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A USER-FRIENDLY THEORY OF ORTHOTROPIC PLASTICITY IN SHEET METALS

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Abstract—The aim is to model some basic aspects of the yielding and plastic flow of textured sheet, especially a specific combination of properties which is relatively common but not covered by existing theories. In constructing a new yield criterion the over-riding considerations here are flexibility and manipulative convenience. The numerical parameters are kept to a minimum and must be determinable readily from data given by standard tests. It is shown that these requirements are met by adding a particular pair of cubic terms to the author's 1948 quadratic.

NOTATION

σ_b	yield stress of a thin sheet under in-plane equibiaxial tension
σ_0, σ_{90}	yield stresses under uniaxial tension at 0° and 90° to the direction of rolling
σ_u	common symbol for σ_0 and σ_{90} when equal
r_0, r_{90}	ratios of transverse to through-thickness increments of logarithmic strain under σ_0 and σ_{90}
r	common symbol for r_0 and r_{90} when equal
ψ_0, ψ_{90}	angles at which a yield locus intersects the coordinate axes in stress space
σ_1, σ_2	stress components parallel and perpendicular to the direction of rolling
$\varepsilon_1, \varepsilon_2$	components of logarithmic strain parallel and perpendicular to the direction of rolling
a, b, c; f, g, h; k, l, m	non-dimensional parameters in some existing yield functions
p, q	non-dimensional parameters in the newly proposed yield function
$\phi(\sigma_1, \sigma_2)$	any function whose contours represent yield loci in stress space

INTRODUCTION

The analysis is relevant to rolled sheets of polycrystalline metal where the texture is fine-grained and each macro-element of material can be treated as if it were homogeneous and structurally orthotropic. The plastic behaviour of such elements is likewise orthotropic, with one axis of symmetry normal to the sheet, a second in the direction of rolling, and the third perpendicular to both. Test specimens in the form of rectangular strips with edges parallel to these axes are considered to be loaded uniformly by in-plane normal stresses (σ_1, σ_2), respectively in the rolling and transverse directions. It is assumed, as usual, that an all-round pressure or tension can be superimposed on the system without effect; conversely, any triaxial loading of a strip, say $(\sigma_1, \sigma_2, \sigma_3)$, has a plane-stress equivalent $(\sigma_1 - \sigma_3, \sigma_2 - \sigma_3)$.

As necessary background, the historical progression of orthotropic yield functions for textured sheet will first be reviewed briefly. The 1948 prototype [1] was a quadratic form:

$$g\sigma_1^2 + f\sigma_2^2 + h(\sigma_1 - \sigma_2)^2 = \sigma_b^2 \tag{1}$$

where σ_b is the yield stress under in-plane equibiaxial tension, f, g and h are non-dimensional numerical parameters and $f + g = 1$ for consistency with $\sigma_1 = \sigma_2 = \sigma_b$. The corresponding locus in (σ_1, σ_2) space is convex (an ellipse) if, and only if, $fg + h > 0$, as becomes evident when Eqn (1) is arranged as a sum of squares:

$$(g\sigma_1 + f\sigma_2)^2 + (fg + h)(\sigma_1 - \sigma_2)^2 = \sigma_b^2.$$

Optimal values of the parameters in any particular case are readily determined, so this simple quadratic has proved helpful in a variety of technological applications. Inevitable limitations were to be expected but did not become apparent for some considerable time, and then only gradually through a succession of well-conceived experiments. However, it was not subsequently found straightforward to devise an improved criterion whose extra flexibility was not outweighed by computational and experimental complexities.

In 1977, for example, a fourth-degree polynomial in σ_1 and σ_2 was tried by Gotoh [2]. This has multiple coefficients but is nevertheless unsuited to representing important types of orthotropic response. In 1979, Hosford [3] advocated a quite different function whose form was more appealing, namely

$$g|\sigma_1|^m + f|\sigma_2|^m + h|\sigma_1 - \sigma_2|^m = \sigma_b^m \tag{2}$$

where $f + g = 1$ with f, g, h all positive and $m = 6$ or 8 according to whether the crystal grains are body-centred or face-centred cubic. In the same year, Hill [4, p. 187] independently suggested a function identical in form, but with *arbitrary* $m > 1$ (integer or non-integer). It was noted, though, that this criterion does not admit so-called "anomalous behaviour" often seen in sheet with planar isotropy (just as Gotoh's does not). A more flexible function was therefore also suggested [4, p. 188], namely

$$g|\sigma_1|^m + f|\sigma_2|^m + h|\sigma_1 - \sigma_2|^m + a|2\sigma_1 - \sigma_2|^m + b|2\sigma_2 - \sigma_1|^m + c|\sigma_1 + \sigma_2|^m = \sigma_b^m \tag{3}$$

where

$$f + g + a + b + 2^m c = 1.$$

Further restrictions to ensure convexity have not been thoroughly investigated when $f \neq g$ and $a \neq b$ (in-plane anisotropy). Anomalous response can be modelled comfortably by this expression so long as the values $m = 2$ or $a = b = c$ or $f = g = h = 0$ are avoided.

More recently, some rather speculative criteria have been proposed when the tensions are *inclined* to the axes of orthotropy. This configuration will not be considered here; the criteria are quoted only in the versions to which they reduce when the tensions are *parallel* to the axes of orthotropy. Thus, as an alternative to Eqn (3), Hill [5] has suggested a function of type

$$|\tfrac{1}{2}(\sigma_1 + \sigma_2)|^m + h|\sigma_1 - \sigma_2|^m + (\sigma_1^2 + \sigma_2^2)^{(m-2)/2}(\sigma_1 - \sigma_2)(k\sigma_1 - l\sigma_2) = \sigma_b^m \tag{4}$$

where $m > 1$ is arbitrary and h, k, l are disposable (subject, as always, to convexity requirements). When $m = 2$ the expression is a rearrangement of Eqn (1) in different notation. In fact, it was this particular variant that prompted the generalization Eqn (4) (and, likewise, its extension when σ_1 and σ_2 are not coaxial with the orthotropy). When there is in-plane isotropy, Eqn (4) simplifies to the frequently used function

$$|\tfrac{1}{2}(\sigma_1 + \sigma_2)|^m + h\,|\sigma_1 - \sigma_2|^m = \sigma_b^m, \tag{5}$$

which is Eqn (3) with f, g, a, b all zero. This was an additional motivation, ensuring that Eqn (4) encompassed isotropic anomalous response. Weixian's function [6], on the other hand, merely reproduces Eqn (2) with an arbitrary exponent, and so offers nothing further of interest in the present context. Lastly, Montheillet *et al.* [7] have recommended a function which can be written as

$$|a\sigma_1 + b\sigma_2|^m + h|\sigma_1 - \sigma_2|^m = \sigma_b^m \tag{6}$$

where $m > 1$, $h > 0$ and $a + b = 1$, necessarily. When $m = 2$, this is a sum of squares which is equivalent to Eqn (1), as already mentioned. The expression in Eqn (6) was adopted in relation to a particular material whose response to loading coaxial with the orthotropy could be modelled in first approximation by Eqn (5) but not by Eqn (2). The in-plane anisotropy was rather weak and could be represented in second approximation with the help of the additional parameters a and b.

OBJECTIVES

Before coming to a new model of orthotropic plastic response, the flexibility of the functions just reviewed will be assessed in one further regard. The question is whether or not they can adequately represent a rather common type of behaviour which, for the most part, has been passed over by theoreticians.

More notation is needed at this point. Let σ_0 and σ_{90} denote the yield stresses in uniaxial tension at 0° and 90°, respectively, to the direction of rolling, and let r_0 and r_{90} denote the corresponding ratios of the transverse to through-thickness increments of logarithmic strain. The behaviour in question may be characterized in an extreme form by the properties $\sigma_0 = \sigma_{90}$ together with $r_0 \neq r_{90}$, or by $r_0 = r_{90}$ together with $\sigma_0 \neq \sigma_{90}$. More generally, either σ_0/σ_{90} or r_0/r_{90} is close to unity while the other is relatively distant. A good example was reported by Stout and Hecker [8] where, in 70–30 brass, the as-received values were $\sigma_0 = 126$ MPa, $\sigma_{90} = 125$ MPa, $r_0 = 1.51$, $r_{90} = 0.37$.

With the function in Eqn (2), and its special case (1), it is found that

$$\left(\frac{\sigma_b}{\sigma_0}\right)^m = g + h, \quad \left(\frac{\sigma_b}{\sigma_{90}}\right)^m = f + h, \quad r_0 = \frac{h}{g}, \quad r_{90} = \frac{h}{f}. \tag{7}$$

With a view to later comparisons, it may be noted that

$$\frac{r_0}{(1 + r_0)}\left(\frac{\sigma_b}{\sigma_0}\right)^m = h = \frac{r_{90}}{(1 + r_{90})}\left(\frac{\sigma_b}{\sigma_{90}}\right)^m \tag{8}$$

independently of f and g. From Eqn (7) it is seen that $\sigma_0 = \sigma_{90} \rightarrow f = g \rightarrow r_0 = r_{90}$ (and conversely). Alternatively from (8) $\sigma_0 = \sigma_{90} \leftrightarrow r_0 = r_{90}$ at once.

With function (3), on the other hand, one can show that

$$\left.\begin{aligned} \frac{r_0}{(1 + r_0)}\left(\frac{\sigma_b}{\sigma_0}\right)^m &= h + 2^{m-1}a + 2b - c, \\ \frac{r_{90}}{(1 + r_{90})}\left(\frac{\sigma_b}{\sigma_{90}}\right)^m &= h + 2a + 2^{m-1}b - c, \end{aligned}\right\} \tag{9}$$

again independently of f and g. These expressions are equal if, and only if, $a = b$; in that event, $\sigma_0 = \sigma_{90} \leftrightarrow r_0 = r_{90}$ and moreover, by Eqn (3) itself, $\sigma_0 = \sigma_{90} \rightarrow f = g$ when $a = b$. On the other hand, if $a \neq b$, there may be scope to model the behaviour in question (provided that the function can be made satisfactory in other respects by simultaneous choice of f, g, h and c).

With function (4) it is evident that $\sigma_0 = \sigma_{90} \leftrightarrow k = l$, whence $r_0 = r_{90}$ necessarily follows by symmetry with respect to interchange of σ_1 and σ_2. A detailed calculation gives

$$\frac{r_0}{(1 + r_0)}\left(\frac{\sigma_b}{\sigma_0}\right)^m = h + \frac{k + l}{m} - \frac{1}{2^m} = \frac{r_{90}}{(1 + r_{90})}\left(\frac{\sigma_b}{\sigma_{90}}\right)^m \tag{10}$$

from which it is apparent at sight that $\sigma_0 = \sigma_{90} \leftrightarrow r_0 = r_{90}$.

Turning finally to function (6), one finds

$$\left.\begin{aligned} \left(\frac{\sigma_b}{\sigma_0}\right)^m = h + a^m, &\qquad \left(\frac{\sigma_b}{\sigma_{90}}\right)^m = h + b^m, \\ r_0 = h/a^{m-1} - b, &\quad r_{90} = h/b^{m-1} - a. \end{aligned}\right\} \tag{11}$$

Equivalent formulae can be extracted from [7] in other notation. It follows that $\sigma_0 = \sigma_{90} \rightarrow a = b \rightarrow r_0 = r_{90}$ (and conversely).

The conclusion is that no existing yield criterion, with the possible exception of Eqn (3), is capable of representing any response where σ_0 is near σ_{90} but r_0 is far from r_{90}. This suggests that a new function should be sought which is *different in kind* from any devised previously. In so doing, a sensible balance should be struck between flexibility and convenience.

A NEW THEORY

Attention is first directed towards textures where $\sigma_0 = \sigma_{90}$ precisely; for convenience this common uniaxial value will be denoted by a single symbol σ_u. According to the normality flow-rule, the logarithmic strain increment associated with a general biaxial state is represented by a vector $(d\varepsilon_1, d\varepsilon_2)$ with the direction and sense of the local outward normal at (σ_1, σ_2) to the current yield locus. In other words, the ratio of the incremental strain components conforms to

$$d\varepsilon_1/d\varepsilon_2 = \frac{\partial\phi}{\partial\sigma_1}\bigg/\frac{\partial\phi}{\partial\sigma_2} = -\,d\sigma_2/d\sigma_1 \tag{12}$$

where, $(d\sigma_1, d\sigma_2)$ is locally tangential to the contour $\phi(\sigma_1, \sigma_2) = $ constant. Under uniaxial tension, in particular, we have

$$\left.\begin{aligned} r_0/(1 + r_0) &= -\,d\varepsilon_2/d\varepsilon_1 = d\sigma_1/d\sigma_2 \quad \text{at } (\sigma_u, 0), \\ r_{90}/(1 + r_{90}) &= -\,d\varepsilon_1/d\varepsilon_2 = d\sigma_2/d\sigma_1 \quad \text{at } (0, \sigma_u) \end{aligned}\right\} \tag{13}$$

since $r_0 = d\varepsilon_2/d\varepsilon_3$ and $r_{90} = d\varepsilon_1/d\varepsilon_3$ with $d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$. The tensions quadrant of a yield locus in the (σ_1, σ_2) plane is depicted schematically in Fig. 1. The orientation of the local tangent and normal in each uniaxial state is defined by an acute angle ψ_0 or ψ_{90} relatively to the coordinate axes as shown. Evidently

$$\left.\begin{aligned} \tan\psi_0 &= r_0/(1 + r_0) = d\sigma_1/d\sigma_2 \qquad \text{at } (\sigma_u, 0), \\ \tan\psi_{90} &= r_{90}/(1 + r_{90}) = d\sigma_2/d\sigma_1 \quad \text{at } (0, \sigma_u). \end{aligned}\right\} \tag{14}$$

These are explicit formulae by which r_0, r_{90}, ψ_0 and ψ_{90} can be calculated directly from any $\phi(\sigma_1, \sigma_2)$.

A simple yield function is now proposed which can model arbitrary values of σ_u, σ_b, r_0 and r_{90}. With applications to thin sheet mainly in mind, the proposal is restricted in the first instance to the tensions quadrant of the (σ_1, σ_2) plane. After trials with several candidate functions the choice fell on a polynomial with just quadratic and cubic terms, and moreover with only specific powers and products of σ_1 and σ_2 present. This is

$$\sigma_1^2 - \left(2 - \frac{\sigma_u^2}{\sigma_b^2}\right)\sigma_1\sigma_2 + \sigma_2^2 + \left\{(p + q) - \frac{(p\sigma_1 + q\sigma_2)}{\sigma_b}\right\}\sigma_1\sigma_2 = \sigma_u^2 \tag{15}$$

where p and q are non-dimensional parameters, positive or negative. Regardless of their values, the equation is seen to be satisfied identically when the stress is uniaxial or

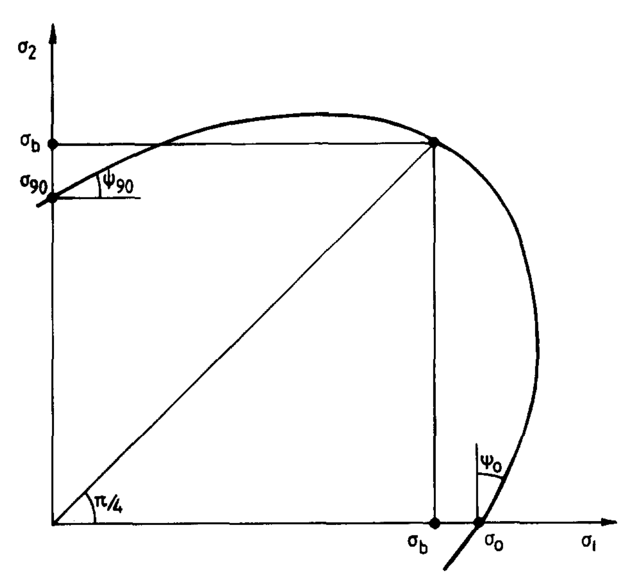

FIG. 1

equibiaxial. Significantly, there is no cubic term in σ_1 or σ_2 alone; consequently, when one is given, the other is obtained by solving only a quadratic. The yield locus in the tensions quadrant can thereby be mapped by taking closely-spaced traverses at constant σ_2 over a certain range; the required intersections are given by positive roots σ_1 of the associated quadratics in turn. It should be noted that Eqn (15), being a cubic, generally has another branch that would be disclosed by traverses beyond the inner oval; this has no physical significance and is disregarded.

Given any measured values of r_0 and r_{90}, we find from Eqn (14) that

$$\left.\begin{aligned} \frac{p\sigma_u}{\sigma_b} - (p+q) &= \frac{2r_0}{1+r_0} - \left(2 - \frac{\sigma_u^2}{\sigma_b^2}\right), \\ \frac{q\sigma_u}{\sigma_b} - (p+q) &= \frac{2r_{90}}{1+r_{90}} - \left(2 - \frac{\sigma_u^2}{\sigma_b^2}\right), \end{aligned}\right\} \tag{16}$$

when $d\sigma_1/d\sigma_2$ is calculated from Eqn (15) in each uniaxial state. These are linear equations for p and q jointly, and a solution always exists since the determinant of coefficients never vanishes when the locus is strictly convex (which requires $\sigma_u < 2\sigma_b$). In particular, the solution is immediate when r_0 and r_{90} are equal (to r, say). It is

$$p = q = \left(\frac{2}{1+r} - \frac{\sigma_u^2}{\sigma_b^2}\right) \bigg/ \left(2 - \frac{\sigma_u}{\sigma_b}\right), \tag{17}$$

for any r and any $\sigma_u/\sigma_b < 2$; correspondingly, the locus is symmetric about the ray $\sigma_1 = \sigma_2$ (mirror reflection). As p and q in Eqn (17) have the sign of the numerator, they are positive when $\sigma_b^2/\sigma_u^2 > \frac{1}{2}(1+r)$; this includes the anomalous regime $\sigma_b/\sigma_u > 1$ with $r < 1$. They are negative, on the other hand, when $\sigma_b^2/\sigma_u^2 < \frac{1}{2}(1+r)$; this includes the anomalous regime $\frac{1}{2} < \sigma_b/\sigma_u < 1$ with $r > 1$. Both p and q are zero when $\sigma_b^2/\sigma_u^2 = \frac{1}{2}(1+r)$, which is the familiar inflexible relationship predicted by Eqn (1) for a sheet with planar isotropy ($f = g$). More generally, when $\sigma_0 = \sigma_{90}$ but $r_0 \neq r_{90}$, the solution of Eqn (16) is

$$\left.\begin{aligned} \left(2 - \frac{\sigma_u}{\sigma_b}\right)p &= \frac{2}{1+r_0} + \left(\frac{2}{1+r_{90}} - \frac{2}{1+r_0}\right)\frac{\sigma_b}{\sigma_u} - \frac{\sigma_u^2}{\sigma_b^2}, \\ \left(2 - \frac{\sigma_u}{\sigma_b}\right)q &= \frac{2}{1+r_{90}} + \left(\frac{2}{1+r_0} - \frac{2}{1+r_{90}}\right)\frac{\sigma_b}{\sigma_u} - \frac{\sigma_u^2}{\sigma_b^2}. \end{aligned}\right\} \tag{18}$$

Equivalently

$$\left.\begin{aligned} \tfrac{1}{2}(p+q) &= \left(\frac{1}{1+r_0} + \frac{1}{1+r_{90}} - \frac{\sigma_u^2}{\sigma_b^2}\right) \bigg/ \left(2 - \frac{\sigma_u}{\sigma_b}\right), \\ \tfrac{1}{2}(p-q) &= \left(\frac{1}{1+r_{90}} - \frac{1}{1+r_0}\right)\frac{\sigma_b}{\sigma_u}, \end{aligned}\right\} \tag{19}$$

which further exemplifies the roles of the two parameters.

Finally, when σ_0 and σ_{90} are distinct, the yield function is extended in an obvious manner to

$$\frac{\sigma_1^2}{\sigma_0^2} - \frac{c\sigma_1\sigma_2}{\sigma_0\sigma_{90}} + \frac{\sigma_2^2}{\sigma_{90}^2} + \left\{(p+q) - \frac{(p\sigma_1 + q\sigma_2)}{\sigma_b}\right\}\frac{\sigma_1\sigma_2}{\sigma_0\sigma_{90}} = 1 \tag{20}$$

where p and q are disposable parameters as before and

$$\frac{c}{\sigma_0\sigma_{90}} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_{90}^2} - \frac{1}{\sigma_b^2}. \tag{21}$$

The criterion is satisfied identically when the stress is uniaxial or equibiaxial. From Eqn (14) applied at $(\sigma_0, 0)$ and $(0, \sigma_{90})$ it is found that

$$\left.\begin{aligned} \frac{p\sigma_0}{\sigma_b} - (p+q) &= \frac{2r_0\sigma_{90}}{(1+r_0)\sigma_0} - c, \\ \frac{q\sigma_{90}}{\sigma_b} - (p+q) &= \frac{2r_{90}\sigma_0}{(1+r_{90})\sigma_{90}} - c, \end{aligned}\right\} \tag{22}$$

by substituting the respective values of $d\sigma_1/d\sigma_2$ given by Eqn (20). These are linear equations for p and q when $\sigma_0, \sigma_{90}, r_0, r_{90}$ and σ_b are all given. The equations are compatible unless the uniaxial and equibiaxial yield-points in the stress plane are collinear; this configuration, however, is excluded by strict convexity. The solution is

$$\left.\begin{aligned} \left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{90}} - \frac{1}{\sigma_b}\right)p &= \frac{2r_0(\sigma_b - \sigma_{90})}{(1+r_0)\sigma_0^2} - \frac{2r_{90}\sigma_b}{(1+r_{90})\sigma_{90}^2} + \frac{c}{\sigma_0}, \\ \left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{90}} - \frac{1}{\sigma_b}\right)q &= \frac{2r_{90}(\sigma_b - \sigma_0)}{(1+r_{90})\sigma_{90}^2} - \frac{2r_0\sigma_b}{(1+r_0)\sigma_0^2} + \frac{c}{\sigma_{90}}. \end{aligned}\right\} \tag{23}$$

The common coefficient on the left is necessarily positive and approaches zero only in the collinear limit. The strain-increment ratio $d\varepsilon_1/d\varepsilon_2$ in any state (σ_1, σ_2) is obtained from Eqns (12) and (20) as

$$\frac{2\sigma_{90}\sigma_1/\sigma_0 + (p+q-c)\sigma_2 - (2p\sigma_1 + q\sigma_2)\sigma_2/\sigma_b}{2\sigma_0\sigma_2/\sigma_{90} + (p+q-c)\sigma_1 - (p\sigma_1 + 2q\sigma_2)\sigma_1/\sigma_b}. \tag{24}$$

Under equibiaxial tension, for example, it is

$$\frac{d\varepsilon_1}{d\varepsilon_2} = \frac{2\sigma_{90}/\sigma_0 - c - p}{2\sigma_0/\sigma_{90} - c - q} \tag{25}$$

which generally differs from unity unless the yield locus is symmetric about the ray $\sigma_1 = \sigma_2$.

A possible analytic continuation of Eqn (20) is perhaps worth mentioning. This extends the yield criterion to negative values of σ_1 and σ_2 and is, accordingly, directed towards thick plates capable of sustaining in-plane compressive loads. The complete locus is assumed to be centred on the stress origin, which is to say that $\phi(\sigma_1, \sigma_2)$ must be an even function. In this respect, the terms of second order in Eqn (20) are acceptable as they stand, but not those of third order. It is proposed, therefore, to replace $(p\sigma_1 + q\sigma_2)\sigma_1\sigma_2$ by

$$(p|\sigma_1| + q|\sigma_2|)\sigma_1\sigma_2, \tag{26}$$

keeping everything else the same. The first partial derivatives of this expression are continuous throughout the entire (σ_1, σ_2) space. The extended locus is thus a centrosymmetric oval with a smoothly turning tangent. On the other hand, when p and q are non-zero, the local *curvature* changes discontinuously as the locus crosses the coordinate axes. This can be seen more clearly when Eqn (26) is re-written as four separate polynomials, one for each quadrant:

$$\left.\begin{array}{lll} \text{(i)} & (p\sigma_1 + q\sigma_2)\sigma_1\sigma_2 & \text{when} \quad \sigma_1 \geqslant 0, \sigma_2 \geqslant 0; \\ \text{(ii)} & (p\sigma_1 - q\sigma_2)\sigma_1\sigma_2 & \text{when} \quad \sigma_1 \geqslant 0, \sigma_2 \leqslant 0; \\ \text{(iii)} & (-p\sigma_1 + q\sigma_2)\sigma_1\sigma_2 & \text{when} \quad \sigma_1 \leqslant 0, \sigma_2 \geqslant 0; \\ \text{(iv)} & -(p\sigma_1 + q\sigma_2)\sigma_1\sigma_2 & \text{when} \quad \sigma_1 \leqslant 0, \sigma_2 \leqslant 0. \end{array}\right\} \tag{27}$$

Such discontinuities in curvature are entirely acceptable when modelling experimental data subject to normal scatter.

CONCLUSIONS

The present aim is to model some basic aspects of the yielding and plastic flow of textured sheet: namely, those observed when combined loads are applied along the in-plane axes of orthotropy. Distinct types of behaviour are known, and no single theory has been found appropriate over the whole range. One behaviour in particular is relatively common but is not predicted by any theory at all. It is characterized by effectively equal yield-points under tension in the rolling and transverse directions, allied to markedly different strain-ratios.

In the paper, attention is initially focussed on modelling this special combination of properties. The main requirements in representing them are considered to be that (i) the yield function and its derivatives should be easy to manipulate algebraically; (ii) no more parameters should be admitted than are strictly necessary in technological applications; (iii) all parameters should be determinable by straightforward tests and, moreover, without excessive computation. This quite demanding prescription is satisfied here by a specific polynomial of third degree, valid in the quadrant of stress space relevant to thin sheet.

The polynomial is then extended to a wide range of behaviours, not just the particular one that prompted its construction in the first place. In brief, it can accommodate any measured values of $\sigma_0, \sigma_{90}, r_0, r_{90}$ and σ_b. In terms of this data, the parameters are given by *explicit* formulae (an extra bonus). It is a reasonable expectation that the entire yield locus will then be reliably predicted, provided its shape is broadly oval.

Lastly, supposing Bauschinger effects to be absent, it is shown how the polynomial can be continued analytically over the whole of the stress space. It is only necessary to change the sign of a term of third degree in passing from one quadrant to another. The composite yield function so generated should be appropriate for thick plates under combined loads of either sign.

It is important to be clear about the physical significance of a locus generated in the way described from the observed values of $\sigma_0, \sigma_{90}, r_0, r_{90}$ and σ_b. These are preferably obtained by back-extrapolation from data collected over the small range of strain needed to establish their values reliably. The locus then characterizes the as-received state, since any changes in texture during the tests themselves will be imperceptible. Specifically, where a ray at constant σ_1/σ_2 intersects the theoretical locus, the values of σ_1 and σ_2 should be close to those that would be obtained by back-extrapolation if the same loading path were to be followed experimentally. The interpretation is not so clear-cut, on the other hand, when values of $\sigma_0, \sigma_{90}, r_0, r_{90}$ and σ_b are determined at strains sufficient to change the initial texture and, especially, when these changes are noticeably path-dependent. In that case, it is suggested that the five quantities should be measured after the same expenditure of work per unit volume in the uniaxial and equibiaxial tests. Then, considered objectively, the theoretical locus represents a *work contour*: it delivers approximate values of σ_1 and σ_2 after equal expenditures of work per unit volume on conceptual paths at any constant σ_1/σ_2. In principle, it cannot be a yield locus when it connects states with different textures, because they cannot possibly lie on a path of *neutral* loading. The precise relationship between work contours and yield loci is for future experiments to elucidate.

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