supervisor for all doctoral dissertations in Serbia until the Second World War and the founder of the first international Serbian mathematical journal.

Mihailo Petrović had a rich, interesting and an unconventional life. It is even difficult to list in one place everything that Petrović was dealing with, let alone to describe it. In addition to the interest in various branches in mathematics, we meet him in many other, often unexpected places. He was a writer of laws and proposals of interstate agreements, but also an inventor and owner of successful and realized patents. Petrović is the creator of an original theory in natural philosophy, mathematical phenomenology and by many he is included in the group of Serbian most important philosophers. He wrote several nice and interesting novels and some of them are a favorite part of Serbian youth literature. He wrote scientific papers and was interested in other natural sciences, e.g. in ichthyology, astronomy, theory of relativity, and chemistry. He was the creator of the coding system and the main cryptographer of the Serbian and later of Yugoslav army. He played violin and led the music band "Suz", which had one of the main places in the bohemian life of Belgrade. Finally, he was a big and passionate fisherman and a great world traveler and sailor in the north and south seas.

Petrović wrote the first paper on relativity under the title "Theory of relativity" already in 1921. The paper had 12 pages and was published in somewhat unusual journal regarding its content, Serbian literary messenger. But this is understandable if one knows that the paper was popular in nature and intended for the general audience. There were not formulas and most of the paper is devoted to special theory of relativity. The significant part of the paper is devoted to the genesis of this theory. Beside the explanation of the basics, he mentioned contributions of various people to this theory, e.g. Michelson - Morley experiment, then Lorentz - FitzGerald contraction, Poincare and Einstein final

foundation of the theory. At the end he mentioned some phenomena related to the general theory of relativity, Mercury orbit precession and the light bending in the gravitational field. In this contexts he mentioned two expeditions to Principe (the northern island of São Tomé and Príncipe) and Sorbel (Brazil) organized respectively by Eddington and Crommelin in 1919. These famous expeditions had the goal to test Einstein prediction, the stars rays bending during the Solar eclipse.

The second paper, "The physical constants in the relativity theory" was published in Глас of the Serbian academy of science in 1927. The paper had 16 pages and was dealing with the measurement of time. The basic idea of the paper is "tacit assumptions", as Petrović wrote, that the physical constant, such as coefficient of electrical conductivity, are invariant from transitions from one inertial coordinate system to the another one. Petrović recalls a device and an experiment produced in 1887 by Gabriel Lippmann which he made for time measuring. The whole idea was that a certain interval time Δt is proportional to the electrical resistance ρ of mercury and that Δt can be measured by Lippman's device. Petrović discussed there two possibilities. The first conclusion was that the measurement of ρ and ρ' in two relativistic systems S and S' would depend not only on the speed of S and S' but also on the orientation of apparatus and even their design. If the absoluteness of ρ is assumed, i.e. $\rho = \rho'$ then we would have $\Delta t = \Delta t'$ what, of course, contradicts the special theory of relativity. Petrović's conclusion was that according to the theory of relativity physical constants do not have always the same values, they depend on how they are measured and they may be accidental. He suggested how to overcome this antinomy by proposing general forms of transformation for values of physical constants in two relativistic system. Petrović most probably met Lippman and was introduced in his ideas during his studies in Sorbonne since Lippmann there was professor of mathematics (since 1883) and physics (since 1886). Finally, we should mention in the favor of Petrovi's paper that the transformation rules for physical values in two relativistic systems in general is still open. Probably the most known example of this type is the relativistic transformation of temperature. M. Planck proposed a transformation formula in 1908, but H. Ott corrected it in the quite opposite way in 1963 (see H. Ott: Zeits. Phys. (1963) 175, 70).

Petrović's third paper on relativity was Physical etalons for time measuring, see Etalons physiques de temps, Publ. de l'Observatorie astronomiquc de l'Université de Belgrade, 1933, t. 11, pp. 5-10. The paper was published in 1933 and in the greatest part it is a compilation of his previous paper on this matter.

3. SIMA MARKOVIĆ

Sima Marković was a talented mathematician, gifted philosopher and a tragic figure at the Yugoslavian political scene and of the communist movement. He finished studies in mathematics with honor in 1911 and immediately became an assistant of Professor Petrović at the mathematical department of the Belgrade University. He completed his doctoral dissertation on Riccati differential equation

in 1913, but the beginning of the First World War broke the academic activities in Serbia and his scientific work as well. After the war he started his political career, was a member of the parliament and in 1920 he was elected together with Filip Filipović, also a mathematician, for the first general secretary of Yugoslav communist party. In the same year the University nominated him for the assistant professor but due to his political activities, the Yugoslav king Alexandar did not want to sign the proclamation for this appointment. Soon he was expelled from the university and even more during twenties he was sent to prison several times where he spent altogether four years. In the thirties he moved to Moscow where he was the scientific associate at the Philosophical institute of the USSR Academy of Sciences, but had high positions in the communist establishment, too. He was also involved in the fractional fights in the party. Seemingly, he was on the wrong side as in 1939, April 19th, in Stalin's purges he was sentenced to death by shooting. He was executed in the same day. Ironically, he did not support offensive or violent methods in politics as he believed in means of parliamentary democracy, in the institutional and legal methods.

Marković's main interest in mathematics was teaching, education and methodic and he wrote several works on these topics. However, his main concern was the political philosophy and to some extent philosophy of science, particularly views based on physics. He wrote several books and booklets on the philosophy of science and related topics. For example, in the book The Principle of Causality and Modern Physics, Marković emphasizes the connection between physics and mathematics. He believed that the key feature of modern physics is its mathematics and that mathematics as all other sciences originates from the experience and from the needs of the everyday life. He also wrote in twenties a short book (80 pages) on the theory of relativity and a longer paper (31 pages) on the same topic published five years later. It interesting that he wrote the book during his stay in prison. The paper is more or less a compilation of the book, so we shall give some comments only on the book.

Sima Marković published his first work on relativity, the book Теорија релативитета, популарна-научна скица (Theory of relativity, a popular-scientific sketch) in Belgrade in 1924 (Figure 2). As the title says, the book is intended to the broad audience, particularly to those readers who are inclined to physics and science in general. It is interesting that the author in the foreword writes: "Even if Theory of relativity is physical by content, by form it still belongs to the most abstract parts of Theoretical mathematics". He glorifies mathematics often and compare it to the highest men's achievements. However, he did not use in the book a mathematical language, except few times some very simple formulas. He explains the main goal of the book as follows: "In this sketch, only the most important facts that characterize the theory of relativity are exposed to the goal to which it has to serve. This sketch represents, therefore, only the skeleton of Relativity Theory, which cannot be revived without mathematics. But, I hope, more careful readers can still get at least an approximate idea of what the Relativity Theory is.". The book consists of two parts:

- Part 1. Special theory of relativity, pages 6 - 45.
- Part 2. General theory of relativity, pages 46 -80.

The first part starts with presentation of views of classical mechanics, Galileo relativity principle and Newton's cosmology: absolute space and absolute time. We could say that Introduction does not differ much from those in modern books, even if the book was written almost a century ago. Rest of the Part 1 is devoted to the explanation of basics of special theory relativity. Relativity of length and time are explained and shown why in this theory is not possible to talk about simultaneous events. Lorentz coordinate transformations, which he called Lorentz formulas, and the contraction factor γ are explained. He presents how special theory of relativity succeeded in the explanation of some old problems in astronomy such as the stellar aberration is. He also explained four-vectors, namely events in the Minkowski space. He ends this part with the very modern view that the physical laws are nothing else but the invariants under coordinate transformation.



Figure 2: Cover page of the Theory of relativity, a popular-scientific sketch book by Sima Marković.

The second part starts with the discussion of the equivalence principle which deals with gravitational and inertial mass and the role of gravitation in General theory of relativity. There are presented virtually then all known facts from this theory. He mentioned main tests of General theory of relativity such as perihelion precession of Mercury's orbit and the bending of light rays in the gravitational fields. There is mentioned the Eddington's measure of star light bending during the Solar eclipse in 1919, just few years before the book appeared. He implicitly

speaks about Riemann space explaining that General theory of relativity assumes a non Euclidian geometry. Some elements of differential geometry are mentioned such as the geodesics and the parallel transport needed for covariant derivative. It must be said that Marković showed the great skill in presenting these highly mathematized content, but not using a single mathematical symbol. He also described a phenomena such as the gravitational time dilation. He discussed also relation between the general theory of relativity and cosmology. He explains that the universe occupies a closed but unbounded space. As an application of the theory he deduced a formula of the size of the universe which depends only on the density ρ of the matter in the universe.

There are no references in the book, but the author mentioned that his book is based on the works of Einstein, Lorentz, Minkowski, Weyl, Riemann, Pauli, Helmholtz and few other physicist. The book is written in a nice and vivid language, and even today it could be interesting to a general reader.

4. MILUTIN MILANKOVIĆ

Milutin Milanković and his work are well known to astronomers. So we will highlight here just few facts from his life and work. Milanković is one of the Serbian greatest and most cited scientists of all times. His theory on the ice ages that accurately explains the change of climate on a large time scale is accepted world-wide. In recognition of his scientific achievements, a crater on the Moon, another one on the Mars and a planetoid were named after him. Besides his the most famous work Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem (published in 1941) and many other scientific papers, he also wrote an excellent book on celestial mechanics and books that popularize science. He is also known as a good civil engineer. Milanković came to Belgrade from Vienna in 1909 to teach applied mathematics at the Faculty of Philosophy of Belgrade University. His coming was a merit of Bogdan Gavrilović and Mihailo Petrović, who both taught mathematics at the University. Milanković was at the Belgrade University till his retirement in 1955, lecturing there applied mathematics. It should be mentioned that theoretical physics, mechanics and astronomy were considered then as the areas of applied mathematics. Milanković also was the first professor who started lecturing celestial mechanics at the University of Belgrade.

Milanković did two short excursions to relativity. He wrote his first paper "O teoriji Michelsonova eksperimenta" (On the theory of Michelson's experiment), Rad JAZU, vol. 190 pp 65-70 already in 1912. He was doing research in this theory in the period 1912-1924. In fact his papers on this matter were on special relativity and both are on Michelson experiment (now known as Michelson-Morley experiment) which gave the strong evidence against ether theory. He discussed there, in the light of the Michelson experiment, the validity of the second postulate of Special theory of relativity, that the speed of light is the same

in all reference frames. Tatomir Anđelić and Andrija Stojković wrote an extensive paper on Milanković's views expressed in theses papers, so we shall not enter into discussion of his works on this matter. We should just mention that he was teaching special theory of relativity, most probably in arrangement with Mihailo Petrović, at the Belgrade university in the twenties of the last century.

5. OTHER SERBIAN SCIENTISTS

There were few other Serbian scientists who were engaged in scientific work or wrote about the theory of relativity. Probably Mileva Einstein - Marić (1875-1948), a mathematician and the first Einstein's spouse, is the most known as a possible contributor to the theory of relativity. There is a big debate if she really had any major contribution to the early Einstein work. We will not enter in this discussion, mainly because we do not know her signed scientific papers. Vladimir Varićak (1865-1942), the well-known mathematician of the Serbian origin, was a professor at the University of Zagreb. He wrote first of all, already in 1910, a paper on special theory of relativity. It interesting that he was a Milanković's high-school teacher. Few physicists and philosophers, Milivoje Dobrosavljević, Ognjen Prica and Dušan Nedeljković also discussed and wrote before the Second World War on the theory of relativity in various contexts. Nikola Tesla, the famous inventers, was strongly against this theory, while Mihailo Pupin, the other famous Serbian electrical engineer ant the professor at the Columbia University in New York, accepted it (Anđelić, Stojković, 1983).

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82 Digitized university textbook of prof. Zaharije Brkić in Virtual Library of Faculty of Mathematics

V. Radović and N. Pejović
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This article presents three digitized works of astronomer and professor at the Department of astronomy of the Faculty of mathematics in Belgrade, Zaharije Brkić (1910-1979). These publications include his doctoral dissertation, which represents the first dissertation in astronomy that had been defended at the Belgrade university. The other two works are university textbooks written jointly with prof. Branislav Ševarlić.

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DIGITIZED UNIVERSITY TEXTBOOK OF PROF. ZAHARIJE BRKIĆ IN VIRTUAL LIBRARY OF FACULTY OF MATHEMATICS

Abstract. In the Virtual Library of the Faculty of Mathematics there are three digitized publications by professor Zaharije Brkić. These publications include his doctoral dissertation, which represents the first dissertation in astronomy that had been defended on Belgrade university. Also, two university textbooks written with prof. Branislav Ševarlić are digitized. In this paper, we give detailed analysis of these works along with explanations of their importance.

1. Introduction



Figure 1. Professor Zaharije Brkić photographed during the observations of total solar eclipse on Hvar (Croatia) in 1961 [3].

For the purpose of keeping and preserving a large number of rare books, textbooks, dissertations and papers, Virtual Library was built as a part of the Digitization project of the Faculty of Mathematics in Belgrade and the Mathematical Institute of the Serbian Academy of Sciences and Arts [1, 2]. Virtual Library is freely available and open via Internet link http://elibrary.matf.bg.ac.rs for general public. This project will allow future generations to better understand works of our prominent scientists, as well as to save past works from oblivion.

In this paper, we present digitized university textbooks written by Zaharije Brkić, who was an astronomer and respected professor at the University of Belgrade. Paper is organized as follows. In the next section we give a short biography of professor Zaharije Brkić. Third, fourth and fifth sections contain detailed analysis of two digitized textbooks and his dissertation.

2. Biography of professor Zaharije Brkić (1910-1979)

Zaharije Brkić (Figure 1) was born in Poljna (central Serbia, near municipality of Trstenik) at 1910. As a young pupil he showed great interest and affinity towards mathematics and physics. After he had finished grammar school in Čačak he started studying at the Faculty of Philosophy in Belgrade, as part of a group for theoretical

mathematics. He graduated at 1936 and two years afterwards he begun to work at the Astronomical observatory Belgrade. At observatory he was first an assistant but soon was promoted to researcher position. Due to his extensive knowledge of astronomical instruments, their usage and application for position determination he started working as an assistant professor on the Faculty of Civil Engineering at 1955. Two years after, on 1957. he was elected as a professor and he remained on that positions until he moved to Faculty of Natural sciences [3, 4].

Zaharije Brkić defended his doctoral dissertation in astronomy at Belgrade university on 26th September 1958. This was the first dissertation to have been defended at Belgrade university in astronomical sciences. His mentor, Vojislav Mišković, was the first person that had obtained doctorate in astronomy. On board for his defense were Tatomir Anđelić, full professor, Vojislav Mišković, full professor and academic, and Konstantin Voronjec as a corespondent member. One year after he got his doctoral degree he moved to department for mechanics and astronomy on Faculty of Natural sciences where he had been working until his retirement on July 1978. During his work at faculty, Zaharije Brkić was on board for multiple doctoral defenses: Branislav Ševarlić (1960), Vasilije Oskanjan (1961), Đorđe Teleki (1964) and Dragutin Đurović (1974) [3, 4].

Zaharije Brkić was one of the founders of the services for (1) time and changing of geographic latitude and (2) for longitude change and movement of Earth poles at the Astronomical observatory. Also, he published 34 scientific papers and 27 professional papers. Besides that, he is coauthor of two university textbooks: Geodetic astronomy and General astronomy. We should mentioned that General astronomy has two editions and it is still in use today. Both textbooks are digitized and available at Virtual Library. More detail about textbooks are given in the following sections.

Zaharije Brkić was married to Marica Marković with who he had two children: daughter Ružica and son Slobodan. Professor Zaharije Brkić died on 24th April 1979 after fighting a difficult desease. Shortly before his death, he was given medal for his scientific achievements and his influence on education for numerous generations of astronomers.

3. Doctoral Dissertation

As it was previous mentioned, doctoral dissetation written by Zaharije Brkić, was the first dissertation in astronomy defended at the University of Belgrade. Its title is "Analiza sistematskih gresaka pasažnog instrumenta i drugih sistematskih uticaja na određivanje vremena" (The analysis of systematic errors of passage instrument and other systematic influences on time determination). His dissertation is publicly available on link: http://elibrary.matf.bg.ac.rs/handle/123456789/22 [7].

Dissertation which was digitized is available in the library of the Faculty of Mathematics where it is classified as museum copy under the catalogue number "Dokt. 4/1". Unfortunately, this copy is damaged and some pages are barely visible (Figure 2).

Dissertation has 120 pages and its content is divided in two parts. First part is a description of chosen problem and historical remarks. Second part is where Zaharije Brkić presents his own scientific work. Whole dissertation is written using typewriter and sufficient copies are obtained using indigo paper. Graphics were multiplied using photosensitive paper which is a reason for gradually degrading of their quality (Figure 2). Historic and scientific significance of this dissertation makes this digitization and its preservation.

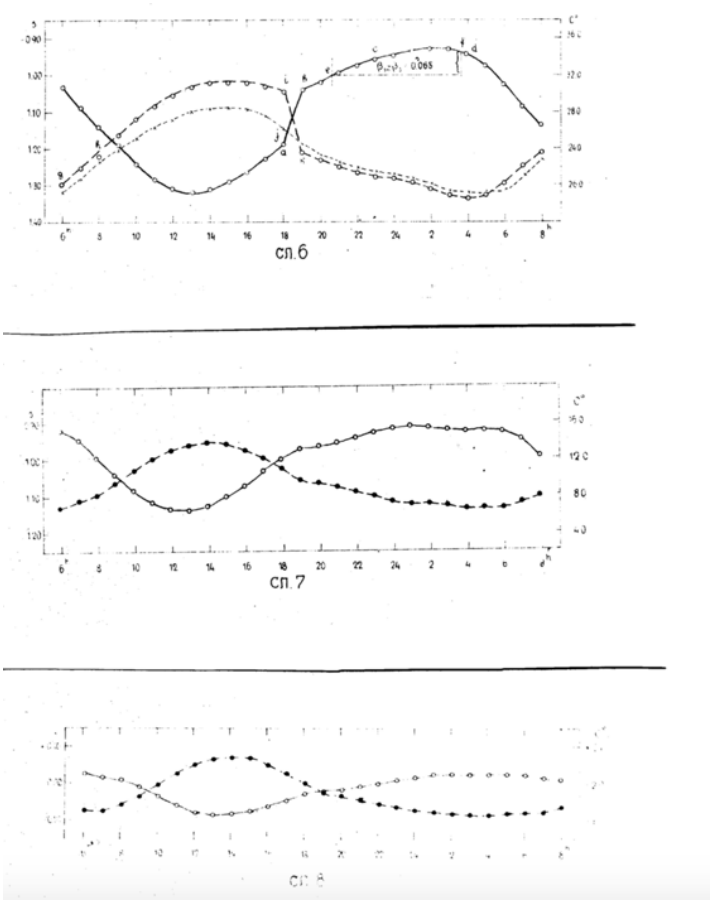


Figure 2. One page at the digitized version of doctoral dissertation of prof. Brkić. On this page, damages are clearly visible.

Authors of this paper have found that Miodrag Dačić has one copy of Zaharije Brkić‘s doctoral dissertation which is not damaged, and its digitization is on the way, and soon it will be available at the same link as already digitized version.

3. Geodetic astronomy

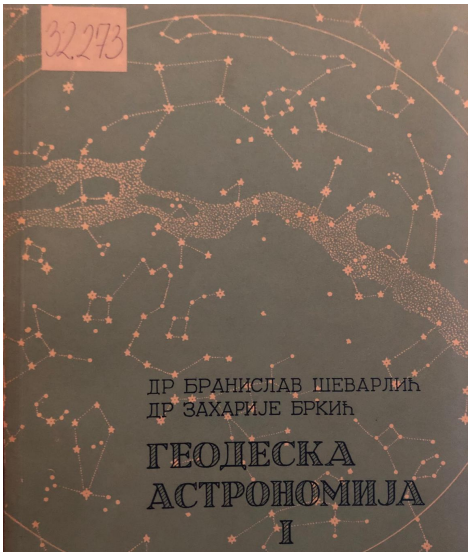


Figure 3. Front page of the textbook Geodetic astronomy.

“Gedeska astronomija” (Geodetic astronomy) was published in 1963. and it represents the first astronomy textbook for the students of the Geodesy department of the Faculty of Civil Engineering. Textbook has 266 pages, with 63 solved problems, 92 illustrations and 33 tables. Solved problems are very useful for students as they help them to overcome and easily accept new terms. In the following paragraphs we are showing some selected parts of this textbook.

On Figure 4 (pages 122 and 123 from textbook) we can see one solved problem (problem no. 49). This problem shows a method for calculation of the time of the Sun’s setting and rising, but accounting for the atmospheric refraction. In this problem, duration of the day is

calculated for Belgrade and Zagreb, and in order to achieve a larger precision, problem is done using two approximations. At that time calculators and computers were not in use, hence extensive calculations had to be done using logarithmic tables or mechanical calculators.

Here, we should mention that completely solved problems with explanations can be found in "Zbirka urađenih zadataka iz Opšte astronomije" (Solved problems in general astronomy) by Vojislav Mišković [5, 6]. The first part of this problem book (published in 1957) and second part (as manuscript) is freely available at Virtual Library (http://elibrary.matf.bg.ac.rs/handle/123456789/650). This book is still the only one problem book in general astronomy.

Discovery and studying of irregularities of the Earth rotation, especially secular slowing down of its rotation led to new astronomical time: ephemeris time (ET). On Figure 5 two pages from the textbook are shown where authors explained ET using graphical and analytical way. Also, the relations between ephemeris time, universal time and sidereal time are given. From the expression 336 ($L_E = L + 1.00273\ \Delta t$) for geographic longitude of ephemeris meridian, we see that the ephemeris meridian is constantly moving away from the Greenwich meridian. This is the consequence of secular slowing down of the Earth rotation period. Determination of a correct longitude is greatly important for the geodesy.

Before determination of the irregularities of Earth rotation, the movements of the Earth poles had been discovered. Therefore, besides changes in longitude we have constant changes in latitude for every place on the Earth. In the Geodetic astronomy authors are deriving the equation of Kostinsky for determination of cartesian coordinates of Earth pole form observational data. It is important to note that the first mathematical model of Earth, and movement of its poles was discovered by our scientist Milutin Milanković. Milutin Milanković gave theoretical derivation of this Earth model in his monumental work "Canon of insolation and ice age problem".

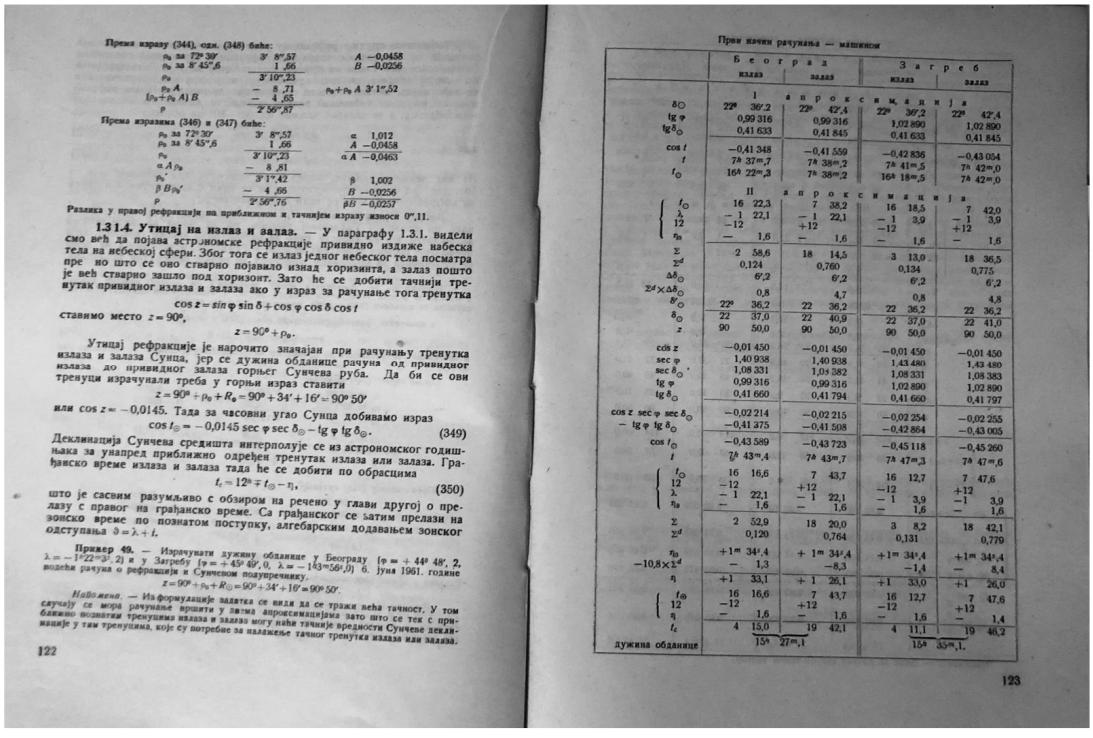
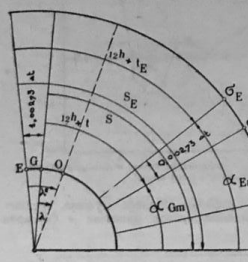


Figure 4. Scan of the pages 122 and 123 from Geodetic astronomy.

1.2.13. Дефинисање јединица за ефемеридско време по аналогији на јединице за звездано и светско (дан средње време) — Практична примена ефемеридског времена може се упростити ако се његове јединице дефинишу по аналогији на јединице светског и звезданог времена. Тада се могу упростити и везе ефемеридског времена са светским и са звезданим временом. У ту сврху уведимо по *Д. Садлеру*, појам *ефемеридског меридијана* који лежи 1,00273 Δt источно од гриничког. Он представља положај у коме би се налазио гринички меридијан да се Земља обртала равномерно. Угао између гриничког и ефемеридског меридијана представља у сваком тренутку збир промена у брзини Земљина обртања, па га зато треба сматрати за тренутан. Географске дужине места на Земљи, рачунате од ефемеридског меридијана, на звaћемо *ефемеридске географске дужине* и обележавати са λ^E. Ефемеридском меридијану одговара на небеској сфери *небески ефемеридски меридијан*. Часовне углове небеских тела рачунате од овог меридијана назваћемо *ефемеридски часовни углови* и обележавати са t^E.



Сл. 40. Везе између ефемеридског, светског и звезданог времена по уопштеним дефиницијама, по *Д. Садлеру*

После увођења ових појмова можемо дефинисати *ефемеридско звездано време* као ефемеридски часовни угао γ тачке. Занемаримо ли изванредно малу дневну промену у положају ефемеридског меридијана, можемо дефинисати *ефемеридски звездани дан* као временски размак између два узастопна пролаза γ тачке кроз ефемеридски меридијан. Ефемеридско звездано време добићемо кад у везе између звезданог и светског времена унесемо место светског времена t_0, ефемеридско време $t_E = t_0 + \Delta t$.

Пролаз једне звезде кроз ефемеридски меридијан догодиће се када је њена ректасцензија једнака ефемеридском звезданом времену:

$$S^E = \alpha + t^E, \qquad t^E = 0, \qquad S^E = \alpha. \tag{324}$$

Пролаз њен кроз меридијан једног места с географском дужином λ, оди. λ^E догодиће се онда у тренутку када је

$$S - \lambda = \alpha, \tag{325}$$

оди.

$$S^E - \lambda^E = \alpha. \tag{326}$$

Из Небеске механике је познато да ректасцензија гриничког средњег сунца по *Њукомбу* износи

$$\alpha_{\odot m} = 18^h\,38^m\,45^s,836 + 8\,640\,184^s,542\,t_s + 0^s,0929\,t_0^2, \tag{327}$$

где је t_0 светско време у јулијанским вековима од 0,0 јануара 1900. г. Ставимо ли место t_0, t_E, оди. $t_0 + \Delta t$, можемо створити појам *ефемеридског средњег сунца* дефинисаног ректасцензијом

$$\alpha_{Em} = 18^h\,38^m\,45^s,836 + 8\,640\,184^s,542\,(t_0 + \Delta t) + 0^s,0929\,(t_0 + \Delta t)^2. \tag{328}$$

Тада можемо *светско време* дефинисати као *гринички часовни угао гриничког средњег сунца увећан за 12h*, а *ефемеридско време* као *ефемеридски часовни угао ефемеридског средњег сунца увећан за 12h*.

На сл. 40. приказан је екваторски пресек Земље и небеске сфере, где су *E*, *G*, *O* респективно тачке на ефемеридском меридијану, гриничком меридијану и меридијану једне посматрачке станице. Тачке σ_E и σ_G представљају ефемеридско и гринички средње сунце, а тачка Σ једну произвољну звезду.

Из постављених дефиниција и са сл. 40. јасно је да постоје везе

$$S = 12^h + t_0 + \alpha_{\odot m}, \tag{329}$$

$$S_E = 12^h + t_E + \alpha_{Em} = 12^h + t_0 + \Delta t + \alpha_{Em}. \tag{330}$$

Разлика $\alpha_{Em} - \alpha_{\odot m}$ своди се практично на

$$8\,640\,184^s,542\,\Delta t \text{ (у јул. вековима)} = +0^s,002\,73\,\Delta t^s \tag{331}$$

када се Δt, које је реда величине 30^s, изрази у јулијанским вековима. Чланови вишег реда у изразу за α_{Em} могу се занемарити. Зато између S и S_E постоји проста веза

$$S_E = S + 1{,}002\,73\,\Delta t. \tag{332}$$

Тада се светско и ефемеридско време могу представити још и изразима

$$t_0 = S - \alpha_{\odot m} - 12^h, \tag{333}$$

$$t_E = S_E - \alpha_{Em} - 12^h, \tag{334}$$

који их везују за гринички звездано, односно за ефемеридско звездано време.

Упоредимо ли најзад изразе (325) и (326) добивамо везу

$$\lambda^E = \lambda + (S_B - S) \tag{335}$$

која, с обзиром на (332) даје везу између географске и ефемеридске географске дужине

$$\lambda^E = \lambda + 1{,}002\,73\,\Delta t, \tag{336}$$

којом је одређен и положај ефемеридског меридијана у односу на гринички, као што је речено на почетку параграфа.

Figure 5. Scan of the pages 112 and 113 from Geodetic astronomy.

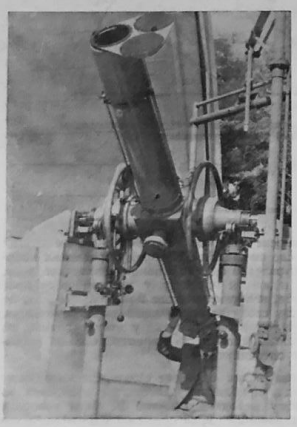
а затим звезде која се одређује:

$$\alpha = T + C_p.$$

Разлика даје у принципу тражену ректасцензију непознате звезде:

$$\boxed{\alpha = \alpha_0 + (T - T_0).} \tag{464}$$

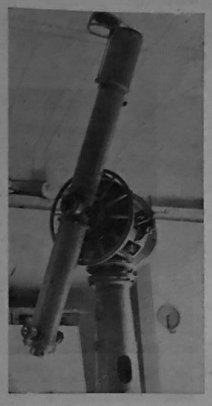
Да бисмо одредили њену деклинацију на релативан начин, довољно је да измеримо њену зенитну даљину у меридијану и да користимо познату географску ширину φ станице одређену из посматрања других звезда. Тада за релативно одређивање опет користимо у принципу једначине (463), само што смо φ добили на други начин.



Сл. 83. Велики пасажни инструмент Астрономске опсерваторије у Београду ($O = 19$ cm, $f = 257{,}8$ cm)

Ректасцензије звезда најтачније се одређују *великим пасажним инструментом* (сл. 83.) на масивном стубу који се изнад пода рачва у два. На њих се ослањају крајеви обртне осовине за коју је учвршћен већи астрономски дурбин (отвора 160—200 mm и жижне даљине око 2 m). Дурбин се поставља тако да при његовом обртању око обртне осовине визура (при идеалној ректификацији инструмента) описује раван меридијана. Пред пролаз звезде кроз меридијан дурбин се приближно доведе на зенитну даљину звезде која ће се посматрати, а кад се њен лик појави у виду светле тачке у видном пољу *бележе се тренуци када он пресеца паралелне вертикалне конце у видном пољу микрометра на дурбину*. За бележење ових тренутака постоји више начина, о којима ће бити говора у другој књизи. Из тих тренутака изведе се рачунски тренутак пролаза звезде кроз средњи идеални конац, а затим обрачунавањем утицаја инструментских константа (које се посебно брижљиво одређују) и сам тренутак T пролаза звезде кроз меридијан, из кога се одређује ректасцензија. За тачнију оцену тренутка пролаза употребљава се тзв. *бељачки микромешар*. У последње време разрађена је на Пулковској опсерваторији, код Љењинграда, метода за одређивање тренутака пролаза кроз меридијан с високом тачношћу помоћу фотоћелије (*Н. Н. Павлов*).

Деклинација се најтачније одређује *великим вертикалним кругом* (сл. 84.). То је астрономски дурбин истих димензија као и велики пасажни инструмент. Поставља се такође на масиван стуб и у меридијан, но може се обртати не само око обртне, већ и око алхидадне осовине (која је приближно вертикална). Поред окуларног микрометра има за мерење зенитне даљине и велики круг у меридијанској равни (обично пречника 0,5—1 m) са нарочито прецизном угловном поделом која се чита помоћу 4—6 микроскопа када се звезда навизира у средишту крста конаца окуларним микрометром. Из мереног вертикалног угла рачунски се добива зенитна даљина у меридијану, потребна, за одређивање деклинације. За брзо и тачно читање круга у последње време користи се прибор са фотоћелијом.



Сл. 84. Велики вертикални круг Астрономске опсерваторије у Београду ($O = 19$ cm, $f = 257{,}8$ cm)

Овакве инструменте има само неколико астрономских опсерваторија на свету. Међутим и ректасцензија и деклинација могу се одређивати једновремено помоћу *меридијанског круга* (сл. 85.). То је уствари велики пасажни инструмент снабдевен још и великим кругом за мерење зенитних даљина. Код свију ових инструмената за постизање високе тачности од нарочите је важности брижљиво испитивање, како мерних органа (либеле, окуларни микрометар, круг, микроскопи за његово читање), тако и крајева обртне осовине у циљу одређивања неправилности њихових радних пресека.

Да би се тачност повећала у последње време се ради на потпуно новим конструкцијама инструмената за одређивање звездаиих екватор-

Figure 6. Scan of the pages 216 and 217 from Geodetic astronomy

On pages 216 and 217 (Figure 6), we can see pictures (photos) of two important astronomical instruments: large passage instrument and large vertical circle from the Astronomical observatory Belgrade. Large passage instrument gives the most precise determination of right accession, and large vertical circle is used for declination determination. Using these two instruments observational catalogues were made. Unfortunately, due to light pollution these instruments are no longer in use at the observatory.

We should mention that for a relative coordinate determination of the equatorial coordinates of stars, meridian circle is used. Meridian circle can estimate both coordinates at the same time. Unhappily, this instrument was destroyed in fire during the NATO bombing of Serbia in 1999.

4. General Astronomy

"Opšta astronomija" (General astronomy) is one of the most significant textbook for students of astronomy. This book is still in extensive use for the first year students as material for the courses General astronomy 1 and General astronomy 2. The first edition of this book was published in 1971., and the second one ten years later (



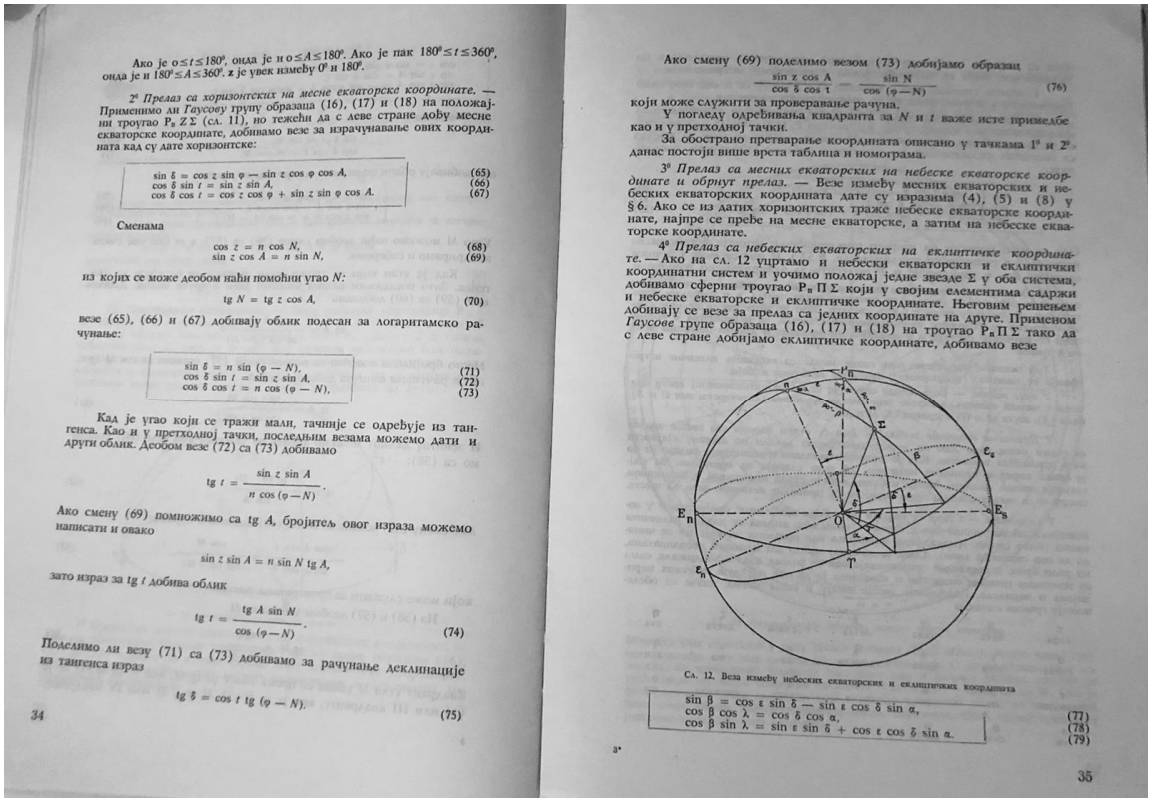
Figure *7*). General astronomy treats the basics of the classical astronomy disciplines such as: spherical, practical, positional and theoretical astronomy, celestial mechanics etc. Here, we should highlight that every chapter in this book starts with historical background and also is followed by graphical representation of the problem.



Figure 7. Front pages of the university textbook General astronomy. Left one is published in 1971., while the right one is the second edition published in 1981.

Second edition of this book is extended with newer data and solved problems. Also, the profit from the sale of this edition went to Fond of prof. Zaharije Brkić whose purpose is to give a reword for the best graduated astronomy students.

On pages 34 and 35 (Figure 8) it is shown how to use equations of spherical trigonometry in order to do transformations between spherical coordinate systems that are used in astronomy. There we can see one image (image number 12 in textbook) of the celestial sphere and spherical triangle that is used to derive the relations between equatorial and ecliptic coordinates. At chapter 70 (Figure 9) determination of true solar time with sundial is shown. With spherical trigonometry it is possible to obtain the relation between the shadow of gnomon and Sun's hour angle. At Belgrade there are a few sundials: in front of a mint of money at Topčider, at the entrance of the Astronomical observatory Belgrade, in front of Serbian clinical centre, in the centre of Zemun and at the representative office of the Hilandar monastery.



Ако је $0 \le t \le 180^0$, онда је и $0 \le A \le 180^0$. Ако је пак $180^0 \le t \le 360^0$, онда је и $180^0 \le A \le 360^0$. **z** је увек између 0^0 и 180^0.

2^0 *Прелаз са хоризонтских на месне екваторске координате.* — Применимо ли Гаусову групу образаца (16), (17) и (18) на положајни троугао $P_n Z \Sigma$ (сл. 11), но тежећи да с леве стране добу месне екваторске координате, добивамо везе за израчунавање ових координата кад су дате хоризонтске:

$$\sin \delta = \cos z \sin \varphi - \sin z \cos \varphi \cos A, \quad (65)$$
$$\cos \delta \sin t = \sin z \sin A, \quad (66)$$
$$\cos \delta \cos t = \cos z \cos \varphi + \sin z \sin \varphi \cos A. \quad (67)$$

Сменама

$$\cos z = n \cos N, \quad (68)$$
$$\sin z \cos A = n \sin N, \quad (69)$$

из којих се може деобом наћи помоћни угао N:

$$\operatorname{tg} N = \operatorname{tg} z \cos A, \quad (70)$$

везе (65), (66) и (67) добивају облик подесан за логаритамско рачунање:

$$\sin \delta = n \sin (\varphi - N), \quad (71)$$
$$\cos \delta \sin t = \sin z \sin A, \quad (72)$$
$$\cos \delta \cos t = n \cos (\varphi - N), \quad (73)$$

Кад је угао који се тражи мали, тачније се одређује из тангенса. Као и у претходној тачки, последњим везама можемо дати и други облик. Деобом везе (72) са (73) добивамо

$$\operatorname{tg} t = \frac{\sin z \sin A}{n \cos (\varphi - N)}.$$

Ако смену (69) помножимо са tg A, бројитељ овог израза можемо написати и овако

$$\sin z \sin A = n \sin N \operatorname{tg} A,$$

зато израз за tg t добива облик

$$\operatorname{tg} t = \frac{\operatorname{tg} A \sin N}{\cos (\varphi - N)}. \quad (74)$$

Поделимо ли везу (71) са (73) добивамо за рачунање деклинације из тангенса израз

$$\operatorname{tg} \delta = \cos t \operatorname{tg} (\varphi - N), \quad (75)$$

Ако смену (69) поделимо везом (73) добијамо образац

$$\frac{\sin z \cos A}{\cos \delta \cos t} = \frac{\sin N}{\cos (\varphi - N)} \quad (76)$$

који може служити за проверавање рачуна.

У погледу одређивања квадранта за N и t важе исте примедбе као и у претходној тачки.

За обострано претварање координата описано у тачкама 1^0 и 2^0 данас постоји више врста таблица и номограма.

3^0 *Прелаз са месних екваторских на небеске екваторске координате и обрнут прелаз.* — Везе између месних екваторских и небеских екваторских координата дате су изразима (4), (5) и (8) у § 6. Ако се из датих хоризонтских траже небеске екваторске координате, најпре се пређе на месне екваторске, а затим на небеске екваторске координате.

4^0 *Прелаз са небеских екваторских на еклиптичке координате.* — Ако на сл. 12 уцртамо и небески екваторски и еклиптички координатни систем и уочимо положај једне звезде Σ у оба система, добивамо сферни троугао $P_n \Pi \Sigma$ који у својим елементима садржи и небеске екваторске и еклиптичке координате. Његовим решењем добивају се везе за прелаз са једних координате на друге. Применом *Гаусове* групе образаца (16), (17) и (18) на троугао $P_n \Pi \Sigma$ тако да с леве стране добијамо еклиптичке координате, добивамо везе

Сл. 12. Веза између небеских екваторских и еклиптичких координата

$$\sin \beta = \cos \varepsilon \sin \delta - \sin \varepsilon \cos \delta \sin \alpha, \quad (77)$$
$$\cos \beta \cos \lambda = \cos \delta \cos \alpha, \quad (78)$$
$$\cos \beta \sin \lambda = \sin \varepsilon \sin \delta + \cos \varepsilon \cos \delta \sin \alpha. \quad (79)$$

Figure 8. Pages 34 and 25 from the textbook General astronomy.

ти све до на око 1 час после Сунчеве кулминације. Правац симетрале угла који заклапају гномонове сенке које додирују исти круг биће приближни правац меридијана. Како има више кругова, биће



Сл. 121. Одређивање положаја подневачке линије гномоном

више оваквих симетрала. Средњи положај свију њих даће нам правац меридијана са већом тачношћу но свака појединa симетрала. Тај правац поклапаће се са правцем најкраће гномонове сенке.

Тренутак када гномонова сенка достиже своју најмању вредност је 12^h правог сунчаног времена за тај датум. Ако пређемо са правог на средње-европско време Tc, у том тренутку, као што смо то чинили у § 23, и ако смо забележили време. То које је показивао наш џепни или ручни часовник у тренутку најкраће гномонове сенке, разлика

$$T_c - T_0 = Cp \qquad (337)$$

даће нам поправку нашег часовника.

Ако смо, сем тога, измерили дужину гномона G и његове најкраће сенке G_0, количник

$$\frac{G}{G_0} = tg\,(90^\circ - Z_\odot) \qquad (338)$$

даће нам зенитну даљину Сунца, а веза

$$\varphi = \delta_\odot + Z_\odot \qquad (339)$$

географску ширину наше станице.

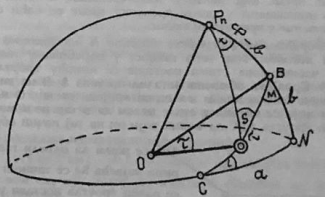
Ако ова мерења вршимо у току целе године можемо одредити и нагиб еклиптике према екватору из везе

$$\varepsilon = \frac{1}{2}(z_{max} - z_{min}) \qquad (340)$$

Стари астрономи одређивали су гномоном још дужину тропске године, дужину сваког годишњег доба и још низ других величина.

2° *Одређивање правог сунчаног времена и часовникова стања сунчаним часовником.* Ако је гномонов стуб вертикалан, положај његове сенке зависи не само од висине, већ и од Сунчеве деклинације. Зато се гномон не може употребљавати за одређивање времена и часовникова стања у произвољном тренутку обданице. Но ако је његов стуб паралелан светској осовини, онда положај сенке не зависи од Сунчеве деклинације. Овакав гномон зове се *сунчани часовник*, јер се може употребити за одређивање времена у општем случају. Ако смо одредили правац меридијана и географску ширину гномоном, неће нам бити тешко да његов стуб поставимо паралелно светској осовини и да од њега направимо сунчани часовник. Остаје још питање како направити његов цифарник.

Претпоставимо најопштији случај — да раван цифарника има произвољан положај у простору. Нека је OP_n правац гномонова стуба, а OBC раван његова цифарника. Она ће бити одређена нагибом *i*



Сл. 122. Рачунање мреже за цифарник сунчаног часовника

према равни хоризонта (сл. 122.) и азимутом *a* од северне тачке; *t* је часовни угао Сунца, а $BOS = \tau$ угао који гномонова сенка опише по цифарнику за време док Сунце пређе часовни угао *t*. Да бисмо нашли поделу на цифарник треба наћи везу између углова τ и *t*. У том циљу ћемо решавањем правоуглог сферног троугла BNC (по *Неперовом* правилу) наћи прво његову страну $BN = b$ и угао $CBN = M$:

$$\operatorname{tg} b = \sin a \operatorname{tg} i,$$

$$\cos M = \cos a \sin i.$$

Применимо ли сада синусну и синусно-косинусну теорему на сферни троугао $P_n BS$, добићемо једначине

Figure 9. Pages 228 and 229 from the textbook General astronomy.

Таблица 18.

Планета	Нагиб	Лонгитуда узлазног чвора	Лонгитуда перихела	Средња лонгитуда	Велика полуоса путање у астр. јед.	Велика полуоса путање у мил. км.
Меркур	7°00'12"	47°30'05"	76°21'59"	20°24'01"	0,387099	57,9
Венера	3 23 38	76 02 59	130 35 10	258 31 50	0,723332	108,1
Земља	0 00 00	—	101 44 12	99 55 39	1,000000	149,5
Марс	1 51 00	49 01 04	334 46 14	276 15 27	1,523688	227,8
Јупитер	1 18 25	39 44 28	13 11 41	68 56 26	5,202803	777,8
Сатурн	2 29 28	113 02 43	91 40 34	273 37 05	9,538843	1426,1
Уран	0 46 22	73 38 28	169 31 47	12 20 01	30,070672	2869,1
Нептун	1 46 35	131 00 31	44 01 07	150 59 03	19,190978	4495,7
Плутон	17 18 48	108 57 16	222 28 34	121 24 14	39,517774	5908,0

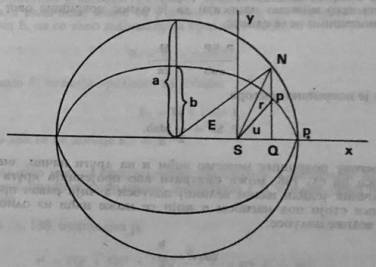
Планета	Ексцентр. путање	Сидеричка револ. у сидер. годинама	Сидеричка револ. у сред. данима	Средње дневно кретање	Средња брз. на путањи у км/сек	Даљина од Земље у мил. км. најмања	Даљина од Земље у мил. км. највећа
Меркур	0,205620	0,24084	87,9693	14732',4199	48,89	82	217
Венера	0,006806	0,61519	224,7008	5767',6698	35,00	40	259
Земља	0,016738	1,00000	365,2564	3548',1928	29,77	—	—
Марс	0,093340	1,88082	686,9797	1886',5186	24,22	56	400
Јупитер	0,048387	11,86178	4332,5879	299',1284	13,07	591	965
Сатурн	0,055786	29,45660	10759,2008	120',4550	9,65	1199	1653
Уран	0,047129	84,01209	30685,93	42',2366	6,80	2586	3153
Нептун	0,008553	164,78201	60187,65	21',5327	5,43	4309	4682
Плутон	0,24684	248,421	90737,2	14',2830	4,74	4303	7517

За оне комете које се крећу параболичким путањама рачунање путањских елемената је простије, јер је увек e једнако јединици, а место велике полуосе a узима се перихелска (најмања) даљина q комете од Сунца. Тако се рачун своди на 5 елемената. Најбољи начин за рачунање кометских путања дао је *Олберс* крајем XVIII века.

91. **Одређивање геоцентричних положаја планете из путањских елемената.** — Ова операција друкчије се назива и рачунање *ефемериде за планету*. Ту се у ствари тражи да се из познатих путањских елемената $i, \Omega, \pi, a, e, t_0$ израчунају њени геоцентрични положаји $(\alpha_1, \delta_1), (\alpha_2, \delta_2), (\alpha_3, \delta_3) \ldots$ у једнаким временским размацима, како би се она могла даље посматрати и проучавати.

Рачунање планетине ефемериде врши се у четири етапе. Најпре се из датих путањских елемената израчунају планетине поларне координате у равни њене путање за дати тренутак. Затим се израчунају њене еклиптичке координате у односу на Сунце (хелиоцентричне координате), даље, њене еклиптичке координате у односу на Земљу (геоцентричне координате) и најзад се ове последње претворе у екваторске.

1° *Рачунање поларних координата планете у равни путање.* Прва етапа рачунања врши се по *Кеплеровој геометријској методи*, која се састоји у овоме. Ако се планета креће по елипси, по *Кеплеровим* законима, њен положај P на путањи у тренутку t може се одредити поларним координатама r и v у односу на Сунце S које је у жижи елипсе. Ове се координате називају *радијус-вектор* и *права аномалија* планете (сл. 138). Да бисмо их израчунали увешћемо два помоћна угла E и M. Угао E се назива



Сл. 138. Рачунање поларних координата планете у равни путање

ексцентрична аномалија планете. Његово теме се налази у средишту елипсе, један крак му је уперен ка перихелу, а други ка тачки N која лежи на кругу описаном из средишта елипсе њеном великом полуосом, и то тамо где га пресеца ордината тачке P у правоуглом координатном систему с почетком у O и апсисном осовином у смеру OP. Угао M се назива *средња аномалија* планете. Њега би заклапао радијус-вектор планете с правцем ка перихелу, када би се планета кретала по кругу равномерно и то *средњом* (дневном) *брзином* планете

$$n = \frac{2\pi}{T}, \qquad (417)$$

где је T њена сидеричка револуција у данима, и пролазила кроз перихел кад и права планета.

Figure 10. Pages 276 and 277 from the textbook General astronomy.

Figure 10 shows another one classical term – Kepler's equation. There we can see that for the calculations of planet's polar coordinates we need Kepler's equation. Kepler's equation: $E - e \sin(E) = M$ is transcendental equation. Until the computer era solving of this equation was too hard.

Acknowledgment

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83 Dela Rudjera Boškovića u Virtuelnoj biblioteci Matematičkog fakulteta

Ž. Mijajlović and N. Pejović
In a book "Rudjer Bošković, preteča moderne nauke"
(D. Antić, ed.), pp. 169-176
Matica srpska u Dubrovniku,
Beograd, 2018. ISBN 978-86-7056-260-8.

This is a reprint of the article "Digitalizovane knjige Rudjera Boškovića" with some additions.

ДЕЛА РУЂЕРА БОШКОВИЋА У ВИРТУЕЛНОЈ БИБЛИОТЕЦИ МАТЕМАТИЧКОГ ФАКУЛТЕТА

Проф. Жарко МИЈАЈЛОВИЋ, проф. Нада ПЕЈОВИЋ
Математички факултет, Универзитет у Београду

Виртуелна библиотека

Општи циљ Виртуелне библиотеке Математичког факултета Универзитета у Београду, http://elibrary.matf.bg.ac.rs, је да се имплементира потпуна и повезана колекција ретро–дигитализованих књига и других дигиталних докумената. Главни део овог пројекта се односи на електронско архивирање, пре свега, старих рукописа, њихово објављивање у електронском фото–типском облику и презентацију широкој публици. Пројекат је у почетку био наклоњен математици, али од 2009. године у Библиотеку се, такође, постављају књиге из других наука и књижевности. Предност, ипак, имају књиге српских аутора и дела која се односе на научни и културни простор Југоисточне Европе.

Неке од књига у Библиотеци су ретке и зна се да их је преостало тек по неколико примерака у штампаном облику. Сматрали смо да је веома важно да се очува њихово постојање у неком облику. Не само као културно или научно наслеђе важно за нашу земљу, већ и као део Светске баштине. Одлучили смо да се оне сачувају и представе широкој публици у електронској, дигитализованој форми. Пројекат је започело 2007. године неколико волонтера. Сада Библиотека има снажну подршку од стране Математичког факултета Универзитета у Београду, Математичког института Српске академије наука и уметности, Националног центра за дигитализацију и у пројектима Министарства за науку Србије.

Виртуелна библиотека Математичког факултета, највећа је база дигитализованих текстова у Србији са слободним приступом. У тренутку писања овог текста, Библиотека садржи близу 2000 књига. У оквиру Библиотеке, постоји неколико важних колекција. На пример, значајна је збирка коју чини 400 докторских дисер-

тација из математичких наука (од којих је највећи део одбрањен на Математичком факултету). Другу важну колекцију чине ретке књиге из XVIII и XIX века. Већи део ових књига су раритети, неке од њих постоје само у једном примерку у штампаном облику и фактички су недоступне јавности. Важан део Библиотеке чине сабрана дела неких водећих српских научника из прошлости: Богдана Гавриловића, Милутина Миланковића, Ђуре Курепе, Ђорђа Станојевића и неколико других. Радови у вези са Виртуелном библиотеком објављују се у часопису НЦД „Преглед", који издаје Математички факултет.

Овде ћемо представити колекцију књига Руђера Бошковића (1711–1787), које се налазе у дигитализованом облику у Виртуелној библиотеци. Ове књиге су прва дела из науке неког писца српског порекла. Напоменимо – да се у Виртуелној библиотеци налазе књиге и других српских писаца из тог раздобља. Наиме, убрзо по објављивању Бошковићевих књига, у другој половини XVIII века, појавила су се прва дела из науке писана на старосрпском:

Василије Дамјановић (1734–1792), *Аритметика*, 1767, Венеција. Ово дело је прва српска књига из математике. Постоји само један познат штампан примерак ове књиге и он се налази у Библиотеци Матице српске.

Захарије Стефановић Орфелин (1726–1785), *Вечни календар*, 1783, Беч. Велики део ове књиге је посвећен описима и објашњењима астрономских феномена. Отуда можемо сматрати – да је ово дело прва српска књига из астрономије. Штампани примерак ове књиге се налази у Библиотеци Астрономске опсерваторије у Београду.

Атанасије Стојковић (1773–1832), *Физика*, 1800, Будим. Ово тротомно дело је прва српска књига из физике. Штампани примерак ове књиге се налази у Библиотеци САНУ.

Дигиталне копије ових књига се налазе у Виртуелној библиотеци.

Споменимо овде једну општу карактеристику ових првих српских научника. Њих је красила универзалност, дакле, астрономија или математика није била њихова једина специјалност. Исто тако, били су успешни и у другим наукама: физици или филозофији, на пример. За њих је наука, пре свега, био поглед на јединствено биће – природу. Језиком науке су се служили да опишу природу тог бића. Поред свега, често су били успешни политичари, уметници, путници, писци романа и песама. Њихов значај за развој српске културе, језика и науке је огроман. Ове књиге су имале примарно просветитељски карактер. Писци су своја дела посвећивали српском народу, или, како то Орфелин на једном

месту у свом *Календару* пише, „на корист Славено–српског народа".

Бошковићева дела у Виртуелној библиотеци

У овој прилици, нећемо се упуштати у биографију Руђера Бошковића (1711. до 1787. године), о томе постоје многа опширна дела. Ипак, споменимо да је Бошковић био највећи математичар и астроном Дубровника и један је од најзначајнијих научника свога доба, уврштен међу 100 најзначајнијих Срба свих времена. Био је професор универзитета, оснивач Миланске опсерваторије и директор Оптичког института француске морнарице. Био је универзалан стваралац: теолог, филозоф, математичар, астроном, физичар, геолог, архитекта, археолог, конструктор, оптичар, дипломата, путописац, професор, песник и полиглота. У историји Католичке цркве је забележен као значајан припадник језуитског реда. Бошковић је рођен 18. маја 1711. као седмо дете трговца Николе Бошковића, Србина из Орахова Дола код Требиња у Херцеговини, и мајке Павле, италијанског порекла, из породице Бара Бетере, познатог дубровачког песника. Живео је углавном у Италији, где је и стекао светску славу. Умро је 13. фебруара 1787. године у Милану.

У Виртуелној библиотеци се налазе четири дигитализована Бошковићева дела. Поред основних библиографских података о тим књигама, дајемо нешто шири осврт на књигу *Elementorum Universae Matheseos*. Напоменимо – да је Руђер Бошковић писао своја дела углавном на латинском и италијанском.

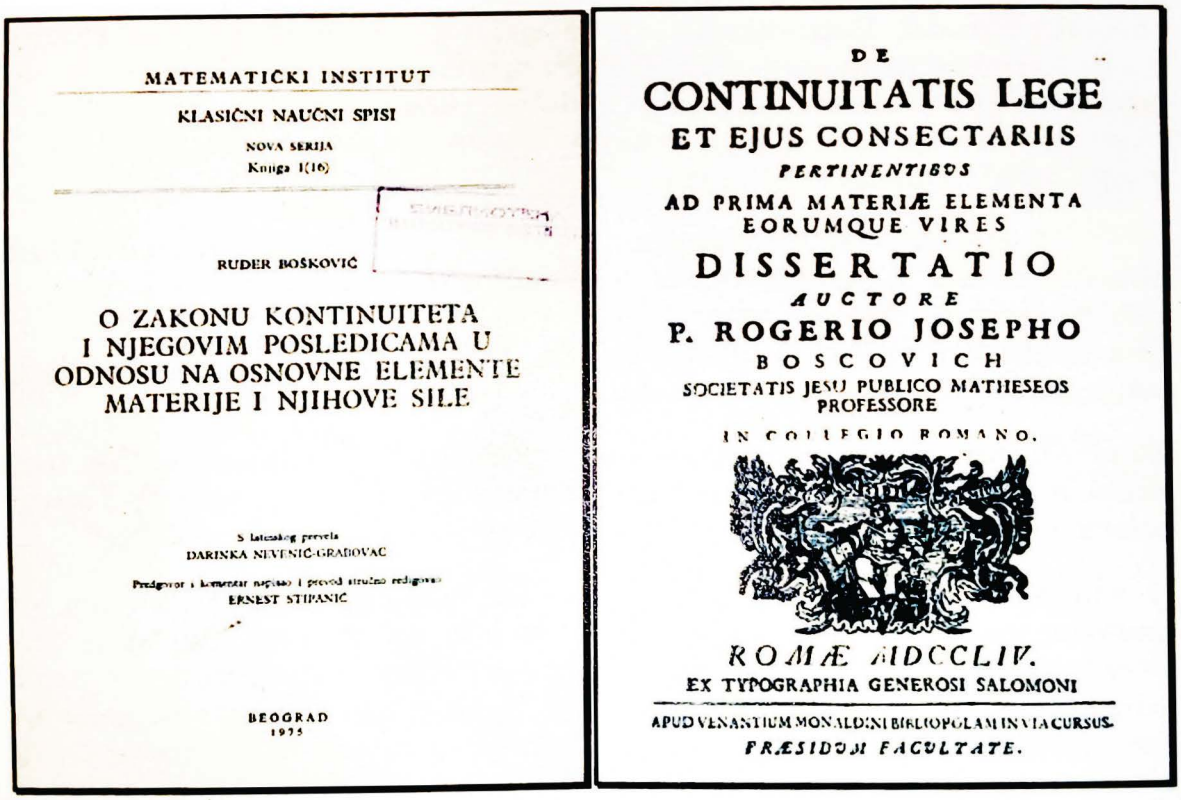
Elementorum Universae Matheseos – Tomus I–III (*Елементи опште математике*), 1757, Venetis. Штампано издање ове тротомне књиге се налази у библиотеци Математичког института САНУ.

О закону континуитета и његовим последицама у односу на основне елементе материје и њихове силе, превод Даринке Невенић-Грабовац, предговор и коментаре написао Ернест Стипанић, 1975, Београд. Ова књига је превод са латинског дела *De continuitatis lege et eius consectariis pertinentibus ad prima materiae elementa eorumque vires dissertatio*, Romae, 1754.

A Theory of Natural Philosophy, Latin-English edition, London, tranls. J. M. Child, *London, 1922, Introduction*: B. Petronijević. Ова књига је упоредно латинско-енглеско издање дела *Theoria philosophiae naturalis*, Venetis, 1763. Штампани примерак лондонског издања Бошковићеве *Теорије* се налази у библиотеци Астрономске опсерваторије у Београду.

Дневник са пута из Цариграда у Пољску (1762), превод Душана Нељековића, 1937, Београд, назив оригинала *Giornale di un viaggio da Constantinopoli in Polonia dell'abate Ruggiero Giuseppe Boscovich, con una sua relazione delle rovine di Troia*, Venetis, 1784.

Рукопис *Закон, или принцип континуитета* је Бошковићу главно полазиште и путоказ у његовој изградњи теорије сила које владају у природи и у вези с тим теорије о саставу материје. Зато је расправа *О закону континуитета и његовим последицама и односу на основне елементе материје и њихове силе* у најужој вези са Бошковићевом теоријом природне филозофије. Она, поред осталих расправа (*О живим силама*, Рим, 1745; *О светлости*, први и други део, Рим, 1748; *О дељивости материје и почелима тела*, написана 1748, објављена у Луки, 1757; *О закону сила што постоје и природи*, Рим, 1755), представља основу из које је произашло главно Бошковићево дело *Теорија природне филозофије сведена на један једини закон сила што постоје у природи* (*Philosophiae naturalis theoria redacta ad unicam legem virium in natura existentium, Viennae*; 1758).


MATEMATIČKI INSTITUT

KLASIČNI NAUČNI SPISI

NOVA SERIJA

Knjiga 1(16)

RUDER BOŠKOVIĆ

O ZAKONU KONTINUITETA I NJEGOVIM POSLEDICAMA U ODNOSU NA OSNOVNE ELEMENTE MATERIJE I NJIHOVE SILE

S latinskog prevela DARINKA NEVENIĆ-GRABOVAC

Predgovor i komentar napisao i prevod stručno redigirao ERNEST STIPANIĆ

BEOGRAD 1975

DE

CONTINUITATIS LEGE

ET EJUS CONSECTARIIS

PERTINENTIBUS

AD PRIMA MATERIÆ ELEMENTA EORUMQUE VIRES

DISSERTATIO

AUCTORE

P. ROGERIO JOSEPHO

BOSCOVICH

SOCIETATIS JESU PUBLICO MATHESEOS PROFESSORE

IN COLLEGIO ROMANO.

ROMÆ MDCCLIV.

EX TYPOGRAPHIA GENEROSI SALOMONI

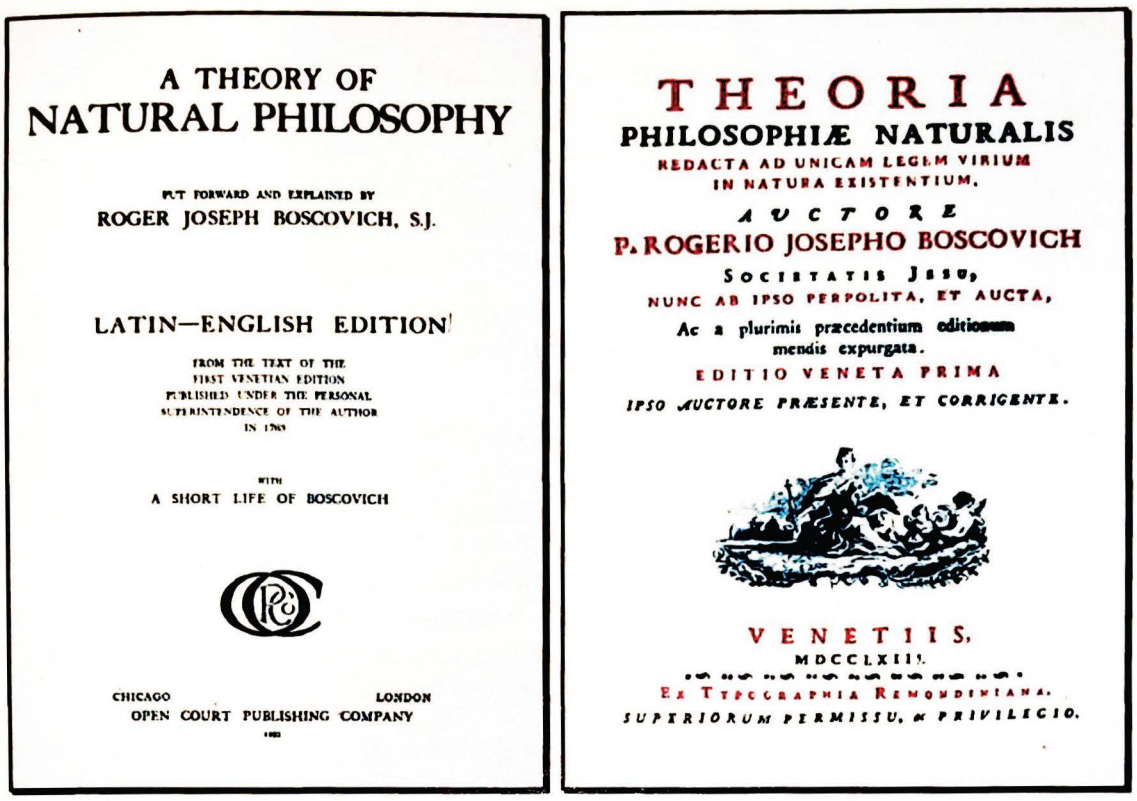
APUD VENANTIUM MONALDINI BIBLIOPOLAM IN VIA CURSUS.

FRÆSIDUM FACULTATE.

Слика 1. *О закону континуитета и његовим последицама у односу на основне елементе*, приредио Е. Стипанић, 1975, Београд.

Отуда, одмах у другом параграфу, Бошковић истиче да је у расправи „читава анализа брижљиво припремљена, да би нас од

најједноставнијих принципа *природе нужним и спонтаним сплетом закључака довела до саме теорије*". То довољно говори од каквог је ова расправа значаја за Бошковићев научни и филозофски опус. Из тог разлога је, према речима приређивача професора Стипанића, Математички институт САНУ објавио превод ове књиге на наш језик, с коментарима.



A THEORY OF
NATURAL PHILOSOPHY

PUT FORWARD AND EXPLAINED BY
ROGER JOSEPH BOSCOVICH, S.J.

LATIN—ENGLISH EDITION

FROM THE TEXT OF THE
FIRST VENETIAN EDITION
PUBLISHED UNDER THE PERSONAL
SUPERINTENDENCE OF THE AUTHOR
IN 1763

WITH
A SHORT LIFE OF BOSCOVICH

CHICAGO LONDON
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THEORIA
PHILOSOPHIÆ NATURALIS
REDACTA AD UNICAM LEGEM VIRIUM
IN NATURA EXISTENTIUM.
AUCTORE
P. ROGERIO JOSEPHO BOSCOVICH
SOCIETATIS JESU,
NUNC AB IPSO PERPOLITA, ET AUCTA,
Ac a plurimis præcedentium editionum
mendis expurgata.
EDITIO VENETA PRIMA
IPSO AUCTORE PRÆSENTE, ET CORRIGENTE.

VENETIIS,
MDCCLXIII.
EX TYPOGRAPHIA REMONDINIANA.
SUPERIORUM PERMISSU, & PRIVILEGIO.

Слика 2. Упоредно латинско–енглеско издање *Теорије природне филозофије*, приредио J. M. Child, Лондон, 1922.

Бошковићев *magnum opus Теорија природне филозофије* је свакако његово најзначајније дело. Прво издање ове књиге је штампано 1758. године у Бечу. У Виртуелној библиотеци, налази се дигитална копија упоредног енглеско – латинско издања, штампаног у Лондону 1922. године. Књигу је приредио James Mark Child према другом издању Бошковићеве књиге штампане у Венецији 1763. године. Друго издање *Теорије природне филозофије*, проширено и употпуњено је у односу на прво издање и штампано је под Бошковићевом супервизијом. У књизи се налази кратак Бошковићев животопис, који је написао угледни српски филозоф Бранислав Петронијевић.

Књига *Елементи опште математике* је по свој прилици, представљала уџбеник намењен Бошковићевим студентима. Да је то случај, уверавамо се на многобројним примерима у књизи и

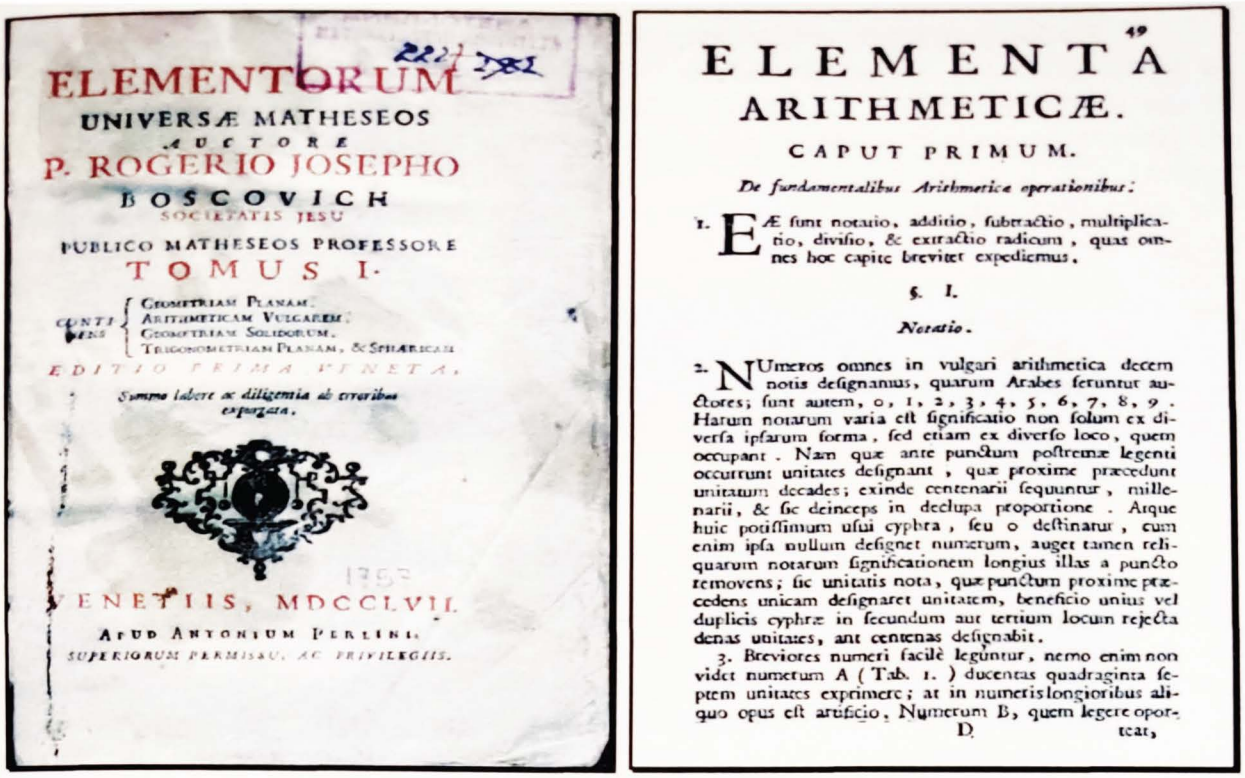
великом броју задатака. Занимљиво је да у многим рукописима и на многим местима на Интернету пише да је књига штампана 1754. у Риму, мада у Бошковићевој библиографији, која се може наћи у поузданим изворима, таквог податка нема. Дигитална копија ове књиге потиче од издања из 1757, штампаног у Венецији (импресум: *Editio Prima Veneta*, MDCCLVII). Исти је случај са дигиталним копијама, које се налазе у великим дигиталним библиотекама *Google books* и *Europeana*.

Део првог тома *Елемената* се односи на елементарну геометрију, с тим да излагање углавном прати *Еуклидове елементе*. У другим поглављима се описују особине основних аритметичких операција, излаже се рачун с рационалним бројевима, особине пропорција, аритметичке и геометријске прогресије логаритама и најзад елементи равне и сферне тригонометрије.

Други том, *Algebra Finitam*, у потпуности је посвећена решавању алгебарских једначина. Најпре се описују особине полинома и рачун с радикалима. Затим се излаже Карданов (Gerolamo Cardano, 1501–1576) метод за решавање кубне једначине и Фераријево (Lodovico Ferrari, 1522–1565) решење алгебарске једначине четвртог степена. У овом рачуну, користи комплексне бројеве (*quantitates imaginarae*), што је била реткост у то време. Бошковић, такође, даје приближне, нумеричке, методе за решавање неких алгебарских једначина већег степена, за које ће се касније испоставити да их и није могуће решити алгебарски, помоћу радикала (Niels Abell, Evariste Galois, прва половина 19. века).

Свакако је најзанимљивији трећи том *Елемената*. Бошковићева теорија кривих другог реда, коју излаже у овом делу *Елемената* била је прва потпуна и систематична теорија коника (конусних пресека). Овде се налазе многе нове идеје, а у излагању се користи чисто геометријским аргументима. Мада је већ увелико био развијен метод Декартовог система координата, Бошковић, у свом делу, не користи овај аналитички приступ.

Специјално, као дефиницију коника, из које изводи њихова својства, изабрао је следећу: коника је геометријско место тачака у равни за које је однос удаљености до фиксиране тачке (фокуса) и дате праве (директрисе) константа. Тај количник је назвао *ratio determinans* конике, за који данас иначе користимо назив ексцентрицитет. Мада је ово својство било познато већ старогрчким математичарима, посебно Папусу Александријском, никад раније није узимано за дефиницију. У савременој аналитичкој геометрији, дефиниција кривих другог реда, у највећем броју случајева, прати управо Бошковићев приступ. Пре Бошковића, за конике су узимане равне криве, које настају пресеком конуса и равни, дакле, у дефиницији се претпоставља тродимензионални простор. Код њега, теорија коника постаје теорија кривих *у равни*.



ELEMENTORUM
UNIVERSÆ MATHESEOS
AUCTORE
P. ROGERIO JOSEPHO
BOSCOVICH
SOCIETATIS JESU
PUBLICO MATHESEOS PROFESSORE
TOMUS I.

CONTINENS
GEOMETRIAM PLANAM.
ARITHMETICAM VULGAREM.
GEOMETRIAM SOLIDORUM.
TRIGONOMETRIAM PLANAM, & SPHÆRICAM.

EDITIO PRIMA VENETA.

Summo labore ac diligentia ab erroribus expurgata.

VENETIIS, MDCCLVII.
APUD ANTONIUM PERLINI.
SUPERIORUM PERMISSU, AC PRIVILEGIIS.

ELEMENTA
ARITHMETICÆ.

CAPUT PRIMUM.

De fundamentalibus Arithmetica operationibus:

1. EÆ sunt notatio, additio, subtractio, multiplicatio, divisio, & extractio radicum, quas omnes hoc capite brevitet expediemus.

§. I.

Notatio.

2. NUmeros omnes in vulgari arithmetica decem notis designamus, quarum Arabes feruntur auctores; sunt autem, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Harum notarum varia est significatio non solum ex diversa ipsarum forma, sed etiam ex diverso loco, quem occupant. Nam quæ ante punctum postremæ legenti occurrunt unitates designant, quæ proxime præcedunt unitatum decades; exinde centenarii sequuntur, millenarii, & sic deinceps in declupa proportione. Atque huic potissimum usui cyphra, seu o destinatur, cum enim ipsa nullum designet numerum, auget tamen reliquarum notarum significationem longius illas a puncto removens; sic unitatis nota, quæ punctum proxime præcedens unicam designaret unitatem, beneficio unius vel duplicis cyphræ in secundum aut tertium locum rejecta denas unitates, aut centenas designabit.

3. Breviores numeri facilè leguntur, nemo enim non videt numerum A (Tab. 1.) ducentas quadraginta septem unitates exprimere; at in numeris longioribus aliquo opus est artificio. Numerum B, quem legere oportet,

D

Слика 3. Елементи опште математике, Венеција, 1757.



РУЂЕР ЈОСИФ БОШКОВИЋ: ДНЕВНИК
са пута из Цариграда у Пољску
издање РАЈКОВИЋ

РУЂЕР ЈОСИФ БОШКОВИЋ

Слика 4. Странице из Бошковићевог *Дневника са пута из Цариграда у Пољску*, превод Д. Недељковић, Београд, 1937.

Увео је и врло користан метод за доказивање разних исказа о коникама. То је позната Бошковићева ексцентрична кружница – кружница с произвољним центром, која има својство да јој је однос полупречника и удаљености средишта до директрисе једнак ексцентритету. Користећи овај појам, Бошковић је описао разне конструкције и доказао многа својства коника – тако што их је изводио из својстава кружница, користећи кореспонденцију између тачака ексцентричне кружнице и конике. У прилогу *De transformatione locorom geometricorum* трећем тому *Елемената*, Бошковић је представио потпуно нову теорију геометријских трансформација, које су у основи колинеације, па се стога може сматрати претечом развоја пројективне геометрије, која настаје тек у XIX веку.

Рукописи других аутора о Бошковићевом делу

У Виртуелној библиотеци се налазе, такође, књиге и рукописи који се односе на живот и дело Руђера Бошковића. Свакако је најзанимљивија ова књига: Коста Стојановић, *Атомистика – један део из философије Руђера Јосифа Бошковића*, Ниш, 1891. Књига је заправо једна компаративна анализа Бошковићеве природне филозофије. Поред ове, у Библиотеци се налазе и друге књиге Косте Стојановића. Стојановић је био свестрани интелектуалац и професор Велике школе у Београду. Поставио је темеље теоријске механике у Србији и написао је *Основе теорије економских вредности*, вероватно најбољу књигу на српском из економских наука. Био је народни посланик и министар у влади Николе Пашића.

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84 Dimitrije Danić and his works in Virtual Library of the Faculty of Mathematics

Ž. Mijajlović and N. Pejović
NCD Review, vol. 35, pp. 12-18, 2019.

This article presents digitized works of Dimitrije Danić (1862-1932), first Serbian doctor of science in the area of mathematics and a prominent professor of the Military Academy in Belgrade. He is educated in Belgrade, Zurich and Berlin. Finally, he got his PhD degree at Jena, Germany, in 1885. Soon after that he became a professor of mathematics at the Military Academy in Belgrade. Danić's books have supplements from the natural sciences that are based on mathematics. These additions that usually came after the basic text were on mechanics, astronomy, geodesy and cartography. This fine feature was certainly of the great benefit to the general student of natural science, or of some technical faculty.

Преглед НЦД 35 (2019), 12–18

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DIMITRIJE DANIĆ AND HIS WORKS IN VIRTUAL LIBRARY OF THE FACULTY OF MATHEMATICS

Abstract. In this paper we present digitized works of Dimitrije Danić (1862-1932), first Serbian doctor of science in the area of mathematics and a prominent professor of the Military Academy in Belgrade. Digital copies of his monographs and text-books are deposited in the Virtual Library of the faculty of Mathematics in Belgrade. These works were digitized in the frame of the long-lasting digitization project *Digitization of scientific and cultural heritage*, run by the Mathematical institute in Belgrade and the Faculty of Mathematics in Belgrade.
Keywords: Dimitrije Danić, digitized works, mathematics, XIX century.

Introduction

This paper belongs to the series of articles on digitized books, doctoral dissertations and other writings deposited in the Virtual Library of the Faculty of Mathematics in Belgrade, (http://elibrary.matf.bg.ac.rs). The library was founded almost 20 years ago, see [**2**], **[3**] and [**4**], and it is one of the largest Internet oriented database in Serbia of digitized books, master and doctoral dissertations and other written texts. It is open to the general public and it is freely accessible. At the time of writing this paper, it contains about 4500 digitized items and several important collections, including about 500 digital copies of PhD theses of Serbian mathematicians and digital copies of several very rare Serbian books, some of them dating the 18th century.

In building this online database we had in mind the preservation in digital form first of all works in mathematical sciences and related texts of Serbian scientists from the past. We consider it as an important part and an approach as well to the preservation of scientific and cultural heritage of the Serbian people. The other and equally important goal is to present these works to the general and scientific audience using the modern information technologies. This work is supported predominantly by the Faculty of Mathematics in Belgrade and projects - funds granted by the Serbian Ministry of Education, Science and Technological Development. It is recognized and supported in certain forms also by the other Serbian scientific and cultural institutions, in particular by the Mathematical Institute SASA (Serbian Academy of Sciences and Arts), SASA itself, University Library in Belgrade and the National Library. These institutions are also the main sources of the material: books, manuscripts and other writings that are digitized and placed in the Virtual Library. However, the most important part of this activity is the work of the participants of the Digitization projects of the Faculty of Mathematics. Without their enthusiasm this library would not exist.

In this occasion we shall present digitized works of Dimitrije Danić, an outstanding Serbian mathematician from the end of XIX century and the beginning of the XX century. At the turn of the 19th to the 20th century appeared in Serbia first mathematical

books that could be considered as the university text-books, see [5]. Let us mention *Algebarska Analiza* (Algebraic Analysis) written by Dimitrija Nešića and published in 1883, and *Analitična Geometrija tačke, prave, kruga i koničnih preseka* (Analytic geometry of point, line, circle and conics) written by Bogdan Gavrilović, published in 1896. Books written by Dimitrije Danić belong to this collection of early Serbian high level text-books in mathematics.

Short biography of Dimitrije Danić



Figure 1. Dimitrije Danić

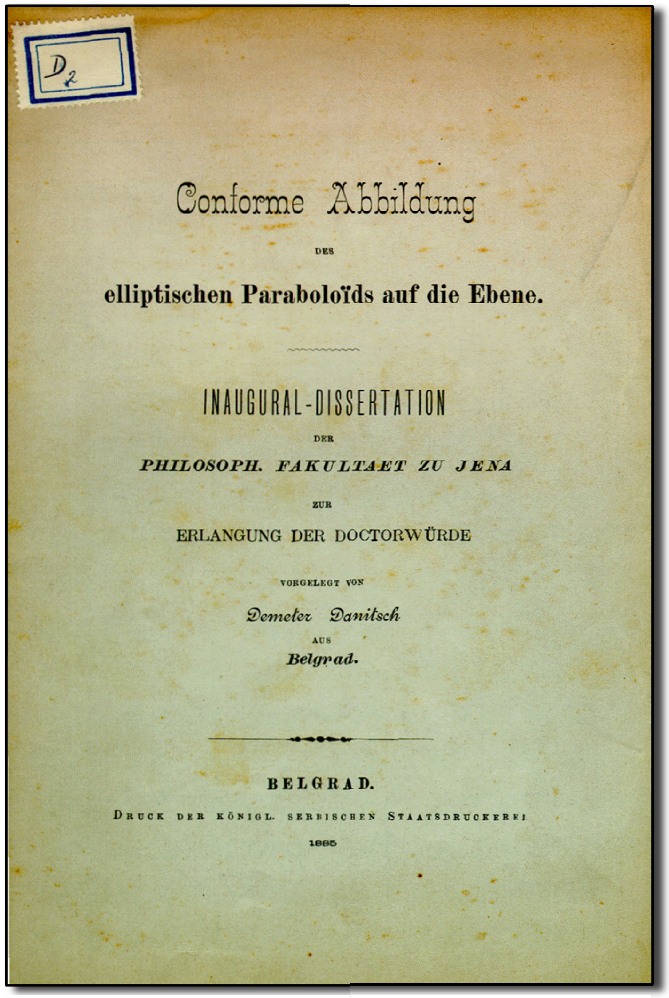
Dimitrije Danić was born on January 21,1862. in Belgrade where he finished elementary education, too. The secondary school he attended in Zurich. After that he completed three semesters at the Polytechnical School at Berlin and seven semesters at the Department of Natural Science and Mathematics of Berlin University, majoring in mathematics. He wrote doctoral dissertation *Conformal Mapping of Elliptic Paraboloid on Plane* at. Jena University, and defended it in 1885. So he became the first Serbian doctor of mathematics. In the same year he applied for the post of a lecturer of lower mathematical analysis at the Belgrade Higher School [1] and in 1887, too, but he was not accepted as full time professor. It is interesting that his opponent for this position was Bogdan Gavrilović, the second Serbian doctor of mathematics and later the rector of the Belgrade University and the president of the Serbian Academy of Sciences. Danić was teaching at the Higher School for a very short time. However, the students boycotted his lectures for some reason. On this occasion, at the request of the Minister of Education, the Rector "sentenced students, thirty of them, to two days in prison". However, this judgment was never enforced. Disappointed, and after this and other controversies, Danić joined the Military Academy where he was elected full time professor of mathematics, very soon, already in December 1888. He lectured there all type of courses in mathematics as a reputable and distinguished professor. According to school curriculum these subjects were of the same quality and level as courses taught at the Belgrade Higher School and later on at the Belgrade University.

[1] The Belgrade Higher School preceded the Belgrade University. This school was transformed to the university in 1905, when the King Petar I signed the decree of founding, *Act on Universities.* The *Higher School is also known as the Great School.* For details concerning the history of Belgrade University, see https://bg.ac.rs/en/university/history.php.

In teaching at the Military Academy, professor Danić was very strict professor and a person of high criterion but fair to his students. Besides his pedagogical work, he was also interested in the problems of military sciences. For example he was involved in problems of inner and outer ballistics of various arms and weapons. He was analyzing the results of combat marksmanship, fire dispersion and marking of the target. In his analysis he applied his knowledge on Jacob Bernoulli's independent experiments. For his distinguished achievements he was decorated with the Order of St.Sava of IV, III and II class.

Danić was teaching at Military Academy till his retirement. He died in Belgrade on March 23, 1932.

Doctoral dissertation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INAUGURAL-DISSERTATION

DER

PHILOSOPH. FAKULTÄET ZU JENA

ZUR

ERLANGUNG DER DOCTORWÜRDE

VORGELEGT VON

Demeter Danitsch

AUS

Belgrad.

BELGRAD.

DRUCK DER KÖNIGL. SERBISCHEN STAATSDRUCKEREI

1885

Figure 2. Cover page of Danić's dissertation

In his doctoral dissertation Dimitrije Danić considered conformal mapping of elliptic paraboloid to plane based on Gauss definition. In his work he used differential and integral calculus for solving obtained differential equations and along this work he had to solve complex elliptical integrals. His other contribution was introduction of elliptical transformations and use of elliptical functions in connection with elliptical integrals. From the modern point of view it could be said that he was working in the theory of complex functions of complex variables. Having in mind the time when this work was done and the level of development of mathematics in Belgrade, then the only center of higher education in Serbia, it could be concluded without any doubt, that his dissertation represented significant contribution to the development of mathematics in Serbia. On 17th March 1885 Danić passed examinations in mathematics, examined by Johannes Thomae, analytical mechanics and physics, examined by professor Sohnke. As a result he was awarded Ph. D. degree. The thesis later was published in Belgrade, but since it was written in German, Danić used a Germanized version of his name *Demeter Danitsch*, as can be seen on Figure 2. The scientific achievements in Danić's dissertation are explained in more details in [7]. Dušan Đurišić, professor of the Military

Academy, translated later, most probably in the second half of XX century, Danić's dissertation into Serbian. Digital copy of this hand-written translation is deposited in the Virtual library [8].

It seems that Danić did not have other scientific contributions or written papers. Ernest Stipanić, professor of Belgrade University and historian of mathematics wrote once [6] that Danić published some papers in Germaine journals, but for this we did not find some evidences.

Here is probably good place to comment in short the question who is the first Serbian doctor of mathematics. The natural candidate is Ruder Boskovic (1711–1787), the polyhistor and the greatest South Slav mathematician and astronomer. Nevertheless, under the circumstances that ruled Serbia until the first half of the nineteenth century, it is not so easy to answer this question. Serbs were conscious of their nationality as early as in the Middle Ages, which by European standards is considered to be very early. However, the spreading the Ottoman Empire and fall of the Serbian Empire stopped their natural development abruptly and any possibility of scientific or cultural development disappeared. By the early 19th century all Serbian countries remained enslaved, and the people were subjected to a persistent and systematic denationalization program. This program can be described "in the style of Aristotle's syllogisms" as follows:

1. The imposition of a foreign religion: Roman Catholic in countries under Austrian, Hungarian or Venetian rule, or Islam in countries under Ottoman's rule,
2. Equalizing the terms Serb and Orthodox.
3. Drawing the "logical conclusion" that people who are not Orthodox may not be Serbs at all.

Some sources as the Soviet dictionary, *Biografičeskij slovar dejatelej v oblasti matematiki*, Kiev, 1979, state that Bošković and even Marin Getaldić were Serbo-Croats. Jovan Dučić claims that the name Boško exists only within Serbs, which clearly indicates the origin of Ruđer Bošković, regardless of his religion [1]. Bošković used to say for himself that he is of Slavic origin. Bošković's father Nikola was an Ortodox Serb, but later he turned to Catholicism. Bošković's mother was of the Italian origin. Therefore we cannot talk with certainty about the nationality of Ruđer Bošković (1711–1787), a Jesuit nun and a Catholic scholar who most of his life spent in Italy and France. What we can say is that Bošković was one of the many of our talented people who had left their homes in search of knowledge, who lived and worked throughout Europe, where they have become immersed in other people's cultures.

Although some parts of Serbian country *de facto* gained independence much earlier, so it was until the Berlin Congress in 1878 when Serbia's independence was recognized *de jure*. Shortly afterwards the first Serb, Dimitrije Danić, became a doctor of mathematics.

Danić's Text-books

Dimitrije Danić wrote eight textbooks and manuals:

1. *Formulas and Theorems in Trigonometry* (1888);
2. *Analytical Geometry on Plane* (1893);
3. *Lectures on Trigonometry with Theory of Logarithm and Complex Numbers* (1889);

4. *Foundations of Infinitesimal Calculus — Differential Calculus* (1920);
5. *Foundations and Theory of Combination and Principles of Probability* (1921);
6. *Foundations of Infinitesimal Calculus and Integral Calculus* (1922);
7. *Analytical Geometry of Plane and Space* (1922);
8. *Formulas and Theorems of Mathematics* (1927).

All those works consist of over 2.000 textbook pages. Some of the textbooks were thematically pioneering works in Serbia, as for example Analytical Geometry of Plane and Analytical Geometry of Space, or Foundations of Theory of Combination and Principles of Probability.

At the end of World War I, Mihailo Petrović Alas, Bogdan Gavrilović and Milutin Milanković founded the Belgrade Mathematicians Club. Danić did not belong to that circle, nor did he attend Club meetings. The reason was probably the circumstances during his early attempts to get a professorship at the High School.

On the other hand, Danić wrote excellent textbooks for the Military Academy students that could be measured both in content and in a style with the best university textbooks. The books were characterized by high precision, completeness and beautiful graphic attachments. It seems that these textbooks were used by students from other faculties of the Belgrade Higher School, later University, too, especially technical ones. The reason was that there were no other textbooks, particularly at the end of the XIX and the beginning of the XX centuries.

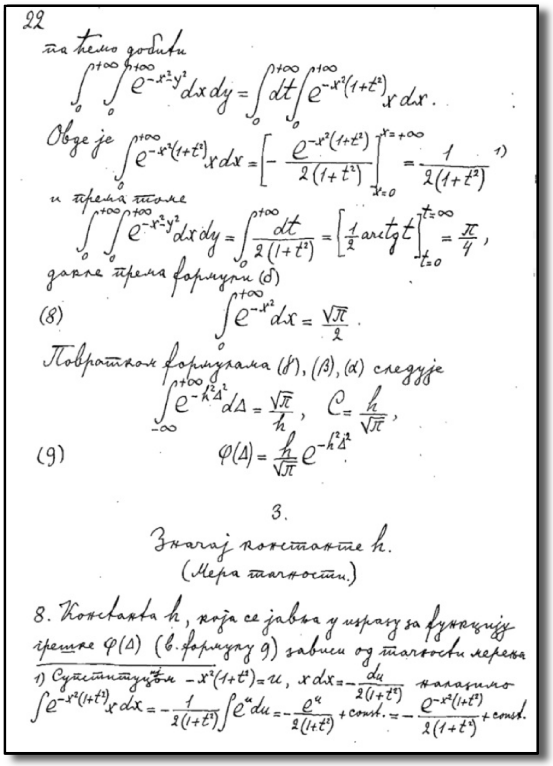
The only professors at the Belgrade Higher School since 1896 to 1909 were Bogdan Gavrilović and Mihailo Petrović. In 1909 came Milutin Milanković for the professor of applied mathematics. Three of them were the only professor and lecturers of mathematics at Belgrade university until the appearance of Mladen Berić and Sima Marković, first mathematicians who earned their doctoral dissertations in Belgrade. The reason for the lack of the text-books is probably the fact that Petrović, a professor at the Faculty of Philosophy, mostly wrote monographs, and his students as a rule learned from his scripts. Gavrilović, who taught at technical colleges, wrote two excellent books, the first in the theory of determinants, the second one in the theory of second-order curves. Both books could be compared to the world's best works in the field. But these books were first of all and foremost monographs and rather difficult to access for the average student. The following somewhat anecdotal story witnesses this state at the University. The first author of this paper owns Gavrilović's book on the second – order curves which consists of two volumes, each having about 450 pages. Fore edges in the text block of about 40 leafs were blackish from the use dirt, while the rest of them in both volumes were quite white. It means that the former owner never opened most of the pages in the book.

Another feature of Danic's books is supplements from the natural sciences that relied on mathematics. These additions that usually came after the basic text were on mechanics, astronomy, geodesy and cartography. This fine feature was certainly of the great benefit to the general student of natural science, or of some technical faculty.

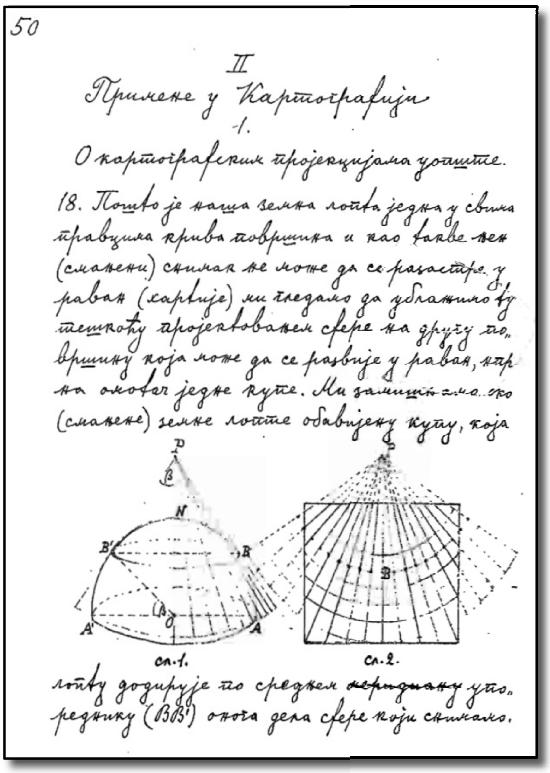
Contents of these books are extensively discussed and in details presented in [7].

Besides the doctoral dissertation, in the Virtual Library [8] there are digital copies of the Danić's books 3, 4, 8. Besides that there are digital copies of two hand written manuscripts:

1. *Теорија конформног снимања и њена примена у Картографији и Вишој Геодезији* (Theory of conformal mapping and its application to Cartography and Higher Geodesy), IV+147, Belgrade, end of XIX Century.
2. *Теорни основи Методе најмањих квадрата* (Foundation of the least square method), II+78, Belgrade, end of XIX Century



Page 50 from the script
Theory of conformal mapping and its application to Cartography and Geodesy



Page 22 from the script
Foundation of the least square method

Both scripts are certainly written before the Great War and are part of the famous collection of 24 manuscripts (see [5]) that belonged to Borivoj J. Pujić, a student of mathematics at that time. This unique collection of manuscripts owns the Mathematical Institute SASA. A reader of these scripts can see that the mathematical tools used in both of them are advanced, while the approach and the presented material is modern for that time.

Conclusion

Dimitrije Danić was the first Serbian doctor of mathematics and also an important and outstanding mathematician and the professor of mathematics at the Military Academy in Belgrade at the end of the XIX and the beginning of XX century. It is considered that he wrote several first-class manuals and text-book on mathematics used at the Academy. We also presented here for the first time scripts of his lectures that witnesses on the high level of his lectures.

Acknowledgment

This work is partially supported by the grant of the Serbian Ministry of education, science and technology development under the project number III 44006. We are very grateful to the anonymous Referee for the constructive critics and valuable suggestions.

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85 O jednom prilogu Mihaila Petrovića Alasa astronomiji

N. Pejović and Ž. Mijajlović
Publ. Astron. Druš. Rudjer Bošković, Beograd
Beograd, vol. 19, pp. 331-341, 2019.
Zb. Konf. "Razvoj astronomije kod Srba X,
Beograd, 22-26 april 2019"
(M. Dimitrijević, ed.).
ISBN 978-86-89035-13-1 (ADRB).

The title of the paper is "ON a contribution of Mihailo Petrović Alas to astronomy". The great Serbina mathematician Mihailo Petrović Alas was the founder of the Serbian mathematical school. Petrović invented a fast numerical procedure which used the Serbian astronomer Vojislav Mišković in his calculations of orbits of rarely observed small planets.

Зборник радова конференције "Развој астрономије код Срба X"
Београд, 22-26. април 2019.
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 19, 2019, 331-341

О ЈЕДНОМ ПРИЛОГУ МИХАИЛА ПЕТРОВИЋА АЛАСА АСТРОНОМИЈИ

НАДЕЖДА ПЕЈОВИЋ и ЖАРКО МИЈАЈЛОВИЋ

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Резиме: Представљен је један нумерички поступак Михаила Петровића Аласа који је Војислав Мишковић користио у својим прорачунима идентификовања мање посматраних малих планета.

Кључне речи: Михаило Петровић Алас, Војислав Мишковић, номограми, апроксимације, мале планете, историја астрономије

*

Прецизно говорећи, Петровић није решио неки одређен астрономски проблем. Заправо дао је апроксимативни нумерички поступак који је у оно време, када нису постојали програмабилни рачунари, омогућио да се битно убрза нумеричка анализа једне класе астрономских података којима се бавио Мишковић. Видећемо како је дошло до формулације проблема и његово решење, а такође упознаћемо се са срдачним и присним односом два великана српске науке 20. века, математичара Михаила Петровића Аласа и астронома Војислава Мишковића.

У првом делу рукописа представићемо краће биографије Михаила Петровића и Војислава Мишковића, као и Клуб београдских математичара који је био главно место професионалног окупљања математичара али и научника из сродних наука, механике, астрономије и геонаука.

Михаило Петровић Алас (1868-1943) по многим одликама био је полимат, научник универзалног занимања за разне области математике и других природних наука. Био је пре свега изванредан математичар и угледан професор Универзитета у Београду, али и рибар, књижевник, филозоф, музичар, светски путник и путописац. Завршио је студије математике на Великој школи у Београду, а 1889. отишао је у Париз где на Сорбони стиче



дипломе из математике, физике и хемије. У 26. години, годину дана по завршетку редовних студија, на истом универзитету брани докторат математичких наука као студент чувених француских математичара Анрија Поенкареа, Шарла Ермитеа и Шарла Емила Пикара. Исте године постаје професор Велике школе и доноси дух француске математике у Београд. Тада почиње његово дуго и плодно бављење науком, док се захваљујући њему Београд у математичким наукама приближава другим великим европским центрима.

Петровић је подједнако добро познавао и добијао изванредне резултате у неколико математичких области, пре свега у диференцијалним једначинама, нумеричкој анализи и геометрији полинома. Занимао се и за природне науке, хемију, физику и биологију у којима је такође објављивао научне радове. Такође је показивао интересовање и за астрономију и објавио је два рукописа из теорије релативности. Објавио је око 400 рукописа, од тога преко триста из математике углавном у водећим светским часописима. Био је ментор свих докторских дисертација (11) из математике које су одбрањене пре Другог светског рата на Београдском универзитету. С обзиром на ову чињеницу, али и на друге заслуге које је имао за српску математику, сматра се оснивачем Српске математичке школе.

Петровић је волео боемски живот. Свирао је виолину и био је оснивач музичке дружине „Суз“ са којом је свирао по београдским кафанама и „рибљим баловима“. Био је учесник шест поморских експедиција по Атланском и Индијском океану и по северним и јужним морима. Све до Другог светског рата био је главни криптограф Југословенске војске и имао је чин потпуковника.

Војислав Мишковић (1892–1976) рођен је у Фужинама у Хрватској. Почео је да студира астрономију на Универзитетима у Будимпешти и Гетингену непосредно пред почетак Првог светског рата. Када је рат почео, дошао је илегално у Србију и постао члан волонтерског вода. После демобилизације отишао је у Француску где завршава студије астрономије



1919, а одмах затим постаје асистент на Астрономској Опсерваторији у Марсеју. Од 1922. године радио је као астроном на Опсерваторији у Ници.

Докторирао је 1924. на Универзитету у Монпељеу. Мишковић је у Француској био веома активан као астроном. Организовао је нове послове у опсерваторијама, био је уредник часописа за астрономију, организовао је и извео астрономска мерења и триангулацију повезујући Корзику са француским Источним Алпима. Том приликом представио је прву варијанту свог оригиналног астрономског инструмента. За ове доприносе добио је медаљу од Француске академије наука.

У Београд је по позиву дошао 1926. године, већ као афирмисан астроном. Исте године именован је за ванредног професора на Филозофском факултету у Београду у оквиру управо основане Катедре за теоријску и практичну Астрономију. Истовремено је постао директор новоосноване Астрономске опсерваторију у Београду. Може се рећи да је доласком Мишковића из Француске у Београду започео развој астрономије у Србији. Мишковић је 1929. изабран за дописног члана Српске краљевске академије, а 1939. постаје редовни члан. У његову част једна мала планета добија име *Мишковић*.

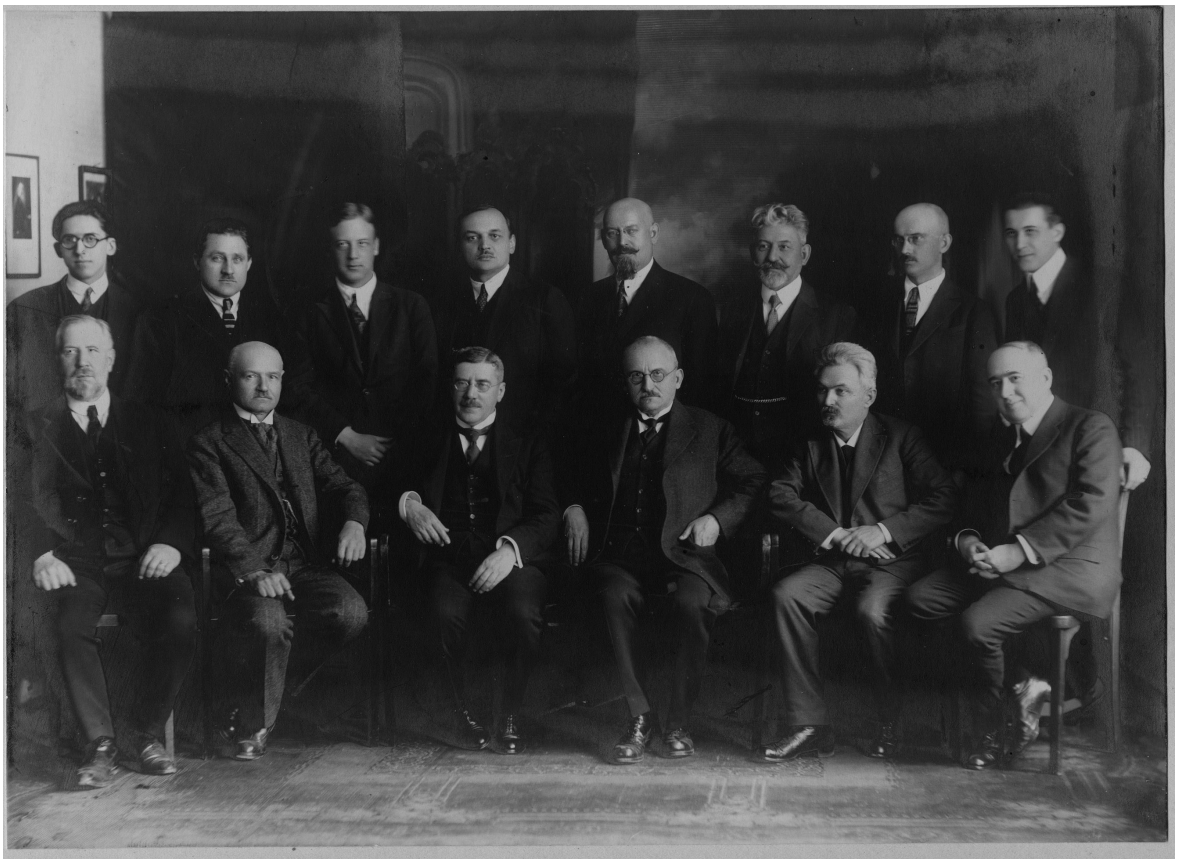
Клуб математичара

У прошлом веку, између два светска рата у Београду је постојао клуб математичара који је окупљао не само математичаре, већ и професоре и асистенте Београдског универзитета који су се бавили примењеном математиком. Под овим су се у оно време подразумевале теоријска механика, астрономија и друге науке које се граниче са математиком. На састанцима клуба чланови су имали прилику да се упознају са конкретним проблемима којима су се бавиле њихове колеге.

Чланови Клуба често су предузимали заједничке подухвате од ширег научног и друштвеног значаја. О томе сведочи, на пример, фотографија из средине двадесетих година прошлог века на којој је представљена експедиција на Фрушку Гору која је била у потрази за местом за нову астрономску опсерваторију,

Проблем

Једном приликом на састанку Клуба, београдски астроном Војислав Мишковић изложио је метод за идентификовање мање посматраних малих планета. Метод је претпостављао једноставну али оптималну рационалну апсоксимацију односа средњих сидеричких дневних кретања планетоида и Земље. Такав нумерички поступак није био познат Мишковићу, а метода је била тешко употребљива без таквог брзог алгоритма.



Историјски снимак: Клуб математичара - Београдска математичка школа 1926 године. Милош Радојчић, Тадија Пејовић, Вјачеслав Жардецки, Антон Билимовић, Петар Зајанковски, Јеленко Михаиловић (сеизмолог), Радивој Кашанин, Јован Карамата (стоје). Никола Салтиков, Михаило Петровић, Павле Поповић (ректор), Богдан Гавриловић, К. Петковић (декан Филозофског факултета), Милутин Миланковић (седе).

Петровић је неколико дана по упознавању са Мишковићевим проблемом представио Мишковићу графички поступак који једноставно и брзо решава тај задатак. У даљем излагању представићемо ово Петровићево решење, „графички рационализатор“.

Решење

Петровић заправо није објавио решење проблема, већ Мишковић и то тек десет година након Петровићеве смрти у чланку

ГРАФИЧКИ РАЦИОНАЛИЗАТОР
Успомена на Михаила Петровића
Зборник радова С.А.Н. XXXV, Математички институт С.А.Н. књ. 3, 1953.

Мишковић је објавио овај спис пре свега у знак сећања на великог математичара и цењеног колегу, Михаила Петровића. Први део Мишковићевог чланка има историјску конотацију. Укратко описује Петровићев лик и његову научну универзалност, али подсећа и на чувени Београдски математички клуб који је имао важну улогу у кругу српских математичара у раздобљу између два светска рата. О томе Мишковић на уводним странама свог чланка пише следеће:

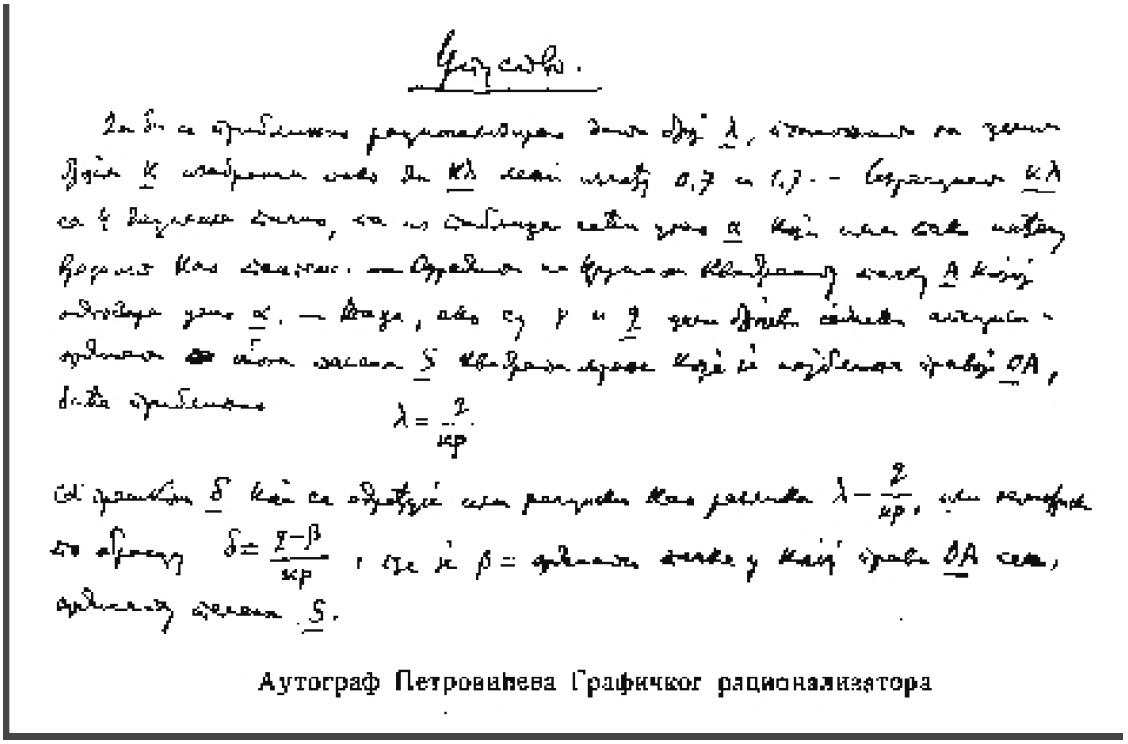
Објављујемо овај мали прилог, мада није био намењен да буде објављен, као успомену на професора Петровића. Писан и његовом руком, оживеће нам у сећању, истина, не Петровића специјалисту за диференцијалне једначине, теорију редова и функција или творца математичких спектара, већ – Петровића који се радо разоноћавао, покаткад, елементарном и нумеричком математиком. А оживеће нам за тренутак у сећању, тих неколико Петровићевих редака, и некадањи клуб математичара Београдског универзитета, који је он толико волео, можда баш зато што није имао никакав формални карактер, ни писане статуте, чак ни неки званични назив, али је радио интензивно, предано и успешно, тако да су се њиме поносили и његови чланови и наш Универзитет.

Ова Петровићева нотица датира из доба првих радних састанака нашег предратног клуба математичара. На једном од њих аутор ових редова приказао је идеју о квазиидентичним опозицијама планетоида и њихову улогу у идентификовању недовољно посматраних тих објеката. За примену идеје требало је одредити периоде, за сваки од познатих планетоида, после којих се они враћају у опозицију са Земљом у исти, или приближно исти положај као и у извесној, произвољно изабраној, почетној опозицији.

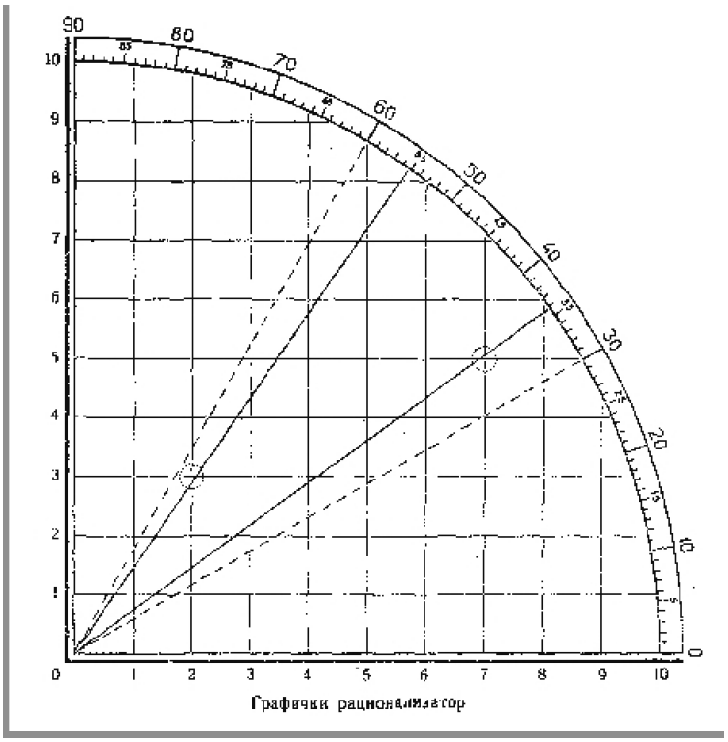


Експедиција београдских астронома и математичара на Фрушку Гору с циљем да се одреди место изградње нове опсерваторије. С лева: Р. Кашанин, Ј. Михаиловић, М. Петровић, П. Поповић, А. Билимовић, М. Миланковић, В. В. Мишковић, Г. Грачанин и водич (Архив САНУ).

Другим речима, требало је, за сваки од познати.х планетоида - а у то време било их је око 1200 познатих – апроксимирати однос средњих сидеричких дневних кретања (или револуција) планетоида и Земље што је могуће простијим разломком, то јест са што мањим апсолутним вредностима и бројиоца и имениоца. Усто је требало још, за сваку апроксимацију, оценити (у данима) и њено отступање од тачне вредности. Дакле, већ и само одређивање периода квазиидентичних опозиција, за све нумерисане планетоиде, претстављало је позамашан нумерички посао. Два-три дана после тога састанка клуба пресрео ме је у Семинару професор Петровић, мало насмејан, са: "Имам за ону вашу ствар згодан графички поступак". И пружио ми је овај листић.



Аутограф Петровићева Графичког рационализатора



Графички рационализатор

Уз листић је била приложена и сличица квадранта са квадратном мрежом. Прочитао сам текст и видео одмах у чему је поступак. Једино ми, у том тренутку, нису биле јасне границе 0,7 и 1,7. И запитах професора Петровића: "Откуд баш 0,7 и 1,7"? Он се насмеја мало, па ће рећи: "Тако му најбоље дође, видићете".

Петровићев графички рационализатор

Петровићево решење је врло једноставно и елементарно, али ипак у себи крије лепу искру изврсне математичке довитљивости. Проблем који је поставио Мишковић може се овако представити.

За дати реалан број λ одредити рационалну апроксимацију $R = q/p$, где су q и p релативно мали бројеви али тако да разлика $\Delta = \lambda - R$ буде што мања.

Овај проблем припада и данас актуелној области диофантовских апроксимација и решава се помоћу развоја броја λ у верижни разломак, што Мишковић и спомиње у свом чланку. Број $\lambda > 0$ добијен је мерењем и наравно у пракси λ је заправо разломак, тј. $\lambda = M/N$, где су M и N произвољни и релативно велики цели бројеви. Петровић решава овај задатак на следећи начин. Посматра праву

$$Y = \lambda X \qquad \text{(L)}$$

која очигледно пролази кроз координатни почетак и има коефицијент правца λ Петровић затим конструише целобројну мрежу са „великим окнима“, рецимо 10x10 и нову праву L'

$$Y = k\lambda X.$$

и тако бира k да L' лежи између две фиксне праве које захватају са апсцисом углове редом од 35° и 60°. У тој мрежи налази тачку S која је најближа правој L'. Ако тачкаS у мрежи има координате (p, q), тада је коефицијент правца праве L' приближно једнак q/p (приметимо да је $q, p < 10$, тј. q и p су релативно мали бројеви), па можемо узети да је

$$k\lambda \approx q/p, \text{ тј.} \lambda = M/N \approx \frac{q}{kp} = R, \text{ одакле } \Delta = \lambda - R = \lambda - \frac{q}{kp}.$$

Мишковић илуструје овај поступак следећим примером.

Објаснићемо је овде на једном нумеричком примеру. Узмимо да треба упростити разломак λ, у којем је $M=2583$, $N=35796$.

По Петровићеву поступку, за фактор k могли бисмо узети било који цео број од 10 до 24. Узећемо, наравно, најмањи, јер је овај и са гледишта рачуна најпогоднији. У том случају је

$$k\lambda=\operatorname{tg}\alpha=0{,}7216, \text{ дакле } \alpha=35^0\,49'\approx 35^0{,}8.$$

Повуцимо из почетка графичког рационализатора праву, под овим углом са апсцисном осом, до саме издељене периферије. Потражимо затим координате темена квадратића мреже кроз који пролази ова права, односно, темена најмање удаљена од те праве. Видимо да је то окружено теме чије су координате $p=7$ и $q=5$. Према томе, тражена приближна вредност датог разломка је

$$\lambda=\frac{M}{N}\approx\frac{q}{kp}=\frac{5}{70}\approx 0{,}0714,$$

са отступањем

$$\frac{M}{N}-\frac{q}{kp}=0{,}0722-0{,}0714\approx 0{,}001.$$

На крају, Мишковић овако коментарише Петровићеву методу.

Петровићев графички рационализатор има две практичне предности. Прво, омогућује да се непосредно види и да ли се дати разломак апроксимира оздо (подбаченом) или озго (пребаченом приближном вредношћу). И, друго, омогућује да се релативно брзо и лако, са графика, не само нађе тражена приближна вредност датог разломка, већ да се и оцени његово приближно отступање од тачне вредности.

Што се тиче степена тачности који се графичким рационализатором може постићи, овај, наравно, зависи од његове величине и прецизности са којом је израђен.

Најзад, још једна напомена. Са графика се види и смисао граница 0,7 и 1,7. То су, уствари, само заокругљене приближне вредности коефицијената праваца, којима је Петровић хтео да ограничи и, на свој начин, ограничио онај сектор на којем ће се координате p и q моћи са подједнаком и највећом тачношћу са графика одређивати.

Прилози

1. Диплома рибарског мајстора Михаила Петровића Аласа.



2.



Google doodle посвећен Михаилу Петровићу поводом 145 година од рођења

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A CONTRIBUTION OF MIHAILO PETROVIĆ ALAS TO ASTRONOMY

Mihailo Petrović invented a numerical method which Vojislav Mišković used in the numerical analysis of data and identification of less observed small planets.

Key words: Mihailo Petrović Alas, Vojislav Mišković, nomogram, numerical approximation, small planets, history of astronomy

86 Digitalizovani univerzitetski udžbenici prof. Zaharija Brkića

V. Radović and N. Pejović
Publ. Astron. Druš. Rudjer Bošković, Beograd
Beograd, vol. 19, pp. 397-405, 2019
Zb. Konf. *Razvoj astronomije kod Srba X, Beograd, 22-26 april 2019*,
(M. Dimitrijević, ed.).
ISBN 978-86-89035-13-1 (ADRB).

The title of the paper is "Digitized university textbooks of Professor Zaharije Brkić". A short biography of Professor Zacharije Brkić and his digitized university textbooks are presented. There are two such textbooks by Professor Brkić and their digitized copies are deposited in the Virtual Library of the Faculty of Mathematics. Titles of the books are "General Astronomy" and "Geodesy". Brkić was the first astronomer who defende his PhD in Serbia, and the digital version of his doctorate is available in the Virtual Library as well.

Зборник радова конференције "Развој астрономије код Срба X"
Београд, 22-26. април 2019.
уредник М. С. Димитријевић
Публ. Астр. друш. "Руђер Бошковић" бр. 19, 2019, 397-405

ДИГИТАЛИЗОВАНИ УНИВЕРЗИТЕТСКИ УЏБЕНИЦИ ПРОФ. ЗАХАРИЈА БРКИЋА

ВИКТОР РАДОВИЋ и НАДЕЖДА ПЕЈОВИЋ

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Резиме: У овом раду биће представљена кратка биографија професора Захарија Бркића, након чега ће бити приказани његови дигитализовани уџбеници и значај истих за развој астрономије у Србији. Два уџбеника професора Бркића су дигитализована и доступна у Виртуелној библиотеци Математичког факултета (elibrary.matf.bg.ac.rs). У питању су уџбеници „Општа астрономија" и „Геодеска астрономија". Поред њих доступна је и дигитализована верзија примерка његове докторске дисертација, што је уједно и први докторат из астрономских наука одбрањен на Београдском универзитету. Уџбеник „Општа астрономија", који је написан у коауторству са професором Б. Шеварлићем, и педесет година након објављивања употребљава се као литература током студија астрономије на Математичком факултету.

Кључне речи: историја астрономије, дигитализација, образовање

1. УВОД

Виртуелна библиотека је развијена у оквиру пројекта дигитализације којим управљају Математички факултет и Математички институт Српске академије науке и уметности (пројекат ИИ 44006). Основна намена виртуелне библиотеке јесте спречавање да књиге, уџбеници и радови наших научника, професора и студената подлегну зубу времена и забораву, а да лако и једноставно буду доступни јавности (Огњановић 2003, Мијајловић, Пејовић 2010).

У овом раду представићемо дигитализоване уџбенике проф. др Захарија Бркића, астронома и уваженог професора Београдског универзитета. У наредној секцији приказана је кратка биографија проф. Захарија Бркића, где су посебно истакнути његови научни доприноси. Треће и четврто поглавље су посвећени анализи дигитализованих уџбеника. Пето поглавље је

посвећено легату и заоставштини професораа Бркића. На крају је дат кратак преглед рада и списак коришћених референци.

2. ЗАХАРИЈЕ БРКИЋ (1910–1979)

Захарије Бркић (Слика 1) рођен је у селу Пољна, које припада општини Трстеник, 8. новембра 1910. године (Dačić, Milogradov-Turin 2011, Споменица 2003). Иако је одрастао у земљорадничкој породици, Захарије је још од најмлађих дана показивао афинитет и интересовање према природним наука, посебно математици. Његови родитељи Милосав и Ленка имали су слуха за синовљеву надареност и омогућили му да заврши школу ван родног места. Захарије Бркић завршио је реалну гимназију у Чачку, а потом прешао у Београд на Филозофски факултет. Дипломирао је 1936. године на групи за теоријску математику.



Слика 1: Професор Захарије Бркић.

Две године након дипломирања почиње да ради као асистент на Астрономској опсерваторији у Београду. Радећи на опсерваторији, показује изузетну способност за рад са астрономским инструментима, те бива веома брзо унапређен. Управо због његовог познавања астрономских инструмената, њихове употребе и коришћења, Захарије Бркић је почео да ради као доцент на Геодетском одсеку Грађевинског факултета 1955. године, где је 1957. унапређен у звање професора. На позицији професора Грађевинског факултета остаје све до преласка на Природно-математички факултет 1959. године, где остаје до одласка у пензију 1978. године.

Захарије Бркић је своју докторску дисертацију под називом „Анализа систематских грешака пасажног инструмента и других систематских утицаја на одређивање времена" одбранио 26. септембра 1958. То је уједно била и прва дисертација из астрономских наука одбрањена на Београдском универзитету. Ментор за израду дисертације био је академик професор Војислав Мишковић. Интересантно је истаћи да је проф. Мишковић био први доктор астрономских наука са наших простора који је докторирао на иностраном универзитету (дисертацију је одбранио 1924. године на универзитету у Монпељеу, Француска). У комисији за одбрану дисертације З. Бркића је поред ментора био Татомир Анђелић, редовни професор и декан, и Константин Ворољец, као дописни члан академије (Vidojevic, Segan 2006). Током рада на факултету Захарије Бркић је био члан комисија за одбрану више докторских дисертација: Бранислав Шеварлић (1960), Василије Оскањан (1961), Ђорђе Телеки (1964) и Драгутин Ђуровић (1974).

Захарије Бркић је један од оснивача неколико служби на Астрономској опсерваторији: Службе времена и промене географске лонгитуде и Службе за промену латитуде и кретања Земљиних полова. Објавио је 34 научна рада и 27 стручних радова. Поред тога, коаутор је два универзитетска уџбеника заједно са својим пријатељем и колегом Бранимиром Шеварлићем.

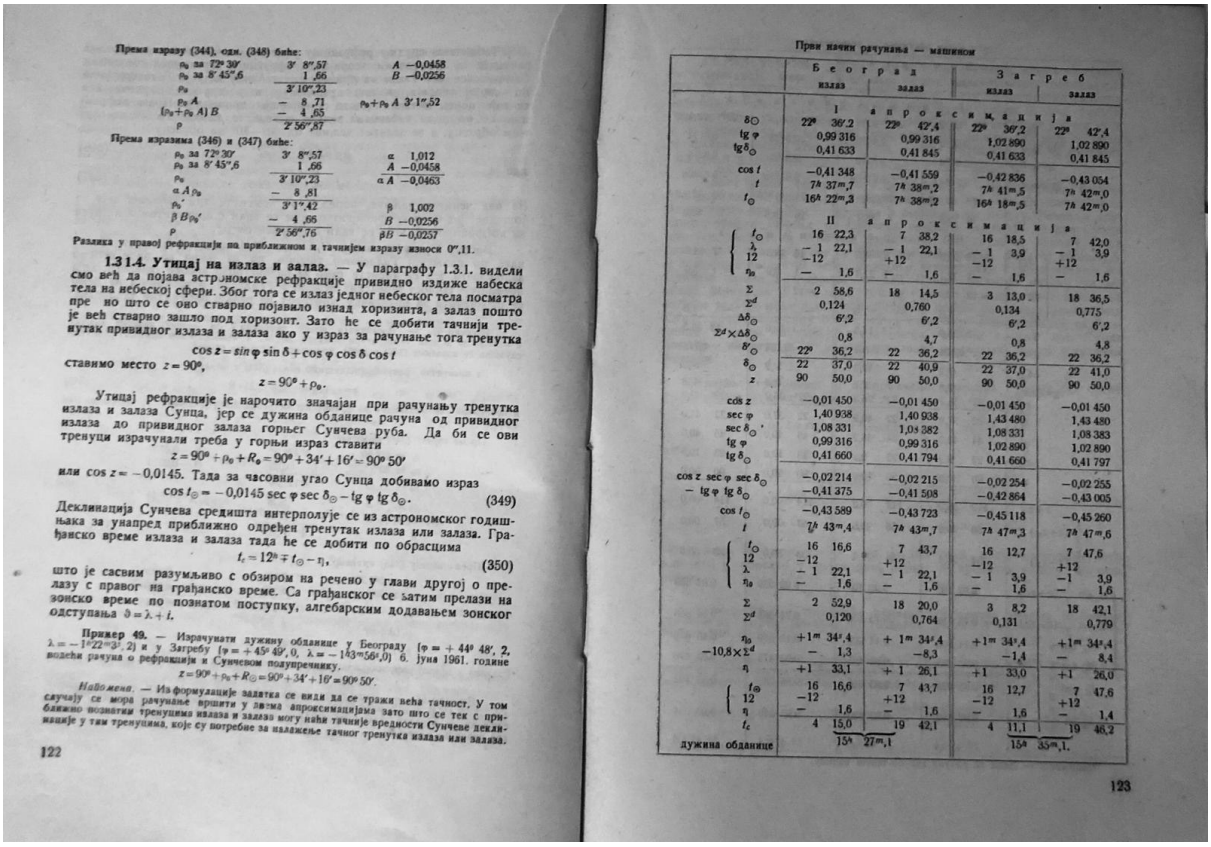
Захарије Бркић је био ожењен Марицом Марковић, са којом је имао двоје деце: ћерку Ружицу и сина Слободана. Након дуготрајне болести, професор Захарије Бркић преминуо је 25. априла 1979. године. Непосредно пред смрт, одликован је Орденом рада са црвеном заставом, који је добио захваљујући доприносима у науци и настави, односно утицају који је оставио на небројане генерације студената и научника.

3. ДИГИТАЛИЗОВАНИ УЏБЕНИК „ГЕОДЕСКА АСТРОНОМИЈА"

Први уџбеник који је професор Захарије Бркић написао заједно са проф. Бранимиром Шеварлићем је „Геодеска астрономија" (Ševarlić, Brkić 1963). Он је објављен 1963. године и представља први уџбеник из астрономије, иако је првенствено намењен студентима Грађевинског факултета. Уџбеник је доступан на адреси: http://elibrary.matf.bg.ac.rs/handle/123456789/1550.

Уџбеник има 266 стана, 63 решених задатака и велики број илустрација и табела. Решени задаци су значајни, јер у домаћој литератури постоји само једна збирка решених задатака из Опште астрономије, коју је написао Војислав Мишковић (Mišković 1956, 2019, Radović, Djošović 2018). Она је такође дигитализована и доступна на адреси http://elibrary.matf.bg.ac.rs/handle/123456789/650. Сама збирка из Опште астрономије се састоји из два дела. Први део је објављен 1957. године и садржи задатке из сферне тригонометрије и координатних система у астрономији. Други део представља необјављени рукопис, који садржи поглавља о рефракцији, небеској механици и привидном годишњем кретању

Сунца. Он је једино доступан на Виртуелној библиотеци (http://elibrary.matf.bg.ac.rs/handle/123456789/650).



Према изразу (344), одн. (348) биће:

ρ_0 за 72° 30′	3′ 8″,57	A −0,0458
ρ_0 за 8′ 45″,6	1 ,66	B −0,0256
ρ_0	3′ 10″,23	
$\rho_0 A$	− 8 ,71	$\rho_0+\rho_0 A$ 3′ 1″,52
$(\rho_0+\rho_0 A)B$	− 4 ,65	
ρ	2′ 56″,87	

Према изразима (346) и (347) биће:

ρ_0 за 72° 30′	3′ 8″,57	α 1,012
ρ_0 за 8′ 45″,6	1 ,66	A −0,0458
ρ_0	3′ 10″,23	αA −0,0463
$\alpha A \rho_0$	− 8 ,81	
ρ_0'	3′ 1″,42	β 1,002
$\beta B \rho_0'$	− 4 ,66	B −0,0256
ρ	2′ 56″,76	βB −0,0257

Разлика у правој рефракцији по приближном и тачнијем изразу износи 0″,11.

1.3.1.4. Утицај на излаз и залаз. — У параграфу 1.3.1. видели смо већ да појава астрономске рефракције привидно издиже набеска тела на небеској сфери. Због тога се излаз једног небеског тела посматра пре но што се оно стварно појавило изнад хоризинта, а залаз пошто је већ стварно зашло под хоризонт. Зато ће се добити тачнији тренутак привидног излаза и залаза ако у израз за рачунање тога тренутка

$$\cos z = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos t$$

ставимо место $z = 90^0$,

$$z = 90^0 + \rho_0.$$

Утицај рефракције је нарочито значајан при рачунању тренутка излаза и залаза Сунца, јер се дужина обданице рачуна од привидног излаза до привидног залаза горњег Сунчева руба. Да би се ови тренуци израчунали треба у горњи израз ставити

$$z = 90^0 + \rho_0 + R_0 = 90^0 + 34' + 16' = 90^0\,50'$$

или $\cos z = -0{,}0145$. Тада за часовни угао Сунца добивамо израз

$$\cos t_\odot = -0{,}0145 \sec\varphi \sec\delta_\odot - \operatorname{tg}\varphi \operatorname{tg}\delta_\odot. \qquad (349)$$

Деклинација Сунчева средишта интерполује се из астрономског годишњака за унапред приближно одређен тренутак излаза или залаза. Грађанско време излаза и залаза тада ће се добити по обрасцима

$$t_c = 12^h \mp t_\odot - \eta, \qquad (350)$$

што је сасвим разумљиво с обзиром на речено у глави другој о прелазу с правог на грађанско време. Са грађанског се затим прелази на зонско време по познатом поступку, алгебарским додавањем зонског одступања $\vartheta = \lambda + i$.

Пример 49. — Израчунати дужину обданице у Београду ($\varphi = +44^0\,48'$, 2, $\lambda = -1^h22^m3^s$, 2) и у Загребу ($\varphi = +45^0\,49'$, 0, $\lambda = -1^h3^m56^s$, 0) 6. јуна 1961. године водећи рачуна о рефракцији и Сунчевом полупречнику.

$$z = 90^0 + \rho_0 + R_\odot = 90^0 + 34' + 16' = 90^0\,50'.$$

Напомена. — Из формулације задатка се види да се тражи већа тачност. У том случају се мора рачунање вршити у двема апроксимацијама зато што се тек с приближно познатим тренуцима излаза и залаза могу наћи тачније вредности Сунчеве деклинације у тим тренуцима, које су потребне за налажење тачног тренутка излаза или залаза.

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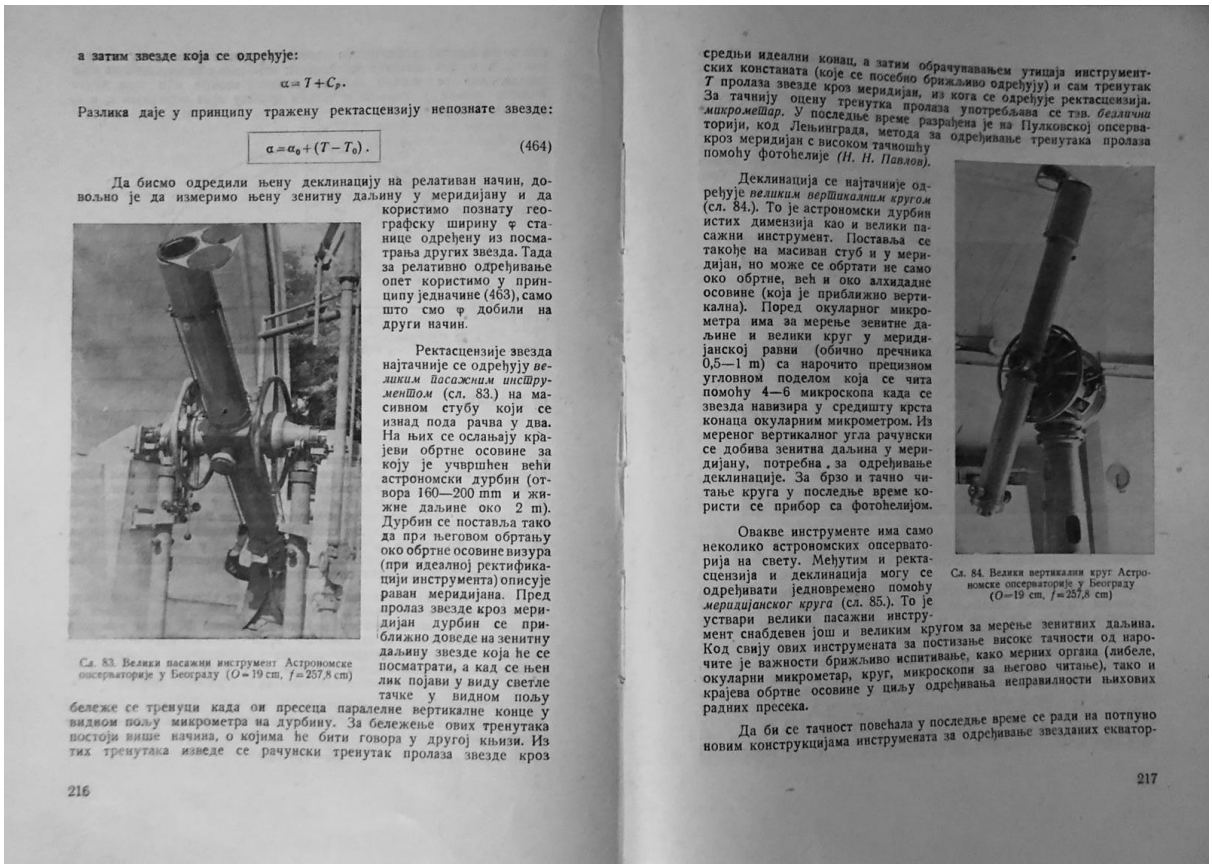
Први начин рачунања — машином

	Београд излаз	Београд залаз	Загреб излаз	Загреб залаз
	I апроксимација			
$\delta_\odot$	22° 36′,2	22° 42′,4	22° 36′,2	22° 42′,4
$\operatorname{tg}\varphi$	0,99 316	0,99 316	1,02 890	1,02 890
$\operatorname{tg}\delta_\odot$	0,41 633	0,41 845	0,41 633	0,41 845
$\cos t$	−0,41 348	−0,41 559	−0,42 836	−0,43 054
t	$7^h\,37^m$,7	$7^h\,38^m$,2	$7^h\,41^m$,5	$7^h\,42^m$,0
$t_\odot$	$16^h\,22^m$,3	$7^h\,38^m$,2	$16^h\,18^m$,5	$7^h\,42^m$,0
	II апроксимација			
$t_\odot$	16 22,3	7 38,2	16 18,5	7 42,0
λ	− 1 22,1	− 1 22,1	− 1 3,9	− 1 3,9
12	−12	+12	−12	+12
η_0	− 1,6	− 1,6	− 1,6	− 1,6
Σ	2 58,6	18 14,5	3 13,0	18 36,5
Σ^d	0,124	0,760	0,134	0,775
$\Delta\delta_\odot$	6′,2	6′,2	6′,2	6′,2
$\Sigma^d\times\Delta\delta_\odot$	0,8	4,7	0,8	4,8
$\delta'_\odot$	22° 36,2	22 36,2	22 36,2	22 36,2
$\delta_\odot$	22 37,0	22 40,9	22 37,0	22 41,0
z	90 50,0	90 50,0	90 50,0	90 50,0
$\cos z$	−0,01 450	−0,01 450	−0,01 450	−0,01 450
$\sec\varphi$	1,40 938	1,40 938	1,43 480	1,43 480
$\sec\delta_\odot$	1,08 331	1,08 382	1,08 331	1,08 383
$\operatorname{tg}\varphi$	0,99 316	0,99 316	1,02 890	1,02 890
$\operatorname{tg}\delta_\odot$	0,41 660	0,41 794	0,41 660	0,41 797
$\cos z \sec\varphi \sec\delta_\odot$	−0,02 214	−0,02 215	−0,02 254	−0,02 255
$-\operatorname{tg}\varphi \operatorname{tg}\delta_\odot$	−0,41 375	−0,41 508	−0,42 864	−0,43 005
$\cos t_\odot$	−0,43 589	−0,43 723	−0,45 118	−0,45 260
t	$7^h\,43^m$,4	$7^h\,43^m$,7	$7^h\,47^m$,3	$7^h\,47^m$,6
$t_\odot$	16 16,6	7 43,7	16 12,7	7 47,6
12	−12	+12	−12	+12
λ	− 1 22,1	− 1 22,1	− 1 3,9	−1 3,9
η_0	− 1,6	− 1,6	− 1,6	− 1,6
Σ	2 52,9	18 20,0	3 8,2	18 42,1
Σ^d	0,120	0,764	0,131	0,779
η_0	$+1^m\,34^s$,4	$+1^m\,34^s$,4	$+1^m\,34^s$,4	$+1^m\,34^s$,4
$-10{,}8\times\Sigma^d$	− 1,3	−8,3	−1,4	− 8,4
η	+1 33,1	+ 1 26,1	+1 33,0	+1 26,0
$t_\odot$	16 16,6	7 43,7	16 12,7	7 47,6
12	−12	+12	−12	+12
η	− 1,6	− 1,6	− 1,6	− 1,4
t_c	4 15,0	19 42,1	4 11,1	19 46,2
дужина обданице	$15^h\,27^m$,1		$15^h\,35^m$,1.	

123

Слика 2: Странице 122 и 123 из уџбеника „Геодеска астрономија” на којима је приказан рачун времена излаза и залаза Сунца са укљученим утицајем рефракције.

Издвојићемо неке интересантне детаље присутне у уџбенику „Геодеска астрономија”. На слици 2 (странице 122 и 123 из уџбеника) приказано је решење проблема израчунавања времена изласка и заласка Сунца (проблем број 49). Овај проблем је значајан због одређивања дужине обданице за неко место. Приступ који је изложен омогућава и обрачун утицаја рефракције и Сунчевог полупречника на излаз и залаз, чиме се постиже већа тачност. Задатак је решен из два основна корака. У првом се обрачунавају приближне екваторске координате Сунца у тренутку када се налази на посматрачевом хоризонту. Тако одређене координате се затим користе у другом кораку ради одређивања средњег времена изласка и заласка Сунца за дато место и датум. Посебно треба истаћи да је за решавање неопходно доста нумеричког рачуна, који је изведен без употребе рачунара и калкулатора, већ коришћењем само логаритамских таблица.



а затим звезде која се одређује:

$$\alpha = T + C_p.$$

Разлика даје у принципу тражену ректасцензију непознате звезде:

$$\boxed{\alpha = \alpha_0 + (T - T_0)}. \qquad (464)$$

Да бисмо одредили њену деклинацију на релативан начин, довољно је да измеримо њену зенитну даљину у меридијану и да користимо познату географску ширину φ станице одређену из посматрања других звезда. Тада за релативно одређивање опет користимо у принципу једначине (463), само што смо φ добили на други начин.

Сл. 83. Велики пасажни инструмент Астрономске опсерваторије у Београду ($O = 19$ cm, $f = 257{,}8$ cm)

Ректасцензије звезда најтачније се одређују *великим пасажним инструментом* (сл. 83.) на масивном стубу који се изнад пода рачва у два. На њих се ослањају крајеви обртне осовине за коју је учвршћен већи астрономски дурбин (отвора 160—200 mm и жижне даљине око 2 m). Дурбин се поставља тако да при његовом обртању око обртне осовине визура (при идеалној ректификацији инструмента) описује раван меридијана. Пред пролаз звезде кроз меридијан дурбин се приближно доведе на зенитну даљину звезде која ће се посматрати, а кад се њен лик појави у виду светле тачке у видном пољу бележе се тренуци када он пресеца паралелне вертикалне конце у видном пољу микрометра на дурбину. За бележење ових тренутака постоји више начина, о којима ће бити говора у другој књизи. Из тих тренутака изведе се рачунски тренутак пролаза звезде кроз средњи идеални конац, а затим обрачунавањем утицаја инструментских константа (које се посебно брижљиво одређују) и сам тренутак T пролаза звезде кроз меридијан, из кога се одређује ректасцензија. За тачнију оцену тренутка пролаза употребљава се тзв. *бездачни микрометар*. У последње време разрађена је на Пулковској опсерваторији, код Љењинграда, метода за одређивање тренутака пролаза кроз меридијан с високом тачношћу помоћу фотоћелије (*Н. Н. Павлов*).

Деклинација се најтачније одређује *великим вертикалним кругом* (сл. 84.). То је астрономски дурбин истих димензија као и велики пасажни инструмент. Поставља се такође на масиван стуб и у меридијан, но може се обртати не само око обртне, већ и око алхидадне осовине (која је приближно вертикална). Поред окуларног микрометра има за мерење зенитне даљине и велики круг у меридијанској равни (обично пречника 0,5—1 m) са нарочито прецизном угловном поделом која се чита помоћу 4—6 микроскопа када се звезда навизира у средишту крста конаца окуларним микрометром. Из мереног вертикалног угла рачунски се добива зенитна даљина у меридијану, потребна за одређивање деклинације. За брзо и тачно читање круга у последње време користи се прибор са фотоћелијом.

Сл. 84. Велики вертикални круг Астрономске опсерваторије у Београду ($O = 19$ cm, $f = 257{,}8$ cm)

Овакве инструменте има само неколико астрономских опсерваторија на свету. Међутим и ректасцензија и деклинација могу се одређивати једновремено помоћу *меридијанског круга* (сл. 85.). То је уствари велики пасажни инструмент снабдевен још и великим кругом за мерење зенитних даљина. Код свију ових инструмената за постизање високе тачности од нарочите је важности брижљиво испитивање, како мерних органа (либеле, окуларни микрометар, круг, микроскопи за његово читање), тако и крајева обртне осовине у циљу одређивања неправилности њихових радних пресека.

Да би се тачност повећала у последње време се ради на потпуно новим конструкцијама инструмената за одређивање звездаиих екватор-

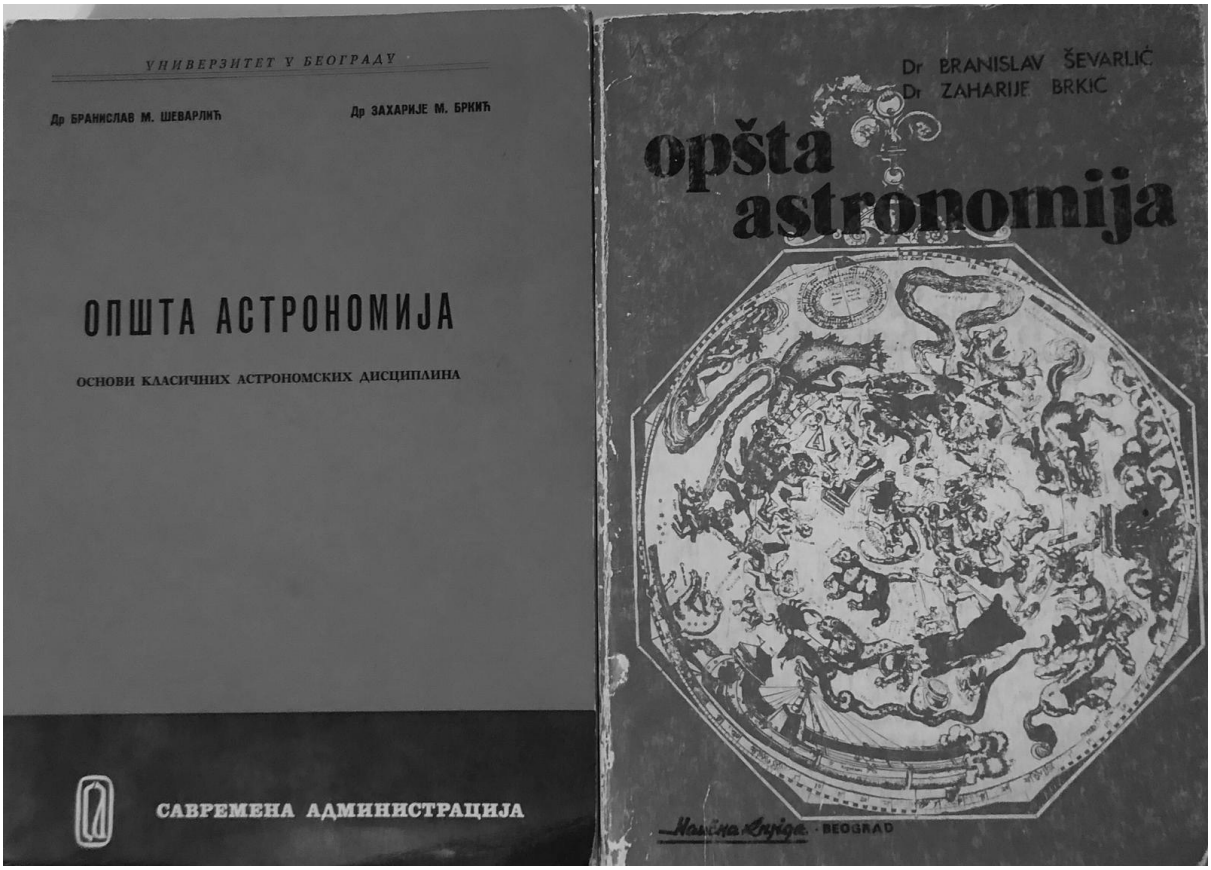
216

217

Слика 3: Странице 216 и 217 из уџбеника „Геодеска астрономија” где су приказани астрономски инструменти који се налазе на Астрономској опсерваторији у Београду (велики пасажни инструмент и велики вертикални круг).

На слици 3 приказане су слике астрономских инструмената које се налазе у уџбенику „Геодеска астрономија” на страни 216 и 217: велики пасажни инструмент и велики вертикални круг. Велики пасажни инструмент омогућава најпрецизније одређивање ректасцензије, док велики вертикални круг омогућава одређивање деклинација звезда. Употребом ова два инструмента, као и великог меридијанског круга који се налазе на Астрономској опсерваторији у Београду прављени су посматрачки каталози који су коришћени за израду фундаменталних каталога. Нажалост, због ширења Београда и све већег светлосног загађења које је последица експанзије града, посматрања се на овим инструментима више не врше и инструменти на опсерваторији су сада само музејски примерци. Треба споменути и велики меридијански круг који омогућава одређивање координата звезда релативном методом и који је такође био део инструмената на опсерваторији. За разлику од пасажног инструмента и вертикалног круга који одређују по једну координату звезде, меридијански круг омогућава одређивање обе координате истовремено. Нажалост, током НАТО бомбардовања Србије 1999. године, меридијански круг је уништен.

4. ДИГИТАЛИЗОВАНИ УЏБЕНИК „ОПШТА АСТРОНОМИЈА”



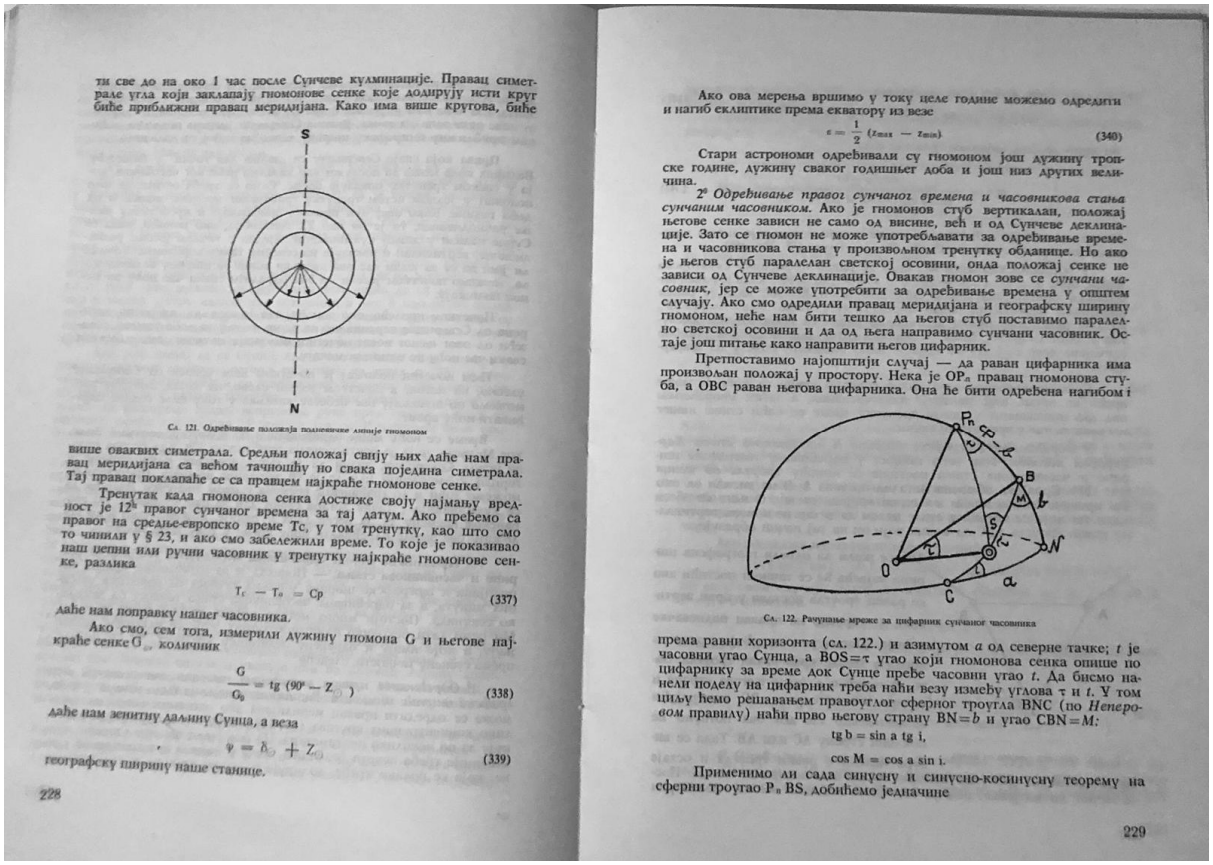
Слика 4: Корице уџбеника „Општа астрономија”. Лева страна представља корицу издања из 1971. године, док је десна корица из 1981. године.

Уџбеник „Општа астрономија” је један од најзначајнијих уџбеника намењених студентима астрономије (Ševarlić, Brkić 1971, 1981). Он је и даље у широкој употреби као основна литература на курсевима „Општа астрономија 1” и „Општа астрономија 2”, који се држе на првој години основних студија Астрономије и астрофизике на Математичком факултету. Прво издање овог уџбеника је објављено 1971. године, а друго 10 година касније. Уџбеник покрива основе класичних астрономских дисциплина: сферна астрономија, привидна и права кретања небеских тела, практична и положајна астрономија, небеска механика и астрономски инструменти. Треба истаћи да је уџбеник написан тако да свако поглавље почиње историјским уводом, чиме читалац стиче добру представу о материји. На сајту Виртуелне библиоте налази се дигитализована верзија првог издања уџбеника (http://elibrary.matf.bg.ac.rs/handle/123456789/485).

Друго издање уџбеника које је објављено 1981. године садржи новије податке, као и исправке уочених грешака. Поред тога, друго издање има и

велики број додатих задатака који се налазе на крајевима одређених области. Нажалост, слог другог издања је значајно лошије урађен у поређењу са првим издањем. Интересантно је напоменути да је друго издање уџбеника изашло две године након смрти професора Захарија Бркића, и да је новац од продаје овог уџбеника искоришћен за фонд који је заснован у част Захарија Бркића. О самом фонду и легату ће бити више речи касније.

Први део уџбеника посвећен је сферној астрономији, где су обрађене основе сферне тригонометрије, небеска сфера, координатни системи, специјални положаји тела на својој путањи као и утицаји који доводе до промена координата неког посматраног тела.



ти све до на око 1 час после Сунчеве кулминације. Правац симетрале угла који заклапају гномонове сенке које додирују исти круг биће приближни правац меридијана. Како има више кругова, биће

Сл. 121. Одређивање положаја подневничке линије гномоном

више оваквих симетрала. Средњи положај свију њих даће нам правац меридијана са већом тачношћу но свака појединa симетрала. Тај правац поклапаће се са правцем најкраће гномонове сенке.

Тренутак када гномонова сенка достиже своју најмању вредност је 12^h правог сунчаног времена за тај датум. Ако пређемо са правог на средње-европско време T_C, у том тренутку, као што смо то чинили у § 23, и ако смо забележили време. То које је показивао наш џепни или ручни часовник у тренутку најкраће гномонове сенке, разлика

$$T_c - T_0 = \text{Ср} \qquad (337)$$

даће нам поправку нашег часовника.

Ако смо, сем тога, измерили дужину гномона G и његове најкраће сенке G_0, количник

$$\frac{G}{G_0} = \text{tg}\,(90^\circ - Z_{\odot}) \qquad (338)$$

даће нам зенитну даљину Сунца, а веза

$$\varphi = \delta_{\odot} + Z_{\odot} \qquad (339)$$

географску ширину наше станице.

Ако ова мерења вршимо у току целе године можемо одредити и нагиб еклиптике према екватору из везе

$$\varepsilon = \frac{1}{2}\,(z_{max} - z_{min}) \qquad (340)$$

Стари астрономи одређивали су гномоном још дужину тропске године, дужину сваког годишњег доба и још низ других величина.

2° *Одређивање правог сунчаног времена и часовникова стања сунчаним часовником.* Ако је гномонов стуб вертикалан, положај његове сенке зависи не само од висине, већ и од Сунчеве деклинације. Зато се гномон не може употребљавати за одређивање времена и часовникова стања у произвољном тренутку обданице. Но ако је његов стуб паралелан светској осовини, онда положај сенке не зависи од Сунчеве деклинације. Овакав гномон зове се *сунчани часовник*, јер се може употребити за одређивање времена у општем случају. Ако смо одредили правац меридијана и географску ширину гномоном, неће нам бити тешко да његов стуб поставимо паралелно светској осовини и да од њега направимо сунчани часовник. Остаје још питање како направити његов цифарник.

Претпоставимо најопштији случај — да раван цифарника има произвољан положај у простору. Нека је OP_n правац гномонова стуба, а OBC раван његова цифарника. Она ће бити одређена нагибом i

Сл. 122. Рачунање мреже за цифарник сунчаног часовника

према равни хоризонта (сл. 122.) и азимутом a од северне тачке; t је часовни угао Сунца, а $BOS = \tau$ угао који гномонова сенка опише по цифарнику за време док Сунце пређе часовни угао t. Да бисмо нашли поделу на цифарник треба наћи везу између углова τ и t. У том циљу ћемо решавањем правоуглог сферног троугла BNC (по *Непером правилу*) наћи прво његову страну $BN = b$ и угао $CBN = M$:

$$\text{tg}\,b = \sin a\,\text{tg}\,i,$$

$$\cos M = \cos a \sin i.$$

Применимо ли сада синусну и синусно-косинусну теорему на сферни троугао $P_n BS$, добићемо једначине

Слика 5: Странице 228 и 229 из уџбеника „Општа астрономија” на којима је приказано одређивање сенке гномонона, односно начин одређивања правог времена употребом сунчевог часовника.

На слици 5 је приказан начин одређивања правог времена употребом сунчевог часовника. Ту је описан поступак како је могуће добити релацију која повезује угао сенке коју баца гномон и часовног угла правог Сунца. Помоћу те везе могуће је конструисати Сунчев часовник за задато место. У Београду постоји неколико Сунчевих часовника: на улазу Завода за израду новчаница и кованог новца на Топчидеру, на улазу астрономске опсерваторије на Звездари, испред Клиничког центра Србије, у центру Земуна и испред задужбине манастира Хиландар.

5. ЛЕГАТ ПРОФЕСОРА ЗАХАРИЈА БРКИЋА

Професор Захарије Бркић је заслужио истакнуто место у историји српске астрономије из више разлога. Први разлог је што је он први доктор астрономских наука који је своју дисертацију одбранио у Србији, на Београдском универзитету, а други разлог је што је основан фонд „Проф. др Захарије Бркић".

Његова доктроска дисертација је дигитализована и доступна је у два облика на Виртуелној библиотеци. Дигитализовани су оштећен и неоштећен примерак дисертације. Оштећен примерак (налази се у библиотеци Математичког факултета) је дигитализован захваљујући професору др Стеви Шегану и др Соњи Видојевић и доступан је на линку: http://elibrary.matf.bg.ac.rs/handle/123456789/22. На тај начин је овај примерак сачуван од зуба времена, које је већ почело да се примећује, посебно на сликама. Неоштећен примерак (налази се у библиотеци Астрономске опсерваторије у Београду) дисертације је дигитализован захваљујући Миодрагу Дачићу који га је имао у својој личној колекцији (http://elibrary.matf.bg.ac.rs/handle/123456789/1921).

У претходном поглављу споменуто је да је новац професора Бркића од продаје другог издања уџбеника „Општа астрономија" искоришћен за оснивање фонда Захарије Бркић. Фонд је основан ради очувања успомене на професора Захарије Бркића, и из њега се награђује најбољи дипломирани студент астрономије и астрофизике у претходној школској години (диплома и новчана награда). Награда је први пут додељена 8. новембра 1981. године. Почетком дведесетих година, током периода инфлације, фонд је изгубио сва средства, а новчану награду је наставио да из сопствених средстава додељује син професора Бркића, Слободан Бркић. Након смрти Слободана Бркића, традицију су наставиле његове ћерке Анђелка, Милица и Верица.

6. ЗАКЉУЧАК

Професор Захарије Бркић је веома задужио српску астрономију својим радом, доприносима и утицајем на велики број генерација дипломираних астронома. Био је први доктор астрономских наука који је докторску дисертацију одбранио на Београдском универзитету, један од оснивача и руководиоца службе времена и службе кретања Земљиних полова, врсни познавалац посматрачких метода, астрономских инструмента и метода обраде посматрачких података итд.

Напоменимо да је у току израда дигиталног легата професора Захарија Бркића, где ће се на једном месту налаизити сви подаци: књиге, научни и стручни радови, информације о фонду еминентног професора. На тај начин ће успомена на њега остати сачувана за будуће генерације.

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DIGITIZED UNIVERSITY TEXTBOOKS OF PROFESSOR ZAHARIJE BRKIC

In the Virtual Library of the Faculty of Mathematics there are three digitized works by professor Zaharije Brkić. These works include his doctoral dissertation, which represents the first dissertation in astronomy that had been defended on Belgrade university. Also, two university textbooks written with prof. Branislav Ševarlić are digitized. In this paper, we give detailed analysis of these works along with explanations of their importance.

Key words: History of Astronomy, digitization, education

87 Professor Jovan Lazović and his digitized university textbooks

N. Pejović and V. Radović
NCD Review, vol. 36, pp. 59-65, 2020.

This article presents a short biography and a book of Jovan Lazović (1931-2019), former professor of astronomy at the Faculty of mathematics in Belgrade. The book is the university textbook "Fundamentals of motion theory of artificial Earth satellites". This textbook was the first one published in Serbia where the motion theory of artificial satellites was shown using clear mathematical and astronomical language. For the reader of the book it is assumed the some knowledge in advanced mathematics, astronomy and celestial mechanics. The digitized version of this book is publicly available in the Virtual Library of the Faculty of Mathematics.

Преглед НЦД 36 (2020), 59–65

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PROFESSOR JOVAN LAZOVIĆ AND HIS DIGITIZED UNIVERSITY TEXTBOOK

Abstract. In this article, we present an analysis of digitized university textbook "Fundamentals of motion theory of artificial Earth satellites" written by professor Jovan Lazović. This textbook was the first one where the motion theory of artificial satellites was shown using clear mathematical and astronomical language. The whole textbook is written precisely and systematically. In order to understand this subject, the previous knowledge in advanced mathematics, astronomy and celestial mechanics are required. The digitized version of this book is publicly available in Virtual Library of the Faculty of Mathematics.

1. Introduction

To keep and preserve a large number of rare books, textbooks, dissertations and papers, Virtual Library was developed as a part of the Digitization project of the Faculty of Mathematics in Belgrade and the Mathematical Institute of the Serbian Academy of Sciences and Arts. Virtual Library is freely available via Internet link http://elibrary.matf.bg.ac.rs for the general public [1, 2]. This project will allow future generations to understand and appreciate the works of our prominent scientists better, as well as to save past works from oblivion.

In this paper, we present a digitized university textbook written by professor Jovan Lazović, who was an astronomer and respected professor at the University of Belgrade. Title of the textbook is "Fundamentals of motion theory of artificial Earth satellites". It was published by "Naučna knjiga" in 1976 [3]. This textbook was the first one where the motion theory of the artificial satellites was shown using clear mathematical and astronomical language. This book preceded a monography "Introduction to Astrodynamics" by Tatomir Anđelić [4]. Together, these are the only books about the motion of satellites, both artificial and natural.

The whole textbook is written precisely and systematically. In order to understand this subject, the previous knowledge in advanced mathematics, astronomy and celestial mechanics are required. This textbook is primarily intended for students of the fourth year of astronomy who already acquired required bits of knowledge. The digitized version of this book is publicly available in Virtual Library of the Faculty of Mathematics.

The article is organized as follows. In the next section we give a short biography of professor Jovan Lazović. Third section contains a detailed analysis of a digitized textbook and final, forth, presents a brief conclusion.

2. Biography of professor Jovan Lazović (1931-2019)

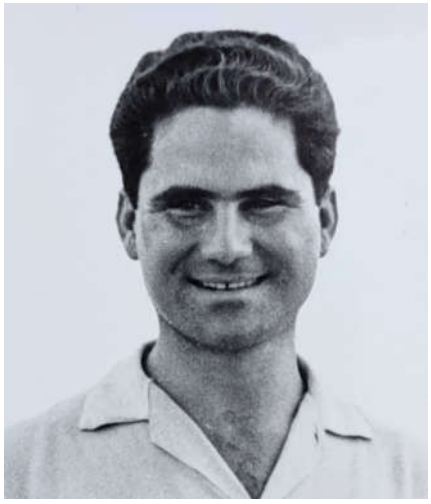


Figure 1. Professor Jovan Lazović (1931-2019).

Jovan Lazović (Figure 1) was born in Belgrade on May 22nd, 1931 in the middle-class family. His parents, mother Leposava and father Petar, cherished the tradition of solid education of the younger generation. Therefore, he attended the French elementary school in Belgrade, but not neglecting the origin and national component of the education. Therefore he was dancing in the national folklore assemble and he was playing bagpipes. He graduated in 1954 at the Faculty of Natural Sciences in Begrade, as part of astronomy group. Ten years after he obtained his bachelor degree, he got his PhD degree (Title: Some essential characteristics in the motion of the quasicomplanar planetoids, 1964). In his dissertation, he analyzed the respective positions of two planetoids with almost the same orbital inclination. Special attention was given to their proximities (positions when the mutual distance is minimal). For these positions, he derived the original method for their determination, and also he examined the mutual gravitational influence of planetoids. On the board for his defense were academicians Radivoje Kašanin, Tatomir Anđelić and Vojislav Mišković. His dissertation is digitized and available at Virtual Library [8, 9] – http://elibrary.matf.bg.ac.rs/handle/123456789/223.

Professor Lazović was working as a math teacher in two high schools in Belgrade after he had graduated from the University. He was elected as teaching assistant at Department of mechanics and astronomy in 1957, and he was teaching various subjects: General astronomy, Positional astronomy, Theoretical astronomy and Theory and practice of numerical calculus. He was promoted in 1967 to an associate professor for celestial mechanics. Therefore, the Department of mechanics and astronomy obtained its permanent teacher for celestial mechanics after the retirement of academician Milutin Milanković in 1955. In 1982 he was elected as a full professor for subjects: Celestial Mechanics and Motion theory of artificial Earth satellites. He was retired on January 1st, 1997.

Professor Lazović wrote university textbook "Fundamentals of motion theory of artificial Earth satellites." in 1976. It was the first one concerning the motion of artificial satellites published in Serbia. Before this work, academician Milutin Milanković wrote textbooks in the same scientific area: "Celestial mechanics" and "Basics of celestial mechanics".

The main topic of J. Lazović research was celestial mechanics, especially the motion of small solar system bodies and their proximities. He published more than 30 scientific papers and ten professional works. Also, he was giving lectures on various national and international conferences. Additionally, he was a very active observational astronomer, and because of that he participated in the creation of ephemeris for the annual publication "Godišnjak našeg neba".

In 1969 the scientific journal "Publications of the Department of Astronomy" was founded with great efforts from professor Lazović. He was the member of editorial office, and he led all unavoidable works regarding publishing and printing of the journal for 12 years (1970-1981). He actively published his original scientific works in the journal in order to promote its influence and usage. Also, he had active role in

calculations for “Godišnjak našeg neba”. Besides his professional activities, professor Lazović was very fond of amateur astronomy.

He had good cooperation with other astronomers at the Department. For example, it is interesting to mention his observations of a total solar eclipse in 1961, which could be seen in our region. He observed that eclipse with his colleague Jovan Simovljević at fortress of Niš. The results of this small expedition were published in two journals; one was a publication of Serbian Academy of Sciences and Arts. He also published several papers with Professor Mike Kuzmanoski which concerned the proximities of small planets of the Solar system [5, 6]. After the Lazović’s retirement professor Kuzmanoski took over the lecturing of Celestial mechanics.

Professor Jovan Lazović was quiet and peaceful men but with the capability to stood up to the authorities when it was necessary. One of the authors of this paper had the honor to have J. Lazović as his professor during the studies. Form the author's experience, J. Lazović was a very professional and great teacher who was deeply in love with the subjects he had been teaching.

On Saturday, August 3rd 2019, our famous professor passed away [10]. He left permanent trace on the Serbian astronomy and on his colleagues and students at Faculty of Mathematics, University of Belgrade. This paper is written in his honor.

3. The Textbook

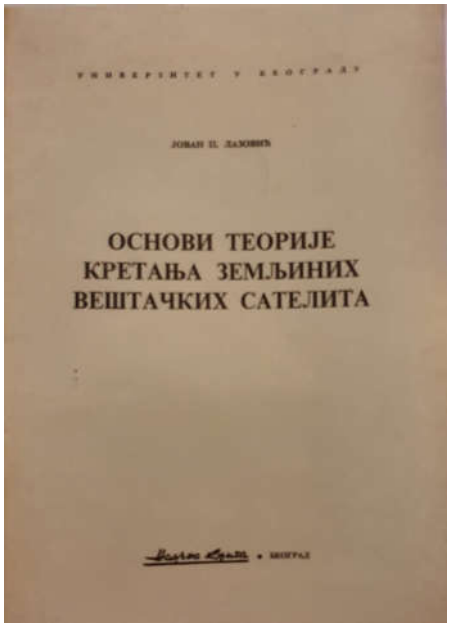


Figure 2. Textbook "Fundamentals of motion theory of artificial Earth satellites" by Jovan Lazović.

"Fundamentals of motion theory of artificial Earth satellites" (Figure 2), published in 1976, is primarily intended for astronomy students on the last, fourth year of their studies at Department of Astronomy [3]. A publisher of this textbook "Naučna knjiga" was established in 1946 and it was one of the oldest publishing corporations in former federal Yugoslavia. It was divided and closed in 1998.

At the time of publishing, it was the only textbook about artificial satellites in the Serbian language. Before, this subject was only explained within Celestial mechanics. In foreign literature, this topic is mainly covered in monographs or chapters of the celestial mechanic's books. This book started the development of this subject in Serbia. For example, Stevo Šegan, lately professor at the Department, got his PhD in this area [7].

The book has one hundred pages with 14 images and two tables. Although it is not extensive, it still demands knowledge of spherical astronomy, celestial mechanic and advanced calculus. In the book, there are many footnotes whose purpose is to give the reader references where he can find the complete derivation of equations which are only given in the textbook. Because of its concision and precision, Lazović textbook was rather difficult for students as it demanded too much time in order to cope with the exposed material. According to references in the book, it was contemporary as it contained all possible facts and information of that time. Considering the given mathematical tools, the "Fundamentals of motion theory of artificial Earth satellites" is an excellent example of a piece from applied mathematics.

In order to discuss the textbook mentioned above, we will divide it into two logically separated parts. The first part (chapters 1, 2 and 3) introduces Laplace's equation, spherical functions, expansion of gravitational potential as well as gravitational acceleration due to Earth flatness. In the second part (chapters 4 and 5) Lazović discusses differential equation of a perturbed motion and secular perturbations of artificial satellites.

3.1. First part. Laplace operator (Laplacian) is introduced using cartesian coordinates in the first chapter. Later in this chapter, we can see definitions of Laplace's equation, gravitational potential, spherical functions, Legendre polynomials and also the expansion of some functions on a spherical surface. Therefore, this chapter gives the required mathematical tools needed for the expansion of gravitational potential.

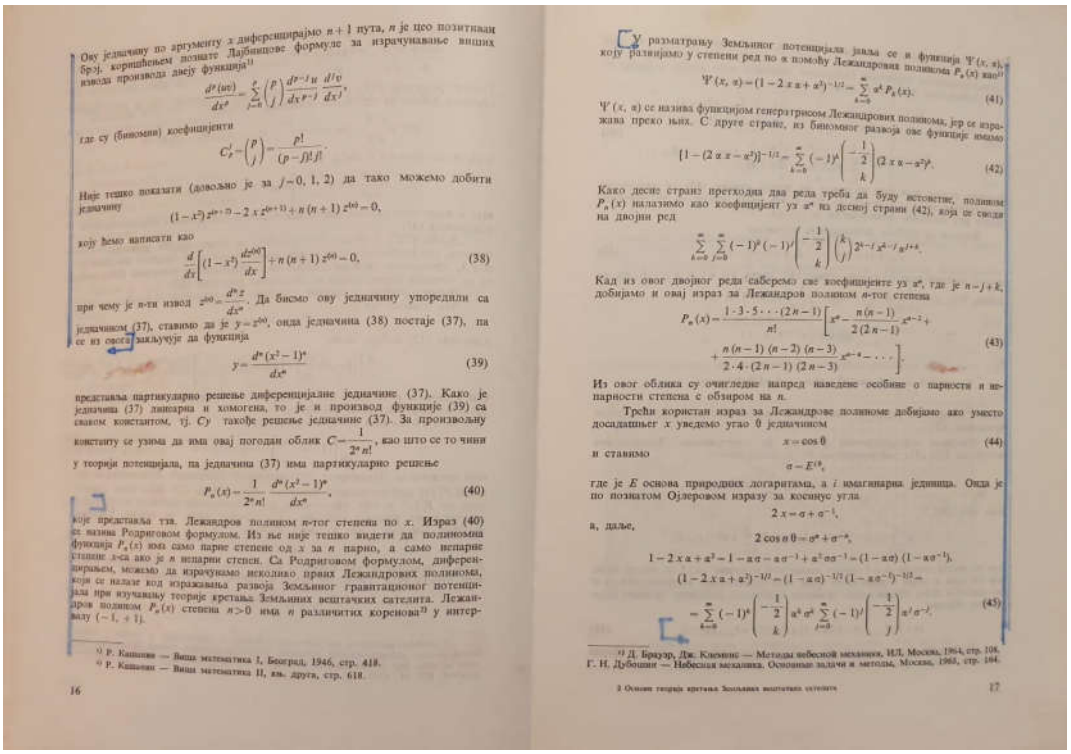
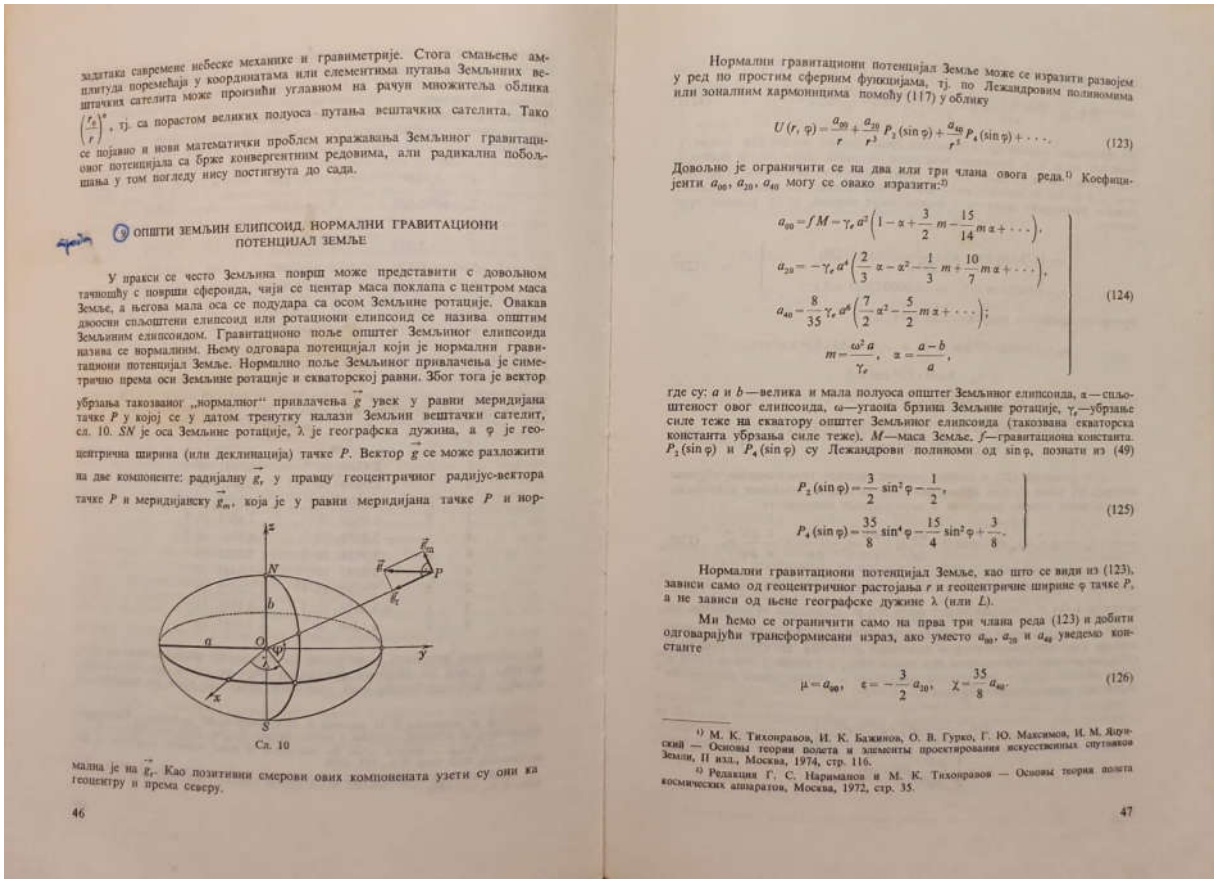


Figure 3. Pages 16 and 17

For instance, on pages 16 and 17 (Figure 3), we can see an example of derivation of Legendre polynomial of the nth degree. Equation number 40 shows Rodrigue's formula or previously called Ivory-Jacobi formula. By differentiation of Rodrigue's formula, a few first Legendre polynomials can be obtained. From this example, it is evident why this book is additionally recognized as a great example of applied mathematics. In footnotes on pages 16 and 17 (Figure 3) J. Lazović gave citations from which he took equations needed for further derivations.

In the second chapter, we can see the expansion of gravitational potential in a spherical harmonics. Next, gravitational potential of a spheroid and gravitation potential of Earth is given. Figure 4 shows pages 46 and 47 from Lazović's textbook. There we could see the general formulae for Earth ellipsoid and standard gravitational potential of Earth. The same figure shows footnotes with literature in Russian on the motion of artificial Earth satellites. In the first table on page 45, a few first values of zonal spherical harmonics are shown.



задатака савремене небеске механике и гравиметрије. Стога смањење амплитуда поремећаја у координатама или елементима путања Земљиних вештачких сателита може произићи углавном на рачун множитеља облика $\left(\frac{r_e}{r}\right)^6$, тј. са порастом великих полуоса путања вештачких сателита. Тако се појавио и нови математички проблем изражавања Земљиног гравитационог потенцијала са брже конвергентним редовима, али радикална побољшања у том погледу нису постигнута до сада.

6 ОПШТИ ЗЕМЉИН ЕЛИПСОИД. НОРМАЛНИ ГРАВИТАЦИОНИ ПОТЕНЦИЈАЛ ЗЕМЉЕ

У пракси се често Земљина површ може представити с довољном тачношћу с површи сфероида, чији се центар маса поклапа с центром маса Земље, а његова мала оса се подудара са осом Земљине ротације. Овакав двоосни сплоштени елипсоид или ротациони елипсоид се назива општим Земљиним елипсоидом. Гравитационо поље општег Земљиног елипсоида назива се нормалним. Њему одговара потенцијал који је нормални гравитациони потенцијал Земље. Нормално поље Земљиног привлачења је симетрично према оси Земљине ротације и екваторској равни. Због тога је вектор убрзања такозваног „нормалног" привлачења $\vec{g}$ увек у равни меридијана тачке P у којој се у датом тренутку налази Земљин вештачки сателит, сл. 10. SN је оса Земљине ротације, λ је географска дужина, а φ је геоцентрична ширина (или деклинација) тачке P. Вектор $\vec{g}$ се може разложити на две компоненте: радијалну $\vec{g}_r$, у правцу геоцентричног радијус-вектора тачке P и меридијанску $\vec{g}_m$, која је у равни меридијана тачке P и нормална је на $\vec{g}_r$. Као позитивни смерови ових компонената узети су они ка геоцентру и према северу.

Сл. 10

Нормални гравитациони потенцијал Земље може се изразити развојем у ред по простим сферним функцијама, тј. по Лежандровим полиномима или зоналним хармоницима помоћу (117) у облику

$$U(r,\varphi)=\frac{a_{00}}{r}+\frac{a_{20}}{r^3}P_2(\sin\varphi)+\frac{a_{40}}{r^5}P_4(\sin\varphi)+\cdots. \tag{123}$$

Довољно је ограничити се на два или три члана овога реда.[1] Коефицијенти a_{00}, a_{20}, a_{40} могу се овако изразити:[2]

$$\left.\begin{aligned} a_{00}&=fM-\gamma_e a^2\left(1-\alpha+\frac{3}{2}m-\frac{15}{14}m\alpha+\cdots\right),\\ a_{20}&=-\gamma_e a^4\left(\frac{2}{3}\alpha-\alpha^2-\frac{1}{3}m+\frac{10}{7}m\alpha+\cdots\right),\\ a_{40}&=\frac{8}{35}\gamma_e a^6\left(\frac{7}{2}\alpha^2-\frac{5}{2}m\alpha+\cdots\right);\\ m&=\frac{\omega^2 a}{\gamma_e},\quad \alpha=\frac{a-b}{a}, \end{aligned}\right\} \tag{124}$$

где су: a и b — велика и мала полуоса општег Земљиног елипсоида, α — сплоштеност овог елипсоида, ω — угаона брзина Земљине ротације, γ_e — убрзање силе теже на екватору општег Земљиног елипсоида (такозвана екваторска константа убрзања силе теже), M — маса Земље, f — гравитациона константа. $P_2(\sin\varphi)$ и $P_4(\sin\varphi)$ су Лежандрови полиноми од $\sin\varphi$, познати из (49)

$$\left.\begin{aligned} P_2(\sin\varphi)&=\frac{3}{2}\sin^2\varphi-\frac{1}{2},\\ P_4(\sin\varphi)&=\frac{35}{8}\sin^4\varphi-\frac{15}{4}\sin^2\varphi+\frac{3}{8}. \end{aligned}\right\} \tag{125}$$

Нормални гравитациони потенцијал Земље, као што се види из (123), зависи само од геоцентричног растојања r и геоцентричне ширине φ тачке P, а не зависи од њене географске дужине λ (или L).

Ми ћемо се ограничити само на прва три члана реда (123) и добити одговарајући трансформисани израз, ако уместо a_{00}, a_{20} и a_{40} уведемо константе

$$\mu=a_{00},\quad \varepsilon=-\frac{3}{2}a_{20},\quad \chi=\frac{35}{8}a_{40}. \tag{126}$$

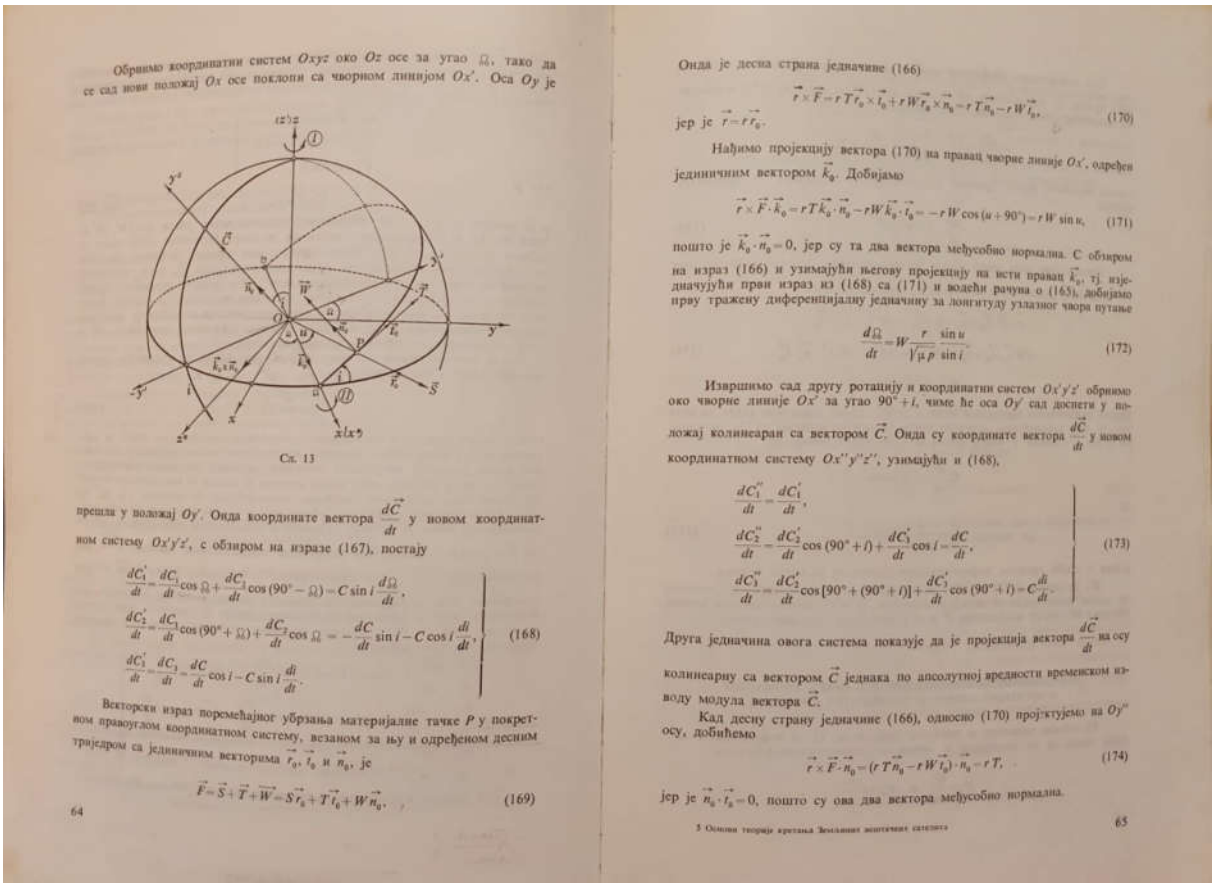
[1] М. К. Тихонравов, И. К. Бажинов, О. В. Гурко, Г. Ю. Максимов, И. М. Ядрицкий — Основы теории полета и элементы проектирования искусственных спутников Земли, II изд., Москва, 1974, стр. 116.

[2] Редакция Г. С. Нариманов и М. К. Тихонравов — Основы теории полета космических аппаратов, Москва, 1972, стр. 35.

Figure 4. Pages 46 and 47

3.2. Second part. The fourth chapter covers the following topics: differential equations of perturbed motion of an artificial satellite and equation of elliptical elements of the satellite (longitude of the ascending node, a parameter of an ellipse, orbital inclination, eccentricity, true anomaly and argument of perihelion). The given system of equations is solved using the method of successive approximations. Calculations using this numerical algorithm are extensive and demand to use powerful computer resources.

Figure 5 allows us to see the vectorial equation of acceleration of a material point in the moving coordinate system. The last, fifth, chapter shows secular perturbation theory of elliptic orbit of an artificial satellite, as well as the influence of Earth flattening. Also, in this chapter, special attention is given to the period of revolution of a satellite around Earth (draconic and sidereal period). Figure 6 displays secular perturbation of Earth's artificial satellite due to influence of Earth flattening on elliptical elements. These formulae reveal all the difficulty of this calculus. This figure, along with all previous figures, designates the importance of mathematics for this subject. Thus, every successful astronomer has to be a primarily great mathematician in order to cope with problems of celestial mechanics and astrodynamics.



Обринимо координатни систем $Oxyz$ око Oz осе за угао Ω, тако да се сад нови положај Ox осе поклопи са чворном линијом Ox'. Оса Oy је

Сл. 13

прешла у положај Oy'. Онда координате вектора $\frac{d\vec{C}}{dt}$ у новом координатном систему $Ox'y'z'$, с обзиром на изразе (167), постају

$$\left.\begin{aligned}
\frac{dC_1'}{dt} &= \frac{dC_1}{dt}\cos\Omega + \frac{dC_2}{dt}\cos(90^\circ-\Omega) = C\sin i\,\frac{d\Omega}{dt},\\
\frac{dC_2'}{dt} &= \frac{dC_1}{dt}\cos(90^\circ+\Omega) + \frac{dC_2}{dt}\cos\Omega = -\frac{dC}{dt}\sin i - C\cos i\,\frac{di}{dt},\\
\frac{dC_3'}{dt} &= \frac{dC_3}{dt} = \frac{dC}{dt}\cos i - C\sin i\,\frac{di}{dt}.
\end{aligned}\right\} \quad (168)$$

Векторски израз поремећајног убрзања материјалне тачке P у покретном правоуглом координатном систему, везаном за њу и одређеном десним триједром са јединичним векторима $\vec{r}_0$, $\vec{t}_0$ и $\vec{n}_0$, је

$$\vec{F} = \vec{S} + \vec{T} + \vec{W} = S\vec{r}_0 + T\vec{t}_0 + W\vec{n}_0, \quad (169)$$

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Онда је десна страна једначине (166)

$$\vec{r}\times\vec{F} = rT\vec{r}_0\times\vec{t}_0 + rW\vec{r}_0\times\vec{n}_0 = rT\vec{n}_0 - rW\vec{t}_0, \quad (170)$$

јер је $\vec{r} = r\vec{r}_0$.

Нађимо пројекцију вектора (170) на правац чворне линије Ox', одређен јединичним вектором $\vec{k}_0$. Добијамо

$$\vec{r}\times\vec{F}\cdot\vec{k}_0 = rT\vec{k}_0\cdot\vec{n}_0 - rW\vec{k}_0\cdot\vec{t}_0 = -rW\cos(u+90^\circ) = rW\sin u, \quad (171)$$

пошто је $\vec{k}_0\cdot\vec{n}_0 = 0$, јер су та два вектора међусобно нормална. С обзиром на израз (166) и узимајући његову пројекцију на исти правац $\vec{k}_0$, тј. изједначујући први израз из (168) са (171) и водећи рачуна о (165), добијамо прву тражену диференцијалну једначину за лонгитуду узлазног чвора путање

$$\frac{d\Omega}{dt} = W\frac{r}{\sqrt{\mu p}}\frac{\sin u}{\sin i}. \quad (172)$$

Извршимо сад другу ротацију и координатни систем $Ox'y'z'$ обринимо око чворне линије Ox' за угао $90^\circ+i$, чиме ће оса Oy' сад доспети у положај колинеаран са вектором $\vec{C}$. Онда су координате вектора $\frac{d\vec{C}}{dt}$ у новом координатном систему $Ox''y''z''$, узимајући и (168),

$$\left.\begin{aligned}
\frac{dC_1''}{dt} &= \frac{dC_1'}{dt},\\
\frac{dC_2''}{dt} &= \frac{dC_2'}{dt}\cos(90^\circ+i) + \frac{dC_3'}{dt}\cos i = \frac{dC}{dt},\\
\frac{dC_3''}{dt} &= \frac{dC_2'}{dt}\cos[90^\circ+(90^\circ+i)] + \frac{dC_3'}{dt}\cos(90^\circ+i) = C\frac{di}{dt}.
\end{aligned}\right\} \quad (173)$$

Друга једначина овога система показује да је пројекција вектора $\frac{d\vec{C}}{dt}$ на осу колинеарну са вектором $\vec{C}$ једнака по апсолутној вредности временском изводу модула вектора $\vec{C}$.

Кад десну страну једначине (166), односно (170) пројектујемо на Oy'' осу, добићемо

$$\vec{r}\times\vec{F}\cdot\vec{n}_0 = (rT\vec{n}_0 - rW\vec{t}_0)\cdot\vec{n}_0 = rT, \quad (174)$$

јер је $\vec{n}_0\cdot\vec{t}_0 = 0$, пошто су ова два вектора међусобно нормална.

5 Основи теорије кретања Земљиног вештачког сателита

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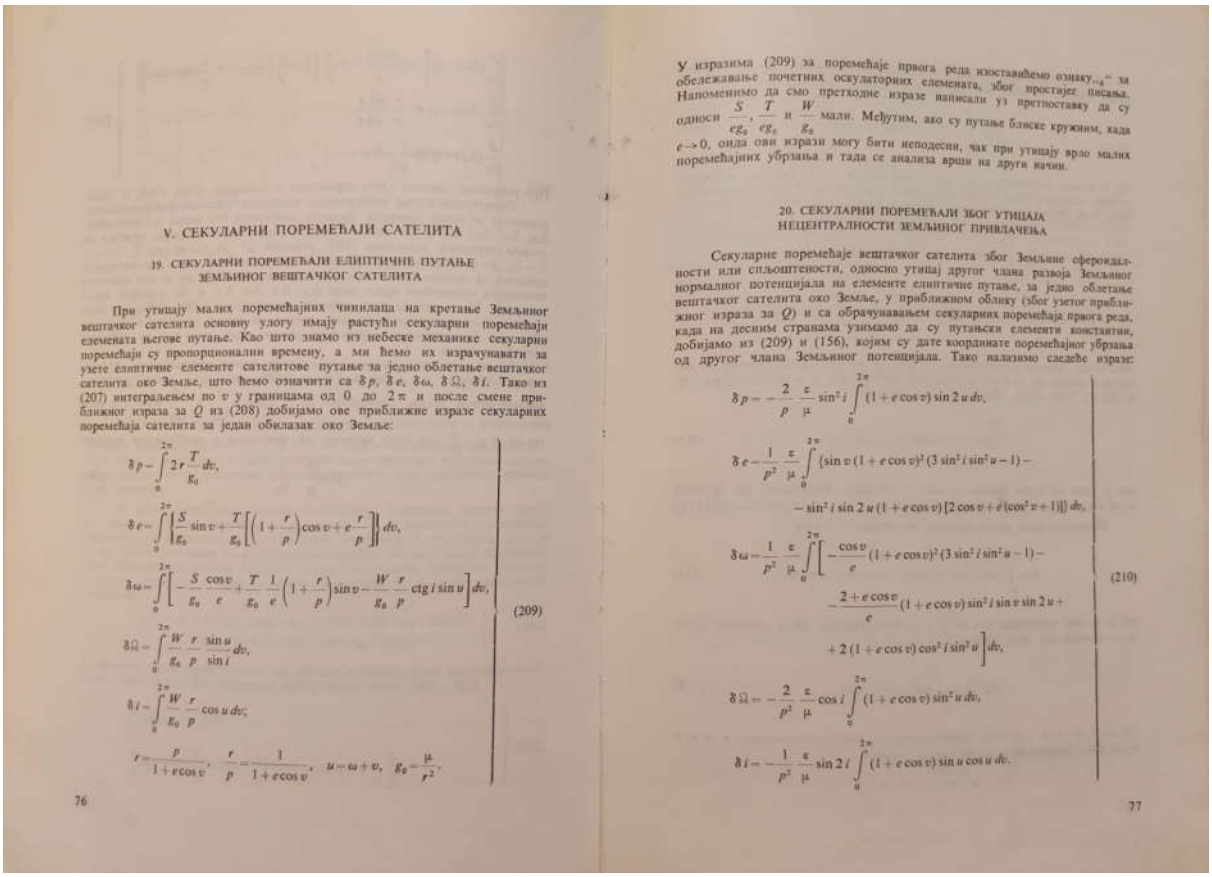
Figure 6. Pages 64 and 65



V. СЕКУЛАРНИ ПОРЕМЕЋАЈИ САТЕЛИТА

19. СЕКУЛАРНИ ПОРЕМЕЋАЈИ ЕЛИПТИЧНЕ ПУТАЊЕ ЗЕМЉИНОГ ВЕШТАЧКОГ САТЕЛИТА

При утицају малих поремећајних чинилаца на кретање Земљиног вештачког сателита основну улогу имају растући секуларни поремећаји елемената његове путање. Као што знамо из небеске механике секуларни поремећаји су пропорционални времену, а ми ћемо их израчунавати за узете елиптичне елементе сателитове путање за једно облетање вештачког сателита око Земље, што ћемо означити са δp, δe, $\delta\omega$, $\delta\Omega$, δi. Тако из (207) интеграљењем по v у границама од 0 до 2π и после смене приближног израза за Q из (208) добијамо ове приближне изразе секуларних поремећаја сателита за један обилазак око Земље:

$$\left.\begin{aligned}
\delta p &= \int_0^{2\pi} 2r\frac{T}{g_0}\,dv,\\
\delta e &= \int_0^{2\pi}\left\{\frac{S}{g_0}\sin v + \frac{T}{g_0}\left[\left(1+\frac{r}{p}\right)\cos v + e\frac{r}{p}\right]\right\}dv,\\
\delta\omega &= \int_0^{2\pi}\left[-\frac{S}{g_0}\frac{\cos v}{e} + \frac{T}{g_0}\frac{1}{e}\left(1+\frac{r}{p}\right)\sin v - \frac{W}{g_0}\frac{r}{p}\operatorname{ctg} i\sin u\right]dv,\\
\delta\Omega &= \int_0^{2\pi}\frac{W}{g_0}\frac{r}{p}\frac{\sin u}{\sin i}\,dv,\\
\delta i &= \int_0^{2\pi}\frac{W}{g_0}\frac{r}{p}\cos u\,dv;\\
r &= \frac{p}{1+e\cos v},\quad \frac{r}{p} = \frac{1}{1+e\cos v},\quad u = \omega + v,\quad g_0 = \frac{\mu}{r^2}.
\end{aligned}\right\} \quad (209)$$

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У изразима (209) за поремећаје првога реда изоставићемо ознаку „$_0$" за обележавање почетних оскулаторних елемената, због простијег писања. Напоменимо да смо претходне изразе написали уз претпоставку да су односи $\frac{S}{eg_0}$, $\frac{T}{eg_0}$ и $\frac{W}{g_0}$ мали. Међутим, ако су путање блиске кружним, када $e\to 0$, онда ови изрази могу бити неподесни, чак при утицају врло малих поремећајних убрзања и тада се анализа врши на други начин.

20. СЕКУЛАРНИ ПОРЕМЕЋАЈИ ЗБОГ УТИЦАЈА НЕЦЕНТРАЛНОСТИ ЗЕМЉИНОГ ПРИВЛАЧЕЊА

Секуларне поремећаје вештачког сателита због Земљине сфероидалности или спљоштености, односно утицај другог члана развоја Земљиног нормалног потенцијала на елементе елиптичне путање, за једно облетање вештачког сателита око Земље, у приближном облику (због узетог приближног израза за Q) и са обрачунавањем секуларних поремећаја првога реда, када на десним странама узимамо да су путањски елементи константни, добијамо из (209) и (156), којим су дате координате поремећајног убрзања од другог члана Земљиног потенцијала. Тако налазимо следеће изразе:

$$\left.\begin{aligned}
\delta p &= -\frac{2}{p}\frac{\varepsilon}{\mu}\sin^2 i\int_0^{2\pi}(1+e\cos v)\sin 2u\,dv,\\
\delta e &= \frac{1}{p^2}\frac{\varepsilon}{\mu}\int_0^{2\pi}\{\sin v(1+e\cos v)^2(3\sin^2 i\sin^2 u - 1) -\\
&\quad - \sin^2 i\sin 2u(1+e\cos v)[2\cos v + e(\cos^2 v + 1)]\}\,dv,\\
\delta\omega &= \frac{1}{p^2}\frac{\varepsilon}{\mu}\int_0^{2\pi}\left[-\frac{\cos v}{e}(1+e\cos v)^2(3\sin^2 i\sin^2 u - 1) -\right.\\
&\quad - \frac{2+e\cos v}{e}(1+e\cos v)\sin^2 i\sin v\sin 2u +\\
&\quad \left. + 2(1+e\cos v)\cos^2 i\sin^2 u\right]dv,\\
\delta\Omega &= -\frac{2}{p^2}\frac{\varepsilon}{\mu}\cos i\int_0^{2\pi}(1+e\cos v)\sin^2 u\,dv,\\
\delta i &= -\frac{1}{p^2}\frac{\varepsilon}{\mu}\sin 2i\int_0^{2\pi}(1+e\cos v)\sin u\cos u\,dv.
\end{aligned}\right\} \quad (210)$$

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Figure 5. Pages 76 and 77

4. Conclusion

A textbook "Fundamentals of motion theory of artificial Earth satellites" by professor Jovan Lazović represent an essential contribution to the existing literature in astronomy in the Serbian language. As we have already mentioned, this textbook is an excellent example of a textbook in the area of applied mathematics. By digitalization of this textbook, we have made this book publicly available to students and the scientific public. Also, we are currently making a great effort in order to collect all available Lazovic's works for making web site where we will honor his legacy.

Acknowledgment

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88 Saturn - a Serbian journal on astronomy from the past

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Astronomical journal *Saturn*, a monthly messenger of the Yugoslav astronomical association "Astronomical Society" founded in Belgrade in 1934, is presented. The journal itself was published in the period 1935 – 1940. All 72 volumes of the journal are digitized by the authors of this article and deposited in the Virtual library of the Faculty of Mathematics in Belgrade, at the Internet address http://elibrary.matf.bg.ac.rs. *Saturn* was the first Serbian and Yugoslav journal on popularization of astronomy and related sciences. The authors note that this open access digital collection could be a valuable source for the history of astronomy in Yugoslavia before WWII

SATURN - A SERBIAN JOURNAL ON ASTRONOMY FROM THE PAST

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Abstract. We present astronomical journal *Saturn* which was published in Belgrade in the period 1935 – 1940. The journal was started as a monthly publication of the newly founded Yugoslav astronomical association "Astronomical Society". All 72 volumes of the journal are digitized and deposited in the Virtual library of the Faculty of Mathematics in Belgrade, http://elibrary.matf.bg.ac.rs.

1. INTRODUCTION

There were not so many astronomical publications in Serbia before WWII, in particular periodicals. Overviews on early astronomical books printed in Serbia, or written by scientists of the Serbian origin, one can find in Pejović and Mijajlović (2011), Martocchia and Marchionni (2013) and Milisavljević et al (2011). Also, a lot of valuable historical information are published in proceedings of biannual conferences *Razvoj astronomije kod Srba* (Development of astronomy among Serbs), organized by professor Milan Dimitrijević. The first Serbian astronomical periodicals appeared in the third decade of the XX century. Belgrade observatory started to publish in that time almost in parallel three periodicals in astronomy: *Annuaire de l'Obs. Astr. Belgrade*, *Mémoires de l'Obs. Astr. Belgrade* and *Godišnjak našeg neba* (Almanac). Each of the first two publications was printed for six years, while the last volume of *Godišnjak* (for 1962) was printed in 1961. However, *Nautički Godišnjak* (Nautical Almanac, published by S. Šegan) may be considered as a successor of *Godišnjak našeg neba*. The publication *Saturn* was the first Serbian astronomical periodical oriented to the general public interested in astronomy, geodesy and meteorology and other related sciences. The main purpose of this article is to announce that this journal is completely digitized and made available, as an open access publication, to the general public. Digital copies of the journal are deposited in the Virtual Library of the Faculty of Mathematics in Belgrade.

2. FOUNDATION

The publication of Saturn started in 1935 as a messenger of the Yugoslav *Astronomical Society*, founded in Belgrade a year before. The original title of the Society was *Academic astronomical Society of the Belgrade University*. It changed once more its name in 1939 to *Yugoslav Astronomical Society*. The intention was to stress the Yugoslav nature of the society. The founder of the journal and the society as well was Djordje M. Nikolić (1908-1971), pro-Yugoslav Serbian science historian. Nikolić got Degree and PhD in Astronomy in France, where he acted as a member of the Resistance during WWII. He wrote rather recognized papers on Rudjer Boković and south-Slav astronomy, as noted by A. Martocchia and S. Marchionni in their paper "Djordje Nikolić' *Yugoslavs in Astronomy*" (2013). The journal was supported by Hypothecary Bank in Belgrade.

This monthly periodical was conceived as a semiprofessional journal with many popular articles intended to astronomy lovers. At the same time the journal was planned to be useful to professional astronomers, surveyors, meteorologists and seismologists, because there were published also scientific articles in these areas. Popular and professional articles were separated into different sections, and there also was a section with short notes and news for people interested in astronomy.

Articles were published in Cyrillic and Latin, reflecting in this way both variants of Serbo-Croatian, following pro-Yugoslav attitude of the founder of the journal, Djordje Nikolić. Slovenian authors published in Slovenian. Some of the leading Yugoslav astronomers and physicists published there, e.g. Serb Milutin Milanković, Croat Stjepan Mohorovičić and Slovenian Lavo Čermelj.

3. Editorial Board

The members of the Editorial board were prominent Yugoslav scientists and professionals in astronomy and related sciences. Most of them were from Belgrade, but there were members from Croatia and Slovenia as well. Some of them had very interesting biographies, while the most of them were holding PhD degree in astronomy obtained in then the leading scientific centers in Europe. Djordje Nikolić was the main editor, while the other members were:

General and academician Stevan Bošković (1869-1957), geodesists and professor at Military Academy in Belgrade. He is known for his very extensive geodetic measurements in Serbia in the first decade of XX century and his translation of the famous Tsinger's three volume book on astronomy. Bošković translated these books not by chance. He used Tsinger's method for the purpose of exact time determination needed for measurements the points of all geographic latitudes in Serbia during his very extensive geodetic measurements. Tsinger's book without doubt was his valuable companion during this exploration. He spent some time in the late XIX century studying astronomy and geodesy at the Pulkovo Observatory. Bošković certainly met Tsinger there who was then the leading Russian geodesists and professor in St. Petersburg, see Pejović and Mijajlović (2011).

Vojislav Grujić (1904-1944) who was holding two doctoral dissertations in mathematics related to astronomy, obtained at Strasbourg University in 1933. Before he went to Strasbourg he worked as an associate at the Astronomical observatory in Belgrade. However, due to some social circumstances he was not admitted to the

Observatory upon his return from Strasbourg. Hence, in spite of his very high education and background in astronomy he had no proper chance to work in astronomy professionally. He tragically lost his life in the eve of the very end of WWII.

Fran Dominko (1903-1987), Slovenian physicist and astronomer, got his Degree and PhD in Bologna, Italy. Worked in Belgrade since 1932, first as an astronomer and later as a gymnasium teacher until WWII. He moved to Ljubljana in 1948 for the university professor of astronomy. More details about him one can find for example in Wikipedia, Slovenian edition.

Ladislav S. Mužinić was the member of the first editorial board. We do not know much about him except that he lived in Zagreb before WWII. He published a short note *Breitenbestimmungsversuch von Zagreb* (Attempt to determine the latitude of Zagreb, 1936) in Astronomical notes, the predecessor of Astronomische Nachrichten. He also wrote articles for "Saturn" on astronomy and geodesy.

Nenad Janković (1911-1997) was jurist, but as professor Milan Dimitrijević described him, "his love and passion and his life's commitment was astronomy". He was the great popularizer of astronomy. For more details see Dimitrijević (1998). On the cover page of the first volume of "Saturn" it is written that Janković is the owner of the journal.

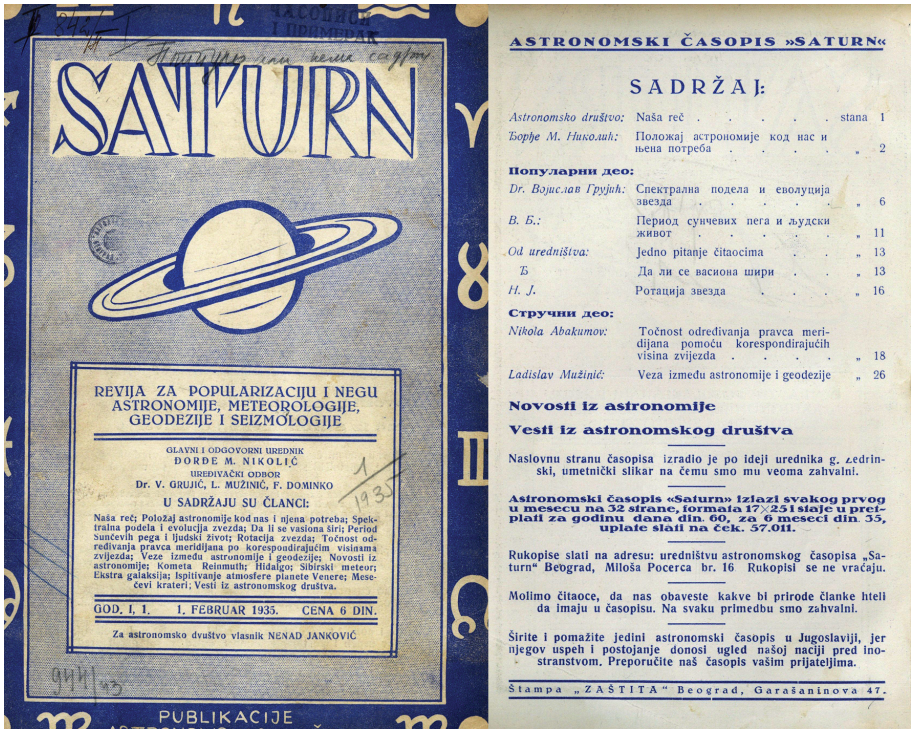
4. Browsing the journal

Many interesting, actual and informative articles were published in *Saturn*. They were in agreement with the current knowledge in astronomy and related sciences and obeyed surprisingly high scientific standards. All contributions, including short notes and reviews are meticulously divided into the "Popular part", "Professional part" and "News and notices". On the other side articles are accordingly classified into subjects: astronomy, meteorology, seismology, geodesy, personal news etc. It was not given priority to any particular discipline. For example, in astronomy articles cover many topics, from comets to time measurement and from supernova to cosmology. The used mathematical apparatus was correct and nontrivial. We can freely say that *Saturn* in many senses could compete with other astronomical journals published in that time in Belgrade. Obviously, editors put a lot of effort in preparing each volume of the journal.

As an illustration we give two examples, short notices printed in the journal. The first one is the obituary of Willem de Sitter (1872 - 1934), a prominent Dutch mathematician, physicist, and astronomer. A short biography and his main contribution to science are presented. Djordje Nikolić published in the same volume the article *Da li se vasiona proširuje* (Does the universe expand) where de Sitter views in cosmology are presented.

The second one is the notice on Hans Ertel's formula $gm^2 + mc = \pi he^2\Lambda$ (*Saturn*, year 1935, vol. 3, page 91) which connects fundamental physical constants: the Newton gravitational constant g, electron mass m, the Planck constant h, the electron charge e, speed of light c and the Einstein cosmological constant Λ. Hans Ertel (1904 - 1971) was a German natural scientist and a pioneer in geophysics, meteorology and hydrodynamics. More information on his biography and contribution to cosmology can be found at *Working group history of geophysics and cosmic physics*.

Here is the translation of the content (main articles) of the first volume of the journal. *Introductory part*: • Introductory word, • Position of astronomy in Yugoslavia



SATURN

REVIJA ZA POPULARIZACIJU I NEGU ASTRONOMIJE, METEOROLOGIJE, GEODEZIJE I SEIZMOLOGIJE

GLAVNI I ODGOVORNI UREDNIK
ĐORĐE M. NIKOLIĆ
UREĐIVAČKI ODBOR
Dr. V. GRUJIĆ, L. MUŽINIĆ, F. DOMINKO

U SADRŽAJU SU ČLANCI:

Naša reč; Položaj astronomije kod nas i njena potreba; Spektralna podela i evolucija zvezda; Da li se vasiona širi; Period Sunčevih pega i ljudski život; Rotacija zvezda; Tačnost određivanja pravca meridijana po korespondirajućim visinama zvijezda; Veze između astronomije i geodezije; Novosti iz astronomije; Kometa Reinmuth; Hidalgo; Sibirski meteor; Ekstra galaksija; Ispitivanje atmosfere planete Venere; Mesečevi krateri; Vesti iz astronomskog društva.

GOD. I, 1. 1. FEBRUAR 1935. CENA 6 DIN.

Za astronomsko društvo vlasnik NENAD JANKOVIĆ

PUBLIKACIJE

ASTRONOMSKI ČASOPIS »SATURN«

SADRŽAJ:

Astronomsko društvo: Naša reč . . . . . stana 1
Ђорђе М. Николић: Положај астрономије код нас и њена потреба . . . . . „ 2

Популарни део:

Dr. Vojislav Grujić: Spektralna podela i evolucija zvezda . . . . . „ 6
В. Б.: Period sunčevih pega i ljudski život . . . . . „ 11
Od uredništva: Jedno pitanje čitaocima . . . „ 13
Б Da li se vasiona širi . . . „ 13
H. J. Rotacija zvezda . . . . „ 16

Stručni deo:

Nikola Abakumov: Tačnost određivanja pravca meridijana pomoću korespondirajućih visina zvijezda . . . . . „ 18
Ladislav Mužinić: Veza između astronomije i geodezije „ 26

Novosti iz astronomije

Vesti iz astronomskog društva

Naslovnu stranu časopisa izradio je po ideji urednika g. Zedrinski, umetnički slikar na čemu smo mu veoma zahvalni.

Astronomski časopis «Saturn» izlazi svakog prvog u mesecu na 32 strane, formata 17×25 i stoje u pretplati za godinu dana din. 60, za 6 meseci din. 35, uplate slati na ček. 57.011.

Rukopise slati na adresu: uredništvu astronomskog časopisa „Saturn“ Beograd, Miloša Pocerca br. 16. Rukopisi se ne vraćaju.

Molimo čitaoce, da nas obaveste kakve bi prirode članke hteli da imaju u časopisu. Na svaku primedbu smo zahvalni.

Širite i pomažite jedini astronomski časopis u Jugoslaviji, jer njegov uspeh i postojanje donosi ugled našoj naciji pred inostranstvom. Preporučite naš časopis vašim prijateljima.

Štampa „ZAŠTITA“ Beograd, Garašaninova 47.

Figure 1: Cover page of the first volume of *Saturn*.

and do we need it. *Popular part*: • Spectral division and evolution of stars, • Sunspots periods and human life, • One question to the readers, • Does the Universe expand, • Star rotation, *Professional part*: • The accuracy in determining the direction of the meridians using the corresponding height of the stars, • Connection between astronomy and geodesy.

5. Digitization

All 72 volumes, about 2100 pages, were digitized during the spring and summer of 2020, and deposited in the Virtual Library of the Faculty of Mathematics in Belgrade. Professor Nadežda Pejović actually initiated this project and organized most of the jobs related to the project. Volumes printed in the period 1935-1937 were digitized by Žarko Mijajlović and in the years 1938 and 1940 by Aleksandar Jovanović, an administrator of the Faculty of Mathematics. Digitization of this periodical for the year 1939 is done by the National Library of Serbia. Mrs Tamara Butigan and Mrs Jasna Majerle from the National Library helped during the realization of this small project and gave us the digital copy of 12 volumes printed in 1939. Milan Jeličić from the *Astronomical Society "Rudjer Bošković"* lent us all hard copies of *Saturn* in possetion of the Society to digitize them and gave us a lot of help and valuable information about this journal. The OCR procedure is utilized on digital copies, so it is easy to search for terms and specific words through all the volumes.

Digital collection of the journal belongs to the division *Periodicals* of the Virtual Library. There are also digital collection of the journal *Godišnjak našeg neba*, see Šegan et al (2009), and digital copies of the significant part of edition of *Vasiona*, the contemporary Serbian popularization astronomical journal.

The National library has all 72 volumes of the journal in the printed form. The *Astronomical Society "Rudjer Bošković"* has all volumes except those printed in 1939.

6. Conclusion

The first Serbian and Yugoslav journal *Saturn* on popularization of astronomy and related sciences is presented. We also announced the digital collection of all issues of the journal which are deposited in the Virtual library of the Faculty of mathematics, University of Belgrade. This open access digital collection could be a valuable source for the history of astronomy in Yugoslavia before WWII.

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PART IV

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89 Complete bibliography of Nadežda Pejović

This bibliography lists all works of professor Nadežda Pejović, published up to the end of November 2020, at the time of coronavirus COVID-19 pandemic.

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90 Bibliography on astronomy

Bibliography of works in astronomy
of professor Nadežda Pejović

This bibliography lists all works of professor Nadežda Pejović in astronomical sciences: geodynamic, astrometry, cosmology and galactic astronomy, published up to the end of November 2020, at the time of coronavirus COVID-19 pandemic.

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This bibliography lists all works in history of science and digitization of professor Nadežda Pejović, published up to the end of November 2020, at the time of coronavirus COVID-19 pandemic.

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